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## Forecasting trend output in the Euro area

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**DISCUSSION PAPER**

# **Forecasting Trend Output in the Euro Area**

Christian Schumacher

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# **Forecasting Trend Output in the Euro Area**

**Christian Schumacher**

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**HWWA DISCUSSION PAPER**

**Edited by the Department  
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# Forecasting Trend Output in the Euro Area

Christian Schumacher\*

## Abstract

This paper estimates and forecasts trend output and output gaps for the Euro area. In the monetary strategy of the European Central Bank (ECB), trend output is used to forecast a reference value for money. For this purpose, trend output must be forecasted as well. In this paper, a permanent-transitory decomposition (PT) based on cointegration restrictions gives an estimate of trend output in the Euro area. Ex-ante forecasts of trend output are generated and to get an impression of the forecast uncertainty, bootstrap simulation is employed to construct prediction intervals that take into consideration estimation uncertainty. The empirical uncertainty around trend output is relatively smaller than the uncertainty of the output gap. The absolute uncertainty of both indicators is quite large and questions their usefulness for monetary policy. When relying on such indicators monetary authorities should clarify this uncertainty.

## 1 Introduction

This paper contains an applied study about forecasting trend output and the output gap in the Euro area. Trend output measures have an important role both in economic theory and policy. For example, the central aim of the European Central Bank (ECB) is to reach and maintain price stability in the Euro area. For this task, the movement of trend or potential output is an important indicator. As an integral part of the monetary strategy of the ECB, a reference value of M3 money growth is forecasted at the end of each year with a one year forecast horizon.<sup>1</sup> Based on the quantity equation of money, for a given non-avoidable or target inflation rate and a trend growth/decline for the velocity of M3, a measure of trend output is needed to pin down the path of the reference value of M3. In addition to the determination of the M3 reference value, trend or potential output serves directly as an indicator when it is related to realized output. In recent theoretical models with imperfect price flexibility, the movement of the output gap, defined as output minus trend output (in logs), leads to inflation movements.<sup>2</sup> Hence, potential output

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<sup>1</sup>See ECB (1998), ECB (1999c).

<sup>2</sup>See for example Yun (1996) or Clarida/Gali/Gertler (2000).

plays a direct role in the monetary transmission mechanism.<sup>3</sup> To measure trend output empirically, many different methods compete with each other and produce a variety of results.<sup>4</sup> This implicates a high degree of uncertainty the monetary policy authority has to face.

At the moment, some measures already exist to measure trend output in the Euro area.<sup>5</sup> In this paper, we use a permanent-transitory (PT) decomposition to decompose output into trend and cycle. This method allows more general dynamics for the trend than the other methods employed before. Within this decomposition, trend output is defined as that part of output that is due to permanent shocks.<sup>6</sup> To determine the decomposition empirically, an econometric model is estimated with the cointegration techniques of Johansen (1988, 1991). The cointegration restrictions help to identify the permanent shocks of the empirical model and hence allow for the determination of trend output. The fully specified and estimated model is then used to generate trend output measures. The forward-looking nature of a central bank's decision process described before requires an examination of ex-ante forecasts for trend output. Therefore, in addition to earlier studies, we construct ex-ante prediction intervals. To investigate the forecast uncertainty surrounding the output gap, forecast intervals based on bootstrapping are generated. The simulated forecast intervals consider the uncertainty that is due to estimation errors.

The paper proceeds as follows: In section 2, the permanent-transitory decomposition is introduced and related to other methods and in section 3 it is shown how forecasts of the components can be calculated. In section 4, trend output and the output gap are estimated and forecasted for Euro area data. Section 5 concludes.

## 2 The permanent-transitory (PT) decomposition

### 2.1 Time series representation

The aim of this paper is to decompose output into a trend component and a cyclical component. Therefore, let us assume that output or GDP is an element of the  $(m \times 1)$  vector  $X_t$ . The decomposition is then

$$X_t = X_t^P + X_t^T \quad \text{or} \quad \Delta X_t = \Delta X_t^P + \Delta X_t^T,$$

where  $X_t^P$  is the permanent or trend component and  $X_t^T$  is the transitory or cycle component. The permanent-transitory (PT) decomposition now defines how trend output can be extracted from observed output. Behind the PT decomposition stands the general belief that behind short-run movements, the economy evolves along a growth path, which is interpreted as the trend. The economy is being affected by two types of shocks: permanent and transitory shocks. The permanent shocks are mainly alterations of technology

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<sup>3</sup>See ECB (2000b), p. 57.

<sup>4</sup>Canova (1998) investigates the effect of various detrending methods on stylized business cycle facts. He finds that the stylized facts differ considerably among the detrending methods.

<sup>5</sup>See Gerlach/Smets (1999) and Bolt/Van Els (2000), ECB (2000c).

<sup>6</sup>This definition of trend output was introduced by Blanchard/Fisher (1989), p. 8.



and improvements of productivity that have a long-run effect on  $X_t$ . The PT decomposition defines that part of output as trend output  $X_t^P$  that is due to the permanent shocks.<sup>7</sup> The short-run fluctuations in output are determined by the transitory shocks. These shocks have no long-run effect on output so that the transitory component  $X_t^T$  is a stationary variable. It fluctuates around the permanent part  $X_t^P$  of output. As mentioned in the introduction, the transitory component of output is often called the output gap. In economic theory, it serves as an indicator for price pressures. In the following, we derive the trend output in first differences,  $\Delta X_t^P$ , since this is the transformation that is most relevant for policy purposes. The trend output enters the derivation of the reference value of M3 growth in first differences. The output gap, instead, is derived in levels,  $X_t^T$ , because theory suggests that only the level affects inflation.

To extract trend output, a representation of output dependent on shocks is required according to the definition above. To derive the PT decomposition, we start with the estimation of a vector error correction model (VECM) making use of Johansen's (1988, 1991) cointegration procedure. The following model is estimated:

$$\Delta X_t = \mu + \alpha\beta'X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t. \quad (1)$$

Here, the endogenous variables are collected in the  $(m \times 1)$  vector  $X_t$ . This vector in first differences,  $\Delta X_t$ , is estimated on a vector of constants  $\mu$ , own  $k-1$  lags of first differences and a linear combination of the levels of  $X_t$ ,  $\beta'X_{t-1}$ , where  $\beta$  is the  $(m \times r)$  matrix of constants that forms the  $r$  cointegration relationships. These cointegration relationships can be interpreted as the long-run equilibria between the variables. The  $(m \times r)$  matrix  $\alpha$  includes the loadings that show how the system reacts to long-run disequilibria or cointegration errors. The  $\varepsilon_t$  are the residuals of the system that will be transformed into the permanent and transitory shocks later. Before the two types of shocks can be derived, output must be expressed as solely dependent on the residuals and deterministic terms. This is the Wold or moving average representation, which can be derived from the VECM by inversion.<sup>8</sup> The Wold representation becomes

$$\begin{aligned} \Delta X_t &= \tau + A(L)\varepsilon_t \\ &= \tau + A_0\varepsilon_t + A_1\varepsilon_{t-1} + A_2\varepsilon_{t-2} + \dots, \end{aligned} \quad (2)$$

where  $\Delta$  is the first difference operator  $\Delta = (1 - L)$  with  $LX_t = X_{t-1}$ .  $\tau$  is the deterministic part, a constant. The Wold representation cuts any stationary time series into a stochastic and a deterministic part. The stochastic part is a function of the error term  $\varepsilon_t$  which is  $\varepsilon_t \sim N(0, \Omega)$ . The variables  $X_t$  are linked to the errors through the lag polynomial  $A(L) = \sum_{i=0}^{\infty} A_i L^i$ , where it is assumed that the parameters decay over time so that  $\sum_{i=0}^{\infty} |A_i| < \infty$ . Above, the first difference operator was used to reach a stationary

<sup>7</sup>See Blanchard/Fisher (1989), p. 8.

