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Statistical Properties of Rates of Return on Shares Listed on the German, French, and Polish Markets – a Comparative Study

Wiesław Dębski¹, Ewa Feder-Sempach², Szymon Wójcik²

ABSTRACT

The financial market and its instruments are subject to numerous studies all over the world. Special attention is duly earned by the multifaceted research and analysis of the behavior of rates of return on shares, as well as shares' other statistical properties including the beta parameter. Stock stability is considered by dividing the whole market into the bull and bear categories, and the rate of return is studied according to different frequencies of measurement.

The purpose of this article is to examine the statistical properties of monthly rates of return for the biggest companies in terms of capitalization and turnover, which were listed on the stock exchange in Frankfurt, Paris, and Warsaw between 2005 and 2015. The basic descriptive statistics of the rates of return as well as the normality and stationarity of their time series undergo analysis. Moreover, hypotheses about equality of the expected value of the rate of return and its variance are subject to verification. This study is also conducted considering the division of the whole market into the bull and bear categories according to two definitions. The obtained results serve to conduct a comparative analysis of the three stock markets enumerated above. The results constitute an introduction to further and more advanced research on the beta parameter and its properties.

KEY WORDS:

rates of return of shares; statistical properties; normality and stationarity; equality of mean and variance; German, French and Polish markets

JEL Classification: G10, G15

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1. Introduction

The mechanisms of the functioning of the financial market and investments have been subject to multifaceted research all over the world for many years. Special attention is duly earned by numerous studies on investment in shares of companies listed on the applicable stock exchange, whether taking into consider-

ation a single country or comparing many countries. In countries where the financial market is highly developed, advanced research has been conducted since the middle of the previous century. In other countries, however, especially countries where political transformation occurred at the end of the previous century, such research may only be conducted in the present day. In certain countries, the present situation includes a quite well-developed financial market and the availability of ample time series for conducting advanced statistical or econometric research, allowing the ob-

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tained results to be generalized. Poland is an example of such a country where the financial market started to emerge at the beginning of the 1990s.

As far as investment in shares is concerned, research and analysis chiefly focus on the mechanisms shaping prices, rates of return, and the beta parameter. Time series characterized by a varied frequency of measurement, e.g., a month, week or day, are most commonly used in studies. Because the share market is subject to large fluctuations in time, the whole sample period in many studies is divided into a bull and a bear market for which it is verified whether, for example, the rates of return have similar statistical properties in these markets or whether the beta parameters are stable. Numerous studies on rates of return on shares also examine their statistical properties, such as the mean value, dispersion, asymmetry or concentration, and verify hypotheses on the normality of distribution and stationarity of the time series of the rates of return. Hypotheses on the equality of the mean and variance undergo verification as well. Such research is most commonly employed in various comparative analyses, mainly in the international context, or serves as an introduction to further, more advanced studies, e.g., studies concerning the beta parameter and its properties.

For the purposes of the latter, the basic statistical properties of the rates of return on the shares under examination need to be known in order to select the method of estimation of the beta parameter so that it will meet the requirements for the best estimator (i.e., unbiased, consistent, and the most effective). It is also significant when the stability of the beta parameter is tested in the case of division of the whole market under analysis into the bull and bear subperiods and when the Sharpe's model (Sharpe, 1963) is estimated separately for each of these two subperiods.

Studies regarding statistical properties of rates of return are mostly concerned with the developed markets and, in the past two decades, emerging markets, as well. These studies point to certain statistical properties shown by time series of the rates of return on shares and the correlations existing between them, as well as their differences in distribution. For example, a basic characteristic is the existence of fat-tails in distributions, which signifies a lack of normality. Other such characteristics are the existence

of autocorrelation and asymmetry and leptokurtosis of distribution (Jajuga, 2000). The characteristic fat-tailed distribution of the rate of return was already described in 1972 by Officer (1972), who analyzed monthly distributions of rates of return of 39 shares listed on the American stock exchange between 1926 and 1968. The authors of numerous studies on rates of return, which have been carried out on emerging markets, find that the distributions of rates of return on shares listed on these markets are different, sometimes considerably so, from the distributions observable on the well-developed markets. For instance, in a study conducted on BRICS countries, it turned out that the distributions of rates of return on shares at different frequencies of measurement have fatter and longer tails than on the developed markets (Adu, Alagidede, & Karimu, 2015). Such findings mean that atypical values of the rates of return on the emerging markets appeared more commonly than on the developed markets and did not depend on the frequency of measurement.

