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Working Paper

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Economics Working Paper Series, No. 18/291

Provided in Cooperation with:

ETH Zurich, CER-ETH – Center of Economic Research

Suggested Citation: Bretschger, Lucas; Vinogradova, Alexandra (2018) : Escaping Damocles' sword: Endogenous climate shocks in a growing economy, Economics Working Paper Series, No. 18/291, ETH Zurich, CER-ETH - Center of Economic Research, Zurich, <https://doi.org/10.3929/ethz-b-000266946>

This Version is available at:

<https://hdl.handle.net/10419/194114>

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Working Paper 18/291
May 2018

Economics Working Paper Series



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Escaping Damocles' Sword: Endogenous Climate Shocks in a Growing Economy

Lucas Bretschger and Alexandra Vinogradova*

May 29, 2018

Abstract

Climate economics has been criticized for ignoring uncertainty, catastrophic changes, and tipping points (Stern 2016). The present paper addresses these issues. We consider multiple climate shocks which are recurring, random, uninsurable, and potentially large. The associated damages and the hazard rate are endogenously driven by the stock of greenhouse gases. We provide closed-form solutions for the optimal climate policy and the growth rate of the economy. The optimal path is characterized by a constant growth rate of consumption and of the capital stock until a shock arrives, triggering a downward jump in both variables. The mitigation policy consists of a simple and intuitive rule which requires spending a constant fraction of output on emissions abatement. In a quantitative assessment we show that under favorable conditions the abatement expenditure represents 0.5% of output, equivalent to \$37 per ton carbon. Under less favorable conditions with respect to abatement technology and damages, coupled with a relative risk aversion which exceeds unity, the abatement propensity increases to 2.9%, equivalent to \$212 per ton carbon, and it jumps to a striking 10% in the pessimistic scenario involving severe shocks and a possible crossing of a tipping point.

JEL Classification: O10, Q52, Q54

Key Words: Climate policy, uncertainty, natural disasters, endogenous growth.

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A draft version of this paper has circulated under the title "Growth and Mitigation Policies with Uncertain Climate Damage".

1 Introduction

1.1 Economics and the Climate

Understanding economic effects of climate change is essential and urgent. An increase in the global temperature is predicted to intensify severity of natural disasters with significant adverse effects on development in many parts of the world. Ferocious tropical hurricanes, massive floods, droughts and landslides cause severe destruction of infrastructure, loss of physical and human capital, and undoubtedly result in a substantial setback in terms of economic growth. According to National Oceanic and Atmospheric Administration (2013), the cost of extreme weather events has risen from about \$20bn in 1980s to almost \$90bn in 2010. The recent Typhoon Haiyan in the Philippines was the strongest recorded storm to make landfall ever (see The Economist 2013). The surge swept away entire cities, at least 11m Filipinos have been affected, many killed, displaced or left homeless. According to media reports on Louisiana flooding in August 2016, quoting The Red Cross: "The catastrophic flood devastating Louisiana is now the worst natural disaster to strike the United States since Hurricane Sandy..."¹ Although climate physicists are not unanimous on whether the frequency of natural disasters will increase in the future or not, the majority agrees that their intensity will rise as the planet warms (IPCC 2014). It is well understood that economic activities cause carbon and other greenhouse gas (GHG) emissions which alter the natural environment and lead to climate change. However, occurrences of climate shocks are not easily predictable and they are typically viewed as random events.

The threat of severe climate catastrophes puts the world economy in a situation similar to the one of Damocles, an obsequious courtier of King Dionysius of Syracuse in the 4th century BC. According to the moral anecdote, Dionysius offered Damocles to sit on his throne in order to taste the fortune of a great man of power and authority, an opportunity which Damocles eagerly accepted. The throne was surrounded by every luxury but the King arranged that a sword should hang above it, held only by a single hair of a horse's tail. The situation of impending tragedies which are restrained by a certain probability can be used as an allegory for the world population facing climate change. The multiplicity and the various degrees of severity of climate disasters may be symbolized by multiple swords of variable sizes hanging above our heads. The analogy ends with two important differences, however. On the one hand, Damocles was allowed to depart from the throne. He could easily escape from the uncomfortable situation unlike the world population which cannot yet move to another planet. On the other hand, Damocles had no power to deal with the imminent threat, while the planet's warming can be controlled by our current policies. The sharpness of the swords and robustness of the hair holding them can be influenced by appropriate mitigation measures reducing the probability and damages of climate catastrophes.

The complexity of both the economic and the ecological parts of the climate-change problem pose considerable modeling challenges, involving long time horizons and various sources of uncertainty. As a consequence, the vast majority of current economy-climate frameworks

¹<http://edition.cnn.com/2016/08/18/us/louisiana-flooding/>

consists of relatively complex numerical simulation models. These have provided many useful insights with respect to the costs and benefits of a climate policy but also produced diverging results (Stern 2016). To gain further insights into the central mechanisms at work - especially those related to the uncertain nature of climate change - a framework of investigation that relies on analytic solutions and provides clear-cut implications for the optimal climate policy in a dynamic economy appears to be desirable.

Within such a framework, a number of important questions need to be addressed. Given the uncertain nature of environmental disasters caused by climate change, how should an economy appropriately balance its production, consumption, investment, and reduction of GHG emissions? What is the optimal rate of output growth and the optimal emissions abatement in the uncertain environment? How do these key variables respond to changes in the underlying economic and climatic fundamentals? In the present paper we examine these questions within a model of a growing economy which features uncertainty about arrivals of climate shocks. We assume that an occurrence of a disaster (also referred to as an "event") follows a random process, and when a disaster strikes, some part of the economy's productive capacity is destroyed. Natural catastrophes induced by climate change are large in scale and have a profound negative impact on both national and global economy. Unlike in the case of relatively small idiosyncratic shocks, the risk of such events cannot be insured.² The magnitude of the damage is assumed to be an increasing function of the stock of greenhouse gases. It follows that the process of accumulation of the productive capacity is both endogenous and stochastic. Our framework also accommodates the concept of risk vulnerability, first proposed by Gollier and Pratt (1996), according to which exposure to risky environments affects an individual's attitude to risk.

In our model, however, the world does not end after an environmental disaster, as it is often assumed in the literature on catastrophic events (see Section 1.3). We consider development with recurring shocks over time, which reflects a likely pattern of climate-induced events in the future.³ Optimal reduction in emissions, and the implied reduction in damages, can be achieved by appropriately balancing two types of activities: capital accumulation and abatement. In an extension of the model we treat *both* damages and the event hazard rate as endogenous and also introduce a "tipping point" scenario which may bring about a substantial detriment to economic activity, including a total collapse.

²In a globalized world, large-scale natural disasters affect not only the economic activity of the country where they strike but also other economies by virtue of either close geographic location or trade relations, FDI, etc. When it comes to relatively small idiosyncratic shocks, they can be insured against by trading insurance claims within a group of regions subject to such shocks. Our focus, however, is on a global economy where an insurance contract against a large natural catastrophe (e.g., Indian Ocean tsunami, meltdown of Greenland ice-sheet, etc.) is challenging to design and implement. Additionally, stability of contractual arrangements on such a large scale seems to be difficult to ensure, especially in light of the long time horizon and asymmetric probabilities of disasters across countries. Insurance companies seem to perceive climate change and the associated disasters as a threat rather than profitable business opportunities. As a prominent example, the strategy of one of the world's leading reinsurance companies, Swiss Re, is to raise awareness about climate-change risks through dialogue with clients, employees, and the public and to advocate a worldwide policy framework for climate change (Swiss Re 2015, p.119,ff.).

³Recurring climate-induced capital destruction may result in a new form of poverty traps for developing countries. See Le Van et al. (2010) for an analysis of poverty traps where the natural environment offers opportunities while in our case it may pose a threat to development.

1.2 Contribution to the Literature

To the best of our knowledge the paper is the first to provide closed-form solutions for the optimal abatement policy and the growth rate of the economy subject to random climate shocks with damages endogenously driven by the accumulation of GHG. We believe that the latter lie at the heart of the climate-change problem. Our climate-policy instrument can be conveniently expressed as a fraction of output which depends on the fundamental characteristics of the climate and the economy. This result is parallel to that of Golosov et al. (2014), who derive a simple formula for the optimal carbon tax, showing that under specific simplifying assumptions on damages and saving propensity the tax is proportional to GDP and depends on just a small number of key parameters. In the present paper we relax some of those assumptions and, in particular, endogenously derive the optimal saving rate of the economy to show how it is affected by exposure to uncertainty associated with climate change. A more frequent occurrence of natural disasters (i.e., higher arrival rate) and a higher damage intensity have a negative impact on the growth rate of consumption and call for more vigorous abatement policies. The dependence of the climate-policy instrument on the hazard rate points to the importance of relying on stochastic models when deriving meaningful and effective policy prescriptions.

Simultaneous determination of the optimal growth rate of consumption is no less important than the abatement policy itself. This is because the growth rate of the economy and the associated accumulation of productive capacity determine (i) the growth of the stock of GHG, (ii) security buffer in the event of a climatic hazard, i.e. how much loss the economy can withstand, and (iii) how much resources can be devoted to mitigation. Thus, with an appropriate choice of the growth rate, less stringent abatement measures may be required. The optimal development of our economy is characterized by a constant growth rate of consumption and capital stock until an event arrives causing a downward jump in both variables. The size of the jump is endogenously determined and depends on a number of key economic and climatic parameters, including the arrival rate of disasters, efficiency of abatement technology, damage intensity, and risk aversion. We show that when preferences are logarithmic, important links to climatic characteristics disappear from the optimal rules. When a random tipping point is introduced, the optimal consumption-to-capital ratio is reduced, while the optimal abatement propensity and the saving rate rise. With log-utility, however, these effects are absent. In a setting where both the hazard rate and the damages respond to the stock of atmospheric GHG, the optimal growth rate may either increase or fall depending on the strength of the precautionary effect, while the abatement policy is unambiguously more stringent as it needs to account for not only the damage size in the event of a climate shock but also for the shocks' frequency.

While the closed-form solution allows us to clearly disentangle the effect of each climatic and economic characteristic of the model, we are also interested in quantitative predictions with respect to the optimal policies. Numerous existing studies proposed required emissions-mitigation schemes (or a carbon tax) to ensure that the global temperature does not exceed a specific threshold (e.g., Pindyck 2012). Other studies asked what the optimal carbon tax (or

willingness to pay) should be, given the various types of uncertainty associated with climate change (Gerlagh and Liski 2017, Golosov et al. 2014, Martin and Pindyck 2015, Pindyck and Wang 2013, van der Ploeg 2014). Our investigation falls into the second category. One of the frequent messages delivered by the recent literature is that the utility discount rate appears to be the main driver of the optimal tax but not a possibility of a climate disaster.⁴ This observation motivates us to quantitatively assess the impact of possible multiple and random climate shocks on the optimal abatement propensity and, by extension, on the price of carbon. We focus on one particular feature of climate change - the dependence of damages and of the hazard rate of natural disasters on the stock of GHG - and we ask (i) what the abatement policy should look like, given those possibilities, and (ii) how the abatement policy should respond, given a change in either economic or climatic fundamentals.

We examine several scenarios based on alternative assumptions about abatement technology, risk aversion, size of damages and probability of occurrence. The gist of our findings is that it matters whether disasters are low-impact (causing a loss of, say, not more than 0.1% of GWP) occurring relatively frequently or high-impact (loss of up to 10% of GWP) even if they occur relatively rarely, say, less than once in a hundred years. Considering the benchmark scenario with log utility, exogenous hazard, no tipping points and low-impact shocks, we find that the optimal abatement expenditure at the global level represents approximately 0.5% of GWP. This turns out to be equivalent to a carbon price of \$36.6 per ton. The average world consumption growth rate under this scenario is 3.475% per year. Our theoretical investigation shows that many crucial climate-change driven effects disappear from the optimal rules when the relative risk aversion (RRA) parameter is set to unity. Adopting the risk-vulnerability hypothesis, we consider higher values of RRA. With $RRA=3$, we find that the optimal abatement propensity increases relatively moderately to 0.7%, although the growth rate falls considerably to 1.15%. On the one hand, low-impact shocks do not require a substantial increase in abatement efforts even with a relatively high RRA. On the other hand, we see that it is the simultaneous reaction of *both* abatement and growth which is decisive. The growth rate of the economy essentially works as a stabilizer of emissions and thus indirectly affects the climate policy instrument. With a lower growth rate, less stringent abatement policy is needed. A second set of scenarios considers high-impact shocks with potential losses of up to 10% of world GDP. The abatement propensity increases considerably to 2.9% and the growth rate falls to 1.11%. Further sensitivity tests show that an increase in the probability of high-impact disasters from 20% in the next 50 years to 20% in the next 10 years increases the abatement propensity from 0.96% to 2.9%. Introducing, in addition, a possibility of a tipping

⁴Golosov et al. (2014) calculate the optimal carbon tax to be \$25.3 and \$489 per ton carbon when the realized damages amount to 0.48% and 30% of GDP (based on Nordhaus 2000), respectively, assuming a 1.5% discount rate and log-utility. With a much lower discount rate of 0.1% (advocated by Stern 2007), the tax jumps up 8-9 fold to \$221 and \$4,263, respectively. In another important contribution, van der Ploeg (2014) considers a possibility of a climate catastrophe, such that a positive carbon feedback occurs at some random (and possibly endogenous) point in time. He finds that the optimal after-catastrophe carbon tax is \$29 and \$216 per ton carbon with the discount rate of 1.5% and 0.1%, respectively. The before-catastrophe tax is slightly lower at \$27.6 and \$198.6. When the hazard rate is endogenous, the tax is only slightly higher. One exception is van den Bijgaart et al. (2016) who show in the context of a deterministic IAM that damage sensitivity plays just as important role as discounting in determining the social cost of carbon.

point doubles the optimal abatement propensity and raises the price of carbon to \$425 per ton. The quantitative section also provides a comparison of our findings with those in the recent studies.

1.3 Related Literature

The present paper contributes to formulating efficient climate policies using an approach which has been identified as an urgent priority in recent assessments of the field. Farmer et al. (2015) conclude that the first of four major issues inadequately addressed by economic models of climate change is uncertainty. An even more stringent view is expressed by Stern (2016) who states that many economic models do not account for catastrophic changes and possible tipping points which would make a crucial difference for policy assessment in his view. Addressing these issues forms the core of our contribution.

Our work relates to several important papers in the field of climate economics. Pindyck and Wang (2013) consider a growing economy subject to random shocks which cause random but exogenously-specified damages to the capital stock and calculate society's willingness to pay for avoiding such shocks.⁵ Müller-Fürstenberger and Schumacher (2015) developed a decentralized version of a neoclassical growth model where agents face stochastic extreme events which are relatively small-scale and thus insurable. A very interesting and policy-relevant result of their paper is that the insurance industry, being the main provider of reactive adaptation, can induce agents to undertake more abatement by signaling the consequences of climate change through insurance premiums.⁶ In a recent paper, Gerlagh and Liski (2017) introduce uncertain events in a climate-economy model where the impacts of climate change are not known but learned over time. As a consequence of the belief updating the optimal carbon tax does not develop in lock-step with income but depends on temperature levels.

There is a growing literature on random catastrophic events causing *irreversible* damage.⁷ Tsur and Zemel (1998) are the first to explicitly analyze reversible events focusing on the optimal steady state policy and transitional dynamics of an economy which is not engaged in any investment activity.⁸ Van der Ploeg (2014) analyzes the optimal carbon tax in an

⁵Pindyck (2012) explicitly models the impact of climate change and economic losses to calculate the willingness to pay to avoid a temperature increase by a specific amount. The analysis proceeds under the assumption that the economy grows at a rate which is a negative function of the global temperature change.

⁶Ikefuji and Horii (2012) examine the optimal growth rate and the carbon tax in an economy where private capital is subject to stochastic depreciation due to climate change. They assume, however, that these stochastic shocks are idiosyncratic and reflect a large number of independent small climate events. Soretz (2007) analyzes efficient pollution taxation within an endogenous growth model where environmental quality has a stochastic impact on factor productivity, which is driven by a Wiener process. Similarly to these contributions we assume that international policy coordination is feasible, as has recently been demonstrated by adoption of the Paris Agreement. If such a mechanism turns out to be inefficient, it has been suggested to replace global carbon taxes by international trading of the rights to exploit fossil-fuel deposits, see Harstad (2012).

