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Agglomeration and growth with and without capital mobility

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DISCUSSION PAPER

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**Richard Baldwin
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Agglomeration and Growth with and without Capital Mobility

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Abstract

This paper presents a simple framework in which the location and the growth rate of economic activities are endogenous and interact. We show that the nature of the equilibrium and of the relation between growth and location depends fundamentally on whether capital is assumed to be mobile (in which case we interpret it as physical capital) or immobile (human capital). In the first case, with constant returns to scale, growth and location are independent and no divergence or convergence process takes place. We show that newly created firms can relocate to the poor region, even though there is always a higher share of firms in the rich region, if the industry is competitive and if the return to capital is low. With immobile capital, a process of convergence between regions takes place when transaction costs on goods are sufficiently high but a process of “catastrophic” agglomeration occurs when these costs are sufficiently high and regional inequality is not affected between regions. With localized technological spillovers, higher spatial concentration of economic activities spurs growth, whether capital is mobile or not. This implies that lowering transaction costs on goods can spur growth but increase regional inequality. Lowering transaction costs on “trade in technologies” between regions may increase both regional equality and growth.

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I INTRODUCTION

Spatial agglomeration of economic activities on the one hand and economic growth on the other hand are processes difficult to separate. Indeed, the emergence and dominance of spatial concentration of economic activities is one of the facts that *Kuznets* (1966) associated with modern economic growth. This strong positive correlation between growth and geographic agglomeration of economic activities has been documented by economic historians (*Hohenberg* and *Lees*, 1985 for example), in particular in relation to the industrial revolution in Europe during the nineteenth century. In this case, as the growth rate in Europe as a whole sharply increased, agglomeration materialized itself in an increase of the urbanization rate but also in the formation of industrial clusters in the core of Europe that have been by and large sustained until now. The role of cities in economic growth and technological progress has been emphasized by urban economists (*Henderson*, 1988, *Fujita* and *Thisse*, 1996), development economists (*Williamson*, 1988) as well as by economists of growth (*Lucas*, 1988). At the other hand of the spectrum, as emphasized by *Baldwin*, *Martin* and *Ottaviano* (2001), the growth takeoff of Europe took place around the same time (end of eighteenth century) as the sharp divergence between what is now called the North and the South. Hence, growth sharply accelerated (for the first time in human economic history) at the same time as a dramatic and sudden process of agglomeration took place at the world level.

Less dramatically and closer to us, *Quah's* results (1996) suggest also a positive relation between growth and agglomeration. He finds that among the Cohesion group of countries (Greece, Spain, Portugal and Ireland, though there are no Irish regional data), the two countries that have achieved a high rate of growth and converged in per capita income terms towards the rest of Europe (Spain and Portugal) have also experienced the most marked regional divergence, Portugal being the country to have exhibited the sharpest increase in regional inequalities. By contrast Greece, which has a low growth rate and has not benefited from a tendency to converge with the rest of Europe, has not experienced a rise in regional inequalities. A recent study by INSEE (1998) shows also that the countries with a per capita GDP level above the European Union average also experience above-average regional disparities. These studies are consistent with the results of *De la Fuente* and *Vives* (1995), for instance, building on the work of *Esteban* (1994), and *Martin* (1998) who suggest that countries have converged in Europe but that this process of convergence between countries took place at the same time as regions inside countries either failed to converge or even diverged.

Hence, these empirical results point to the interest of studying growth and the spatial distribution of economic activities in an integrated framework. From a theoretical point of view, the interest should also be clear. There is a strong similarity between models of endogenous growth and models of the “new economic geography”. They ask questions that are related: one of the objectives of the first field is to analyze how new economic activities emerge through technological innovation; the second field analyzes how these economic activities choose to locate and why they are so spatially concentrated. Hence, the process of creation of new firms/economic activities and the process of location should be thought as joint processes. From a methodological point of view, the two fields are quite close as they both assume (in some versions) similar industrial structures namely, models of monopolistic competition.

In this chapter, we will attempt to clarify some of the links between growth and agglomeration. Partly, the interest of such connection will be to explain the nature of the observed correlation between growth and agglomeration. We will analyze how growth alters the process of delocation. In particular, and contrary to the fundamentally static models of the “new economic geography”, we will see how spatial concentration of economic activities may be consistent with a process of delocation of firms towards the poor regions. We will also show that the growth process can be at the origin of a catastrophic agglomeration process. One of the surprising features of the *Krugman* (1991) model, was that the introduction of partial labor mobility in a standard “new trade model” could lead to catastrophic agglomeration. We will show that the introduction of endogenous growth in the same type of standard “new trade model” could lead to the same result. The advantage is that all the results are derived analytically in the endogenous growth version.

In this first part, the causality link will be from growth to location: growth may be at the origin of an agglomeration process. The relation between growth and agglomeration depends crucially on capital mobility. Without capital mobility between regions, the incentive for capital accumulation and therefore growth itself is at the heart of the possibility of spatial agglomeration with catastrophe. In the absence of capital mobility, some results are in fact familiar to the New Economic Geography (*Fujita, Krugman and Venables, 1999*): a gradual lowering of transaction costs between two identical regions first has no effect on economic geography but at some critical level induce catastrophic agglomeration. In the model presented in this chapter, in the absence of migration, “catastrophic” agglomeration means that agents in the south have no more private in-

centive to accumulate capital and innovate. We show that capital mobility eliminates the possibility of catastrophic agglomeration and is therefore stabilizing in this sense. This is in sharp contrast with labor mobility which we know to be destabilizing. However, capital mobility also makes the initial distribution of capital between the two regions a permanent phenomenon so that both the symmetric and the Core-Periphery equilibria are always stable. One interesting finding in this chapter is that a common very simple threshold level of transaction costs determines i) when the symmetric equilibrium loses stability and when the Core-Periphery gains stability in the absence of capital mobility or with imperfect capital mobility: ii) the direction of capital relocation between the poor and the rich country with capital mobility.

In a second section of this chapter, we will concentrate on the opposite causality running from spatial concentration to growth. For this, we will introduce localized spillovers which will imply that the spatial distribution of firms will have an impact on the cost of innovation and the growth rate.

This chapter uses modified versions of *Baldwin (1999)*, *Baldwin, Martin and Ottaviano (2000)* and *Martin and Ottaviano (1999)*. The first two papers analyze models of growth and agglomeration without capital mobility. In contrast to the first paper which uses an exogenous growth model, this chapter analyses endogenous growth. In contrast to the second paper, we restrict our attention to the case of global technology spillovers. The last paper presents a model of growth and agglomeration with perfect capital mobility.

II THE CASE WITHOUT LOCALIZED SPILLOVERS: GROWTH MATTERS FOR GEOGRAPHY

1 The basic framework of growth and agglomeration

Consider a world economy with two regions (north and south) each with two factors (labor L and capital K) and three sectors: manufactures M , traditional goods T , and a capital-producing sector I . Regions are symmetric in terms of preferences, technology, trade costs and labor endowments. The Dixit-Stiglitz M -sector (manufactures) consists of differentiated goods where production of each variety entails a fixed cost (one unit of K) and a variable cost (a_M units of labor per unit of output). Its cost function, therefore, is $\pi + w a_M x_i$, where π is K 's rental rate, w is the wage rate, and x_i is total output of a typical firm. Traditional goods, which are assumed to be homogenous, are produced by

the T-sector under conditions of perfect competition and constant returns. By choice of units, one unit of T is made with one unit of L .

Regional labor stocks are fixed and immobile, so that we eliminate one possible source of agglomeration. Each region's K is produced by its I-sector. I is a mnemonic for innovation when interpreting K as knowledge capital, for instruction when interpreting K as human capital, and for investment-goods when interpreting K as physical capital. One possible interpretation of the difference between the situation of capital mobility and one of capital immobility is that in the first case we view K as physical capital (mobility then means the delocation of plants) or as knowledge capital that can be marketable and tradable through patents. The second case, capital immobility, would be more consistent with the interpretation of human capital. In this case, labor immobility implies capital immobility. The I-sector produces one unit of K with a_I units of L , so that the marginal cost of the I sector, F , is $w a_I$. Note that this unit of capital in equilibrium is also the fixed cost of the manufacturing sector. To individual I-firms, a_I is a parameter, however following *Romer* (1990) and *Grossman* and *Helpman* (1991), we assume a sector-wide learning curve. That is, the marginal cost of producing new capital declines (i.e., a_I falls) as the sector's cumulative output rises. Many justifications of this learning are possible. *Romer* (1990), for instance, rationalizes it by referring to the non-rival nature of knowledge. We can summarize these assumptions by the following:

$$\dot{K} = \frac{L_I}{a_I} \quad ; \quad F = w a_I \quad ; \quad a_I = 1/K^W \quad ; \quad K^W = K + K^* \quad (1)$$

where K and K^* are the northern and southern cumulative I-sector production levels. Note that we assume that spillovers are global: the North learns as much from an innovation made in the south than in the north¹. In section II of this chapter, we will introduce localized technological spillovers. Following *Romer* (1990) and *Grossman* and *Helpman* (1991), depreciation of knowledge capital is ignored. Finally, the regional K 's represent three quantities: region-specific capital stocks, region-specific cumulative I-sector production, and region-specific numbers of varieties (recall that there is one unit of K per variety). The growth rate of the number of varieties, on which we will focus, is therefore: $\dot{K} / K = g$.

¹ The next section analyzes the case of localized spillovers.

We assume an infinitely-lived representative consumer (in each country) with preferences:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \ln Q dt \quad ; \quad Q = C_Y^{1-\alpha} C_M^\alpha \quad ; \quad C_M = \left(\int_{i=0}^{K+K^*} c_i^{1-1/\sigma} di \right)^{\frac{1}{1-1/\sigma}} \quad (2)$$

where ρ is the rate of time preference, and the other parameters have the usual meaning. Utility optimization implies that a constant fraction of total northern consumption expenditure E falls on M-varieties with the rest spent on Y . Northern optimization also yields unitary elastic demand for T and the CES demand functions for M varieties. The optimal northern consumption path also satisfies the Euler equation which requires $\dot{E}/E = r - \rho$ (r is the north's rate of return on investment) and a transversality condition. Southern optimization conditions are isomorphic.

