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FINANCIAL ECONOMICS | RESEARCH ARTICLE CAPM with various utility functions: Theoretical developments and application to international data

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1 $\!\!\!^{1\ddagger}$

Abstract: This paper presents an extension of the Capital Assets Pricing Model (hereafter CAPM) where various utility functions are applied. Specifically, we propose an overall CAPM beta that accounts for the higher order moments and reflects the investor preferences and attitudes toward risk. We particularly develop CAPM betas for different classes of utility function: the negative exponential utility function, power utility function or "Constant Relative Risk Aversion (CRRA) Utilities" and hyperbolic utility function or "HARA Utilities" (hyperbolic absolute risk aversion). In order to validate our theoretical results, we analyze the impact of investors' preferences on the valuation equation. Applying the International CAPM, the results indicate that our utilities-based betas differ largely from the traditional CAPM betas. Moreover, the results confirm the importance of higher order moments on the pricing equation. Finally, the results both empirically and theoretically post to the consistent effect of the risk aversion degree on our utilities-based CAPM.

Subjects: Applied Mathematics; Quantitative Finance; Statistics for Business, Finance & Economics

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PUBLIC INTEREST STATEMENT

Since its appearance in 1964, the Capital Asset Pricing Model of Sharpe has raised much debate among academicians and practitioners (Black, 1972; Fama & French, 1993; Fama & French, 2015; Fama & Macbeth, 1973; Roll, 1977 among others). The CAPM relies on several simplifying assumptions that render its applicability questionable. Among its assumptions, the quadratic utility function which supposes that investors are risk-averse and maximize the mean-variance criterion. This paper is an extension of the CAPM where various utility functions are applied. Using the truncated Taylor approximation, we extract risk measures from various utility functions. We, then, replace the variance by our new risk measures extracted from utility functions in the systematic risk (beta) of the CAPM. In order to validate our theoretical findings, we use a sample of indexes listed in the MSCI classification on a daily basis. Our results both empirically and theoretically post to the consistent impact of the risk aversion degree on our utilitiesbased CAPM.

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Keywords: truncated Taylor approximation; negative exponential utility function; power utility function; hyperbolic utility function; risk measure; assets pricing; International CAPM

JEL classifications: C02; C13; C15; G12; G15; G23

1. Introduction

The traditional version of the CAPM of Sharpe (1964) and Lintner (1965) assumes that assets' returns are a linear function of their equivalent systematic risk measured by $\beta_{CAPM'}$ the slope from the regression of securities' returns on the market risk premium.

The CAPM relies on several restrictive assumptions. In particular, the validity of the model supposes verifying the normality of stock returns and investors' homogeneous anticipations. Under The first condition, the expected utility can be expressed with an exact function of the mean and variance of the returns' distributions. However, the second one is necessary to legitimate the formalization problem of the investor choice in a risky situation. For the mean variance approach to be valid, we require a guadratic utility function. This means that returns? distribution is fully described by the two first moments, i.e. the mean and the variance. This makes the theoretical derivation of the CAPM, as well as its applicability, questionable at best. Indeed, many studies (see, for instance: Fama (1965), Arditti (1971), Singleton and Wingender (1986), and, more recently, Chung, Johnson and Schill (2006)) show that stock returns do not follow the Gaussian assumption, since assets returns distributions are asymmetrical with heavy tails. Several studies, then (see, for instance, Roll (1977), Fama and French (1992) among others) have announced daringly the death of the CAPM. Specifically, the unability of the ho_{CAPM} to explain the cross section of expected stock returns. Although a large body of empirical literature, developed in the following decades, provide varying results and the validity of CAPM is still between offenders and defenders. In fact, Shah, Abdullah, Khan, and Khan (2011) compare the performance of the CAPM and the Fama and French (1993) three factor model. Their results favor the use of the CAPM model to estimate expected returns. Fama and French (2015) propose a five-factor model aims at capturing the size, value, profitability, and investment patterns in average stock returns. They find that by the inclusion of profitability and investment factors, the value factor (book to market) of the FF three-factor model becomes redundant for describing average returns. This is not strange since, the $\beta_{\it CAPM}$ is based on traditional risk measures. Traditionally, Markowtiz (1952) proposes the variance as a measure of risk (see Bouchaud & Potters, 1997; Duffie & Richardson, 1991) and the "mean-variance" approach to determine the optimal portfolio, minimizing the variance or maximizing returns. Nevertheless, this model is valid only on a quadratic utility function framework and supposes that returns follow a normal distribution. To solve this problem, Markowitz (1959) suggests the semi-variance to account for the downside risk. Other risk measures are proposed, such as the partial order moments and the value-at-risk (see Bouchaud & Selmi, 2001; Coombs & Lehner, 1981, 1984; Fishburn, 1982, 1984; Luce, 1980; Pollatsek & Tversky, 1970; Sarin, 1987; Stone, 1973).

However, these attempts do not appear to bring definitive solutions. Indeed, it seems that the question is quite controversial. In fact, modeling the non linear distribution suffers from the lack of a pertinent risk measure to capture for investors preferences. This problem is getting worse when cumulated together with the necessity to specify a utility function, because only the investors utility function can determine their preferences.

Bell (1988) proposed an exponential utility function plus a linear function $U(x) = ax - be^{-cx}$ with $a \ge 0, b > 0$ and c > 0 in order to determine a measure of risk in the following form $E[e^{-c(x-E(x))}]$. Bell (1995) applied the same technique for measuring risk using other types of utility functions. Always in the utility function context Heston (1993) proposed a general risk measure of the form, U(E(X)) - E(U(X)). In the same context, Jia and Dyer (1996) suggested a general risk measure in the form: R = -E[U(X - E(X))]. Bellalah and Selmi (2002) showed that that a maximization program of the expected utility can be equivalent under certain conditions, to the minimization of some risk measure. Yet their risk measure is valid only for some utility functions classes. Particularly, the application of this measure to a power utility function leads to a zero risk which is very far from reality. Nevertheless, these risk measures are a quite vague. Indeed, since they are not developed from utility functions, one cannot expect that they fully describe investors' preferences. In these sense, our objective is threefold: first, we develop several risk measures based on diverse classes of utility functions. More concretely, we propose risk measures for the negative exponential utility function or "CARA Utilities" (constant absolute risk aversion), power utility function or "CARA Utilities" (constant absolute risk aversion), power utility function or "CARA Utilities" (constant absolute risk aversion). We, then determine the CAPM systematic risk (β_{CAPM}) based on the risk measures extracted from utilities. For robustness check, we, finally, apply the CAPM with various risk attitudes to international data.

The structure of the paper is as follows. In Section 2, we develop our theoretical risk measures based on utility functions. In Section 3, we determine a general CAPM beta that can be applied to different utility functions as well as betas for the negative exponential utility function, the power utility function, and hyperbolic utility function. In Section 4, we apply these betas to international data and compare them with the traditional CAPM betas. Section 5 concludes the paper.

2. Risk measures and utilities functions: Theoritical developments

In this section, we theoretically determine risk measures based on investors' utility functions that accounts for the moments of order three and four (the skewness and the kurtosis), which are indicators of the asymmetry and peakedness of the probability distribution. Bellalah and Selmi (2002) showed that a program of the expected utility maximizing can be equivalent under certain conditions, to the minimization of some risk measure (they used cubic and negative exponential utility functions to test their approach). However, this risk measure can be used only for some defined utility functions. Specifically, the application of this measure to a power utility function leads to a zero risk which is very far from reality. To overcome these limitations, remaining in the context of the expected utility approach, we determine an overall risk measure for different classes of utility function or "CRRA Utilities" (constant relative risk aversion), and hyperbolic utility function or "HARA Utilities" (hyperbolic absolute risk aversion).

2.1. The expected utility approach

2.1.1. The approach presentation

We consider that each agent has at time *t* an initial wealth *W* and a *von Neumann–Morgenstern* utility function, strictly increasing and concave. We further assume that the utility function of the investor is continuously differentiable and satisfies the following properties:

$$U^{(1)} > 0$$
, $U^{(2)} < 0$, $U^{(3)} > 0$, $U^{(4)} < 0$

with $U^{(i)}$ the *i*-th derivative of the utility function.

