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Employment and Growth Effects of Tax Reforms in a Growth-Matching Model

ABSTRACT

This paper explores how revenue-neutral tax reforms impact employment and economic growth in models of exogenous and endogenous growth and search frictions on the labor market. We show that (i) a cut in the payroll tax financed by an increase in the wage tax lowers both equilibrium employment and the equilibrium growth rate, that (ii) a higher energy tax combined with a cut in wage taxes boosts employment but has an ambiguous effect on growth, and that (iii) a higher energy tax combined with a cut in payroll taxes enhances employment but mitigates economic growth.

JEL-Classification: E6; H2; J6; O4

Keywords: search unemployment, growth, tax reform

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1 Introduction

The exploration of the employment effects of tax swaps has become an important issue in the academic and policy debate at least since the emergence of the European unemployment problem. The idea that one might be able to reduce equilibrium unemployment by shifting between different types of taxes is high on the research agenda (see, e.g., Sorensen 1997; Pissarides 1998). However, an almost neglected issue in this context is the impact of these reforms on economic growth. If there is a trade-off between employment and growth as put forward, for instance, by Aghion and Howitt (1992), and Eriksson (1997), an employment boosting tax reform has a negative impact on economic growth. Taking the growth issue into account may lead to different policy conclusions concerning the recommendation or dismission of a specific tax reform. The contribution of this paper is to analyze the employment and growth effects of revenue-neutral tax reforms in a search equilibrium model à la Pissarides (1990) which we extent by introducing capital accumulation and economic growth. Distinguishing between models of exogenous and endogenous growth, we consider taxes on wage income, the payroll, and on an imported factor of production.

Our analytical framework merges three strands of literature. First is the literature on employment-enhancing tax reforms. Starting with the contributions of Hersoug (1984) and Lockwood and Manning (1993), it has been established that an increasing degree of income-tax progression may be good for employment (e.g. Koskela and Vilmunen 1996). From the point of view of trade unions the trade-off between wage increases and employment becomes less attractive inducing a wage moderation. As our focus is on the interaction between employment and growth effects, we will confine the analysis to proportional-tax systems. Moreover, most work on environmental tax reform, e.g. Bovenberg and de Mooij (1994), and Bovenberg and van der Ploeg (1998), has shown that using the proceeds of a higher energy tax in order to lower the payroll tax does not always boost employment. Preexisting distortions may be augmented by the increase in the energy tax. A variety of factors has taken into account to sign the employment effect, in particular the properties of the pre-reform equilibrium. For an excellent survey of the literature on the double-dividend issue see Goulder (1995). Pissarides (1998) has shown that the modelling of the labor market imperfections is of minor importance for the sign and size of the employment effect of a tax cut. Consequently, it is of second-order importance whether one assumes a union wage bargaining model, an efficiency wage

model or a search equilibrium model. But since we have to take a stand, we assume a framework in the spirit of Pissarides (1990), where unemployment is the result of search frictions in the labor market. Another choice concerns the issue of real-wage resistance. If the ratio of unemployment benefits to wages is fixed, the effect of tax cuts is mainly on wages, whereas if these benefits are indexed to prices, there may be considerable scope for employment gains (Pissarides, 1998; Pflüger, 1997). Despite we recognize that real-wage resistance is a significant feature of many economies, unemployment benefits are assumed to be indexed to the net wage (constant net replacement ratio). Summarizing the literature, Layard et al. (1991) argue that real wage resistance is likely to be temporary rather than permanent, that is, in the long run, rises in the labor tax do not affect real labor costs. Furthermore, the assumption of a constant (net) replacement ratio preserves balanced growth, since the relative attractiveness of being employed compared to being unemployed does not change along the equilibrium growth path.

The second strand of research is on the growth effects of tax policies. In the Solow model, in which (exogenous) labor-augmenting technical progress is the main determinant of the growth rate, tax policies have an impact only on the long-run per-capita income level (see, for instance, Carlberg 1988) but not on the long-run growth rate itself. To derive the possibility that the government influences the long-run growth rate, more recent models of endogenous growth are needed. Using an AK-based growth model Turnovsky (2000) discusses the role of income and consumption taxes in enhancing economic growth. Kim (1998) develops an endogenous growth model which allows for the assessment of the extent to which differences in the tax systems account for the difference in the actual growth rates across countries. He finds that about 30% of the difference of growth rates between the United States and a set of East Asian countries can be explained by differences in the tax systems. Bovenberg and de Mooij (1997) discuss how an environmental tax reform impacts economic growth, but they abstract from labor as an input factor. Our framework distinguishes between models of exogenous and endogenous growth, and one finding is that the choice of the growth model is not crucial, that is, the change in the long-run per-capita income level is a good proxy for the change in the long-run growth rate.

The third strand of research we refer to is on the interaction between employment and growth. If growth comes through creative destruction (Aghion and Howitt, 1992), the flow of workers into the pool of unemployed and thus the equilibrium

unemployment rate is a positive function of the growth rate of the economy. A higher equilibrium growth rate, on the other hand, induces higher future revenues and thus rising vacancies that lead to more employment. For this reason current job creation and equilibrium employment is increasing in the growth rate (so-called capitalization effect, see Bean and Pissarides, 1993). Overall, the relationship between employment and growth is difficult to sign (Aghion and Howitt, 1994).

While the models just discussed have their focus on analyzing either taxes and equilibrium unemployment or taxes and growth, our model analyzes the issues of equilibrium unemployment, economic growth and different tax systems in a unified framework. The only work, at least to our knowledge, which uses a similar set up is Daveri and Tabellini (2000), and Eriksson (1997). Eriksson (1997) presents an endogenous growth model of the AK-type in which unemployment is caused by search frictions. He finds that an increase in the capital income tax reduces the incentive to save and due to the capitalization effect reduces the equilibrium growth rate. Daveri and Tabellini (2000) develop an overlapping generations endogenous growth model where wages are set by monopolistic trade unions. They show that a higher labor income tax is met by a higher bargained wage forcing firms to cut employment. This in turn lowers the income of the young and thus savings. But there is another mechanism. Because of the initial rise in the capital/labor ratio the marginal product of capital and hence the incentive to save declines enforcing the negative impact on capital accumulation and hence on economic growth. For a critical assessment of this model see Nickell and Layard (1999).

In contrast to much of the mentioned literature, we assume that tax reforms must be ex post revenue-neutral in the sense that they are budget neutral after all adjustments in the economy have taken place. For a similar but static framework see Michaelis and Pflüger (2000).

The model we set up in the next section frames a small open economy that produces and exports a homogeneous good and imports a productive factor (energy for instance). To keep the model as simple as possible we impose the condition of a balanced trade account. Our model can be reduced to two equilibrium conditions, the efficient factor allocation function showing equilibrium in the factor markets for labor, energy and capital, and the capital accumulation function depicting the equilibrium growth path. The intersection of these curves determines the steady state values of labor market tightness (employment) and capital per effective worker (equilibrium growth rate). We will show that (i) a cut in the payroll tax financed by

an increase in the wage tax will lower both equilibrium employment and the growth rate, that (ii) a higher energy tax combined with a cut in the wage tax has an ambiguous effect on growth but boosts employment, and that (iii) a higher energy tax combined with a cut in the payroll tax is good for employment and for almost all parameter constellations bad for growth.

The rest of paper is organized as follows. Section 2 and 3 present the model and the analysis of the steady state solution, respectively. The tax reform analysis is performed in Section 4 (Solow growth model) and Section 5 (endogenous growth). Section 6 concludes.

