



Finite-sample distributions of self-normalized sums

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Finite-sample distributions of self-normalized sums *

Abstract

Logan et al. (1973) analyze the limit probability distribution of the statistic $S_n(p) = \sum_{i=1}^n X_i / \left(\sum_{i=1}^n |X_i|^p \right)^{1/p}$ as $n \rightarrow \infty$, when X_i is in the domain of attraction of a stable law with stability index α . By simulations, we provide quantiles of the usual critical levels of the finite-sample distributions of the Student's t-statistic as ${}_{\alpha}t_{\xi}(n) = S_n(p) \left[(n-1) / (n - S_n^2(p)) \right]^{1/2}$ with $p=2$. The response surface method is used to provide approximate quantiles of the finite-sample distributions of the Student's t-statistic.

Zusammenfassung

Logan u.a. (1973) untersuchen die Grenzwahrscheinlichkeitsverteilung der Statistik, $S_n(p) = \sum_{i=1}^n X_i / \left(\sum_{i=1}^n |X_i|^p \right)^{1/p}$ mit $n \rightarrow \infty$, für den Fall, dass X_i im Anziehungsbereich eines stabilen Gesetzes mit Stabilitätsindex von α liegt. Mit Hilfe einer Simulation werden Quantile der üblicherweise verwendeten kritischen Niveaus für Verteilungen endlicher Stichprobenumfänge der Studentischen t-Statistik, ${}_{\alpha}t_{\xi}(n) = S_n(p) \left[(n-1) / (n - S_n^2(p)) \right]^{1/2}$ mit $p=2$ ermittelt. Die Antwort-Oberfläche-Methode gibt die approximierten Quantile der Verteilungen endlicher Stichprobenumfänge der t-Statistik in einer übersichtlichen Form an.

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1 Introduction

Econometricians have long been aware that the tails of a great deal of economic data are thicker than that of the Gaussian distribution. Therefore, it is of interest in hypothesis testing to specify both the limiting distribution and the finite-sample distributions of the Student's t -statistic

$${}_\alpha t(n) := {}_\alpha S(n) \left(\frac{n-1}{n - {}_\alpha S^2(n)} \right)^{\frac{1}{2}}, \quad (1)$$

with

$${}_\alpha S(n) := \frac{\sum_{i=1}^n X_i}{\sqrt{\sum_{i=1}^n X_i^2}}, \quad (2)$$

where the random sequence $\{X_i\}_1^n$ is in the domain of attraction (DA) of an α -stable law with index $\alpha \in (0, 2]$. The DA condition is equivalent to $P(|X| > x) = x^{-\alpha}L(x)$, $x > 0$, where $L(z)$ is a slowly varying function,¹ and $\lim_{x \rightarrow \infty} \frac{P(X > x)}{P(|X| > x)} = r$; $\lim_{x \rightarrow \infty} \frac{P(X < -x)}{P(|X| > x)} = l$, for some $r, l \geq 0$. When $r = l$, the random variable X_i is symmetric, and thus the statistic in (2) is self-normalized in the sense that it has zero mean and unit variance. A random variable X is said to be stable if for any positive numbers A and B , there is a positive number C and a real number D such that $AX_1 + BX_2 \stackrel{d}{=} CX + D$, where X_1 and X_2 are independent random variables with $X_i \stackrel{d}{=} X, i = 1, 2$; and “ $\stackrel{d}{=}$ ” denotes equality in distribution. Moreover, $C = (A^\alpha + B^\alpha)^{1/\alpha}$ for some $\alpha \in (0, 2]$, where the exponent α is called index of stability. When $0 < \alpha < 2$, the tails of the distribution are thicker than those of the normal distribution. The tails become thicker as α decreases such that moments of order α or higher do not exist. A stable random variable, X , with index α is called α -stable. The α -stable distributions are described by four parameters denoted by $S(\alpha, \beta, \gamma, \delta)$. The shape of the α -stable distribution is determined by the stability parameter α . For $\alpha = 2$ the α -stable distribution reduces to the normal distribution, the only member of the α -stable family with finite variance; for $\alpha < 2$ it has infinite variance. Skewness is governed by $\beta \in [-1, 1]$. When $\beta = 0$, the distribution is symmetric. The location and scale of the α -stable distributions are denoted by γ and δ . The standardized version of the α -stable distribution is given by $S((x - \gamma)/\delta; \alpha, \beta, 1, 0)$. The α -stable distribution is an interesting error-distribution candidate, because only the α -stable distribution can serve as a limiting distribution of sums of independent, identically distributed random variables. This is an appealing property, given that disturbances can be viewed as random variables which represent the sum of all