In contrast, in a study concerning rates of return on shares listed on 5 developed European markets, Corhay and Tourani Rad (1994) concluded that time series of daily rates of return are characterized by the lack of autocorrelation and that daily rates of return on the indices of these markets are marked with heteroscedasticity. The authors of the study also indicated that GARCH-type models are the most suitable in statistical terms for analyzing daily rates of return on shares on these markets.

Another study examining the statistical properties of rates of return tested the stationarity of the rates of return on stock indices on ten markets in Central and Eastern Europe between 1994 and 2006. Its authors (Harrison, Lupu, & Lupu, 2010) concluded that these time series at daily measurement of the rates of return are stationary and that there is a significant cross-correlation among the rates of return on indices in these countries. Examination of a cross-correlation among companies was also carried out for the rates of return on shares listed on the Chinese stock exchange between 1999 and 2011. It was observed that the correlations increased significantly during the crashes of the market in China in 2001 and 2008 (Ren & Zhou, 2014).

Other examples of studies of the statistical properties of rates of return on shares at various frequen-

cies of measurement of the rate of return may be, among others, (Poon & Taylor, 1992; Ren, Guo, & Zhou, 2009; Zumbach, Fernandez, & Weber, 2014). Research studies on the statistical properties of the rates of return on shares listed on the German and French market were based, however, on data collected at a high frequency of measurement (daily and minute-based data) and were conducted, among others, by (Kramer & Runde, 1996; Qiu, Zhenga, Rena, & Trimper, 2007; Teiletche, 1998).

In Poland the statistical properties of rates of return on shares were also examined in numerous studies. For instance, based on a sample of data collected from the biggest companies listed on the Warsaw stock exchange between 2000 and 2006, Kompa and Matuszewska-Janica (2008) concluded that for the majority of shares under analysis, the distributions of the rates of return are asymmetric, that kurtosis is significantly different from the normal distribution and that the estimated variances for the bull and bear sub-periods are significantly different in statistical terms. In another study, Piontek (2007) also stated that the majority of distributions of the rates of return on shares of several dozen companies listed on the Warsaw stock exchange were fat-tailed and asymmetric.

The main objective of the article is examination of the statistical properties of monthly rates of return of companies listed on the stock exchange in Frankfurt, Paris, and Warsaw between 2005 and 2015, which were the biggest in terms of capitalization and turnover. This analysis covers the descriptive statistics of the rates of return as well as normality of distribution and stationarity of their time series. Moreover, hypotheses about the equality of the expected value of the rate of return and equality of its variance undergo examination. This study is also conducted for a division of the whole market into the bull and the bear segments according to two definitions used in the relevant literature. The results that have been obtained serve to carry out a comparative analysis of the largest companies on the German, French, and Polish markets. We want to compare the Polish market with the two largest markets in the EU.

The study may be particularly useful for Poland, whose stock market has operated for 26 years and is considered to be relatively large and well-developed in Europe (although has been little diagnosed so far),

and where an important role is played by international institutional investors. Knowledge acquired from the comparative analysis that is part of this study will carry cognitive value not only for Poland but for investors from the European Union as well, who may use it in the course of making investment decisions on these three stock markets under examination. Such comprehensive research comparing the properties of the rates of return on shares listed on the abovementioned stock markets has not been conducted before.

