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Asymmetric competition, risk, and return distribution

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Asymmetric competition, risk, and return distribution

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Abstract

We propose a parsimonious statistical model of firm competition where structural differences in the strength of competitive pressure and the magnitude of return fluctuations above and below the system-wide benchmark translate into a skewed Subbotin or asymmetric exponential power (AEP) distribution of returns to capital. Empirical evidence from US data illustrates that the AEP distribution compares favorably to popular alternative models such as the symmetric or asymmetric Laplace density in terms of goodness of fit when entry and exit dynamics of markets are taken into account.

Keywords: Return on capital, maximum entropy, asymmetric Subbotin distribution

JEL classification: C16, D21, L10, E10, C12
1. Introduction

In Smith’s (1776) classical view on competition, the competitive process is an inherently dual phenomenon. While generating incessant fluctuations of individual rates around the economy-wide norm, the perpetual reallocation of capital in search of higher returns implies a general tendency for profit rate equalization. Integrating this Smithian view with the maximum entropy principle developed by Jaynes (1957) and advanced in economics by Foley (1994), a series of recent studies including Alfarano and Milaković (2008), Alfarano et al. (2012), and Scharfenaker and Semieniuk (2016) approach this classical notion of competition from a probabilistic perspective. A key prediction common to these studies is that the dual properties of competition translate into a stationary Laplace distribution of returns to capital. In response to this view, this paper argues that neither the symmetric nor the asymmetric version of the Laplace density is sufficiently flexible to describe the empirical return distribution in the presence of firm entry and exit. This is because the Laplace distribution implies that the intensity of competitive pressure acting on each individual firm is independent of its current business performance and identical across all firms, which is not true for an economy subject to entry and exit dynamics. Hence, employing the method of constrained entropy maximization, we generalize the extant modeling framework to encompass variations in the nature of the competitive environment contingent on firms’ current profitability. Our model predicts a skewed Subbotin or asymmetric exponential power distribution of returns to capital, and establishes a link between return volatility and the intensity of competitive pressure. As our subsequent empirical analysis shows, the AEP distribution provides a significantly better fit to the observed asymmetry in the tails of the return distribution than the symmetric or asymmetric Laplace distribution prevalent in recent studies of firm profitability.

1 Throughout the study we use the terms return on assets, return to capital, and profit rate interchangeably.
2. A model of asymmetric competition

In line with the previous work of Alfarano and Milaković (2008), our model of firm competition rests on the idea that the dispersion of individual returns around the economy-wide benchmark reflects the idiosyncratic activities of profit-seeking firms under the general tendency for profit rate equalization. A general way to formalize this dispersion is the $L_\alpha$-norm distance from the benchmark return $m \in \mathbb{R}$,

$$
\sigma = \left( \mathbb{E}|x - m|^\alpha \right)^{1/\alpha},
$$

where $\alpha \in \mathbb{R}^+$ is interpreted as a phenomenological measure of the intensity of competition (Alfarano et al., 2012), and $\sigma$ reflects the notion of business risk since it contains information on the volatility of returns.

While the different flavors of Laplacian models of competition proposed in the extant literature implicitly assume that all firms in the population operate in the same competitive environment, a large body of theoretical and empirical work suggests that both risk and the intensity of competition should vary between the two groups of incumbent and entering/exiting companies. Among various ways of rationalizing this variation, the role of financial markets for the sustainability of firms’ business operations serves as a useful reference. Numerous studies of financial market imperfections emphasize that a firm’s net worth position and balance sheet condition, both of which crucially depend on the firm’s performance, significantly affect its financial capacity to support business operations (see, e.g., Bernanke et al., 1996; Fazzari et al., 1988; Greenwald and Stiglitz, 1993; Kiyotaki and Moore, 1997). According to this literature, large and long-lived corporations with sound balance sheet conditions are more likely to exhibit low bankruptcy risk and are, therefore, less exposed to pressures from creditors and investors in external capital markets than small and young firms. A corollary of this observation is that, due to the high bankruptcy risk reflecting vulnerable financial conditions, the latter firms are tied down to more stringent

