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Three papers on optimal product positioning

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Three Papers on Optimal Product Positioning

Klaus Brockhoff, edt.

Alternative Approaches for Optimal Product Positioning

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1. Some Research Results

The application of various multidimensional scaling techniques leads to a representation of ideals and real product perceptions in an attribute space. Consider that data from only two hundred customers who have an identical evoked set of seven products to serve a certain segment of a market have been collected. In addition to that, each customer reports on his ideal product perception. Through the application of certain multi-dimensional scaling procedures let all these data be represented in a n-dimensional space. If this space is to be communicated to a product planner it may be separated into n(n-1)/2 two-dimensional spaces. Thus, the planner looks at 1600 * n(n-1)/2 individual points. He may than try to find that n-vector of coordinates that would represent an optimal product position. Obviously, this is too puzzling a task to be solved without substantial support from planning models.

In the formulation of such models we can discriminate between heuristic models and optimization models. In the following, I am concerned with optimization models.

Three optimization models seem to be of particular interest:

(a) the model by Shocker and Srinivasan (1974) that I call SS in the following,

(b) the model by Simon (1977), called SI, and

(c) the model by Brockhoff and Rehder (1977) that is labelled BR. (The model by F. Zufryden in this volume (1977) differs from the one considered here although it uses the same criterion for product choice.)

Their basic difference is in the assumption on the buying probability. This is most easily shown by the following Fig. 1 . Be d_i the distance between the customers' ideal point and the ith brand, $i \varepsilon I$, and be $d^* = \min \{d_i | i \varepsilon I\}$. Then, the nonnegative probability of purchasing a new product, π , to be located at a distance of D from the ideal product is expressed by

$$(1) \quad \pi_{SS} = a/D^b \qquad (a, b > 0),$$

$$(2) \quad \pi_{SI} = \begin{cases} 1, \text{ if } D \leq d^\ell \leq d^* \\ P(D), \text{ if } d^\ell \leq D \leq d^u \quad , \quad (a, b > 0) \\ 1 < \varepsilon \text{ , if } d^* \leq d^u \leq D \quad (\text{with } \varepsilon > 0 \text{ a small number}) \end{cases}$$

$$(3) \qquad \text{with } P(D) = \left(\frac{\max D-d^*}{\max D-d^\ell}\right)^\gamma ,$$

$$(4) \qquad \gamma \geq \ell n\, \varepsilon / (\ell n(\max D-d^u) - \ell n(\max D-d^\ell))$$

$$(5) \quad \pi_{BR} = \begin{cases} 1, \text{ if } D \leq d^* \\ 0, \text{ otherwise} \end{cases}$$

- Fig. 1 about here -

Quite obviously, π_{SI} has been constructed to bridge the gap between the single choice assumption that underlies π_{BR} and the assumption on π_{SS}. However, no empirical evidence ist presented to determine the new parameters d^u, d^ℓ, and ε. Furthermore,D is explicitly constrained to a finite range of levels of each attribute as can be read from (3). This is not requested in the other approaches.

The π_{SS} and π_{SI} seem to be more "flexible" to adapt to the real world than the single choice model (5) as they use at least two parameters. In (1), one of them is a scaling constant (a), and is therefore not considered further. However, b weights the decay of π_{SS} with increasing D. The following observations can be made on b:

(a) The optimal product positions are quite sensitive with respect to the level of b. This can be concluded from tests that were run on models based on (1) and (5).

(b) If a company enters a new market, and if $b \to \infty$, the probability of purchase approaches monotonically the single choice model (Shocker, Srinivasan, 1974, p. 931).

(c) If the company that introduces a new product has existing products in the market, and if $b \to \infty$, the solutions for $\pi(b)$ do not necessary approach monotonically to the single choice solution (see Fig. 2).

- Fig. 2 about here -

This may lead to problems for numerical reasons (Albers, Brockhoff, 1977): An upper bound b^u may exist for b, beyond which $\pi(b \geq b^u)$ cannot be calculated on a certain computer. The desired degree of approximation of π_{SS} to π_{BR} may, however, necessitate a choice of $b > b^u$. Thus, upper and lower bounds of b may be in conflict such that solutions become impossible. A relaxation of the lower bound leads invariably to a less satisfying approximation of π_{SS} to π_{BR}.

(d) The single choice model has at least some empirical support (Braun, Srinivasan, 1976; Pessemier et al., 1971), such that the approximation $b \to \infty$ may be necessary in some instances.

(e) The economic interpretation of b rests implicitly on the assumption of stable preferences over time (Day, 1967). Furthermore, b is assumed to be stable for certain markets (Pessemier et al., 1971). Both assumptions are subject to obvious criticism. Neither is it plausible to assume that preferences remain stable nor can it be assumed that b would not differ between customers.

One could try to model these influences explicitly. In the first case this would call for a dynamic model. With respect to the second problem it may become an empirical research question to what extent b_k can be discriminated from individually different saliences of a bundle of attributes.

The models differ furthermore with respect to the way they incorporate the buying decision. In SS and SI it is incorporated in the objective function:

$$\text{maximize} \sum_{k\varepsilon K} q_k \cdot s_k$$

with K : the set of customers

s_k : revenue from the k-th customer, $k \varepsilon K$,

$$q_k = \frac{\pi_{SS,k} + \sum_{i\varepsilon I'} \pi_{SS,i,k}}{\pi_{SS,k} + \sum_{i\varepsilon I} \pi_{SS,i,k}},$$

where $I' \subseteq I$ ist the set of the firm's existing products and I the set of all products in the evoked set of the customers. Furthermore, $\pi_{SS,i,k} = a/d^b_{i,k}$ and $\pi_{SS,k} = a/D^b_k$, for $k\varepsilon K$, $i\varepsilon I$. In BR, the buying decision appears in the constraints:

$$\text{maximize} \sum_{k\varepsilon K} x_k s_k$$

subject to

$$D_k - d^*_k \leq (1-x_k)M$$

$$x_k \ \varepsilon \ \{0,1\}$$

Where M is a finite upper bound on D_k, $k \ \varepsilon \ K$;
$d^*_k = \min \{d_{ki} = [\sum_{j\varepsilon J} W_{kj} (c_{kj} - e_{kij})^m]^{1/m} \mid i\varepsilon I\}$;

with J : the set of attributes,

W_{kj}: the saliences on $j \ \varepsilon \ J$ as evaluated by $k \ \varepsilon \ K$,

c_{kj}: the ideal product perception on $j \ \varepsilon \ J$ by $k \ \varepsilon \ K$,

e_{kij} : the perception of the i-th product, $i \ \varepsilon \ I$, on $j \ \varepsilon \ J$ by $k \ \varepsilon \ K$,

m : a metric parameter, mostly $m = 2$;

$$D_k = \left| \sum_{j \varepsilon J} W_{kj} \ (C_{kj} - y_j)^m \right|^{1/m} ,$$

where y_j, $j \varepsilon J$, is the coordinate of the new product that is to be optimized. This optimization model is a convex problem which is not guaranteed for either SS or SI. Thus, it should be possible to solve it optimally. This can be achieved by a specialized branch and bound procedure, which is an extension of the PROPOSAS algorithm (Albers, Brockhoff, 1977; Albers, 1977b), while SS or SI rely on more or less heuristic procedures of search through grids or gradient methods. The computing time (CPU time) for the solution of the BR model has been reduced considerably (that is by a factor of 2 to 3) as compared with the report on first results (Albers, Brockhoff, 1977). This is due to the construction of a sharper bound in the extension to the original algorithm.

Another difference between models concerns the definition of the attribute space. All the approaches considered up til now assume a continuous attribute space and do not restrict the ideal product perception of a customer. The single choice model by F. Zufryden (1977) uses the assumptions of a finite number of attribute levels or - in case of continuous attribute scales - that of a linear proportionality between attribute levels and utility. Both assumptions are rather restrictive.

2. Further Research

(a) All three models are static. It should be tried to find multi-period solutions. Attempts to do this in the BR framework are promising, as the index set K can be extended to cover customers at different points in time. This is computationally feasible due to the improvements of the algorithm just mentioned. It could be discussed to what extend the image of a new product can be varied and should be allowed to vary from one period to the next. It may turn out that the planned variation of a product image is an attribute in itself or is at least connected with an attribute. This could certainly introduce a nonlinear objective function and thus decrease the solvability of the BR-model.

(b) The linear objective function of the BR-model will also become nonlinear, if positioning cost and price that depend on the new product position are introduced. It remains to be seen to what extent this can be solved analytically. The same argument holds for the SS or SI models. - The approach necessitates the development of planning and control instruments that relate R & D and marketing activities to attribute changes rather directly than to sales, profits etc. Their ultimate test may provide more information for the development of models that would incorporate dynamic aspects and uncertainty.

(c) The measurement of distances between product perceptions relies on the assumption of some metric m . It can be shown empirically that the change in the metric parameter influences the optimal location of a new product and the value of the objective function. A specific example is demonstrated in Fig. 3. This explains in part, why empirical analyses have mostly referred to metric parameters of one or two. However, further investigations into their correct de-

- Fig. 3 about here -

termination would be valuable, specially if the metric could be tied in a normative way to product and customer characteristics.

(d) Furthermore, it is interesting to investigate, whether weights that differentiate between attributes above or below an ideal perception occur in reality and how they influence optimal solutions. This introduces the question whether saliences are constant for each attribute and each customer or whether they change relative to the position of the ideal of the same customer.

(e) In applied research one may want to study the interplay of segmentation criteria, the choice process and product positioning.

The multi-product introduction can be modelled and solved within the framework of all three models.(Within the BR framework see: Albers, 1977a). This is another way, whereby specific aspects of integration between short term marketing tactics and the long term strategy that extents to the R & D laboratory may be integrated on the basis of product attributes.

Furthermore, it may be tried to introduce certain segmentation criteria (such as usage situation) via the multi-product approach into the model.