2. Research sample and methodology

The study examines logarithmic monthly rates of return on shares (measured in %) of the biggest companies listed on stock markets in Frankfurt (28 companies included in the DAX index portfolio), Paris (36 companies included in the CAC40 index portfolio), and Warsaw (37 companies included in the WIG20 and mWIG40 index portfolio) between 2005 and 2015 (132 observations) and calculated according to the closing rates on the last day of the month. Data were obtained from the Thomson Reuters service. The rates of return were calculated without taking into account income from dividends.

For the rates of return defined in such a way, the basic descriptive statistics are estimated, and the normality and stationarity of their time series are analyzed in statistical terms. The following questions are examined as well: a) Is the expected value of the rate of return on a share of a company under analysis significantly different from zero? b) Does a statistically significant difference exist between the rate of return on a share of a company and the rate of return of a portfolio of the stock index the company belongs to? c) Is there a statistically significant difference between volatility of the rate of return on a share of a company and the volatility of the rate of return of the portfolio of the relevant stock index? This study was also carried out considering the division of the whole market into the bull and bear segments, which was done according to the behavior of the STOXX Europe 600 Index (containing companies from 17 European states) throughout the period of the sample and taking into consideration the recommendations proposed in the contemporary relevant literature, e.g., by Pagan and Sossounov (2003). This division was made according to the following two

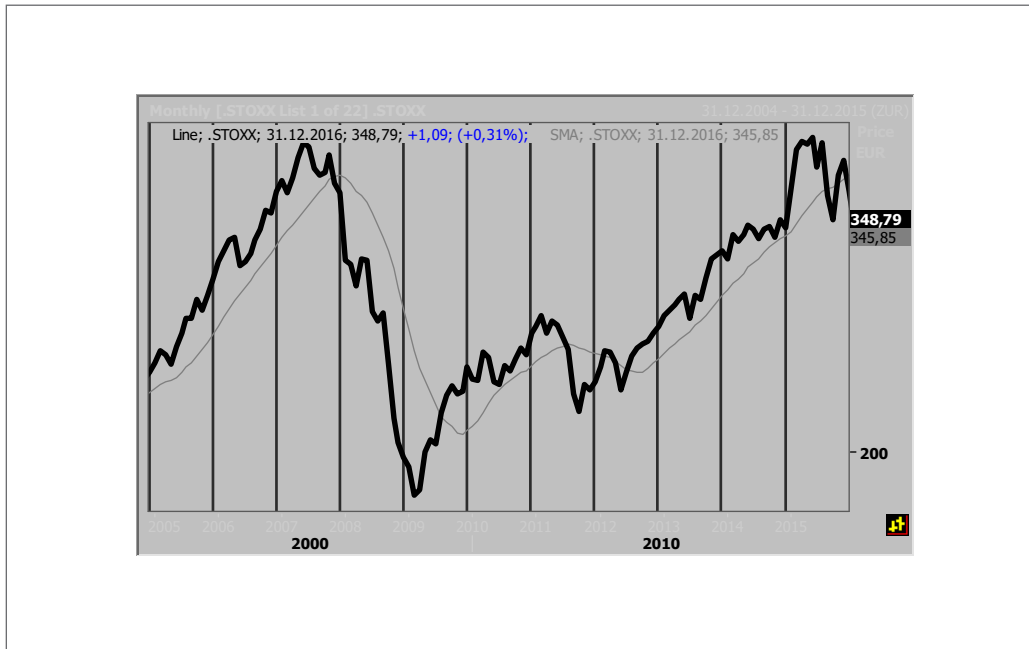


Figure 1. STOXX Europe 600 Index in 2005-2015

definitions that are most often used in the literature (Dębski & Feder-Sempach, 2012):

1. Definition 1 is based on the crossover of a 14-day moving average with the index, which is illustrated below in Figure 1 presenting the STOXX Europe 600 Index, where the index is marked with a line in bold and the thin line indicates the moving average (the average from a longer period was a straight line, and the bull and bear markets were difficult to indicate).
2. Definition 2 – the period of the STOXX Europe 600 Index from the bottom (the smallest value of the index) to the top (the highest value of the index) is the bull market, and the period between the top and the bottom is the bear market.