Under these assumptions, the utility function can be expanded in Taylor series in the neighborhood of the expected future wealth $\mathbb{E}(W)$:

$$U(W) = \sum_{i=0}^{N} \frac{1}{n!} U^{(i)}(\mathbb{E}(W)) \mathbb{E}\left[(W - \mathbb{E}(W))^{i} \right] + \xi_{N+1}(W)$$
(1)

where $\xi_{n+1}(W)$ is defined as follows :

$$\xi_{N+1}(W) = \frac{U^{(N+1)}(\zeta)}{(N+1)!} \left[W - \mathbb{E}(W) \right]^{(N+1)}$$

with:

$\zeta \in]W, \mathbb{E}(W)[siW < \mathbb{E}(W)ou\zeta \in]\mathbb{E}(W), W[siW < \mathbb{E}(W)etN \in N^*$

Assuming that the Taylor approximation of U in the neighborhood of $\mathbb{E}(W)$ is convergent and the distribution F(W) is only determined by its moments, and supposing that n tends to the infinity of the expected value of the Equation (7), we get :

$$\mathbb{E}(U(W)) = \int_{-\infty}^{+\infty} \left\{ \lim_{N \to \infty} \left[\sum_{i=0}^{N} \frac{1}{i!} U^{(i)}(\mathbb{E}(W)) \mathbb{E}\left[(W - \mathbb{E}(W))^{i} \right] + \xi_{N+1}(W) \right] \right\} dF(W)$$
(2)

which implies:

$$\mathbb{E}[U(W)] = U(\mathbb{E}(W)) + \frac{1}{2}U^{(2)}(\mathbb{E}(W)) \mathbb{E}\left[(W - \mathbb{E}(W))^2\right] \left[+ \frac{1}{3!}U^{(3)}(\mathbb{E}(W)) \mathbb{E}\left[(W - \mathbb{E}(W))^3\right] \right] \times \left[+ \frac{1}{4!}U^{(4)}(\mathbb{E}(W)) \mathbb{E}\left[(W - \mathbb{E}(W))^4\right] \right] \left[+ \sum_{i=5}^N \frac{1}{n!}U^{(i)}(\mathbb{E}(W)) \mathbb{E}\left[(W - \mathbb{E}(W))^i\right] \right]$$
(3)

with

 $\lim_{N\to\infty}\xi_{N+1}(W)=0$

Let $\sigma^2 = \mathbb{E}\left[(W - \mathbb{E}(W))^2\right]$, $\lambda_3 = \mathbb{E}\left[(W - \mathbb{E}(W))^3\right]$ and $\lambda_4 = \mathbb{E}\left[(W - \mathbb{E}(W))^4\right]$ be, Respectively, the variance, the third- and the fourth-order moments. Equation (9) becomes:

$$\mathbb{E}[U(W)] = U(\mathbb{E}(W)) + \frac{1}{2}U^{(2)}(\mathbb{E}(W))\sigma^{2} + \frac{1}{3!}U^{(3)}(\mathbb{E}(W))\lambda_{3} \\ \times \left[+\frac{1}{4!}U^{(4)}(\mathbb{E}(W))\lambda_{4} + \sum_{i=5}^{N}\frac{1}{n!}U^{(i)}(\mathbb{E}(W))\mathbb{E}\left[(W - \mathbb{E}(W))^{i} \right] \right]$$
(4)

This equation takes into account all centered moments of the probability distribution of the random wealth, namely the third (*skewness*) and the fourth (*kurtosis*) moments, and not only the first two moments. It shows that the investor has a preference for the mean and the asymmetry (positive) and an aversion to variance and kurtosis.

2.1.2. Why an expected utility approach?

This approach can be justified by two reasons. On the one hand, the Markowitz "mean- variance" approach assumes that the probability density of wealth is Gaussian, meaning that it is perfectly defined by its first two moments. However, several empirical studies have shown that the mean and variance are not sufficient to fully define the probability distribution. This issue has led many authors to propose alternative forms of the probability density function than the Gaussian. For example, Simaan (1993) proposed a non-spherical distribution, Adcock and Shutes (1999) assumed that the financial asset returns distribution follows a multivariate Skew-Normal, Rachev and Mitnik (2000) defined a Levy-Pareto stable distribution for returns. It seems thus necessary to define an approach that accounts for the first four moments. On the other hand, we need to consider particular utility functions because only utilities can describe investors preferences. For example, assume that investor preferences are represented by a quadratic utility function of the form (see Maillet & Jurczenko, 2006):

$$U(W) = \alpha_0 + \alpha_1 W + \alpha_2 W^2 + \alpha_3 W^3 + \alpha_4 W^4$$
with $\alpha_i \in R^*, i = 1, 2, 3, 4.$
(5)

By applying the mathematical mean, we get:

$$\mathbb{E}(U(W)) = \alpha_0 + \alpha_1 \mathbb{E}(W) + \alpha_2 \mathbb{E}(W^2) + \alpha_3 \mathbb{E}(W^3) + \alpha_4 \mathbb{E}(W^4)$$
(6)

while:

$$\begin{cases} \mathbb{E}(W^2) = \sigma^2(W) + \mathbb{E}(W)^2 \\ \mathbb{E}(W^3) = \lambda_3(W) + 3\mathbb{E}(W)\sigma^2(W) + \mathbb{E}(W)^3 \\ \mathbb{E}(W^4) = \lambda_4(W) + 4\mathbb{E}(W)\lambda_3(W) + 6\mathbb{E}(W)^2\sigma^2(W) + \mathbb{E}(W)^4 \end{cases}$$

This allows us to get:

$$\mathbb{E}(U(W)) = \alpha_0 + \alpha_1 \mathbb{E}(W) + \alpha_2 \mathbb{E}(W)^2 + \alpha_3 \mathbb{E}(W)^3 + \alpha_4 \mathbb{E}(W)^4 + \left[\alpha_2 + 3\alpha_3 \mathbb{E}(W) + 6\alpha_4 \mathbb{E}(W)^2\right] \sigma^2(W) + \left[\alpha_3 + 3\alpha_4 \mathbb{E}(W)\right] \lambda_3(W) + \alpha_4 \lambda_4(W)$$
(7)

It remains to verify the stochastic dominance of this utility function. That is

$$U^{(1)} > 0, U^{(2)} < 0, U^{(3)} > 0, U^{(4)} < 0$$

The first four derivatives from the Equation (5) are:

$$U^{(1)} = \alpha_1 + 2\alpha_2 W + 3\alpha_3 W^2 + 4\alpha_4 W^3$$

$$U^{(2)} = 2\alpha_2 + 6\alpha_3 W + 12\alpha_4 W^2$$

$$U^{(3)} = 6\alpha_3 + 24\alpha_4 W$$

$$U^{(4)} = 24\alpha_4$$

whereas, we know that $U^{(3)} > 0$ et $U^{(4)} < 0$, so we have:

$$\begin{cases} \alpha_3 > 0 \\ W < -\left(\frac{\alpha_3}{4\alpha_4}\right) \\ \alpha_4 < 0 \end{cases}$$

We note that the second derivative of the utility function is a simple second-order equation that must satisfy these conditions¹:

$$\left\{ \begin{array}{l} \alpha_2 < \frac{3\alpha_3^2}{8\alpha_4} & \text{if } \Delta < 0 \\ W < -\left(\frac{\alpha_3}{4\alpha_4}\right) + \frac{\sqrt{(9\alpha_3^2 - 24\alpha_2\alpha_4)}}{12\alpha_4} \\ W < -\left(\frac{\alpha_3}{4\alpha_4}\right) - \frac{\sqrt{(9\alpha_3^2 - 24\alpha_2\alpha_4)}}{12\alpha_4} \\ 0 < \alpha_2 \ge \frac{3\alpha_3^2}{8\alpha_4} & \text{if } \Delta < 0 \end{array} \right.$$

It remains to verify that the first derivative of the quartic utility function is positive. We note that the first derivative is a simple third- order equation which shall, after some calculations, verify the following three conditions:

$$\begin{cases} \alpha_{1} > 0 \\ \left(-\frac{\alpha_{3}^{2}}{16\alpha_{4}^{2}} + \frac{\alpha_{2}}{6\alpha_{4}}\right)^{3} + \left(\frac{\alpha_{3}^{3}}{16\alpha_{4}^{2}} - \frac{\alpha_{2}\alpha_{3}}{16\alpha_{4}^{2}} + \frac{\alpha_{1}}{8\alpha_{4}}\right)^{2} > 0 \\ W < -\left(\frac{\alpha_{3}}{4\alpha_{4}}\right) + \frac{A^{2} + (9\alpha_{3}^{2} - 24\alpha_{2}\alpha_{4})}{12\alpha_{4}A} \\ A = \left(\frac{B + \sqrt{-108(3\alpha_{3}^{2} - 8\alpha_{4}\alpha_{2})^{3} + B^{2}}}{2}\right)^{\frac{1}{3}} \\ B = \left(-54\alpha_{3}^{3} - 432\alpha_{2}^{2}\alpha_{1} + 216\alpha_{4}\alpha_{3}\alpha_{2}\right) \end{cases}$$