¹ $L(z)$ is a slowly varying function as $z \rightarrow \infty$, if for every constant $c > 0$, $\lim_{x \rightarrow \infty} \frac{L(cx)}{L(x)}$ exists and is equal to 1. See Ibragimov and Linnik (1971, p. 394) for more details on slowly varying functions.

external effects ignored by the model. Therefore, the hypothesis test based on the α -stable distributed disturbances is directly related to the statistic in (1). For more details on the α -stable distributions and discussions of the role of the α -stable distribution in financial market modelling, see Zolotarev (1986), Samorodnitsky and Taqqu (1994) and McCulloch (1996).

Logan et al. (1973) consider the limit distribution of the statistic in (2). When $0 < \alpha < 2$, the statistic in (2) is a pseudo statistic because the second moment for the α -stable random variable does not exist. This pseudo statistic, however, still has a limit probability distribution. Logan et al. (1973) show that for $0 < \alpha < 2$, the limiting density, $f(y)$, is a complicated form involving an integral of a ratio of parabolic cylinder functions with $Y := n^{-\frac{1}{\alpha}} \sum_{i=1}^n X_i$ as:

$$f(y) = \lim_{s \rightarrow iy} \text{Real} \left[\frac{1}{\pi} \int_0^{\infty} \phi(t) e^{-st} dt \right], \quad (3)$$

where $\text{Real}[\]$ denotes the real part of a complex number and $\phi(t)$ is the characteristic function of a stable random variable as follows:

$$\phi(t) = \int_{-\infty}^{\infty} e^{iut} dP(X < u) = \begin{cases} -\delta |t|^\alpha [1 - i\beta \text{sign}(t) \tan \frac{\pi\alpha}{2}] + i\gamma t, & \text{for } \alpha \neq 1, \\ -\delta |t| [1 + i\beta \frac{\pi}{2} \text{sign}(t) \ln |t|] + i\gamma t, & \text{for } \alpha = 1. \end{cases}$$

This limit distribution is a calculable expression, and quantiles of the usual critical levels of the distribution are given in Tables 2a-c at the end of the paper. Furthermore, the limiting distribution has the properties that the tails of the cumulated density function are Gaussian-like at $\pm\infty$ and that the density function has finite (infinite) singularities at ± 1 for $1 < \alpha < 2$ ($0 < \alpha < 1$). This is because the sums in the numerator and denominator in (2) are essentially determined by a few summands of largest modulus for $\alpha < 2$. As α increases to 2, the singularities vanish and the density function tends to the normal density. This singularity, called the bimodality for $1 < \alpha < 2$, has been conformed in several subsequent works (see, for example, Phillips and Hajivassiliou, 1987).

2 Finite-sample αt -distributions

In this section, we perform simulations to approximate quantiles of the distributions of the Student's t -statistic in (1) with finite degrees of freedom for α -stable variables. By doing that, we concentrate on the most empirically-relevant case, namely $1 \leq \alpha < 2$ and $\beta = \gamma = 0$, i.e., the underlying stable random variables are symmetric about zero. Specifically, we consider α -values from 1.0 to 1.9 in steps of 0.1; the degrees of freedom ranged from 1 to 30 in steps of 1. For each of the resulting 300 (α, n) -combinations, 100000 replications are generated.²

²The α -stable random variables are generated with the algorithm of Weron (1996).

