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## Article

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## FINANCIAL ECONOMICS | RESEARCH ARTICLE

# Hopf bifurcation in an open monetary economic system: Taylor vs. inflation targeting rules (Malaysian case)

Mohd Naim Bin Mohd Johari<sup>1</sup> and Adem Kilicman<sup>2\*</sup>

**Abstract:** The main objectives of this research are to analyze the trends of expectations condition within Malaysian economic system and investigate the existence of Hopf bifurcation in the economic dynamical system's policy in order to examine the existence of periodic solutions. The study uses two types of monetary policy rules which are namely: Taylor rule and inflation targeting rule. The results reveal that the patterns of expectations condition for Malaysia economic system from 2004 until 2014 are quite similar except for exchange rate case. Furthermore, it shows that Hopf bifurcation occurs within the policy's variables in both forms of rules in Malaysian open economic system.

**Subjects:** Development Studies; Economics, Finance, Business & Industry; Education

**Keywords:** Hopf bifurcation; Taylor rule; inflation targeting rule; Malaysian economic system

### 1. Introduction

Most of us accept that the outside noise is the vital source of the volatility and randomness in the dynamic system's behaviour in majority of the economic models. However, we can have different sources for such kind of behaviour that has shown by the chaos revolution which extremely related to non-linearity (see Mohd Roslan, Salleh, & Kilicman, 2010). In other terms, nonlinear system can be more suitable in econometric analysis with linear systems and random disturbances may be



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### PUBLIC INTEREST STATEMENT

*Bifurcation theory* is the study of points in a mathematical system at which system's behavior changes extremely. On either side of these points, the qualitative behavior of the system is slightly different. In fact, the behavior of bifurcation can demonstrate the chaos' occurrence in system of dynamic. The main objectives of this research are to analyze the trends of expectation condition within Malaysian economic system and to investigate the existence of Hopf bifurcation in the economic dynamical system's policy. The study uses two types of monetary policy rules which are known Taylor rule and inflation targeting rule. The results reveal that the patterns of expectation condition for Malaysia economic system from 2004 until 2014 are quite similar except in the exchange rate case. Furthermore, it shows that Hopf bifurcation occurs within the policy's variables in both forms of rules in Malaysian open economic system.

insufficient. When the chaotic behaviour happens, forecast of the economy becomes extremely tough and limits predictability of the future behaviour from the history. Hence, economic analyst and designer of the policy found it hard to adapt this theory (Grandmont, 1985).

Barnett and He (2002) studied the relevant bifurcation policy in the models of macroeconomic. Barnett and Duzhak (2008) used New Keynesian models instead of Euler equations models that chosen by Barnett and He (2002). Barnett and Eryilmaz (2012) then extended the results of Barnett and Duzhak (2008) to the open economy case. They used an open economy New Keynesian Model which is Gali and Monacelli (2005) model in order to show the existence of Hopf bifurcation in an open economy. Barnett and Eryilmaz (2013) examined another mainstream New Keynesian model instead of Gali and Monacelli (2005) model in the open economy tradition. The model is Clarida, Gali and Gertler (2002) model.

The main objectives of this research are to analyze the trends of expectations condition within Malaysian economic system and to investigate the monetary policy that related to Hopf bifurcation in an open monetary economic system. Meanwhile, the specific objective is to study the Hopf bifurcation in an economic system with two different types of rules in monetary policy which are Taylor rule and inflation targeting rule.

The rest of this paper organized as follows. Section 2 explains about Hopf bifurcation. Then, we go through Malaysia economic system and the model's structure. After that, we explore the application of basic model in Malaysian economic system and Hopf bifurcation in the models. Finally, the last part expresses the conclusion remarks.

## 2. Hopf bifurcation

The word Poincar'e-Andronov-Hopf bifurcation (in short Hopf bifurcation) is an equilibrium periodic solution's family under variation of a parameter. In a differential equation, when a complex conjugate pair of eigenvalues of a system becomes purely imaginary at a fixed point, a Hopf bifurcation exists.