<sup>8</sup>For this purpose, the error correction model is transformed into a VAR in levels and then presented in companion form. This can be interpreted as a first order VAR that can be transformed into the moving average model. See Pesaran/Shin (1996). The constant is defined as  $\tau = A(1)\mu$ .

series, because we are concerned with output that is often assumed to be integrated of order one, or  $I(1)$ .

In the Wold representation, the endogenous variable is linked only to a constant and the stochastic errors. Therefore, the parameters can be interpreted as multipliers. An error or shock in period  $t$ ,  $\varepsilon_t$ , has an impact on  $\Delta X_t$  of  $A_0$ . After one period the shock in period  $t$  causes  $\Delta X_{t+1}$  to change with  $A_1$ , after two periods it has an impact of  $A_2$  and so on. The level of the process can be written after  $n$  periods as

$$X_{t+n} = X_{t-1} + \Delta X_t + \Delta X_{t+1} + \dots + \Delta X_{t+n},$$

since the vector can be understood as the sum of cumulated differences starting from an initial value  $X_{t-1}$ . In multiplier notation, holding shocks other than  $\varepsilon_t$  constant, the effect of  $\varepsilon_t$  on  $X_{t+n}$  is

$$\begin{aligned} \frac{\partial X_{t+n}}{\partial \varepsilon_t} &= 0 + \frac{\partial(\Delta X_t)}{\partial \varepsilon_t} + \dots + \frac{\partial(\Delta X_{t+n})}{\partial \varepsilon_t} \\ &= A_0 + A_1 + \dots + A_n = \sum_{i=0}^n A_i, \end{aligned}$$

which can be interpreted as the long-run impact of a shock in period  $t$ . Letting the forecast interval become very large, we get the long-run multiplier  $A(1) = \sum_{i=0}^{\infty} A_i = A_0 + A_1 + \dots$ .  $A(1)$  is the value of  $X_t$  due to shocks that is reached after all transitional dynamics have died out. If one now divides the matrix polynomial into a long-run and a short-run part, that is  $A(L) = A(1) + \tilde{A}(L)(1-L)$ , the first difference of the endogeneous variables can then be expressed as

$$\begin{aligned} \Delta X_t &= \tau + A(L)\varepsilon_t = \tau + A(1)\varepsilon_t + \tilde{A}(L)(1-L)\varepsilon_t \\ &= \tau + \beta_{\perp}(\alpha'_{\perp}(I - \sum_{i=1}^{k-1} \Gamma_i)\beta_{\perp})\alpha'_{\perp}\varepsilon_t + \tilde{A}(L)(1-L)\varepsilon_t, \end{aligned}$$

where  $\beta_{\perp}(\alpha'_{\perp}(I - \sum_{i=1}^{k-1} \Gamma_i)\beta_{\perp})$  is the long-run effect of the  $(m-r)$  permanent shocks  $\alpha'_{\perp}\varepsilon_t$ .<sup>9</sup> and  $\beta_{\perp}$  and  $\alpha_{\perp}$  are full rank  $(m \times (m-r))$  orthogonal complements to the cointegration vectors and the matrix of the loadings, respectively.<sup>10</sup> The decomposition explains the first difference of the endogeneous variables as the long-run effect of permanent shocks and the resulting stationary combination of shocks. Since the PT decomposition defines the trend as the whole part of output that is due to permanent shocks and not only their long-run impact, one must find an expression where the MA representation is related to the permanent shocks  $\alpha'_{\perp}\varepsilon_t$ . The moving average lag polynomial is to be decomposed into<sup>11</sup>

$$\begin{aligned} \Delta X_t &= \tau + A(L)\varepsilon_t = \tau + A^P(L)\varepsilon_t^P + A^T(L)\varepsilon_t^T \\ &= \tau + A(L)\bar{\alpha}_{\perp}\alpha'_{\perp}\varepsilon_t + A(L)\bar{\gamma}\gamma'\varepsilon_t. \end{aligned} \tag{3}$$

<sup>9</sup>For a detailed derivation, see Johansen (1995), p. 41.

<sup>10</sup>The orthogonal complement is defined as  $\alpha'\alpha_{\perp} = 0$ . When  $\alpha$  is  $m \times r$  dimensional,  $\alpha_{\perp}$  has dimensions  $[m \times (m-r)]$ .

<sup>11</sup>The derivation follows Yang (1998).

In the second line, the first part is the component of output due to permanent shocks,  $\alpha'_\perp \varepsilon_t$ . The matrix  $\bar{\alpha}_\perp$  is unknown as well as  $\bar{\gamma}$  and  $\gamma$ . All matrices must now be constructed so that the permanent and transitory part add up to the MA polynomial  $A(L)\varepsilon_t$ . This implies

$$\begin{pmatrix} \bar{\alpha}_\perp & \bar{\gamma} \end{pmatrix} = \begin{pmatrix} \alpha_\perp \\ \gamma \end{pmatrix}^{-1}.$$

To identify this matrix, it is assumed that permanent and transitory shocks are uncorrelated, that is  $E[(\alpha'_\perp \varepsilon_t)(\gamma' \varepsilon_t)'] = \alpha'_\perp \Omega \gamma \stackrel{!}{=} 0$ . This restriction is fulfilled by the matrix

$$\gamma = \alpha - \alpha_\perp (\alpha'_\perp \Omega \alpha_\perp)^{-1} \alpha'_\perp \Omega \alpha.$$

Given this matrix, the permanent part of output is

$$\Delta X_t^P = \tau + A^P(L)\varepsilon_t^P = \tau + [A(L)\bar{\alpha}_\perp] \alpha'_\perp \varepsilon_t. \quad (4)$$

Hence, trend output of a VECM according to the PT decomposition can be derived once a model is estimated where the corresponding output variable enters the endogeneous variables  $X_t$ , cointegration can be found to pin down the permanent part of the model and the structural shocks are assumed to be uncorrelated.

As mentioned before, the output gap, defined as output minus trend output, can serve as a more direct indicator for inflation pressures. In the PT decomposition, output minus trend output is the transitory part  $X_t^T$ . Up to now, it is expressed in first differences:

$$\Delta X_t^T = A^T(L)\varepsilon_t^T = [A(L)\bar{\gamma}] \gamma' \varepsilon_t. \quad (5)$$

To obtain the gap as defined before, this term must be aggregated over time to get

$$X_t^T = \sum_{i=1}^t A^T(L)\varepsilon_t^T = \sum_{i=1}^t [A(L)\bar{\gamma}] \gamma' \varepsilon_i = \tilde{A}^T(L)\gamma' \varepsilon_t, \quad (6)$$

ignoring a starting value for simplicity and redefining the lag polynomial  $\tilde{A}^T(L) = \tilde{A}_0^T + \tilde{A}_1^T + \dots$  according to

$$\tilde{A}_i^T = \begin{cases} A_0^T & \text{for } i = 0 \\ \tilde{A}_{i-1}^T + A_i^T = \sum_{j=0}^i A_j^T & \forall i > 0 \end{cases}.$$

## 2.2 Relationship to other methods

Evans/Reichlin (1994) estimate vector autoregressive (VAR) models, too, to decompose output into trend and cycle. The trend of output in their paper is defined by  $\Delta X_t^{P,ER} = \tau + A(1)\varepsilon_t$ . Hence, the level of trend output  $X_t^P$  is a random walk. This implies that shocks that have a permanent effect on output immediately alter trend output with their full long-run impact measured by  $A(1)$ . This definition of trend ignores possible partial adjustments after a permanent shock occurred. Trend output is then the imaginary level of output that is reached after all transitional dynamics have worked themselves out.