According to definition 1, there are three periods of bull market (101 observations) and two periods of bear market (31 observations), whereas according to definition 2, there are three periods of bull market (97 observations) and three periods of bear market (35 observations).

3. Discussion of the results

During analysis of the basic statistical properties of monthly rates of return enjoyed by the largest companies on the German, French, and Polish markets, their basic descriptive statistics were estimated first, which are the arithmetic mean (\bar{R}), standard deviation (S), standard semideviation (SS), skewness (A), and kurtosis (K). The results are presented in Table 1 as a percentages for the distinguished ranges of these statistics for the whole sample and taking into account the division into the bull and bear markets according to the definitions provided above.

These results demonstrate that the mean of the largest number of companies from all the three markets (from 54% to 64%), considering the whole sample, fits within the range [0; 1]; that is, it is not very high. The mean of the biggest number of companies that are to be found on the Polish market (35%) is above 1, and that on the French market is negative (25%). On the bull market, the overwhelming majority of companies achieve a mean of above 1 (from 67% to 76% accord-

Table 1. Consolidated information about descriptive statistics of CAC40, DAX and WIG20+40 companies

Value of statistic form a given range	Share of the companies (in %)														
	Whole sample			Definition 1						Definition 2					
				Bull market			Bear market			Bull market			Bear market		
	CAC40	DAX	WIG20+40	CAC40	DAX	WIG20+40	CAC40	DAX	WIG20+40	CAC40	DAX	WIG20+40	CAC40	DAX	WIG20+40
$(-\infty; 0)$	25	14	11	3	7	3	94	96	87	3	0	3	100	100	100
\bar{R} $[0; 1]$	64	64	54	30	18	21	6	4	8	8	11	5	0	0	0
$(1; +\infty)$	11	22	35	67	75	76	0	0	5	89	89	92	0	0	0
$[0; 10]$	78	72	46	94	96	76	39	36	11	89	89	62	61	50	27
S $[10; 15]$	22	21	49	6	4	21	39	36	51	11	11	35	33	32	62
$(15; +\infty)$	0	7	5	0	0	3	22	28	38	0	0	3	6	18	11
$[0; 7]$	72	64	38	100	96	92	97	86	92	100	100	92	100	82	100
SS $[7; 10]$	28	18	57	0	4	5	3	14	8	0	0	5	0	18	0
$(10; +\infty)$	0	18	5	0	0	3	0	0	0	0	0	3	0	0	0
$(-\infty; -0.5)$	42	61	33	3	14	3	28	43	24	8	4	3	44	68	57
A $[-0.5; 0.5]$	58	39	62	94	82	84	69	50	68	75	79	57	53	32	43
$(0.5; +\infty)$	0	0	5	3	4	13	3	7	8	17	18	41	3	0	0
$(-\infty; -1)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
K $[-1; 1]$	50	25	32	81	79	54	83	68	65	75	71	51	83	61	49
$(1; +\infty)$	50	75	68	19	21	46	17	32	35	25	29	49	17	39	51

ing to definition 1 and approximately 90% according to definition 2), whereas on the bear market, a substantial majority of companies (from 87% to 96% according to definition 1) or all of them (according to definition 2) have a negative mean. On the Polish bear market according to definition 1, 5% of the companies have a mean higher than 1, and 8% of companies have a mean within the range $[0; 1]$. Companies listed on the French (78%) and German (72%) markets pose the smallest investment risk measured by standard deviation, and the biggest risk is posed by companies on the Polish market in which only 46% of companies pose risk lower than 10, and 49% ranged from $[10; 15]$.