*Definition 2.1* Specifically, *Hopf bifurcation* is a local bifurcation where loses in stability occurs at a fixed point of a dynamical system, as a complex plane imaginary axis is crossed by a pair of complex conjugate eigenvalues (see Moosavi Mohseni & Kilicman, 2014).

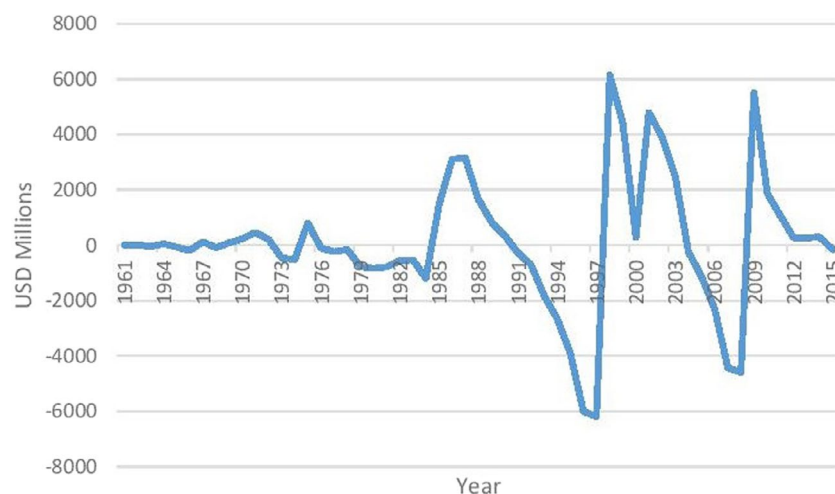
## 3. Malaysian economic system

### 3.1. Malaysia output gap

Figure 1 depicts the output gap that occurred in Malaysia. The output gap is an economic measure of the difference between the real GDP of an economy and its potential GDP. From the figure, the

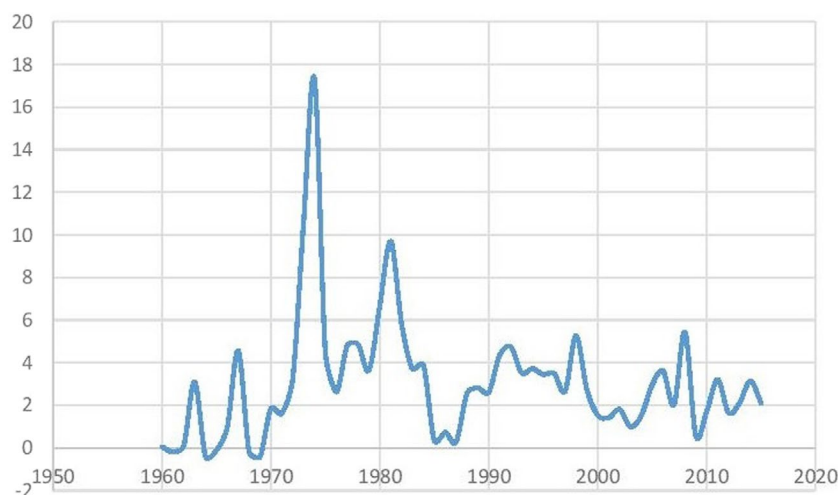
**Figure 1. Malaysia output gap.**

Source: Authors' calculation.



**Figure 2. Malaysia inflation rate.**

Source: World Development Indicators.



**Figure 3. Malaysia real exchange rate.**

Source: World Development Indicators.



largest output gap in Malaysia is in 1998 with 6144.50 USD million and the smallest output gap is in 1997 with −6196.14 USD million.

### 3.2. Malaysia inflation rate

Figure 2 indicates Malaysia inflation rate from 1960 until 2015 with the lowest rate of −0.158346529 in 1968 and the highest rate of 17.32898098 in 1974.

### 3.3. Malaysia real exchange rate

Figure 3 illustrates the real exchange rate in Malaysia. It refers to the purchasing power of a currency relative to another at current exchange rates and prices. According to the graph, it is recorded high of 3.9244 in 1998 and recorded low of 2.1884 in 1979.

## 4. Model's structure

In this model, a standard version of Walsh model is being used (Carl, 2003). It is then modified for the open monetary economic system.