⁷For early theoretical contributions see Clarke and Reed (1994), Tsur and Zemel (1996) and for recent numerical models Lemoine and Traeger (2014), Lontzek et al. (2015).

⁸De Zeeuw and Zemel (2012) provide a dynamic characterization of an optimal emission policy when the time of the regime switch from low to high damage is uncertain. One of their key findings is that, due to precautionary reasons, emissions in the low-damage regime may be lower than in the case where the system is already in the high-damage regime.

economy subject to a random shock which reduces the nature's capacity to absorb greenhouse gases. The analysis considers the possibility of the hazard rate being a function of accumulated pollution. Although this ingredient of the model requires a partial equilibrium approach (with exogenous output and no capital accumulation), it allows to disentangle the components of the optimal tax which are driven by the presence of uncertainty and by endogeneity of the hazard rate.⁹ We believe that irreversibility may well characterize behavior of some ecosystems but is a rather extreme assumption in the context of the global economy. The chances of one severe shock associated with a total capital loss at the global level are rather low, while recurring shocks, with potentially large although non-destructive damages, constitute a more realistic scenario.

We add to this literature by providing clear-cut analytical solutions for the optimal growth rate of consumption and for the abatement policy in a general equilibrium model featuring random recurring shocks with damages being driven by investment and abatement decisions. We stress the importance of the simultaneous consideration of climate policy and growth for three reasons. First, climate change is a long-run phenomenon so that development of the economy within the same time frame is consequential. Second, optimal climate policy has an impact on economic growth, since climate change may negatively affect the stock of productive capacity.¹⁰ Third, if economic expansion is associated with an increasing stock of GHG, the optimal climate policy is affected, i.e. a higher growth rate in equilibrium requires more stringent policy measures.

The remainder of the paper is organized as follows. Section 2 develops our baseline framework. In Section 3, we present the main results with respect to the optimal growth rate, abatement, and saving propensity. Sections 4 and 5 provide two extensions of the baseline model by introducing a tipping point and an endogenous hazard rate. Section 6 offers quantitative implications. Section 7 concludes.

2 The Framework

2.1 Baseline Model

We consider a global economy which produces a composite consumption good under constant returns to scale using as input broadly defined capital, denoted by K_t . The production process is polluting: every instant t a flow of greenhouse gas (GHG) emissions, E_t , is released into the atmosphere. The stock of GHG, denoted by P_t , is thus augmented every instant by E_t and reduced by αP_t , where $\alpha \in [0, 1)$ represents the natural absorption rate of greenhouse gases (e.g., by deep oceans) and is assumed to be very small.¹¹ Pollution causes deterioration

⁹In a model with uncertain pollution stock dynamics, Athanassoglou and Xepapadeas (2012) find that investment in damage control is increasing in the degree of uncertainty and that control might be a substitute for mitigation when it is sensitive to changes in uncertainty.

¹⁰Dell et al. (2012) provide empirical evidence for the link between temperature and economic growth. Their findings support the idea that the dynamic dimension of climate damages constitutes an important element in the analysis of growth and the environment.

¹¹Nordhaus and Boyer (2000) use a more elaborate representation of the carbon cycle, with atmosphere, upper ocean layers and deep oceans as the three main carbon reservoirs. They calibrate the transfer rate from the atmo-

of the natural environment and an increase in the global temperature, leading to a random occurrence of natural disasters. We assume that an arrival of a natural disaster (we shall also refer to it as an "event") follows the Poisson process with the mean arrival rate λ . In Section 5 we extend our model to include endogenous arrivals, while for the moment we assume that λ is constant. When an event occurs, an endogenously-determined amount of the existing capital stock is destroyed. We denote by $\omega_t \in (0, 1)$ the fraction of capital which survives the shock. In fact, recent floods, as the one in Pakistan in 2010 or in the Philippines in 2013, had a profound effect on infrastructure and the capital stock (both physical and human). According to the predictions of climate sciences, the magnitude of the damage is likely to increase in the future due to climate change and hence we model it as a positive function of the stock of GHG or, equivalently, the higher is the pollution stock the lower is the after-shock share of capital, $\frac{\partial \omega_t}{\partial P_t} < 0$. In addition, we assume that the damages are larger, the larger is the amount of capital exposed to destruction, i.e. $\frac{\partial \omega_t}{\partial K_t} < 0$, capturing the notion of the value at risk (Bouwer 2011).

The output, denoted by $Y_t(K_t)$, can be either spent on consumption, C_t , or invested. There are two types of non-consumption spending: (i) investment to augment the capital stock and (ii) financing of emissions abatement. Specifically, we assume that an endogenous fraction, θ_t , of output is spent on the latter, so that abatement expenditure is given by $I_t = \theta_t Y_t$. The remaining share $(1 - \theta_t)Y_t$ is split between consumption and capital accumulation. Total abatement, $Z(I_t)$, is a positive function of the abatement expenditure, $Z'(I_t) > 0$. The total per period emissions are then given by emissions stemming from the economic activity minus abatement. We assume that one unit of output causes ϕ units of pollution,¹² so that total emissions are given by $E_t = \phi Y_t - Z(I_t)$.

The economy's objective is to maximize the expected discounted utility over an infinite planning horizon with respect to consumption, C_t , and the share of output devoted to abatement, θ_t , subject to the stochastic capital accumulation process and the dynamics of the pollution stock. Specifically, the planner's programme is

$$\max_{C_t, \theta_t} \mathbb{E}_0 \left\{ \int_0^\infty U(C_t, P_t) e^{-\rho t} dt \right\} \quad (1)$$

$$\text{s.t.} \quad dK_t = [(1 - \theta_t)Y_t(K_t) - C_t]dt - [1 - \omega_t(P_t, K_t)] K_t dq_t, \quad K_0 \text{ given}, \quad (2)$$

$$dP_t = (E_t - \alpha P_t)dt, \quad P_0 \text{ given}, \quad (3)$$

$$E_t = \phi Y_t(K_t) - Z(I_t), \quad (4)$$

$$I_t = \theta_t Y_t(K_t), \quad (5)$$

where \mathbb{E}_0 is the expectations operator, dq_t is an increment of the Poisson process, and $\rho > 0$ is the constant rate of time preference. The utility function is twice continuously differentiable in both arguments. We also require that the capital stock, pollution stock, consumption and

sphere to the upper ocean layer as 0.333 per decade and from the upper oceans to the deep oceans as 0.115 per decade, implying an indirect transfer rate from the atmosphere to deep oceans of approximately 0.0038 per year, which is proxied by α in our model.

¹²With appropriate choice of units, polluting intensity ϕ can be restricted to lie between zero and unity.

emissions rates are non-negative and $\theta_t \in [0, 1)$.

2.2 Assumptions

We now introduce some useful functional forms and explain their motivation and consequences in the model.

1. Production Technology

We assume constant returns to scale in aggregate production and as capital is the only input, output is produced with an AK technology, a frequently adopted specification in climate-economy models (see, e.g., Müller-Fürstenberger and Schumacher 2015, Pindyck and Wang 2013). Parameter A denotes the constant factor productivity and K_t is interpreted as a broad measure of capital in the economy, including physical and human capital, intangibles, etc.¹³

$$Y_t = AK_t. \quad (6)$$

2. Abatement Technology

We also assume constant returns in abatement, i.e. total abatement is directly proportional to the resources allocated to emissions control, with the proportionality parameter $\sigma > 0$ representing the efficiency of abatement technology:

$$Z(I_t) = \sigma I_t. \quad (7)$$

3. Damages

Natural disasters destroy a share of the economy's productive capacity. We assume that the survived share of capital, ω , can be represented by a function of a single argument, v , which depends on P and K . In particular, we assume that $v = P^\xi K^\eta$, where the constants $\xi > 0$ and $\eta > 0$ govern the relative importance of the climate change component and the exposure component, respectively. We shall also choose a negative exponential function for ω , so that

$$\omega(v) = e^{-\delta v} \in (0, 1), \quad (8)$$

where the parameter $\delta > 0$ can be interpreted as the damage intensity. This specification has several desirable characteristics. First, it ensures that ω is indeed a fraction, i.e. lies between zero and unity. Second, ω is decreasing in P and K , i.e. $\frac{\partial \omega}{\partial P} = \xi \omega \ln \omega / P < 0$, $\frac{\partial \omega}{\partial K} = \eta \omega \ln \omega / K < 0$ to capture the effect of climate change and the capital exposure, respectively. In general, any function satisfying these properties would be suitable for describing ω .

4. Preferences

How to represent society's preferences over consumption and environmental quality has long been a cornerstone question for economists. Over the last few years of research three main

¹³Despite its formal simplicity, the AK model unites all the desirable properties of an aggregate production function in a dynamic climate model. It generates sustained growth endogenously, results in the same implications for investment and growth as if we included different capital components such as physical, human, and knowledge capital separately, and is consistent with the empirically observed strong positive relationship between investment rates and growth rates across countries and time periods (see McGrattan 1998).

trends have emerged. First, the non-expected utility and homothetic preferences of Epstein-Zin type (Epstein and Zin 1989) have gained momentum (Barro 2015, Lemoine and Traeger 2014, Pindyck and Wang 2013, van der Ploeg and de Zeeuw 2016) due to their ability to disentangle the elasticity of intertemporal substitution (EIS) from the relative risk aversion (RRA) coefficient.¹⁴ Second, logarithmic preferences have been widely used due to their tractability (Golosov et al. 2014, Müller-Fürstenberger and Schumacher 2015). The third option, adopted here (as well as in Barrage (2014) and van der Ploeg and de Zeeuw (2016)), is to assume a constant relative risk aversion (CRRA) utility function, also encompassing the logarithmic version as a special case. We assume that utility is additively-separable in consumption and environmental quality, with the latter being inversely related to the stock of GHG. Finally, the utility function is increasing and concave in consumption and decreasing and concave in pollution stock, exhibiting risk aversion with respect to both arguments:

$$U(C, P) = \frac{C^{1-\varepsilon}}{1-\varepsilon} - \chi \frac{P^{1+\beta}}{1+\beta}, \quad \varepsilon > 0, \beta > 0, \chi > 0 \quad (9)$$

where χ is the relative weight of pollution. We will show in Sections 3 and 6 the importance of considering a general CRRA structure, as opposed to its knife-edge case of log-utility, which ignores important effects in a climate-growth context.

Recent empirical literature on attitude to risk in the presence of disasters has largely confirmed the "risk vulnerability" hypothesis first proposed by Gollier and Pratt (1996). The hypothesis states that agents operating in risky environments characterized by a possibility of a loss on average, i.e. agents who are exposed to unfair risks, tend to exhibit a more risk-averse behavior than otherwise (see, e.g. Harrison et al. 2007, Guiso and Paiella 2008, Cameron and Shah 2015). Moreover, risk aversion also increases in the magnitude of damages (Cameron and Shah 2015). We include this property in our model by linking the parameters of the damage function to the parameters governing attitude to risk in the utility function: $\varepsilon = \varepsilon(\eta)$ and $\beta = \beta(\xi)$ with $\varepsilon'(\eta) = \beta'(\xi) = \gamma > 0$, i.e. the mappings are linear with equal slope such that the relative importance of the climate change component and the exposure component is preserved.

¹⁴This property, however, comes at a cost. It has been recently shown that Epstein-Zin preferences are non-monotonic with respect to the first-order stochastic dominance meaning that under some parameter constellations they lead to choices of dominated strategies (Bommier and Le Grand 2014). This is less of an issue for numerically simulated models and may be accommodated with the aid of numerical methods. It is a considerable challenge, however, for theoretical models involving dynamic stochastic optimization. In our view, giving up the first-order stochastic dominance monotonicity is a fairly high price to pay in the context of our model, where our main concern is the optimal choice of consumption and investment decisions.

2.3 Solution

Denoting by $V(K, P)$ the value function associated with the optimization problem described in (1) - (5), the Hamilton-Jacobi-Bellman (HJB) equation may be written as

$$\rho V(K, P) = \max_{C, \theta} \left\{ U(C, P) + V_k[(1 - \theta)Y - C] + V_p[(\phi - \theta\sigma)Y - \alpha P] + \right. \\ \left. + \lambda [V(\tilde{K}, P) - V(K, P)] \right\}, \quad (10)$$

where $V(\tilde{K}, P)$ is the value function after the occurrence of an event which depends on the new capital stock $\tilde{K} = \omega(P, K)K$. For convenience, we use the notation $V_k \equiv \partial V(K, P)/\partial K$, $V_p \equiv \partial V(K, P)/\partial P$, $\tilde{V}_k \equiv \partial V(\tilde{K}, P)/\partial \tilde{K}$, $\tilde{V}_p \equiv \partial V(\tilde{K}, P)/\partial P$. Time subscripts are omitted when there is no ambiguity. The first-order conditions with respect to the control and the state variables consist of

$$C : U_c - V_k = 0, \quad (11)$$

$$\theta : -V_k Y - V_p \sigma Y = 0, \quad (12)$$

$$K : \rho V_k = V_{kk}[(1 - \theta)Y - C] + V_k(1 - \theta)A + V_p(\phi - \sigma\theta)A + V_{pk}[(\phi - \sigma\theta)Y - \alpha P] + \\ + \lambda \left(\tilde{V}_k \frac{d\tilde{K}}{dK} - V_k \right), \quad (13)$$

$$P : \rho V_p = U_p + V_{pp}[(\phi - \sigma\theta)Y - \alpha P] + V_{kp}[(1 - \theta)Y - C] + \lambda \tilde{V}_k \frac{d\tilde{K}}{dP} - \alpha V_p + \\ + \lambda (\tilde{V}_p - V_p). \quad (14)$$

We relegate the detailed derivations to the Appendix, while focusing on the key results in the main text. The value function can be found explicitly as

$$V(K, P) = \frac{\psi^{*-\varepsilon} K^{1-\varepsilon}}{1-\varepsilon} - \frac{x P^{1+\beta}}{1+\beta}.$$

The constants ψ^* and x depend on the parameters of the model and x can be obtained explicitly as

$$x = \frac{\chi}{(1 + \beta)\alpha + \rho}, \quad (15)$$

while ψ^* , representing the optimal ratio of consumption to capital, is an implicit solution to the following equation:

$$\varepsilon \psi = \rho + (\varepsilon - 1)A \left(1 - \frac{\phi}{\sigma} \right) - \lambda (\omega^{1-\varepsilon} - 1), \quad (16)$$

where $\omega(\psi) = e^{-\delta[\psi^\varepsilon x \sigma]^{-1/\gamma}}$ is constant at the optimum. In the rest of the analysis we shall assume that the parameter γ , which governs how strongly risk attitude reacts to damages, is equal to unity. This is done for simplicity, as the magnitude of this parameter does not have any qualitative effects on the results. In the numerical part of the paper we do consider alternative values of γ as robustness checks.

Eq. (16) may have zero or one root if $\varepsilon \leq 1$ and zero, one or two roots if $\varepsilon > 1$. In the latter case, if the parameter constellation is such that two roots exist, the equilibrium is given by the largest root (see Appendix A.2).

Computing the differential of V_k and applying the Itô's formula for jump processes allows us to obtain the law of motion of the consumption rate:

$$\frac{dC}{C} = \frac{1}{\varepsilon} \left\{ A \left(1 - \frac{\phi}{\sigma} \right) - \rho + \lambda [\omega^{1-\varepsilon}(1 + \varepsilon \ln \omega) - 1] \right\} dt + (\omega - 1) dq. \quad (17)$$

Given that $\tilde{K} = \omega K$, while $C = \psi^* K$ and $\tilde{C} = \psi^* \tilde{K}$, ω also represents the ratio of post- to pre-shock consumption rates. Therefore, the last term on the RHS of (17) is negative, reflecting the downward jump in consumption at the time of a disaster. The first term on the RHS, multiplying dt , represents what we label as "trend" consumption growth rate, g^* , which prevails in-between climate shocks:

$$g^* \equiv \frac{1}{\varepsilon} \left\{ A \left(1 - \frac{\phi}{\sigma} \right) - \rho + \lambda (\zeta - 1) \right\}, \quad (18)$$

where $\zeta = \omega^{1-\varepsilon}(\psi^*)(1 + \varepsilon \ln \omega(\psi^*))$. The expression has a familiar Keynes-Ramsey form albeit with some additional elements. The standard Keynes-Ramsey formula states that the growth rate of consumption equals the difference between the real interest rate (usually the marginal product of capital) and the rate of pure time preference, adjusted by the elasticity of intertemporal consumption substitution. First, note that in Eq. (18) the economy's implicit real interest rate, given by the first term inside the parentheses, is not equal to just the marginal productivity of capital but is reduced by the emission intensity of output, adjusted by the abatement efficiency, i.e., the term ϕ/σ . It follows that in our framework pollution has an unambiguously negative effect on the real interest rate. It may be dampened by either increasing the abatement efficiency, σ , or decreasing the polluting intensity, ϕ .