2 The Model

2.1 Flows in the Labor Market

Aggregate labor endowment of households is constant and denoted by $\overline{L}$. At every instant, labor is either employed or unemployed; the employed workers are denoted as E and the unemployed as U. Thus, the labor force is represented by

$$\overline{L} = E + U. \tag{1}$$

The labor market is characterized by search frictions with firms looking for jobless workers filling vacancies and unemployed searching for a job. Both sides of the market have incomplete information about the opposite market side. The level of search activities is represented by the number of vacancies V, the number of unemployed U and the number of matches M formed per time unit. If no frictions were present, laid-off workers would find immediately new jobs and equilibrium unemployment would not exist.

The Matching Function

Matching takes place between newly created vacancies and unemployed workers. The underlying matching technology is of the usual constant returns to scale variety:[1]

$$M = m(V, U) = V^{1-\beta}U^{\beta}, \quad 0 < \beta < 1 \tag{2}$$

where β is the matching-elasticity of U. Let $\theta := V/U$ denote labor market tightness. The matching-probability for searching workers is then obtained as

$$p(\theta) := M/U = \theta^{1-\beta}; \tag{3}$$

[1]See Petrongolo and Pissarides (2001) for further details on matching functions.

the tighter the labor market, the easier to find a job. The matching-probability for the firm is

$$q(\theta) := M/V = \theta^{-\beta}; \tag{4}$$

the tighter the labor market, the more difficult to fill a vacancy. Note that $p(\theta) = \theta q(\theta)$.

Flow Equilibrium

The change in employment is determined by inflows in and outflows out of unemployment. The inflows into unemployment are characterized by the separation of existing job-matches at any point in time and are described by the exogenously given separation rate ν times the number of workers E. Thus, inflows depict the number of unproductive jobs which generate layoffs.[2] On the other hand, outflows are represented by the flow of newly formed job-matches and, therefore, by the matching-function $m(V, U)$. Taking outflows and inflows together, the dynamics of employment result as the difference between both and can be expressed as $\dot{E} = m(V, U) - \nu E$. In the steady state (flow equilibrium) employment is constant, $\dot{E} = 0$, so

$$\theta^{-\beta} V = \nu E. \tag{5}$$

Now we can use (1), (3) and (5) to solve for employment and unemployment:

$$e(\theta) := \frac{E}{\bar{L}} = \frac{p(\theta)}{\nu + p(\theta)}, \quad e_\theta > 0 \tag{6}$$

$$u(\theta) := \frac{U}{\bar{L}} = \frac{\nu}{\nu + p(\theta)}, \quad u_\theta < 0. \tag{7}$$

In equilibrium, employment and unemployment are determined by the transition rates. The higher the separation rate ν, the lower (higher) the steady-state employment (unemployment) rate. Furthermore, the tighter the labor market (higher θ), the higher the matching-probability $p(\theta)$ and the higher employment (the lower unemployment).

[2] For an exogenous separation rate see also Pissarides (1990) as well as Postel-Vinay (1998) and for an endogenous rate see Mortensen and Pissarides (1994, 1998).

2.2 Firms

Each firm uses capital K, an imported factor Z and labor in efficiency units λE to produce a homogenous good X. In the following we shall term Z as energy, however, one can think of it as raw material, too. The technology is of the Cobb-Douglas type with constant returns to scale:

$$X = F(K, Z, \lambda E) := K^{\alpha} Z^{\gamma} [\lambda E]^{\varepsilon} \tag{8}$$

where α, γ and $\varepsilon := 1 - \alpha - \gamma$ denote the production elasticity of capital, energy and labor in efficiency units, respectively. Labor productivity grows with the exogenous rate g, that is, $\lambda := \lambda_0 e^{gt}$. By constant returns to scale, the production function can be rewritten in efficiency units as

$$x = k^{\alpha} z^{\gamma}, \tag{9}$$

where $x := X/\lambda E$, $k := K/\lambda E$ and $z := Z/\lambda E$ denote real output, capital and energy per efficiency unit of labor (or: per effective worker). Note that $F_K = \alpha x/k; F_Z = \gamma x/z$ and $F_E = \varepsilon \lambda x$ with F_j denoting the partial derivative of $F(\cdot)$ with respect to $j = K, E, Z$.

Firms maximize the present-discounted value of expected profits with respect to investments I, energy Z and the creation of job vacancies V. Each vacancy induces gross hiring costs c, which are assumed to depend on the producer wage: $c = \eta\,(1 + t_{pw})\,w$, where w is the wage rate, t_{pw} is the payroll tax, and η is a constant. The current flow of profits amounts to output minus gross factor payments minus gross search expenditures. The factor payments consists of capital costs rK, labor costs $(1 + t_{pw})wE$ and costs for the imported energy $(1 + t_z)p_z Z$, where r is the interest rate, p_z is the energy price (determined at the world market) and t_z is the tax levied on the use of energy. Taking these aspects into consideration, the representative firm faces the following intertemporal optimization problem:

$$\max_{I,V,Z} \int_0^{\infty} \{F(K, Z, \lambda E) - rK - (1 + t_{pw})\,wE - (1 + t_z)\,p_z Z - \eta\,(1 + t_{pw})\,wV\} e^{-rt} dt$$

$$\begin{aligned} \text{s.t.} \quad & \dot{E} = V^{1-\beta} U^{\beta} - \nu E \\ & \dot{K} = I \\ & K(0), E(0) \quad \text{given.} \end{aligned}$$

In a steady state the first-order conditions for labor, capital and energy are given by (see Appendix A):

$$F_E(\cdot) = (1+t_{pw})\, w \left[1 + \frac{\eta}{1-\beta}\,(r+\nu-g)\,\theta^{\beta}\right] \tag{10}$$

$$F_K(\cdot) = r \tag{11}$$

$$F_Z(\cdot) = (1+t_z)\, p_z, \tag{12}$$

The last term in the squared bracket in Eq. (10) represents the present value of expected net hiring costs. A higher separation rate ν and a higher interest rate r means that the expected present value of a successful matching falls. An increase in the rate of technological progress g means an increase in the growth rate of wages, which is equivalent to a decline in the costs of current recruiting activities. Moreover, the tighter the labor market (higher θ), the lower is the probability of filling the firm's vacancies and the higher are the expected hiring costs.

2.3 Wage Determination[3]

The wage rate for a job is bargained between the firm and the worker after they meet. They share the rent of a realized job match, i.e. the sum of the expected search costs for the firm and the worker. Let $V_J$denote the expected present value of an occupied job and $V_V$the expected present value of a vacant job. Then the value functions are:

$$rV_J = \lambda x - (1+t_{pw})w - r\lambda k - (1+t_z)\lambda z - \nu(V_J - V_V) \tag{13}$$

$$rV_V = -\eta(1+t_{pw})w + q(\theta)\cdot(V_J - V_V) \tag{14}$$

Eq. (13) states that the expected present value of a filled job is the worker's real output minus labor costs, capital costs and energy costs as well as minus the loss from the destruction of the job. Following Pissarides (1990) we assume that there are no quasi-rents from a fixed capital stock, i.e. in the case of a job destruction capital can be sold at the second-hand market. As can be seen from (14) the value of a vacancy is the gain $V_J - V_V$ received with probability $q(\theta)$ minus hiring costs. With free entry of new vacancies, $V_V = 0$, Eq. (14) shows that in equilibrium, the expected profits from a filled job have to cover the hiring costs: $q(\theta)\cdot V_J = \eta(1+t_{pw})w$.

[3]See also Nickell (1999) and Zanchi (2000) for a recent discussion of the wage determination in search models.