Similar relations are observed on the bull market where the overwhelming majority of companies pose risk smaller than 10, and the fewest companies with such risk are found on the Polish market. On the bear market, there are relatively high fractions of companies with risk ranging $[10; 15]$ or even exceeding 15. In terms of risk measured by standard semideviation for the whole sample, companies on the French market have the best scores, and companies on the Polish market, where 57% have a semideviation within the range $[7; 10]$, have the worst scores. It is noteworthy that 18% of companies on the German market have a semideviation higher than 10, which is relatively high. On the

bull market, 92% or more of the companies have a standard semideviation within the range $[0; 7)$, and on the bear market, these fractions are slightly smaller; there are at least 14% companies with a semideviation ranging from $[7; 10]$ on the German market.

For the whole sample, the largest number of companies with low asymmetry (range $[-0.5; 0.5]$) is found on the Polish market (62%); on the French market, it is 58%, and on the German market, it is 39%. A relatively large number of companies have negative asymmetry lower than -0.5 ; on the German market, it is as much as 61% (on the French market it is 42%), and only 5% of companies exclusively on the Polish market have positive asymmetry higher than 0.5 . This could signify that the distribution of rates of return on the markets that are more developed than the Polish one have the so-called fat tails. Admittedly, approximately 60% of companies on the Polish and French market are characterized by very low asymmetry (or show no asymmetry at all). On the bull market, the vast majority of companies from all the markets show asymmetry within the range $[-0.5; 0.5]$, and the fractions of companies are larger when using definition 1 rather than definition 2. On the Polish market, (according to definition 2) 41% of companies show right-sided asymmetry higher than 0.5 . In contrast, on the bear market, relatively high fractions of companies with left-sided asymmetry below -0.5 may be observed, and these fractions are higher for definition 2 than for definition 1.

The results obtained for kurtosis (estimated as excess kurtosis, which is equal to 0 for a normal distribution) indicate that on all the markets under examination for the whole research sample and a relatively large number of companies, the values of kurtosis exceed 1 (75% of companies on the German market, 68% on the Polish market, and 50% on the French market), which testifies to the fact that the empirical distribution of logarithmic monthly rates of return of these companies is thinner than normal, that is, leptokurtic. There is, however, a relatively wide group of companies (on the French market amounting to as much as 50%) whose kurtosis takes the values from the range $[-1; 1]$, which may signify that their empirical distribution of the rate of return coincides with the normal rate. On the French and German bull markets, there are substantially more companies whose kurtosis ranges $[-1; 1]$ than companies whose kurtosis

exceeds 1. On the Polish market, the fractions of kurtosis from these ranges fluctuate around 50%. On the bear market, the situation is similar, and according to definition 2, 49% of companies on the Polish market show kurtosis ranging from $[-1; 1]$, and 51% of companies have kurtosis higher than 1. Altogether, this means that there are a relatively high number of companies (and this is mainly true for the French and German markets – the ones that are more developed than the Polish market) whose empirical distributions of the rates of return may be convergent with the normal distribution. This may confirm a thesis often encountered in the relevant literature, which states that on the developed markets, empirical distributions of monthly rates of return display a tendency to coincide with a normal distribution.

One of the fundamental statistical properties of the rates of return on shares, in particular in analyses based on time series, is an answer to the question of whether an empirical distribution of a rate of return under examination is a normal distribution. The study may be carried out on the basis of several tests, and the most commonly used are the Jarque-Bera test and the Shapiro-Wilk test. The values of these statistics are presented in Table 2 (as p-values) for the rates of return on shares of companies on the German, French, and Polish markets.