Second, expression in (18) accounts for the effect of uncertainty, represented by the last term $\lambda(\zeta - 1)$, which includes the exposure effect (-1) and the pollution-stock effect (ζ). We are especially interested in the sign of $(\zeta - 1)$. If it is positive, then the presence of uncertainty speeds up consumption growth. In this case, the optimal stochastic consumption path is tilted counterclockwise, as compared to the consumption path in a deterministic Keynes-Ramsey model. Therefore, the economy starts with a relatively low consumption rate at the beginning of the planning horizon, which implies the presence of the precautionary-saving motive, including saving for financing of emissions control.¹⁵ The peculiarity of the current setting is that the gross savings are endogenously split between two purposes: capital accumulation and abatement, both of which serve to protect the economy from climate disasters. It is clear that abatement reduces emissions and therefore unambiguously contributes to a reduction in damages. Capital accumulation, however, has a double-sided effect. On the one hand, more capital implies more output and more emissions. On the other hand, having more capital creates an "emergency buffer" for the rainy days - when a disaster strikes. We discuss in more detail the

¹⁵The outcome is analogous to what has been found in the literature on precautionary savings under uncertainty in other contexts (see, e.g., Wälde 1999, Steger 2005).

economy's optimal saving rate and how it is affected by climatic parameters in Section 3.3. For the moment we shall focus our attention on the precautionary effect and its relevance for the optimal growth rate.

When the term $(\zeta - 1)$ in (18) is negative, the risky environment contributes to a growth slowdown, tilts the consumption profile clockwise and thus implies a precautionary *dissaving* motive. Note that $\zeta = \omega^{1-\varepsilon}(1+\varepsilon \ln \omega)$ is composed of two terms. First, the ratio of the marginal utilities of consumption after and before the shock, $u'(\tilde{C})/u'^{-\varepsilon}$, which is unambiguously larger than unity since $\tilde{C} < C$. Second, the term $d\tilde{K}/dK = \omega(1+\varepsilon \ln \omega)$ which represents the reaction of the post-shock capital stock to a change in the pre-shock capital stock. The latter includes the direct positive effect of K on \tilde{K} , given by $\omega > 0$, and an indirect negative effect, given by a change in ω itself, $K(\partial\omega/\partial K) = \omega\varepsilon \ln \omega < 0$. Whether the direct effect dominates the indirect effect, depends on the magnitude of ε .¹⁶ If $\varepsilon < -(\ln \omega)^{-1}$, the direct effect dominates, implying that $d\tilde{K}/dK > 0$. The threshold $-(\ln \omega)^{-1}$ may take values between approximately 3.47 and 99 for $\omega \in [0.75, 0.99]$. Clearly, the condition is largely satisfied for the relevant range of ω and empirically-plausible range of ε (see figure 1a).¹⁷

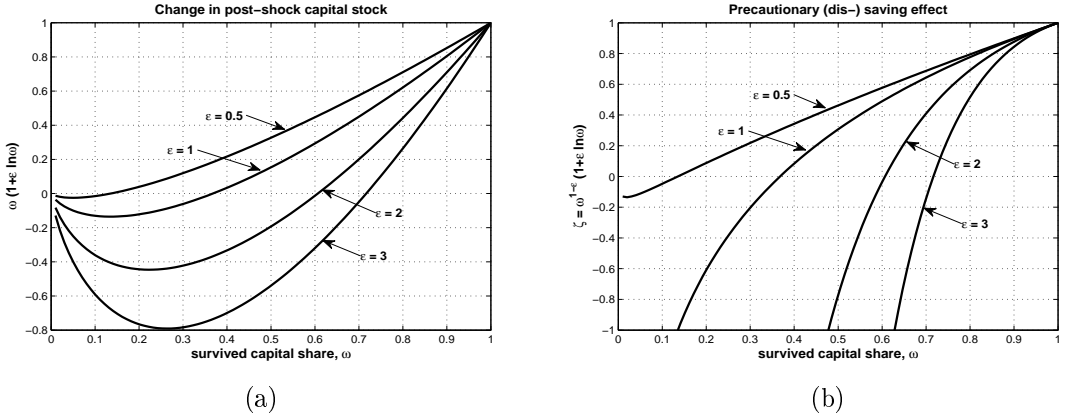


Figure 1: Precautionary effect and risk aversion.

Turning our focus to the relevant range of ω , we still need to find out whether the presence of uncertainty contributes to a faster or slower optimal consumption growth. So far we have established that an increase in K will lead to an increase in the post-event capital stock but less than proportionally since $\omega(1 + \varepsilon \ln \omega) < 1$ (recall that $\omega \in (0, 1)$ and $\ln \omega < 0$). The

¹⁶With the assumption $\varepsilon = \eta$ we can also relate the magnitudes of the direct and indirect effects of K on \tilde{K} to the parameter governing the strength of the exposure component in the damage function. Recall that if $\eta < 1$, the survived share of capital is convex in K , i.e., as capital stock grows its marginal vulnerability to shocks declines. Alternatively, if η exceeds a specific threshold, the marginal vulnerability increases. The threshold is given by $(1 + \ln \omega)^{-1}$. For relatively small damages, say 1% ($\omega = .99$), the threshold is approximately unity. For relatively large damages, say 25% ($\omega = .75$), the value is around 1.4. Given that for the relevant range of the damage magnitude the threshold values are close to one, we shall distinguish primarily between two possibilities, $\eta < 1$ and $\eta > 1$. This is also convenient for our further analysis since we need to consider alternative values of ε , with the usual cut-off value of one.

¹⁷Only for extremely large damages, exceeding 25% of GDP, the condition $\varepsilon < -(\ln \omega)^{-1}$ might be violated, implying that $d\tilde{K}/dK$ may become negative. Consequently, the term $(\zeta - 1)$ in (18) may become negative as well, implying a precautionary dissaving motive. The intuition here is clear, since the overall effect of an increase in the capital stock on \tilde{K} is negative, the optimal response of the economy is to decrease its growth and saving rates.

valuation of this increase is given by the ratio of marginal utilities $\omega^{-\varepsilon} > 1$. The total effect, $\zeta = \omega^{1-\varepsilon}(1 + \varepsilon \ln \omega)$, can thus be larger or smaller than unity, depending on the magnitude of ε . We provide a detailed analysis in the appendix, while here we summarize the results as follows. If (i) $\varepsilon \in (0, 1)$, the expression is unambiguously less than unity, while if (ii) $\varepsilon > 1$, a priori an ambiguity exists. We show, however, that this ambiguity can be resolved and ζ is less than unity for the relevant range of parameters (see figure 1b). We conclude that $(\zeta - 1)$ is negative and hence the presence of uncertainty contributes to a lower consumption growth rate than in the deterministic model. This finding points to the presence of a specific type of consumption "smoothing" such that the economy aims at reducing the slope of its consumption profile and at controlling the size of consumption jumps, in fact the percentage drop is stabilized at a constant value. By reducing the slope and by stabilizing the jumps, the economy achieves the optimal consumption trajectory which is "smoothed out" as much as possible, leaving only the unpredictable component - the timing of jumps - affect its evolution. Even though a somewhat smoother profile can be implemented, *perfect* consumption smoothing is not achievable because of random *timing* of events.

The smoothing effect can be confirmed by comparing the optimal consumption growth rate of our economy with the one prevailing in a stochastic environment but with shocks of exogenous size. Suppose that random climate disasters arrive at the same Poisson rate but the survived share of the capital stock is exogenous and constant at $\bar{\omega} \in (0, 1)$. Assume further that the exogenous percentage drop in the capital stock is the same as the optimal one of our baseline model, so that $\bar{\omega} = \omega$. Then the trend growth rate of consumption is given by $\bar{g} = \frac{1}{\varepsilon} \left\{ A \left(1 - \frac{\phi}{\sigma} \right) - \rho + \lambda \left[\frac{U'(\bar{C})}{U'(C)} - 1 \right] \right\}$, where the ratio of marginal utilities is simply $\omega^{-\varepsilon} > 1$. It follows immediately that $\bar{g} > g^*$. Even though the drops in consumption are identical under both scenarios, the optimal consumption path associated with \bar{g} is steeper than that associated with g^* . The reason is that in the former case the economy, being unable to affect the size of the damage, is forced to choose a relatively high growth rate in order to build up an emergency buffer. In the latter case the economy endogenously controls the size of the jump and simultaneously chooses a lower trend rate as a precautionary measure. Such a growth path of the economy is also fundamentally different from its "expected" version which may be interpreted as a hypothetical extreme case of consumption smoothing. The expected growth rate in our baseline model, denoted by g^e , is given by

$$g^e \equiv \frac{d\mathbb{E}_t C_t / dt}{C_t} = \frac{1}{\varepsilon} \left\{ A \left(1 - \frac{\phi}{\sigma} \right) - \rho + \lambda \left[\omega^{1-\varepsilon} (1 + \varepsilon \ln \omega) - 1 + \varepsilon (\omega - 1) \right] \right\}. \quad (19)$$

We are now in a position to state the following

Lemma 1: *The solution to the maximization problem (1) - (9) is characterized by the optimal trend consumption growth rate which is (i) lower than that of the deterministic model without climate change induced disasters; (ii) lower than that of a stochastic model with fixed-damage disasters; but (iii) higher than the expected growth rate.*

Proof: Parts (i) and (ii) follow from the discussion above. Part (iii) follows from the fact that since $\omega < 1$, the last term inside the square brackets in (19) is negative and thus $g^e < g^*$. ■

As an illustration of the optimal path in the baseline model, we show in Figure 2 the consumption rate as a function of time. The solid line represents the stochastic path, which exhibits a trend growth rate g^* in the absence of climate events. At times t_1 and t_2 , negative environmental shocks are assumed to occur causing an immediate downward jump, followed by a subsequent period of growth at the previous rate. The dashed line shows the time profile of consumption under the *expected* growth scenario. There is a fundamental difference between the dashed and the solid curves in that the former smoothes out the jumps and discontinuities of the latter, creating an illusion of a perfect consumption smoothing and thereby ignoring the crucial effects of uncertainty.

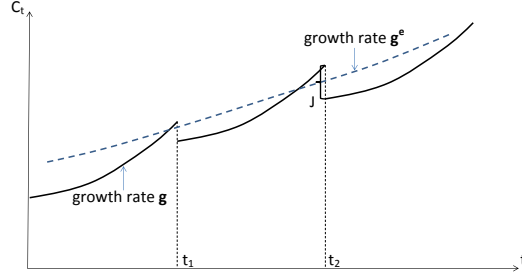


Figure 2: Optimal path (solid) vs. expected path (dashed).

We now turn to the optimal abatement policy. How much of the current resources to devote to emissions control is a key policy question. It turns out that it is optimal to allocate a specific constant fraction of output to abatement activities. Condition (14), together with the value function, allows us to obtain θ^* as a function of the parameters:

$$\theta^* = \frac{\phi}{\sigma} - \frac{\lambda \omega^{1-\varepsilon} \ln \omega}{A}. \quad (20)$$

The first term on the RHS represents abatement of current emissions. The second term represents additional abatement to reduce the stock of green-house gases already existing in the atmosphere. The latter is inversely related to the level of TFP and positively related to the arrival rate of disasters.

Proposition 1: *The solution of the maximization problem described by (1) - (9) is characterized by the following:*

- (i) *optimal consumption rate is a constant fraction of the capital stock;*
- (ii) *optimal abatement expenditure is a constant fraction of output;*
- (iii) *consumption, capital stock, output, and the overall abatement grow at the same constant rate in-between climate shocks.*

Proof: Given the value function, (i) follows directly from (11), so that consumption-to-capital ratio is constant and equal to ψ . The statement in (ii) follows from (20) and the fact that ψ is constant. Then g^* is the growth rate of C , K , and Y . Since abatement share is constant, total abatement expenditure grows at the same rate as well, which completes the proof. ■

Corollary 1: *In the case of logarithmic utility consumption-to-capital ratio, ψ^* , is independent of climate parameters.*

With log utility ($\varepsilon = 1$), Eq. (16) can be solved explicitly and after substitution into (18) and (20) we obtain:

$$\psi^* = \rho, \quad \theta^* = \frac{\delta\lambda}{\rho A x \sigma} + \frac{\phi}{\sigma}, \quad g^* = A(1 - \theta^*) - \rho.$$

Since ψ^* does not respond to any climatic parameters, the post-to-pre shock consumption ratio, ω , is independent of the hazard rate. Consequently, the optimal growth rate responds to λ only through its direct effect, $\partial g / \partial \lambda$ but not through the jump effect, $(\partial g / \partial \omega)(\partial \omega / \partial \psi)(\partial \psi / \partial \lambda)$. The direct effect reflects the precautionary "dissaving" motive and is negative (equal to $-\frac{\delta}{x\sigma\rho}$), contributing to a growth slowdown, while the indirect effect may either reinforce the direct effect or counteract it, depending on the value of ε . When log utility is assumed, the latter element disappears from the general picture. This observation motivates us to depart from the log-utility case in the quantitative assessment of the model presented in Section 6. One of our questions of interest will be to quantify the discrepancy in the optimal policy rules under unitary ($\varepsilon = 1$) and alternative values of the relative risk aversion (RRA) coefficient.

3 Effects of Economic and Climatic Fundamentals

3.1 The Abatement Propensity

In order to study the responses of the economy's optimal abatement policy, characterized by θ^* , to changes in the fundamental parameters of the model, we totally differentiate the system of Eqs. (16) and (20). Our key "climatic" parameters of interest are λ and δ which reflect the expected frequency and damage intensity of climate shocks, respectively. Among the "economic" parameters we consider σ , representing the efficiency of abatement technology, and ϕ , representing polluting intensity of production.

$$\overbrace{\begin{pmatrix} \varepsilon [1 - \lambda(1 - \varepsilon)\omega^{1-\varepsilon}\psi^{-1} \ln \omega] & 0 \\ \varepsilon\omega^{1-\varepsilon}\psi^{-\varepsilon-1}\lambda\delta [1 + (1 - \varepsilon) \ln \omega] & xA\sigma \end{pmatrix}}^M \times \overbrace{\begin{pmatrix} d\psi \\ d\theta \end{pmatrix}}^z = \underbrace{\begin{pmatrix} \Delta_{\psi\lambda} & \Delta_{\psi\delta} & \Delta_{\psi\sigma} & \Delta_{\psi\phi} \\ \Delta_{\theta\lambda} & \Delta_{\theta\delta} & \Delta_{\theta\sigma} & \Delta_{\theta\phi} \end{pmatrix}}_{\Delta} \times \underbrace{\begin{pmatrix} d\lambda \\ d\delta \\ d\sigma \\ d\phi \end{pmatrix}}_y,$$

where the exact expressions for Δ 's can be found in the Appendix. We can compactly rewrite the system as $Mz = \Delta y$. It is useful for future analysis to find the determinant of M :

$$|M| = M_{11}M_{22} = \varepsilon [1 + \lambda(1 - \varepsilon)\omega^{1-\varepsilon}\psi^{-\varepsilon-1}\delta(x\sigma)^{-1}] xA\sigma = \varepsilon xA\sigma \left[1 - \frac{\lambda(1 - \varepsilon)\omega^{1-\varepsilon} \ln \omega}{\psi} \right].$$

Since M_{22} is unambiguously positive, the sign of $|M|$ hinges on the sign of M_{11} , which is in general ambiguous. If $\varepsilon \in (0, 1]$, M_{11} is positive, while if $\varepsilon > 1$, M_{11} may become negative

under some constellations of parameter values, in particular, if $\varepsilon > \varepsilon^{M_{11}} \equiv 1 - \frac{\psi\omega^{\varepsilon-1}}{\lambda \ln \omega} > 1$. As noted by Pindyck (2013), if ε is viewed as a parameter reflecting opinions and objectives of policymakers, its estimates may range from 1 to 3. In order to get a rough idea whether $\varepsilon^{M_{11}}$ is anywhere near the empirically relevant range, we compute it assuming a plausible range of values for λ , ω and ψ . It turns out that $\varepsilon^{M_{11}}$ is beyond the empirically-relevant range of ε and thus we shall exclude the case $M_{11} < 0$.¹⁸

For the moment we are interested in the effects of the arrival rate (λ), damage intensity (δ), abatement efficiency (σ), and polluting intensity (ϕ). The effects of the remaining parameters can be computed in a similar manner. Applying the Cramer's rule, we have: $\frac{d\theta}{di} = \frac{|M_{\theta i}|}{|M|}$, where $i = \lambda, \delta, \sigma, \phi$, and $M_{\theta i}$ is the matrix M with the second column replaced by the column of Δ corresponding to i . Similarly, $\frac{d\psi}{di} = \frac{|M_{\psi i}|}{|M|}$, where $M_{\psi i}$ is the matrix M with the first column replaced by the column of Δ corresponding to i .