If the investor preferences are represented by a quartic utility function, it is easy to verify that the agent has a preference for the mean and the asymmetry and an aversion to variance and kurtosis.

$$\begin{cases} \frac{\partial \mathbb{E}[U(W)]}{\partial \mathbb{E}(W)} = \alpha_1 + 2\alpha_2 \mathbb{E}(W) + 3\alpha_3 \mathbb{E}(W^2) + 4\alpha_4 \Big[\mathbb{E}(W^3) + \mathbb{E}(W)^3 \Big] > 0 \\ \frac{\partial \mathbb{E}[U(W)]}{\partial \alpha_2^2(W)} = \alpha_2 + 3\alpha_3 \mathbb{E}(W) + 6\alpha_4 \mathbb{E}(W)^2 < 0 \\ \frac{\partial \mathbb{E}[U(W)]}{\partial \lambda_4(W)} = 4\alpha_4 \mathbb{E}(W) > 0 \\ \frac{\partial \mathbb{E}[U(W)]}{\partial \lambda_4(W)} = \alpha_4 < 0 \end{cases}$$

2.2. Risk measures and utility functions

2.2.1. A Jia and Dyer (1996) risk measure

Jia and Dyer (1996) have proposed a risk measure which is consistent with the utility theory. Specifically, they showed that there is a negative relationship between investor preferences and risk. In this section, we present this "pure" risk measure within Jia and Dyer framework.

Definition 2.2.1 Let *P* be a convex set of all probability distribution of the set of lotteries {X, Y, Z, ...}. Let P^0 , the all normal probability distributions set, which is a subset of *P* defined by²:

 $P^{0} = \{ X' | X' = X - \mathbb{E}(X), X \in P \}$

We call P^0 "the risk set" of the probablity distribution of X' the standard risk of the lottery X.

Definition 2.2.2 Let $X', Y' \in P^0$, we have $X' >_p Y'$ if and only if $Y' >_p X'$.

We refer by $>_{p}$ to a binary relation of risk and by $>_{p}$ a binary relation of preference in P^{0} .

It is important to note that if the preference relation $>_p$ satisfies the von Neumann-Morgenstern (1944) expected utility axioms, then the agent preferences can be represented by an expected utility function.

THEOREM 2.2.1 For all X', Y' $\in P^0$, X' >_R Y' if and only if R(X') > R(Y'), with R(X') a risk measure:

 $R(X') = -\mathbb{E}\left[U(X - \mathbb{E}(X))\right]$

(8)

where U is a von Neumann-Morgenstern (1944) utility function.

Remark 2.2.1 This risk measure is called general because it imposes no restriction, neither on the shape of the probability distribution nor on the utility function of the lottery.

According to this theorem, Jia and Dyer assume that there is a negative relationship between the preference of the investor as measured by $\mathbb{E}\left[U(X - \mathbb{E}(X))\right]$ and the risk measure denoted *R*. They assume, in addition, that this risk measure must satisfy two conditions. The first concerns derivatives that is, $U^{(2n)} < 0$ and $U^{(2n+1)} > 0$. In other words, the utility function should check stochastic dominance of order *n*. The second deals with the investor risk aversion, preferring even-order moments and being adverse to odd-order moments. This result is well known: Markowitz (1952)³ showed that an individual having a concave utility function for low values of the lottery and convex for high values of the lottery, would prefer the mean and skewness. Therfore, he prefers the right side of asymmetric distributions (gain) and hates the variance and kurtosis that is the left side of asymmetric distributions (loss).

Another important point is that the risk measure defined by Equation (8) is applied to the centered variable $R - \mathbb{E}(X)$. If we play again in the lottery, the mathematical mean $\mathbb{E}(X)$ may serve as a

preference value. In this case, $\widetilde{X} = X - \mathbb{E}(X)$ is a centered lottery that reflects the "pure" risk of the original lottery *R*.

We will now give examples of risk measures for some utility functions.

The exponential utility function

If the investor's preferences are represented by an exponential utility function of the form:

$$U(R) = -e^{-\theta R}$$

with $\theta > 0$, the risk measure corresponding to this function is given by (see Jia & Dyer, 1996):

$$\mathcal{R} = -\mathbb{E}(U(\widetilde{R})) = -\mathbb{E}[U(R - \mathbb{E}(R))] = \mathbb{E}\left[e^{-\theta(R - \mathbb{E}(R))}\right]$$

This risk measure is confused with that of Bell (1988) for an exponential utility function as well as linear utility function of the form:

$$U(R) = aR - be^{(-cR)}$$

with $a \ge 0, b > 0$ et c > 0.

The quadratic utility function

If the investor's preferences are represented by a quadratic utility function of the form:

$$U(R) = \alpha R - \theta R^2$$

with $\alpha > 0$ and $\theta > 0$, the risk measure corresponding to this utility function is:

$$\mathcal{R}(\widetilde{R}) = \theta \mathbb{E}\left[\left(R - \mathbb{E}(R) \right)^2 \right]$$

Remark 2.2.2 This is nothing other than the variance of *W*. This result is already seen in the previous paragraph where we justified the Markowitz "mean-variance" approach.

Quadratic utility functions pose two problems. The first is that these functions are decreasing beyond a certain value of $\tilde{R} > \frac{\alpha}{2\theta}$ and the second is that they are characterized by an increasing risk aversion function. To overcome this problem, Levy (1969) proposed a cubic utility function of the form:

$$U(R) = \alpha R - \theta R^2 + \gamma R^3$$

with α , θ , and γ are positive and constant parameters.

Remark 2.2.3 Note that the choice of parameters α , θ , and γ is not arbitrary. For example, if this function must be increasing, it is necessary that $\theta^2 < 3\alpha\gamma$ and to ensure the concavity, it is necessary that $R < \frac{\theta}{3\gamma}$.

The risk measure according to Equation (14) is:

$$\begin{aligned} \mathcal{R} &= \mathbb{E}\Big[(R - \mathbb{E}(R)^2) \Big] - \gamma' \, \mathbb{E}\Big[(R - \mathbb{E}(R))^3 \Big] \end{aligned}$$
with $\gamma' = \frac{\gamma}{\theta} > 0$

Note that this risk measure is a linear combination of the second-order moment (variance) and the third-order moment (the skewness). It is clear that this risk measure is a decreasing function of the asymmetry (since $\gamma' = \frac{\gamma}{\theta} > 0$). In other words, a transformation of the weight to the right side of the distribution (that is the gain side) reduces the risk.

The quartic utility function

We assume now that the investor's preferences are represented by a quartic utility function of the form:

$$U(R) = \alpha R - \theta R^2 + \gamma R^3 - \delta R^4$$

with $\alpha > 0$ $\theta < 0$ $\gamma > 0$ and $\delta < 0$. Applying Equation (14) for this utility function, we obtain:

$$\mathcal{R} = \theta \mathbb{E}\left[\left(R - \mathbb{E}(R)^2 \right) \right] - \gamma \mathbb{E}\left[\left(R - \mathbb{E}(R) \right)^3 \right] + \delta \mathbb{E}\left[\left(R - \mathbb{E}(R) \right)^4 \right]$$

We remark that this risk measure is a linear combination of the second-order moment, the thirdorder moment, and the fourth-order moment. It is clear that it depends negatively of the skewness and positively of the kurtosis, as it should. This reinforces the intuition according to which the risk is actually dependent to the loss and not to the profit and it is related to the great fluctuations rather than the small ones.

2.2.2. Theoretical developments of risk measures based on utilities functions

Our aim is to determine risk measures for various classes of utility functions and, hence, for diverse degrees of risk aversion. We, particularly, determine risk measures for the the negative exponential utility function (constant risk aversion), power utility function (decreasing risk aversion), and hyperbolic utility function (the risk aversion is dependent to the function parameters). To extract risk measures from utility functions, we use the same technique as in Jia and Dyer (1996) but the truncated Taylor approximation of order four in return.