The worker's expected returns are given by the value functions

$$rV_E = (1 - t_w)w - \nu(V_E - V_U) \tag{15}$$

$$rV_U = B + \theta q(\theta) \cdot (V_E - V_U) \tag{16}$$

where V_E and V_U denote the expected present value of being employed and unemployed, respectively. The permanent income of an employed individual is the net wage $(1-t_w)w$ minus the loss associated with a transition to unemployment. Finally, the expected return from unemployment amounts to the (indefinitely available) unemployment benefits, B, plus the gain in income if a job is found. For the sake of simplicity we restrict the analysis to the case of a constant net replacement ratio: $h \equiv B/(1 - t_w)w$. Any change in the net wage, caused either by a change in the wage rate w or by a change in the wage tax, leads to a proportionate adjustment of the level of benefits.

The (representative) firm and worker choose the wage w that maximizes the Nash product $(V_E - V_U)^{\phi} \cdot (V_J - V_V)^{1-\phi}$, where ϕ stands for the bargaining power of the worker. The first-order-condition reads:

$$V_E - V_U = \frac{\phi(1 - t_w)}{(1 - \phi)(1 + t_{pw})}(V_J - V_V). \tag{17}$$

By making use of the definition of the net replacement ratio h, the free entry condition, $V_V = 0$, and the asset equations, we get from eqs. (13) - (17) the bargained real wage

$$w = \frac{\phi}{[1 - (1 - \phi)\, h - \phi\theta\eta]\,(1 + t_{pw})} F_E(\cdot) \tag{18}$$

as a share of the marginal product of labor. This share depends on the model parameters in a very intuitive way: firstly, the higher the workers' bargaining strength ϕ and the net replacement ratio h, the higher is the share and thus the real wage. Secondly, because of a higher rent from a job match, the wage is increasing in the hiring costs captured by η. Thirdly, a tighter labor market (higher θ) improves the chance of an unemployed to find a job and lowers the chance of a firm to fill a vacancy. This raises the bargaining position of the worker and thus the real wage. Fourthly, any reduction in the payroll tax t_{pw} (which corresponds to an increase in the firm's profits) will be answered by a one-to-one increase in the wage rate, so the producer real wage $(1 + t_{pw})w$ does not depend on the payroll tax. And lastly,

since the assumption of a constant *net* replacement ratio ensures that the relative attractiveness of being employed compared to being unemployed does not change with the wage tax t_w, any change in t_w is neutral for the bargained real wage.[4]

2.4 Government Budget, Trade Account and Savings

The government controls four policy parameters: a wage income tax, t_w, a payroll tax, t_{pw}, a tax on the imported factor, t_z, and unemployment benefits, B. The tax bases are as follows: wage tax: wE, payroll tax: $wE + \eta wV$, energy tax: $p_z Z$. The tax base of the payroll tax is larger than that on wage income because it covers also hiring costs. The tax revenues are completely spent for paying the unemployment benefits, BU. Therefore, the government budget constraint reads

$$BU = t_w wE + t_{pw} wE + t_{pw} \eta wV + t_z p_z Z. \tag{19}$$

The economy under consideration imports the productive factor Z and exports tradeables Ex. Domestic producer have no market power, so both the export price p_{Ex} and the import price p_z are exogenously determined on the world market. We assume a balanced trade account:

$$p_{Ex} Ex = p_z Z. \tag{20}$$

Domestic firms elastically supply their goods on the world market and sell the quantity necessary to finance the value of imports. Note that we do not allow for any trade in capital. This is clearly a shortcut which can be defended, for instance, by the Horioka/Feldstein-puzzle, which states that the capital markets are far from perfectly integrated; domestic investment and thus domestic capital accumulation is highly correlated with domestic savings. The real interest rate is not determined on the world capital market but primarily influenced by domestic factors like the saving rate.

[4]Note that the wage equation (18) does not make economic sense in the cases where a party dominates the bargain. If the firm sets the wage ($\phi = 0$), the wage is driven down to zero, since we do not assume any income or utility from leisure activities. If the worker sets the wage ($\phi = 1$), one can show that this wage will be greater than the marginal product of labor. In this case, however, the firm does not cover the hiring costs, it will make losses. When the firm anticipates such a scenario, it will not engage in any job creation. In other words, to fulfill the firm's participation constraint, i.e. the zero profit condition, the bargaining strength ϕ must not be too large.

Domestic savings S are a constant fraction s of national income Y, which, in turn, amounts to real output minus the value of imports. Savings are used to finance investments and pay for the costs of vacancies:

$$S = sY = s(X - p_z Z) = I + \eta w V. \tag{21}$$

3 Steady-State Solution

To determine the overall steady-state solution, the model will be reduced to three equations – the efficient factor allocation function showing equilibrium in factor markets, the capital accumulation function depicting the equilibrium growth path, and the government budget constraint. These three equations can be solved for (the change in) three endogenous variables: labor market tightness θ, capital per effective worker k, and a tax rate.

3.1 Efficient Factor Allocation

By combining the first-order conditions (10) - (12) for labor, capital and energy, and by using the wage equation (18) and the production function (9), we yield after some manipulations the *efficient factor allocation function* (see Appendix B):

$$k^{\frac{\varepsilon}{1-\gamma}} = \frac{\eta\alpha\phi}{(1-\beta)} \left(\frac{\gamma}{(1+t_z)\, p_z} \right)^{\frac{\gamma}{1-\gamma}} \left\{ \frac{1}{\theta^{-\beta} \left[(1-\phi)(1-h) - \eta\phi\theta \right] - \frac{\eta\phi}{1-\beta} (\nu - g)} \right\}. \tag{22}$$

It represents all combinations of capital per effective worker and labor market tightness where all factor markets are in equilibrium. The properties of (22) are summarized in:

Proposition 1 *(i) The efficient factor allocation function is positively sloped in the* (k, θ)*-space.*
(ii) A higher payroll tax t_{pw} *and a higher wage tax* t_w *leaves the efficient factor allocation unaffected.*
(iii) In the (k, θ)*-space, a higher tax on the imported factor leads to a downward shift of the efficient factor allocation function , i.e., ceteris paribus, capital per effective worker is decreasing in* t_z*.*

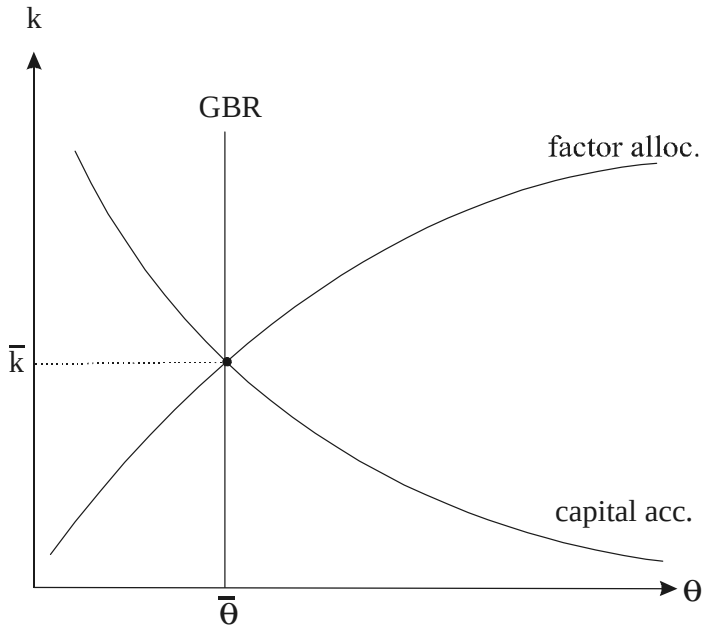

Figure 1: Steady State of the Economy.

Proof. All parts follow from (22) by derivation. ■

Proposition (1) can be made intuitive by considering the first order conditions and the wage equation. An increase in capital per effective worker k leads c.p. to a decline in the interest rate, which in turn generates an incentive to create new jobs via lower hiring costs (see Eq. (10)). In a new equilibrium the labor market is tighter (higher θ). Part (ii) reflects the fact that the producer wage $(1 + t_{pw})w$ and thus labor costs are independent of t_{pw} and t_w. A higher energy tax reduces z which in turn causes a decline in the marginal product of capital and thus a decline in the profit-maximizing capital per effective worker.