This assumption is in stark contrast to the widely held view that technological innovations have transitional dynamics. Lippi/Reichlin (1994) declare the random walk assumption of trend output as inconsistent with standard views about the dynamics of productivity shocks that are justified with adjustment costs on capital and labour, learning-by doing processes and time to build. The PT decomposition takes these processes into consideration. The difference can be shown as follows:<sup>12</sup> In the PT decomposition trend output is defined as  $\Delta X_t^P = \tau + A^P(L)\varepsilon_t^P$  where the lag polynomial can be decomposed into

$$\Delta X_t^P = \tau + A^P(1)\varepsilon_t^P + \tilde{A}^P(L)\varepsilon_t^P,$$

with  $\tilde{A}^P(L) = A^P(1) - A^P(L)$ . Since only the permanent shocks have a long-run effect on  $X_t$  by definition,  $A^P(1)\varepsilon_t^P = A(1)\varepsilon_t$  holds. Hence the difference between the trend in Evans/Reichlin (1994),  $\Delta X_t^{P,ER}$ , and the trend of the PT decomposition is  $\Delta X_t^{P,ER} - \Delta X_t^P = \tilde{A}^P(L)\varepsilon_t^P$ , the transitory effects of permanent shocks on output. The same argument holds for the so called unobserved components (UC) models. For example, Gerlach/Smets (1999) estimate a state space model using the Kalman filter where trend output is an unobserved variable. Although the model allows for structural equations to be estimated, an additional equation to define trend output must be supplied. Here, as is usually done in comparable models,<sup>13</sup> trend output is restricted to follow a random walk, too. The UC approach in general has the potential to implement richer trend dynamics. The main difference in comparison with the PT approach is that the modelling strategy is more restricted, since a general-to-specific procedure is not applicable due to the computational burden of the iterative Maximum Likelihood estimation.

The approach of Dupasquier/Guay/St.-Amant (1999) is more closely related to is the PT decomposition applied here. In their paper, these authors use VAR models to determine the permanent part of output under consideration of the transitory dynamics of permanent shocks, too. They call their approach LRRO, because long-run restrictions are imposed on shocks to output. This is in general also in accordance with the PT decomposition, but what differs from this paper is the way in which the long-run restrictions are imposed. Dupasquier/Guay/St.-Amant (1999) suggest to estimate the VAR in a restricted form when cointegration is present. In their paper, a two step strategy is used. At the first step, the cointegration vectors are determined. In the second step, output in first differences, the cointegration errors  $\beta'X_t$  and other variables enter a vector of endogenous stationary variables that is used to form a VAR model. Then, direct restrictions on the long-run matrix of shocks serve to identify permanent and transitory shocks.<sup>14</sup> For example, one restricted VAR model is defined in the vector  $Z_t$  and structural shocks  $\eta_t$  according to

$$Z_t = \begin{pmatrix} \Delta y_t \\ (y_t - c_t) \\ \Delta i_t \end{pmatrix}, \quad \eta_t = \begin{pmatrix} \eta_t^P \\ \eta_t^{T1} \\ \eta_t^{T2} \end{pmatrix},$$

<sup>12</sup>See Dupasquier/Guay/St.-Amant (1999), p. 583.

<sup>13</sup>See Apel/Jansson (1999).

<sup>14</sup>Long-run restrictions are a widely used tool applied in structural vector autoregressions (SVAR) to identify structural shocks. See Amisano/Giannini (1997).

where  $\Delta y_t$  denotes the first difference of logged output,  $(y_t - c_t)$  a cointegration error between output and consumption and  $\Delta i_t$ , the first difference of a short-term interest rate. The structural shocks  $\eta_t$  are  $\eta_t^P$ , which is the permanent shock, and  $\eta_t^{Ti}$  for  $i = 1, 2$  that denote two transitory shocks. The VAR in  $Z_t$  is estimated, inverted into the MA representation and the long-run matrix restricted with zero elements to identify the structural shocks. The zero restrictions are imposed to transform the long-run matrix of the structural MA representation into a triangular one. For example, in the first line, the elements of the long-run matrix that relate output to transitory shocks are set to zero. One objection can be stated against this identification scheme. The system has  $m = 3$  variables and  $r = 1$  cointegration vector. Hence, there should be  $m - r = 2$  permanent shocks in the system. Hence, the cointegration restrictions of the first step of the LRRO approach is not correctly taken over into the second step.

In the PT decomposition applied here, the cointegration restrictions are fully taken into account. Once the cointegration vectors are estimated, the assumed non correlation of permanent and transitory shocks leads to a uniquely defined permanent part of output. Since we are interested only in the decomposition into permanent and transitory shocks, a more detailed decomposition like triangularization within the group of permanent and transitory shocks is not required. The two-step procedure of the LRRO approach is less efficient than the PT decomposition applied here, because the explicit restrictions in the first step model are not taken into account in the restricted VAR estimation.<sup>15</sup> Although it is possible to restrict the long-run matrix of a restricted VAR correctly in principle, the PT method applied here is more direct.<sup>16</sup> Of course, this advantage holds only if cointegration can be found. If not, a-priori long-run restrictions have to be used to identify permanent and transitory shocks. An example for a trend decomposition of this kind is Astley/Yates (1999). But since the data set we will use later in the empirical application shows common trends, the PT decomposition seems to be the appropriate method. It allows an empirical decomposition into trend and cycle based only on the assumption of uncorrelated permanent and transitory shocks.

To conclude, in relationship to the methods discussed above, the PT decomposition based on cointegration restrictions employed here relies on weaker assumptions concerning the time series properties of the trend part and a more direct identification strategy. There are theoretical justifications to allow for richer trend dynamics and up to now no comparable applications for Euro area trend output exist. These features motivate the measurement of Euro area trend output with the PT decomposition in this paper.

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<sup>15</sup>See Vlaar (1998), p. 16, who emphasizes this point in the context of impuls response functions.

<sup>16</sup>Warne (1993) estimates a restricted VAR, transforms it into a VMA of the level variables in first differences and poses the appropriate number of common trends but however doesn't use the restrictions of the cointegration vectors in his general identification scheme.