\footnote{Notice that within the broader family of Subbotin distributions both the symmetric and asymmetric Laplace are nested as special cases with a shape parameter $\alpha$ equal to unity. Hence, both statistical models are consistent with the notion of a constant intensity of competition for all firms.}
covenant measures in financial contracts than large corporations, which provides a pronounced signal of heavier pressures from external capital markets imposed on these firms. This implies that there exist structural differences in the strength of competitive pressure and the nature of return fluctuations between the right and left tail of the return distribution, the latter of which reflects primarily the activity of entering and exiting companies with low or even negative returns.

To underline these features of the competitive process, we define the conditional measures of volatility $\sigma_l^{\alpha_l} = E|\omega - m|^\alpha_l$ for $x < m$, and $\sigma_r^{\alpha_r} = E|x - m|^\alpha_r$ for $x > m$, where $\alpha_l, \alpha_r, \sigma_l, \sigma_r \in \mathbb{R}^+$, $x \in \mathbb{R}$ is the return to total capital, and $l$ and $r$ refer to the left and right tail, respectively, of the return distribution. Given that our entire knowledge on the competitive system is encoded in these two conditional moment constraints, we can mathematically derive the probability distribution of returns that is achieved in the most evenly distributed number of ways using the principle of maximum entropy. Formally, this view on the competitive system of interacting firms leads to an equilibrium distribution maximizing the information entropy

$$\max_{f(x) \geq 0} \left\{ \frac{\alpha_l}{\sigma_l} \theta(m - x) \right\} = 1$$

subject to the natural constraint that normalizes the probability density function $f(x)$

$$\int_{-\infty}^{+\infty} f(x) dx = 1$$

and the two conditional moment constraints

$$\int_{-\infty}^{+\infty} f(x) \left| \frac{x - m}{\sigma_l} \right|^{\alpha_l} \theta(m - x) dx = 1,$$

and

$$\int_{-\infty}^{+\infty} f(x) \left| \frac{x - m}{\sigma_r} \right|^{\alpha_r} \theta(x - m) dx = 1,$$

where $\theta(z)$ is the Heaviside theta function that returns 0 for $z \leq 0$ and 1 for $z > 0$.\(^3\) In this formalization, the latter two constraints prescribe the qualitative differences between the left and right tail of the return distribution due to entry and exit dynamics.

\(^3\)For simplicity, we consider a Heaviside function that returns the value one only for strictly positive $z$. Of course, the removal of $z = 0$ is irrelevant to the value of an integral due to the measure-zero property of a single point in the case of a continuous measure.
We show in Appendix A that the solution to the maximization program (2)-(5) is given by the asymmetric exponential power distribution introduced by Bottazzi and Secchi (2011)

\[
f(x) = \frac{1}{C} \exp \left( -\frac{1}{\alpha_l} \left| \frac{x-m}{\sigma_l} \right|^{\alpha_l} \theta(m-x) \right) \exp \left( -\frac{1}{\alpha_r} \left| \frac{x-m}{\sigma_r} \right|^{\alpha_r} \theta(x-m) \right),
\]

(6)

where \( \sigma_l, \sigma_r \) are the scale (or dispersion) parameters for the two tails of the distribution, \( \alpha_l, \alpha_r \) denote the shape parameters, and \( C = \sigma_l^{\alpha_l/\alpha_l} \Gamma(1+1/\alpha_l) + \sigma_r^{\alpha_r/\alpha_r} \Gamma(1+1/\alpha_r) \) is a normalization constant with the Gamma function \( \Gamma(\cdot) \).