#### 4.1. Basic model

The basic model used in this research is composed of these three equations:

$$y_t = E_t y_{t+1} + \alpha_1 (i_t - E_t \pi_{t+1}) + \alpha_2 e_t; \alpha_1 < 0, \alpha_2 < 0 \quad (1)$$

$$\pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 y_t; 0 < \beta_1 < 1, \beta_2 < 0 \quad (2)$$

$$e_t = \gamma_0 + \gamma_1 (E_t \pi_{t+1}^* - E_t \pi_{t+1}); \gamma_0 > 0, \gamma_1 > 0 \quad (3)$$

where  $y_t$  is the actual output gap from potential its output (steady state output),  $\pi_t$  denotes the domestic inflation rate,  $e_t$  represents the real exchange rate,  $\pi_t^*$  is the world inflation rate and  $E_t$  represents the mathematical expectation condition on period  $t$  information.

The first equation denotes the forward looking rational expectation of the IS curve which is the demand side of the economy. The rational expectation hypothesis refers to the prediction should be similar to the data generation model where all information available at that time. Meanwhile, the second equation shows the supply side of the economy that represents expectations forward looking. It is a modified version of Phillips curve that related to the supply curve. Last but not least, the third equation of the model is the purchasing power parity (PPP). This equation helps to investigate the effect of the economy in foreign side.

#### 4.2. Rules of policy

The first rule is the Taylor rule:

$$i_t = \lambda_1 \pi_t + \lambda_2 y_t; \lambda_1 > 0, \lambda_2 > 0 \quad (4)$$

where  $\lambda_1$  and  $\lambda_2$  denote the coefficients for inflation and output gap respectively. This rule is a non-optimized rule of monetary policy.

The second monetary policy rule that has been employed in this study is the inflation targeting rule. It is in the form of current looking inflation targeting rule where the formula is stated as follows:

$$i_t = \Phi_1 \pi_t \quad (5)$$

Equation (5) is a linkage between the monetary policy instrument and the major target of the monetary authority which is the inflation rate. Whereas, Equations (1) to (3), in combination with either Taylor rule or inflation targeting rule represent a small open macroeconomic model. We can rewrite these systems of equations as follows:

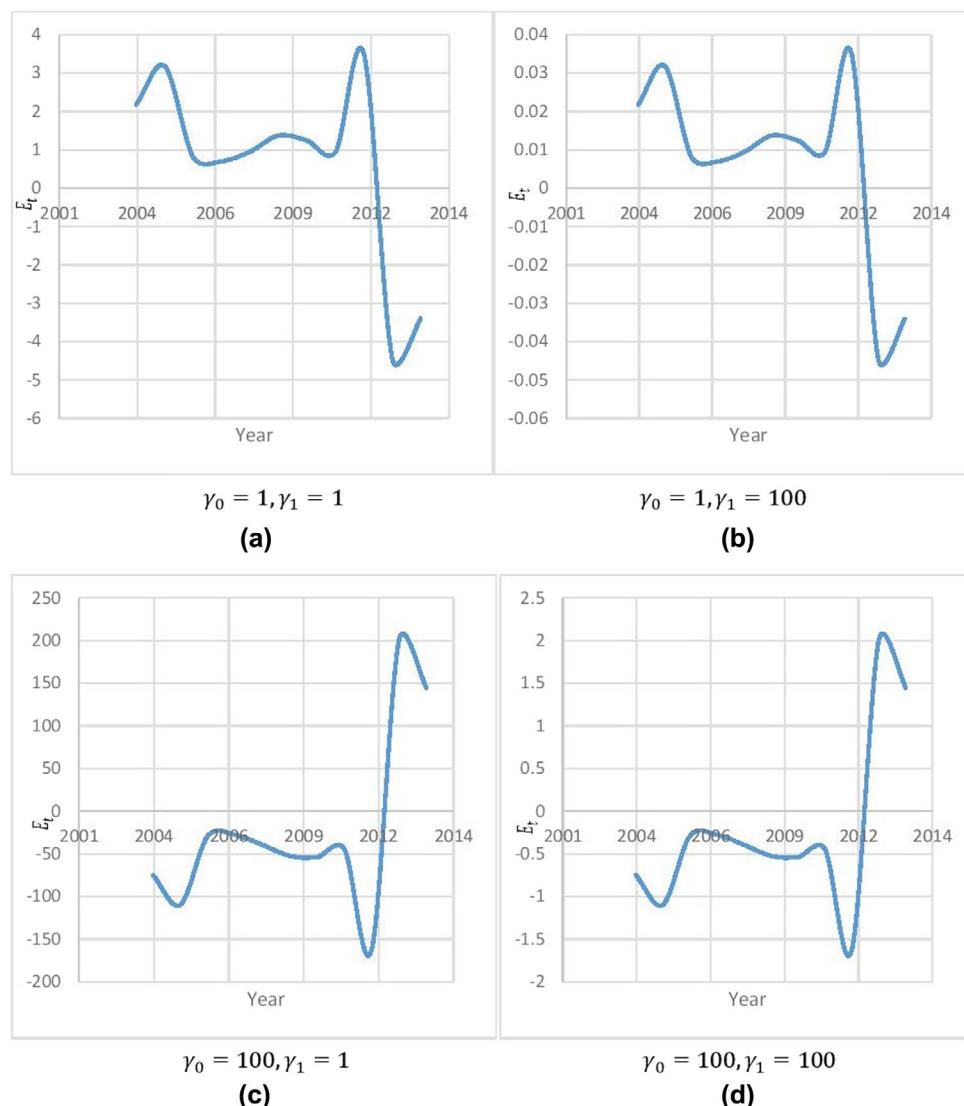
$$\Psi E_t Z_{t+1} = \Phi Z_t + \bar{C} \quad (6)$$