The table demonstrates that for the whole sample, fractions rejecting the null hypothesis in both tests that have been performed are relatively high, which means that empirical distributions of the rates of return from all the three markets are not normal (at the level of significance of 0.05 in line with the Jarque-Bera test, it is true in the case of over 78% companies on the German market, over 64% companies on the Polish market, and over 63% on the French market). Everything changes completely when the market is divided into the bull and bear segments because fractions rejecting the null hypothesis are considerably smaller for definition 1 and slightly higher for definition 2. For example, when the Jarque-Bera test is applied at significance level of 0.05, this fraction does not exceed 30% according to definition 1 and 55% according to definition 2 (and the highest fractions are found on the Polish market). This signifies that there is no basis to reject the null hypothesis for the remaining companies, that is, the empirical distributions of the rates of return on shares

Table 2. Fraction of rejections of the null hypothesis in the normality tests of returns of CAC40, DAX and WIG20+40 companies (in %)

	Index	Jarque-Bera test			Shapiro-Wilk test			
		pval<0.01	pval<0.05	pval<0.1	pval<0.01	pval<0.05	pval<0.1	
Whole sample	CAC40	41.67	63.89	77.78	41.67	55.56	72.22	
	DAX	67.86	78.57	85.71	64.29	78.57	82.14	
	WIG20+40	51.35	64.86	70.27	59.46	62.16	75.68	
Definition 1	Bull market	CAC40	2.78	13.89	22.22	5.56	16.67	22.22
		DAX	17.86	21.43	28.57	17.86	25.00	25.00
		WIG20+40	16.22	29.73	45.95	18.92	32.43	45.95
	Bear market	CAC40	5.56	11.11	19.44	2.78	13.89	16.67
		DAX	10.71	25.00	32.14	3.57	14.29	28.57
		WIG20+40	8.11	24.32	29.73	8.11	13.51	27.03
Definition 2	Bull market	CAC40	19.44	30.56	36.11	13.89	25.00	38.89
		DAX	25.00	28.57	35.71	21.43	28.57	32.14
		WIG20+40	29.73	54.05	64.86	32.43	51.35	62.16
	Bear market	CAC40	11.11	22.22	30.56	13.89	25.00	25.00
		DAX	25.00	39.29	46.43	25.00	35.71	42.86
		WIG20+40	24.32	48.65	59.46	37.84	54.05	62.16

of the companies under examination are normal. The conclusion is that a relatively large number of companies on the bull and the bear markets (which might be over 70% on the French market, more than 60% on the German market, and over 45% on the Polish market) may have distributions of rates of return that are convergent with the normal distribution. It is also observable that on the better-developed markets, these fractions are higher than on the Polish market.

Another desirable property, which is particularly important in analyses and modeling of financial time-series, is that their stationarity is confirmed. An Augmented Dickey-Fuller test (ADF) was adopted in order to examine the stationarity of time series of rates of return of companies from the markets under analysis (Charemza & Deadman, 1997, pp. 110-122). The study was performed for a case in which there was so-called

drift observed in a time series of a rate of return on shares, which in the random walk testifies to the existence of a steady trend, as well as for a case in which a deterministic trend was additionally taken into account. The results of this study are presented in Table 3.

The outcome of the first test indicated that at all three significance levels, the time series of all the companies in all the segments of the markets under analysis are stationary. In the second test, at the significance level of 0.05, there are three cases (one of which is encountered on the French market and two on the German market) where the number of stationary time series of the rates of return reaches over 96%. These results indicate that there should be no major complications in the process of correct – in terms of properties – estimation of the beta parameter and its fundamental characteristics, e.g., dispersion.

Table 3. Fraction of rejections of the null hypothesis in the unit root tests of returns of CAC40, DAX and WIG20+40 companies (in %)