The smaller \( \alpha_l (\alpha_r) \), the heavier becomes the left (right) tail of the distribution. The case \( \alpha_l = \alpha_r = 1 \) and \( \sigma_l \neq \sigma_r \) coincides with an asymmetric Laplace distribution, while \( \alpha_l = \alpha_r = 1 \) and \( \sigma_l = \sigma_r \) is consistent with a symmetric Laplace density.

3. Evidence from US firms

The empirical application of our model rests on standardized items from the accounting books as reported in the Thomson Reuters Datastream Worldscope database. Our sample covers the period 2007-2016 and consists of 20,030 publicly traded companies from all sectors except the banking industry. Since the sample is unbalanced, these firms are not only heterogeneous with respect to the market in which they operate, but also in terms of the year they enter and exit the population surveyed by the database. For all these firms we compute the return to capital as the ratio of the flow of operating income (i.e. sales minus total operating expenses) to the stock of total assets at the end of the fiscal year, which serves as an approximation of the profit rate.

Figure 1 illustrates the cross-sectional return distribution for ten consecutive years between 2007 and 2016. Visual inspection suggests that this distribution

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4Zhu and Zinde-Walsh (2009) discuss various statistical features, including the maximum entropy property, of a differently parametrized version of the asymmetric exponential power distribution.

5In line with previous work, we exclude entities with SIC codes 60 and 61 (“banks”) because of structural differences in the composition of their balance sheets compared to non-financial firms.
Figure 1: Cross-sectional distribution of the return on assets for all publicly traded non-banking firms in the Datastream Worldscope database. The dashed lines represent a fit of the asymmetric exponential power distribution in (6) to the data.
Table 1: Estimated parameters of the asymmetric exponential power distribution and implied conditional mean absolute deviation for each year between 2007 and 2016. Standard errors are shown in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_l$</th>
<th>$\alpha_r$</th>
<th>$\sigma_l$</th>
<th>$\sigma_r$</th>
<th>$m$</th>
<th>$d_l$</th>
<th>$d_r$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>0.207</td>
<td>0.525</td>
<td>0.381</td>
<td>0.059</td>
<td>0.07</td>
<td>0.65</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>2008</td>
<td>0.207</td>
<td>0.505</td>
<td>0.417</td>
<td>0.061</td>
<td>0.07</td>
<td>0.74</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>2009</td>
<td>0.279</td>
<td>0.483</td>
<td>0.378</td>
<td>0.059</td>
<td>0.06</td>
<td>0.68</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>2010</td>
<td>0.275</td>
<td>0.485</td>
<td>0.363</td>
<td>0.059</td>
<td>0.06</td>
<td>0.65</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>2011</td>
<td>0.281</td>
<td>0.585</td>
<td>0.416</td>
<td>0.064</td>
<td>0.05</td>
<td>0.68</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>2012</td>
<td>0.274</td>
<td>0.492</td>
<td>0.382</td>
<td>0.060</td>
<td>0.06</td>
<td>0.68</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>2013</td>
<td>0.269</td>
<td>0.478</td>
<td>0.370</td>
<td>0.052</td>
<td>0.06</td>
<td>0.70</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>2014</td>
<td>0.282</td>
<td>0.509</td>
<td>0.350</td>
<td>0.056</td>
<td>0.06</td>
<td>0.61</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>2015</td>
<td>0.293</td>
<td>0.569</td>
<td>0.356</td>
<td>0.058</td>
<td>0.05</td>
<td>0.57</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>2016</td>
<td>0.286</td>
<td>0.471</td>
<td>0.304</td>
<td>0.052</td>
<td>0.06</td>
<td>0.53</td>
<td>0.03</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: Parameters are estimated with the maximum likelihood method using the software SUBBOTOOLS (Bottazzi, 2004). $d_l$ and $d_r$ refer to the conditional mean absolute deviation defined in (B.1) in Appendix B. The last column reports the $p$-value of the likelihood ratio test with null hypothesis $H_0: \alpha_l = \alpha_r = 1$. Zero entries in the last column imply $p < 5 \times 10^{-3}$.