On the supply side, free trade in Y equalizes nominal wage rates as long as both regions produce some T (i.e. if α is not too large). Taking home labor as numeraire and assuming Y is freely traded, we have $w=w^*=1$. As for the M-sector, we choose units such that $a_M = 1-1/\sigma$ so that we get the usual pricing rules. With monopolistic competition, equilibrium operating profit is the value of sales divided by σ . Using the goods market equilibrium and the optimal pricing rules, the operating profits are given by:

$$\begin{aligned} \pi &= B \left(\frac{\alpha E^w}{\sigma K^w} \right) \quad ; \quad B \equiv \left[\frac{s_E}{s_n + \phi(1-s_n)} + \frac{\phi(1-s_E)}{\phi s_n + 1 - s_n} \right] \\ \pi^* &= B^* \left(\frac{\alpha E^w}{\sigma K^w} \right) \quad ; \quad B^* \equiv \left[\frac{\phi s_E}{s_n + \phi(1-s_n)} + \frac{1-s_E}{\phi s_n + 1 - s_n} \right] \end{aligned} \quad (3)$$

Where $s_E \equiv E/E^w$ is north's share of world expenditure E^w . s_n is the share of firms which are located in the north. When capital is immobile, this share is the share of capital owned by the Northern region: s_K . ϕ is the usual transformation of transaction costs. Also, B is a mnemonic for the 'bias' in northern M-sector sales since B measures the extent to which the value of sales of a northern variety exceeds average sales per variety worldwide (namely, $\alpha E^w/K^w$).

There are many ways to determine optimal investment in a general equilibrium model. Tobin's q-approach (Tobin, 1969) is a powerful, intuitive, and well-known method for doing just that. Baldwin and Forslid (2000) have shown how to use Tobin's q in the

context of open economy endogenous growth models. The essence of Tobin's approach is to assert that the equilibrium level of investment is characterized by the equality of the stock market value of a unit of capital – which we denote with the symbol v – and the replacement cost of capital, F . *Tobin* takes the ratio of these, so what trade economists would naturally call the M-sector free-entry condition (namely $v=F$) becomes Tobin's famous condition $q = v/F=1$.

Calculating the numerator of Tobin's q (the present value of introducing a new variety) requires a discount rate. In steady state, $\dot{E}/E = 0$ in both nations², so the Euler equations imply that $r=r^* = \rho$. Moreover, the present value of a new variety also depends upon the rate at which new varieties are created. In steady state, the growth rate of the capital stock (or of the number of varieties) will be constant and will either be the common $g=g^*$ (in the interior case), or north's g (in the core-periphery case). In either case, the steady-state values of investing in new units of K are:

$$v = \frac{\pi}{\rho + g} \quad ; \quad v^* = \frac{\pi^*}{\rho + g} \quad (4)$$

It can be checked that the equality, $v=F$, is equivalent to the arbitrage condition present in endogenous growth models such as Grossman and Helpman (1991). The condition of no arbitrage opportunity between capital and an asset with return r implies: $r = \frac{\dot{v}}{v} + \frac{\pi}{v}$.

On an investment in capital of value v , the return is equal to the operating profits plus the change in the value of capital. This condition can also be derived by stating that the equilibrium value of a unit of capital is the discounted sum of future profits of the firm with a perpetual monopoly on the production of the related variety. The free entry condition in the innovation sector ensures that the growth rate of the value v of capital is equal to growth rate of the marginal cost of an innovation, F , which due to intertemporal spillovers is $-g$. With $r = \rho$, we get the regional q 's:

$$q = \frac{\pi}{F(\rho + g)} \quad ; \quad q^* = \frac{\pi^*}{F(\rho + g)} \quad (5)$$

2 To see this, use the world labor market equilibrium: $2L = \alpha E^w \left(\frac{\sigma - 1}{\sigma} \right) + (1 - \alpha)E^w + g$ which

says that world labor supply can be used either in the manufacturing sector, the traditional sector or the innovation sector. It implies that a steady state with constant growth only exists if E^w itself is constant.

Using the definition of F , the marginal cost of innovation, Tobin's qs are:

$$q = \frac{\pi K^w}{(\rho + g)} ; q^* = \frac{\pi^* K^w}{(\rho + g)} \quad (6)$$

Note that in the case of global spillovers, the common growth rate is easy to find as it does not depend on geography. For this, we can use the world labor market equilibrium:

$$2L = \alpha E^w \left(\frac{\sigma - 1}{\sigma} \right) + (1 - \alpha) E^w + g$$

, which states that labor can be used either in the manufacturing sector (remember that the unit labor requirement in this sector is normalized to $(\sigma - 1)/\sigma$), in the Y sector or in the innovation sector (\dot{K}^w is the production of the sector per unit of time and $F = 1/K^w$ is the labor requirement in the innovation sector). The world level of expenditure is simply given by: $E^w = 2L + \rho$ which states that, with unit intertemporal elasticity of substitution, world expenditure is equal to world labor income plus ρ times steady-state world wealth, $FK^w = 1$. To find the growth rate, we therefore do not need to know anything about the location of firms or the distribution of capital. Using these equations, the growth rate of the number of varieties and of the world capital stock is given by:

$$g = 2L \frac{\alpha}{\sigma} - \frac{\sigma - \alpha}{\sigma} \rho \quad (7)$$

Using equations (6) and (7) as well as the definition of world income, it is easy to check that $q = B$ and $q^* = B^*$.

Finally, a simple equilibrium relation exists between s_E and s_K , the northern share of expenditures and the northern share of capital. It can be shown that optimizing consumers set expenditure at the permanent income hypothesis level in steady state. That is, they consume labor income plus ρ times their steady-state wealth, $FK = s_K$, and, $FK^* = (1 - s_K)$ in the north and in the south respectively. Hence, $E = L + \rho s_K$, and $E^* = L + \rho(1 - s_K)$. Thus, we get:

$$s_E \equiv \frac{E}{E^w} = \frac{L + \rho s_K}{2L + \rho} = 1/2 + \frac{\rho(2s_K - 1)}{2(2L + \rho)} \quad (8)$$

This relation between s_E and s_K , can be thought as the optimal savings/expenditure function since it is derived from intertemporal utility maximization. The intuition is simply that an increase in the northern share of capital increases the permanent income in the north and leads therefore to an increase in the northern share of expenditures.

From now on two roads are open:

1) we can let capital owners decide where to locate production. Capital is mobile even though capital owners are not, so that profits are repatriated in the region where capital is owned. In this case, s_n , the share of firms located in the north and s_K , the share of capital owned by the north, may be different. s_n is then endogenous and determined by an arbitrage condition that says that location of firms is in equilibrium when profits are equalized in the two regions. Because of capital mobility, the decision to accumulate capital will be identical in both regions so that the initial share of capital owned by the north, s_K , is permanent and entirely determined the initial distribution of capital ownership between the two regions.

2) a second solution is to assume that capital is immobile. Presumably, this would be the case if we focus on the interpretation of capital being human (coupled with immobile agents). In this case, the location of production, s_n , is pinned down by capital ownership: $s_n = s_K$.

Because the case of capital mobility eliminates the possibility of a “catastrophe” similar to the new economic geography model and from that point of view is simpler, we start with it.

2 Perfect capital mobility

With perfect capital mobility, operating profits have to be the same in both regions which also implies that the value of capital has to be the same in both regions. Hence, $\pi = \pi^*$ and $q = q^* = 1$. This, together with the assumption of constant returns to scale, and the assumption of global spillovers (implying that the cost of innovation is the same in both regions) means that the two regions will accumulate capital at the same constant rate so that any initial distribution of capital is stable and no “catastrophic” scenario can unfold (see *Martin and Ottaviano, 1999*). The reason is that the return to capital accu-

mulation is the same in both regions and therefore the incentive to accumulate are identical in the two regions when capital is perfectly mobile.

With capital mobility, an obvious question arises: where does capital locate? Capital owned in one region can be located elsewhere. We have that $n+n^* = K+K^*$, but n (n^*) does not need to be equal to K (K^*). Again, the arbitrage condition, which implies that profits across regions need to be equal for firms to be indifferent between the two locations, pins down the equilibrium location of firms. Using equation (3), we get that there is no more incentive for relocation when the following relation between s_n and s_E is verified:

$$s_n = 1/2 + \frac{(1+\phi)(2s_E - 1)}{2(1-\phi)} \quad (9)$$

This is an example of the “home market” effect: firms locate in the large market (the market with the highest share of expenditure) because of increasing returns in the monopolistic competition sector. Using equation (8), we get the equilibrium relation between the share of firms located in the north (s_n) and the share of capital owned by the north (s_K):

$$s_n = 1/2 + \frac{\rho(1+\phi)(2s_K - 1)}{2(1-\phi)(2L + \rho)} \quad 0 \leq s_n \leq 1 \quad (10)$$

Note also that if the initial distribution of capital in the north is such that $s_K > 1/2$, then more firms will be located in the north than in the south: $s_n > 1/2$. An increase in the share of capital in the north, s_K , induces relocation to the north as it increases expenditure and market size there. Note also that lower transaction costs (higher ϕ) will reinforce the home market effect, implying that an unequal distribution of capital ownership will translate in an even more unequal distribution of firms. Remember that, because of free capital mobility, the growth rate of capital is the same in both regions so that s_K is entirely determined by the initial exogenous distribution of capital and is constant through time. It also implies that the share of income and expenditures in the north does not depend on the location of firms. This eliminates a potential linkage that will prove crucial when we relax the assumption of perfect capital mobility. Hence, the equilibrium described by (10) is always stable. In particular, the symmetric equilibrium where $s_n = s_K = 1/2$, is always stable for any level of transaction costs on trade in goods. To see this, one can analyze the effect of an exogenous increase in s_n , by a small amount and

check the impact of this perturbation on the ratio of profits in the north to profits in the south. That is, ask the question whether an increase in geographic concentration in the north decreases or increases the incentive to relocate in the north. The symmetric equilibrium is stable, if and only if $\partial(\pi/\pi^*)/\partial s_n$ is negative. Indeed this is the case for all positive levels of transaction costs since, evaluated at the equilibrium geography:

$$\frac{\partial(\pi/\pi^*)}{\partial s_n} = -\frac{(1-\phi)^2}{(1+\phi)^2} \frac{1}{s_E(1-s_E)} < 0$$

Evaluated at the equilibrium given by (10), an exogenous increase in the share of firms located in the north always decreases relative profits there, so that it leads firms to go back to the south. The location equilibrium determined in (10) is always stable. The reason is that when more firms locate in the north, this increases competition there (and decreases it in the south). We will come back to this **local competition effect**.