Before discussing the main results, some observations regarding the finite-sample distributions are in order. First, by a given sample size, the finite-sample distributions tend more and more, as is expected, to become bimodal as the stability index α decreases. Second, by a given low α , say $\alpha < 1.5$, the bimodality of finite-sample distributions still remain as $n \rightarrow \infty$. When $1.5 < \alpha < 2$, the bimodality vanishes quickly as n increases, but the degeneration to the two points with mass at ± 1 still remains asymptotically. Third, the tails of the finite-sample densities tend to become thinner, as n increases. These phenomena are summarized in Figure 1 by the empirical distributions of the simulated ${}_{\alpha}t$ with $n = 2, 10$ and 30 ; $\alpha = 1.8, 1.5$ and 1.2 .

The first column of Figure 2 shows the simulated response surfaces for percent points $\xi = 0.9, 0.95$ and 0.99 , illustrating the dependence of the quantiles, denoted by ${}_{\alpha}t_{\xi}(n)$, on α and n . The difference of the quantiles of the ${}_{\alpha}t$ -distribution from those of the usual t -distribution increases as α drops below 2, but the increase is rather smooth and well-behaved. The second column shows the fitted response surfaces for percent points $\xi = 0.9, 0.95$ and 0.99 .

Rather than simply tabulating specific quantiles of the simulated ${}_{\alpha}t$ -distributions for selected (α, n) -combinations we employ response surface techniques to present all simulation results in a compact fashion.³ In fact, we fit a joint response surface reflecting the dependence of the quantiles not only on α and n , but also on the percent point ξ . In the estimation, we use three percent points, namely 0.9, 0.95 and 0.99. Combining these with the 300 (α, n) -pairs, the joint response surface is derived from simulated quantiles of 900 (α, n, ξ) -combinations. Because of the smoothness of the transition when α drops below 2, i.e., when moving from the usual t -distribution to the ${}_{\alpha}t$ -distribution, we estimate the response surface in terms of deviations of the quantiles of the ${}_{\alpha}t$ -distribution from those of the usually tabulated t -distribution⁴ i.e.,

$$\Delta_{\alpha}t_{\xi}(n) := {}_2t_{\xi}(n) - {}_{\alpha}t_{\xi}(n).$$

³Response surface methodology has been used in various statistical and econometric applications, see Myers et al. (1989) for more on this topic.

⁴Alternatively, one can employ the method of Peizer and Pratt (1968) to approximate the quantiles of the t -distribution

$${}_2t_{\xi}(n) = \sqrt{n \left[\exp \left\{ \left(n - \frac{6}{5} \right) \left(\frac{z(\xi)}{n - \frac{2}{3} - \frac{1}{10n}} \right)^2 \right\} - 1 \right]},$$

where $z(\xi)$ is the ξ -quantile of the standard normal distribution.