is remarkably stable over time, testifying to the presence of a robust distributional regularity. The distribution is strongly skewed to the left and exhibits considerable excess kurtosis, i.e. heavy tails, particularly for returns below the mode. This impression is confirmed by the parameter estimates in Table 1. The fitted shape parameters are consistently below unity, indicating departure from the Laplace distribution. Comparing the size of the parameter estimates, we find that $\alpha_l < \alpha_r$ holds for all years, which clearly confirms that the left tail is more leptokurtic than the right one. Hence, our empirical results support the conjecture of qualitative differences in the competitive environment and the strength of competitive pressure between entering/exiting and incumbent firms, for which it has been shown that the cross-sectional return distribution is well approximated by the symmetric Laplace (see, e.g., Alfarano et al., 2012). As (1) shows, these differences in the Subbotin shape parameter translate into dif-
ferent norm distances, captured by the scale parameters $\sigma_l$ and $\sigma_r$. To assess and compare the profit rate volatility in the two tails of the return distribution, we consider the mean absolute deviation from $m$ over the entire support as a function of the parameters $(\alpha_l, \alpha_r, \sigma_l, \sigma_r)$, and decompose it into the sum of the respective deviations in the two tails of the distribution, i.e. $d \equiv d_l + d_r$. In Appendix B we provide the closed-form solution of this expression. Comparing the estimates $\hat{d}_l$ and $\hat{d}_r$, we confirm that the volatility of returns to the left of the mode exceeds the volatility in the right tail by at least one order of magnitude, implying that shorter-lived companies that operate below the system-wide norm are exposed to disproportionately higher risks than their competitors. In sum, we obtain strong empirical support for the hypothesis that the cross-sectional return distribution for the entire population of firms, including small and large entities, is skewed Subbotin rather than symmetric or asymmetric Laplace. Results of the likelihood ratio test firmly corroborate this impression.

4. Discussion and conclusion

The asymmetric exponential power distribution has been introduced in industrial dynamics as a tool for describing economic data that exhibit heavy skewness and leptokurtosis. Despite its relevance in statistical applications, however, an economic justification for this distribution has been largely unavailable and unidentified in previous studies. Against this background, the central contribution of our work lies in providing an economic underpinning for the AEP distribution in a statistical model of competition that captures variations in the competitive environment and risk between entering/exiting and incumbent companies. Compared to the existing Laplacian approaches to firm profitability, a key advantage of our methodology manifests itself in its ability to consider the nexus between these two spheres of competitive activity.

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6We are fully aware that the notion of risk is not uniquely associated with the mean absolute deviation of firm profitability. In the field of corporate governance, however, some form of deviation measure for return on assets plays a significant role in capturing firms’ risk characteristics. On this conceptual issue of risk, see, for example, Faccio et al. (2011), and John et al. (2008).
Since the maximum entropy principle informs us merely about the general nature of the binding constraints that give rise to a particular distributional regularity under the most decentralized form of economic activity, our results are not limited to a particular economic mechanism but are consistent with a plethora of economic interpretations that go beyond the one briefly sketched in this study. At the same time, we argue that our approach building on the maximum entropy principle is sufficiently general to be applied to a wide range of economic phenomena outside corporate profitability that share similar distributional characteristics. For example, Fagiolo et al. (2010) report that consumption expenditures obey the AEP distribution, while Bottazzi et al. (2014) and Reichstein and Jensen (2005) discuss applications to the firm size and growth rate distribution, respectively. Skewed and thick-tailed distributions are also observed for earnings and wealth (Benhabib and Bisin, 2018) as well as for the returns to total wealth, and it would certainly be worthwhile to investigate the implication of the distributional regularity in the latter on the dynamics of inequality. This would require to study the process governing the returns under the observed asymmetry. We will address this question in future research.