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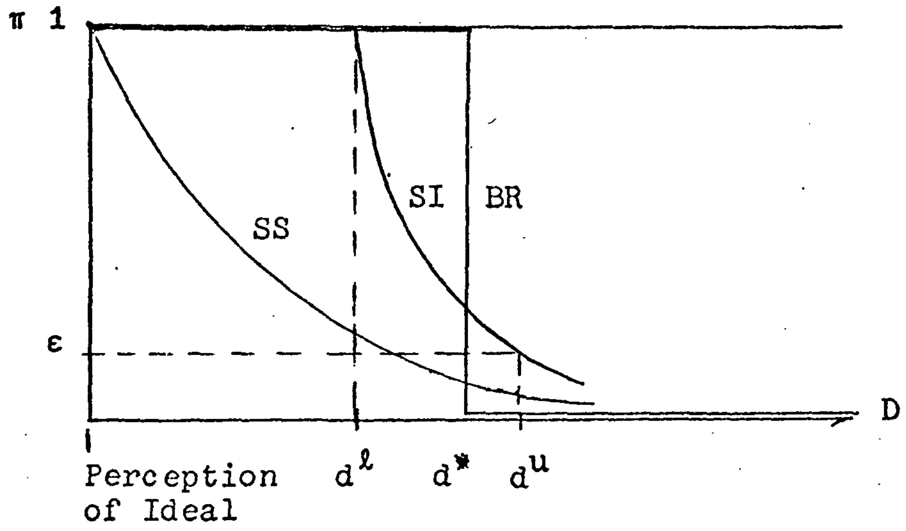

Fig. 1 : Purchase Probabilities

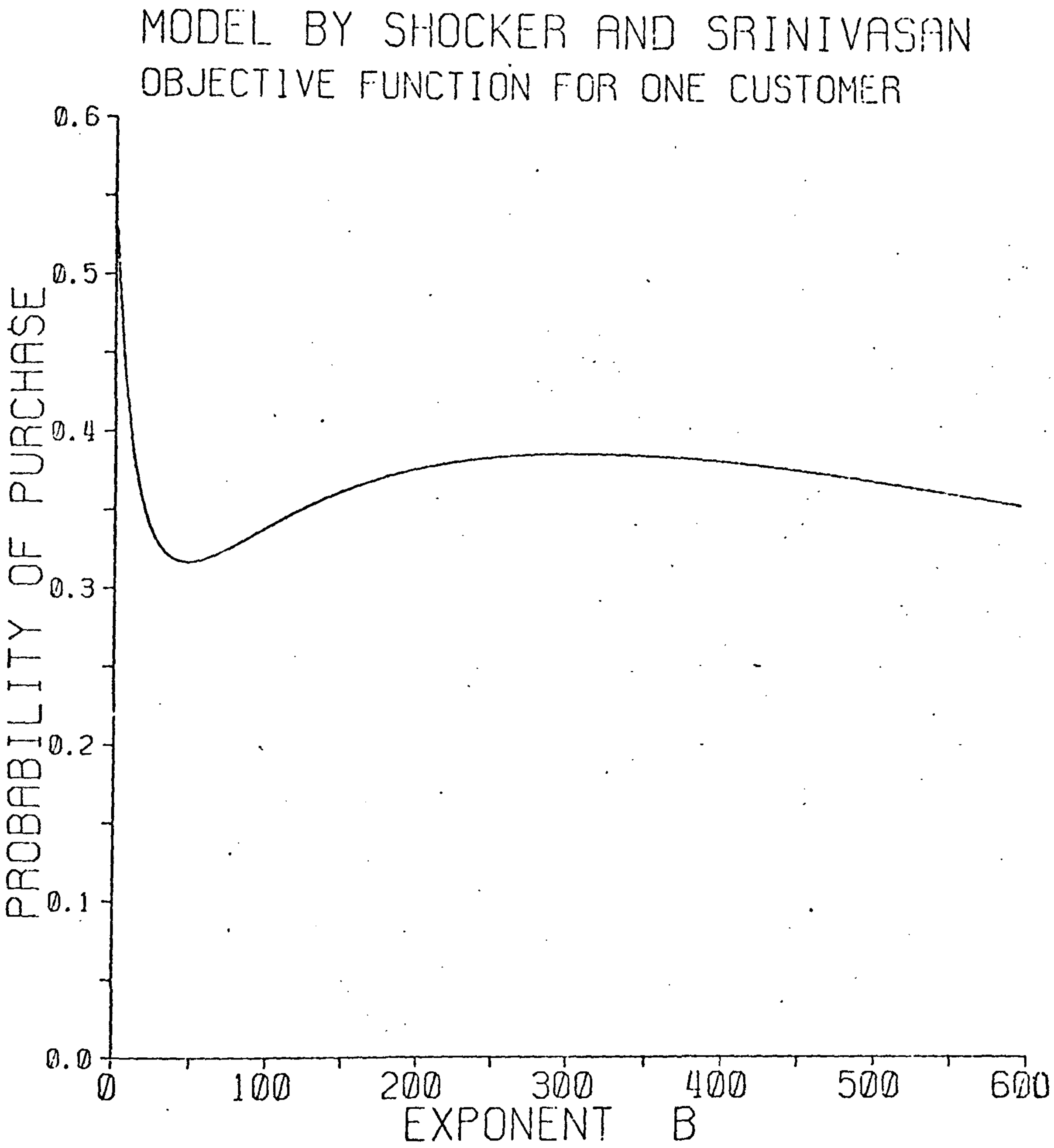

Fig. 2 : Example of the development of the purchase probability $q_{SS}(b)$

Problem Nr.		1 (SSØ4)	2 (ABM3)	3 (ABM2)
Total Nr. of customers		20	20	20
Nr. of Attributes		3	2	2
Nr. of buyers in the optimal solution	m=1	10	13	9
	m=2	11	13	9
	m=9	11	13	8
Coordinates of the optimal solution	m=1	- .0164; .0006; .024	.0025; - .0021	.0170; .0161
	m=2	- .0140; .0003; .002	- .0081; - .0061	.0172; - .0106
	m=9	- .0130; - .0001; .004	- .0134; - .0018	.0270; - .046
CPU-Time (sec)	m=1	.336	.520	.425
	m=2	.350	.358	.670
	m=9	.355	.546	.609
Individual buyers that prefer the new product	m=1	1,3,4,5,6,7,8,9,11,12	1,2,3,5,6,7,8,13,14,15,16,17,18	2,3,6,7,8,13,14,15,16
	m=2	1,2,3,4,5,6,7,8,9,10,11	1,2,3,4,5,6,7,8,13,14,15,17,18	1,2,3,6,7,8,15,16,18
	m=9	1,2,3,4,5,6,7,8,9,10,11	1,2,3,4,6,7,8,11,13,14,15,17,18	1,2,3,5,7,8,9,16

Fig. 3 : Sample Results from PROPOSAS (new version) for five real product perceptions and equal saliences

A Comparison of two Approaches to the Optimal Positioning of a New Product in an Attribute Space

by

S. Albers and K. Brockhoff

Abstract

This paper is concerned with the optimal positioning of a new product in an attribute space. A deterministic optimization model that relies on the axiom of choice is compared with the model by Shocker and Srinivasan that claims to incorporate a probabilistic measure of choice. Investigating the assumptions and the construction of both models, it is shown that the probability function by Shocker and Srinivasan is based on a controversial assumption and incorporates a parameter that has not been given an economic explanation. Furthermore, both models are applied to complex configurations. It is demonstrated that the optimal positions for the new product resulting from the model by Shocker and Srinivasan depend heavily on the parameter just mentioned and may run contrary to plausible consequences of the axiom of choice.

Contents

1. The Problem and Approaches to its Solution

Multidimensional scaling has been used extensively during recent years to structure product markets. Irrespective of the particular input data to such models their result is a configuration of locations of existing brand perceptions and of ideal product perceptions in an attribute space. This information, although only a static snapshot of the "real" market, may be used to identify those regions or locations where a new product should be placed to optimize some objective function. The possibility to recognize "niches" in a so-called psychological market model was suggested and studied theoretically even before the advent of powerful multidimensional scaling procedures [Spiegel, 1961].

Whenever the attribute space has more than two dimensions the optimization problem becomes much too complex to be solved by inspecting the corresponding graphical output, even if the problem is restricted to the case of the potential introduction of just one new product. Therefore, the optimal positioning of new products requires analytical methods. There exist some heuristic approaches to the solution of this problem that either search for satisfactory solutions, i.e.[Pessemier, 1975; Urban, 1975], or work with a set of prespecified locations [Sabel, 1971]. In this paper we investigate optimization models to solve the product positioning problem. Two models seem to be of particular relevance. The first one was developed in conjunction with a multidimensional scaling technique (LINMAP, which is not of interest in this paper) by Shocker and Srinivasan [1974]. It could be solved by a gradient method which, however, provides only solutions that are local maxima. The second one was formulated by one of the authors in [Brockhoff, Rehder, 1976]. An algorithm to solve problems of considerable size economically has been described in [Albers, Brockhoff, 1977].

We intend to compare the assumptions and results of these two optimization models. This may serve to arrive at a judgment on the suitability and applicability of both approaches. To simplify the comparison, the discussion is restricted to the problem that arises with the introduction of just one new product, where all locations in the attribute space are technologically feasible. The problem of introducing multiple products is discussed in [Shocker, Srinivasan, 1974] and in [Albers, 1976]; the latter reference includes an algorithm for simultaneously positioning multiple products. The input data are the same for both models. The following notation is used throughout the paper:

- K : a set of customers,
- I : a set of existing brands,
- $\hat{I}$: $\hat{I} \subset I$, a set of already existing brands supplied by the firm that intends to introduce the new product,
- J : a set of mutually independent attributes which are considered as potentially relevant to constitute brand preferences; the attribute space is $\mathbb{R}^{\bar{J}}$, where $\bar{J}$ is the number of elements (cardinality) in J,
- e_{kij} : an estimate of the k-th ($k \varepsilon K$) customer's perception of the existing brand $i \varepsilon I$ as expressed by the coordinate value of the j-th ($j \varepsilon J$) coordinate,
- c_{kj} : an estimate of the k-th ($k \varepsilon K$) customer's perception of his ideal product as expressed by the coordinate value of the j-th ($j \varepsilon J$) coordinate,
- w_{kj} : salience (or weight), measuring the relative importance of each attribute $j \varepsilon J$ to the user $k \varepsilon K$,
- s_k : revenue of the firm from the new product sales to the k-th ($k \varepsilon K$) customer.

The outputs of both models, and hence the decision variables, are denoted by:

y_j : the coordinate value on each dimension $j \in J$ for the location of the new product.

2. An Axiom of Choice-Model

The model by Brockhoff and Rehder [1976] was developed to conform with the well known axiom of choice. A recent formulation of this axiom is as follows:

"Alternatives that are closer to the anchor are preferred to those that are farther away. To be as close as possible to the perceived anchor point is the rationale of human choice" [Zeleny, 1976, p. 18].

The perceived anchor point is identical to the "ideal product" in this paper and similar to the "ideal alternative" in Festinger [1964, p. 132].