There are several interesting questions that we can analyze in this framework. First, in this model with growth meaning with constant creation of new firms, do we have relocation of firms towards the north or towards the south? In economic geography models without growth, industrial concentration implies that firms are destroyed in the south and relocated in the north. Here, the relocation story is richer. To see what is the direction of relocation we need to look at the difference between the share of capital owned by the north and the share of firms located in north. This is given by:

$$s_K - s_n = \frac{[L(1-\phi) - \rho\phi][2s_K - 1]}{(1-\phi)(2L + \rho)} \quad (11)$$

In the symmetric equilibrium, where both regions are endowed originally with the same amount of capital there is no relocation of course. If the initial distribution of capital is such that $s_K > 1/2$, so that the north is richer than the south, then the direction of the capital flows is ambiguous and depends on the sign of the following expression: $L(1-\phi) - \rho\phi$. If this expression is positive, then $s_K > s_n$ so that some of the capital owned by the north relocates to the south. The reason of the ambiguity of the direction of location is that two opposite effects are present: a **local competition effect** that makes the poor capital region attractive because firms (each using one unit of capital) installed there face less competition; a **capital income effect** that makes the rich region attractive because due to its high level of income and expenditure the rich region represents a larger

market. The first effect dominates when transaction costs are high (ϕ is low) because then, the **local competition effect** is strong as the southern market is protected from northern competition. Also if the number of workers is high, the share of income that comes from capital is low relative to labor income so that the **capital income effect** is small. On the contrary, when the rate of time preference is high, the return to capital is high also which makes the capital rich region more attractive. There is a threshold level of transaction costs that determines the direction of capital flows. It is given by:

$$\phi^{CP} = \frac{L}{L + \rho} \quad (12)$$

When transaction costs are below this level, relocation takes place towards the south and vice-versa. The reason why we attach CP (for Core-Periphery) to this threshold will become clear later when we analyze a version of the Core-Periphery model, as we will see that this threshold value comes back again and again.

An interesting feature here is that concentration in the north (s_K and $s_n > 1/2$), is compatible with relocation of firms from north to south ($s_K < s_n$) when $\phi < \phi^{CP}$. This comes from the introduction of growth and the fact that a larger number of newly created firms are created and owned by the north than by the south; the competition effect then kicks in and tends to drive s_n below s_K .

A second interesting question we can ask is the following: when is that that when all capital is owned by the north, all firms are also located in the north and no relocation occurs towards the south? That is, when is it that when $s_K = 1$, then $s_n = 1$? We can already think of this situation as a Core-Periphery one. Using equation (10), it is easy to see that this will be the case when $\phi > \phi^{CP}$ as defined in equation (12). Hence, with capital mobility, when transaction costs are low enough the Core-Periphery is a stable equilibrium in the sense that if all the capital is owned by the north, all firms are also located in the north.

3 No capital mobility

Restricting capital mobility (together with the assumptions of labor mobility) has two implications. First, the number of firms and the number of units of capital owned in a region are identical: $s_n = s_K$. Second, because the arbitrage condition of the previous section does not hold, profits may be different in the two regions. This in turn implies that, contrary to the previous section, the two regions may not have the same incentive to accumulate capital so that the initial ownership of capital does not need to be permanent. This means that the analysis will be quite different from the previous section. We will ask the following questions which are the usual ones in the “new economic geography” models. Starting from an equal distribution of capital, the symmetric equilibrium, we will determine under which circumstances it remains a stable equilibrium. Then we will look at the Core-Periphery equilibrium and again ask when this equilibrium is stable.

3.1 Stability of the symmetric equilibrium

We first consider interior steady states where both nations are investing, so $q = 1$ and $q^* = 1$. Using (3) and (6) in (7), $q = q^* = 1$ and imposing $s_n = s_K$ we get:

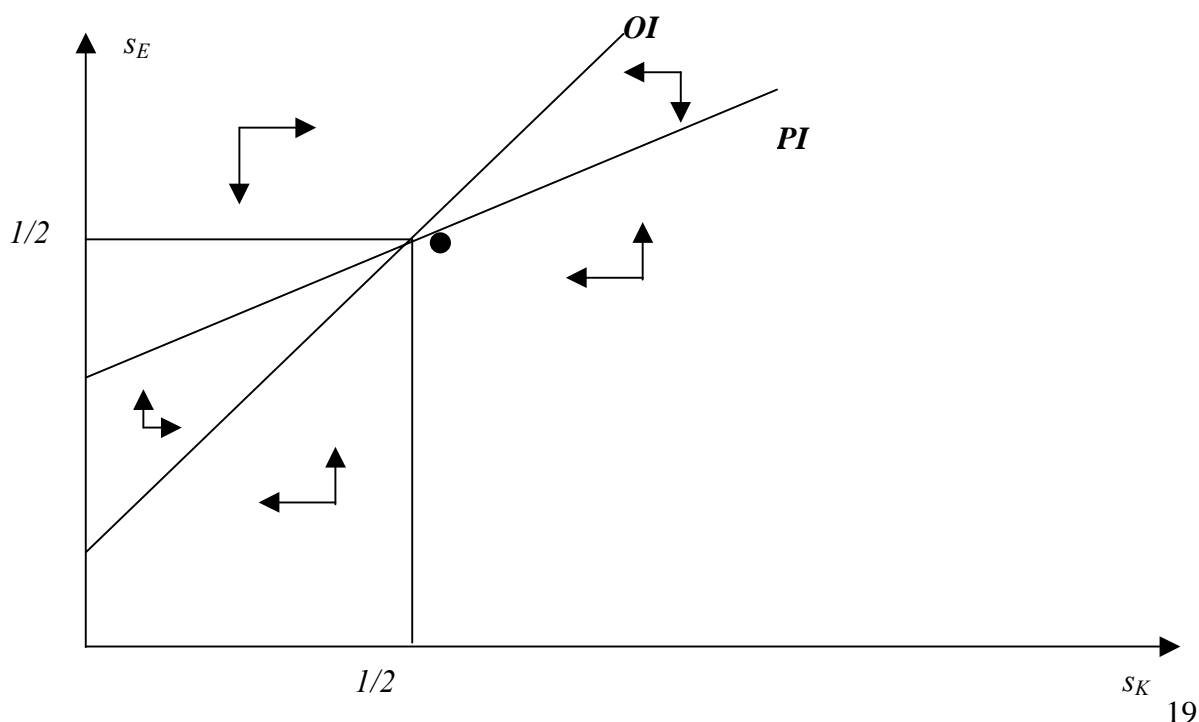
$$s_K = 1/2 + \frac{(1 + \phi)(2s_E - 1)}{2(1 - \phi)} \quad (13)$$

Note that this is the same relations as the one in (9) except that it now determines the location of capital ownership and not only the location of production. Together with equation (8), this defines a second positive relation between s_E and s_K . The intuition is that an increase in the northern share of expenditure raises demand for locally produced manufactured goods more than for goods produced in the south. This relative increase in northern demand increases profits in the north and therefore the marginal value of an extra unit of capital. In other words, the numerator of Tobin’s q increases in the north. Hence, this raises the incentive to innovate there and the north indeed increases its share of capital s_K . The intuition is therefore very close to the “home market effect” except that it influences here the location of capital accumulation. Together with the optimal saving relation of (8), it is easy to check that the symmetric solution $s_E = s_K = 1/2$ is always an equilibrium, in particular it is an equilibrium for all levels of transaction costs. The symmetric equilibrium is the unique equilibrium for which both regions accumulate

capital ($q = q^* = 1$). However, the fact that there are two positive equilibrium relations between s_E and s_K , the share of expenditures and the share of capital in the north, should warn us that the symmetric equilibrium may not be stable. Indeed, in this model a 'circular causality' specific to the presence of growth and capital immobility tends to destabilize the symmetric equilibrium. It can be related to the well-known demand-linked cycle in which production shifting leads to expenditure shifting and *vice versa*. The particular variant present here is based on the mechanism first introduced by *Baldwin* (1999).

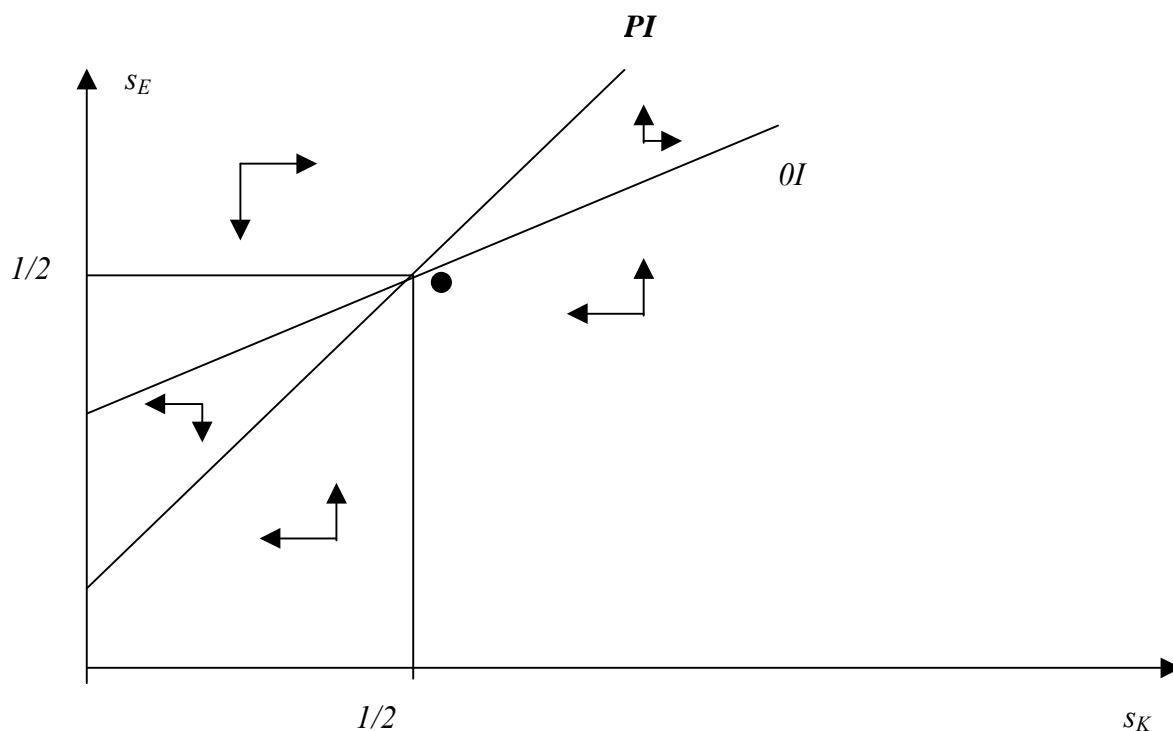
There are several ways to study the symmetric equilibrium's stability. We can first graph the two equilibrium relations between s_E and s_K , the "Permanent Income" relation (call it PI) given by equation (8) and the "Optimal Investment" relation (call it OI) given by equation (13). In the case where the slope of the PI relation is less than the OI relation we get graph 1. At the right of the permanent income relation, s_E , the share of expenditures in the north, is too low given the high share of capital owned by the north (agents do not consume enough). The opposite is true at the left of the PI relation. At the right of the optimal investment relation, s_K , the share of capital in the north, is too high given the low level of s_E , the share of expenditures in the north (agents invest too much). The opposite is true is at the left of the OI relation. This graphical analysis suggests that in this case the symmetric equilibrium is stable.