2.2.2.1. The general framework. We assume that the von Neumann–Morgenstern utility function is strictly increasing, concave and at least fourth-order differentiable. We assume further that the probability distribution of W is entirely determined by its moments. It is important to note that the utility function shall satisfy these four properties:

$$U^{(1)} > 0$$
, $U^{(2)} < 0$, $U^{(3)} > 0$, $U^{(4)} < 0$

with $U^{(i)}$ the *i*-th derivative of the utility function.

Under these assumptions, the mathematical mean applied to the Taylor series, truncated to order four, in the neighborhood of $\mathbb{E}(R)$ leads to the following approximation:

$$\mathbb{E}[U(R)] = U(\mathbb{E}(R)) + \frac{\sigma^2}{2}U^{(2)}(\mathbb{E}(R)) + \frac{\lambda_3}{3!}U^{(3)}(\mathbb{E}(R)) + \frac{\lambda_4}{4!}U^{(4)}(\mathbb{E}(R))$$
(9)

where σ^2 , λ_3 and λ_4 are, respectively, the variance, the skewness, and the kurtosis.

Maximizing the expected utility applied to the final wealth, $\mathbb{E}(U(R))$, is equivalent to maximizing the following quantity:

$$U(\mathbb{E}(R)) + \frac{\sigma^2}{2} U^{(2)}(\mathbb{E}(R)) + \frac{\lambda_3}{3!} U^{(3)}(\mathbb{E}(R)) + \frac{\lambda_4}{4!} U^{(4)}(\mathbb{E}(R))$$

THEOREM 2.2.2 We consider a probability space $(\Omega, \mathbb{F}, \mathbb{P})$. Let Γ be a set of random variables defined on $(\Omega, \mathbb{F}, \mathbb{P})$. A random variable $X \in \Gamma$ is identified to the future income of a given act.

One places oneself within the expected utility hypothesis. An agent forms choices on the set Γ according to the following criterion:

X is preferred to Y, denoted $X \geq Y$, if and only if:

$$\mathbb{E}\big[U(X)\big] \ge \mathbb{E}\big[U(Y)\big]$$

where $U:\mathbb{R} \to \mathbb{R}$ is a strictly increasing and concave utility function.

According to this theorem, the lottery X is less risky than Y (having the same mathematical mean), this implies that:

$$\frac{1}{2} \Big(\sigma_X^2 - \sigma_Y^2 \Big) U^{(2)}(\mathbb{E}(R)) + \frac{1}{3!} \Big(\lambda_3^X - \lambda_3^Y \Big) U^{(3)}(\mathbb{E}(R)) + \frac{1}{4!} \Big(\lambda_4^X - \lambda_4^Y \Big) U^{(4)}(\mathbb{E}(R)) > 0$$

This leads to consider the following quantity as a measure of risk denoted \mathcal{R} :

$$\mathcal{R} = -\sigma^2 U^{(2)}(\mathbb{E}(R)) - \frac{1}{3}\lambda_3 U^{(3)}(\mathbb{E}(R)) - \frac{1}{12}\lambda_4 U^{(4)}(\mathbb{E}(R))$$
(10)

This risk measure is consistent with the risk mesure of Markowitz (1952) and that of Scott and Horvath (1980) who have shown that a rational investor would prefer the odd-order moments and hate even ones. Indeed,

$$\begin{aligned} \frac{\partial \mathcal{R}}{\partial \sigma^2} &= -U^{(2)}(\mathbb{E}(R)) > 0\\ \frac{\partial \mathcal{R}}{\partial \lambda_3} &= -U^{(3)}(\mathbb{E}(R)) < 0\\ \frac{\partial \mathcal{R}}{\partial \lambda_4} &= -U^{(4)}(\mathbb{E}(R)) > 0 \end{aligned}$$

In reality, this risk measure coincides with that of Jia and Dyer (1996). According to them, if we play once the lottery, $\mathbb{E}(R)$ can serve as a reference value. In this case, $\widetilde{R} = R - \mathbb{E}(R)$ is a centered lottery that reflects the pure risk. Under this condition, the risk measure \mathcal{R} defined by Equation (10) becomes:

$$\mathcal{R} = -\sigma^2 U^{(2)}(0) - \frac{1}{3}\lambda_3 U^{(3)}(0) - \frac{1}{12}\lambda_4 U^{(4)}(0)$$
(11)

We consider the risk measure defined by Equation (16) as a *general* risk measure for all utility functions in order to determine theoretically risk measures applicable to all types of utility functions.

In the remainder of this subsection, we will theoretically determine for each of the different types of utility functions a corresponding risk measure.

2.2.2.1. A risk measure for the Negative Exponential Utility Function. An agent that has preferences described by a negative exponential utility function is a risk-averse agent. Moreover, his risk aversion is constant with respect to the amount of wealth.

$$U(R) = -\exp\left(-\theta R\right)$$

where $\theta > 0$ is the absolute risk aversion, which is constant.

It is necessary to verify if this utility function satisfies the four stochastic dominance properties that is: $U^{(1)} > 0$, $U^{(2)} < 0$, $U^{(3)} > 0$, $U^{(4)} < 0$ with $U^{(i)}$ the *i*-th derivative of the utility function.

 $\left\{ \begin{array}{l} U^{(1)} = \theta \exp{(-\theta R)} > 0 \\ U^{(2)} = -\theta^2 \exp{(-\theta R)} < 0 \\ U^{(3)} = \theta^3 \exp{(-\theta R)} > 0 \\ U^{(4)} = -\theta^4 \exp{(-\theta R)} < 0 \end{array} \right.$

The Taylor approximation truncated to the fourth order applied to the expected utility of this function implies:

$$\mathbb{E}\left[U(R)\right] = \exp\left(-\theta E(R)\right) \left[-1 - \frac{1}{2}\sigma^2\theta^2 + \frac{1}{3!}\lambda_3\theta^3 - \frac{1}{4!}\lambda_4\theta^4\right]$$

Maximizing this quantity is equivalent to minimize the risk defined as follows:

$$\mathcal{R}_{exp} = \sigma^2 - \frac{1}{3}\lambda_3\theta + \frac{1}{12}\lambda_4\theta^2 \tag{12}$$

As required, this risk measure is a linear combination of the variance, the skewness and the kurtosis as it is fourth-order differentiable. We note in addition that the moments weights depend on risk aversion. More specifically, this provides an opportunity for investors to weight weakly or heavily the wealth distribution tails according to their attitude toward risk. It is worth noting that if the final wealth follows a Gaussian law, our risk measure is reduced to the variance. Indeed, this result is well known under the assumption of the model of Markowitz (1952).

An agent that has a power utility function is risk-averse with a decreasing absolute risk aversion and a constant relative risk aversion with reference to the amount of wealth. This utility function is called *CRRA*⁴ because it has a constant relative risk aversion function equal to γ . This utility function has been the subject of several theoretical and empirical studies, such as Rubinstein (1976), Coutant (1999), Bliss and Panigirtzoglou (2004), Guidolin and Timmermann (2005a, 2005b) and Jondeau and Rockinger (2003, 2005, 2006) among others.

$$U(R) = \frac{1}{1-\gamma} R^{1-\gamma}$$

This function is translation invariant, we can write as follows: $U(W) = \frac{1}{1-\gamma}R^{1-\gamma} = \exp\left\{\frac{\log(R)(1-\gamma)}{1-\gamma}\right\}$ that tends to $\log(R)$ when $\gamma \to 1$. As for the negative exponential utility function and before determining the corresponding risk measure of the power utility function, we must check if this function satisfies the four following properties:

$$\begin{cases} U^{(1)} = R^{-\gamma} > 0\\ U^{(2)} = -\gamma R^{-\gamma-1} < 0\\ U^{(3)} = \gamma (1+\gamma) R^{-\gamma-2} > 0\\ U^{(4)} = -\gamma (1+\gamma) (2+\gamma) R^{-\gamma-3} < 0 \end{cases}$$

Similarly, if $\gamma = 1$, we have $U^{(1)} > 0$, $U^{(2)} < 0$, $U^{(3)} > 0$ and $U^{(4)} < 0$

The Taylor approximation truncated to the fourth order applied to the expected utility of this function implies:

$$\begin{cases} \mathbb{E}\left[\frac{1}{1-\gamma}R^{1-\gamma}\right] \simeq (1-\gamma)^{-1}\mathbb{E}(R)^{(1-\gamma)} - \frac{\gamma}{2}\mathbb{E}(R)^{-(1+\gamma)}\sigma^2 \\ + \frac{\gamma(1+\gamma)}{3!}\mathbb{E}(R)^{-(\gamma+2)}\lambda_3 - \frac{\gamma(1+\gamma)(2+\gamma)}{4!}\mathbb{E}(R)^{-(\gamma+3)}\lambda_4 \\ \mathbb{E}\left[\log(R)\right] \simeq \log(\mathbb{E}(R)) - \frac{1}{2}\sigma^2\mathbb{E}(R)^{-2} + \frac{2}{3!}\lambda_3\mathbb{E}(R)^{-3} - \frac{6}{4!}\lambda_4\mathbb{E}(R)^{-4} \end{cases}$$