Suppose, the firm offers no other products in the relevant market ($\hat{I}=\emptyset$). Then, according to the axiom of choice, the k-th customer ($k \in K$) buys the new product, iff

$$(1) \qquad \left[\sum_{j \in J} w_{kj}(c_{kj}-y_j)^2\right]^{1/2} = D_k \leq d_k^* ,$$

where

$$(2) \qquad d_k^* = \text{Min}\ \{d_{ki} = \left[\sum_{j \in J} w_{kj}(c_{kj}-e_{kij})^2\right]^{1/2} \mid i \in I\}.$$

It should be noted that the distances D_k and d_{ki} could be generalized to other Minkowski-metrices. Both models under discussion could be extended to cover such metrices easily, without sacrificing solvability.

If we introduce the binary variables

$$(3) \qquad x_k = \begin{cases} 1, \text{ if (1) holds,} \\ 0, \text{ if (1) does not hold} \quad , \end{cases} \qquad (k \varepsilon K),$$

and if we assume that the maximization of the revenue of the firm is a valid objective, we arrive at the following model:

$$(4) \qquad \sum_{k \varepsilon K} x_k \cdot s_k \quad \Rightarrow \text{Max !}$$

subject to

$$(5) \qquad D_k - d_k^* \leq (1-x_k) \cdot M \quad , \qquad (k \varepsilon K),$$

$$(6) \qquad x_k \;\varepsilon\; \{0,1\} \quad , \qquad (k \varepsilon K),$$

where M is a finite upper bound on D_k $(k \varepsilon K)$.

If $\hat{I} \neq \emptyset$, $\hat{I} \subset I$, that is the firm is already operating in the market, it will concentrate on those customers $k \;\varepsilon\; \hat{K}$ only, who buy from one of its competitors.

Defining

$$(7) \qquad d_k^{*e} = \text{Min } \{d_{ki} | i \varepsilon \hat{I}\},$$

$$(8) \qquad d_k^{*s} = \text{Min } \{d_{ki} | i \varepsilon I \setminus \hat{I}\}$$

a customer k buys one of the products $i \;\varepsilon\; \hat{I}$ of the firm if $d_k^{*e} < d_k^{*s}$, and he buys one of the products $i \;\varepsilon\; I \setminus \hat{I}$ from anyone of the competitors, if $d_k^{*e} > d_k^{*s}$.

Hence $\hat{K}$ is given by:

$$(9) \qquad \hat{K} = \{k \varepsilon K | d_k^{*e} < d_k^{*s}\}.$$

To cover the cases where $\hat{I} \neq \emptyset$ and $\hat{K} \subset K$ analytically, the set K in the relations (4), (5), (6) would have to be replaced by $\hat{K}$.

Models of the type (4) through (6) are often called first choice or single choice models, because they assume that each consumer chooses consistently the most preferred brand [i.e.: Pessemier, et al., 1971, p. B-378].

Such a model is formulated by Brockhoff and Rehder [1976] as a mixed integer nonlinear programming problem with the real variables y_j ($j \epsilon J$), which are in turn determined by D_k ($k \epsilon K$), and the binary variables x_k ($k \epsilon K$). The problem is solved by the algorithm "PROPOSAS" which is described in [Albers, Brockhoff, 1976] for the special case: $s_k = 1$ ($k \epsilon K$) and $w_{kj} = 1$ ($k \epsilon K$, $j \epsilon J$). However, it also works for the general case.

The present version of the model may be characterized as deterministic.

3. A So-called "Probabilistic" Choice Model

3.1. The Model by Shocker and Srinivasan

In the Shocker - Srinivasan model [1974, pp. 931 et seq.], the following (unconstrained) objective function is maximized:

$$(10) \qquad \sum_{k \epsilon K} q_k \cdot s_k \Rightarrow \text{Max!} \quad ,$$

where

$$(11) \qquad q_k = \frac{\pi_k + \sum_{i \epsilon \hat{I}} \pi_{ki}}{\pi_k + \sum_{i \epsilon I} \pi_{ki}}$$

is the ratio of the individual's likelihood of purchasing one of the products of the firm (including the new product) to the likelihood of purchasing all products (including the new product). Furthermore,

(12) $\pi_k = \frac{a}{D_k^b}$,

(13) $\pi_{ki} = \frac{a}{d_{ki}^b}$, $a > 0$, $b > 0$, [1)]

with D_k and d_{ki} defined as in (1) and (2).

In this version, π_{ki} has been called "a measure of an individual's probability of purchasing the i-th product".

The parameter a may be constrained such that π_{ki} can only take on values $0 \leq \pi_{ki} \leq 1$. Therefore, π_{ki} has been called the probability of purchasing the i-th product. As a consequence, a may be set equal to one if used in (11) without loss of generality.

It should be noted that the objective function (10) is not concave so that maximizing (10) only provides local maxima. No sufficient conditions for identifying a global maximum for (10) have been given.

Observing the relations (12) and (13), we find that the parameter b is of special importance to the model. Therefore, the rôle of b is to be investigated more closely.

1) In the original publication b is only restricted to be non-negative. We do not allow $b = 0$ because this does not lead to a proper optimization problem, as q_k from (11) becomes independent from the location of the new product.

3.2. The Rôle of b : Its Substantial Meaning

While the introduction of an additional parameter into a model increases its potential "fit" to real world data, the parameter itself has to be determined before using the model. Therefore,the parameter needs to be interpreted and its effects have to be explored.

"Empirical research by Pessemier,et al. [1971], suggests that its value [b] depends primarily upon the product-market and less upon individual differences" [Shocker, Srinivasan, 1974, p. 931; additions in [·] by the authors]. This quotation is correct, indeed. However, the parameter b we talk about may not be exactly the same. In Pessemier, et al.,[1971, pp. B-373 et seq.] b is applied to adjust "expected relative frequencies of purchase" as derived from preference judgments to observed relative frequencies of purchase. The following may be concluded from this article:

(a) We do not find an economic explanation of b. Indeed, there appear slight inconsistencies in the paper on which Shocker and Srinivasan base their arguments.

(b) The probability of purchase RFP_{ki} given in Pessemier, et al.,[1971] is not necessarily uniformly related to the distances d_{ki} used in the model by Shocker and Srinivasan [1974]. The value RFP_{ki} is a function of preferences between brands, from which overall preference scales are derived through pairwise comparisons. As the ideal point is only implicit in this preference scale, it may not fully coincide with the one used in Shocker and Srinivasan [1974].

(c) Pessemier,et al.,[1971] have determined b for different markets. They have not reported on the question whether b is likely to change if a new product is introduced into a market. However, this is the situation studied in those models that try to solve the product positioning problem.

In a more recent paper [Srinivasan, 1975] it seems to be implied that b has some relationship to the so-called "difficulty parameter". This parameter is used in Kuehn and Day's publication [1962] to measure a consumer's difficulty to evaluate differences with respect to a product's attribute. If the implication holds, however, it meets the following criticisms:

- it can not be guaranteed that b is equal for all customers,
- Zeleny [1976] has argued, that difficulties to evaluate differences along an attribute scale can be captured in the saliences - which occur in addition to b in [Srinivasan, 1975].

An even more basic point of criticism relates to the question, whether the "probabilities of purchase" have to be related to the deterministic distance measures and the parameter b that is varied to achieve a good fit to the actual frequencies of purchases. In this point the analyses refer to an article by Day [1965].

In this article it is assumed that the preferences of a consumer "remain reasonably stable over time" [Day, 1965, p. 407], and that the consumer is not "capable of recognizing his >>most preferred<< level every time he is faced with a choice between products with different levels of the attribute" [Day, 1965, p. 407].

It becomes quite clear that the observations of the purchases occur over a set of points in time, while the preferences are looked at only once, assuming that they do not change. In our opinion it seems at least as plausible to assume that preference functions do not remain stable over time. Thus, the axiom of choice is assumed to be valid at each point in time. It is the assumption of stable preferences that obscures its recognition.

If we assume that the preferences change over time, then we should expect that the distance measures are stochastic. Shocker, Srinivasan [1974], however, based on the assumption of stable preferences over time, take the distances between ideal points and real brands as constants. The "stochastic" character of their model is in fact an artefact of the assumption of stable preferences over time.

The Brockhoff, Rehder [1976] model may be extended to cover stochastically variating distance measures. One aspect is to take the time path of observations explicitly into account, the other is to take care of difficulties to evaluate differences with respect to a product's attributes via the introduction of saliences (w_{kj} in this paper). However, this will not be demonstrated here.

3.3. The Rôle of b: An Analysis of its Limiting Case

If the parameter b in the model by Shocker and Srinivasan [1974, p. 931] tends to infinity the model theoretically approaches a single choice model under certain conditions.

This does not imply that one could use the model as a single choice model for all practical purposes, if b would be chosen "large enough". Consider a firm that already supplies four products (i = 1,2,3,4) to a market, where eight brands are offered altogether. The additional brands (i = 5,6,7,8) are offered by the competitors. Simplifying the total model for expository purposes, we look at one customer (k = 1) only. The following data may have been observed:

Distance of the new product to the ideal product:	D_1 = 1.200
Distances of the existing products (offered by the firm under consideration) to the ideal product	Distances of the existing products (offered by the competitors) to the ideal product
d_{11} = 1.100	d_{15} = 1.050
d_{12} = 1.300	d_{16} = 1.400
d_{13} = 1.045	d_{17} = 1.060
d_{14} = 1.900	d_{18} = 1.044

- Fig. 1 about here -

According to the single choice assumption the customer would not buy the new product. Rather, he would choose product $i = 8$, as $d_{18} = \text{Min}\ \{d_{1i} |\ i = 1,2,\ldots,8\}$. One should assume that the curve $q_1(b)$ approaches monotonely to zero with increasing values of b, if the model by Shocker and Srinivasan approaches a single choice model. Looking at Fig. 1, however, we find a curve drawn from the data given above that is not monotonely approaching the limiting case. Only if $b > 306$, the probability $q_1(b)$ decreases monotonely with increasing b. Therefore, $b = 306$ establishes a lower bound on b for an application of this model as a single choice model.

Furthermore, Fig. 1 shows a function which is only evaluated within the limits $1 \leq b \leq 595$. For $b > 595$, $q_1(b)$ cannot be computed. The termination is caused by a "floating overflow", which means that the calculations within arithmetic operations lead to values of variables too high to be represented in our computer[1)]. Therefore, $b = 595$ establishes an upper bound on b for practical reasons. This may occur in other examples also: However, the upper bound may vary from one set of data to the other.

Thus, the model leads to the following problems:

(a) There might arise a lower bound on b, possibly greater than the upper bound.

(b) Even if it is feasible to choose b smaller than its upper bound, the degree of approximation to the single choice model may be poor. This can be demonstrated again by looking at Fig. 1.