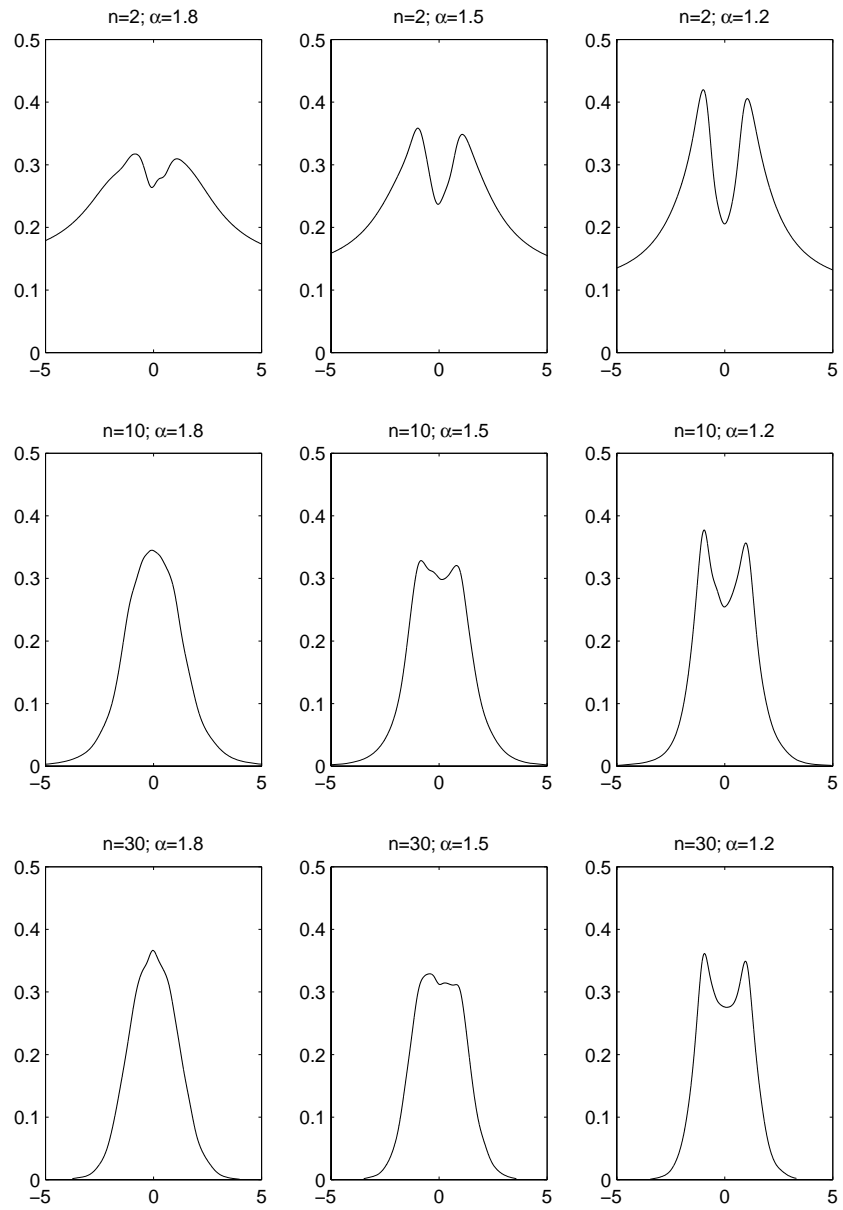


Figure 1: Simulated densities of ${}_a t$ random variable for selected n and α

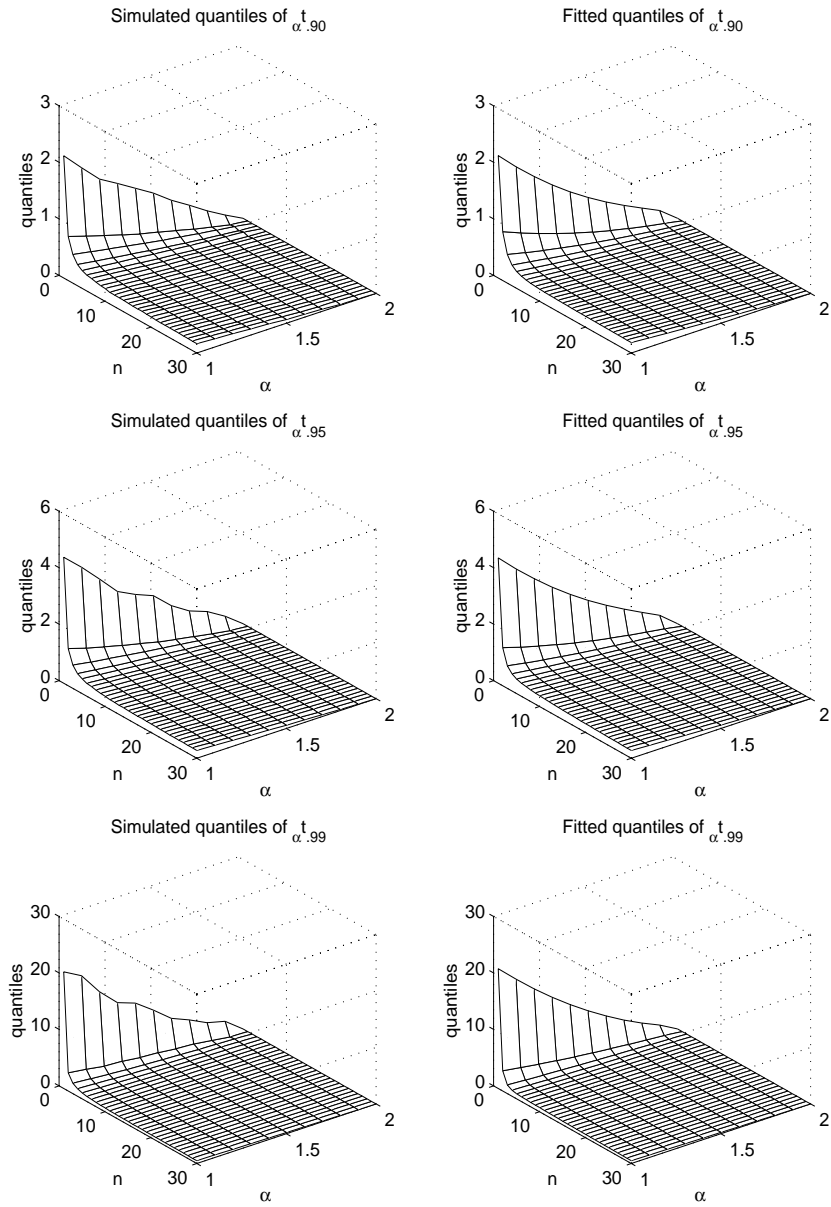


Figure 2: Simulated and fitted quantiles of αt -distributions

For the response surface, we specify the polynomial

$$\Delta_{\alpha}t_{\xi}(n) = \sum_{i=0}^2 \sum_{j=1}^4 \sum_{k=1}^4 a_{ijk}(-\ln(1-\xi))^i(2-\alpha)^{\frac{j}{2}}n^{-\frac{k}{2}} + u_{\alpha,n,\xi}, \quad (4)$$

which ensures that $\Delta_{\alpha}t_{\xi}(n) \rightarrow 0$ as $\alpha \rightarrow 2$ and/or $n \rightarrow \infty$. When $n \rightarrow \infty$, the response surface is specified as

$$\Delta_{\alpha}t_{\xi} = \sum_{i=0}^2 \sum_{j=1}^4 b_{ij}(-\ln(1-\xi))^i(2-\alpha)^{\frac{j}{2}} + u_{\alpha,\xi}, \quad (5)$$

which ensures that $\Delta_{\alpha}t_{\xi}(n) \rightarrow 0$ as $\alpha \rightarrow 2$.

Estimating (4) and (5) with the least squares method, it turns out that only a subset of regressors is needed to fit the simulated quantiles. We selected the subset by maximizing the adjusted- R^2 value. The estimated coefficients are presented in Table 1a for the regression (4) and 1b for the regression (5) at the end of the paper. The adjusted- R^2 values for the regression (4) and (5) are 0.9902 and 0.9959, respectively. Various additional measures of goodness of fit, namely the standard deviation (0.1170 for (4), 0.0061 for (5)) and the absolute mean value (0.0317 for (4), 0.0047 for (5)) of the residuals, also suggest a good fit.

Based on the estimates in (4) the response surfaces for percent points $\xi = 0.9, 0.95$ and 0.99 are illustrated in the second column in Figure 2. For selected α -values response surface approximations for the 0.9-, 0.95-, and 0.99-quantiles of the $_{\alpha}t$ -distribution are reported in Tables 2a-c at the end of the paper. The first column of each table corresponds to the usual t -distribution, and the last row of each table corresponds to the limit probability distribution in (3). Due to the singularity effect, absolute quantiles of the percentage points outside of ± 1 decrease as α decreases.

3 Summary

We presented an extension of the usual t -distribution for normally distributed variables to $_{\alpha}t$ -distributions for heavy-tailed random variable Quantiles of finite degrees-of-freedom distributions for α -stable variables were simulated and compactly summarized in terms of a fitted response surface. The approximated critical values can be used to perform t -type tests when residuals are α -stable distributed.