Graph 1: The northern shares of expenditure and capital: the stable case



In the case where the slope of the PI relation is steeper than the OI, then the same reasoning leads to graph 2. This suggests that in this case, the symmetric equilibrium is unstable.

Graph 2: The northern shares of expenditure and capital: the unstable case



According to this graphical analysis, the transaction cost below which the symmetric equilibrium becomes unstable is exactly the one for which the slope of the PI curve equals the slope of OI curve. This turns to be the threshold level ϕ^{CP} given by equation (12).

To gain more intuition on this result, we can also study the symmetric equilibrium's stability in a different and more rigorous way. We can analyze the effect of an exogenous increase s_K , by a small amount and check the impact of this perturbation on Tobin's q , allowing expenditure shares to adjust according to (8). The symmetric equilibrium is stable, if and only if $\partial q / \partial s_K$ is negative: in this case, an increase in the share of northern capital lowers Tobin's q in the north (and therefore the incentive to innovate) and raises it in the south (by symmetry $\partial q / \partial s_K$ and $\partial q^* / \partial s_K$ have opposite signs). Thus when $\partial q / \partial s_K < 0$, the perturbation generates self-correcting forces in the sense that the

incentive to accumulate more capital in the north falls and increases in the south. If the derivative is positive, the increase in the share of capital in the north reinforces the incentive to accumulate more capital in the north: the symmetric equilibrium is unstable in this case. Differentiating the definition of q with respect to s_K , we have:

$$\left(\frac{\partial q/q}{\partial s_K}\right)_{s_K=1/2} = 2\left(\frac{1-\phi}{1+\phi}\right)\left(\frac{\partial s_E}{\partial s_K}\right)_{s_K=1/2} - 2\frac{(1-\phi)^2}{(1+\phi)^2} \quad (14)$$

This expression illustrates the two forces affecting stability. The first term is positive, so it represents the destabilizing force, namely the demand-linked one, which can also be interpreted as a **capital income effect** as an increase in the capital share of the north increases its capital income and its expenditure share. This effect was absent in the case of capital mobility. The negative second term reflects the stabilizing **local-competition effect** which was the only present in the capital mobility case. Clearly, reducing trade costs (an increase in ϕ) erodes the stabilizing force more quickly than it erodes the destabilizing demand-linkage.

Using (8) to find $\partial s_E/\partial s_K = \rho/[2L+\rho]$, the critical level of ϕ at which the symmetric equilibrium becomes unstable is defined by the point where (14) switches sign. It is easy to check that again this critical level is given by ϕ^{CP} of equation (12). The appendix uses standard stability tests involving eigenvalues and derives the same result.

When trade costs are high the symmetric equilibrium is stable and gradually reducing trade costs produces standard, static effects – more trade, lower prices for imported goods, and higher welfare. There is, however, no impact on industrial location, so during an initial phase, the global distribution of industry appears unaffected. As trade freedom moves beyond ϕ^{CP} , however, the equilibrium enters a qualitatively distinct phase. The symmetric distribution of industry becomes unstable, and northern and southern industrial structures begin to diverge; to be concrete, assume industry agglomerates in the north. Since s_K cannot jump, crossing ϕ^{CP} triggers transitional dynamics in which northern industrial output and investment rise and southern industrial output and investment fall. Moreover, in a very well defined sense, the south would appear to be in the midst of a 'vicious' cycle. The demand linkages would have southern firms lowering employment and abstaining from investment, because southern wealth is falling, and southern wealth is falling since southern firms are failing to invest. By the same logic, the north would appear to be in the midst of a 'virtuous' cycle.

3.2 *The Core-Periphery equilibrium*

In addition to the symmetric equilibrium, a core-periphery outcome ($s_K = 0$ or 1, but we will focus only on the second one where the north gets the core) can also exist. For $s_K = 1$ to be an equilibrium, it must be that $q = 1$ and $q^* < 1$ for this distribution of capital ownership. Continuous accumulation is profitable in the north since $v = F$, but $v^* < F^*$ so no southern agent would choose to setup a new firm. Defining the Core-Periphery equilibrium this way, it implies that it is stable whenever it exists. Using (3), (6) and (7), (8), q^* with $s_K = 1$ simplifies to:

$$q^* = \frac{(1 + \phi^2)L + \phi^2 \rho}{(2L + \rho)\phi} \quad (15)$$

If q^* is less than 1 when $s_K = 1$, then the Core-Periphery equilibrium exists and is stable as there is no incentive for the south to innovate in this case. The threshold ϕ that solves $q^* = 1$ defines the starting point of the core-periphery set. Again, this threshold is ϕ^{CP} of equation (12). This implies that at the level of the transaction costs for which the symmetric equilibrium becomes unstable, the Core-Periphery becomes a stable equilibrium.

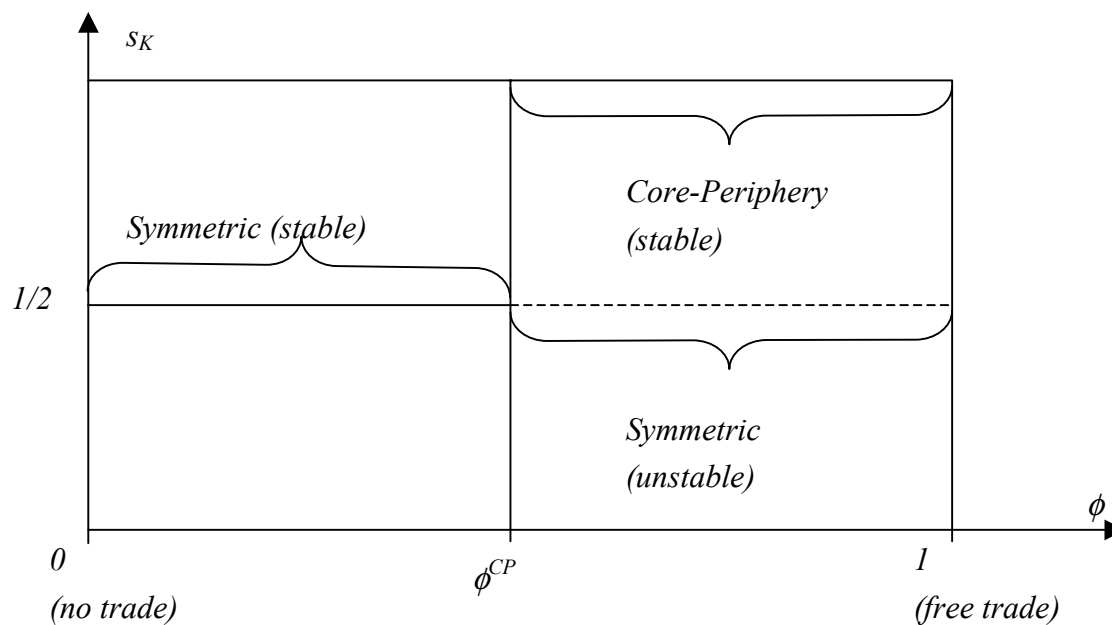
When transaction costs are high enough, the Core-Periphery equilibrium is not a stable equilibrium: in this case the south would have an incentive to innovate because the profits in the south are high enough. This is because even though the southern market is small in this case (it has no capital income in the Core-Periphery equilibrium), it is protected from northern competition thanks to high transaction costs. When transaction costs are low enough, this protection diminishes and the fact that the market in the south is small becomes more important: in this case, above the threshold ϕ^{CP} , it becomes non profitable to operate a firm in the south.

Using $s_K = 1$, the remaining aspects of the core-periphery steady state are simple to calculate. In particular, since $s_K = 1$, $q = 1$, and $q^* < 1$, we have that no labor is used in the innovation or manufacturing sectors in the south and all innovation is made in the north.

Note that the core-periphery outcome ($s_K = 1$) is reached only asymptotically. This is because the stock of capital in the south does not depreciate and once the level of ϕ^{CP} is crossed, stays constant, whereas the stock of capital in the north keeps growing at rate g .

Graph 3 summarizes the model's stability properties in a diagram with ϕ and s_K on the axes:

Graph 3: Stability properties of equilibria



Following the tradition of the “new economic geography” we have analyzed here the existence and stability conditions of the symmetric and Core-Periphery equilibria. In this simple model we can go further and analyze what would happen if we started from a situation in which the north had more capital than the south ($1/2 < s_K < 1$). It can be checked, using equations (3), (6) and (7) that in this case $q < 1$ (and $q^* > 1$) if:

$$(1 - s_K)(s_K - 1/2)(1 - \phi)[-L(1 - \phi) + \rho\phi] < 0$$

that is if $\phi < \phi^{CP}$. Hence, in this case, the north would not innovate (the large stock of capital implies a high degree of competition) and the south would innovate. Hence, if we start from such an interior asymmetric equilibrium then one would converge back to the symmetric equilibrium as long as transaction costs are high enough. If $\phi < \phi^{CP}$, then the economy converges to the core-periphery equilibrium.