3.2 Capital Accumulation

In a next step, we have to analyze how the economy evolves over time. The increase in the capital stock at a point in time equals investment and the amount invested equals the amount saved minus hiring costs: $\dot{K} = I = sY - \eta wV$. The change in capital per efficiency unit of labor, $\dot{k}$, can be derived as $\dot{k} = sy - (g + \dot{E}/E)k - \eta wV/\lambda E$, where $y = Y/\lambda E$ is income per effective worker. In the steady state capital per effective worker as well as employment is constant, $\dot{k} = \dot{E} = 0$, so the capital

stock is the solution to

$$sy = gk + \eta w \frac{\nu \theta^{\beta}}{\lambda}. \tag{23}$$

which can be rewritten as (see Appendix C)

$$k^{\frac{\varepsilon}{1-\gamma}} = \frac{1}{g}\left(\frac{\gamma}{(1+t_z)\,p_z}\right)^{\frac{\gamma}{1-\gamma}} \left\{\frac{s(1-\gamma+t_z)}{1+t_z} - \frac{\varepsilon\eta\nu\phi\theta^{\beta}}{[1-(1-\phi)\,h-\phi\theta\eta]\,(1+t_{pw})}\right\} \tag{24}$$

The properties of the capital accumulation function (24) are stated in

Proposition 2 *In a growth equilibrium, the steady-state level of capital per efficiency unit of labor is (i) decreasing in labor market tightness, (ii) increasing in the payroll tax, (iii) independent of the wage tax, and (iv) ambiguous in the tax on the imported factor.*

Proof. Properties (i) - (iii) follow from (24) by derivation. (iv) The partial derivative of k with respect to t_z is given by $\frac{\partial k}{\partial t_z} = D\,[s(1-\gamma)r - (1+t_z)\alpha g] \gtreqless 0$ where D is a positive constant. ■

Part (i) replicates a result already obtained by Pissarides (1990): the capital accumulation function is negatively sloped in the (k, θ)-space (see Figure 1). If the labor market gets tighter, the number of vacancies V and the bargained real wage w increases. This makes recruitment costs $\eta w V$ higher, which in turn lowers capital accumulation (see (21)). Similarly, a higher payroll tax t_{pw} lowers the bargained wage w, so hiring costs decrease and thus capital usage increases. In Figure 1 the capital accumulation function shifts up. The wage tax t_w leaves the wage and thus hiring costs and investments unaffected. A higher energy tax t_z has no influence on hiring costs but its impact on income, $y = x - p_z z$, and thus on savings and investment is twofold: for a given real output the reduced imports lead to an increase in y, however, output declines because of a lower z. The net effect on y (and savings and capital accumulation) is parameter dependent.

3.3 The Government Budget Constraint (GBR)

When discussing the employment and growth effects of tax reforms we impose the condition of ex-post revenue-neutrality. Tax reforms must be budget neutral after adjustments of the wage rate, employment, vacancies etc. have taken place. In order

to take these adjustments into account, we rearrange (19) to obtain the government budget constraint as a function of tax rates and of the two endogenous variables, k and θ. As shown in Appendix D, this leads to

$$\frac{\gamma t_z}{1+t_z} = \frac{\varepsilon\phi}{(1+t_{pw})\left[1-(1-\phi)\,h-\eta\phi\theta\right]}\left[(1-t_w)\,h\nu\theta^{\beta-1}-t_w-t_{pw}\left(1+\eta\nu\theta^{\beta}\right)\right]. \tag{25}$$

Capital per effective worker does not appear in (25), so the government budget is neutral with respect to changes in k. In the (k,θ)-plane, the GBR is a vertical line (see Figure 1). The reason is that for a given θ and thus for a given level of employment, vacancies and unemployment, a higher k corresponds to a higher wage and thus to higher tax revenues. But on the other hand, due to the assumption of a constant net replacement ratio, unemployment benefits increase, too. Since the additional expenditures equal the additional tax revenues, a higher k is budget neutral. Turn now to the shift parameters of the GBR-line. By the assumption of Laffer-efficiency higher tax rates lead to additional tax revenues. To restore a balanced budget, labor market tightness θ has to decrease, since this corresponds to a lower level of employment (lower tax base) and a higher level of unemployment (higher expenditures) and thus to a leftward shift of the GBR-line in Figure 1.

4 Tax Reforms

Eqs. (22), (24), and (25) form a system of three equations with three endogenous variables: the tightness parameter θ, capital per effective worker k, and a tax rate. Due to the non-linearity of all three equations, we log-linearize the model around a steady-state[5] denoting relative changes by a tilde. The log-linearized versions of these equations read

$$\begin{bmatrix} a_1 & -a_2 & 0 & 0 & a_5 \\ b_1 & b_2 & 0 & -b_4 & -b_5 \\ 0 & c_2 & c_3 & c_4 & c_5 \end{bmatrix} \begin{bmatrix} \tilde{k} \\ \tilde{\theta} \\ \tilde{t}_w \\ \tilde{t}_{pw} \\ \tilde{t}_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{26}$$

where the coefficients are constants defined in Appendix E.

[5]We omit the proof of existence, uniqueness and stability of this steady state. This proof is available on request from the authors.

4.1 Switch from Payroll to Wage Taxes

The first tax reform we are interested in is a budget neutral substitution of wage taxes for payroll taxes. Assuming that the tax on the imported factor of production will be held constant, $\tilde{t}_z = 0$, the employment and growth effects are given by the solution of

$$\begin{bmatrix} a_1 & -a_2 & 0 \\ b_1 & b_2 & 0 \\ 0 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} \partial\tilde{k}/\partial\tilde{t}_{pw} \\ \partial\tilde{\theta}/\partial\tilde{t}_{pw} \\ \partial\tilde{t}_w/\partial\tilde{t}_{pw} \end{bmatrix} = \begin{bmatrix} 0 \\ b_4 \\ -c_4 \end{bmatrix} \tag{27}$$

The properties of (27) can be summarized in

Proposition 3 *A budget-neutral shift from payroll to wage taxes unambiguously reduces both labor market tightness and capital per effective worker.*

Proof. The reaction of capital per effective worker and labor market tightness are given by $\partial\tilde{k}/\partial\tilde{t}_{pw} = |A|^{-1}(a_2 b_4 c_3) > 0$ and $\partial\tilde{\theta}/\partial\tilde{t}_{pw} = |A|^{-1}(a_1 b_4 c_3) > 0$, respectively, where $|A| := a_1 b_2 c_3 + a_2 b_1 c_3 > 0$. The assumption of Laffer-efficiency ensures $\partial\tilde{t}_w/\partial\tilde{t}_{pw} < 0$. ■

This result can be explained as follows. The decrease in the payroll tax will be answered by a one-to-one increase in the bargained gross wage w, hence there is no impact on the sum of labor costs per worker and hiring costs per vacancy, $(1 + t_{pw})w + \eta(1 + t_{pw})w$. On the other hand, the increase in the wage tax, t_w, necessary to finance the cut in the payroll tax, has no repercussions on the gross wage. As aforementioned, due to the assumption of a constant net replacement ratio the unemployed "participate" by a decline in the level of benefits, so that the relative attractiveness of being unemployed compared to being employed and thus the bargained wage remains unaffected. Consequently, the efficient factor allocation function is not affected by the tax swap under consideration.

However, the rise in the wage rate implies a rise in hiring costs and thus a decline in investments. In Figure 2, the capital accumulation function shifts down and to the left, the new equilibrium is at point B. It should be clear that the cut in the payroll tax shifts the vertical government-budget-constraint line (not depicted) to the right, whereas the increase in the wage tax shifts this line to the left. As the algebraic solution indicates, the net effect is a leftward shift, point B is the intersection of all three curves.