Maximizing this quantity is equivalent to minimize the risk defined as follows:

$$\begin{cases} \mathcal{R}_{iso(\gamma\neq1)} = \sigma^2 - \frac{(1+\gamma)}{3} \mathbb{E}(R)^{-1} \left[\lambda_3 - \frac{(2+\gamma)}{4} \lambda_4 \mathbb{E}(R)^{-1} \right] \\ \mathcal{R}_{iso(\gamma=1)} = \sigma^2 - \frac{2}{3} \lambda_3 \mathbb{E}(R)^{-1} + \frac{1}{2} \lambda_4 \mathbb{E}(R)^{-2} \end{cases}$$
(13)

As far as the risk measure of the previous function, this measure is likewise a linear combination of the variance, the skewness, and the kurtosis. We note in addition that the moments weights depend, this time, on the relative risk aversion (γ) and the final wealth mean.

2.2.2.3. A risk measure for the Hyperbolic Utility Function. We assume that the investor's preferences are specified by the hyperbolic utility function or HARA "Hyperbolic absolute risk aversion"⁵ of the form⁶:

$$U(R) = \frac{\gamma}{1-\gamma} \left(\theta + \frac{\alpha}{\gamma}R\right)^{(1-\gamma)}$$

with:

$$\begin{cases} \theta + \frac{\alpha}{\gamma} R > 0\\ \frac{1}{\gamma} > -\frac{1}{2}\\ \alpha > 0 \quad \theta \ge 0 \end{cases}$$

It is easy to show that this function checks the four stochastic dominance properties.

$$\begin{cases} U^{(1)} = \alpha \left(\theta + \frac{\alpha}{\gamma}R\right)^{-\gamma} > 0\\ U^{(2)} = -\alpha^2 \left(\theta + \frac{\alpha}{\gamma}R\right)^{-(1+\gamma)} < 0\\ U^{(3)} = \alpha^3 \left(\frac{\gamma+1}{\gamma}\right) \left(\theta + \frac{\alpha}{\gamma}R\right)^{-(2+\gamma)} > 0\\ U^{(4)} = -\alpha^3 \left(\frac{(\gamma+1)(\gamma+2)}{\gamma^2}\right) \left(\theta + \frac{\alpha}{\gamma}R\right)^{-(3+\gamma)} < 0 \end{cases}$$

The Taylor approximation truncated to the fourth order applied to the expected utility of this function implies:

$$\begin{split} \mathbb{E} \Big[U(R) \Big] &\simeq \frac{\gamma}{1 - \gamma} \Bigg[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R) \Bigg]^{1 - \gamma} - \frac{\alpha^2}{2} \Bigg[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R) \Bigg]^{-(1 + \gamma)} \sigma^2 \\ &+ \lambda_3 \frac{\alpha^3}{3!} \frac{(\gamma + 1)}{\gamma} \Bigg[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R) \Bigg]^{-(2 + \gamma)} \\ &- \lambda_4 \frac{\alpha^4}{4!} \frac{(\gamma + 1)(\gamma + 2)}{\gamma^2} \Bigg[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R) \Bigg]^{-(3 + \gamma)} \end{split}$$

Maximizing this function is equivalent to minimize the risk defined as follows:

$$\max \mathbb{E}\left[U(R)\right] \Leftrightarrow \min - \left[-\sigma^{2} + \lambda_{3}\frac{\alpha}{3}\frac{(\gamma+1)}{\gamma}\left[\theta + \frac{\alpha}{\gamma}\mathbb{E}(R)\right]^{-1} - \lambda_{4}\frac{\alpha^{2}}{12}\frac{(\gamma+1)(\gamma+2)}{\gamma^{2}}\left[\theta + \frac{\alpha}{\gamma}\mathbb{E}(R)\right]^{-2}\right]$$

That is to minimize this quantity:

$$\mathcal{R}_{\text{HARA}} = \sigma^2 - \frac{\alpha}{3} \frac{(\gamma+1)}{\gamma} \left[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R) \right]^{-1} \left[\lambda_3 - \lambda_4 \frac{\alpha}{4} \frac{(\gamma+2)}{\gamma} \left[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R) \right]^{-1} \right]$$
(14)

We can remark that this risk measure depends on the final wealth *R* and the parameters (α , θ , γ). It is a linear combination of the variance, the skewness, and the kurtosis. Their weights depend on all utility function parameters and on the final wealth. We obviously note that our risk measure negatively depends on the skewness and positively on the variance and the kurtosis, but we cannot judge on the variation of this measure in relation to various utility function parameters, namely (α , θ , γ).

Our three risk measures include the two, third- and fourth-order moments. They give a good idea on the dissymmetry and tails of the distribution that inform us on the frequency of extreme movements.

3. Model specifications: Utilities-based CAPM

The CAPM is based on a certain number of simplifying assumptions making it applicable. These assumptions are presented as follows:

- The markets are perfect and there are neither taxes nor expenses or commissions or asymmetric information of any kind;
- All the investors are risk-averse and maximize the mean-variance criterion;
- The investors have homogeneous anticipations concerning the distributions of the returns probabilities (Gaussian distribution);

The aphorism behind this model is as follows: the return of an asset is equal to the risk-free rate raised with a risk premium which is the risk premium average multiplied by the systematic risk coefficient of the considered asset. Thus, the expression is a function of:

- The systematic risk coefficient which is noted as β_i ;
- The market return noted $E(R_M)$;
- The risk-free rate (Treasury bills), noted R_f.

This model is explained as follows:

$$E(R_i) = R_f + \beta_i \Big[E(R_M) - R_f \Big]$$
(15)

where $E(R_M) - R_f$ represents the risk premium and β_i corresponds to the systematic risk coefficient of the considered asset.

From a mathematical point of view, this one corresponds to the ratio of the covariance of the assets' returns and that of the market and the variance of the market returns.

$$\begin{cases} \beta_i = \frac{\operatorname{cov}(R_i; R_M)}{V(R_M)} \\ \beta_i = \frac{\partial \sigma_M}{\partial \sigma_i} \frac{\sigma_i}{\sigma_M} \end{cases}$$
(16)

where σ_M represents the standard deviation of the market return (market risk) and σ_i is the standard deviation of the assets' returns.

Subsequently, if an asset has the same characteristics as those of the market (representative asset), then, its equivalent β will be equal to 1. Conversely, for a risk-free asset, this coefficient will be equal to 0.

The β coefficient is the back bone of the CAPM. Indeed, the beta is an indicator of profitability since it is the relationship between the assets volatility and that of the market. Volatility is related to the returns variations which are an essential element of profitability. Moreover, it is an indicator of risk,

since if this asset has a β coefficient which is higher than 1, this means that if the market is in recession, the return on the asset drops more than that of the market and less than it if this coefficient is lower than 1.