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T able1a.
P arameterEstimates of Fitted Response Surface^a

i	k j	a_{ijk}			
		1	2	3	4
0	3	–	–	-25.720 (-4.920)	41.699 (7.570)
1	1	–	–	–	-0.282 (-2.706)
	3	1.415 (5.229)	-10.047 (-5.604)	41.057 (8.769)	-45.278 (-11.688)
2	1	–	–	-0.154 (-2.332)	0.390 (5.671)
	2	0.109 (5.308)	–	–	–
	3	-0.332 (-4.614)	2.815 (6.229)	-10.530 (-10.752)	10.500 (15.364)
	4	–	–	0.301 (4.573)	–

^at-value are reported in parentheses.

T able 1b.
P arameterEstimates of Fitted Response Surface^a

j i	b_{ij}			
	1	2	3	4
0	0.410 (1.839)	-1.132 (-2.368)	1.214 (2.024)	-0.746 (-2.592)
1	-0.296 (-2.393)	0.481 (3.249)	–	–
2	0.067 (3.747)	-0.098 (-4.466)	–	0.025 (6.554)

^at-value are reported in parentheses.

Table 2a.

 $\alpha t_{.90}$ -quantiles for selected α -values

α	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0
n											
1	6.31	6.07	5.92	5.77	5.60	5.42	5.21	4.98	4.73	4.45	4.15
2	2.92	2.86	2.80	2.74	2.67	2.59	2.50	2.40	2.30	2.18	2.06
3	2.35	2.32	2.29	2.25	2.20	2.16	2.11	2.05	1.99	1.92	1.85
4	2.13	2.11	2.08	2.05	2.02	1.99	1.95	1.91	1.86	1.82	1.77
5	2.02	2.00	1.97	1.95	1.92	1.89	1.86	1.83	1.79	1.75	1.72
6	1.94	1.93	1.90	1.88	1.86	1.83	1.80	1.78	1.74	1.71	1.68
7	1.89	1.88	1.86	1.84	1.81	1.79	1.77	1.74	1.71	1.68	1.65
8	1.86	1.84	1.82	1.80	1.78	1.76	1.74	1.71	1.68	1.66	1.63
9	1.83	1.82	1.80	1.78	1.76	1.74	1.71	1.69	1.66	1.64	1.61
10	1.81	1.80	1.78	1.76	1.74	1.72	1.70	1.67	1.65	1.62	1.60
11	1.80	1.78	1.76	1.74	1.72	1.70	1.68	1.66	1.63	1.61	1.58
12	1.78	1.77	1.75	1.73	1.71	1.69	1.67	1.65	1.62	1.60	1.57
13	1.77	1.76	1.74	1.72	1.70	1.68	1.66	1.64	1.61	1.59	1.56
14	1.76	1.75	1.73	1.71	1.69	1.67	1.65	1.63	1.61	1.58	1.56
15	1.75	1.74	1.72	1.70	1.69	1.67	1.64	1.62	1.60	1.58	1.55
16	1.75	1.73	1.72	1.70	1.68	1.66	1.64	1.62	1.59	1.57	1.55
17	1.74	1.73	1.71	1.69	1.67	1.65	1.63	1.61	1.59	1.57	1.54
18	1.73	1.72	1.70	1.69	1.67	1.65	1.63	1.61	1.58	1.56	1.54
19	1.73	1.72	1.70	1.68	1.66	1.64	1.62	1.60	1.58	1.56	1.53
20	1.72	1.71	1.70	1.68	1.66	1.64	1.62	1.60	1.58	1.55	1.53
21	1.72	1.71	1.69	1.68	1.66	1.64	1.62	1.60	1.57	1.55	1.53
22	1.72	1.70	1.69	1.67	1.65	1.63	1.61	1.59	1.57	1.55	1.53
23	1.71	1.70	1.69	1.67	1.65	1.63	1.61	1.59	1.57	1.55	1.52
24	1.71	1.70	1.68	1.67	1.65	1.63	1.61	1.59	1.57	1.54	1.52
25	1.71	1.70	1.68	1.66	1.65	1.63	1.61	1.59	1.57	1.54	1.52
26	1.71	1.69	1.68	1.66	1.64	1.63	1.61	1.59	1.56	1.54	1.52
27	1.70	1.69	1.68	1.66	1.64	1.62	1.60	1.58	1.56	1.54	1.52
28	1.70	1.69	1.67	1.66	1.64	1.62	1.60	1.58	1.56	1.54	1.52
29	1.70	1.69	1.67	1.66	1.64	1.62	1.60	1.58	1.56	1.54	1.51
30	1.70	1.69	1.67	1.65	1.64	1.62	1.60	1.58	1.56	1.54	1.51
∞	1.65	1.64	1.63	1.62	1.60	1.59	1.56	1.55	1.54	1.52	1.50