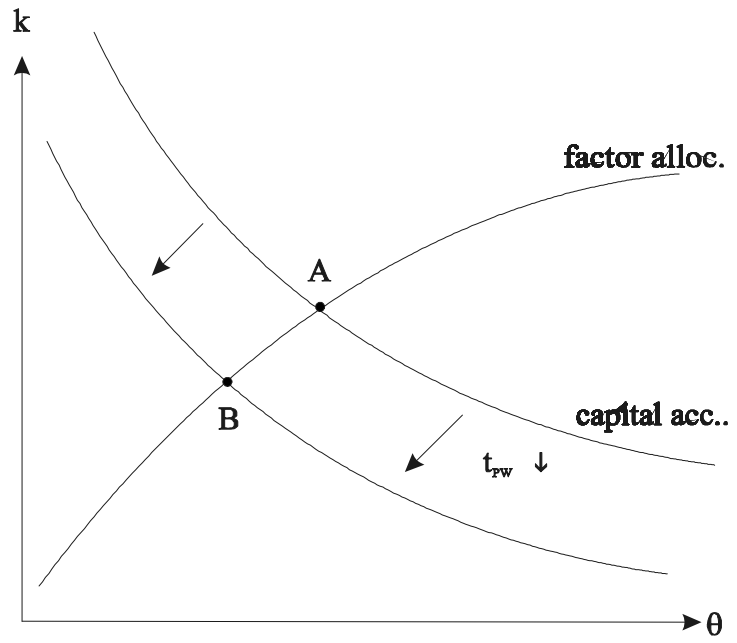

Figure 2: Switch from Payroll to Wage Taxes

4.2 Switch from Wage Taxes to Energy Taxes

The employment and growth effects of a revenue-neutral substitution of taxes on the imported factor for wage taxes are described by the solution of

$$\begin{bmatrix} a_1 & -a_2 & 0 \\ b_1 & b_2 & 0 \\ 0 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} \partial\tilde{k}/\partial\tilde{t}_z \\ \partial\tilde{\theta}/\partial\tilde{t}_z \\ \partial\tilde{t}_w/\partial\tilde{t}_z \end{bmatrix} = \begin{bmatrix} -a_5 \\ b_5 \\ -c_5 \end{bmatrix} \tag{28}$$

Proposition 4 *A revenue-neutral shift from wage taxes to taxes on the imported factor unambiguously raises labor market tightness; capital per effective worker may increase or decrease.*

Proof. The reaction of capital per effective worker and labor market tightness are given by $\partial\tilde{k}/\partial\tilde{t}_z = |A|^{-1}(a_2b_5c_3 - a_5b_2c_3) \gtreqless 0$ and $\partial\tilde{\theta}/\partial\tilde{t}_z = |A|^{-1}(a_1b_5c_3 + a_5b_1c_3) > 0$, respectively, where $|A| := a_1b_2c_3 + a_2b_1c_3 > 0$. The assumption of Laffer-efficiency ensures $\partial\tilde{t}_w/\partial\tilde{t}_z < 0$. ■

A higher energy tax reduces the marginal product of capital and thus k via a lower z. In Figure 3 this corresponds to a downward shift of the factor allocation function. Concerning capital accumulation we stated in part (iv) of Proposition

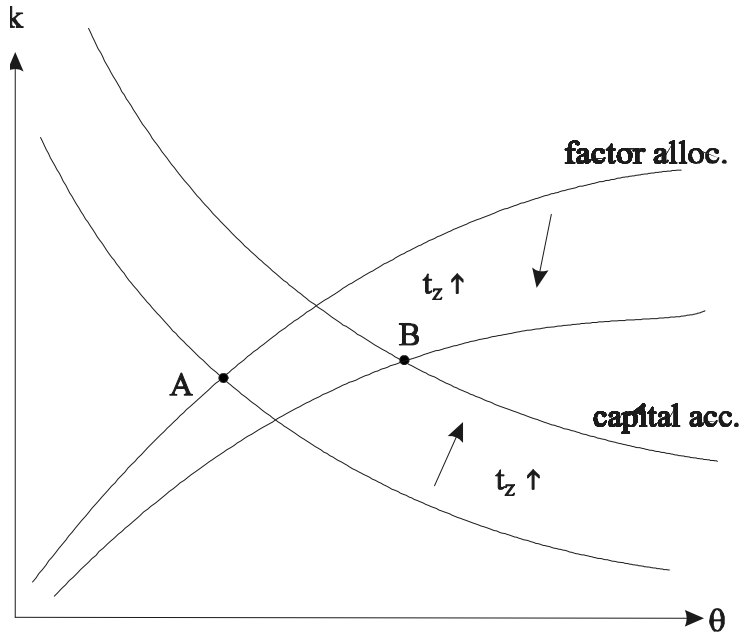

Figure 3: Switch from Wage to Energy Taxes

2 that a higher t_z has an ambiguous effect on capital per effective worker. The capital accumulation function may shift to the left or to the right. In the former case this enhances the decline in k and mitigates the positive impact on θ, whereas in the latter case (which we assume in Fig. 3) the decline in k is mitigated and the positive effect on labor market tightness is enhanced. Since the endogenously determined decline in the wage tax has no impact on both effective factor allocation and capital accumulation, point B is the new equilibrium. Note that even in the case of a leftward shift of the capital accumulation function, point B must always lie to the right of the initial equilibrium (point A): the algebraic solution unambiguously indicates an increase in labor market tightness and thus an increase in employment.

4.3 Switch from Payroll Taxes to Energy Taxes

If the additional tax revenues of a higher tax on the imported factor are used to finance a cut in payroll taxes, the employment and growth effects are described by the solution of

$$\begin{bmatrix} a_1 & -a_2 & 0 \\ b_1 & b_2 & -b_4 \\ 0 & c_2 & c_4 \end{bmatrix} \begin{bmatrix} \partial\tilde{k}/\partial\tilde{t}_z \\ \partial\tilde{\theta}/\partial\tilde{t}_z \\ \partial\tilde{t}_{pw}/\partial\tilde{t}_z \end{bmatrix} = \begin{bmatrix} -a_5 \\ b_5 \\ -c_5 \end{bmatrix} \tag{29}$$

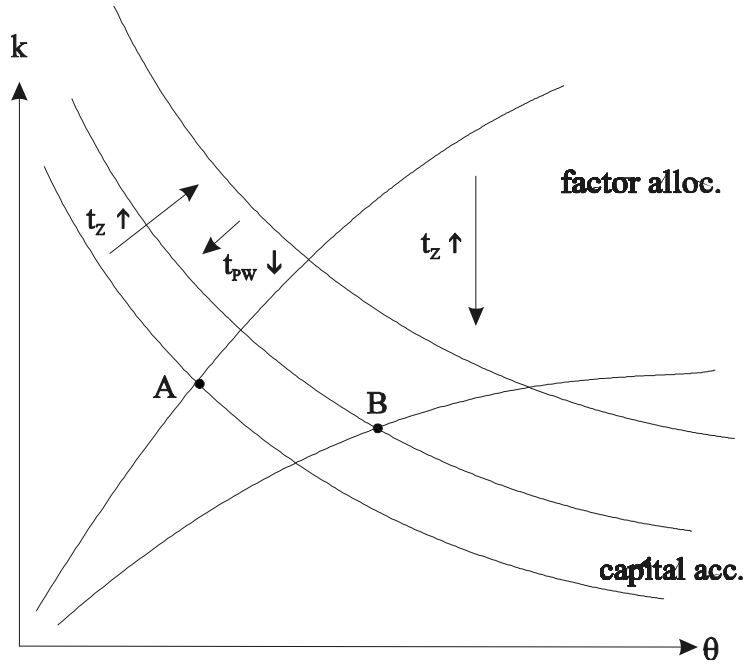

Figure 4: Switch from Payroll to Energy Taxes

Proposition 5 *A revenue-neutral shift from payroll taxes to taxes on the imported factor unambiguously raises labor market tightness; capital per effective worker may increase or decrease.*