The covariance can be written as follows:

$$cov(R_{i};R_{M}) = \frac{V(R_{i} + R_{M}) - V(R_{i}) - V(R_{M})}{2}$$
(17)

Hence, the β of a particular asset *i* becomes:

$$\beta_i = \frac{V(R_i + R_M) - V(R_i)}{2V(R_M)} - \frac{1}{2}$$
(18)

Therefore, the CAPM Model can be written as follows:

$$E(R_i) = R_f + \left[\frac{V(R_i + R_M) - V(R_i)}{2V(R_M)} - \frac{1}{2}\right]E(R_M - R_f)$$
(19)

Genaral-based utility CAPM

Considering our general risk measure, the CAPM is expressed as follows:

$$E(R_{i}) = R_{f} + \left[\frac{\Re(R_{i} + R_{M}) - \Re(R_{i})}{2\Re(R_{M})} - \frac{1}{2}\right]E(R_{M} - R_{f})$$
(20)

Replacing \mathfrak{R} by its experssion in Equation (10), the eta becomes:

$$\beta = \frac{1}{2} \left[\frac{E(R_i) \left(\sigma_{R_i}^2 - \sigma_{R_i + R_M}^2\right) U^2 + \frac{1}{3} (\lambda_3^{R_i} - \lambda_3^{R_i + R_M}) U^3 + \frac{1}{12} (\lambda_4^{R_i} - \lambda_4^{R_i + R_M}) U^4 + E(R_M) \left(-\sigma_{R_i + R_M}^2 U^2 - \frac{1}{3} \lambda_3^{R_i + R_M} U^3 - \frac{1}{12} \lambda_4^{R_i + R_M} U^4\right)}{E(R_M) \left(-\sigma_{R_M}^2 U^2 - \frac{1}{3} \lambda_3^{R_M} U^3 - \frac{1}{12} \lambda_4^{R_M} U^4\right)} - 1 \right]$$
(21)

so, the CAPM equation based on our general risk measure is:

$$E(R_{i}) = R_{f} + \frac{1}{2} \left[\frac{E(R_{i}) \left(\sigma_{R_{i}}^{2} - \sigma_{R_{i}+R_{m}}^{2}\right) U^{2} + \frac{1}{3} (\lambda_{3}^{R_{i}} - \lambda_{3}^{R_{i}+R_{m}}) U^{3} + \frac{1}{12} (\lambda_{4}^{R_{i}} - \lambda_{4}^{R_{i}+R_{m}}) U^{4} + E(R_{m}) \left(-\sigma_{R_{i}+R_{m}}^{2} U^{2} - \frac{1}{3} \lambda_{3}^{R_{i}+R_{m}} U^{3} - \frac{1}{12} \lambda_{4}^{R_{i}+R_{m}} U^{4}\right)}{E(R_{m}) \left(-\sigma_{R_{m}}^{2} U^{2} - \frac{1}{3} \lambda_{3}^{R_{m}} U^{3} - \frac{1}{12} \lambda_{4}^{R_{m}} U^{4}\right)} - 1 \right] E(R_{m} - R_{f}) \left[\frac{E(R_{i}) \left(-\sigma_{R_{i}+R_{m}}^{2} U^{2} - \frac{1}{3} \lambda_{3}^{R_{m}} U^{3} - \frac{1}{12} \lambda_{4}^{R_{m}} U^{4}\right)}{E(R_{m}) \left(-\sigma_{R_{m}}^{2} U^{2} - \frac{1}{3} \lambda_{3}^{R_{m}} U^{3} - \frac{1}{12} \lambda_{4}^{R_{m}} U^{4}\right)} \right] \right]$$

$$(22)$$

Negative exponential utility function CAPM

Now consider an investor whose preferences are described by a negative exponantial utility function, his β is :

$$\beta = \frac{1}{2} \left[\frac{\sigma_{R_i + R_M}^2 - \frac{1}{3}\lambda_3^{R_i + R_M}\theta + \frac{1}{12}\lambda_4^{R_i + R_M}\theta^2 - \sigma_{R_i}^2 - \frac{1}{3}\lambda_3^{R_i}\theta + \frac{1}{12}\lambda_4^{R_i}\theta^2}{\sigma_{R_M}^2 - \frac{1}{3}\lambda_3^{R_M}\theta + \frac{1}{12}\lambda_4^{R_M}\theta^2} - 1 \right]$$
(23)

$$\Rightarrow \beta = \frac{1}{2} \left[\frac{\sigma_{R_i + R_M}^2 - \sigma_{R_i}^2 - \frac{1}{3}\theta \left(\lambda_3^{R_i + R_M} - \lambda_3^{R_i} \right) + \frac{1}{12}\theta^2 \left(\lambda_{R_i + R_M}^4 - \lambda_{R_i}^4 \right)}{\sigma_{R_M}^2 - \frac{1}{3}\theta \lambda_{R_M}^3 + \frac{1}{12}\theta^2 \lambda_4^{R_M}} - 1 \right]$$
(24)

Hence, his CAPM valuation equation is presented as follows:

$$CAPM_{exp}:E(R_{i}) = R_{f} + \frac{1}{2} \left[\frac{\sigma_{R_{i}+R_{M}}^{2} - \sigma_{R_{i}}^{2} - \frac{1}{3}\theta\left(\lambda_{3}^{Ri+RM} - \lambda_{3}^{Ri}\right) + \frac{1}{12}\theta^{2}\left(\lambda_{R_{i}+R_{M}}^{4} - \lambda_{Ri}^{4}\right)}{\sigma_{R_{M}}^{2} - \frac{1}{3}\theta\lambda_{R_{M}}^{3} + \frac{1}{12}\theta^{2}\lambda_{4}^{R_{M}}} - 1 \right] (E(R_{M}) - R_{f})$$
(25)

We can note from Equation (24) that our beta is divided into three risks: the volatility risk, the skewness risk, and the kurtosis risk. The volatility effect is positive which means that higher volatility (of both the asset and the market return) leads to a higher risk premium. This is a direct consequence of risk-averse investors who require higher return for supporting higher risk. Whereas, the skewness effect is negative which means that returns are left skewed. The negative effect proves that investors hate downside movements. This is a normal behavior for risk-averse investors. Meanwhile, investors have preferences for the kurtosis (positive effect) indicating that they prefer central values. We can conclude that investors prefer even moments and hate odd ones. The moment effect depends also from the weights, that is as higher as the moments order is as lower as the effect on beta will be.

Power utility function CAPM

If the investor has a power utility function, then his valuation equation differs with refrence to the paramater γ as follows:

$$\begin{aligned} \mathsf{CAPM}_{\mathsf{iso}(\neq 1)} &: \mathcal{E}(R_i) = R_f + \frac{1}{2} \left[\frac{\sigma_{R_j + R_M}^2 - \frac{(1+\gamma)}{3} \mathbb{E}(R_i + R_M)^{-1} \left[\lambda_3^{R_j + R_M} - \frac{(2+\gamma)}{4} \lambda_4^{R_j + R_M} \mathbb{E}(R_i + R_M)^{-1} \right] - \sigma_{R_j}^2 - \frac{(1+\gamma)}{3} \mathbb{E}(R_i)^{-1} \left[\lambda_3^{R_j} - \frac{(2+\gamma)}{4} \lambda_4^{R_j} \mathbb{E}(R_i)^{-1} \right]}{-2c_{R_j}^2 - \frac{(1+\gamma)}{3} \mathbb{E}(R_j)^{-1} \left[\lambda_3^{R_j} - \frac{(2+\gamma)}{4} \mathbb{E}(R_j)^{-1} \right]} - 1 \right] (\mathcal{E}(R_M) - R_f) \\ \mathsf{CAPM}_{\mathsf{iso}(\gamma=1)} &: \mathcal{E}(R_i) = R_f + \frac{1}{2} \left[\frac{\sigma_{R_j + R_M}^2 - \frac{2}{3} \lambda_3^{R_j + R_M} \mathbb{E}(R_j + R_M)^{-1} + \frac{1}{2} \lambda_4^{R_j + R_M} \mathbb{E}(R_j + R_M)^{-2} - \sigma_{R_j}^2 + \frac{2}{3} \lambda_3^{R_j} \mathbb{E}(R_j)^{-1} - \frac{1}{3} \lambda_4^{R_j} \mathbb{E}(R_j)^{-2}}{\sigma_{R_M}^2 - \frac{2}{3} \lambda_3^{R_j} \mathbb{E}(R_M)^{-1} + \frac{1}{2} \lambda_4^{R_M} \mathbb{E}(R_M)^{-2}} - 1 \right] (\mathcal{E}(R_M) - R_f) \end{aligned} \tag{26}$$

Equation (26) indicates that the moments effect is the same as in the negative exponential utility function. Higher volatility and kurtosis lead to higher risk premium. However, higher skewness results in lower risk premium.