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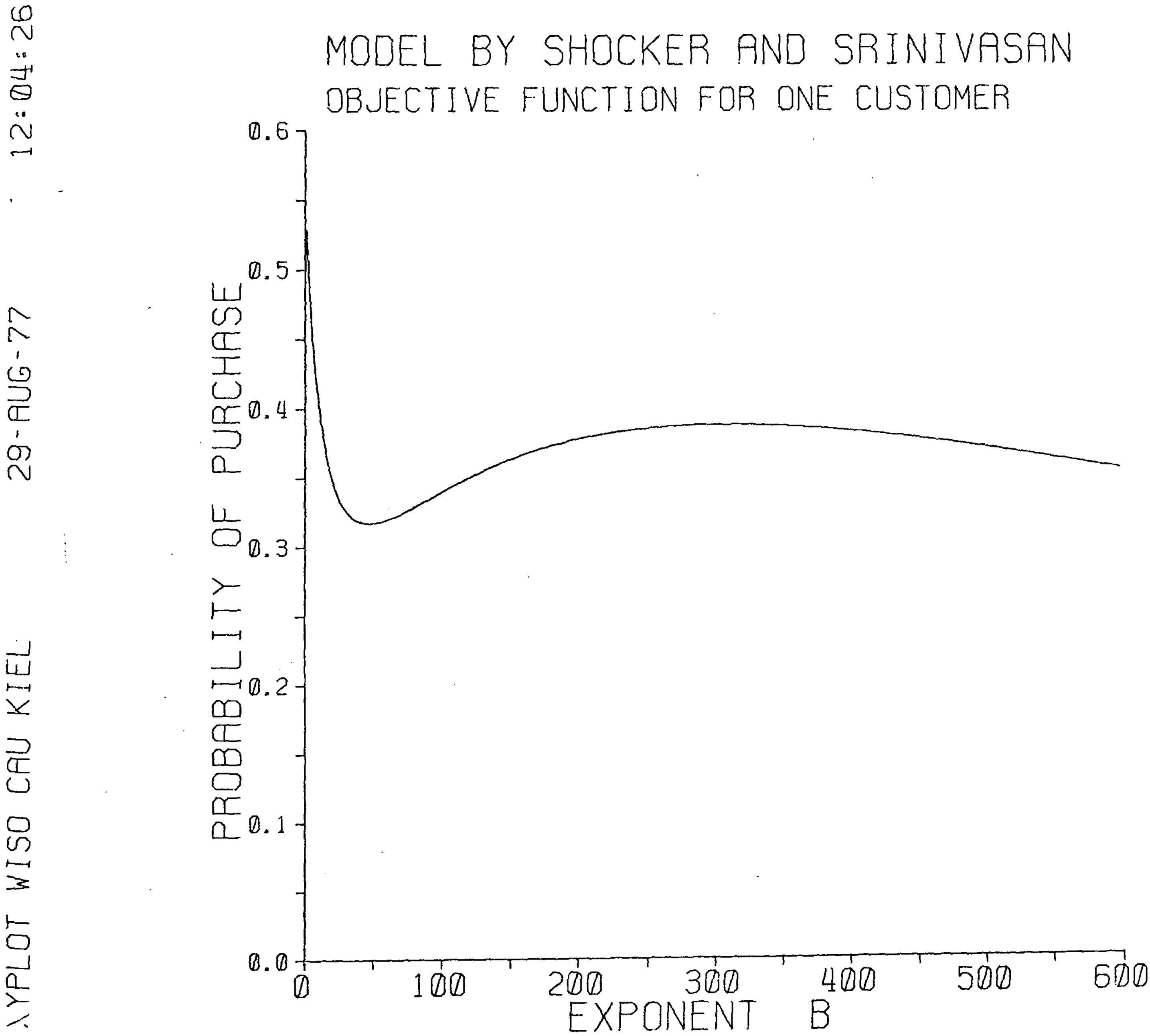
1) We use a pdp 10 digital computer, on which the magnitude of real constants must lie approximately within the range $0.14 \cdot 10^{-38}$ to $1.7 \cdot 10^{38}$.


MODEL BY SHOCKER AND SRINIVASAN
OBJECTIVE FUNCTION FOR ONE CUSTOMER
PROBABILITY OF PURCHASE
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
100
200
300
400
500
600
EXPONENT B
XYPLOT WISO CAU KIEL 29-AUG-77 12:04:26

(c) As D_k varies within the optimization process and as non-monotone functions $q_k(b)$ may have to be aggregated, the degree of approximation cannot be evaluated without knowledge of the single choice solution.

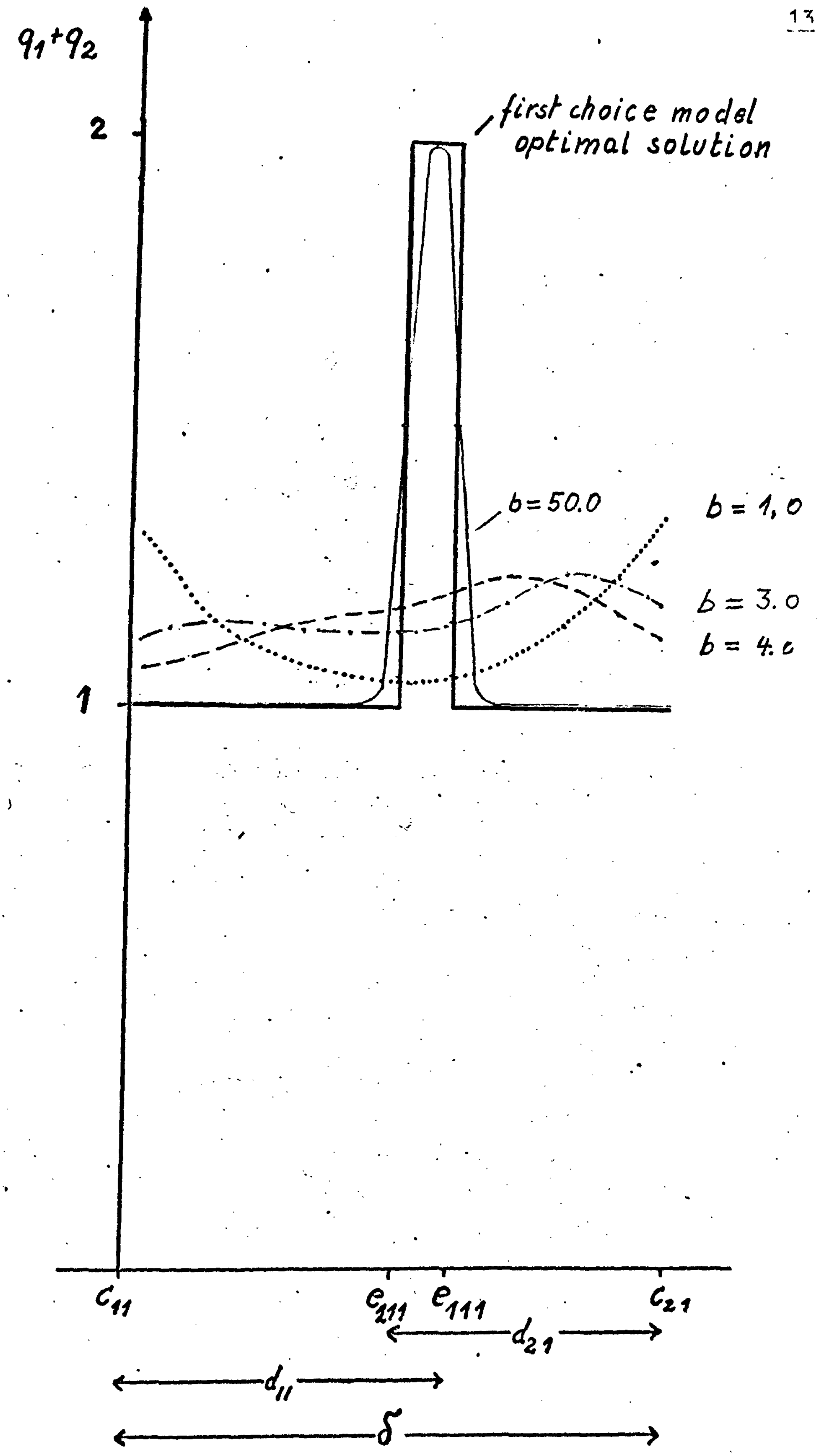
4. A Comparison of both Models

If the models by Shocker, Srinivasan [1974] and Brockhoff, Rehder [1976] are applied to a set of data, the comparison of the resulting optimal positions for the new product provides further insight.

4.1. Analysis of Another Simple Configuration

Consider a configuration of two ideal products (c_{11} and c_{21}), each perceived by one customer in an one-dimensional attribute space (see Fig. 2). Only one brand exists already in the market, which is offered by a competitor. This brand is perceived at e_{111} and e_{211}, such that the resulting sum of the distances $d_1^* = |c_{11}-e_{111}|$ and $d_2^* = |c_{21}-e_{211}|$ is greater than $\delta = |c_{11}-c_{21}|$.

Fig. 2 demonstrates that in this case the model by Shocker and Srinivasan approaches the single choice model with increasing b. The maximum of the function $q = q_1 + q_2$ moves from c_{21} to the region between e_{211} and e_{111} with increasing b . On the other hand, this movement shows clearly that for relatively small values of b the axiom of choice is not observed.



q_1+q_2
2
first choice model
optimal solution
b=50.0
b = 1,0
b = 3.0
b = 4.0
1
c_{11}
e_{211}
e_{111}
c_{21}
d_{21}
d_{11}
δ

Fig. 2

4.2. Analysis of Complex Configurations

Numerous complex configurations of ideal and real product perceptions in an attribute space have been studied.

If the configurationsof ideal products are distributed randomly to occur with equal probability everywhere in the attribute space, the locations for one new product derived from both models do not differ extremly so that such examples do not fit to show significant effects of the rôle of b in the model by Shocker, Srinivasan. However, this does not seem to be a very realistic case, either. Therefore, configurationshave been constructed to contain two distinct clusters of ideal products. In this paper we analyse three examples in detail. For expository reasons, each case will be presented graphically. This restricts the examples to two dimensions of the attribute space. To avoid confusion, the graphics only contain 20 points representing ideal products (symbol: *) from the respective number of customers and some points representing real products (symbol: #) which are assumed to be perceived unequivocally by each customer, that means $e_{k_1ij} = e_{k_2ij}$ $(k_1 \neq k_2;\ k_1, k_2 \varepsilon K)$. To simplify the evaluation of the results of both models, we assume that the revenue s_k of the firm obtained from each customer, $k \varepsilon K$, and the individual's saliences of the attributes w_{kj} $(k \varepsilon K,\ j \varepsilon J)$ are equal.