Concluding remarks:

If we compare the case of perfect capital mobility and no capital mobility, we can get the following conclusions:

- when $\phi < \phi^{CP}$, the absence of capital mobility leads to convergence between the two regions: if one region starts with more capital than the other then, the two regions converge to the symmetric equilibrium. On the contrary, with capital mobility, any initial distribution of capital ownership becomes permanent. However, some of the firms owned by the north will relocate and produce in the south. This will produce some sort of convergence in terms of GDP but not in terms of GNP.
- when $\phi > \phi^{CP}$, the absence of capital mobility leads to divergence between the two regions: asymptotically, whatever the initial distribution of capital (with some advantage to the north though), all the capital is accumulated and owned by the north. With capital mobility, as long as all the capital is not entirely in the north, some firms will still produce in the south. However, some of the southern capital will be delocated in the north.

Hence, in the case of mobile capital (physical or tradable innovations such as patents), the key parameter for regional income distribution is the “exogenous” initial distribution of capital. In the case of immobile (human) capital, the key parameter is the level of transaction costs. The regional distribution of capital affects the long term regional income distribution “only” to the extent that it determines which region becomes the core, through a small initial advantage in capital endowments for example. To simplify matters we have used a model where only one type of capital exists. To make it more realistic, in particular for the European case, it would be interesting to extend it and take into account the different natures of capital so that part of the capital is mobile and part is not.

We have analyzed the polar cases of no capital mobility and full capital mobility and have shown that they are very different. A natural question is whether our analysis can be extended to the intermediate case where some finite transaction costs exist on capital movements. In short, the answer is that with small transaction costs the analysis is identical to the case of no capital mobility. To see this, remember that the innovation sector is perfectly competitive so that in equilibrium if a region has an innovation sector the

marginal cost of an innovation is equal to its marginal value, that is the value of the discounted profits of a monopolistic firm: there are no profits to be made in equilibrium in the innovation sector. If a firm relocates its production from one region to another it will necessarily make a loss: the reason is that the value of discounted profits in the region where it may relocate is pinned down to F , the common marginal cost, due to free entry in that region. Hence, paying the transaction, however small, will imply a loss. Hence, if we restrict ourselves to the case of positive transaction costs on capital movements, the equilibrium with imperfect capital mobility is essentially identical to the case of no capital mobility. Does this imply that the case of perfect capital mobility, which is the only one in which actual capital movements can take place in this framework, is too special to be relevant? Remember that the reason why transaction costs on capital movements eliminate any trade in capital is that capital production is perfectly competitive and that capital is a homogenous good. Trade in capital could appear to be destabilizing despite the presence of such transaction costs if these assumptions are relaxed, for example along the lines of the model of *Martin and Rey (1999)*.

III THE CASE WITH LOCALIZED SPILLOVERS: GEOGRAPHY MATTERS FOR GROWTH (AND VICE VERSA)

In the previous section, we showed that growth could dramatically alter economic geography in the sense that the process of accumulation of capital could lead to catastrophic agglomeration. However, geography had no impact on growth. This was due to the fact that we assumed global spillovers: the learning curve, which as in any endogenous growth model, was at the origin of sustained growth, was global in the sense that the north and the south would learn equally from an innovation made in any region. In this section, we analyze how localized spillovers give a role in growth to the geography of production and innovation activities.

The presence of localized spillovers has been well documented in the empirical literature. Studies by *Jacobs* (1969) and more recently by *Jaffe et al.* (1993), *Coe and Helpman* (1995 and 1997), *Ciccone and Hall* (1996) provide strong evidence that technology spillovers are neither global nor entirely localized. The diffusion of knowledge across regions and countries does exist but diminishes strongly with physical distance which confirms the role that social interactions between individuals, dependent on spatial proximity, have in such diffusion.

Introducing localized technological spillovers requires a minor modification to one of the assumptions made in the previous section. Equation (1) that described the innovation sector assumed global spillovers in the sense that the marginal cost of an innovation, identical in both regions, was: $F = w a_I = I/K^W$, so that it was decreasing in the total stock of existing capital. Hence, this was similar to the assumption of *Grossman and Helpman* (1991) that past inventions (the stock of K) had a positive effect on the productivity of R&D. Now suppose that these spillovers are localized in the sense that the cost of R&D in one region also depends on the location of firms (capital). Hence, the northern cost of innovation depends more on the number of firms located in the north than in the south so that equation (1) becomes (taking into account that the wage rate is equal to (1):

$$F = a_I \quad ; \quad a_I \equiv \frac{1}{K^W A} \quad ; \quad A \equiv s_n + \lambda(1 - s_n) \quad (16)$$

where λ measures the degree of localization of technology spillovers. The lower λ the more localized these spillovers are or put it differently, the higher the transaction costs

on the mobility of ideas, technologies, innovations... If $\lambda=1$, we go back to the case of global spillovers. The cost function of the innovation sector in the south is isomorphic.

Again the case of perfect capital mobility is easier than the case without capital mobility. Hence, we will start with the former following some of the analysis of *Martin and Ottaviano* (1999) and then describe the model without capital mobility following *Baldwin, Martin and Ottaviano* (2000).

1 The case of perfect capital mobility

There are three endogenous variables that we are interested in: g , the growth rate which is common to both regions; s_n , the share of firms that are producing in the north; and s_E , the share of expenditure in the north which also can be thought as a measure of income inequality between north and south. Remember that with perfect capital mobility, s_K the share of capital in the north is given by the initial distribution of capital as the stocks of capital in both regions are growing at the same rate. We want to find the different equilibrium relations between these three endogenous variables.

Due to localized spillovers, it is less costly to innovate in the region with the highest number of firms (which represent also capital or innovations). This implies that, because of perfect capital mobility, all the innovation will take place in the region with a higher number of firms. Remember that due to perfect competition the value of an innovation is equal to its marginal cost. The shares of firms are perfectly tradable across regions (perfect capital mobility) so the value of capital (or firms) cannot differ from one region to another and no innovation will take place in the south. But the south will be able to simply buy (without transaction costs) innovations or capital produced in the north. Hence, in the case when $s_K > 1/2$, that is when the initial stock of capital is higher in the north than in the south, we know from the previous section that this will imply that more firms will be located in the north ($s_n > 1/2$) so that all innovation will take place in the north. In this case the world labor market equilibrium will be given by:

$$2L = \frac{g}{s_n + \lambda(1-s_n)} + \alpha \frac{\sigma-1}{\sigma} E^W + (1-\alpha)E^W \quad (17)$$

Remember also that world expenditure is given by: $E^W = 2L + \rho FK^W$. The value and marginal cost of capital is given by F in (16). The reason again is that the R&D sector is

perfectly competitive so that in equilibrium the marginal cost of an invention (F) must be equal to its value v . Using this and equation (17), we get the growth rate of capital g as a function of s_n , our first equilibrium relation:

$$g = \frac{2\alpha L}{\sigma} [s_n + \lambda(1 - s_n)] - \rho \frac{\sigma - \alpha}{\sigma} \quad 1/2 < s_n \leq 1 \quad (18)$$

Compared to the growth rate derived in the previous section, this one differs because of the presence of localized spillovers: spatial concentration of firms (a higher s_n) implies a lower cost of innovation and therefore a higher growth rate. Note also that for a given geography of production (a given s_n), less localized spillovers (a higher λ) also implies a lower cost of innovation in the north (as the innovation sector in the north benefits more from spillovers of firms producing in the south) and a higher growth rate.

The arbitrage condition consistent with the assumption of perfect capital mobility requires profits to be equalized in the two locations so that $\pi = \pi^* = \alpha E^W / (\sigma K^W)$. This gives the same relation between s_n and s_E as in the previous section (equation 9), which we called the home market effect and which is the second equilibrium relation we will use:

$$s_n = 1/2 + \frac{(1 + \phi)(2s_E - 1)}{2(1 - \phi)} \quad (19)$$

To find the third equilibrium relation, one between s_E and g , remember that due to intertemporal optimization, $E = L + \rho v K$ where v is the value of capital which itself is equal to the discounted value of future profits. Using these relations, it is easy to get the last equilibrium relation:

$$s_E = 1/2 + \frac{\alpha \rho (2s_K - 1)}{2\sigma(g + \rho)} \quad (20)$$

Note that as long as $s_K > 1/2$, that is, as long as the north is initially better endowed in capital than the south, then $s_E > 1/2$, that is, income per capita is higher in the north than in the south. Note also that income inequality is decreasing in the growth rate. The reason is that the value of capital is lower with higher growth because of more future competition due to faster entry of new firms. If the north is relatively rich in capital

($s_K > 1/2$), the level of capital income declines more in the north than in the south, leading to decreasing income inequality.

The equilibrium characterized by these three relations is stable for the same reasons as in the case of perfect capital mobility of the previous section. Capital mobility allows southerners to save and invest buying capital accumulated in the north (in the form of patents or shares). Hence, the lack of an innovation sector does not prevent the south from accumulating capital: the initial inequality in wealth does not lead to self-sustaining divergence. No “circular causation” mechanism which would lead to a core-periphery pattern, as in the “new geography” models of the type of *Krugman* (1991), will occur.

Using equations (17), (18) and (19), the equilibrium is the solution to a quadratic equation given in appendix I. The equilibrium growth rate follows from equation (17). One can find the transaction cost such that relocation goes from north to south in the case where $s_K > 1/2$ (which implies also that $s_n > 1/2$). $s_K > s_n$ if:

$$\phi < \frac{\lambda L(1-s_K) + Ls_K}{\lambda L(1-s_K) + Ls_K + \rho}$$

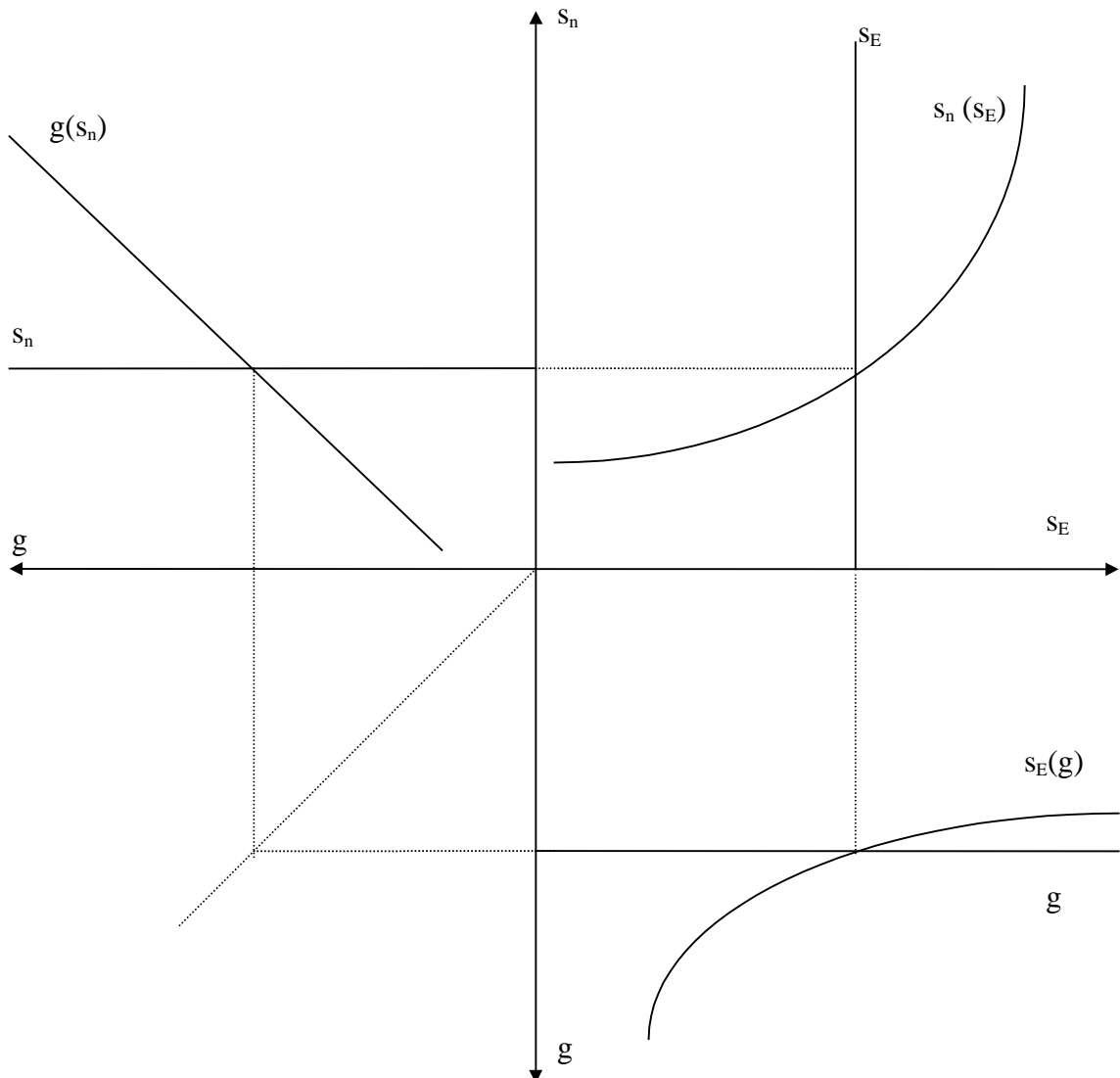
Note that when all the capital is owned by the north ($s_K = 1$), then the threshold level of transaction cost is again ϕ^{CP} given in the previous section. Note also that in the less extreme case where $s_K < 1$, less localized spillovers imply, everything else constant, that relocation will take place towards the south. The reason is that less localized spillovers imply a lower cost of innovation in the north, and therefore a lower value of capital of which the north is better endowed with. Hence, less localized spillovers generate, for a given distribution of capital, a more equal distribution of incomes and expenditures and therefore attract firms in the south. Another way to say it is that less localized spillovers weaken the **capital income effect** which was described in the previous section.

One could analyze the properties of this equilibrium by analyzing the equilibrium location s_n given in appendix. However, it is more revealing to use a graphical analysis.

Equation (19) provides a positive relation between s_n and s_E , the well known “home market” effect. On graph 4, this relation is given by the curve $s_n(s_E)$ in the NE quadrant. Equation (18) provides a positive relation between g and s_n . This is the localized spill-

overs effect: when industrial agglomeration increases in the region where the innovation sector is located, the cost of innovation decreases and the growth rate increases. This relation is given by the line $g(s_n)$ in the NW quadrant. Finally, equation (20) provides a negative relation between s_E and g . This is the “competition” effect: the monopoly prof-

Graph 4: Equilibrium growth, agglomeration and income inequality



its of existing firms decrease as more firms are created. As the north is more dependent on this capital income, the northern share of income and, therefore, of expenditures decreases. This relation is described by the curve $s_E(g)$ in the SE quadrant.

We will use later on this graphical tool to analyze various public policies. However, it is easy to see already that for example an increase in regional inequality in capital endowments s_K shifts to the right the $s_E(g)$ in the SE quadrant. The impact is therefore an increase in income inequality and an increase in spatial inequality in the sense that s_n increases. However, because the economic geography becomes less dispersed, the growth rate g is higher.

It is also easy to analyze the impact of lower transaction costs on goods (higher ϕ). For a given income disparity, it increases spatial inequality (see equation (19) so that the schedule $s_n(s_E)$ shifts up in the NE quadrant. This in turn increases the growth rate which leads to lower income inequality, an effect that mitigates the initial impact on spatial inequality. Overall even though spatial inequality has increased, the growth rate has increased and nominal income disparities have decreased.

It is also interesting to analyze the effects of an increase in λ , that is less localized technology spillovers. This can be interpreted as lowering transaction costs on ideas and information such as telecommunication costs. This shifts the $g(s_n)$ to the left in the NW quadrant so that growth increases for a given geography of production. This lowers income disparities between the two regions as monopolistic profits are eroded by the entry of new firms. This in turn brings a decrease in spatial inequality on the geography of production as s_n decreases.

Welfare implications

The structure of the model is simple enough so that it is fairly easy, at least compared to the other models, to present some welfare implications. One question we can ask is whether the concentration of economic activities, generated by market forces, is too small or too important from a welfare point of view. As noted in *Martin and Ottaviano (1999)*, on top of the usual inefficiency present in monopolistic competition models, there are several market failures. A standard distortion, which is not linked to economic geography, is due to the presence of inter-temporal technological spillovers. Because current research diminishes the cost of future research, the market equilibrium will display too little research activity in equilibrium (*Grossman and Helpman, 1991*). Two distortions, which are directly linked to economic geography and are therefore of more interest to us also exist. First, when investors choose their location they do not take into

account the impact of their decision on the cost of innovation in the north where the innovation sector is located. Localized positive spillovers are not internalized in the location decision and from that point of view the “market” economic geography will display too little spatial concentration. Second the location decision also has an impact on the welfare of immobile consumers which is not internalized by investors. This happens for two reasons. On the one hand an increase in spatial concentration affects negatively the cost and therefore the value of existing capital so that the wealth of capital owners in both regions decreases. This affects more the north than the south. On the other hand, when spatial concentration in the north increases, consumers in the north gain because of the lower transport costs they incur. Symmetrically, consumers in the south loose. V and V^* , the indirect individual utilities of north and south respectively, as a function of the spatial concentration s_n and of the growth rate g are given by:

$$V = C + \frac{1}{\rho} \ln \left(1 + \frac{\rho s_K}{L s_n} \right) + \frac{\alpha}{\rho(\sigma-1)} \ln [s_n + \phi(1-s_n)] + \frac{\alpha g}{\rho^2(\sigma-1)} \quad (21)$$

$$V^* = C + \frac{1}{\rho} \ln \left[1 + \frac{\rho(1-s_K)}{L s_n} \right] + \frac{\alpha}{\rho(\sigma-1)} \ln [1-s_n + \phi s_n] + \frac{\alpha g}{\rho^2(\sigma-1)}$$

where C is a constant. We can analyze how a change in the spatial concentration s_n affects welfare in both regions:

$$\frac{\partial V}{\partial s_n} = \frac{2L\alpha^2(1-\lambda)}{\rho^2\sigma(\sigma-1)} - \frac{s_K}{L s_n^2 + \rho s_K s_n} + \frac{\alpha}{\rho(\sigma-1)} \frac{1-\phi}{s_n + \phi(1-s_n)} \quad (22)$$

$$\frac{\partial V^*}{\partial s_n} = \frac{2L\alpha^2(1-\lambda)}{\rho^2\sigma(\sigma-1)} - \frac{1-s_K}{L s_n^2 + \rho(1-s_K)s_n} - \frac{\alpha}{\rho(\sigma-1)} \frac{1-\phi}{1-s_n + \phi s_n}$$

There are three welfare effects of a change in spatial concentration. The first term is identically positive in both regions: an increase in spatial concentration increases growth because, through localized spillovers, it decreases the cost of innovation. The second term is negative in both regions: the decrease in the cost of innovation also diminishes the value of existing firms and therefore diminishes the wealth of capital owners. Because the north owns more capital than the south, this negative effect is larger in the north than in the south. Finally, the last term represents the welfare impact of higher

concentration on transaction costs. This welfare effect is positive in the north and negative in the south.

To analyze whether the market geography displays too much or too little concentration in the north implies to evaluate these two equations at the market equilibrium. It can be checked that as long as λ is sufficiently small (technological spillovers are sufficiently localized), the effect of an increase in spatial concentration is always positive on the north. It is interesting that the north will gain less by an increase in geographical concentration if it owns a larger share of the capital. Another way to say this is that capital owners may lose from geographical concentration in the north. Geographical concentration in the north may improve welfare in the south. This is in stark contrast with standard economic geography models without growth where the southerners always lose following an increase in concentration in the north. Here the positive effect on growth may more than compensate the negative impact of concentration on transaction costs and on wealth. This will be so if λ is sufficiently small (technological spillovers are sufficiently localized), and if transaction costs are low enough.