where  $Z_t$  = vector of state variables,  $\bar{C}$  = vector of exogenous variables and  $\Psi, \Phi$  = matrices of parameters.

#### 5. Application of basic model in Malaysian economic system

The patterns of expectations condition for Malaysia economic system from 2004 until 2014 are quite similar for the first and second equations in the basic model. Nevertheless, it is different for the third equation case. By assuming  $\gamma_0 = 1$ , the graphs are expressed like Figure 4(a) and (b). Both figures show that the condition is expected to become maximum in the year of 2012 and minimum in the year of 2014. However, the graphs are changed drastically when the value of  $\gamma_0$  changes from 1 to 100 like that have been illustrated in Figure 4(c) and (d). Both graphs depict that the expected maximum condition is in the year of 2013 and minimum in the year of 2012. In summary, the drastic changes in the trends of the graphs in Figure 4 depends heavily on the changes in value of  $\gamma_0$ . This might be due to the sharp volatility in exchange rate.

**Figure 4. The graph on mathematical expected condition from 2004 until 2014 based on the third equation in the basic model.**



## 6. Theoretical explanation

In order to investigate whether a Hopf bifurcation occurs in the models, this study employs the methodology that used by Barnett and Duzhak (2008). In a two-dimensional system, the following well-known existence theorem of Hopf bifurcation is commonly practiced by using a  $2 \times 2$  Jacobians (Galdonfo, 1996).

### 6.1. Existence theorem of Hopf bifurcation: Two-dimensions

Consider the class of two-dimensional first-order difference equation systems produced by the map  $y \mapsto f(y, \Phi), y \in \mathbb{R}^2$ , with vector of parameters  $\Phi \in \mathbb{R}^n$ . Assume for each  $\Phi$ , there exists a local fixed point,  $y^* = y^*(\Phi)$ , in the relevant interval at which the eigenvalues of the Jacobian matrix, evaluated at  $(y^*(\Phi), \Phi)$ , are complex conjugates,  $\lambda_{1,2} = a(y, \Phi) \pm ib(y, \Phi)$ , suppose that for one of those equilibria, there is a critical value for one of those parameters,  $\Phi_c$ , that is satisfy the following properties:

- (1)  $\text{mod}(\lambda_1) = \text{mod}(\lambda_2) = \sqrt{a^2 + b^2} = 1$ ;  $\lambda_i^j \neq 1 \forall i = 1, 2$  and for  $j = 1, 2, 3, 4$
- (2)  $\left. \frac{\partial \text{mod}[\lambda_i(\Phi)]}{\partial \Phi_j} \right|_{\Phi=\Phi^*} \neq 0 \forall i = 1, 2$

Then, there exists a Hopf bifurcation at the equilibrium point  $(y^*(\Phi^*), \Phi^*)$ .