	Index	ADF test with drift			ADF test with drift and trend			
		(p-value)			(p-value)			
		pval<0.01	pval<0.05	pval<0.1	pval<0.01	pval<0.05	pval<0.1	
Whole sample	CAC40	100.00	100.00	100.00	100.00	100.00	100.00	
	DAX	100.00	100.00	100.00	100.00	100.00	100.00	
	WIG20+40	100.00	100.00	100.00	100.00	100.00	100.00	
Definition 1	Bull market	CAC40	100.00	100.00	100.00	100.00	100.00	100.00
		DAX	100.00	100.00	100.00	100.00	100.00	100.00
		WIG20+40	100.00	100.00	100.00	100.00	100.00	100.00
	Bear market	CAC40	100.00	100.00	100.00	88.89	97.22	100.00
		DAX	100.00	100.00	100.00	89.29	96.43	100.00
		WIG20+40	100.00	100.00	100.00	91.89	100.00	100.00
Definition 2	Bull market	CAC40	100.00	100.00	100.00	100.00	100.00	100.00
		DAX	100.00	100.00	100.00	100.00	100.00	100.00
		WIG20+40	100.00	100.00	100.00	100.00	100.00	100.00
	Bear market	CAC40	100.00	100.00	100.00	94.44	100.00	100.00
		DAX	100.00	100.00	100.00	92.86	96.43	100.00
		WIG20+40	100.00	100.00	100.00	97.30	100.00	100.00

Next, the study proceeds to statistically verify the hypotheses regarding whether the expected rate of return on shares of a company under analysis is significantly different from zero and whether this expected rate of return is significantly different from the expected rate of return on the stock index that the company belongs to. In other words, the null hypothesis in the former case was formulated as follows: $H_0 : E(R_i) = 0$, and that in the latter case was $H_0 : E(R_i) = E(R_{index})$. The values of the statistics (as p-values) for both hypotheses that undergo verification are provided in Table 4.

The presented estimations demonstrate that in the majority of cases (for the whole sample), there is no basis to reject the null hypothesis regarding the equality of the expected rate of return on shares of companies being tested neither with zero nor with the expected

rate of return on the index that the company is included in. For instance, the fractions rejecting the null hypothesis (at $\alpha = 0.05$) are 14.29% on the German market, 8.11% or 0% on the Polish market, and 5.56% on the French market. The results obtained for a division of the whole market into the bull and bear segments are very diversified and allow no general conclusion to be drawn. For example, on the bull market according to definition 1, at the significance level of 0.05, the fractions rejecting the null hypothesis of equality of the rate of return of a company being tested with zero are as follows: 67.86% (the German market), 45.95% (the Polish market), and 41.67% (the French market), whereas on the same market according to definition 2, the fractions are considerably higher. Moreover, a very broad difference between the obtained fractions reject-

Table 4. Fraction of rejections of the null hypothesis of equality to zero of returns and of equality of index return and company return of CAC40, DAX and WIG20+40 companies (in %)

	Index	$H_0 : E(R_i) = 0$			$H_0 : E(R_i) = E(R_{index})$			
		pval<0.01	pval<0.05	pval<0.1	pval<0.01	pval<0.05	pval<0.1	
Whole sample	CAC40	0.00	5.56	8.33	0.00	5.56	11.11	
	DAX	3.57	14.29	17.86	7.14	14.29	17.86	
	WIG20+40	2.70	8.11	16.22	0.00	0.00	8.11	
Definition 1	Bull market	CAC40	11.11	41.67	66.67	0.00	2.78	2.78
		DAX	28.57	67.86	75.00	7.14	17.86	17.86
		WIG20+40	13.51	45.95	56.76	21.62	56.76	72.97
	Bear market	CAC40	2.78	5.56	11.11	0.00	8.33	16.67
		DAX	0.00	0.00	10.71	0.00	0.00	7.14
		WIG20+40	0.00	0.00	8.11	0.00	0.00	5.41
Definition 2	Bull market	CAC40	55.56	77.78	88.89	2.78	11.11	16.67
		DAX	64.29	78.57	85.71	14.29	17.86	28.57
		WIG20+40	62.16	86.49	86.49	89.19	94.59	94.59
	Bear market	CAC40	30.56	63.89	69.44	0.00	22.22	25.00
		DAX	21.43	42.86	67.86	10.71	25.00	32.14
		WIG20+40	29.73	70.27	81.08	89.19	100.00	100.00

ing the null hypothesis emerges between the bear market segments distinguished according to definitions 1 and 2. Regarding verification of the null hypothesis about equality of a rate of return on a share of a company being tested with the rate of return on the index portfolio the company is part of, the fractions rejecting this hypothesis both on the bull and the bear markets are significantly varied. In particular, such differences occur between the Polish market and the French German markets and between the adopted definitions that divide the market into the bull and bear segments.