T able 2b.
 $\alpha t_{.95}$ -quantiles for selected α -values

α	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0
n											
1	12.71	12.20	11.88	11.56	11.21	10.82	10.40	9.93	9.42	8.87	8.27
2	4.30	4.18	4.09	3.99	3.88	3.76	3.63	3.48	3.32	3.14	2.95
3	3.18	3.12	3.06	3.00	2.94	2.86	2.78	2.69	2.60	2.50	2.39
4	2.78	2.73	2.69	2.64	2.58	2.53	2.47	2.40	2.33	2.25	2.17
5	2.57	2.53	2.49	2.45	2.40	2.35	2.30	2.24	2.18	2.12	2.05
6	2.45	2.41	2.38	2.34	2.29	2.25	2.20	2.15	2.09	2.04	1.98
7	2.36	2.33	2.30	2.26	2.22	2.18	2.13	2.09	2.04	1.98	1.93
8	2.31	2.28	2.24	2.21	2.17	2.13	2.09	2.04	1.99	1.94	1.89
9	2.26	2.23	2.20	2.17	2.13	2.09	2.05	2.01	1.96	1.91	1.86
10	2.23	2.20	2.17	2.14	2.10	2.06	2.02	1.98	1.94	1.89	1.84
11	2.20	2.18	2.14	2.11	2.08	2.04	2.00	1.96	1.92	1.87	1.83
12	2.18	2.15	2.12	2.09	2.06	2.02	1.98	1.94	1.90	1.86	1.82
13	2.16	2.14	2.11	2.07	2.04	2.01	1.97	1.93	1.89	1.85	1.81
14	2.14	2.12	2.09	2.06	2.03	1.99	1.96	1.92	1.88	1.84	1.80
15	2.13	2.11	2.08	2.05	2.02	1.98	1.95	1.91	1.87	1.83	1.79
16	2.12	2.10	2.07	2.04	2.01	1.97	1.94	1.90	1.87	1.83	1.79
17	2.11	2.09	2.06	2.03	2.00	1.97	1.93	1.90	1.86	1.82	1.78
18	2.10	2.08	2.05	2.02	1.99	1.96	1.93	1.89	1.86	1.82	1.78
19	2.09	2.07	2.04	2.02	1.99	1.95	1.92	1.89	1.85	1.81	1.78
20	2.09	2.06	2.04	2.01	1.98	1.95	1.92	1.88	1.85	1.81	1.77
21	2.08	2.06	2.03	2.01	1.98	1.95	1.91	1.88	1.84	1.81	1.77
22	2.07	2.05	2.03	2.00	1.97	1.94	1.91	1.88	1.84	1.81	1.77
23	2.07	2.05	2.02	2.00	1.97	1.94	1.91	1.87	1.84	1.81	1.77
24	2.06	2.04	2.02	1.99	1.96	1.93	1.90	1.87	1.84	1.80	1.77
25	2.06	2.04	2.02	1.99	1.96	1.93	1.90	1.87	1.84	1.80	1.77
26	2.06	2.04	2.01	1.99	1.96	1.93	1.90	1.87	1.84	1.80	1.77
27	2.05	2.03	2.01	1.98	1.96	1.93	1.90	1.87	1.83	1.80	1.77
28	2.05	2.03	2.01	1.98	1.95	1.93	1.90	1.87	1.83	1.80	1.77
29	2.05	2.03	2.00	1.98	1.95	1.92	1.89	1.86	1.83	1.80	1.77
30	2.04	2.02	2.00	1.98	1.95	1.92	1.89	1.86	1.83	1.80	1.77
∞	1.96	1.94	1.93	1.92	1.89	1.87	1.82	1.80	1.78	1.74	1.72