## 6.2. Taylor rule

First of all, we can rewrite our first system in the form of (6) after necessary substitution. In notation of matrix:

$$\begin{bmatrix} 1 & -\alpha_2\gamma_1 - \alpha_1 \\ 0 & \beta_1 \end{bmatrix} E_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 - \alpha_1\lambda_2 & -\alpha_1\lambda_1 \\ -\beta_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} -\alpha_2\gamma_0 - \alpha_2\gamma_1 E_t \pi_{t+1}^* \\ 0 \end{bmatrix} \quad (7)$$

Then, the Jacobian matrix of the above system can be represented as:

$$J = A^{-1} \cdot B \quad (8)$$

$$\text{where } A = \begin{bmatrix} 1 & -\alpha_2\gamma_1 - \alpha_1 \\ 0 & \beta_1 \end{bmatrix}, \text{ and } B = \begin{bmatrix} 1 - \alpha_1\lambda_2 & -\alpha_1\lambda_1 \\ -\beta_2 & 1 \end{bmatrix}.$$

The Jacobian of the economic model defined above is:

$$J = \begin{bmatrix} 1 - \frac{\alpha_1\beta_1\lambda_2 + \alpha_2\beta_2\gamma_1 + \alpha_1\beta_2}{\beta_1} & \frac{-\alpha_1\beta_1\lambda_1 + \alpha_1 + \alpha_2\gamma_1}{\beta_1} \\ -\frac{\beta_2}{\beta_1} & \frac{1}{\beta_1} \end{bmatrix} \quad (9)$$

The characteristic Equation of (9) is represented as:

$$\left| \begin{bmatrix} 1 - \frac{\alpha_1\beta_1\lambda_2 + \alpha_2\beta_2\gamma_1 + \alpha_1\beta_2}{\beta_1} & \frac{-\alpha_1\beta_1\lambda_1 + \alpha_1 + \alpha_2\gamma_1}{\beta_1} \\ -\frac{\beta_2}{\beta_1} & \frac{1}{\beta_1} \end{bmatrix} - \Lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0 \quad (10)$$

The determinant is  $\Lambda^2 + b\Lambda + c = 0$  where  $b = -\frac{1}{\beta_1}[1 + \beta_1(1 - \alpha_1\lambda_2) - \beta_2(\alpha_1 + \alpha_2\gamma_1)]$  and  $c = \frac{1}{\beta_1}[1 - \alpha_1(\beta_2\lambda_1 + \lambda_2)]$ . In order to solve a pair of complex conjugate eigenvalues, we assume that  $\Delta < 0$  where:

$$\Delta = b^2 - 4ac = \left\{ -\frac{1}{\beta_1}[1 + \beta_1(1 - \alpha_1\lambda_2) - \beta_2(\alpha_1 + \alpha_2\gamma_1)] \right\}^2 - \frac{4}{\beta_1}[1 - \alpha_1(\beta_2\lambda_1 + \lambda_2)] \quad (11)$$

As a result, we get  $\Lambda_1 = h + iv$  and  $\Lambda_2 = h - iv$ , where  $h = -\frac{b}{2}$  and  $v = \frac{\sqrt{\Delta}}{2}$ . From this finding, the model's parameters are  $\Phi = [\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_0, \gamma_1, \lambda_1, \lambda_2]$ . However, the parameters that relevant for bifurcation are  $\lambda_1$  and  $\lambda_2$  which denote the coefficients of monetary policy rule.