We want to stress that these assumptions should not be taken as an indication of limited capability of the algorithm presented elsewhere [Albers, Brockhoff, 1976].

The first configuration (see Fig. 3) shows a cluster A with 14 ideal product perceptions in the first quadrant and a cluster B with 6 ideal product perceptions in the third quadrant. The competitors offer two products which

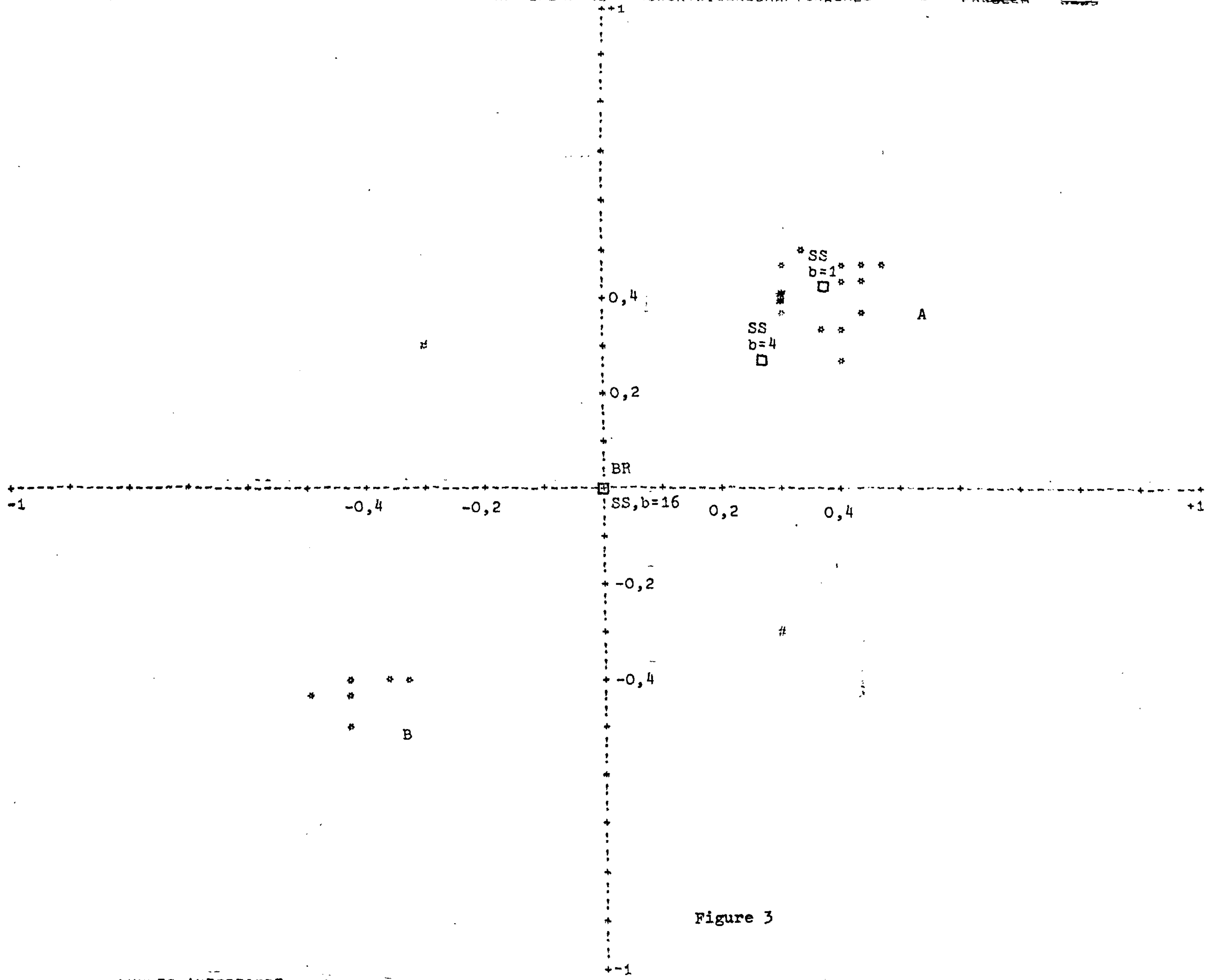


Figure 3

*** ALLE PUNKTE AKZEPTIERT ***
* = IDEALPRODUKTPOSITION
X = MEHRFACH *
\# = REALPRODUKTPOSITION
% = MEHRFACH #
$ = * UND # IN EINEM PUNKT

- Fig. 3 about here -

are perceived to be located in the second and the fourth quadrants. Using the algorithm by Albers, Brockhoff to optimize this example according to the axiom of choice model, we obtain the location (-0.004/+0.012) for the new product. This would cause all customers to buy the new product. If we apply the model by Shocker, Srinivasan with small values for the parameter b we arrive at "optimal" positions within cluster A or nearby, i.e. for b = 1 => (0.37/0.41) and for b = 4 => (0.26/0.29). If the axiom of choice holds, only the customers of cluster A would buy a new product which is positioned there. Taking b = 16 the model by Shocker, Srinivasan provides an optimal location (0.001/0.028) that is almost identical to the solution of the model by Brockhoff, Rehder.

These results indicate that, taking small values for b, the Shocker-Srinivasan-model serves to identify regions close to a set of ideal product perceptions, with no real brands nearby. As a result one may suppose that Shocker, Srinivasan search for locations corresponding to the proverb: "a bird in hand is worth two in the bush". Irrespective of the theoretical discussion of uncertainty (see 3.2) this assumption is disproved in the following configuration (see Fig. 4):

In Fig. 4 one recognizes two clusters, each containing 10 ideal products. As two existing brands offered by competitors are already located within cluster A in the first quadrant, the model by Brockhoff, Rehder provides (-0.66/-0.57) within cluster B in the third quadrant as the optimal location for the new product. The new product is to be located closer to the ideal products belonging to cluster B than to the other existing brands which are offered by competitors and are perceived to

- Fig. 4 about here -

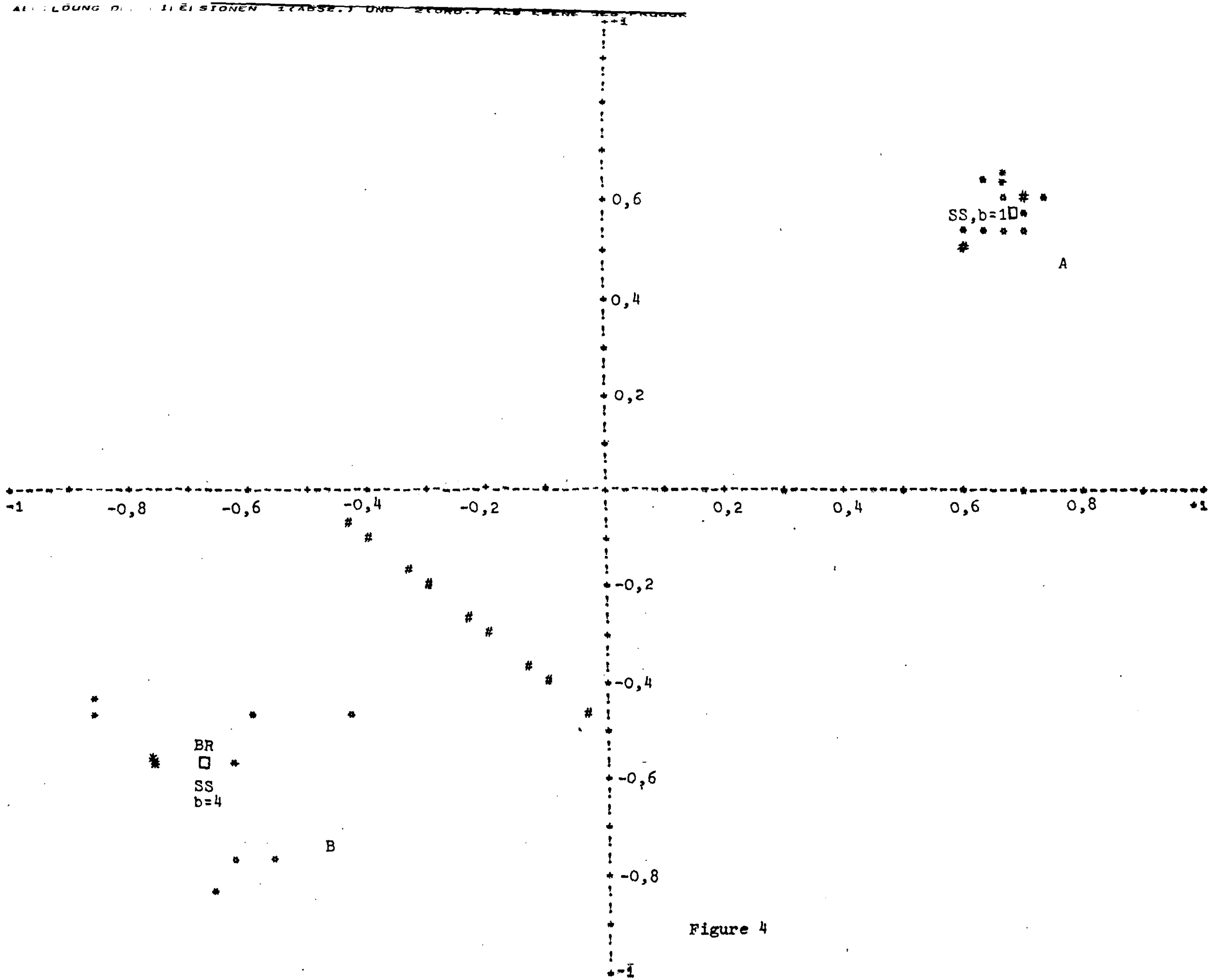


Figure 4

*** ALLE PUNKTE AKZEPTIERT ***
* = IDEALPRODUKTPOSITION
X = MEHRFACH *
# = REALPRODUKTPOSITION
% = MEHRFACH #
$ = * UND # IN EINEM PUNKT

be positioned in the third quadrant, but distant from cluster B. Taking b small, i.e. b = 1, the model by Shocker and Srinivasan, however, provides an optimal location (0.68/0.60) within cluster A and close to two existing brands. It should be noted that for b = 4 the result of the model by Shocker and Srinivasan is nearly identical to the first choice model.

Contrary to the supposition made above, both examples show that, if b is "too small", the model by Shocker and Srinivasan leads to an optimal position for a new product within a cluster of ideal products

- which are distributed around a center with a small variance, and
- from which the sum of distances to the existing brands is small - irrespective of the location of any existing brand.