Hyperbolic utility function CAPM We assume that the investor's preferences are specified by the hyperbolic utility function or HARA "Hyperbolic absolute risk aversion". His β is as follows :

$$\begin{split} \beta &= \frac{1}{2} \\ x \Bigg[\frac{\sigma_{R_{+}R_{M}}^{2} - \frac{\alpha}{3} \frac{(y+1)}{\gamma} \Big[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R_{i} + R_{M}) \Big]^{-1} \Big[\lambda_{3}^{R_{i} + R_{M}} - \lambda_{4}^{R_{i} + R_{M}} \frac{\alpha}{\gamma} \frac{(y+2)}{\gamma} \Big[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R_{i} + R_{M}) \Big]^{-1} \Big] - \sigma_{R_{i}}^{2} - \frac{\alpha}{3} \frac{(y+1)}{\gamma} \Big[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R_{i}) \Big]^{-1} \Big[\lambda_{3}^{R_{M}} - \lambda_{4}^{R_{M}} \frac{\alpha}{4} \frac{(y+2)}{\gamma} \Big[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R_{i}) \Big]^{-1} \Big] \\ \sigma_{R_{M}}^{2} - \frac{\alpha}{3} \frac{(y+1)}{\gamma} \Big[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R_{M}) \Big]^{-1} \Big[\lambda_{3}^{R_{M}} - \lambda_{4}^{R_{M}} \frac{\alpha}{4} \frac{(y+2)}{\gamma} \Big[\theta + \frac{\alpha}{\gamma} \mathbb{E}(R_{i}) \Big]^{-1} \Big] \\ (27) \end{split}$$

Besides, his CAPM valuation equation is:

We note from Equation (27), like the negative exponential and the power utility functions, in the hyperbolic power utility function, investors are variance-kurtosis seekers. As we can see, the beta determined from utility functions depends not only on the mean and the variance but also on the third and the fourth moments.

Table 1. Estimated values of the constant coefficient of relative risk aversion					
Study	CRRA range				
Arrow (1971)	1				
Friend and Blume (1975)	2				
Hansen and Singleton (1982, 1984)	0-1				
Mehra and Prescott (1985)	55				
Ferson and Constantinides (1991)	0-12				
Epstein and Zin (1991)	0-12				
Cochrane and Hansen (1992)	40–50				
Normandin and Saint-Amour (1998)	<3				
Sophie Coutant (1999)	0-11.404				
Aït-Sahalia and Lo (2000)	12.7				
Guo and Whitelaw (2001)	3.52				
Bliss and Panigirtzoglou (2004)	0.37-15.97				

4. Application and results discussion

4.1. Data and methodology

We collect returns of indexes listed in the MSCI classification including the MSCI World Index, the MSCI Emerging Markets Index, the MSCI European Index, the MSCI South Africa, as well as specific market indexes, the bel 20, CAC 40, DAX, SP500, Tunindex, on a daily basis from January 2003 through January 2014. All empirical investigations are conducted on the CAPM. The MSCI World Index is used as a proxy for the global market portfolio and the three-month US Treasury bill is used as a proxy for the risk-free rate. We calculate, firstly, the traditional CAPM beta, then our betas extracted theoretically from various utility functions. We, afterward, compare the estimated beta to not only the traditional CAPM beta but also to our different betas developed from various utility functions.

To assess the risk of different utility function, we use the risk aversion parameters (θ and γ) as in the literature summarized in Table 1.

4.2. Results and discussion

We, firstly, present the results related to the calculation of the traditional beta. Then, an application of our different betas extracted theoretically from utility functions is done. We, finally, compare the estimated value of beta to their calculated values from utilities and the traditional version as well. To apprehend the distribution of our sample, we use descriptive statistics presented in Table 2.

Table 2 reports the summary statistics of daily returns for the following indexes: BEL 20, CAC40, DAX, TUNINDEX, SP500, MSCI Emerging Markets, MSCI Europe, MSCI South Africa, and MSCI World Index. The results indicate that the mean of all indexes lies on both sides of 0 with a highest value of 0.00022 for the MSCI South Africa index. All returns exhibit negative skewness which indicates that the distribution of returns is shifted on the right of the median, and thus the tail of the distribution is left- skewed. All indexes have high kurtosis with the biggest value is found for the TUNINDEX (2,432.70). The high values of kurtosis indicate that returns are highly concentrated around the mean, due to lower variations within observations. The high values of the distribution (min and max). This means that all indexes are left-skewed, have heavy tails and are picked which is an explicit departure from the normality assumption, which implies that returns are dispersed and

Table 2. The sample descriptive statistics								
Indexes	Mean	Median	Standard deviation	Skewness	Kurtosis	Min	Max	
CAC40	-0.00012	0.00021	0.02350	-26.65	1,136.70	-1	0.11180	
DAX	0.00017	0.00067	0.02340	-27.08	1,160.60	-1	0.11400	
BEL20	-0.00012	0.00022	0.02260	-29.96	1,326.60	-1	0.09780	
SP500	-0.00002	0.00048	0.02240	-30.94	1,382.00	-1	0.11580	
Tunindex	0.00018	0.00010	0.01950	-47.33	2,432.70	-1	0.04190	
MSCI Europe	0.00000	0.00056	0.02370	-26.27	1,113.60	-1	0.11290	
MSCI Emerging Markets	0.00014	0.00130	0.02470	-23.09	932.44	-1	0.13680	
MSCI South Africa	0.00022	0.00062	0.02240	-30.99	1,384.00	-1	0.06140	
MSCI World	-0.00004	0.00078	0.02150	-35.14	1,635.10	-1	0.09520	

Table 3. Calculation of the traditional CAPM beta					
Indexes	β				
CAC40	1.023				
DAX	1.017				
BEL20	0.978				
SP500	1.007				
Tunindex	0.758				
MSCI Europe	1.044				
MSCI Emerging Markets	1.022				
MSCI South Africa	0.907				

therefore the risk is important, which leads us to conclude that the distribution is leptokurtic creating a vulnerability to the risks of extreme loss.

4.2.1. Traditional beta

Table 3 reports the values of betas calculated from the CAPM equation for different market indexes with reference to the global market index (MSCI World). The beta of an index gives its sensitiveness to the market movements. It gives the contribution of a specific index to the overall market risk and represents the systematic risk. The results indicate that the beta is always high ranging between 0.758 and 1.044. The Tunindex and the MSCI South Africa has the lowest value of beta (respectively, 0.758 and 0.907). Whereas, the highest value are found for the MSCI indexes and the European indexes. This is not strange since African countries remain on the edge of the global market which explains their relatively low values of beta.

However, since the beta is calculated basing on the normality assumption and hence determined from only the first two moments of the distribution, the conclusion is not straightforward. In fact, basing on the descriptive statistics, our study sample diverge from the Gaussian distribution. Hence, a risk measure based on simply the mean and the variance may lead to a misinterpretation of the results. So, a pertinent risk measure should rather include further the higher order moments.

4.2.2. Utilities-based beta

Results from Table 4 indicate firstly the remarquable difference of the values of the traditional beta (Table 3) and the negative exponetional utilility beta. This finding highlights the impact of different risk aversion degrees. Secondly, we remark that the values of beta based on the negative exponential utility function⁷ increase with the aversion parameter β for different indexes. In other words, the

Table 4. Calculation of the CAPM beta extracted from the negative exponential utility function 10^{-2}							
β	0.5	1	2	10	20		
CAC40	-49.90	-49.87	-49.67	-47.35	-40.41		
DAX	-49.9	-49.87	-49.76	-45.35	-40.40		
BEL20	-49.91	-49.87	-49.76	-47.35	-40.41		
SP500	-49.9	-49.87	-49.76	-47.35	-40.41		
Tunindex	-49.92	-49.88	-49.77	-47.36	-40.42		
MSCI Europe	-49.9	-49.87	-49.76	-47.34	-40.40		
MSCI Emerging Markets	-49.9	-49.87	-49.76	-47.34	-40.40		
MSCI South Africa	-49.91	-49.87	-49.77	-47.35	-40.41		

systematic risk increases with the absolute risk aversion. We obviously remark from this table the effect of changes in our CAPM beta value according to β . It is important to note that the Tunindex presents the lowest value of beta resulting from the negative exponential utility. This consolidates the results previously found with the traditional CAPM beta. Though, the traditional systematic risk measures do not reflect the investor preferences.

Since the negative exponential beta includes extreme values of the distribution. That's why this latter exhibits high kurtosis and negative skewness. It's not strange to find that the negative exponential betas differ largely from the traditional ones. Indeed, according to Equation (13), our CAPM beta is a combination of the variance, the skewness and the kurtosis. It positively depends on the variance and kurtosis (λ_4) and negatively on the skewness ($-\lambda_3$). If $\beta \to +\infty$, β ($-\lambda_3$) increases (because $\lambda_3 < 0$) and $\beta^2 \lambda_4$ increases also, which explains this variation of negative exponential beta according to the traditional one. The effect of higher moments is much bigger than the volatility effect. All indexes have low volatility and high skewness and kurtosis. Moreover, the weights of those moments depend on the risk aversion (β) and are bigger than the weight of the volatility. That's why the negative exponential betas are negative with reference to the traditional betas.