Appendix A. Maximum entropy program

The Lagrangian for the maximum entropy program (2)-(5) reads

\[ L = -\int_{-\infty}^{+\infty} f(x) \log f(x) dx \]
\[ -\lambda_N \left( \int_{-\infty}^{+\infty} f(x) dx - 1 \right) \]
\[ -\lambda_l \left( \int_{-\infty}^{+\infty} f(x) \left| \frac{x - m}{\sigma_l} \right|^{\alpha_l} \theta_l dx - 1 \right) \]
\[ -\lambda_r \left( \int_{-\infty}^{+\infty} f(x) \left| \frac{x - m}{\sigma_r} \right|^{\alpha_r} \theta_r dx - 1 \right), \]

where \( \theta_l \equiv \theta(m - x) \), \( \theta_r \equiv \theta(x - m) \), and \( \lambda_i \) are the Lagrange multipliers for \( i = N, l, r \). The first-order necessary condition for the maximum requires

\[ \frac{\partial L}{\partial f(x)} = -\log f(x) - \xi - \lambda_l \left| \frac{x - m}{\sigma_l} \right|^{\alpha_l} \theta_l - \lambda_r \left| \frac{x - m}{\sigma_r} \right|^{\alpha_r} \theta_r = 0, \]

where \( \lambda_i \) are the Lagrange multipliers for \( i = N, l, r \).
where \( \xi \equiv 1 + \lambda N \). Straightforward manipulation shows that the solution is of the form
\[
 f(x) = \exp(-\xi) \exp\left(-\lambda_l \frac{|x - m_l|^{\alpha_l}}{\sigma_l} \right) \exp\left(-\lambda_r |x - m_r|^{\alpha_r} \right).
\] (A.3)

To determine each multiplier as a function of the parameters \((\alpha_l, \alpha_r, \sigma_l, \sigma_r)\), we use the inverted forms of natural and moment constraints in the maximization program. Substituting (A.3) into each constraint, integrating by substitution, and using the properties of the Gamma function, we finally obtain the unique pair
\[
 \lambda_l = \frac{1}{\alpha_l}, \quad \lambda_r = \frac{1}{\alpha_r}.
\] (A.4)

Substituting these multipliers into the inverted natural constraint, we recover the partition function
\[
 \exp(-\xi) = \frac{1}{\sigma_l \alpha_l^{\frac{1}{\alpha_l}} \Gamma\left(1 + \frac{1}{\alpha_l}\right) + \sigma_r \alpha_r^{\frac{1}{\alpha_r}} \Gamma\left(1 + \frac{1}{\alpha_r}\right)}
\] (A.5)
that normalizes the probability density. Finally, substituting (A.5) and the multipliers in (A.4) into (A.3) yields the result (6) in the main text.

**Appendix B. Mean absolute deviation**

For the probability distribution in (6), the mean absolute deviation from \(m\) over the entire support, \(d \equiv E[|x - m|]\), is given by
\[
 d = \int_{-\infty}^{\infty} |x - m| f(x) dx
 = \frac{1}{C} \int_{-\infty}^{m} (m - x) \exp\left(-\frac{1}{\alpha_l} \left(\frac{m - x}{\sigma_l}\right)^{\alpha_l}\right) dx
 + \frac{1}{C} \int_{m}^{\infty} (x - m) \exp\left(-\frac{1}{\alpha_r} \left(\frac{x - m}{\sigma_r}\right)^{\alpha_r}\right) dx
 = \frac{1}{C} \sigma_l^2 \alpha_l^{\frac{2}{\alpha_l}} \Gamma\left(2 \frac{2}{\alpha_l}\right) + \frac{1}{C} \sigma_r^2 \alpha_r^{\frac{2}{\alpha_r}} \Gamma\left(2 \frac{2}{\alpha_r}\right)
 = d_l + d_r,
\] (B.1)

where \(d_i \equiv \frac{1}{C} \sigma_i^2 \alpha_i^{\frac{2}{\alpha_i}} \Gamma\left(2 \frac{2}{\alpha_i}\right)\) for \(i = l, r\).
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