2 The case without capital mobility: the possibility of take-off and agglomeration

Here we follow the analysis of *Baldwin, Martin and Ottaviano* (2000) and simply sketch the nature of the solution. The model is identical to the one described in the previous section except for the introduction of localized spillovers as described in the previous section. This has several consequences: the geography of production has now an impact on the cost of innovation so that as in the previous section, the global growth rate is affected by geography. The value of capital, which can differ in the two regions as capital mobility is absent, is itself affected by geography because the innovation sector is perfectly competitive. Hence, the marginal cost of capital and innovation is equal to its value. In turn, this affects wealth and expenditures in the two regions so that profits will depend on geography in this way too. This implies that the two relations between the share of capital in the north (s_K) and the share of expenditures in the north (s_E) are going to be much more complex than in the case without localized spillovers. The optimal savings/expenditure function derived from intertemporal utility maximization, which we interpreted as a permanent income relation in the previous section (equation 8) becomes:

$$s_E = 1/2 + \frac{\rho\lambda(2s_K - 1)}{2\{2LAA^* + \rho[A(1-s_K) + A^*s_K]\}} \quad (23)$$

where A is given in (16) and A^* is the symmetric. The permanent income relation is such that s_E is always increasing in s_K : an increase in the northern share of capital increases the northern share of expenditures. When we consider interior steady states where both nations are investing (innovating), so that $q = 1$ and $q^* = 1$, the second relation between s_E and s_K , which we called the optimal investment one (see equation 13 in the previous section), becomes, in the presence of localized spillovers:

$$s_E = 1/2 + \frac{(2s_K - 1)(\lambda + \lambda\phi^2 - 2\phi)}{2(1 - \phi^2)[A(1-s_K) + A^*s_K]} \quad (24)$$

Note of course that $s_E = s_K = 1/2$, the symmetric equilibrium is a solution to the two equilibrium relations (23) and (24). Two other solutions to this system may exist which are given by:

$$s_K = 1/2 \pm \frac{1}{2} \sqrt{\left(\frac{1+\lambda}{1-\lambda}\right)\left(\frac{1+\lambda\Lambda}{1-\lambda\Lambda}\right)} \quad ; \quad \Lambda \equiv \left[1 - \frac{2\rho\phi(1-\lambda\phi)}{L(\lambda + \lambda\phi^2 - 2\phi)}\right]^{-1} \quad (25)$$

Both s_E and s_K converge to $1/2$ either as λ approaches 1 or as ϕ approaches the value:

$$\phi^{cat} = \frac{L(1+\lambda) + \rho - \sqrt{(1-\lambda^2)[L(1+\lambda) + \rho]^2 + \lambda^2\rho^2}}{\lambda[L(1+\lambda) + 2\rho]} \quad (26)$$

from above. For levels of ϕ below ϕ^{cat} , these two solutions are imaginary. In addition, for levels of ϕ above another critical value:

$$\phi^{CP'} = \frac{2L + \rho - \sqrt{(2L + \rho)^2 - 4\lambda^2 L(L + \rho)}}{2\lambda(L + \rho)} \quad (27)$$

one of the solutions is negative and the other one is above unity. Since both violate boundary conditions for s_K , the corresponding steady state outcomes are the corner solutions $s_K = 0$ and $s_K = 1$. Note that for $\lambda = 1$, $\phi^{cat} = \phi^{CP'} = \phi^{CP}$ as defined in the previous section. It is possible to show that $\phi^{cat} < \phi^{CP'} < \phi^{CP}$. Hence, localized spillovers make the catastrophic agglomeration possible for higher transaction costs. As in the case without

localized spillovers, we can study the stability of the Core-Periphery equilibrium by analyzing the value of q^* at $s_K=1$:

$$q^* \Big|_{s_K=1} = \frac{\lambda[L(1+\phi^2) + \rho\phi^2]}{\phi(2L + \rho)} \quad (28)$$

When $q^* < 1$, we know that then the Core-Periphery equilibrium is stable as the south has no incentive to innovate any more. It is easy to check that $q^* < 1$, when $\phi > \phi^{CP}$.

The stability of the symmetric equilibrium can be studied following the same method as in the case without localized spillovers. We turn to signing $\partial q / \partial s_K$ evaluated at the symmetric equilibrium. Differentiating the definition of q with respect to s_K , we have:

$$\left(\frac{\partial \bar{q} / \bar{q}}{\partial s_K} \right) \Big|_{s_K=1/2} = 2 \left(\frac{1-\phi}{1+\phi} \right) \left(\frac{ds_E}{ds_K} \right) \Big|_{s_K=1/2} + \frac{4}{1+\lambda} \frac{1+\phi^2}{(1+\phi)^2} \left[1 - \lambda - \frac{(1-\phi)^2}{1+\phi^2} \right] \quad (29)$$

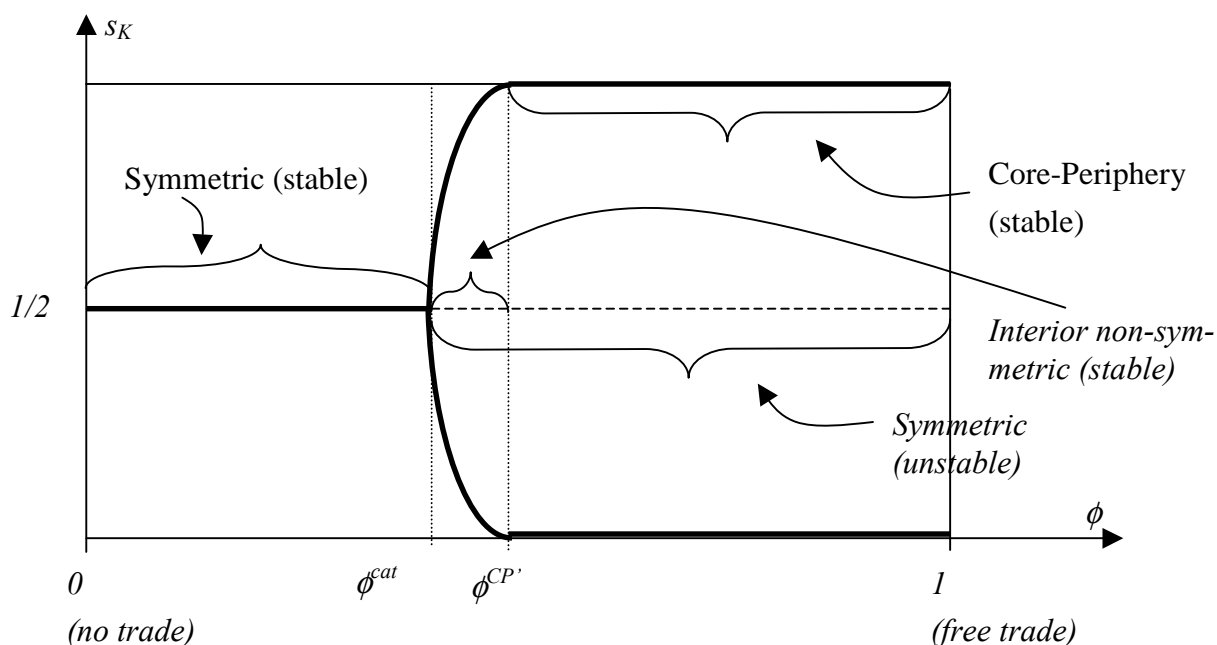
Using (23) to find $\partial s_E / \partial s_K = \frac{2\lambda\rho}{(1+\lambda)[L(1+\lambda) + \rho]}$, when evaluated at $s_K = 1/2$, we see

that the system is unstable (the expression in (29) is positive) for sufficiently low trade costs (i.e. $\phi \approx 1$). The two effects discussed in the previous section in the case without localized spillovers are still present. The first positive term is the **capital income effect**: an increase in s_K increases north's capital income, expenditure share and local profits so that the value of an innovation (the numerator of Tobin's q) increases. The last negative term is the stabilizing **local competition effect**: an increase in s_K increases local competition and reduces profits in the north. The second (positive) term is new and can be thought of as the **localized spillovers effect**: an increase in s_K implies a lower cost of innovation in the north (the denominator of Tobin's q) and therefore increase the incentive to innovate in the north.

The critical level at which the expression in (29) becomes positive is ϕ^{cat} . The appendix uses standard stability tests involving eigenvalues and derives the same result. One can also check that ϕ^{cat} is also the critical level beyond which the slope (evaluated at $s_K = 1/2$) of the permanent income relation (equation 23) becomes larger than the slope of the optimal investment relation (equation 24).

Graph 5 summarizes the model's stability properties in a diagram with ϕ and s_K on the axes. It shows that up to ϕ^{cat} , only the symmetric equilibrium exists and is stable. Between ϕ^{cat} and ϕ^{CP} , the symmetric steady state loses its stability to the two neighboring interior steady states, which are thus saddle points by continuity. This is called a "supercritical pitchfork bifurcation". After ϕ^{CP} , only the Core-Periphery equilibria are stable. Note that these can be attained only asymptotically because, due to the absence of capital depreciation, the south share of capital never goes to zero even after it stops investing (i.e. after ϕ^{CP}).

Graph 5: Falling transaction costs on goods: Stability properties of equilibria in the presence of localized spillovers



Introducing localized technology spillovers implies that economic geography affects the global growth rate and the model generates endogenous stages of growth. There are different stages of growth in the sense that if we think that transaction costs are lowered with time (in line with the "new economic geography" literature), then as economic geography is altered in a non linear way, the growth rate itself changes in a non linear manner. When transaction costs are high so that $\phi < \phi^{cat}$, the equilibrium economic geography is such that industry is dispersed between the two regions. This implies that spillovers are minimized and the cost of innovation is maximum. Using the optimal in-

vestment condition $q = q^* = 1$, and the fact that $s_K = 1/2$, it is easy to find the growth rate (see also equation (18) using $s_K = s_n = 1/2$) in that first stage:

$$g = \frac{\alpha L(1 + \lambda)}{\sigma} - \rho \frac{\sigma - \alpha}{\sigma} \quad (30)$$

The growth rate of course increases with λ . Asymptotically, when $s_K = 1$, spillovers are maximized so that the cost of innovation is minimized. Again using equation (18) with $s_K = s_n = 1$, the growth rate is in that stage:

$$g = \frac{2\alpha L}{\sigma} - \rho \frac{\sigma - \alpha}{\sigma} \quad (31)$$

The growth rate in that final stage is higher than the growth rate in the first stage when transactions costs are high. In the former stage, innovation has stopped in the south which then is entirely specialized in the traditional good. In the intermediate stage, which we call the take-off stage, i.e. when transaction costs are such that $\phi^{cat} < \phi < \phi^{CP'}$, the growth rate cannot be analytically found. However, it can be characterized as a take-off stage as the pitchfork bifurcation properties of the system entail that the economy leaves a neighborhood of the symmetric steady state equilibrium to reach a neighborhood of the asymmetric steady state equilibrium in finite time.

We have seen that a gradual lowering of transaction costs on goods (an increase in ϕ) leads, once the transaction cost passes a certain threshold, to a catastrophic agglomeration characterized by a sudden acceleration of innovation in one region (take-off) mirrored by the sudden halt of innovation in the other region. The north (the take-off region) enters a virtuous circle in which the increase in its share of capital expands its relative market size and reduces its relative cost of innovation which in turn induces further innovation and investment. In contrast, the south enters a vicious circle in which lower wealth leads to lower market size and lower profits for local firms. It also leads to an increase in the cost of innovation so that the incentive to innovate diminishes.