### 3 Forecasts and forecast errors of the PT components

Up to now, we are able to calculate ex-post estimates of the PT components. To derive ex-ante forecasts and forecast errors, we set an arbitrary forecast horizon of  $h$  periods into the future. The permanent component of output at period  $t + h$  is

$$\begin{aligned}\Delta X_{t+h}^P &= \tau + A^P(L)\varepsilon_{t+h}^P \\ &= \tau + A_0^P\varepsilon_{t+h}^P + A_1^P\varepsilon_{t+h-1}^P + A_2^P\varepsilon_{t+h-2}^P + \dots\end{aligned}$$

Conditional on information available at time  $t$ , a forecast  $h$  periods ahead is

$$\Delta X_{t+h|t}^P = \tau + A_h^P\varepsilon_t^P + A_{h+1}^P\varepsilon_{t-1}^P + A_{h+2}^P\varepsilon_{t-2}^P + \dots$$

because future expectations of the errors are zero. The forecaster has no information about the size of future shocks, only about the distribution of the shocks. The forecast error is the difference between the realization of the process at  $t + h$  and the forecasted value

$$\Delta X_{t+h}^P - \Delta X_{t+h|t}^P = A_0^P\varepsilon_{t+h}^P + A_1^P\varepsilon_{t+h-1}^P + \dots + A_{h-1}^P\varepsilon_{t+1}^P.$$

The forecast error variance or mean square error (MSE) is

$$\begin{aligned}\text{MSE}_{\Delta X_{t+h|t}^P} &= \text{E} \left[ \left( \Delta X_{t+h}^P - \Delta X_{t+h|t}^P \right) \left( \Delta X_{t+h}^P - \Delta X_{t+h|t}^P \right)' \right] \\ &= \text{E} \left[ \left( A_{h-1}^P\varepsilon_{t+1}^P + \dots + A_0^P\varepsilon_{t+h}^P \right) \left( A_{h-1}^P\varepsilon_{t+1}^P + \dots + A_0^P\varepsilon_{t+h}^P \right)' \right] \\ &= \sum_{i=0}^{h-1} A_i^P \Omega^P A_i^{P'},\end{aligned}\tag{7}$$

where absence of autocorrelation of the residuals is assumed so that the cross products vanish.  $\Omega^P$  is the covariance matrix of the permanent part and is defined as  $\Omega^P = \alpha'_\perp \Omega \alpha_\perp$ . The transitory or cyclical component of the PT decomposition projected  $h$  periods ahead is derived similarly. The future transitory part is  $X_{t+h}^T = \tilde{A}^T(L)\gamma'\varepsilon_{t+h}^T$  with the conditional forecast  $X_{t+h|t}^T = \tilde{A}_h^T\varepsilon_t^T + \tilde{A}_{h+1}^T\varepsilon_{t-1}^T + \tilde{A}_{h+2}^T\varepsilon_{t-2}^T + \dots$ . The MSE is

$$\begin{aligned}\text{MSE}_{X_{t+h|t}^T} &= \text{E} \left[ \left( X_{t+h}^T - X_{t+h|t}^T \right) \left( X_{t+h}^T - X_{t+h|t}^T \right)' \right] = \sum_{i=0}^{h-1} \tilde{A}_i^T \Omega^T \tilde{A}_i^{T'} \\ &= \sum_{i=0}^{h-1} \left( \sum_{j=0}^i A_j^T \right) \Omega^T \left( \sum_{j=0}^i A_j^T \right)',\end{aligned}\tag{8}$$

where  $\Omega^T$  is the covariance of the transitory component,  $\Omega^T = \gamma'\Omega\gamma$ . We assume that the errors are normally distributed,  $\varepsilon_t \sim N(0, \Omega)$ . Further, they are serially uncorrelated. The forecast error of the permanent component  $\Delta X_{t+h}^P - \Delta X_{t+h|t}^P$  is a linear transformation of these normal vectors and, therefore, also normal:

$$\Delta X_{t+h}^P - \Delta X_{t+h|t}^P = \sum_{i=0}^{h-1} A_i^P \varepsilon_{t+h-i}^P \sim N(0, \text{MSE}_{\Delta X_{t+h|t}^P}).$$

The error of the cyclical component  $X_{t+h}^T - X_{t+h|t}^T$  is also a linear transformation of  $\varepsilon_t$ :

$$X_{t+h}^T - X_{t+h|t}^T = \sum_{i=0}^{h-1} A_i^* \varepsilon_{t+h-i} \sim N(0, \text{MSE}_{X_{t+h|t}^T}).$$

Normalizing the errors on their standard deviations leads to standard normal forecast errors. Then, forecast intervals could be easily computed.<sup>17</sup>

## 4 Estimation and forecasting trend output in the Euro area

### 4.1 Estimation of the VECM, ex-post trend output and the ex-post output gap

We now follow closely Evans/Reichlin (1994) and Dupasquier/Guay/St.-Amant (1999) who decompose U.S. output into trend and output gap with similar methods, to estimate the PT decomposition for the Euro area empirically. In both of these papers, VAR models with cointegration restrictions are estimated. Evans/Reichlin (1994) include time series into the data set which are expected to be good forecasters of GDP. This stems from the fact that the long-run value of permanent shocks, as represented by the long-run multiplier matrix  $A(1)$ , can be interpreted as the long-run forecast of the underlying series.<sup>18</sup> The use of variables that help to explain output movements good may therefore improve the PT decomposition. They use GDP, consumption, the unemployment rate, a composite leading indicator and a coincident indicator. Dupasquier/Guay/St.-Amant (1999) use GDP, consumption which is justified by having a long-run relationship with output according to the permanent-income hypothesis, and a short-term interest rate to decompose output into trend and the output gap. Because of data limitations, some of these variables are not available for an investigation of Euro area trend output.

In this paper, we use time series of real GDP, real consumption, the OECD leading indicator, and a money market rate. The time series are quarterly and seasonally adjusted and the sample range is from 1977:1 to 1999:4. As unit root tests show, all time series are integrated of order one. The sources of the data set are Eurostat, OECD Main Economic Indicators and IMF International Financial Statistics. Since Eurostat publishes only very short time series of aggregated Euro area data, time series of the member countries must be aggregated. More detailed information about the data set, especially the aggregation scheme for Euro area output, can be found in the data appendix. Due to the lack of consistent data for the Euro area right now, the proposed empirical model should be interpreted as a baseline example that illustrates the general possibilities of the PT decomposition applied here. Data extension and data revisions in the future may alter the empirical model. Although the estimation results should to be taken with care, the need for trend output measures from policy institutions justifies the empirical investigation.

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<sup>17</sup>See Lütkepohl (1993), chapter 2.2.3.

<sup>18</sup>See Evans/Reichlin (1994), p. 234.

multivariate tests	autocorrelation		normality
	LM <sub>1</sub>	LM <sub>4</sub>	
	16.63 (0.41)	19.06 (0.27)	20.61 (0.01)

univariate tests	heteroscedasticity		normality
	ARCH <sub>5</sub>	JB <sub>2</sub>	
LGDP	9.53 (0.09)	0.13 (0.94)	
SR	1.81 (0.86)	19.21 (0.00)	
LLI	2.59 (0.76)	2.07 (0.36)	
LC	3.63 (0.60)	3.98 (0.14)	

Table 1: p-values in brackets; LDGP = gross domestic product, SR = short term interest rate, LLI = leading indicator, LC = private consumption, each variable is in natural logarithms.

In contrast to the papers cited above, we use Johansen cointegration tests to estimate the empirical model. This procedure involves three steps. First, the overall fit of the model is investigated with univariate and multivariate tests without any cointegration restrictions. Second, the cointegration rank  $r$  that is equal to the number of cointegration equations is determined. Third, the corresponding cointegration vectors are determined that allow to identify the permanent and transitory part of the model.