We obtain similiar results if we analyse configurations that consider brands which are offered by the firm that intends to introduce a new product. Looking at Fig. 5 one recognizes a cluster A of 13 ideal product perceptions in the first quadrant and a small cluster B of 7 ideal product perceptions in the third quadrant. As the firm already offers a product that is perceived to be located within cluster A, the optimal position (-0.43/-0.41) corresponding to the model by Brockhoff, Rehder is given within cluster B. This way the firm offers a new product that is closer to the ideal products of 7 customers who until now prefer the existing brands of its competitors located in the third quadrant. If one takes b = 4 in the model by Shocker and Srinivasan the optimal position

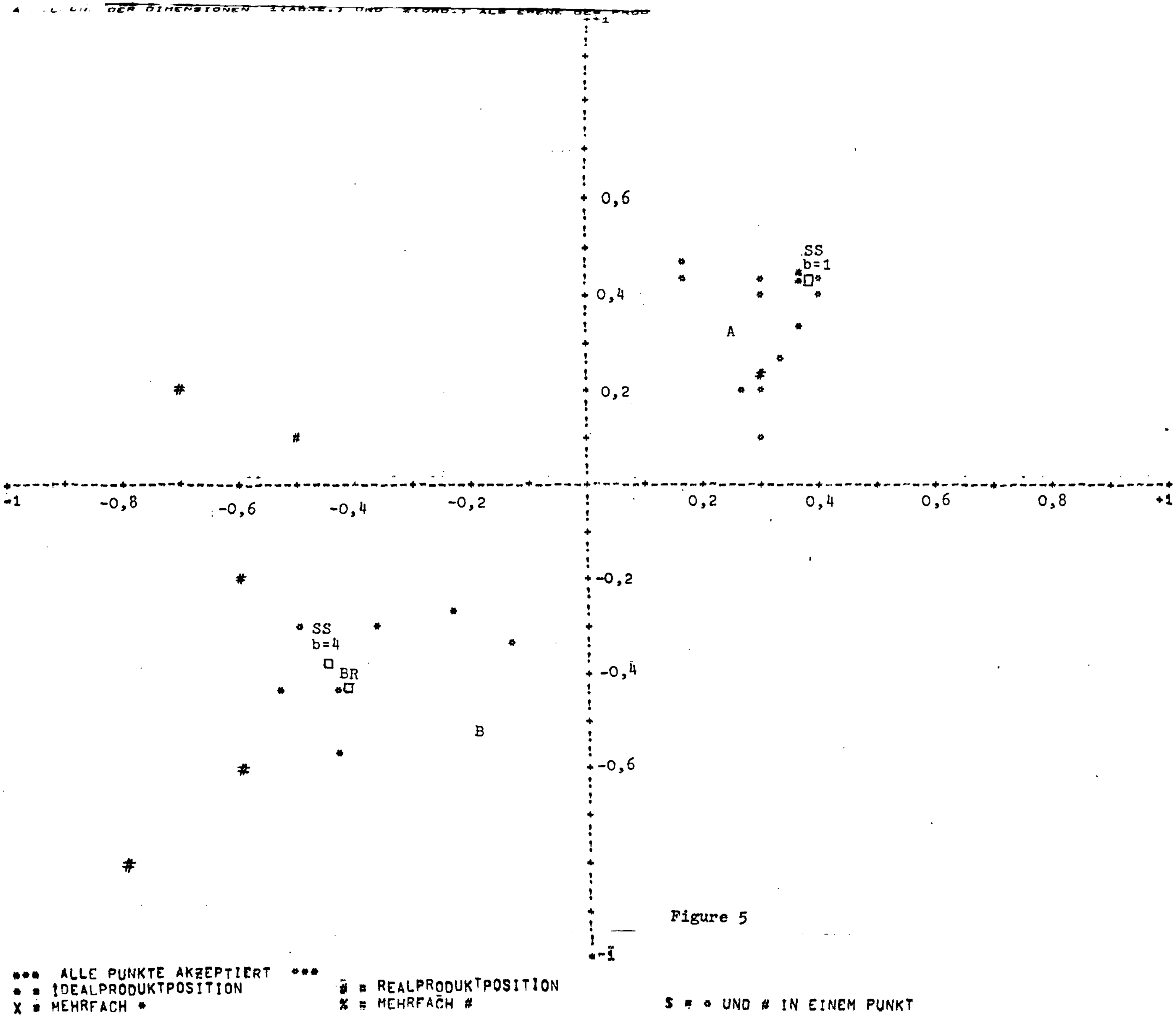

SS
b=1
A
SS
b=4
BR
B
0,6
0,4
0,2
-0,2
-0,4
-0,6
-1
-0,8
-0,6
-0,4
-0,2
0,2
0,4
0,6
0,8
+1
*** ALLE PUNKTE AKZEPTIERT ***
* = IDEALPRODUKTPOSITION
X = MEHRFACH *
# = REALPRODUKTPOSITION
% = MEHRFACH #
$ = * UND # IN EINEM PUNKT

Figure 5

(-0.45/-0.37) is nearly identical to the optimal solution derived from the model by Brockhoff, Rehder. If, however, one takes b = 1 one gets an optimal position at (0.38/0.42) close to the product already offered by the firm so that both products of the firm would be in close competition, a result that is not desired.

Concluding the analysis of the three examples we find that if b is small, the optimal position of a new product derived from the model by Shocker and Srinivasan is far from the optimal position as determined in the single choice model.

Taking b large, the solutions to the three examples are nearly identical for both models. But it should be mentioned that Shocker and Srinivasan cannot prove global optimality for their results, which is possible under the PROPOSAS algorithm.

As the solutions are quite sensitive with respect to changing b values, and as the degree of approximation to the single choice model cannot be evaluated prior to its solution, it seems to be advisable to use the PROPOSAS algorithm for the solution of the single choice model immediately. Aside from theoretical and empirical results presented above, one may take in mind that CPU-time requirements to solve the Shocker and Srinivasan model are considerably higher than for the solutions by the algorithm of Albers and Brockhoff.[1]

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1 To optimize the model by Shocker and Srinivasan we applied a gradient method using 30 starting points. The CPU-times, required for solving the three examples mentioned above, range from 166 to 1456 seconds. On the other hand, all examples were solved by the algorithm of Albers and Brockhoff in less than one second of CPU-time. All computations were carried out on a pdp-10 computer at the University of Kiel.

It is interesting to note that Pessemier, et al. [1971, pp. 378-381] already arrived at results that lent credibility to the first choice model with respect to the "absolute average forecast error"-criterion (as opposed to the variance). Referring to this article, Shocker and Srinivasan [1974, p. 931] go even further: "The limited research to date (involving frequently purchased consumer nondurables) indicates this simpler single choice model gives predictive results approximatly as good as the more complex models."

5. Conclusions

A model for optimal product positioning in an attribute space that relies on the axiom of choice [Brockhoff, Rehder, 1976] and the corresponding algorithm [Albers, Brockhoff, 1977] have been compared with the optimization model by Shocker and Srinivasan [1974].

It is shown that the latter model is based implicitly on the controversial assumption of stable preferences over time. The "stochastic character" of this model appears as an artefact of this assumption. Examples illustrate that the results of this model are not credible for any value of b, if credibility is judged against the single choice model.

One may argue that the Shocker-Srinivasan-model provides the more flexible approach as it approaches the single choice model if b is taken to be large. However, the possible existence of a lower bound on b and - for practical reasons - an upper bound as well devaluates this idea. Even if a feasible value for b exists, the degree of approximation to the single choice model cannot be evaluated without prior knowledge of the proper single choice solution. Furthermore, the Shocker-Srinivasan-model does not guarantee global maxima, because the objective function is not concave. Finally, it was found that the PROPOSAS algorithm uses considerably less CPU-time than solutions of the Shocker-Srinivasan-model via a gradient method for optimization.

These results indicate that the single choice model and the PROPOSAS algorithm described elsewhere offers advantages over the other, seemingly more flexible procedure. However, the Brockhoff-Rehder-model should be extended to cover uncertainty and dynamic aspects of choice behavior in a proper way. It seems possible to solve this task in future, if only at the price of extended data collection.

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Zur optimalen Produktpositionierung bei Unsicherheit

von

Klaus Brockhoff

Zusammenfassung: Das Modell der optimalen Ausstattung von Produkten mit kaufrelevanten Produkteigenschaften ist bisher auf statische Betrachtungen beschränkt. Hier wird eine Ausweitung auf die mehrperiodige Betrachtungsweise hin vorgenommen, um so auch Aspekten der Unsicherheit Rechnung zu tragen.

1. Problemstellung

Unter der Neuproduktplanung wird hier die folgende Aufgabe verstanden: Gegeben sei ein Raum voneinander unabhängiger und kaufrelevanter Produkteigenschaften, in dem die Käuferperzeptionen von realen Produkten und je Käufer einem Idealprodukt nach einem geeigneten Verfahren der multidimensionalen Skalierung angeordnet sind. Gesucht wird in diesem Raum ein Punkt, in dem ein dort angeordnetes neues Produkt eine Zielfunktion optimiert (vgl. [5]).

Eine Lösung für dieses Problem und ein wirtschaftlich arbeitender Lösungsalgorithmus sind bekannt [2]. Der Algorithmus ist für die simultane Positionierung mehrerer Produkte erweitert worden [1]. Die Überlegenheit des Planungskonzepts gegenüber einem Verfahren von Shocker und Srinivasan [11] konnte demonstriert werden [3]. Gegenüber diesem Ansatz,wie einem Modell von Simon [12], weist unser Ansatz den Vorteil auf, nicht auf heuristische Lösungsverfahren angewiesen zu sein, da er als konvexes Optimierungsproblem konzipiert ist. Die Kaufentscheidung wird, wie in der algorithmisch allerdings zusätzliche Annahmen erfordernden und u.a. deshalb weniger befriedigenden Arbeit von Zufryden [16], auf dasjenige Produkt konzentriert, das der Vorstellung vom Idealprodukt am nächsten kommt (single choice model). Alle genannten Ansätze sind statisch und, abgesehen von impliziten Annahmen, auch allein auf eine Periode bezogen. Deshalb erscheint es sinnvoll,jetzt ein Problem zu behandeln, das bisher weder in heuristischen Planungsverfahren noch in Optimierungsmodellen einer Lösung zugeführt wurde; seine Existenz ist allenfalls durch fragwürdige Annahmen überdeckt worden (vgl. die Kritik an [6] in [3]): Es handelt sich um die adäquate Berücksichtigung der Unsicherheit in den für die Modelle verwendeten Daten.