Proof. The reaction of capital per effective worker and labor market tightness are given by $\partial\tilde{k}/\partial\tilde{t}_z = |A|^{-1}\left(-a_5(b_2c_4 + b_4c_2) - a_2b_4c_5 + a_2b_5c_4\right) \lessgtr 0$ and $\partial\tilde{\theta}/\partial\tilde{t}_z = |A|^{-1}\left(a_1b_5c_4 + a_5b_1c_4 - a_1b_4c_5\right) > 0$, respectively, where $|A| := a_1(b_2c_4 + b_4c_2) + a_2b_1c_4 > 0$. The assumption of Laffer-efficiency ensures $\partial\tilde{t}_{pw}/\partial\tilde{t}_z < 0$. ■

As indicated above, a higher tax on imports lowers the profit-maximizing k, shifting the factor allocation function downwards. The decrease in the payroll tax is neutral to the optimal factor inputs, the factor allocation function is not affected. Turning to the capital accumulation function we observe a leftward shift due to the decline in t_{pw} and an ambiguous shift due to the increase in t_z. In Figure 4, the new equilibrium B lies to the right of the initial equilibrium A showing a higher θ and thus an increase in equilibrium employment. Capital per effective worker may increase, but for almost all parameter constellations we observe a decline in k. For this tax reform we yield a trade off between employment and growth; employment boosts whereas capital per effective worker probably declines.

5 Endogenous Growth

In this section we are interested to see how tax reforms affect the growth rate itself, i.e. we relax the assumption of an exogenous growth rate by setting up an endogenous growth model. Following Romer (1986) we introduce endogenous growth by assuming positive learning and knowledge spillovers working through the economy's capital stock per worker, $\bar{k} = K/E$. The production function for firm i takes the Cobb-Douglas form,

$$X_i = \bar{k}^{\delta} K_i^{\alpha} Z_i^{\gamma} E_i^{\epsilon}, \tag{30}$$

where $\alpha + \delta = 1$ and $\alpha + \gamma + \epsilon = 1$. In per capita terms we get $x_i = \bar{k}^{\delta} k_i^{\alpha} z_i^{\gamma}$, which simplifies to

$$x = kz^{\gamma} \tag{31}$$

in a symmetric equilibrium, where $k_i = \bar{k} = k$. At the firm level, the technology exhibits constant returns to scale in the private inputs, K_i, Z_i and E_i. At the aggregate level, however, there are constant returns in capital and increasing returns in the inputs K, Z and E. The private marginal products in the representative firm are $F_K = \alpha x/k, F_Z = \gamma x/z$ and $F_E = \epsilon x$. As usual the optimization conditions for firms entail equality between the marginal products and marginal costs:

$$\alpha \frac{x}{k} = r \tag{32}$$

$$\gamma \frac{x}{z} = (1 + t_z) p_z \tag{33}$$

$$F_E = (1 + t_{pw})\, w \left[1 + \frac{\eta}{1 - \beta} \left(r + \nu - \hat{w} \right) \theta^{\beta} \right] \tag{34}$$

Since in a steady state the bargained real wage is a constant share of the marginal product of labor, and F_E is a constant share of output, we can conclude that the growth rate of wages equals the growth rate of output per worker, $\hat{w} = \hat{x}$. In a steady state the real interest rate r is assumed to be constant, so we have $\hat{x} = \hat{k}$ from Eq. (32). The production function (31) implies that the growth rate of x is $\hat{x} = \hat{k} + \gamma \hat{z}$. Combining these results yields $\hat{z} = 0$, i.e. in a steady state the use of energy will be constant. To ensure this outcome, we have to assume that the price of energy rises at the growth rate of output per worker. This follows from the steady-state version of Eq. (33): $\hat{p}_z = \hat{x} - \hat{z} = \hat{x}$.

Now turn to the optimizing behavior of households. The infinitely-lived household j is assumed to maximize utility

$$U_j = \int_0^\infty e^{-\rho t} \cdot \frac{(c_j)^{1-\sigma} - 1}{1-\sigma} dt, \tag{35}$$

subject to the constraint $\dot{k}_j = rk_j + I_j - c_j$, where I_j is the non-capital income of household j (j's share of aggregate profits plus the wage rate if employed and unemployment benefits if unemployed). The parameters $\rho > 0$ and $\sigma > 0$ denote the rate of time preference and the elasticity of marginal utility of consumption, respectively. Focusing on a symmetric equilibrium, the optimal growth rate of consumption can be derived as

$$\hat{c} = \frac{\dot{c}}{c} = \frac{1}{\sigma}(r - \rho). \tag{36}$$

Any tax reform influencing the real interest rate has an impact on the growth rate of consumption. If the interest rate rises, the rate of return to savings rises, households shift some consumption to the future, they accumulate more capital leading to a higher growth rate of capital, output and consumption.

¿From the resource constraint of the overall economy (expressed in per capita terms)

$$\dot{k} = x - p_z z - \eta w \frac{V}{E} - c \tag{37}$$

follows $\hat{k} = \hat{c}$ in the steady state (see Appendix F). This constraint states that the increase in the capital stock equals output minus the value of imports minus hiring costs minus consumption. Let g denote the (endogenous) optimal growth rate, then we have $\hat{k} = \hat{x} = \hat{p}_z = \hat{w} = \hat{c} \equiv g = \frac{1}{\sigma}(r - \rho)$.

If we combine the first-order conditions (32) through (34) using $\hat{w} = g$, the wage equation (18) and the production function (31), the result is

$$(1-\phi)(1-h) - \phi\eta\theta = \frac{\phi\eta}{1-\beta}\left(\alpha\left(\frac{\gamma k}{(1+t_z)p_z}\right)^{\frac{\gamma}{1-\gamma}} + \nu - g\right)\theta^\beta \tag{38}$$

This equation is the analogue to the factor allocation function (22) in Section 3.1. It shows all combinations of labor market tightness and the growth rate that are consistent with optimizing firms which interact on competitive markets. The log-linearized version of (38) reads

$$d_1\tilde{g} = d_2\tilde{\theta} - d_5\tilde{t}_z \tag{39}$$

where the coefficients are positive constants defined in Appendix E. The factor allocation function is positively sloped in the (g, θ)-space.[6] A lower g is equivalent to a lower growth rate of wages implying a rise in the costs of current recruiting activities. This is an incentive to push recruiting activities to the future, labor market tightness θ falls. Since the producer wage $(1 + t_{pw})w$ and thus labor costs are independent of both the payroll tax and the labor tax, neither t_{pw} nor t_w occur in Eq. (39). A higher energy tax t_z reduces via a lower profit-maximizing z the interest rate $r = \alpha z^{\gamma}$ which in turn causes a rise in the expected present value of a successful matching and thus a rise in labor market tightness; the factor allocation function shifts to the right.