Table 1 shows the multivariate and univariate goodness-of-fit tests for the VECM with cointegration rank set to  $r = m$ . The model is estimated with four lags of differenced endogeneous variables.<sup>19</sup>

The goodness-of-fit statistics indicate sufficient statistical properties of the model.<sup>20</sup> Multivariate tests for autocorrelation of the residuals can't reject the null of no autocorrelation. Moreover, there is no sign of heteroscedasticity in the univariate statistics. However, the test statistic for multivariate normality is 20.61, which corresponds to a probability value of 0.01. This rejects the null of normality. As the univariate tests show, it is the equation of the money market interest rate that has non-normal residuals. But a graphical inspection of the residuals shows that the rejection of the null is only due to an outlier that is not captured by the model. Since the Johansen test for cointegration does not strictly depend on the normality assumption, we use this specification for further investigations.<sup>21</sup> Moreover, to estimate and forecast trend output later we won't rely on the normality assumption. The results of the cointegration rank test are given in table 2.

Compared with the critical values of Osterwald-Lenum (1992), the null of no cointegration and cointegration rank  $r = 1$  are rejected. The null of two cointegration relations cannot be rejected. Hence, one can confirm that two cointegration vectors exist between the variables. Hence, we found an empirical model with two cointegration equations.

<sup>19</sup>The lag order was chosen according to the Akaike and Schwarz information criteria as well as sequential likelihood ratio tests.

<sup>20</sup>For a detailed description of the statistics, see Hansen/Juselius (1995).

<sup>21</sup>See Lütkepohl (1993), p. 359, remark 5.



eigenvalue	trace	95 per cent critical value	$H_0 : r$	
0.297	61.90	47.21	0	
0.198	31.23	29.68	1	
0.118	12.07	15.41	2	
0.013	1.13	3.76	3	

cointegration		coefficients of $\beta'$			
equation	LGP	SR	LLI	LC	
1	1.000	-0.764	-0.782	-0.452	
2	1.000	1.244	-0.002	-0.789	

Table 2: Critical values are from Osterwald-Lenum (1992). - The variables are LDGP = gross domestic product, SR = short term interest rate, LLI = leading indicator, LC = private consumption, each variable is in natural logarithms.

Again, as in Evans/Reichlin (1994), we leave the two cointegrations unrestricted in the following. The given cointegration properties of the system now allow to derive trend output and the output gap.

Trend output growth and the output gap are depicted in the figures (1) and (2).

Trend output growth shows some variability in the short-run. This result is in accordance with other studies that analyze the relative importance of permanent shocks. Although the importance of the permanent part of output varies in detail, almost all estimates show that permanent shocks matter and imply at least some short-run fluctuations in output.<sup>22</sup> Therefore, trend output should be distinguishable from a linear trend. The results of the model applied here have this feature. The ratio of the variances of the first differenced gap in relation to the trend,  $V(\Delta X_t^T)/V(\Delta X_t^P)$ , is 2.20, which means that approximately two thirds of the output variance reflect fluctuations in the gap.

The output gap is, as expected by construction, stationary. It replicates important business cycle phases that were observed in the big countries of the Euro Area. The economic upturn due to German unification as well as the following recession in 1993 are replicated. After 1993, the economic recovery takes place quite steady interrupted by the small downturns in 1996 and 1998, whereas the later is mainly due to the recent crisis in East Asia and some Eastern European countries such as Russia.

## 4.2 Ex-ante forecasting

The forecast errors derived before were an approximation based on the assumption that the parameters are known. In empirical work, the model's parameters usually have to be estimated. To determine the size of estimation uncertainty, asymptotic theory can be used. But because the model is dynamic, it must be assumed that the sample size of the estimated model goes to infinity to reach asymptotic test statistics. In most empirical

<sup>22</sup>See for example King/Plosser/Stock/Watson (1991).

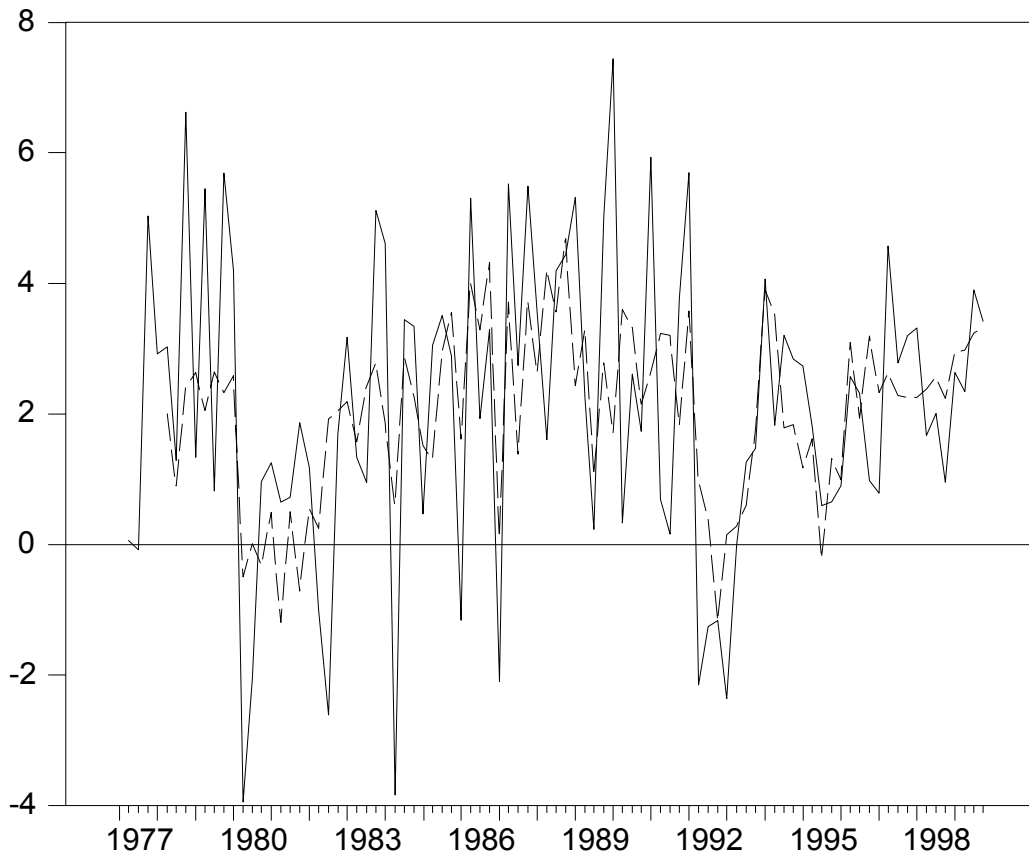


Figure 1: Trend output (solid line), output (dashed line) in first differences at annual rates

applications, this assumption is generally not fulfilled. Especially for the Euro area, one has to cope with short time series. In our case, we rely on roughly 90 observations. Therefore, one has to pose the question how good the approximations of the parameter distributions are.