The model can also be used to analyze the gradual lowering of transaction costs on ideas, such as communication costs. In our model, this could be interpreted as an increase in λ , that is an increase in the extent to which an innovation (which is the same thing as one new unit of capital in our model) in one region decreases the cost of further innovation in the other region. It is possible to show that both ϕ^{cat} and $\phi^{CP'}$ are increas-

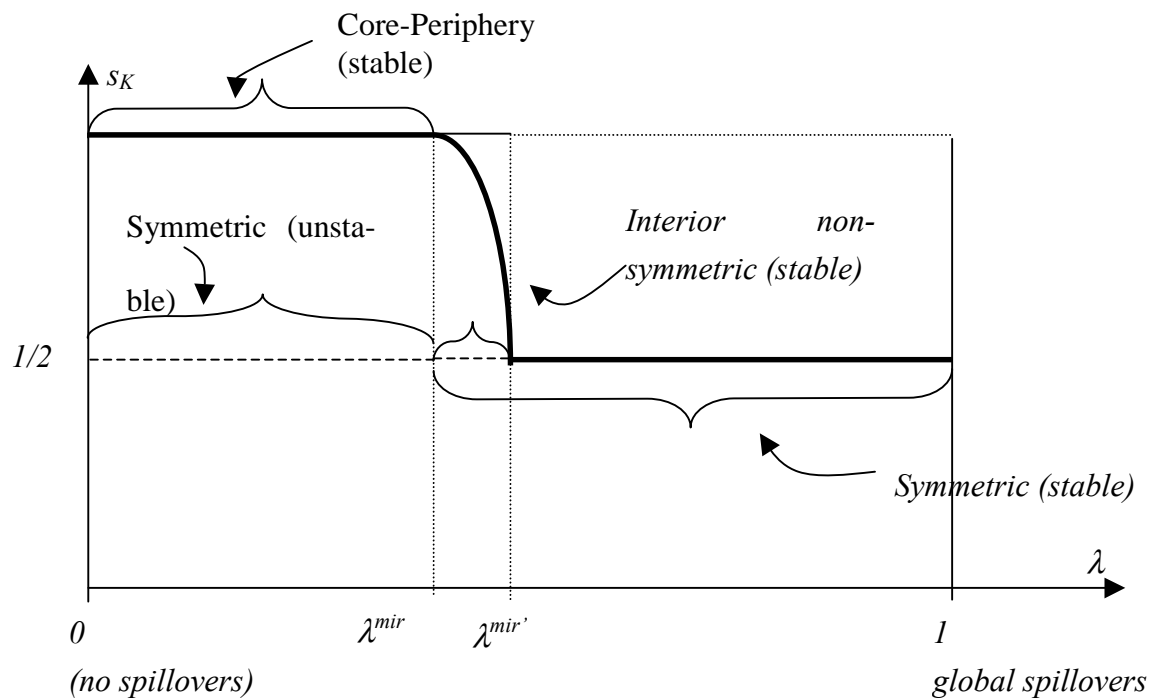
ing in λ . The intuition is that as spillovers are becoming more global, an increase in the northern share of capital does not decrease much the relative cost of innovation in the north (a destabilizing effect), so that the capital income effect (the stabilizing effect based on lower transaction costs on goods) must be stronger. One important implication is that from a situation with full agglomeration in the north ($s_K = 1$) and fixed transaction costs on goods, a gradual increase in λ (more globalized spillovers due for example to falling telecommunication costs) initially has no impact on southern industry. However, because the cost of innovation in the south decreases with λ , Tobin's q in the south increases with λ . At some point, when λ is high enough, q^* becomes more than 1, and the south begins to innovate. The value of this threshold level which we call λ^{mir} (for "miracle") is :

$$\lambda^{mir} = \frac{\varphi(2L + \rho)}{L(1 + \varphi^2) + \rho\varphi^2} \quad (32)$$

As in the case of falling transaction costs on goods, there is a second critical value where the symmetric equilibrium becomes stable. This value, denoted as $\lambda^{mir'}$ is the level of λ such that $\partial q/\partial s_K$ evaluated at the symmetric equilibrium becomes negative. As with the north take-off, the "miracle" in the south would appear as a virtuous circle: as it starts investing, its wealth and permanent income rise so that market size in the south and profits made by local firms increase. In turn, as the number of innovations made in the south increases, the cost of future innovations decreases. This "miracle" implies a jump in the investment rate, as Tobin's q in the south is more than 1, and rapid industrialization. Also incomes between the south and the north converge. Graph 6 below describes the effect of an increase in λ on the model's stability properties in a diagram with λ and s_K on the axes.

The main focus in the "new economic geography" literature has been on the consequence of falling transaction costs on trade in goods. We have shown that in a dynamic model with endogenous growth and localized spillovers, lower transaction costs on goods have an effect on industry location but also on the growth rate. These effects can be "catastrophic" or not, depending on the mobility of capital. The results also point out to a stark difference between lowering transaction costs on goods and lowering transaction costs on ideas. Lower transaction costs on goods may foster divergence in incomes if it triggers an agglomeration process. However, lowering transaction costs on ideas has

Graph 6: Falling transaction costs on ideas: Stability properties of equilibria in the presence of localized spillovers



the opposite effect as it can make the core-periphery equilibrium unstable and trigger a sudden industrialization in the south which leads to convergence. In our model, the distinction between transaction costs on goods and transaction costs on ideas is an easy one. However, in reality trading goods often implies exchanging ideas in the process so that the processes that govern the evolution of the two types of transaction costs are certainly intertwined.

Appendix I:

The equilibrium location in the case of perfect capital mobility is given by:

$$s_n = \frac{1}{2} + \frac{\sqrt{\Delta} - (1 - \phi)(L + L\lambda + \rho)}{8L(1 - \lambda)(1 - \phi)}$$

$$\Delta = 4(2s_K - 1)(1 - \phi^2)(1 - \lambda)L\rho + (1 - \phi)^2(L\lambda + L + \rho)^2$$

Appendix II:

Local stability of the symmetric equilibrium can be studied in terms of the eigenvalues of the Jacobian matrix associated with the system. These eigenvalues are:

$$e_1 = L(1 + \lambda) + \rho ; \quad e_{2,3} = \frac{b \pm \sqrt{b^2 - 4c\sigma(1 + \lambda)}}{2\sigma(1 + \lambda)} ;$$

$$b = \sigma[e_1(1 - \lambda) + 2\lambda\rho] + 4\frac{\alpha\phi e_1(1 - \lambda\phi)}{(1 + \phi)^2} ; \quad c = -2\alpha e_1 \left\{ e_1 \left[\lambda - 2\frac{\phi(1 + \lambda)}{(1 + \phi)^2} \right] - \frac{\lambda\rho(1 - \phi)}{1 + \phi} \right\}$$

The first eigenvalue is real and positive, the second and third give rise to different cases. By inspection, it can be checked that at sufficiently low levels of ϕ eigenvalues are all real. In this case, the eigenvalue that adds the radical - call this e_2 - is always positive. The third eigenvalue changes sign at the point where $c=0$. Solving ϕ for this, we get ϕ^{cat} as defined by equation (26).

References:

- Baldwin, R. (1999)*
Agglomeration and Endogenous Capital, *European Economic Review*, Vol. 43, No. 2, pp. 253-280
- Baldwin, R. and Forslid, R. (1997a)*
- Trade liberalization and endogenous growth: A q-theory approach, *Journal of International Economics*, Vol. 50, No. 2, pp 497-517, 2000
 - The Core-Periphery Model and Endogenous Growth: Stabilising and De-Stabilising Integration", *Economica*, 2000, Vol. 67, Nr. 3, pp. 307-324
- Baldwin, R.; Martin, P. and Ottaviano, G. (1998)*
Global Income Divergence, Trade and Industrialization: The Geography of Growth Take-Off', NBER Working Paper No. W6458. Forthcoming in *Journal of Economic Growth*
- Ciccone, Antonio and Hall, Robert (1996)*
Productivity and the Density of Economic Activity, *American Economic Review*, Vol. 87, No. 1, pp. 54-70
- Coe, David and Helpman, Elhanan (1995)*
International R&D Spillovers, *European Economic Review*, Vol. 39, pp. 859-887
- Coe, David; Helpman, Elhanan and Hoffmaister, Alexander (1997)*
North-South R&D Spillovers, *The Economic Journal*, Vol. 107, No.440, pp. 134-149
- Fujita, M., Krugman, P. and Venables, A. (2000)*
The Spatial Economy: Cities, Regions and International Trade, MIT Press
- Grossman, G. and Helpman, Elhanan (1991)*
Innovation and Growth in the World Economy, MIT Press, Cambridge USA
- Jacobs, J. (1969)*
The Economy of Cities. Vintage , New York
- Jaffe, A.; Trajtenberg, M. and Henderson, R. (1993)*
Geographic Localization of Knowledge Spillovers as Evidenced by Patent Citations, *Quarterly Journal of Economics*
- Krugman, P. (1981)*
Trade, Accumulation and Uneven Development, *Journal of Development Economics*, Vol. 8, pp. 149-161
(1991a) Increasing Returns and Economic Geography, *Journal of Political Economy*, Vol. 99, No. 2, pp 483-499
(1991b) History versus Expectations, *Quarterly Journal of Economics*, CVI
- Krugman, P. and Venables, A. (1995)*
Globalization and the inequality of nations, *Quarterly Journal of Economics*, Vol. 110, pp 857-880

Martin, P. and Ottaviano, G. (1999)

Growing Locations: Industry Location in a Model of Endogenous Growth, *European Economic Review*, Vol. 43, No. 2, 281-302

(2000) Growth and Agglomeration, *International Economic Review*, forthcoming, also on <http://www.enpc.fr/ceras/martin/TAKE28.pdf>

Martin, P. and Rey, H. (1999)

Financial Super-markets: Size Matters for Asset Trad, CEPR DP No. 2232

Rivera-Batiz, L., and Romer, P. (1991)

Economic Integration and Endogenous Growth, *Quarterly Journal of Economics*, Vol. 106, pp. 531-555

Romer, P. (1990)

Endogenous Technological Change, *Journal of Political Economy*, 98.5, part II, pp. S71-S102

Tobin, J. (1969)

A General Equilibrium Approach to Monetary Theory, *Journal of Money, Credit and Banking*, Vol. 1, pp. 15-29