**Proposition 6.1** There is Hopf bifurcation existed in the model if and only if  $\Delta < 0$  and  $\lambda_1 = \Psi_2\lambda_2^2 + \Psi_1\lambda_2 + \Psi_0$  where:

$$\begin{aligned} \bullet \Psi_0 &= -\frac{[1 - \beta_2(\alpha_1 + \alpha_2\gamma_1)]^2}{2\alpha_1\beta_1\beta_2} + \frac{(\alpha_1 + \alpha_2\gamma_1)}{\alpha_1} + \frac{\beta_1}{2\alpha_1\beta_2} \\ \bullet \Psi_1 &= \frac{\beta_1 - \beta_2(\alpha_2\gamma_1 + \alpha_1)}{\beta_2} \\ \bullet \Psi_2 &= -\frac{\alpha_1\beta_1}{2\beta_2} \end{aligned}$$

**Proof** By referring to the first condition of the existence of Hopf bifurcation theorem:

$$\text{mod}(\Lambda_1) = \text{mod}(\Lambda_2) = \sqrt{h^2 + v^2} = 1.$$

Substituting  $h = -\frac{b}{2}$  and  $v = \frac{\sqrt{\Delta}}{2}$  into this equation we get:



$$\left( \left\{ \frac{\frac{1}{\beta_1}[1 + \beta_1(1 - \alpha_1\lambda_2) - \beta_2(\alpha_1 + \alpha_2\gamma_1)]}{2} \right\}^2 + \frac{\left\{ -\frac{1}{\beta_1}[1 + \beta_1(1 - \alpha_1\lambda_2) - \beta_2(\alpha_1 + \alpha_2\gamma_1)] \right\}^2}{4} - \frac{\frac{4}{\beta_1}[1 - \alpha_1(\beta_2\lambda_1 + \lambda_2)]}{4} \right)^{\frac{1}{2}} = 1. \quad (12)$$

After solving  $\lambda_1$  from Equation (12), the parameter's critical value is as stated in Proposition 1. When this parameter is in its critical value, the derivative of the modulus with respect to  $\lambda_1$  is as followed:

$$\left. \frac{\partial \text{mod}(\Lambda_1)}{\partial \lambda_1} \right|_{\lambda_1^c} = \left. \frac{\partial \text{mod}(\Lambda_2)}{\partial \lambda_1} \right|_{\lambda_1^c} = \frac{\alpha_1\beta_2}{2\beta_1} \quad (13)$$

Since  $\alpha_1\beta_2 < 0$  and  $0 < \beta_1 < 1$ , then  $\frac{\alpha_1\beta_2}{2\beta_1} < 0$ . This shows that  $\frac{\alpha_1\beta_2}{2\beta_1} \neq 0$ . Thus, the second condition for the Hopf bifurcation is satisfied. Hence, this proved both conditions of the first proposition.

### 6.3. Inflation targeting rule

At first, we can rewrite our first system in the form of (6) after necessary substitution. In notation of matrix:

$$\begin{bmatrix} 1 & -(\alpha_1 + \alpha_2\gamma_1) \\ 0 & \beta_1 \end{bmatrix} E_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -\alpha_1\phi \\ -\beta_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} - \begin{bmatrix} \alpha_2\gamma_0 + \alpha_2\gamma_1 E_t \pi_{t+1}^* \\ 0 \end{bmatrix} \quad (14)$$

The Jacobian of the economic model defined above is:

$$J = \begin{bmatrix} 1 - \frac{\alpha_1\beta_2 + \alpha_2\beta_2\gamma_1}{\beta_1} & \frac{\alpha_1 + \alpha_2\gamma_1 - \alpha_1\beta_1\phi}{\beta_1} \\ -\frac{\beta_2}{\beta_1} & \frac{1}{\beta_1} \end{bmatrix} \quad (15)$$

The characteristic Equation of (15) is represented as:

$$\left| \begin{bmatrix} 1 - \frac{\alpha_1\beta_2 + \alpha_2\beta_2\gamma_1}{\beta_1} & \frac{\alpha_1 + \alpha_2\gamma_1 - \alpha_1\beta_1\phi}{\beta_1} \\ -\frac{\beta_2}{\beta_1} & \frac{1}{\beta_1} \end{bmatrix} - \Lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0 \quad (16)$$