Proposition 2: *The abatement propensity is:*

- (i) *an increasing function of the arrival rate,*
- (ii) *an increasing function of the damage intensity,*
- (iii) *a decreasing function of the abatement efficiency and*
- (iv) *an increasing function of the polluting intensity.*

Proof:

$$\begin{aligned} \frac{d\theta^*}{d\lambda} &= -A^{-1}\omega^{1-\varepsilon} \{ \ln \omega - \lambda[(1-\varepsilon) \ln \omega + 1] \varepsilon \psi^{-1} \ln \omega (1 - \omega^{1-\varepsilon}) M_{11}^{-1} \} > 0, \\ \frac{d\theta^*}{d\delta} &= \lambda \omega^{1-\varepsilon} [(1-\varepsilon) \ln \omega + 1] \varepsilon \psi^{-\varepsilon} |M|^{-1} > 0, \\ \frac{d\theta^*}{d\sigma} &= -\frac{\phi}{\sigma^2} + \frac{\lambda \omega^{1-\varepsilon} \ln \omega}{A\sigma} [(1-\varepsilon) \ln \omega + 1] \left[1 - \frac{\varepsilon(1-\varepsilon)\theta^* A}{\psi M_{11}} \right] < 0, \\ \frac{d\theta^*}{d\phi} &= \frac{\varepsilon}{\sigma M_{11}} [1 + \lambda \omega^{1-\varepsilon} (1-\varepsilon)^2 (\ln \omega)^2 \psi^{-1}] > 0. \end{aligned}$$

Corollary 2: *In the case of logarithmic utility function ($\varepsilon = 1$), the abatement propensity is an increasing linear function of the hazard rate, of the damage intensity, and of the polluting intensity, and a decreasing convex function of the abatement efficiency.*

3.2 The Growth Rate

The effects on the growth rate are found as $\frac{dg^*}{di} = \frac{\partial g^*}{\partial i} + \frac{\partial g^*}{\partial \omega} \frac{\partial \omega}{\partial \psi} \frac{d\psi}{di}$.

¹⁸We are interested in the minimum possible threshold and thus need to consider only a minimum value for ψ . The share of consumption in the total capital stock, ψ , is computed as consumption share in GDP, multiplied by the total factor productivity, assumed to be 5%. With the consumption share as low as 60%, this gives a value of 0.03. Historically, high-impact events (small ω) are associated with rare occurrence (small λ) and low-impact events are more frequent. We consider a range of possibilities, from fairly common disasters to rare catastrophes. Consider first the former category. When the maximum value of λ is equal to 0.5 and the corresponding damage is 1% ($\omega = 0.99$), the threshold value of ε , denoted by $\varepsilon^{M_{11}}$, is above 6.5. For a more destructive but at the same time less frequent event ($\omega = 0.95$, $\lambda = 0.1$), $\varepsilon^{M_{11}} > 5.5$. In the case of very rare and extremely damaging events the threshold remains in a similar range. For $\lambda = 0.004$ and $\omega = 0.7$, $\varepsilon^{M_{11}} > 5.3$. For $\lambda = 0.001$ and $\omega = 0.5$, $\varepsilon^{M_{11}} > 4.5$. All of these values are beyond the empirically relevant range.

Proposition 3: *The optimal trend consumption growth rate is:*

- (i) *a decreasing function of the arrival rate,*
- (ii) *a decreasing function of the damage intensity,*
- (iii) *an increasing function of the abatement efficiency and*
- (iv) *a decreasing function of the polluting intensity.*

Proof:

$$\begin{aligned}
\frac{dg^*}{d\lambda} &= \frac{1}{\varepsilon} \left\{ \omega^{1-\varepsilon} (1 + \varepsilon \ln \omega) - 1 - \lambda \varepsilon \psi^{-1} \omega^{1-\varepsilon} (1 - \omega^{1-\varepsilon}) \ln \omega M_{11}^{-1} \Gamma \right\} < 0, \\
\frac{dg^*}{d\delta} &= -\ln \omega (\lambda \omega^{1-\varepsilon})^2 (1 - \varepsilon) \psi^{-\varepsilon-1} A |M|^{-1} \Gamma < 0, \\
\frac{dg^*}{d\sigma} &= \frac{A\phi}{\sigma^2} [1 + \lambda \omega^{1-\varepsilon} (\ln \omega)^2 (1 - \varepsilon)^2 \varepsilon \psi^{-1}] - (\lambda \omega^{1-\varepsilon} \ln \omega)^2 \Gamma \frac{(1 - \varepsilon)}{\psi \sigma} > 0, \\
\frac{dg^*}{d\phi} &= -\frac{\varepsilon A}{\sigma M_{11}} [1 + \varepsilon \lambda \omega^{1-\varepsilon} (1 - \varepsilon)^2 (\ln \omega)^2 \psi^{-1}] < 0,
\end{aligned}$$

where $\Gamma \equiv [(1 - \varepsilon)(1 + \varepsilon \ln \omega) + \varepsilon] > 0$.

While the results (i), (ii), and (iv) are self-explanatory, the statement in (iii) deserves a short interpretation. In general, the effect of the abatement efficiency on g^* is ambiguous. The first term in $\frac{dg^*}{d\sigma}$ is clearly positive, while the sign of the second term depends on ε . If $\varepsilon = 1$ it vanishes and if $\varepsilon > 1$ it is unambiguously positive. The fact that a higher abatement efficiency has an ambiguous bearing on economic growth is due to two effects - the interest-rate effect and the jump-smoothing effect - which work in opposite directions. On the one hand, an improvement in efficiency of abatement increases the economy's real interest rate and thus enhances the growth rate through the first term in Eq. (18). On the other hand, it increases the post-event consumption rate, shrinking the pre- to post-event consumption gap and thus contributes to a growth slowdown through the last term in Eq. (18). It turns out that with a relatively high RRA the latter effect is dominated by the former.

Polluting intensity also affects the growth rate through two channels. The first represents the direct effect stemming from a decline in the real interest rate. The second, the indirect effect, takes into account the change in the consumption jump, ω , through ψ . A higher polluting intensity requires a higher abatement share (Proposition 2(iv)) and thus it reduces the share of consumption in total capital. Both direct and indirect effects are negative and contribute to a growth slowdown.

Corollary 3: *In the case of logarithmic utility function the optimal trend consumption growth rate is a decreasing linear function of the damage intensity, of the arrival rate, of the polluting intensity, and an increasing concave function of the abatement efficiency.*

3.3 Saving propensity

We define the propensity to save, s , as the non-consumption share of output, i.e. a share of output spent on augmenting the capital stock plus on abatement (the latter being equal to

θ^*): $s = 1 - \psi/A$. It follows that the effects of all our parameters of interest $(\lambda, \delta, \sigma, \phi)$ on s are simply the opposite of those on ψ , divided by $A > 0$.

Proposition 4: *The propensity to save is:*

- (i) *an increasing function of the arrival rate,*
- (ii) *an increasing function of the damage intensity,*
- (iii) *a decreasing function of the abatement efficiency and*
- (iv) *an increasing function of the polluting intensity.*

Proof:

$$\begin{aligned}\frac{ds}{d\lambda} &= -(1 - \omega^{1-\varepsilon})x\sigma|M|^{-1} > 0, \\ \frac{ds}{d\delta} &= -\lambda\omega^{1-\varepsilon}(1 - \varepsilon)\psi^{-\varepsilon}|M|^{-1} > 0, \\ \frac{ds}{d\sigma} &= (1 - \varepsilon) [xA\phi + \lambda\omega^{1-\varepsilon}\psi^{-\varepsilon}\delta] (\sigma|M|)^{-1} < 0, \\ \frac{ds}{d\phi} &= -xA(1 - \varepsilon)|M|^{-1} > 0.\end{aligned}$$

Corollary 4: *In the case of logarithmic utility function, the saving propensity reduces to $1 - \frac{\rho}{A}$ and is therefore independent of the arrival rate, damage intensity, abatement efficiency and polluting intensity.*

In the case of log utility, all the parameters of interest lose their relevance (the derivatives of ψ and thus s become zero). An increase in the abatement efficiency σ causes a decrease in θ^* , implying that with unchanged s the share of output devoted to augmenting the capital stock increases. A better abatement technology in an economy with logarithmic preferences results in a smaller abatement share but a larger capital stock. On the other hand, an increase in the polluting intensity causes θ^* to rise, while s remains unchanged, implying that capital investment must fall. The same is true for an increase in the arrival rate of disasters and their damage intensity.

Note that when $\varepsilon \neq 1$ these effects might be mitigated because of the impacts on s which may go in the same direction as those on θ^* . For instance, an increase in the polluting intensity increases both s and θ^* . It can be shown that an increase in s is smaller than that in θ^* , implying that the abatement propensity rises at the expense of the investment share, which falls. The reduction in capital investment is, however, smaller than under logarithmic preferences.

The key endogenous variables, θ^* , g^* and s , respond non-linearly to changes in the economic and climatic fundamentals when we depart from log utility, although the direction of the effects is intuitively clear. A worsening in the shock's characteristics, i.e. an increase in their expected frequency and/or damage intensity, increases θ^* and s and reduces g^* . An improvement in abatement efficiency or a reduction in polluting intensity decreases θ^* , s , and raises g^* . The non-linearities in responses are important for a quantitative assessment of the optimal policy which we present in Section 6.

4 Tipping Point

So far we have considered natural disasters which arrive repeatedly and destroy a share of the economy's stock of capital. In the aftermath of a disaster the economy suffered a decline in its consumption but it had a possibility to grow and recover. Some authors point to the fact that a rise in the global temperature may cause substantial economic damages or even put the global economy into a state from which recovery will no longer be possible, passing a so-called "tipping point" (Tsur and Zemel 1996, Lenton and Ciscar 2013, Lemoine and Traeger 2014, de Zeeuw and Zemel 2014, van der Ploeg 2014, van der Ploeg and de Zeeuw 2016).

In natural sciences, a tipping point is a threshold at which a small disturbance in external conditions causes a rapid irreversible change of an ecosystem. In economics, the interpretation of the threshold has been adapted to refer to a point where the world economy suffers a substantial decline in GWP. A rise in the temperature above a (possibly uncertain) threshold may trigger irreversible changes in natural and geophysical processes such as, for example, melting of Greenland Ice Sheet, collapse of Atlantic thermohaline circulation, disintegration of West-Antarctic Ice Sheet, dieback of Amazon rainforest, etc. The exact economic impact and the time frame of these changes is not yet fully understood. Climate experts agree that they represent "high-consequence" outcomes which might result in at least a 25% loss of global income. Nordhaus and Boyer (2000) calculate a percentage loss of income in various regions to lie between 22.1 and 44.2%, assuming a 30% global loss and varying vulnerabilities among regions. The probability of such losses is estimated to be 1.2 percent when the temperature increases by 2.5°C and 6.8 percent for a 6°C increase.

The exact relationship between economic damages and either a temperature increase or an absolute temperature level is still, however, a matter of debate (Lenton and Ciscar 2013). Neither economists nor natural scientists converge to an unambiguous view on when and under which circumstances (e.g. level of carbon in the atmosphere) the world economy may hit one or several irreversibility thresholds and which ones. Although *crossing* of a tipping point may depend on the stock of GHG, the inflicted *damages* may not. It is likely that damages will be long-lasting and will no longer depend on the *contemporaneous* temperature (Lenton and Ciscar 2013). Since the link between the extent of damages and stock of GHG in the atmosphere is not yet clearly established, our modeling approach in this section will consist of treating damages as a random variable. They are assumed to be severe or even destructive with certain probabilities. We shall start by assuming exogenous arrivals in order to gain first insights into the behavior of the economy in terms of its optimal growth rate and abatement policy. In the next section we shall endogenize arrivals of natural disasters and a possible crossing of a tipping point. We believe it is an important property of our model to show how the economy develops after the occurrence of a tipping point, which so far has been neglected in the literature.

We assume that a catastrophic event follows a second Poisson process, labeled $q_t^{(2)}$, independent of $q_t^{(1)}$, where $q_t^{(1)}$ is the same process as in our benchmark model of Section 2. The arrival rates are labeled λ_2 and λ_1 , respectively, while the survived share of productive capacity

associated with $q_t^{(2)}$ is random and labeled $\bar{\beta}$. Formally,

$$\bar{\beta} = \begin{cases} \beta^d & \text{with probability } \pi \text{ ("destructive" tipping point),} \\ \beta^s > \beta^d & \text{with probability } 1 - \pi \text{ ("severe" tipping point), } \pi \in (0, 1). \end{cases}$$

The random variable $\bar{\beta}$ can take the value β^d with probability π or the value β^s with probability $1 - \pi$. In the former case, the event is "high-damage" and is associated with total economic collapse, in the sense that almost entire or possibly entire capital stock at time t is destroyed (β^d can be close to zero). In the latter case, the event is "severe-damage" (but non-destructive), in the sense that only a fraction $1 - \beta^s \in (0, 1)$ of the capital stock is destroyed. The fraction β^s can be small enough to represent a substantial loss of income if a collapse in one of the above-mentioned geophysical processes is triggered. Although we constrain β^s to lie between zero and unity, we have in mind damages which do not exceed 45% of GDP (keeping in mind the highest threshold used by Nordhaus and Boyer (2000) of 44.2%).

The HJB equation of the maximization problem may now be written as

$$\begin{aligned} \rho V(K, P) = \max \big\{ & U(C, P) + V_k [(1 - \theta)Y - C] + V_p [(\phi - \sigma\theta)Y - \alpha P] + \\ & + \lambda_1 [V(\bar{K}, P) - V(K, P)] + \mathbb{E}_{\bar{\beta}} \{ \lambda_2 [V(\bar{K}, P) - V(K, P)] \} \big\}, \end{aligned}$$

where $\mathbb{E}_{\bar{\beta}}$ is the expectation operator with respect to the distribution of $\bar{\beta}$, $\bar{K} = \omega K > 0$ as before, and $\bar{K} = \bar{\beta}K \geq 0$ is the remaining capital stock after a tipping point has been triggered. Given the distribution of $\bar{\beta}$, the last term can be simply written as $\lambda_2 [\pi V(\beta^d K, P) + (1 - \pi)V(\beta^s K, P) - V(K, P)]$. In the rest of this Section we denote the equilibrium values with an overbar (except for $\bar{\beta}$ which has been defined earlier). Following similar steps as in Section 2, one may verify that consumption-to-capital ratio, denoted by $\bar{\psi}$, is constant and is the solution of

$$\varepsilon\psi = \rho - (1 - \varepsilon)A \left(1 - \frac{\phi}{\sigma} \right) - \lambda_1(\omega^{1-\varepsilon} - 1) - \lambda_2 \left[\pi(\beta^d)^{1-\varepsilon} + (1 - \pi)(\beta^s)^{1-\varepsilon} - 1 \right], \quad (21)$$

where $\omega \equiv e^{-\delta[\psi^\varepsilon x \sigma]^{-1/\gamma}}$. The last term in Eq. (21) is unambiguously positive if $\varepsilon > 1$, which implies that $\bar{\psi} < \psi^*$, i.e. the economy of the benchmark model enjoys a higher consumption-capital ratio than the economy facing a possible tipping point. This is even more so the higher is the chance of crossing a destructive tipping point ($\pi \rightarrow 1$) and the smaller is the survived capital stock in the case of crossing ($\beta^d \rightarrow 0$).