To solve this problem, one can use simulation techniques, which can be applied to a fully specified probability time series model. One simulation method is the bootstrap approach. The bootstrap approach treats the empirical residuals of the model under investigation as interchangeable over time given they are serially uncorrelated. When a large number of random errors is drawn from their empirical distribution with replacement, it is possible to simulate realizations of the model's endogeneous variables. For each set of the residuals, the autoregressive structure of the time series model allows for calculating the model's endogeneous variables. With these realizations, the model can be reestimated and used for forecasting. If a large number of forecasts is replicated, one gets an empirical distribution of the baseline forecast. In contrast to asymptotic approximations, both the estimation and forecast errors are taken into account. Now, we use bootstrapping to generate forecast intervals of the PT components. The empirical set of residuals of the

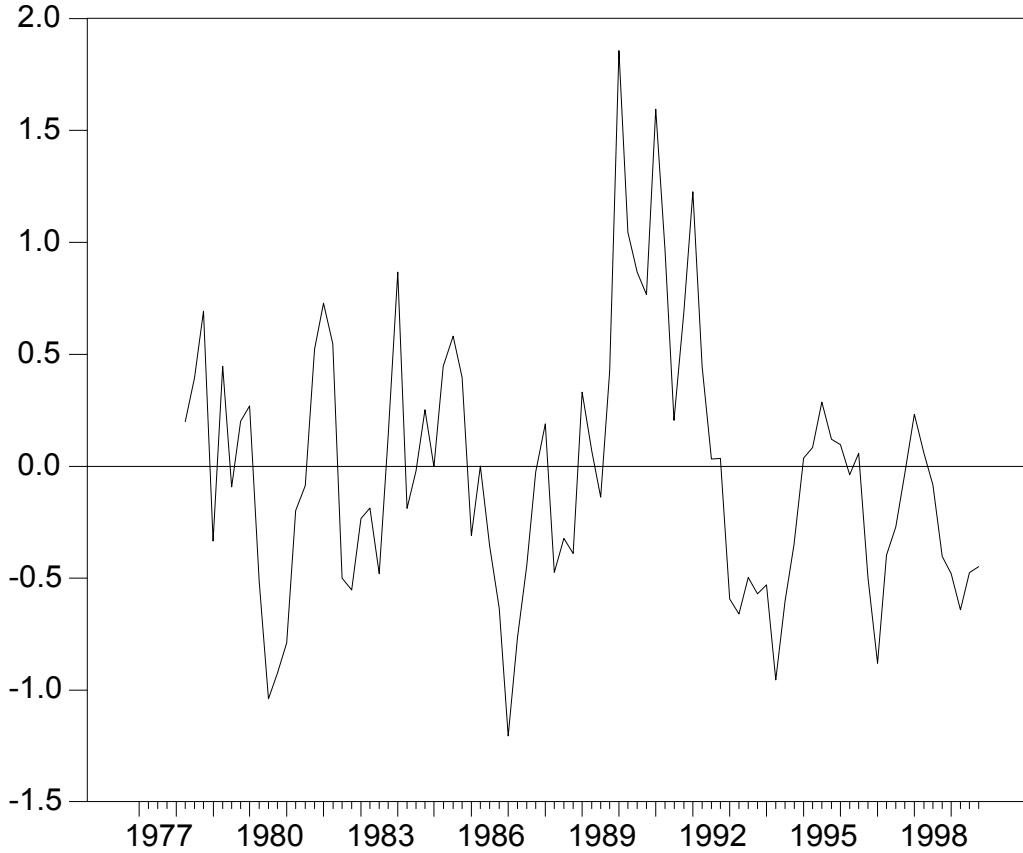


Figure 2: Output gap in percent

baseline model is used to replicate  $b = 5000$  artificial vector time series. The simulation experiment proceeds as follows:

1. Given the estimated VECM, a baseline PT decomposition is performed ex post. Then, the forecast of the permanent part is generated,  $\Delta \widehat{X}_{t+h|t}^P$ . Moreover, the transitory part  $X_{t+h|t}^T$  is calculated to give a measure of the output gap.
2. From the empirical distribution of  $\widehat{\varepsilon}_t$ ,  $b = 5000$  residuals  $\widehat{\varepsilon}_{b,i} \forall i = 1, \dots, t$  are drawn with replacement. Since  $\widehat{\varepsilon}_t$  is a vector of the equation's random errors, the contemporaneous correlations among these residuals should be considered. Applying the Cholesky decomposition to the covariance of the residuals  $\widehat{\Omega}$  gives the lower triangular matrix  $P$  such that  $\widehat{\Omega} = PP'$ . The residuals can then be transformed into independent errors  $\widehat{\xi}_t = P^{-1}\widehat{\varepsilon}_t$ , because  $E(\widehat{\xi}_t \widehat{\xi}_t') = E[P^{-1}\widehat{\varepsilon}_t (P^{-1}\widehat{\varepsilon}_t)'] = P^{-1}\widehat{\Omega}P^{-1'} = I$  holds by construction. After drawing  $\widehat{\xi}_{b,t}$  replications with the uniform distribution, the corresponding residuals are  $\widehat{\varepsilon}_{b,i} = P\widehat{\xi}_{b,t}$ .<sup>23</sup>

<sup>23</sup>For an application, see Pesaran/Shin (1996), p. 140.

3. The estimated VECM is transformed into the corresponding VAR in levels in order to simulate sequences of the endogeneous variables  $\widehat{X}_{b,t}$  for each set of the residuals.
4. The model is reestimated for each replication. This gives a set of parameters  $\widehat{A}_b^P(L)$ .
5. Forecasts of the permanent part of the PT,  $\Delta\widehat{X}_{b,t+h|t}^P$ , are then calculated in two ways: (A) conditional on the baseline values of  $\Delta\widehat{X}_i^P \forall i = 1, \dots, t$  up to the end of the estimation sample and (B) unconditional are then calculated. The model to forecast trend output relies on the parameter set  $\widehat{A}_b^P(L)$  and the bootstrapped residuals  $\widehat{\varepsilon}_{b,i} \forall i = 1, \dots, t$ . The transitory component is forecasted the same way.
6. The empirical distribution of  $\Delta\widehat{X}_{b,t+h}^P$  and  $X_{b,t+h}^T$  give forecast intervals around the initial forecast  $\Delta\widehat{X}_{t+h}^P$  and  $\widehat{X}_{t+h}^T$ , respectively. The lower and the upper bound of the forecast interval are the 5 and 95 percentiles of the empirical forecast distributions, respectively.

The way the simulation experiment is constructed closely to Clements/Taylor (1999) who generate bootstrapped ex-ante forecast intervals for stationary VAR models. A main difference arises in step 5, where both conditional forecasts (A) and unconditional forecasts (B) are generated. When forecasting observed time series, bootstraps of conditional forecasts are often recommended because only they use the information available at the forecast origin. Once a new observation of the time series becomes available, no uncertainty is left in that current point of time. When bootstraps of the forecasts are not conditioned on this information set, the forecast is uninformative, because the forecast interval spans a range similar to that of ex-post values. From this point of view, it seems plausible to generate conditional forecasts formed on the basis of the last calculated trend output and output gap values as has been done in step 5 (A). This proceeding can be found in other studies about output gap measurement, too. For example, in unobserved component (UC) models a time series model is put into state-space representation and solved via the Kalman filter. As a by-product of this filtering, the error variance of the unobserved state vector is calculated conditional on information in the previous period.<sup>24</sup> Although authors who apply UC models don't give detailed information about how the uncertainty bands are generated, it can be expected, that this elegant advantage of the Kalman Filter is used.<sup>25</sup>

Although conditional forecasts of trend output are widely applied, it gives rise to one main objection. Conditional forecasts are grounded on observed past values of time series. A conditional forecast of trend output as an unobservable variable is not certain even in the past. It is estimated based on information available in the current period. Therefore, estimation errors can occur. Hence, the value of trend output the ex-ante forecast is based on is not certain and should be indicated by an uncertainty interval, too. To take into consideration this objection, we calculated a unconditional forecast

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<sup>24</sup>See Harvey (1989).