To pin down the equilibrium growth rate and equilibrium labor market tightness we now turn to the capital accumulation function. This function can be derived from the resource constraint of the economy, Eq. (37). If we divide this equation by k, substitute for $p_z z$ from (33) and for V/E from (5), observe $\hat{k} = g$, we get $g = \frac{x}{k}\frac{1-\gamma+t_z}{1+t_z} - \frac{\eta\nu\theta^{\beta}w}{k} - \frac{c}{k}$. Now insert the wage equation (18), recall that $\frac{x}{k} = \frac{r}{\alpha}$ from (32) and $r = \rho + \sigma g$ from (36), make use of $\hat{c} = \hat{k}$ so that in a steady state the ratio c/k is a constant given by the initial equilibrium values $c(0)/k(0)$, we arrive at

$$g = \frac{\rho + \sigma g}{\alpha}\left[\frac{1-\gamma+t_z}{1+t_z} - \frac{\eta\nu\phi\epsilon\theta^{\beta}}{(1+t_{pw})(1-(1-\phi)h-\phi\eta\theta)}\right] - \frac{c(0)}{k(0)} \tag{40}$$

Log-linearizing leads to

$$e_1\tilde{g} = -e_2\tilde{\theta} + e_4\tilde{t}_{pw} + e_5\tilde{t}_z \tag{41}$$

where the coefficients are constants defined in Appendix E. The capital accumulation function (41) is negatively sloped in the (g, θ)-space. As θ rises, hiring costs rise, and therefore capital accumulation and the growth rate falls. A higher payroll tax t_{pw} is growth-enhancing, since the wage w and thus hiring costs fall. These resources are (partly) used to accumulate capital. A higher energy tax t_z reduces imports, so that income and capital accumulation increase. Compared to the Solow model of Section 3, the government budget constraint does not change at all, it is still given by Eq. (19).

[6] In the following the analysis will be carried out in terms of the factor allocation function and the capital accumulation function in the (g, θ)-space. However, in order to save space we omit separate figures and refer to Figure 1 through 4 where the growth rate g has to be substituted for capital per effective worker k.

With the help of Eqs. (39), (41) and (19), i.e.

$$\begin{bmatrix} d_1 & -d_2 & 0 & 0 & d_5 \\ e_1 & e_2 & 0 & -e_4 & -e_5 \\ 0 & c_2 & c_3 & c_4 & c_5 \end{bmatrix} \begin{bmatrix} \tilde{g} \\ \tilde{\theta} \\ \tilde{t}_w \\ \tilde{t}_{pw} \\ \tilde{t}_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

we are now able to see how the endogenous variables g, θ and a tax rate respond to changes in the tax structure.

Proposition 6 *A budget-neutral shift from payroll to wage taxes reduces both the growth rate and labor market tightness.*

Proof. The reaction of the growth rate and labor market tightness are given by $\partial\tilde{g}/\partial\tilde{t}_{pw} = |A|^{-1}(d_2e_4c_3) > 0$ and $\partial\tilde{\theta}/\partial\tilde{t}_{pw} = |A|^{-1}(d_1e_4c_3) > 0$, respectively, where $|A| := d_1e_2c_3 + d_2e_1c_3 > 0$. The assumption of Laffer-efficiency ensures $\partial\tilde{t}_w/\partial\tilde{t}_{pw} < 0$. ■

The impact of this tax reform is fully described by the impact of the lower payroll tax on the capital accumulation function. In the (g, θ)-space this function shifts inwards, i.e. for a given labor market tightness the growth rate declines. This is due to the increase in the bargained real wage w and the corresponding increase in hiring costs, which in turn reduces the resources available for capital accumulation. The lower growth rate (of wages) implies a rise in the costs of current recruiting activities, so that labor market tightness decreases.

Proposition 7 *A revenue-neutral shift from wage taxes to taxes on the imported factor raises labor market tightness, the growth rate may increase or decrease.*

Proof. The reaction of labor market tightness and the growth rate are given by $\partial\tilde{\theta}/\partial\tilde{t}_z = |A|^{-1}(d_1e_5c_3 + d_5e_1c_3) > 0$ and $\partial\tilde{g}/\partial\tilde{t}_z = |A|^{-1}(d_2e_5c_3 - d_5e_2c_3) \gtreqless 0$, respectively, where $|A| := d_1e_2c_3 + d_2e_1c_3 > 0$. The assumption of Laffer-efficiency ensures $\partial\tilde{t}_w/\partial\tilde{t}_z < 0$. ■

Since the decline in the labor tax t_w is neutral with respect to both capital accumulation and the optimal factor allocation, it is solely the increase in the energy tax which causes changes in g and θ. A hike in t_z means that due to a lower z and thus a lower interest rate the costs of a vacancy fall, labor market tightness goes

up. The growth rate falls as the marginal product of capital falls. On the other hand, a higher energy tax leads to lower imports and these resources can be used for additional vacancies and the accumulation of capital. This reinforces the increase in θ and countervails the decline in the growth rate. The total effect on g is unclear.

Proposition 8 *A revenue-neutral shift from payroll taxes to taxes on the imported factor unambiguously raises labor market tightness; the growth rate may increase or decrease.*

Proof. The reaction of labor market tightness and the growth rate are given by $\partial\tilde{\theta}/\partial\tilde{t}_z = |A|^{-1}(d_1e_5c_4 - d_1e_4c_5 + d_5e_1c_4) > 0$ and $\partial\tilde{g}/\partial\tilde{t}_z = |A|^{-1}(-d_5(e_2c_4 + e_4c_2) - d_2e_4c_5 + d_2e_5c_4) \gtreqless 0$, respectively, where $|A| := d_1e_2c_4 + d_1e_4c_2 + d_2e_1c_4 > 0$. The assumption of Laffer-efficiency ensures $\partial\tilde{t}_{pw}/\partial\tilde{t}_z < 0$. ■

As just described, a higher energy tax has a positive effect on labor market tightness and an ambiguous effect on the growth rate. The decrease in the payroll tax is neutral to the optimal factor inputs and thus to the factor allocation function. However, the capital accumulation function shifts inwards, that is we observe a negative effect on θ as well as g. It is straightforward to show that the overall effect on labor market tightness is positive, whereas the growth rate probably declines.

6 Conclusions

In this paper we have analyzed the employment and growth effects of different revenue-neutral tax reforms. The main results are stated in both the abstract and the introduction, so there is no need to repeat them here. Is there a "first-best tax reform", that is a tax reform which boosts employment as well as growth? Our analysis suggests that such a reform exists: cut the labor tax and - in order to fulfill the budget constraint - increase the payroll tax. Given the reasonable assumption of a constant net replacement ratio, the cut in the labor tax is neutral to wage formation whereas the increase in the payroll tax lowers the bargained real wage. This stimulates employment and reduces hiring costs. Since savings are used to finance investments and hiring costs, a larger fraction of savings is now available for capital accumulation. If a cut in the payroll tax (labor tax) is financed by an increase in the energy tax, we observe a positive employment effect but no clear-cut growth effect, that is, there may be a trade-off between employment and growth.

Lastly, let us mention two limitations of our framework. We do not have any criterion which allows us to analyze meaningfully the welfare implications of alternative policies. In particular, if the employment and growth effects show different signs, an unambiguous ranking of the tax instruments is not possible and thus the policy conclusions are only vague. A related point is concerned with our focus on analytical results. The method of log-linearization restricts us to small changes in the policy parameters. In order to evaluate large policy shocks and/or to get a numerical assessment of the employment and growth effects, a calibration of the model would be necessary.