Since the marginal utility of consumption is independent of the carbon stock, the optimality condition with respect to P is not affected by the presence of the tipping point possibility. Consequently, the optimal abatement share, labeled $\bar{\theta}$, has the same expression as in Eq. (20), although now it depends on $\omega(\bar{\psi})$, which is smaller than $\omega(\psi^*)$. Whether $\bar{\theta}$ is larger or smaller than θ^* depends on whether $d\bar{\theta}/d\bar{\omega} = -\lambda\bar{\omega}^{-\varepsilon}[1 + (1 - \varepsilon)\ln \bar{\omega}]/A$ is negative or positive, respectively. It can be verified that the expression in the square brackets is always positive for the relevant range of $\bar{\omega}$ and therefore $d\bar{\theta}/d\bar{\omega} < 0$. This result is also very intuitive: the smaller is the share of the capital stock which is expected to "survive" a disaster the larger is the

incentive to abate. We may thus conclude that $\bar{\theta} > \theta^*$.

The economy deals with the tipping risk not only by choosing a higher abatement share but also by adjusting its optimal rate of growth. The latter can be found from the optimality conditions with respect to consumption and capital stock and by applying the Itô's Lemma for jump processes on the differential of V_k :

$$\bar{g} = \frac{1}{\varepsilon} \left\{ \left(1 - \frac{\phi}{\sigma} \right) A - \rho + \lambda_1 [\bar{\omega}^{1-\varepsilon} (1 + \varepsilon \ln \bar{\omega}) - 1] + \lambda_2 \left[\pi (\beta^d)^{1-\varepsilon} + (1 - \pi) (\beta^s)^{1-\varepsilon} - 1 \right] \right\}. \quad (22)$$

We know that $\bar{\psi} < \psi^*$ implies that $\omega(\bar{\psi}) < \omega(\psi^*)$ and thus the term multiplying λ_1 in Eq. (22) is smaller than the term multiplying λ in Eq. (18). If $\varepsilon < 1$, the last term in (22) is non-positive and hence $\bar{g} < g^*$. If, however, $\varepsilon > 1$, the last term is positive, in which case \bar{g} might exceed g^* in spite of the fact that $\bar{\psi} < \psi^*$. The optimal time path of consumption is then rotated counterclockwise, with the initial consumption rate being smaller than in the baseline. This result is parallel to the precautionary saving motive. The intuition underlying this optimal strategy is related to the economy's willingness to accept a drop in consumption some day in the future. When the intertemporal elasticity of substitution is relatively low ($\varepsilon > 1$), the economy values relatively more a smoother time-profile of consumption as opposed to the total consumption possibilities over the planning horizon. This is why with $\varepsilon > 1$ the precautionary saving motive is stronger than with $\varepsilon \leq 1$. The resulting relatively high growth rate and the associated increase in emissions call for a more stringent climate policy. By adopting a high-saving and high-abatement policy the economy aims at reducing the damages from possible future shocks thus avoiding large discontinuities in its consumption path. We summarize the results in the following:

Proposition 5: *When the economy is facing a prospect of a severe and possibly total destruction of its productive capacity, then, compared to the economy without the tipping point risk,*

- (i) *its optimal consumption-to-capital ratio is smaller,*
- (ii) *its saving rate is higher,*
- (iii) *its abatement share is larger, and*
- (iv) *its growth rate is lower if $\varepsilon < 1$ and may become larger if $\varepsilon > 1$.*

What is the effect of a higher probability of a total economic collapse on the incentives to abate? We see that the RHS of Eq. (21) is increasing in π if $\varepsilon \in (0, 1)$ and is decreasing in π if $\varepsilon > 1$. Therefore, with a higher probability of a total collapse, the optimal consumption-to-capital ratio, $\bar{\psi}$, increases in the former case and decreases in the latter. Consequently, $d\bar{\theta}/d\pi$ is negative and, respectively, positive when $\varepsilon \leq 1$. The lower willingness to accept large swings in consumption (high ε) leads the economy to choose a higher abatement share and a lower consumption rate in anticipation of a possible large drop in consumption in the future. Alternatively, when the elasticity of intertemporal substitution is relatively high, a higher chance of a significant consumption cut in the future leads to exactly the opposite - an increase in the consumption rate and a decrease in the abatement share. A similar analysis can be conducted for an increase in the damages, i.e. a decline in either β^d or β^s . The optimal abatement share is increased only when the elasticity of intertemporal consumption

substitution is relatively low and decreased otherwise. With logarithmic utility function ($\varepsilon = 1$) all these effects disappear completely, so that neither the abatement propensity, nor the growth rate respond to a possibility of a large-scale climate catastrophe.

Corollary 5: *In the case of logarithmic utility function a possibility of a random crossing of a tipping point has no effect on the consumption-capital ratio, on the abatement share, on the growth rate, and on the saving propensity.*

In the current setting, even though a tipping point is entirely random, in the sense that neither its arrival nor damages can be influenced directly, the abatement policy is nevertheless more stringent than in the baseline model. The intuition is that a possibility of tipping affects consumption allocation across time and therefore it also affects the growth rate of the capital stock, which in turn determines the change in the pollution stock. In the next section we endogenize the arrival rate of disasters, including tipping points, and show an additional reason to increase abatement efforts.

Overall, our qualitative conclusions of this section are very much in line with the literature studying the impact of a tipping point possibility on climate policy. There is a general consensus, in both the theoretical contributions (Tsur and Zemel 1998, de Zeeuw and Zemel 2014) and in numerical models (Lemoine and Traeger 2014, Lontzek et al. 2015), that the policy should be more stringent, with the stringency being positively related to the economic damages and the probability of occurrence. We also establish that the increase in stringency only emerges when we depart from log utility, confirming that the strength of relative risk aversion constitutes an important element to be taken into consideration when formulating policy prescriptions.

5 Endogenous Arrivals

We have mentioned in the introductory section that climate scientists have diverging opinions on whether the frequency of natural disasters will increase in the future due to global warming or not. The IPCC report (2014) explicitly states that such a possibility exists. Recent contributions by van der Ploeg (2014), van der Ploeg and de Zeeuw (2016), and Zemel (2015) model the hazard rate endogenously, as an increasing function of the stock of carbon in the atmosphere.¹⁹ In this section we explore the implications of introducing an endogenous disaster arrival rate in our benchmark model. For the moment we shall write a general function $\hat{\lambda} = \hat{\lambda}(P_t, K_t)$ with both partial derivatives being positive: $\hat{\lambda}_k \equiv \partial \hat{\lambda} / \partial K > 0$, $\hat{\lambda}_p \equiv \partial \hat{\lambda} / \partial P > 0$

¹⁹Using the Duffie-Epstein stochastic differential utility framework, van der Ploeg and de Zeeuw (2016) find that the optimal carbon price increases in the face of a pending catastrophe to make the shock less imminent and that adjustments to saving are needed to smooth consumption. They also confirm numerically that assuming a RRA which is different from the inverse of the elasticity of intertemporal consumption substitution does not affect the results significantly. Zemel (2015) studies dynamic interactions between mitigation and adaptation activities, where the former reduces the risk of a harmful event, while the latter reduces the damage inflicted in case an event occurs nonetheless.

to capture the climate change and the exposure component. We use a " $\hat{\cdot}$ " to indicate that the arrival rate is endogenous and to distinguish it from the constant λ of our baseline model. We subsequently specify a possible functional form for this relationship. We continue to assume that if a disaster strikes, there is a positive probability $\pi \in (0, 1)$ that this disaster is a tipping point. Crossing a tipping point involves a detriment to some economic activity so that only a fraction $0 \leq \hat{\beta} < \omega$ of capital survives. Here we do not make a distinction between β^d and β^s as we did in the previous section and consider instead only one type of tipping with full destruction included as a special case $\hat{\beta} = 0$.

The HJB equation of the problem is similar to (10), except that now (i) the arrival rate depends on the stock of carbon and on the stock of capital; (ii) we need an expectation operator in front of the last term to capture a possible occurrence of a tipping point. The optimality conditions with respect to consumption and the abatement propensity remain unchanged. Only the conditions with respect to K and P are augmented by a term representing the change in the value function due to a change in the arrival rate and by a term capturing the survived capital in the aftermath of tipping. The exact expressions can be found in the appendix.

In order to make further progress, we need to specify how the arrival rate of climatic hazards is affected by economic activity. We assume that the arrival rate is an increasing and possibly non-linear function of a single variable, which captures the exposure and the climate-change effects through a Cobb-Douglas combination of K and P , similarly to how we modeled damages in our baseline model. Denoting this variable by \hat{v} , we may write $\hat{\lambda} = \hat{\lambda}(\hat{v})$, $\hat{\lambda}'(\hat{v}) > 0$, $\lim_{\hat{v} \rightarrow 0} \hat{\lambda} \geq 0$ and constant, where $\hat{v} = (K^\eta P^\xi)^\mu = v^\mu$ and $\mu > 0$ is used to differentiate the effect on the arrival rate from the effect on damages. We relegate the detailed derivations to the Appendix, while focusing only on the final results in the main text. We use a " $\hat{\cdot}$ " above a variable to indicate the equilibrium value (except for $\hat{\beta}$ which is constant).

The consumption-to-capital ratio, $\hat{\psi}$, is implicitly given in

$$\varepsilon \hat{\psi} = \rho - (1 - \varepsilon)A \left(1 - \frac{\phi}{\sigma}\right) - \hat{\lambda}(\hat{v}) \left[(1 - \pi)\hat{\omega}^{1-\varepsilon} + \pi\hat{\beta}^{1-\varepsilon} - 1\right], \quad (23)$$

where $\hat{v} = \left(\hat{\psi}^\varepsilon x \sigma\right)^{-\mu/\gamma}$ in equilibrium. It is possible that (23) has multiple solutions for some functional forms, especially non-linear, of $\hat{\lambda}$.²⁰ We discuss in the Appendix the conditions under which multiple solutions occur. When the value function is increasing in $\hat{\psi}$, the relevant solution is given by the largest root of (23).

Comparing Eq. (23) with (16), we may conclude that if $\hat{\lambda} \geq \lambda$ and $\varepsilon > 1$, the right-hand side of (23) is unambiguously smaller than that of (16), implying that $\hat{\psi} < \psi^*$. Moreover, since $d\hat{v}/d\hat{\psi} < 0$ and $\hat{\lambda}'(\hat{v}) > 0$, a smaller consumption-capital ratio increases the arrival rate, which reinforces our initial conclusion. If the utility function is logarithmic, the solution is unique: $\hat{\psi} = \psi^* = \rho$.

The optimality condition with respect to the pollution stock (see Appendix) allows us to

²⁰Van der Ploeg and de Zeeuw (2013) use a linear and a quartic specification, while van der Ploeg (2014) uses a linear function and three alternatives (quadratic, cubic and quartic) to calibrate the hazard function. The latter paper also shows that the results with quadratic, cubic and quartic specifications do not differ significantly from the results with a simple linear function (see Table 1, p. 38).

solve for the optimal abatement share:

$$\hat{\theta} = \frac{\phi}{\sigma} - \frac{(1-\pi)\hat{\lambda}\hat{\omega}^{1-\varepsilon}\ln\hat{\omega}}{A} - \frac{\hat{v}\hat{\lambda}'\mu}{A} \left[\frac{(1-\pi)\hat{\omega}^{1-\varepsilon} + \pi\hat{\beta}^{1-\varepsilon} - 1}{1-\varepsilon} \right], \quad (24)$$

Comparing Eq. (24) with Eq. (20), we note two main differences. First, in the expression for $\hat{\theta}$ the arrival rate depends on the consumption-to-capital ratio $\hat{\psi}$ through \hat{v} , while in the expression for θ^* the arrival rate is fixed. Second, the last term in (24) does not appear in (20). Similarly to van der Ploeg (2014), and adopting his terminology, the last term represents the "risk-averting effect," while the middle term is the tipping-adjusted "raising-the-steaks effect." To understand the intuition behind the expression for $\hat{\theta}$, consider first the simpler case without a tipping point by setting π to zero. Then the second term in (24) is similar to the second term in (20), except that the function ω^* has ψ^* as its argument, while the function $\hat{\omega}$ has $\hat{\psi}$. We have established that for $\hat{\lambda} \geq \lambda$ we have $\hat{\psi} < \psi^*$. Moreover, $d[\hat{\omega}^{1-\varepsilon} \ln \hat{\omega}]/d\hat{\psi} > 0$, so that the second term in (24) is smaller than that of (20), working to increase $\hat{\theta}$ compared to θ^* . Let us now turn to the last term in (24) involving $\hat{\lambda}'$. Clearly, its presence is warranted by the fact that the arrival rate is endogenous. Moreover, this marginal effect on the arrival rate is weighted by its contribution to the marginal change in the value of the program, represented by the expression in the square brackets. Without tipping points ($\pi = 0$), this is given by the value of the survived unit of capital relative to the status quo, the term $(\hat{\omega}^{1-\varepsilon} - 1)/(1 - \varepsilon)$. With a possible tipping point, this value is increased since $\hat{\beta} < \hat{\omega}$ by our assumption that damages from a tipping point are larger than from a weather hazard. Thus, the last term is negative if $\varepsilon > 1$ and, being subtracted from the first two, it contributes to an increase in $\hat{\theta}$. We would thus expect $\hat{\theta}$ to be larger than θ^* when $\hat{\lambda}$ is at least as large as λ , which seems to be plausible in light of the predictions from climate physicists (IPCC 2014).

We turn next to the optimal growth rate. With the optimality conditions with respect to C and K and application of the Itô's Lemma on the differential of V_k we obtain

$$\hat{g} = \frac{1}{\varepsilon} \left\{ A \left(1 - \frac{\phi}{\sigma} \right) - \rho + \hat{\lambda} \left[(1-\pi)\hat{\omega}^{1-\varepsilon}(1 + \varepsilon \ln \hat{\omega}) + \pi\hat{\beta}^{1-\varepsilon} - 1 \right] + \hat{\lambda}'\varepsilon\mu\hat{v} \left[\frac{(1-\pi)\hat{\omega}^{1-\varepsilon} + \pi\hat{\beta}^{1-\varepsilon} - 1}{1-\varepsilon} \right] \right\}. \quad (25)$$

Comparing (25) with (18), we see that the term $A \left(1 - \frac{\phi}{\sigma} \right) - \rho$, representing the standard Keynes-Ramsey component, is the same. The third term in (25) is similar to the respective term in (18), while the last term in (25) has no equivalent in the benchmark model. Assume for the moment that $\pi = 0$ (no tipping) and $\hat{\lambda} \geq \lambda$. Then, since $\hat{\psi} < \psi^*$ and $\hat{\omega} < \omega$, the third term inside the curly braces in (25) is smaller (i.e. more negative) than the respective term in (18) and hence it contributes to a growth slowdown. However, if the damages from tipping are sufficiently large ($\hat{\beta}$ is very small), this term may become positive and the effect may be reversed, that is the growth rate will be increased. Our numerical experiments show that such a scenario occurs only under very implausible parameter constellation. Turning to the last term in (25), which appears due to the endogeneity of the hazard rate and has no equivalent

in the benchmark model, we see that it is unambiguously negative for $\varepsilon > 1$, hence reducing the growth rate. A higher probability of tipping (larger π) and the associated damages (lower $\hat{\beta}$) increase the absolute value of this term, contributing to an even slower growth rate. We summarize the results in the following

Proposition 6: *When the disaster arrival rate is endogenous and is at least as large as the exogenous rate of the benchmark model,*

- (i) the optimal consumption-to-capital ratio is smaller,*
- (ii) the saving rate is higher,*
- (iii) the abatement share is larger, and*
- (iv) the growth rate is lower.*

The evidence from IPCC (2014) on rising frequencies of climate-driven natural disasters suggests that our near future might be characterized by arrival rates $\hat{\lambda}$ which are larger than, say, the known historical average λ . If this is the case, we conclude that a larger abatement propensity is warranted and the optimal growth rate of the world economy will have to be lower. The intuition behind is entirely driven by the precautionary considerations which dictate a lower growth rate of polluting input and a more aggressive abatement in order to reduce the probability of disasters and the associated damages.