The determinant is  $\Lambda^2 + b\Lambda + c = 0$  where  $b = \frac{1}{\beta_1}[-1 - \beta_1 + \beta_2(\alpha_1 + \alpha_2\gamma_1)]$  and  $c = \frac{1}{\beta_1}[1 - \alpha_1\beta_2\phi]$ . In order to solve a pair of complex conjugate eigenvalues, we assume that  $\Delta < 0$  where:

$$\Delta = b^2 - 4ac = \left\{ \frac{1}{\beta_1}[-1 - \beta_1 + \beta_2(\alpha_1 + \alpha_2\gamma_1)] \right\}^2 - \frac{4}{\beta_1}(1 - \alpha_1\beta_2\phi) \quad (17)$$

As a result, we get  $\Lambda_1 = h + iv$  and  $\Lambda_2 = h - iv$ , where  $h = -\frac{b}{2}$  and  $v = \frac{\sqrt{\Delta}}{2}$ . Then, the parameter of the models are obviously  $\kappa = [\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_0, \gamma_1, \phi]$ , but the parameter that relevant for bifurcation is  $\phi$  which denote the coefficients of inflation targeting rule.

**Proposition 6.2** There is Hopf bifurcation existed in the model if and only if  $\Delta < 0$  and

$$\phi = \frac{\beta_1}{\alpha_1\beta_2} + \frac{1}{\alpha_1\beta_2} - \frac{[-1 - \beta_1 + \beta_2(\alpha_1 + \alpha_2\gamma_1)]^2}{2\alpha_1\beta_1\beta_2}.$$

**Proof** By referring to the first condition of the existence of Hopf bifurcation theorem:

$$\text{mod}(\Lambda_1) = \text{mod}(\Lambda_2) = \sqrt{h^2 + v^2} = 1.$$



Substituting  $h = -\frac{b}{2}$  and  $v = \frac{\sqrt{\Delta}}{2}$  into this equation we get:

$$\left( \left\{ \frac{-\frac{1}{\beta_1}[-1 - \beta_1 + \beta_2(\alpha_1 + \alpha_2\gamma_1)]}{2} \right\}^2 + \frac{\left\{ \frac{1}{\beta_1}[-1 - \beta_1 + \beta_2(\alpha_1 + \alpha_2\gamma_1)] \right\}^2}{4} - \frac{\frac{4}{\beta_1}(1 - \alpha_1\beta_2\phi)}{4} \right)^{\frac{1}{2}} = 1 \quad (18)$$

After solving  $\phi$  from Equation (18), the parameters' critical value is as followed:

$$\phi = \frac{\beta_1}{\alpha_1\beta_2} + \frac{1}{\alpha_1\beta_2} - \frac{[-1 - \beta_1 + \beta_2(\alpha_1 + \alpha_2\gamma_1)]^2}{2\alpha_1\beta_1\beta_2} \quad (19)$$

The second condition of the Hopf bifurcation theorem is represented as:

$$\frac{\partial \text{mod}(\Lambda_1)}{\partial \phi} \Big|_{\phi^c} = \frac{\partial \text{mod}(\Lambda_2)}{\partial \phi} \Big|_{\phi^c} = \frac{\alpha_1\beta_2}{2\beta_1} \neq 0. \quad (20)$$

Thus, this proved both conditions of the second proposition.

## 7. Conclusion

In conclusion, the patterns of expectations condition for Malaysia economic system from 2004 until 2014 are quite similar except for exchange rate case. This might be due to the sharp volatility in exchange rate itself. Furthermore, the bifurcation boundaries existed in the parameter spaces of two forms of different monetary policy rules and the Hopf bifurcation detected on both systems of linear economy. It does not lead to clear argument on which rule is more sensitive to bifurcation even though we compare these two systems with each other. Besides, the robustness of the dynamical inference is extremely damage when a bifurcation boundary crosses into the confidence region of the mode parameters. The findings support the results of Torre (1977), Grandmont, Barnett and Duzhak (2008), Barnett and He (2002) as well as Barnett and Eryilmaz (2012, 2013). Thus, the monetary policy maker might be careful with the boundary relationships between parameters of bifurcation that describe the policy rule.

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