Die am Ausgangspunkt der Betrachtung eingesetzten multidimensionalen Skalierungsverfahren verwenden Urteile über die Ähnlichkeit von Produkten, direkte Präferenzurteile oder Beurteilungen von Produkteigenschaften [5]. In der psychologischen Literatur hat die Beobachtung, daß solche Reaktionen auf die als Produkte dargebotenen Stimuli einer Streuung unterliegen, zur Entwicklung von Versuchsanordnungen zur Erfassung dieser Streuungen und zur Ableitung von Modellen zur Verarbeitung dieser Information, etwa bei der Ableitung von Präferenzordnungen, geführt [14]. Der Stimulus-Sampling-Ansatz zur Erklärung von Kaufwahrscheinlichkeiten mag als ein hier besonders relevanter Hinweis darauf dienen, daß auch das Marketing mit ähnlichen Überlegungen vertraut ist [4, 9, bes. S. 43 ff.; 13, S. 185 ff.].

Es muß als natürlich erscheinen, wenn man annimmt, daß Abweichungen des Kaufverhaltens von punktuell ermittelten Kaufwahrscheinlichkeiten insbesondere auf Präferenzveränderungen der Käufer in der Zeit zurückgehen. Damit wird die Annahme gleichbleibender Präferenzen zurückgewiesen, die zu ihrer Unterstützung Wahrscheinlichkeitsfunktionen für die Beschreibung der Abweichungen zwischen dem tatsächlichen Kaufverhalten und einer historischen Präferenzordnung bemühen muß [6]. Planungsmodelle, in denen auf solche Wahrscheinlichkeitsfunktionen zurückgegriffen wird, unterstellen rationales, d.h. der Verwirklichung der Präferenzordnung entsprechendes Käuferverhalten nur für den Zeitpunkt der Feststellung der Präferenzordnung, sonst aber nicht. Während es viele Erklärungen für die Veränderung von Präferenzordnungen gibt, ist eine überzeugende Erklärung für die gerade dargestellte Unterstellung zeitweise irrationalen Käuferverhaltens nicht erkennbar. Sogenannte "probabilistische" Modelle der optimalen Positionierung neuer Produkte, die diesem Konzept folgen, werden deshalb hier nicht weiter behandelt (vgl. [3]).

Veränderungen der Reaktionen auf Stimuli schlagen sich in den Ergebnissen der multidimensionalen Skalierungsmodelle nieder [8]. Erstens können sie zur Veränderung von Realproduktpositionen führen, zweitens zur Veränderung von Ideal-produktpositionen im Eigenschaftsraum, drittens zur Veränerung individueller Gewichte der einzelnen Dimensionen dieses Raumes. Durch diese drei Einflüsse werden die Abstandsmaße zwischen den einzelnen Produktpositionen im Raum berührt. Unterstellt man, wie speziell im Modell der einfachen Wahl (single choice model), daß dasjenige Produkt beim Kauf ausgewählt werde, dessen Abstand vom Idealprodukt im Eigenschaftsraum minimal ist, so kann die Veränderung der Abstandsmaße auf Veränderungen von Kaufentscheidungen hinweisen. (Vgl. hierzu das sogen. Axiom der Wahl, z.B. bei [15].) Damit wird es interessant, auch neue Produkte unter Berücksichtigung dieser Effekte zu positionieren.

2. Ein deterministisches Planungsmodell

Eine einfache Version des bisher von uns verwendeten deterministischen Planungsmodells lautet wie folgt:

(1) $$\max \sum_{k \varepsilon K} s_k \, x_k$$

(2) $$D_k(y) - d_k \leq (1-x_k)M, \quad k \varepsilon K$$

(3) $$x_k = \{0,1\}.$$

Es handelt sich um ein nichtlineares (wegen der Abstandsmaße in (2)), gemischt-ganzzahliges Optimierungsmodell mit:

K - der Menge der Käufer (oder einem repräsentativen Sample dieser Menge), $k \varepsilon K$,

d_k - dem Maß für den Abstand zwischen der Idealproduktperzeption und der nächstliegenden Realproduktperzeption des Käufers $k \varepsilon K$ bei gegebenem Abstandsmaß, z.B. der euklidischen Norm,

$D_k(y)$ - dem Maß für den Abstand zwischen der Idealproduktperzeption und der als Variable aufgefaßten Position des neuen Produkts y'
$y' = \{y_j \mid j \in \mathbb{R}^m\}$ im m-dimensionalen Eigenschaftsraum für den Käufer $k \in K$,

M - einer großen Zahl,

s_k - der Absatzmenge je Periode an den Käufer $k \in K$.

Verschiedene Aspekte dieses Ansatzes sind der Literatur zu entnehmen: [1; 2; 3; 5]. Hier interessieren allein Möglichkeiten zur Berücksichtigung der Unsicherheit in d_k und in $D_k(y)$; letzteres, weil die Idealproduktperzeption nicht als sicher angenommen werden kann, was nichts mit dem Charakter von y als Vektor von Entscheidungsvariablen zu tun hat. Unsicherheiten in s_k sollen uns hier nicht beschäftigen.

3. Aspekte der Einbeziehung der Unsicherheit

Die modellmäßige Behandlung der Ungewißheit hängt nun in starkem Maße von den verfügbaren Informationen ab. Häufig wird unterstellt, daß Erwartungswert und Streuung für jede normalverteilte unsichere Größe auf der Grundlage empirischen Wissens geschätzt werden könnten. Angenommen, dies würde hier zutreffen. Dann könnte in (1) s_k durch den Erwartungswert $\bar{s}_k$ derselben Größe ersetzt werden. Weiter müßte (2) in der Form einer wahrscheinlichkeitsbeschränkten Nebenbedingung formuliert werden. Das deterministische Ersatzprogramm würde dann an Stelle von (2) Bedingungen enthalten, die auf der linken Seite einen positiven Ausdruck aufweisen würden, der vom geforderten Sicherheitsniveau und den Streuungen abhängig wäre. Ein besonderes Problem liegt noch darin, daß d_k in (2) als Minimum aus den Abständen aller Realprodukte zu den Idealprodukten bestimmt war. Diese Operation ist im Falle stochastischer Produktpositionen natürlich nicht mehr eindeutig festgelegt, weshalb man sich schon hier z.B. mit der Betrachtung von Erwartungswerten behelfen müßte, was den Wert der folgenden Rechnungen dann beeinträchtigen würde.

Trotz dieser Bedenken erscheint es interessant, sich die Konsequenzen einer solchen Konzeption vor Augen zu führen (vgl. Abb. 1) . In Abb. 1 seien I_k, $k = \{1,2,3\}$, Idealproduktperzeptionen und d_k die als Erwartungswerte aufgefaßten Abstände zu einer bisher realisierten Realproduktperzeption. Als Lösungsgebiet für die Positionierung eines neuen Produkts kommt bei Annahme euklidischer Normen als Abstandsmaße das senkrecht schraffierte Gebiet in Frage. Die Berücksichtigung der Unsicherheit führe allein zur Einführung eines durch die Schwankungen in Realproduktperzeptionen bedingten Sicherheitsabschlags, so daß sich d_k auf d_k' verkürzen. Das Lösungsgebiet schrumpft dann hier zu dem im Schnittpunkt der Kreise mit den Radien d_k' bestimmten Punkt zusammen. Die Zielfunktionswerte stimmen hier noch überein. Weitere Unsicherheit bezüglich der Idealproduktperzeptionen würde tendenziell zu $d_k'' < d_k'$ führen, wodurch im Beispiel nur noch Neuproduktpositionierungen bestimmt würden, die einen geringeren Zielfunktionswert beim vorgegebenen Sicherheitsniveau realisieren als in der Ausgangslage einer Planung auf der Grundlage von Erwartungswerten.

In der Regel wird keine ausreichende Information zur Abschätzung der Erwartungswerte und der Varianzen vorliegen. Auch daran könnte, will man sich nicht auf subjektive Schätzungen zurückziehen, der bisher diskutierte Ansatz scheitern. Da auch subjektive Schätzungen nur vorliegende Beobachtungen verarbeiten, ist zu prüfen, ob diese Information nicht auch direkt in die Planung eingehen kann. Solche Information könnte aus wenigen, in verschiedenen Zeitpunkten von denselben Personen erfragten und aufbereiteten Real- und Idealproduktperzeptionen bestehen. Für die einzelnen Datensätze sei der Index $t \in T$ eingeführt.

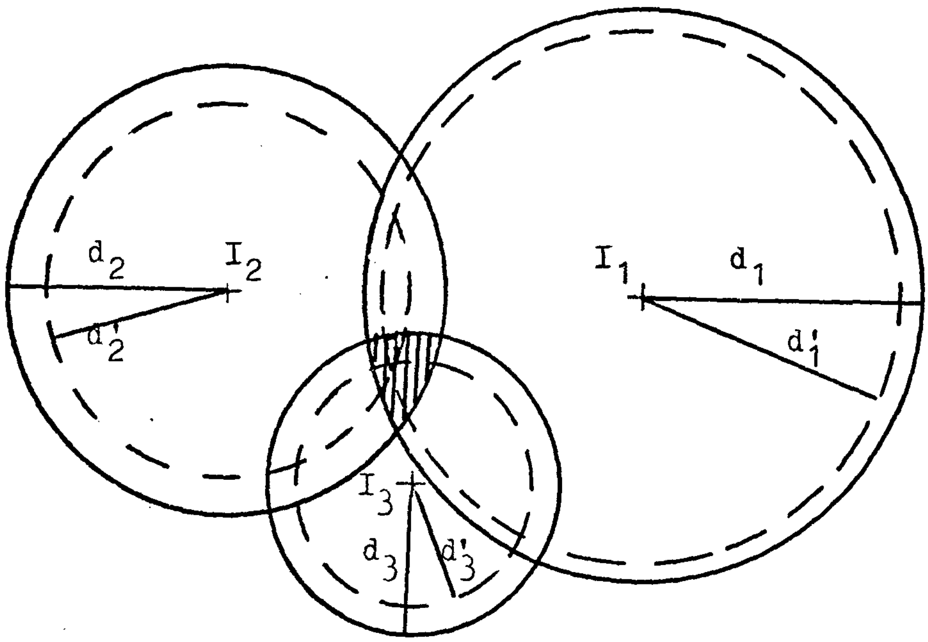

Abb. 1

Ziel der Produktpositionierung kann es nun sein, nicht allein in einem Zeitpunkt mit der dann gültigen Präferenzstruktur einen maximalen Umsatz zu ermöglichen, sondern für möglichst viele Präferenzstrukturen der Käufer die vorgezogene Produktalternative zu bieten. Die Unsicherheit äußert sich in zufällig um einen unbekannten Punkt im Eigenschaftsraum streuende Produktperzeptionen.