6 Quantitative Implications

In this Section we explore the quantitative implications of our model and compare them with recent findings in the literature. Our overarching objective is two-fold: to quantitatively assess the optimal abatement propensity and the growth rate of the economy; and to provide a sensitivity analysis with respect to the key economic and climatic characteristics. In particular, we look at the sensitivity of the results to variations in risk aversion, efficiency of abatement technology, and characteristics of natural hazards. We also ask what the implications for the abatement policy are when an event is relatively common and low-impact vs. rare but high-impact. The latter question is to a large extent motivated by Stern's critique (Stern 2016) of current economy-climate models which, in his view, fail to adequately take into account the possibility of large-scale climate shocks. In our context, this task is essentially equivalent to assessing the responsiveness of the climate-policy instrument to changes in the severity and in the frequency of natural disasters.

The role of events' frequencies and damages has been recently considered by several authors. In an important contribution featuring a multi-catastrophe environment, Martin and Pindyck (2015) provide estimates of the society's willingness to pay (WTP) to avoid one or several disasters (e.g., mega-virus, nuclear and bioterrorism, climate change, etc.). Taking the climate catastrophe in isolation and assuming that in the next 50 years there is a 20% chance of experiencing a 20% reduction in GDP, the willingness to pay to avoid such an event is found to be 4.8% of consumption when $\varepsilon = 2$ and 18% when $\varepsilon = 4$. This example shows that WTP is

highly sensitive to the choice of RRA.²¹ One reason for the large jump in WTP as ε increases from 2 to 4 is that the model assumes a constant growth rate of consumption (also equal to the discount rate). We have shown earlier that the growth rate itself depends on ε and, in particular, it falls when ε rises, thus calling for a smaller WTP. Since our model allows for endogenous determination of the optimal consumption growth rate, we can elucidate the role of ε for both g and θ .

In the context of climate-policy analysis it is more common to look at the carbon price or carbon tax rather than WTP. In a recent article developing a climate DSGE model, Golosov et al. (2014) calculate the range of the optimal carbon tax of \$56.9 to \$496 per ton carbon in 2010 assuming alternative discount rates.²³ For a similar range of discount rates van der Ploeg (2014) finds that the optimal tax should be roughly a half, between \$29 and \$216 per ton. There is quite some divergence in the estimates of the optimal tax, with the main conclusion from the recent research being that the size of economic damages induced by climate change or a possibility of crossing a tipping point play a secondary role, while the discount rate seems to be the key parameter driving policy prescriptions.²⁴ This is one of the reasons why we chose to explicitly focus on recurring random catastrophic events as the main source of economic damages in order to elucidate their role in shaping the abatement policy. In our framework it is possible to infer the price of a carbon unit by dividing the abatement expenditure by total emissions in a given year.²⁵ The abatement expenditure is obtained by multiplying GWP with the optimal abatement propensity from Eq. (20).

6.1 Calibration

The reference unit of time is set to one year. In the benchmark calibration we assume the rate of time preference of 1.5% per year, as in Dietz and Stern (2015), Golosov et al. (2014), Nordhaus (2008) and van der Ploeg (2014).²⁶ These authors also assume a CRRA utility

²¹RRA parameter exerts two opposing effects on future welfare and thus on current climate policy. On the one hand, a larger ε implies that the marginal utility drops more quickly with an increase in consumption. If consumption is expected to grow, one extra unit in the future will yield a smaller marginal utility. On the other hand, ε reflects aversion to risk. So if future welfare is uncertain, its value will be smaller the larger is ε . Pindyck (2013) writes:²² "Most models show that unless risk aversion is extreme (e.g., η is above 4), the first effect dominates, which means an increase in η (say, 1 to 4) will reduce the benefits from an abatement policy." This sounds like rather bad news for climate policy if one uses an empirically plausible calibration for ε . However, the above reasoning does not take into account two important considerations. First, the optimal growth rate of the economy depends on ε and, second, it is determined *jointly* with the optimal climate policy. A higher ε leads to a decline in the optimal consumption growth and therefore to smaller future consumption rates as compared to the case where the dependency of the growth rate on ε is not taken into account. Since g falls, the former effect (the fall in marginal utility) - which is supposed to dominate unless ε is extremely high - is mitigated.

²³A tax range of \$28 to \$55/tC has been obtained by Barrage (2014), see e.g. Figure S.9 of her paper, after having introduced TFP growth, CRRA utility function and capital depreciation in the model of Golosov et al. (2014).

²⁴Dietz and Stern (2015) amend the standard DICE model with damages to the capital stock and to the total factor productivity. They show that even with the discount rate of 1.5% the extended DICE can produce considerably higher carbon taxes than the original DICE model. See also van den Bijgaart et al. (2016).

²⁵This measure of a carbon unit is not perfect since we do not know how much of emitted carbon has been abated in a given year. Thus our measure represents an upper bound.

²⁶The discount rate of 1.5% is a commonly-used value in the literature. We adopt this parametrization solely for comparison purposes and invite the reader to investigate the social and philosophical foundations of discounting in Stern (2015).

function. Golosov et al. (2014) use $\varepsilon = 1$ (log utility), which we adopt here as a starting point for the purpose of having a meaningful comparison. The relative weight on pollution in the utility function, χ , is set to unity. The parameter governing the curvature of the disutility of pollution, β , is also set to unity, which implies a quadratic disutility, often used in the literature. The carbon absorption capacity of natural sinks is set at $\alpha = 0.0038$ (see footnote 6).

To calibrate output emission intensity, ϕ , we take the data from the World Bank series " CO_2 emissions per GDP" (World Bank 2016), which reports average values for the period 2011-15 ranging from 0.1 kg per dollar of GDP for Sweden and France, 0.2 for Germany and the UK, 0.4 for the US, Canada and Brazil and up to 2.1 for China. We use a world average value of 0.4 kg, so that $\phi = 0.0004$ tons CO_2 per dollar of GDP.

As for abatement efficiency, σ , empirical studies (Hood 2011, McKinsey 2009) show that various abatement activities are inexpensive and thus relatively efficient; in the residential sector it applies to electronics, appliances, and insulation retrofit, in transportation, e.g., to hybrid cars, and in agriculture to tillage management. The marginal costs of these activities are reported to be negative or slightly positive amounting to less than \$5 per ton of CO_2 . Extending abatement activities through further policies, e.g. in the power sector and with reforestation, increases the costs substantially, although Hood (2011) concludes that "a significant level of emission abatement could be achieved with existing technologies at carbon prices of less than \$50 per ton of CO_2 ". With the highest value of the marginal cost curve of \$50 per ton, the average value lies in the range of \$10 to \$15. However, for reaching the 2°C target further emission reductions might become necessary in the future, including carbon capturing and sequestration, whose costs are estimated to lie between \$50 and \$100 per ton of CO_2 . One should note, however, that there are considerable learning effects in these new technologies, especially over a long time horizon. We therefore choose an average value of the various abatement measures and aim to include dynamic effects (learning) by setting $\sigma = 0.08$ in the benchmark calibration, which corresponds to \$12.5 per ton of abated CO_2 . We shall also consider an alternative value of \$20 ($\sigma = 0.05$) when technology development is viewed in a more pessimistic way. For the total factor productivity, A , we adopt a moderate value of 5%.

Statistics for large-scale natural catastrophes over the last few decades suggest varying arrival rates and damages for different types of disasters. The Indian Ocean Tsunami in 2004 caused at least \$10 bn worth of damage and affected six countries: Indonesia, India, Maldives, Sri Lanka, Somalia, and Thailand. The damage amounted to 0.86% of the sum of GDPs in 2004 of the affected countries (Somalia not included due to lacking GDP data in WDI). Hurricane Katrina in 2005 caused \$108 bn damage which amounted to 0.825% of the US GDP. Typhoon Haiyan in the Philippines in 2013 caused \$2.8 bn damage, equivalent to 1.05% of GDP. Cavallo et al. (2013) count 2597 natural disasters (floods and storms, including hurricanes) worldwide during the period 1970–2008, which implies an average annual arrival rate of 0.34. Assuming that 10 percent of the shocks are climate-related events yields $\lambda = 0.034$. With respect to larger shocks, Pindyck and Martin (2015) refer to the IPCC to assume that there is a 20% chance that a catastrophic-climate outcome occurs in the next 50 years, which implies that $\lambda = 0.004$.

In our quantitative assessment we shall consider two scenarios: (i) relatively low damage

intensity of disasters ("low" δ) and (ii) relatively high damage intensity of disasters ("high" δ).²⁷ Within each scenario we distinguish among three arrival rates, two abatement efficiencies, and two values of the relative risk aversion parameter. The arrival rates correspond to a 20% chance of a disaster occurring in the next 50, 20, and 10 years, corresponding to $\lambda = 0.004$, $\lambda = 0.01$ and $\lambda = 0.02$, respectively. The abatement efficiencies are $\sigma = 0.08$ (\$12.5/t CO_2) and $\sigma = 0.05$ (\$20/t CO_2), as discussed earlier.

The relative risk aversion (RRA) coefficient, ε , is a subject of an ongoing debate in the theoretical and empirical literature. Recall that we have assumed, following the risk-vulnerability literature, that preferences for risk-taking are affected by exposure to damages. In particular, RRA is proportional to the parameter which governs the exposure effect in the damage function, η , with a constant proportionality coefficient γ . Calibrating risk aversion seems to be more straightforward from the data availability point of view. This is why instead of calibrating η , for which the data are not readily available, we calibrate ε and then perform sensitivity analysis with respect to both ε and γ . This approach also allows us to meaningfully compare our results with those in the existing studies. We consider two calibrations for ε and check sensitivity of the results to variations in γ (in the Appendix), with the benchmark values set to unity for both (log utility). For any given ε , a value of γ larger (resp., smaller) than unity would indicate a reduction (resp., increase) in η and therefore a reduction (resp., increase) in damages. The benchmark parameter values are summarized in Table 1.

ρ	χ	β	α	ε	A	ϕ	σ	λ	δ	γ
0.015	1	1	0.0038	1	0.05	0.0004	0.08	0.02	$1e - 6$	1

Table 1: Benchmark values of parameters.

6.2 Low-impact Events

Let us analyze the results pertaining to the first scenario (low δ), summarized in Table 2. In the left-hand panel ($\varepsilon = 1$), with a more optimistic abatement efficiency ($\sigma = 0.08$) and a 20% chance of experiencing a climate-change driven disaster in the next 50 years ($\lambda = 0.004$), we find the optimal abatement propensity of 0.5% and the optimal growth rate of 3.475%.

To calculate the associated carbon price, we multiply θ^* with the world output and divide

²⁷The intensities $\delta = 1e - 6$ ("low") and $\delta = 1e - 5$ ("high") correspond to damages of less than 0.1% of gross world product (GWP) and 5 – 10% of GWP, respectively. Recall that in our model damages are endogenous and hence depend not only on δ but on all other parameters. Calibrating δ proved to be a challenging task as there is no direct mapping to this parameter in the data. In order to circumvent this issue we chose the values which deliver average worldwide damages from natural disasters over the last five years. Several sources converge on those damages being in the range of several hundred billion USD for 2011-2015 (see http://www.economist.com/blogs/dailychart/2011/03/natural_disasters, <http://www.theonebrief.com/the-impact-of-natural-disasters-on-the-global-economy>, <https://www.weforum.org/agenda/2015/12/how-much-do-natural-disasters-cost-the-world>, http://www.kit.edu/kit/english/pi_2016_058_natural-disasters-since-1900-over-8-million-deaths-and-7-trillion-us-dollars-damage.php). In particular, our lower value of δ corresponds to an average worldwide damage of approximately \$370 bn which reflects the value in the CATDAT database.

	$\varepsilon = 1$		$\varepsilon = 3$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda = 0.004$				
θ^*	0.500	0.800	0.541	0.867
g^*	3.475	3.459	1.158	1.152
$\lambda = 0.01$				
θ^*	0.500	0.801	0.603	0.967
g^*	3.475	3.459	1.157	1.151
$\lambda = 0.02$				
θ^*	0.501	0.801	0.706	1.136
g^*	3.475	3.459	1.155	1.148

Table 2: The optimal policy with low damage intensity: $\delta = 1e - 6$ (up to 1% of GWP).

by tons of carbon emissions using the latest available data. Reuters and World Bank report that global emissions in 2014 amounted to 10.7 bn tons of carbon so that, with the global world output in 2014 at \$78.28 trillion, the implied world carbon price amounts to \$36.6 per ton.²⁸ This value is robust to changes in the disaster arrival rate. Specifically, increasing the hazard rate from 0.004 to 0.02 has no significant impact on the optimal abatement propensity and growth. We shall show shortly that this outcome is strictly linked to the logarithmic utility assumption and, to some extent, to the low damage intensity of climate disasters. By contrast, changing abatement efficiency from a relatively high value ($\sigma = 0.08$) to a lower value ($\sigma = 0.05$) brings about an increase in the optimal abatement propensity from 0.5% to 0.8% and the corresponding carbon price rises to \$58.52 per ton which is comparable to the baseline value \$56.9/tC obtained by Golosov et al. (2014). Reduced abatement efficiency induces an only slightly lower optimal growth rate of 3.46%.

Exposure to climate risks may lead to a higher degree of risk aversion according to the risk-vulnerability hypothesis. This suggests that the unitary value of ε may be too low. We thus consider a higher value: $\varepsilon = 3$. We find that under the benchmark calibration the optimal abatement propensity increases slightly from 0.5% to 0.54%.²⁹ The optimal growth rate, however, drops from 3.47 to 1.16%. When abatement efficiency is less favorable, θ^* rises from 0.8 to 0.87% and the growth rate is significantly reduced from 3.46 to 1.15%. To examine the sensitivity of the results to variations in γ , we replicate Table 2 for $\gamma = 0.9$ and $\gamma = 1.1$ in Appendix D.1. A higher (lower) value of γ for a given ε implies a lower (higher) exposure to damages and hence a lower (higher) optimal abatement propensity. Comparing the results

²⁸The caveat of this approach is that we use yearly emissions, which are in fact net emissions, that is after some abatement has taken place in a given year. We therefore treat our estimated carbon price as indicative and focus on the abatement propensity as our main policy variable of interest.

²⁹This result is consistent with Barrage (2014) who finds that an increase in RRA from 1 to 2 increases the optimal carbon tax only marginally, see Figure S.10 and Figure S.11 of her paper, from about \$56/tC to slightly over \$60/tC.

	$\varepsilon = 1$		$\varepsilon = 3$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda = 0.004$				
θ^*	0.502	0.802	0.956	1.589
g^*	3.475	3.459	1.149	1.139
$\lambda = 0.01$				
θ^*	0.504	0.806	1.662	2.846
g^*	3.475	3.459	1.137	1.114
$\lambda = 0.02$				
θ^*	0.508	0.812	2.905	5.178
g^*	3.475	3.454	1.114	1.068

Table 3: The optimal policy with high damage intensity: $\delta = 1e - 5$ (up to 10% of GWP).

from Table 2 with those in either Table 6 ($\gamma = 0.9$) or Table 7 ($\gamma = 1.1$), we find no significant changes in either θ^* or g^* for $\varepsilon = 1$ and only a few percentage points differences in θ^* (but not in g^*) for $\varepsilon = 3$. We anticipate that climate shocks with a larger damage intensity may profoundly alter the optimal policy. In addition, the impact of a higher ε has to be examined more carefully in the context of more severe disasters.

6.3 High-impact Events

We consider next Table 3 with the same parameter constellation except for δ which is increased 10-fold to deliver damages of up to 10% of GWP.³⁰ First note that with log-utility the optimal abatement propensity and the optimal growth rate are not affected in a major way. However, when we set ε to 3, the picture changes significantly. First, looking at the top panel ($\lambda = 0.004$) and the optimistic abatement efficiency case ($\sigma = 0.08$), we already find that the optimal abatement propensity rises significantly from 0.541% to 0.956%, which corresponds to an increase in the carbon price from about \$40 to about \$70 per ton. The growth rate is reduced only marginally from 1.16 to 1.15%. Second, moving to the less optimistic abatement efficiency scenario ($\sigma = 0.05$) increases the abatement propensity from 0.96 to 1.6%. Third, with a more frequent disasters ($\lambda = 0.02$), the abatement propensity jumps 4-5 fold from 0.706 and 1.136 (for $\sigma = 0.08$ and $\sigma = 0.05$ in Table 2, resp.) to 2.9 and 5.18%, respectively. The latter implies a carbon price of \$379 per ton. At the same time the growth rate is reduced from 1.16 and 1.15 to 1.11 and 1.07%, respectively.