The last hypothesis that undergoes verification concerns the equality of variance of a rate of return on a share of a company with variance of a rate of return on the stock index in which the company is included. Verification of the hypothesis formulated in such a way

is carried out based on the Fisher-Snedecor test statistic. The values of this statistic are provided as p-values in Table 5.

The table shows that for the whole sample, in the majority of cases and on all the markets (e.g., at significance level of 0.05), variance of the rate of return of a company is significantly different from variance of the rate of return of the whole stock index portfolio (on the Polish market, the fraction rejecting the null hypothesis is approximately 97%). On the Polish bull market (according to both definitions), the fraction rejecting the null hypothesis even amounts to 100% (this is also true for the German bull market distinguished in line with definition 1). On the French market, the fraction is slightly smaller and reaches 89%. On the bear market, the fractions rejecting the null hypothesis

Table 5. Fraction of rejections of the null hypothesis of equality of variances of returns of CAC40, DAX and WIG20+40 companies (in %)

		Index	$H_0 : S^2_i = S^2_{index}$		
			pval<0.01	pval<0.05	pval<0.1
Whole sample		CAC40	77.78	80.56	83.33
		DAX	67.86	71.43	78.57
		WIG20+40	91.89	97.30	100.00
Definition 1	Bull market	CAC40	83.33	88.89	88.89
		DAX	89.29	100.00	100.00
		WIG20+40	97.30	100.00	100.00
	Bear market	CAC40	52.78	61.11	66.67
		DAX	42.86	46.43	53.57
		WIG20+40	40.54	64.86	72.97
Definition 2	Bull market	CAC40	77.78	83.33	88.89
		DAX	85.71	96.43	96.43
		WIG20+40	97.30	100.00	100.00
	Bear market	CAC40	47.22	63.89	63.89
		DAX	46.43	50.00	60.71
		WIG20+40	64.86	75.68	83.78

are slightly smaller and range from 61%-65% according to definition 1 and 50%-76% according to definition 2. This means that on the bull and bear market segments of all the three markets under examination, there are also differences in the volatility of the rates of return of a company and the rates of return on an index portfolio the company is included in for the majority of companies.

4. Conclusions

To summarize the findings of the study and the comparative analysis of the biggest companies listed on two developed (German and French) markets and a third emerging (Polish) market, it may be concluded that they are interesting in cognitive terms. The obtained results indicate that there are quite significant differ-

ences in the statistical properties of the rates of return on shares of companies between both types of markets as well as between the distinguished bull and bear market segments. In contrast, their common characteristics are low asymmetry and kurtosis in the case of a relatively large number of companies, especially when the division into the bull and bear markets is adopted, which could suggest convergence between the empirical distributions of the rates of return under examination and normal distribution. This has also been confirmed by tests that were adopted to verify the hypothesis concerning the normality of empirical distributions of the rates of return of companies under analysis; however, for the whole sample, there are definitely more cases in which the rates of return have no normal distribution. An undoubtedly positive – from

the point of view of statistics – property of nearly all time series under examination is their stationarity. Verification of the hypothesis concerning equality of the expected rate of return on shares of a company being tested with zero or the expected rate of return on the index portfolio to which the company belongs has demonstrated that for the vast majority of companies (for the whole sample), the equality is present, whereas for the bull and bear segments, the outcome is highly equivocal, especially with the division made according to the adopted definitions. Verification of the hypothesis concerning equality of investment risk of the rate of return on shares of a company being tested with the risk posed by the stock index portfolio to which the company belongs reveals that the risks are predominantly different. The study also demonstrated that the market has rightly been divided into the bull and bear segments since the results obtained for these segments are occasionally significantly different from the results for the whole sample.

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