Damit erhalten wir den folgenden Ansatz:

(4) $$\max \sum_{k \in K} \sum_{t \in T} \bar{s}_{k,t}\, x_{kt} ,$$

(5) $$D_{k,t}(y) - d_{k,t} \leq (1-x_{k,t}) \cdot M , \quad k \in K,\ t \in T,$$

(6) $$x_{k,t} = \{0,1\}, \qquad k \in K,\ t \in T.$$

Hierbei ist in (4) $\bar{s}_{k,t}$ zusätzlich mit t indiziert, da nun die Periode, für die die Absatzmenge geschätzt wird, einmal in ihrer Länge variieren kann, zum zweiten aber auch bei gleicher Länge die Absatzmenge wegen typischer zeitlicher Nachfragefiguren unterschiedliche Werte aufweisen kann. Letzteres ist vor allem dann interessant, wenn auch entsprechend typische Schwankungen in den Präferenzbildern auftreten. Die gewählte Indizierung erlaubt es auch, durch entsprechende Wertebestimmung implizit die Wahrscheinlichkeit eines Kaufs eines der Angebote im betrachteten Markt im Zeitpunkt $t \epsilon T$, in dem eine bestimmte Präferenzstruktur besteht, für eines der betrachteten Produkte zu berücksichtigen. Deshalb wird hier eine solche Wahrscheinlichkeit nicht explizit gemacht.

In Abb. 2 wird gezeigt, daß das Lösungsgebiet für das Positionierungsproblem (4) bis (6) gegenüber einer für $t = 1$ oder $t = 2$ ermittelten Lösung (senkrecht bzw. waagerecht schraffiert) eingeschränkt wird. Hierbei entspricht der Zielfunktionswert der Summe der Zielfunktionswerte für zwei auf die beiden Beobachtungspunkte bezogene Planungsprobleme unter der Voraussetzung $s := s_{k,t}$. Sobald die Überschneidung der Lösungsgebiete verlorengeht, gilt diese Aussage über die Zielfunktionswerte nicht mehr.

Wir haben damit ein Unsicherheit berücksichtigendes Konzept gewonnen, das in einer gewissen Grundlinie mit dem Ansatz der sogenannten "flexiblen Planung" verwandt ist: In den Nebenbedingungen werden alle bekannten Zustände abgebildet. In jedem Zustand kann nach dem "single choice-Modell" bei gegebenem y über Kauf oder Nichtkauf entschieden werden. In der Zielfunktion kann über den Erwartungswert $\bar{s}_{k,t}$ das Eintreten der einzelnen Zustände berücksichtigt werden, d.h., die Wahrscheinlichkeit dafür, daß in $t \epsilon T$ überhaupt gekauft wird.

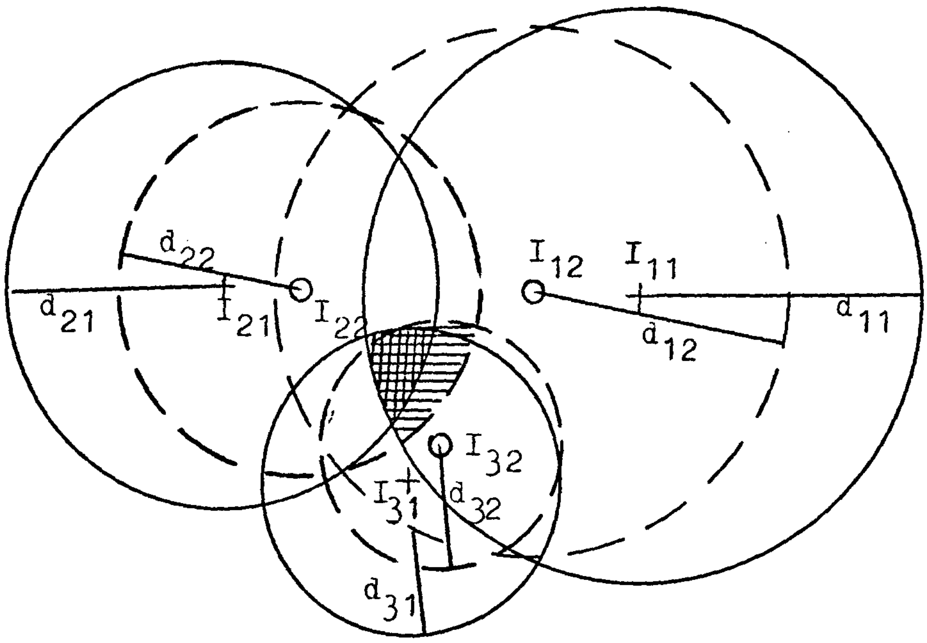

Abb. 2

Kürzlich hat Hax [7] argumentiert, daß die Nebenbedingungen eines solchen Systems dann die Form von Wahrscheinlichkeitsrestriktionen haben sollten, wenn keine vollständige Abbildung der Zustände in ihnen vorgenommen werde. Diesem Argument kann allerdings nur im Sinne einer Problemreduktion gefolgt werden, wo aus den vollständig bekannten Zuständen die Parameter zur Formulierung der Sicherheitsabschläge in diesen Nebenbedingungen bekannt werden. Hier ist die Situation aber gerade so, daß das Bekannte zugleich das ausschließlich Bekannte ist und deshalb die Möglichkeit der Ableitung solcher Parameter bezweifelt wird.

Nehmen wir nun einmal $\bar{s} := \bar{s}_{k,t}$ an. Man erkennt dann leicht, daß der Ansatz zu einer Lösung führt, die Indifferenz gegenüber der N-maligen Realisation der "first choice" durch das neue Produkt bei M Käufern oder der M'-maligen Realisation bei N'Käufern, M, N > 0 (nur $M \neq N$ ist nicht trivial), wenn nur $M \cdot N = M' \cdot N'$. Ist man hier nicht indifferent, so stößt man an Grenzen des zugrundeliegenden Konzepts. Es müßten nämlich nun Lerneffekte als Präferenzen bildend in den Ansatz aufgenommen werden. Diese könnten über kumulierte Absatzmengen der Vergangenheit zum Ausdruck kommen. Eine solche Dynamisierung soll nicht Gegenstand dieser Betrachtung sein, zumal der Modellansatz hierdurch erheblich kompliziert wird.

Einfacher ist die Situation, wenn die in der Zeit sich verändernden Produktperzeptionen - und damit ggf. auch die Präferenzen - allein dazu benutzt werden, eine Folge optimaler Produktpositionen festzulegen. Das Erklärungsmodell für diese Datenänderungen wird hierbei nicht expliziert, was sicher ein Mangel ist, sondern im Grunde durch eine rein zeitabhängige Betrachtung ersetzt, was andererseits gerade in strategischen Planungsmodellen nicht ungewöhnlich ist.

Bezeichnen wir nun mit u ein Element aus der Indexmenge U, $U \subseteq T$; es bezeichnet einen Zeitpunkt, für den eine optimale Produktpositionierung vorgenommen werden soll. Wir erhalten dann die Aufgabe (4) mit (6) und

$$(7) \qquad D_{k,t}(y_u) - d_{k,t} \leq (1-x_{k,t}) \cdot M, \quad k \in K,\ t \in T,\ u \in U .$$

Für praktische Planungsprobleme ist nun davon auszugehen, daß die Positionen y_u, und y_{u+1}, $u=1,2,\ldots,u',u'+1,\ldots,\bar{u}$, durch Ausreißerwerte in den stochastischen Präferenzfunktionen nicht zu weit voneinander entfernt sein dürfen, da der geplante Übergang von der einen zur anderen Position nur beschränkt möglich erscheint. Zusätzlich zu (4), (6), (7) erscheinen deshalb Nebenbedingungen des Typs

$$(8) \qquad \delta(y_u, y_{u+1}) \leq \bar{\delta} \quad , \quad u \varepsilon U,$$

mit

$$(9) \qquad \delta(y_u, y_{u+1}) := \sqrt{\sum_{j=1}^{m} (y_{j,u} - y_{j,u+1})^2}$$

zweckmäßig. Durch (8) wird die Positionierung y_{u+1} auf ein Gebiet beschränkt, das in einer Kugel mit dem Radius $\bar{\delta}$ um y_u eingeschlossen ist. Zur Bestimmung von $\bar{\delta}$ wird man Kosten der Veränderung von Produktpositionen zu berücksichtigen haben.

Diese Überlegung führt zu der Frage, wie unterschiedliche Schwierigkeiten (und damit Kosten) der Veränderung einer Positionierung parallel zu einzelnen Achsen des Raumes der die Produktgruppe kennzeichnenden Eigenschaften berücksichtigt werden könnten. Um die vom Lösungsalgorithmus [2] ausgenutzte Eigenschaft, daß die Schnittmenge konvexer Körper eine konvexe Lösungsmenge bildet, weiter benutzen zu können, erscheint der folgende Ersatz von (9) zweckmäßig:

$$(10) \qquad \delta(y_u, y_{u+1}) := \sqrt{\sum_{j=1}^{m} w_j (y_{j,u} - y_{j,u+1})^2} \quad .$$

Hierbei sind w_j Gewichtungsfaktoren. Sie werden relativ niedrig angesetzt für solche j, in deren Richtung eine Veränderung der Produktposition relativ groß sein kann; je höher die Werte von w_j im Vergleich zu anderen, desto relativ niedriger kann eine Veränderung der Produktposition entlang dieser Dimension sein. Die Nebenbedingung (8) wird unter Verwendung von (10) in Richtung der Koordinaten des Eigenschaftsraums umgekehrt proportional zu den w_j verzerrrte Kugeln um $(y_{1,u}, y_{2,u}, \dots, y_{m,u})$ erzeugen, durch die die Positionen für die Wahl von $(y_{1,u+1}, y_{2,u+1}, \dots, y_{m,u+1})$ eingeschränkt werden.

Selbstverständlich unterliegt dieser Ansatz eines Partialmodells den üblichen Einwendungen gegen solche Modelle: hier insbesondere, daß keine simultane und dynamische Planung der Gewinne vorgesehen ist. Ohne entsprechenden Versuchen der Weiterentwicklung vorzugreifen, kann festgestellt werden, daß schon der vorliegende Planungsansatz über die praktischen Problemlösungen hinausreicht, wesentliche Einflüsse auf die Produktpositionierung zu explizieren anregt, und ohne Rückgriff auf heuristische Techniken grundsätzlich analytisch zu optimieren ist.

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