Finally, we can assess the difference between high-impact rare events and more common low-impact events. To this end we compare the results from the top panel of Table 3 with the

³⁰Since damages are endogenous in our model, they vary between 5 and 10% of GWP, depending on the values of other parameters.

results from the bottom panel of Table 2. This corresponds to moving from a scenario with a 20% chance of experiencing a 0.05% reduction in GDP in the next 10 years to a scenario with a 20% chance of experiencing a 5% drop in GDP in the next 50 years. Under log utility there is almost no change in either the abatement propensity or the growth rate, regardless of the value of σ . With $\varepsilon = 3$ the growth rate reacts relatively moderately by falling by about half a basis point. The abatement propensity, by contrast, reacts more strongly with an increase of 25 basis points (from 0.706 to 0.956). An even stronger increase of 45 basis points is observed for a less optimistic abatement technology. Even if we constrain the expected damage to be exactly identical in both cases (we reduce λ down to 0.002073), the abatement propensity is still higher ($\theta = 1.205$) and the growth rate is lower ($g = 1.145$). We conclude that a possibility of rare but high-impact events calls for a more stringent abatement policy as compared to relatively frequent but low-impact events (keeping expected damages identical). Our results provide strong support of Stern's hypothesis that optimal climate policy becomes more stringent once rare high-impact events are taken into consideration.

6.4 Tipping Point

We turn next to a quantitative assessment of the model which includes a tipping point scenario. We are aware of the fact that an attempt to calibrate such a scenario is doomed to be controversial. Nonetheless, rough assessment remains feasible when based on recent studies from Earth system sciences and expert elicitation. It has been identified that five tipping elements might be triggered by the year 2200 under three alternative temperature corridors (low=0.5-2°C, medium=2-4°C, high>4°C), which are meltdown of Greenland ice sheet, disintegration of the West Antarctic ice sheet, reorganization of the Atlantic thermohaline circulation, increased amplitude of the El Niño Southern Oscillation, and dieback of the Amazon rainforest (Kriegler et al. 2009). The probability of tipping at least one element under the medium warming scenario is assessed at 16% minimum and under high warming scenario at over 56% (Lenton and Ciscar 2013). We calibrate the arrival rate of a tipping point in the former, "optimistic", case as a 16% tipping chance over the next 190 years, $\lambda_2 = 0.16/190 = 0.0008421$, and in the latter, "pessimistic" case as $\lambda_2 = 0.56/190 = 0.0029473$.

Calibration of damages is even more disputable. As argued by Lenton and Ciscar (2013), there are at least three points to be taken into consideration. First, different tipping elements have a different time frame of associated damages. Some may occur within a year, others within a millennium. Second, damages are likely to have a regional aspect. Third, some tipping elements might be reversible, at least in theory. Lenton et al. (2008) thoroughly document nine tipping elements with their critical values, associated global temperature level, transition timescale and key impacts. The key impacts are extensively described by the authors, although they do not provide any even rough estimates of potential damages in terms of GWP loss or similar. We shall thus make the following simple assumption about our betas: the destructive damage is equal to 90% loss of productive capacity ($\beta^d = 0.1$) and the non-destructive damage is the loss of 30% ($\beta^s = 0.7$). The former loss may appear to be large. It can be justified if we think of long-lasting damages spread over decades if not centuries. The latter value is the

	$\lambda_2 = 0.00084$		$\lambda_2 = 0.00294$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda_1 = 0.004$				
$\bar{\theta}$	0.547	0.877	0.568	0.911
\bar{g}	1.324	1.319	1.740	1.735
$\lambda_1 = 0.01$				
$\bar{\theta}$	0.618	0.992	0.671	1.080
\bar{g}	1.323	1.317	1.739	1.732
$\lambda_1 = 0.02$				
$\bar{\theta}$	0.737	1.186	0.844	1.365
\bar{g}	1.321	1.314	1.736	1.727

Table 4: The optimal policy with low damage intensity: $\delta = 1e - 6$.

same as in Nordhaus and Boyer (2000) and Golosov et al. (2014). The chance that a tipping point happens to be destructive is assumed to be 5% ($\pi = 0.05$). The rest of the parameters are set at our benchmark values and $\varepsilon = 3$. We present the estimates of $\bar{\theta}$ and \bar{g} in Table 4 (low δ) and Table 5 (high δ).

Focusing first on Table 4, we find that our estimates are similar to those of Table 2 regardless of the value of λ_2 , with $\bar{\theta}$ and \bar{g} being slightly higher. By contrast, when environmental shocks are characterized by a higher degree of severity (see Table 5), the abatement propensity increases significantly. With relatively frequent weather hazards ($\lambda_1 = 0.02$) and a relatively high efficiency of abatement, $\bar{\theta}$ reaches 3.3% of GWP or \$244t/C, which is close to the carbon tax of \$229 found by Dietz and Stern (2015) in their "high damage" to capital stock scenario with random climate sensitivity parameter. With a less efficient abatement technology $\bar{\theta}$ rises to 6.1% or \$447t/C which is close to the \$489 estimate found by Golosov et al. (2014) in a scenario with catastrophic damages (defined as loss of 30% of GDP). In our "pessimistic" tipping scenario but with a more favorable abatement productivity θ remains relatively high at 5.12%, as compared to 2.9% in Table 3, but jumps to a striking 11% when abatement productivity is less favorable.

Our quantitative analysis leads to four conclusions. First, log-utility assumption in models of climate change is innocuous only if environmental shocks are "not too severe", i.e. they are characterized by a relatively low damage intensity. This is likely not to be the case for climate change induced natural disasters. Damages from such catastrophes have amounted to approximately 1% of GDP of countries where they happened to strike. Intensity of disasters will worsen even further due to the planet's warming, according to predictions of climate physicists. Second, efficiency of abatement technology plays an important role as well and more so when disasters are high-impact. Decreasing abatement efficiency by about one third requires an increase in the abatement propensity by more than a half if shocks are low-impact and by three quarters when they are high-impact. Third, a prospect of rare but high-impact

	$\lambda_2 = 0.00084$		$\lambda_2 = 0.00294$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda_1 = 0.004$				
$\bar{\theta}$	1.030	1.729	1.312	2.289
\bar{g}	1.315	1.302	1.726	1.706
$\lambda_1 = 0.01$				
$\bar{\theta}$	1.858	3.232	2.622	4.880
\bar{g}	1.299	1.272	1.701	1.650
$\lambda_1 = 0.02$				
$\bar{\theta}$	3.336	6.113	5.123	10.989
\bar{g}	1.272	1.214	1.651	1.513

Table 5: The optimal policy with high damage intensity: $\delta = 1e - 5$.

events calls for a more stringent mitigation policy than of low-impact frequent events with the same expected damage. Finally, a prospect of triggering a tipping point requires a further increase in the abatement propensity by at least a factor of two. In a high-temperature corridor this may represent a substantial expenditure on mitigation of up to 10% of GWP.

7 Conclusions

An increase in the global temperature is predicted to render natural disasters, e.g. tropical storms, hurricanes, tsunamis, floods, droughts, etc., more severe and intense. Such calamities have a profound negative impact on the economy's infrastructure, physical and human capital, and they undoubtedly represent a set-back in terms of economic growth and development. An efficient and timely climate policy is necessary in order to limit damages from such devastating shocks.

In the present article we propose a model of a growing economy subject to random natural disasters, which destroy part of the economy's productive input. An important feature of our model is consideration of recurring shocks where the extent of the damage is endogenously determined through the interaction between capital accumulation process and an appropriate emissions abatement policy. We deliver clear analytical solutions and emphasize the importance of *simultaneous* consideration of the optimal dynamic behavior of the economy and of the climate policy.

A higher hazard rate of natural disasters and a larger damage intensity unambiguously reduce the economy's growth rate and call for a more stringent climate policy. Arrivals of rare but high-impact events necessitate a more vigorous abatement policy as compared to relatively frequent but low-impact events with identical expected damages. When a risk of a tipping point is introduced the optimal abatement propensity increases, while the growth rate may rise or fall depending on the strength of the precautionary effect.

In the case of a unitary elasticity of marginal utility (logarithmic preferences), often used in the literature on the grounds of better tractability, the consumption-to-capital ratio and the propensity to save are independent of the climatic parameters. Also, with log-preferences a possibility of a tipping point does not alter the optimal mitigation expenditure. In light of this, policy-relevance and applicability of findings stemming from models based on the log-utility assumption seem to suffer from some limitations. Assuming a non-unitary relative risk aversion reveals important macroeconomic interdependencies and alters policy conclusions significantly.

We also provide quantitative results by calibrating our model to the recent data on global carbon emissions, output, frequency of large natural catastrophes and their damages. With log utility, the share of output which should be devoted to emissions control is approximately 0.5% of GWP. This number is comparable to what has been found in the recent studies which relied on log-utility assumption. However, when we use a higher value for the relative risk aversion parameter, we find that the abatement propensity increases significantly, reaching 3-5% of GWP, as we consider higher degrees of severity of environmental shocks. These values are equivalent to a carbon price in the range \$212-\$370 per ton. With an additional possibility of hitting a tipping point the abatement propensity rises to over 6% of gross world output in a best-case (medium-temperature corridor) scenario and to 13.8% in the most pessimistic case (high-temperature corridor). If one takes the side of climate physicists who believe that climate change will cause an increase in disaster frequency - in addition to the damage intensity - then an even more stringent climate policy becomes warranted.

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Appendices

A Appendix to Section 2

A.1 Solution to the baseline model

Hamilton-Jacobi-Bellman equation may be written as

$$\rho V(K, P) = \max_{C, \theta} \left\{ U(C, P) + V_k[(1 - \theta)Y - C] + V_p[(\phi - \theta\sigma)Y - \alpha P] + \lambda [V(\tilde{K}, P) - V(K, P)] \right\}, \quad (\text{A.1})$$

and the first-order conditions with respect to the control and the state variables consist of

$$C : U_c - V_k = 0, \quad (\text{A.2})$$

$$\theta : -V_k Y - V_p \sigma Y = 0, \quad (\text{A.3})$$

$$K : \rho V_k = V_{kk}[(1-\theta)Y - C] + V_k(1-\theta)A + V_p(\phi - \sigma\theta)A + V_{pk}[(\phi - \sigma\theta)Y - \alpha P] + \lambda \left(\tilde{V}_k \frac{d\tilde{K}}{dK} - V_k \right), \quad (\text{A.4})$$

$$P : \rho V_p = U_p + V_{pp}[(\phi - \sigma\theta)Y - \alpha P] + V_{kp}[(1-\theta)Y - C] + \lambda \tilde{V}_k \frac{d\tilde{K}}{dP} - \alpha V_p + \lambda (\tilde{V}_p - V_p). \quad (\text{A.5})$$

Suppose the value function is of the form

$$V(K, P) = \frac{\psi^{-\varepsilon} K^{1-\varepsilon}}{1-\varepsilon} - \frac{x P^{1+\beta}}{1+\beta},$$

then from (A.2) and (A.3),

$$C = \psi K \quad (\text{A.6})$$

$$(\psi K)^{-\varepsilon} = x \sigma P^\beta. \quad (\text{A.7})$$

Verification of the value function proceeds by substituting the optimal policy (A.6)-(A.7) into the HJB equation. The constants ψ and x are then found by the method of undetermined coefficients.

Substitution of the optimal policy into (A.5) allows us to solve for θ , while (13) allows us to obtain g .

A.2 Solution for ψ

The implicit solution for ψ is provided in

$$\varepsilon \psi = \rho + (\varepsilon - 1)A \left(1 - \frac{\phi}{\sigma} \right) - \lambda (\omega^{1-\varepsilon}(\psi) - 1), \quad (\text{A.8})$$

where $\omega(\psi) = e^{-\delta \psi^{-\varepsilon} (x\sigma)^{-1}}$. Consider the functions on the left-hand side and on the right-hand side and call them $LHS(\psi)$ and $RHS(\psi)$, respectively. The former is a simple straight line from the origin with the slope ε . The latter is a non-linear function with the slope:

$$\frac{dRHS}{d\psi} = \lambda(1-\varepsilon)\varepsilon\psi^{-1}\omega^{1-\varepsilon} \ln \omega \geq 0 \Leftrightarrow \varepsilon \geq 1$$

and the second derivative

$$\frac{d^2 RHS}{d\psi^2} = -\lambda(1-\varepsilon)\varepsilon\psi^{-2}\omega^{1-\varepsilon} \ln \omega [(1-\varepsilon)\varepsilon \ln \omega + 1 + \varepsilon] \geq 0 \Leftrightarrow \varepsilon \leq 1.$$

It follows that when $\varepsilon < 1$, $RHS(\psi)$ is decreasing and convex in ψ . Thus, if a solution to Eq. (A.8) exists, it is unique. The solution exists provided that $\lim_{\psi \rightarrow 0} RHS > 0$, that is: $\rho - (1 - \varepsilon)A \left(1 - \frac{\phi}{\sigma}\right) + \lambda > 0$. When $\varepsilon > 1$, $RHS(\psi)$ is increasing and concave with the limit equal to minus infinity as $\psi \rightarrow 0$. Thus, zero, one or two solutions may exist. If the slope of RHS is larger than ε for any ψ , then a unique solution is guaranteed. This amounts to showing that $\psi < \lambda(\varepsilon - 1)\omega^{1-\varepsilon}(-\ln \omega)$. Recall that ψ on the left-hand side of the inequality is between zero and one since it represents a fraction of consumption in total capital stock. As far as the right-hand side is concerned, although the term $\omega^{1-\varepsilon}$ is larger than unity, the other terms, namely λ , $(\varepsilon - 1)$ and $-\ln \omega$ can be smaller than unity, so that the inequality is ambiguous and a possibility of 2 or 0 roots cannot be ruled out. Possible solutions to (A.8) are illustrated graphically in figure 3.

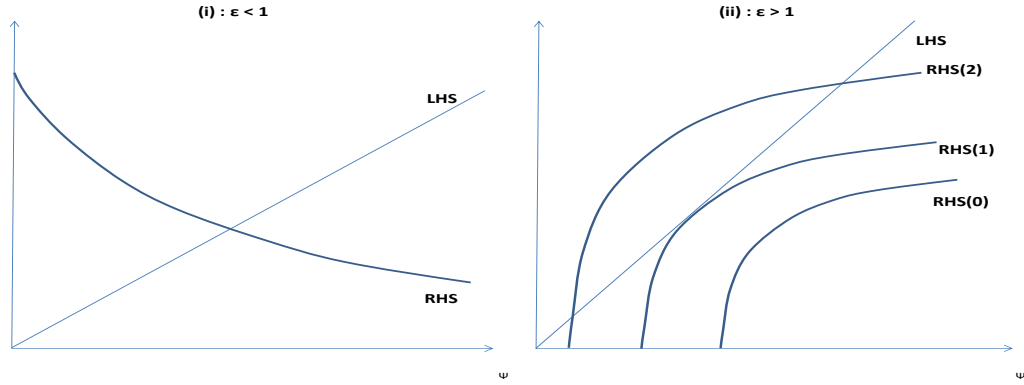


Figure 3: Possible solutions for ψ .

It can be shown that for $\varepsilon \geq 1$ the value function increases in ψ . First note that, ignoring jumps (i.e. when $dq = 0$):

$$\frac{dV(K, P)}{d\psi} = \frac{(\psi K)^{-\varepsilon}}{\varepsilon - 1} \frac{K}{\psi} \left[\varepsilon + (\varepsilon - 1) \frac{dK}{d\psi} \frac{\psi}{K} \right] + \frac{\varepsilon (\psi K)^{-\varepsilon} \psi^{-\varepsilon - 1}}{\sigma(x\sigma)^{1/\beta}} \left[1 + \frac{dK}{d\psi} \frac{\psi}{K} \right] > 0,$$

where $dK/d\psi = tK \frac{\lambda}{\varepsilon} [1 + (1 - \varepsilon)\varepsilon \ln \omega] \omega^{-\varepsilon} \frac{d\omega}{d\psi} > 0$. Since $d\omega/d\psi > 0$, the jump term also increases in ψ . Therefore the value function increases in ψ , so that in the case of two roots only the second one is relevant.