T able 2c.
 $\alpha t_{.99}$ -quantiles for selected α -values

α	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0
n											
1	63.66	61.91	60.50	58.91	57.14	55.17	53.01	50.65	48.11	45.37	42.45
2	9.92	9.58	9.32	9.05	8.76	8.44	8.09	7.70	7.28	6.82	6.33
3	5.84	5.69	5.56	5.44	5.31	5.17	5.02	4.86	4.68	4.49	4.29
4	4.60	4.50	4.41	4.32	4.23	4.13	4.02	3.91	3.79	3.66	3.53
5	4.03	3.95	3.87	3.79	3.71	3.62	3.53	3.43	3.33	3.23	3.12
6	3.71	3.63	3.56	3.48	3.41	3.32	3.24	3.15	3.06	2.96	2.86
7	3.50	3.43	3.36	3.29	3.21	3.13	3.05	2.97	2.88	2.79	2.69
8	3.36	3.29	3.22	3.15	3.08	3.00	2.92	2.84	2.75	2.67	2.57
9	3.25	3.19	3.12	3.05	2.98	2.90	2.83	2.75	2.66	2.58	2.49
10	3.17	3.11	3.04	2.98	2.91	2.83	2.76	2.68	2.60	2.52	2.43
11	3.11	3.05	2.98	2.92	2.85	2.78	2.70	2.63	2.55	2.47	2.38
12	3.05	3.00	2.94	2.87	2.80	2.73	2.66	2.59	2.51	2.43	2.35
13	3.01	2.96	2.90	2.83	2.77	2.70	2.63	2.56	2.48	2.40	2.32
14	2.98	2.92	2.86	2.80	2.74	2.67	2.60	2.53	2.46	2.38	2.30
15	2.95	2.89	2.84	2.78	2.71	2.65	2.58	2.51	2.44	2.36	2.29
16	2.92	2.87	2.81	2.75	2.69	2.63	2.56	2.49	2.42	2.35	2.28
17	2.90	2.85	2.79	2.73	2.67	2.61	2.55	2.48	2.41	2.34	2.27
18	2.88	2.83	2.77	2.72	2.66	2.60	2.53	2.47	2.40	2.33	2.26
19	2.86	2.81	2.76	2.70	2.64	2.58	2.52	2.46	2.39	2.32	2.26
20	2.85	2.80	2.75	2.69	2.63	2.57	2.51	2.45	2.39	2.32	2.25
21	2.83	2.78	2.73	2.68	2.62	2.56	2.50	2.44	2.38	2.32	2.25
22	2.82	2.77	2.72	2.67	2.61	2.56	2.50	2.44	2.38	2.31	2.25
23	2.81	2.76	2.71	2.66	2.61	2.55	2.49	2.43	2.37	2.31	2.25
24	2.80	2.75	2.70	2.65	2.60	2.54	2.49	2.43	2.37	2.31	2.25
25	2.79	2.74	2.70	2.65	2.59	2.54	2.48	2.43	2.37	2.31	2.25
26	2.78	2.74	2.69	2.64	2.59	2.53	2.48	2.42	2.37	2.31	2.25
27	2.77	2.73	2.68	2.63	2.58	2.53	2.48	2.42	2.36	2.31	2.25
28	2.76	2.72	2.68	2.63	2.58	2.53	2.47	2.42	2.36	2.31	2.25
29	2.76	2.72	2.67	2.62	2.57	2.52	2.47	2.42	2.36	2.31	2.25
30	2.75	2.71	2.66	2.62	2.57	2.52	2.47	2.42	2.36	2.31	2.25
∞	2.58	2.50	2.47	2.44	2.43	2.36	2.33	2.28	2.25	2.18	2.11

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