<sup>25</sup>The paper of Gerlach/Smets (1999) is a recent example where trend output is estimated with a UC model.

of trend output in step 5 (B). A disadvantage of this unconditional forecasting is that once there are no values of past trend output that can pin down the ex-ante forecast, the forecast itself becomes less informative. As said before, the forecast interval covers a range similar to that of all ex-post values. This forecast interval has asymptotically the same boundaries in the forecast horizon. Despite this caveat, Dupasquier/Guay/St.-Amant (1999) generate ex-post confidence intervals that have an unconditional shape. Although the estimated output gap shows a high variance, this is not reflected in the confidence bands that are virtually horizontal.

A detailed comparison of the forecast uncertainty in the papers above is not possible, because the authors don't give exact information in their papers on what they are conditioning as well as whether and how the estimation uncertainty is implemented in the construction of confidence intervals.<sup>26</sup> Because the methods in the applied literature are not obvious at this point, we calculated both conditional and unconditional forecasts under consideration of estimation uncertainty. Moreover, the discussion shows that the construction of forecast intervals depends on a large number of options the forecaster can choose. In the context of trend output and output gaps in the Euro area, this variety of methods should lead policy makers to make clear statements about the method they apply. Unclear methods of forecasting don't allow to judge the forecast accuracy of the underlying model and prevent model comparisons.

Figures (3) and (4) show the point forecasts and the simulated forecast intervals. The sample size in the graphics was set to 1991, first quarter, to 1999, fourth quarter.

The forecasts regions of trend output and the output gap are indicated by the forecast intervals calculated with bootstrap simulations. The broader intervals in both graphs are the ones belonging to the unconditional forecast (B). The prediction intervals for trend and the gap show a considerable degree of uncertainty independent of conditionality. For example, the conditional forecast of trend output is not significantly different from its long run value of 0.5 percent per quarter after two quarters.

An interesting result is that the forecast uncertainty of trend output growth is smaller than output gap uncertainty.<sup>27</sup> The following numbers concern the conditional forecast interval. In the fourth quarter of 2000, after a 4-step forecast, the output gap forecast interval covers 1.35 percentage points, while the trend output growth uncertainty covers 1.01 percentage points. At the end of 2001, the intervals cover 1.78 percentage points for the gap and 1.30 for trend output, at the end of 2002 2.01 and 1.32, respectively. This discrepancy implies that forecasting trend output growth is more certain than the output gap which implies that the usefulness for these indicator concepts is different although both are derived from the same empirical model. There may be at least one reason for this. The observed relationship between the ex-ante forecast errors can be interpreted as

$$X_{t+h}^T - X_{t+h|t}^T > \Delta X_{t+h}^P - \Delta X_{t+h|t}^P,$$

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<sup>26</sup>Forecast uncertainty in state space models is usually incorporated using the Monte Carlo simulation technique of Hamilton (1986). In the trend output context, this is employed by Kuttner (1994). Another method to consider estimation uncertainty is applying the bootstrap to state space models directly. This would be more comparable to the approach chosen here. See Berkowitz/Kilian (1996).

<sup>27</sup>A similar result is found by De Brouwer (1998) and in ECB (2000c).



Figure 3: Trend output in first differences (dashed line), output (solid line), forecast intervals (dashed lines)

which first tells us that the gap is estimated in levels and the gap is in first differences. The forecast error of the output gap can be decomposed according to  $X_{t+h}^T - X_{t+h|t}^T = X_{t+h-1}^T - X_{t+h-1|t}^T + \Delta X_{t+h}^T - \Delta X_{t+h|t}^T$ . The forecast errors can then be written as

$$X_{t+h-1}^T - X_{t+h-1|t}^T + \Delta X_{t+h}^T - \Delta X_{t+h|t}^T > \Delta X_{t+h}^P - \Delta X_{t+h|t}^P.$$

The MSE form of this relation is

$$+\text{MSE}_{X_{t+h-1|t}^T} + 2\text{E} \left[ \left( X_{t+h-1}^T - X_{t+h-1|t}^T \right) \left( \Delta X_{t+h}^T - \Delta X_{t+h|t}^T \right) \right] > \text{MSE}_{\Delta X_{t+h|t}^P}.$$

Since the permanent and transitory part of output are uncorrelated at all leads and lags, the mean square errors of the permanent and transitory part are uncorrelated, too. This implies

$$\text{MSE}_{\Delta X_{t+h|t}} = \text{MSE}_{\Delta X_{t+h|t}^P} + \text{MSE}_{\Delta X_{t+h|t}^T},$$

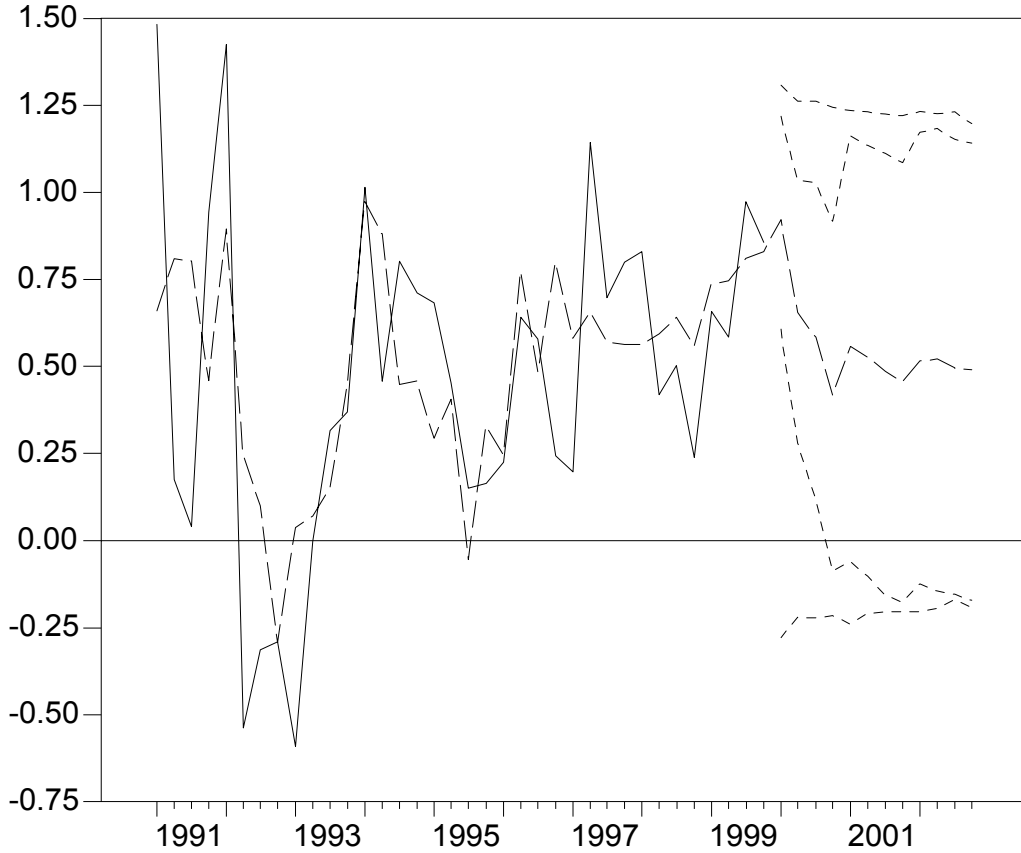


Figure 4: Output gap in percent (thin line), conditional und unconditional forecast intervalls (dashed lines)

and

$$+\frac{1}{2}\text{MSE}_{X_{t+h-1|t}^T} + \text{E} \left[ \left( X_{t+h-1}^T - X_{t+h-1|t}^T \right) \left( \Delta X_{t+h}^T - \Delta X_{t+h|t}^T \right)' \right] \frac{\text{MSE}_{\Delta X_{t+h|t}^T}}{2} > \frac{1}{2}\text{MSE}_{\Delta X_{t+h|t}}.$$