B Appendix to Section 3

B.1 Comparative Statics

$$\overbrace{\begin{pmatrix} \varepsilon [1 - \lambda(1 - \varepsilon)\omega^{1-\varepsilon}\psi^{-1} \ln \omega] & 0 \\ \varepsilon\omega^{1-\varepsilon}\psi^{-\varepsilon-1}\lambda\delta [1 + (1 - \varepsilon) \ln \omega] & xA\sigma \end{pmatrix}}^M \times \overbrace{\begin{pmatrix} d\psi \\ d\theta \end{pmatrix}}^x = \underbrace{\begin{pmatrix} \Delta_{\psi\lambda} & \Delta_{\psi\delta} & \Delta_{\psi\sigma} & \Delta_{\psi\phi} \\ \Delta_{\theta\lambda} & \Delta_{\theta\delta} & \Delta_{\theta\sigma} & \Delta_{\theta\phi} \end{pmatrix}}_{\Delta} \times \underbrace{\begin{pmatrix} d\lambda \\ d\delta \\ d\sigma \\ d\phi \end{pmatrix}}_y,$$

where

$$\Delta' = \begin{pmatrix} 1 - \omega^{1-\varepsilon} & \delta\psi^{-\varepsilon}\omega^{1-\varepsilon} \\ \lambda(1 - \varepsilon)\omega^{1-\varepsilon}\psi^{-\varepsilon}(x\sigma)^{-1} & \lambda\psi^{-\varepsilon}\omega^{1-\varepsilon} [1 - \delta(1 - \varepsilon)\psi^{-\varepsilon}(x\sigma)^{-1}] \\ \frac{\varepsilon-1}{\sigma^2} (A\phi + \lambda\omega^{1-\varepsilon}\delta\psi^{-\varepsilon}x^{-1}) & \frac{1}{\sigma} [\lambda\delta\psi^{-\varepsilon}\omega^{1-\varepsilon} [(1 - \varepsilon)\delta\psi^{-\varepsilon}(x\sigma)^{-1} - 1] - \phi xA] \\ (1 - \varepsilon)\frac{A}{\sigma} & xA \end{pmatrix}$$

and

$$|M| = M_{11}M_{22} = \varepsilon [1 - \lambda(1 - \varepsilon)\omega^{1-\varepsilon}\psi^{-1} \ln \omega] xA\sigma.$$

Consumption-Capital Ratio

$$\begin{aligned} \frac{d\psi}{d\lambda} &= \frac{|M_{\psi\lambda}|}{|M|} = (1 - \omega^{1-\varepsilon})xA\sigma|M|^{-1} \begin{cases} > 0, \text{ if } \varepsilon \in (0, 1) \text{ or } \varepsilon > 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta} \\ \leq 0, \text{ if } 1 \leq \varepsilon < 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta}. \end{cases} \\ \frac{d\psi}{d\delta} &= \frac{|M_{\psi\delta}|}{|M|} = \lambda\omega^{1-\varepsilon}A(1 - \varepsilon)\psi^{-\varepsilon}|M|^{-1} \begin{cases} > 0, \text{ if } \varepsilon \in (0, 1) \text{ or } \varepsilon > 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta} \\ \leq 0, \text{ if } 1 \leq \varepsilon < 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta}. \end{cases} \end{aligned}$$

$$\begin{aligned} \frac{d\psi}{d\sigma} &= \frac{|M_{\psi\sigma}|}{|M|} = (\varepsilon - 1) [xA\phi + \lambda\omega^{1-\varepsilon}\psi^{-\varepsilon}\delta] A(\sigma|M|)^{-1} \begin{cases} < 0, \text{ if } \varepsilon \in (0, 1) \text{ or } \varepsilon > 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta} \\ \geq 0, \text{ if } 1 \leq \varepsilon < 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta}. \end{cases} \\ \frac{d\psi}{d\phi} &= \frac{|M_{\psi\phi}|}{|M|} = xA^2(1 - \varepsilon)|M|^{-1} \begin{cases} > 0, \text{ if } \varepsilon \in (0, 1) \text{ or } \varepsilon > 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta} \\ \leq 0, \text{ if } 1 \leq \varepsilon < 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta}. \end{cases} \end{aligned}$$

Growth Rate

$$\begin{aligned} \frac{dg}{d\lambda} &= \frac{\partial g}{\partial \lambda} + \frac{\partial g}{\partial \omega} \left[\frac{\partial \omega}{\partial \psi} \frac{d\psi}{d\lambda} + \frac{\partial \omega}{\partial \lambda} \right] = \frac{1}{\varepsilon} \left\{ \omega^{1-\varepsilon}(1 + \varepsilon \ln \omega) - 1 - \lambda\varepsilon\psi^{-1}\omega^{1-\varepsilon}(1 - \omega^{1-\varepsilon}) \ln \omega M_{11}^{-1} \Gamma \right\} < 0, \\ \frac{dg}{d\delta} &= \frac{\partial g}{\partial \delta} + \frac{\partial g}{\partial \omega} \left[\frac{\partial \omega}{\partial \psi} \frac{d\psi}{d\delta} + \frac{\partial \omega}{\partial \delta} \right] = -\ln \omega (\lambda\omega^{1-\varepsilon})^2 (1 - \varepsilon)\psi^{-\varepsilon-1} A|M|^{-1} \Gamma < 0, \\ \frac{dg}{d\sigma} &= \frac{\partial g}{\partial \sigma} + \frac{\partial g}{\partial \omega} \left[\frac{\partial \omega}{\partial \psi} \frac{d\psi}{d\sigma} + \frac{\partial \omega}{\partial \sigma} \right] = \frac{A\phi}{\sigma^2} [1 + \lambda\omega^{1-\varepsilon}(\ln \omega)^2(1 - \varepsilon)^2\varepsilon\psi^{-1}] - (\lambda\omega^{1-\varepsilon} \ln \omega)^2 \Gamma \frac{(1 - \varepsilon)}{\psi\sigma} > 0, \\ \frac{dg}{d\phi} &= \frac{\partial g}{\partial \phi} + \frac{\partial g}{\partial \omega} \left[\frac{\partial \omega}{\partial \psi} \frac{d\psi}{d\phi} + \frac{\partial \omega}{\partial \phi} \right] = -\frac{\varepsilon A}{\sigma M_{11}} [1 + \varepsilon\lambda\omega^{1-\varepsilon}(1 - \varepsilon)^2(\ln \omega)^2\psi^{-1}] < 0, \end{aligned}$$

where $\Gamma \equiv [(1 - \varepsilon)(1 + \varepsilon \ln \omega) + \varepsilon] > 0$.

Abatement Propensity

$$\begin{aligned}
\frac{d\theta^*}{d\lambda} &= \frac{|M_{\theta\lambda}|}{|M|} = -A^{-1}\omega^{1-\varepsilon} \ln \omega \{1 - \lambda[(1-\varepsilon) \ln \omega + 1]\varepsilon\psi^{-1}(1 - \omega^{1-\varepsilon})M_{11}^{-1}\} > 0, \\
\frac{d\theta^*}{d\delta} &= \frac{|M_{\theta\delta}|}{|M|} = \lambda\omega^{1-\varepsilon}[(1-\varepsilon) \ln \omega + 1]\varepsilon\psi^{-\varepsilon}|M|^{-1} > 0, \\
\frac{d\theta^*}{d\sigma} &= \frac{|M_{\theta\sigma}|}{|M|} = -\frac{\phi}{\sigma^2} + \frac{\lambda\omega^{1-\varepsilon} \ln \omega}{A\sigma}[(1-\varepsilon) \ln \omega + 1] \left[1 - \frac{\varepsilon(1-\varepsilon)\theta^*A}{\psi M_{11}}\right] \geq 0, \\
\frac{d\theta^*}{d\phi} &= \frac{|M_{\theta\phi}|}{|M|} = \frac{\varepsilon}{\sigma M_{11}} [1 + \lambda\omega^{1-\varepsilon}(1-\varepsilon)^2(\ln \omega)^2\psi^{-1}] > 0.
\end{aligned}$$

C Endogenous Arrival

C.1 Solution of the extended model

The HJB equation is given by

$$\begin{aligned}
\rho V(K, P) &= \max \{U(C, P) + V_k[(1-\theta)Y - C] + V_p[(\phi - \sigma\theta)Y - \alpha P] + \\
&\quad + \lambda(K, P)\mathbb{E} [V(\tilde{K}, P) - V(K, P)]\},
\end{aligned}$$

where $\tilde{K} = \omega K$ with probability $(1-\pi)$ and $\tilde{K} = \hat{\beta}K$ with probability π , $\hat{\beta} \in [0, \omega]$.

The optimality conditions with respect to the control and the state variables consist of:

$$C : \quad U_c - V_k = 0, \tag{C.1}$$

$$\theta : \quad -V_k Y - V_p \sigma Y = 0, \tag{C.2}$$

$$\begin{aligned}
K : \quad \rho V_k &= V_{kk}[(1-\theta)Y - C] + V_k(1-\theta)A + V_p(\phi - \sigma\theta)A + V_{pk}[(\phi - \sigma\theta)Y - \alpha P] + \\
&\quad + \hat{\lambda} \left[(1-\pi) \frac{dV(\omega K, P)}{dK} + \pi \frac{dV(\hat{\beta}K, P)}{dK} - V_k \right] + \\
&\quad + \hat{\lambda}_k \left[(1-\pi)V(\omega K, P) + \pi V(\hat{\beta}K, P) - V(K, P) \right], \tag{C.3}
\end{aligned}$$

$$\begin{aligned}
P : \quad \rho V_p &= U_p + V_{pp}[(\phi - \sigma\theta)Y - \alpha P] + V_{kp}[(1-\theta)Y - C] - \alpha V_p + \\
&\quad + \hat{\lambda} \left[(1-\pi) \frac{dV(\omega K, P)}{dP} + \pi \frac{dV(\hat{\beta}K, P)}{dP} - V_p \right] + \\
&\quad + \hat{\lambda}_p \left[(1-\pi)V(\omega K, P) + \pi V(\hat{\beta}K, P) - V(K, P) \right]. \tag{C.4}
\end{aligned}$$

Solution proceeds along similar lines as in Section 2. First, we guess that the value function is of the same form as in Section 2. Then we derive the optimal policy from the first two optimality conditions. Recall that we also assumed $\hat{\lambda} = \hat{\lambda}(\hat{v})$, where $\hat{v} = (K^\eta P^\xi)^\mu$. Substituting the latter, along with the policy, into HJB equation, we find the constants of the value function, $\hat{\psi}$ and x . Condition (C.4) is then used to find the optimal $\hat{\theta}$, while (C.3) allows us to obtain \hat{g} .

C.2 Analysis of Multiple Solutions for $\hat{\psi}$

Multiple solutions to Eq. (23), which we reproduce here for convenience

$$\varepsilon\psi = \rho - (1 - \varepsilon)A \left(1 - \frac{\phi}{\sigma}\right) - \hat{\lambda}(\psi) \left[(1 - \pi)\hat{\omega}^{1-\varepsilon}(\psi) + \pi\hat{\beta}^{1-\varepsilon} - 1\right] \quad (\text{C.5})$$

are feasible when the elasticity of marginal utility is above unity ($\varepsilon > 1$). Define the function $LHS(\psi) = \varepsilon\psi$ as the left-hand side of the equation and, similarly, the function $RHS(\psi)$ as the right-hand side of the equation. The LHS is a straight line with the slope equal to ε . The RHS is, in general, a non-linear function of ψ . Differentiating it with respect to ψ , we obtain

$$\frac{dRHS}{d\psi} = -\frac{d\hat{\lambda}}{d\hat{v}} \frac{\partial \hat{v}}{\partial \psi} \left[(1 - \pi)\hat{\omega}^{1-\varepsilon}(\psi) + \pi\hat{\beta}^{1-\varepsilon} - 1\right] - \hat{\lambda} \left[(1 - \pi)(1 - \varepsilon)\hat{\omega}^{-\varepsilon} \frac{\partial \hat{\omega}}{\partial \psi}\right] \geq 0 \Leftrightarrow \varepsilon \geq 1,$$

so that RHS is strictly increasing in ψ . The second derivative is of ambiguous sign which means that the function has concave and convex segments and inflection point(s). Examining its asymptotic behavior, we see that $\lim_{\psi \rightarrow 0} RHS = \pm\infty \Leftrightarrow \varepsilon \leq 1$ and $\lim_{\psi \rightarrow 1} RHS$ is constant. Possible solutions to (C.5) are illustrated below.

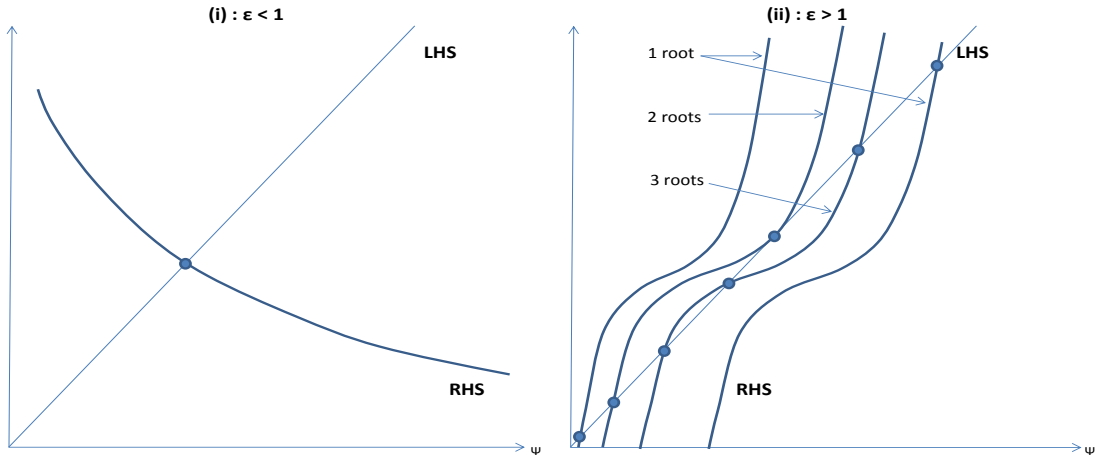


Figure 4: Possible solutions for $\hat{\psi}$.

D Robustness Checks

D.1 Sensitivity to γ .

$\gamma = 0.9$	$\varepsilon = 1$		$\varepsilon = 3$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda = 0.004$				
θ	0.50015	0.80024	0.62005	1.00844
g	3.47499	3.45998	1.15589	1.14903
$\lambda = 0.01$				
θ	0.50038	0.80060	0.80158	1.32557
g	3.47497	3.45995	1.15218	1.14247
$\lambda = 0.02$				
θ	0.50075	0.80121	1.10810	1.86662
g	3.47494	3.45990	1.14593	1.13128

Table 6: Optimal policy with $\delta = 1e - 6$ (up to 1% of GWP) and $\gamma = 0.9$.

$\gamma = 1.1$	$\varepsilon = 1$		$\varepsilon = 3$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda = 0.004$				
θ	0.50015	0.80024	0.51710	0.82648
g	3.47499	3.45999	1.15810	1.15298
$\lambda = 0.01$				
θ	0.50038	0.80060	0.54278	0.86626
g	3.47499	3.45998	1.15776	1.15244
$\lambda = 0.02$				
θ	0.50075	0.80121	0.58565	0.93276
g	3.47497	3.45996	1.15718	1.15155

Table 7: Optimal policy with $\delta = 1e - 6$ (up to 1% of GWP) and $\gamma = 1.1$.

D.2 Sensitivity to δ and β .

$\gamma = 1$	$\delta = 1e - 7$		$\beta = 2$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda = 0.004$				
θ^*	0.504	0.807	0.548	0.878
g^*	1.158	1.153	1.157	1.152
$\lambda = 0.01$				
θ^*	0.510	0.816	0.621	0.996
g^*	1.158	1.153	1.156	1.150
$\lambda = 0.02$				
θ^*	0.520	0.833	0.742	1.194
g^*	1.158	1.153	1.154	1.147

Table 8: Optimal policy with $\delta = 1e - 7$ (left-hand panel) and $\beta = 2$ (right-hand panel). Other parameters are at benchmark values and $\varepsilon = 3$.

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