Table 5 reports the values of β extracted from the power utility function for different values of the parameter γ . The values of β are almost positive for all indexes except for the MSCI Europe index which means that index returns vary in the same direction as the global market portfolio. However for the MSCI Europe, the β is negative indicating that the variation is inversely related to the market portfolio. We note that the value of β tend to increase with the increase of the risk aversion parameter γ . This result means that the higher the risk aversion the higher the risk premium is. For the MSCI Europe, the results are a bit striking since it goes beyond the theoretical presumption: in fact, it is found that the risk premium is negative which means that we require lower return for higher risk. Hence, we note the remarquable difference of the CAPM beta based on the power utility function and the one based on the negative exponential utility function which highlight the impact of considering various risk aversion degrees.

Table 5. Calculation of the CAPM beta extracted from the power utility function 10^4							
γ	0.5	2	3	4	10	0	
CAC40	2.9301	9.3767	15.628	23.442	103.15	1.5626	
DAX	4.8733	15.5594	25.99	38.984	171.53	2.5991	
BEL20	3.1618	10.118	16.864	25.296	111.31	1.6862	
SP500	6.9744	22.319	37.199	55.798	245.52	3.7195	
Tunindex	4.6299	14.815	24.692	37.037	162.96	2.4693	
MSCI Europe	-6,161.31	-19,716	-32,860	-49,290	-216,880	-3,286	
MSCI Emerging Markets	7.8865	25.236	42.06	63.09	277.59	4.2062	
MSCI South Africa	4.6299	14.815	24.692	37.037	162.96	2.4693	

Table 6. Calculation of the CAPM beta extracted from the hyperbolic utility function 10 ⁻²							
Indexes	$\gamma = -10; \ \beta = 2$		$\gamma = -3; \alpha$	= 1	$\gamma = 10; \beta = 2$		
	$\alpha = 1$	$\alpha = 10$	$\beta = 1$	$\beta = 2$	$\alpha = 1$	$\alpha = 10$	
CAC40	-49.91	-49.36	-49.91	-49.9	-49.9	-48.99	
DAX	-49.91	-49.35	-49.92	-49.9	-49.9	-48.99	
BEL20	-49.91	-49.36	-49.92	-49.9	-49.9	-48.99	
SP500	-49.91	-49.36	-49.92	-49.9	-49.9	-48.99	
Tunindex	-49.92	-49.37	-49.93	-49.91	-49.91	-49	
MSCI Europe	-49.91	-49.35	-49.91	-49.9	-49.9	-48.99	
MSCI Emerging Markets	-49.91	-49.35	-49.91	-49.9	-49.9	-48.99	
MSCI South Africa	-49.91	-49.36	-49.92	-49.91	-49.91	-48.99	

Table 6 shows the calculations of our CAPM beta based on the HARA utility function.⁸ It illustrates the variation in our CAPM beta based on the HARA utility function for different values of γ , α and β . All things being equal, we note that beta is undergoing positive variation in terms of α for different indexes. We also remark that the systematic risk variation in terms of α is not affected by the choice of the parameters γ and β . Contrary to the positive variation of beta according to α , the absolute risk aversion for a HARA utility function is decreasing according to α . Moreover, this table shows the variation of beta based on the HARA utility function according to γ . These variations are not negligible, if we increase the parameter α all things being equal for β . Nevertheless, this variation of the beta based on the HARA utility function according to γ is almost zero if we opt for different choices of the parameter β while keeping the parameter α constant. As for the absolute risk aversion of a HARA utility function, it also has two regimes: (i) decreasing if $\gamma > 0$, in this scenario the value of risky assets held by the fund manager tends to increase and (ii) increasing for $\gamma < 0$, thus it presents different attitudes toward risk depending on the parameter γ . Furthermore, the variation in the beta based on the HARA utility function depending on the parameter β , indicates that, unlike the other parameters α and γ , beta remains often constant as a function of β for all indexes. This means that β has no effect on the quantification of systematic risk. The betas based on the HARA utility and negative exponential one converge when $\gamma = +\infty$. In fact, the negative exponential utility is a particular case of the HARA specifically when $\gamma = +\infty$ and $\beta = 1$.

4.2.3. Estimation of the CAPM: Utilities based versus traditional CAPM

Systematic risk is the risk that is correlated with the return to the market; when the return to the market goes up, systematic return should also increase. Since the largest component of the variance is the sum of squares, many say that the R^2 is the proportion of variance that is explained by the regression model. When we translate this approximation to the CAPM model, then the R-squared is

Table 7. Regression results of the CAPM equation							
Index	Constant term		CAP	R ²			
	Value	p-value	Value	p-value			
CAC40	9.3010×10^{-5}	0.5539	0.9983	0	0.9991		
DAX	$-1.91339.3010 \times 10^{-4}$	0.223	0.9971	0	0.9991		
BEL20	7.1362 × 10 ⁻⁵	0.6503	1.0005	0	0.9991		
SP500	-1.5909×10^{-5}	0.8842	0.9989	0	0.9996		
Tunindex	-2.2774×10^{-4}	0.3038	1.0016	0	0.9982		
MSCI Europe	-2.6437×10^{-5}	0.8516	0.9981	0	0.993		
MSCI Emerging Markets	-1.7314×10^{-4}	0.4155	0.9984	0	0.9984		
MSCI South Africa	-2.5250×10^{-4}	0.2296	0.9992	0	0.9984		

an approximate measure of the amount of systematic risk contained in the total variation. According to the CAPM the non-systematic risk can be diversified away. Table 7 shows that the R^2 for all indexes is of 0.99, then about 99% of all risk in this stock is systematic, meaning non-diversifiable. That also means that 1% of the risk displayed of returns appears to be diversifiable. The P-Value of the t- statistic is less than 0.05, then the model is estimated with sufficient confidence to use. This means that over our estimation period, the price for the firm has been influenced by the systematic component in the estimation. Moreover, the estimated betas are near the traditional ones which is not surprising till both of them is based on the sample Linear Model.

5. Conclusion

The aim of this paper is to develop a new CAPM extracted from various utility functions. We particularly develop, theoretically, new risk measures extracted from the negative exponential utility function, the power utility and the hyperbolic utility. We, then, replace the traditional beta of the CAPM by our new betas based on our risk measures that include the higher order moments. It's found that our beta is divided into three risks: the volatility risk, the skewness risk and the kurtosis risk. It's found, that higher volatility leads to a higher risk premium which confirms risk aversion. Moreover, we find that investors have preferences for even moments and hate odd ones.

We empirically tested our approach on a sample data consisting of daily returns of the following indexes : BEL20 CAC40, SP500, DAX, TUNINDEX, MSCI World, MSCI Europe, MSCI Emerging markets, MSCI South Africa, and the one month US treasury Bills. The results indicate that our utility-based betas outstrip the traditional CAPM beta in the way that they capture the real investor's preferences. The results show also that our betas are in roughly all cases negative (except the power utility) which is not the case of the traditional beta. In fact, the inclusion of higher moments has shifted the systematic risk to negative. This typically logic since the risk premium required depends on the investors risk tolerance and their preferences and is not only determined linearly.

Our finding is crucial for academicians and practitioners because it underlines the real investors? behavior in determining their pricing equation. In fact, we have demonstrated both theoretically and empirically, that the risk measure and the pricing equation differ with regards to investors? aversion degree.

Finally, our approach may be beneficial for investors in the market, since it gives a more powerful tool to estimate the risk based beta and to check how returns co-vary with the global market when considering their preferences because only utility function can determine investor's preferences.

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Notes

- 1. The resolution of this equation depends on the sign of Δ . As a first step, we assume that Δ is negative so the sign of the equation is the same as α_4 (negative) and as a second step, we assume that Δ is positive so we can determine the conditions for the various parameters.
- 2. It is easy to show that the set P^0 is a convex set if the set P is convex.
- 3. See also Scott and Horvath (1980) who demonstrated the same result as Markowitz.
- 4. In the case where γ = 0, the investor is risk neutral.
 5. This function is called hyperbolic, because its risk aversion
- function is increasing for $\gamma > 0$ and decreasing for $\gamma < 0$. 6. If $\gamma > 0$ and $\theta = 0$, we have the power utility function
- and if $\gamma \rightarrow +\infty$ and $\theta = 1$, we have the negative exponential utility function.

- This function characterizes a risquophobe individual whose aversion toward risk is constant regardless of wealth and the value of his risky assets will remain constant.
- 8. for the case $\gamma > 0$ and $\beta = 0$, we find a power utility function and for the case $\gamma = +\infty$ and $\beta = 1$, we find the negative exponential utility function.

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