This inequality implies that the higher the MSE of the first difference of the output gap is in relation to the MSE of observed output, the more likely is the higher observed forecast uncertainty of the output gap. If the forecast horizon goes to infinity, the MSE converges to the unconditional variance of the process under consideration. Hence, the higher uncertainty of the output gap level could be due to the higher importance of the transitory shocks in relation to the permanent shocks. However, this result holds only if the mixed covariance in the second line of the left hand side of the inequality does not overcompensate the effect of the higher MSE. An overcompensation is possible because not all terms in the covariance have to be positive. Especially, expressions of autocorrelations of  $\Delta X_{t+h}^T$  with negative sign may arise. Because there are no unique exploitable expressions between the forecast errors, the empirical results may vary when

another empirical model or data set would be used. Nevertheless, we could give an intuitive reason why the gap uncertainty is higher than trend output uncertainty.

## 5 Conclusions

Estimating and forecasting trend output and output gaps is a difficult task. The unobservable character of these variables give rise to a high degree of estimation and forecast uncertainty. In this paper, a PT decomposition is used to estimate a trend output measure and an output gap for the Euro area. Especially, the forecasting properties of the PT components were investigated. The output gap measures are relatively more uncertain than trend output growth measures. This implies that trend output is a more reliable indicator for policy purposes as the output gap. In addition to this relative uncertainty, the simulated forecast intervals indicate considerable uncertainty surrounding trend output and the output gap. Although other studies often reach to the same conclusion, this result is quite unsatisfactory from a practitioner's point of view. This has the following implications:

- First, output gap uncertainty may affect monetary policy rules.<sup>28</sup> In the monetary strategy of the ECB, the reference value of M3 is derived without recognizing the uncertainty surrounding the trend output forecast.<sup>29</sup> However, the existing uncertainty implies that deviations from the reference value mustn't necessarily indicate a target missing and, hence, not necessarily a need for policy actions. This implies that the transparency of monetary policy could be improved when the uncertainty of the central bank's decision tools is made public.
- Second, the considerable uncertainty questions the usefulness of trend output and output gap measures as reliable indicators for monetary policy in general. If the output gap is hardly distinguishable from zero as the results indicate, inflationary pressures cannot be derived. This indicates further investigations to reduce the estimation uncertainty and the reliability of trend output in relation to other indicators for monetary policy.

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<sup>28</sup>See Smets (1998).

<sup>29</sup>Other sources of uncertainty can arise from the forecast of the trend growth/decline of M3 velocity.



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## A Data appendix

The empirical model used in this paper uses time series of real GDP, real consumption, the OECD leading indicator, and a money market rate. The time series are quarterly and seasonally adjusted except the interest rate. The sample range is from 1977:1 to 1999:4. The main data source for aggregated Euro area national accounts data is Eurostat. This institution provides time series in accordance with the European System of National Accounts. Unfortunately, the time series of real GDP, real consumption are only available from 1991 up to now. This data series would imply only a very small sample size for econometric testing. To get longer time series and therefore increase the degrees of freedom, one solution of this problem is to aggregate national time series. For the GDP and private consumption time series, we use data from OECD, Main Economic Indicators. Since these time series are measured in national currencies, the time series of the member countries must be converted. Aggregation requires a conversion of each member countries' GDP into a single currency. Here, the method of the ECB that uses fixed exchange rates to aggregate past money data is employed.<sup>30</sup> Output at PPP exchange rates for 1997 of the OECD are used to construct the weights of the national series.<sup>31</sup> The resulting series is then linked to the GDP series of Eurostat at the first quarter of 1991. The fixed weights ensure that the dynamics of the national real output series uphold. The resulting series is measured in Euro. For estimation, the GDP and consumption time series are transformed into natural logarithms.

The OECD leading indicator for the Euro area is already available for a sufficient time span and is directly used for estimation after taking logarithms.

The money market rate is the 3-month deposits interest rate provided by the ECB. The series starts in the first quarter of 1994. A longer time series can be generated by aggregation again. National short-term interest rates are provided by the IMF, International Financial Statistics. We use the money market rate (line 60B). The weights for aggregation are the same as for the GDP and consumption data. The resulting series is linked to the 3-month deposits interest rate in the first quarter of 1994.

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<sup>30</sup>See ECB (1999c), p. 42.

<sup>31</sup>A detailed discussion of the alternative weighting schemes can be found in Fagan/Henry (1999).

	ADF	PP	KPSS	order of integration
LGDP	-2.53 (t,4)	-1.91	0.16** (tau)	I(1)
SR	-3.53** (t,10)	-1.94	0.19** (tau)	I(1)
LLEADIND	-1.74 (t,4)	-2.04	0.11** (tau)	I(1)
LC	-2.72 (t,4)	-1.69	0.18** (tau)	I(1)
$\Delta$ LGDP	-3.14** (c,3)	-8.81**	0.09 (mu)	I(0)
$\Delta$ SR	-3.15** (c,8)	-7.46**	0.15 (mu)	I(0)
$\Delta$ LLEADIND	-5.13** (c,3)	-3.96**	0.11 (mu)	I(0)
$\Delta$ LC	-2.59* (c,3)	-10.25**	0.10 (mu)	I(0)

Table 3: \*\* (\*) denotes rejection of the null at the 5% (10%) level. - LGDP = gross domestic product, SR = short term interest rate, LLI = leading indicator, LC = private consumption, each variable is in natural logarithms. - ADF is the Dickey-Fuller t-test. The first position in brackets determines the deterministic components: t is a constant plus a time trend, c denotes only a constant. The second term in brackets denotes the lag length, which was determined applying a general-to-special reduction. - PP is the Phillips-Perron t-test where the deterministic specification is the same as in the ADF test. - The tau and mu abbreviations in the KPSS test column denote the inclusion of a constant and deterministic trend or only the inclusion of a constant, respectively.

Since in this paper cointegration techniques are applied, the integration properties of the time series should be investigated. In table 3, results for the augmented Dickey-Fuller (ADF), Phillips-Perron PP and KPSS tests are reported.<sup>32</sup> The null of the ADF and PP test is that the time series is non-stationary, the null of the KPSS test assumes a stationary time series. The majority of the test results tend towards the non-stationarity of the variables in levels.

<sup>32</sup>For details about the PP and ADF test, see Hamilton (1994), ch. 17. The critical values of the KPSS test are from Kwiatkowski/Phillips/Schmidt/Shin (1992), p. 166.