7 Appendix

A: Derivation of the First-order Conditions for Labor, Capital and Energy

In order to solve the optimization problem, a present-value Hamiltonian function $\mathcal{H}$ with two state variables, E respectively K, the control variables I and V as well as costate variables μ_1 and μ_2 is set up:

$$\begin{aligned} H &= e^{-rt}\left[F(K,Z,\lambda E) - rK - (1+t_{pw})wE - (1+t_z)p_z Z - \eta(1+t_{pw})wV\right] \\ &\quad + \mu_1(V^{1-\beta}U^{\beta} - \nu E) + \mu_2 I, \end{aligned} \tag{A1}$$

The Hamiltonian conditions are

$$\frac{\partial \mathcal{H}}{\partial V} = 0 \iff -\eta\,(1+t_{pw})\,we^{-rt} + \mu_1(1-\beta)\theta^{-\beta} = 0 \tag{A2}$$

$$-\dot{\mu}_1 = \frac{\partial \mathcal{H}}{\partial E} \iff -\dot{\mu}_1 = e^{-rt}[F_E(\cdot) - (1+t_{pw})\,w] - \mu_1\nu \tag{A3}$$

$$\dot{E} = \frac{\partial \mathcal{H}}{\partial \mu_1} \iff \dot{E} = V^{1-\beta}U^{\beta} - \nu E \tag{A4}$$

$$\frac{\partial \mathcal{H}}{\partial I} = 0 \iff \mu_2 = 0 \tag{A5}$$

$$-\dot{\mu}_2 = \frac{\partial \mathcal{H}}{\partial K} \iff -\dot{\mu}_2 = e^{-rt}[F_K(\cdot) - r] \tag{A6}$$

$$\dot{K} = \frac{\partial \mathcal{H}}{\partial \mu_2} \iff \dot{K} = I \tag{A7}$$

$$\frac{\partial \mathcal{H}}{\partial Z} = 0 \iff -e^{-rt}[F_Z(\cdot) - (1+t_z)\,p_z] = 0 \tag{A8}$$

with the transversality conditions

$$\lim_{t\to\infty} \frac{\partial \mathcal{H}}{\partial E} E = \lim_{t\to\infty} \frac{\partial \mathcal{H}}{\partial K} K = 0.$$

The first-order conditions for capital and energy, Eqs. (11) and (12) in the text, immediately follow from (A5),(A6), and (A8). Solving Eq. (A2) for μ_1 and differentiating the result with respect to time yields

$$\dot{\mu}_1 = \frac{\eta\,(1+t_{pw})\,e^{-rt}}{1-\beta} w\theta^{\beta} \left(\frac{\dot{w}}{w} + \beta \frac{\dot{\theta}}{\theta} - r \right) \tag{A9}$$

By inserting (A2) and (A9) into (A3) we obtain

$$F_E(\cdot) = (1+t_{pw})\, w \left[1 + \frac{\eta}{1-\beta} \left(r + \nu - \frac{\dot{w}}{w} - \beta \frac{\dot{\theta}}{\theta} \right) \theta^{\beta} \right]. \tag{A10}$$

Since in the steady state labor market tightness is constant, $\dot{\theta} = 0$, and wages grow at the rate of technological progress, $\dot{w}/w = g$, the first-order condition for labor simplifies to (10) in the text.

B: Derivation of the Efficient Factor Allocation Function

If we substitute the wage equation (18) into the firm's optimum condition for labor (10), we get

$$\frac{(1-\phi)(1-\beta)(1-h)}{\eta\phi} = (1-\beta)\theta + \theta^{\beta}(r+\nu-g) \tag{B1}$$

This equation determines labor market tightness at a given interest rate. Inserting the expressions for the marginal products of capital and energy, $F_K = \alpha x/k$ and $F_Z = \gamma x/z$, respectively, into the first-order conditions (11) and (12), and taking the production function (9) into account, we can find the interest rate to be

$$r = \alpha \left(\frac{\gamma}{(1+t_z)p_z} \right)^{\frac{\gamma}{1-\gamma}} k^{\frac{-\epsilon}{1-\gamma}}. \tag{B2}$$

Combining (B2) and (B1) and rearranging yields the efficient factor allocation function as stated in the text.

C: Derivation of the Capital Accumulation Function

When we insert $F_z = \gamma x/z$ into the optimum condition for the imported factor (12), the value of imports can be written as $p_z z = \gamma x/(1+t_z)$. If we substitute this into the definition of national income, we get $y = x - p_z z = \frac{1-\gamma+t_z}{1+t_z}x$. In a next step plug this result as well as the wage equation (18) into (23) to arrive at

$$s\left(\frac{1-\gamma+t_z}{1+t_z}\right)x = gk + \frac{\eta\nu\phi\theta^{\beta}F_E(\cdot)}{\lambda\,(1+t_{pw})\,[1-(1-\phi)\,h-\phi\theta\eta]} \tag{C1}$$

Now, use $F_E(\cdot) = \varepsilon\lambda x$ and $rk = \alpha x$ to get

$$\frac{s(1-\gamma+t_z)}{1+t_z} = g\frac{\alpha}{r} + \frac{\varepsilon\eta\nu\phi\theta^{\beta}}{(1+t_{pw})\,[1-(1-\phi)\,h-\phi\theta\eta]} \tag{C2}$$

Finally, by observing Eq. (B2) and rearranging we yield (24) in the text.

D: Derivation of the Government Budget Constraint

Recalling the definitions of labor market tightness, $\theta = V/U$, and the net replacement ratio, $h = B/(1-t_w)w$, the unemployment benefits can be expressed as $BU = (1-t_w)\,whV/\theta$. In view of this the government budget constraint (19) can be rewritten in efficiency units of labor as

$$(1-t_w)\,h\nu\frac{w}{\lambda}\theta^{\beta-1} = t_w\frac{w}{\lambda} + t_{pw}\frac{w}{\lambda} + t_{pw}\eta\nu\frac{w}{\lambda}\theta^{\beta} + t_z p_z z. \tag{D1}$$

where use has been made of the flow equilibrium $V/E = \nu\theta^{\beta}$ (see Eq. (5)). The value of imports is $p_z z = \gamma x/(1+t_z)$, thus

$$\left[(1-t_w)\,h\nu\theta^{\beta-1} - t_w - t_{pw}\left(1+\eta\nu\theta^{\beta}\right)\right]\frac{w}{\lambda} = \frac{t_z\gamma x}{1+t_z}. \tag{D2}$$

Now making use of the wage equation (18) and $F_E(\cdot) = \varepsilon\lambda x$, the steady-state balanced budget function, Eq. (25) in the text, is implied.

E: Parameter definitions

$a_1 := \frac{\varepsilon\theta^{\beta}r}{(1-\beta)(1-\gamma)}$; $a_2 := \theta + \frac{\beta\theta^{\beta}}{1-\beta}(r+\nu-g)$; $a_5 := \frac{\gamma\theta^{\beta}r}{(1-\beta)(1-\gamma)}$; $b_1 := \frac{\varepsilon}{1-\gamma}$
$b_2 := \frac{r}{\alpha g}\frac{\varepsilon\eta\nu\phi\theta^{\beta}}{1+t_{pw}}\left(\frac{\beta N+\phi\eta\theta}{N^2}\right)$; $b_4 := \frac{r}{\alpha g}\frac{\varepsilon\eta\nu\phi\theta^{\beta}}{(1+t_{pw})N}$; $b_5 := \frac{\gamma sr}{(1+t_z)\alpha g} - \frac{\gamma}{1-\gamma}$
$c_2 := \phi\varepsilon\eta\nu\beta t_{pw}\theta^{\beta} + \phi\varepsilon h\nu(1-\beta)(1-t_w)\theta^{\beta-1} - \frac{\eta\phi\gamma(1+t_{pw})t_z\theta}{1+t_z}$;

$c_3 := \phi\varepsilon(1-t_{pw})(1+h\nu\theta^{\beta-1})$; $c_4 := (1+t_{pw})\left(\frac{\gamma t_z}{1+t_z}N + \phi\varepsilon(1+\eta\nu\theta^{\beta})\right)$;
$c_5 := \frac{\gamma(1+t_{pw})N}{1+t_z}$; $d_1 := \frac{g}{1-\beta}$; $d_2 := \theta^{1-\beta} + \frac{\beta}{1-\beta}(r+\nu-g)$; $d_5 := \frac{\gamma r}{(1-\beta)(1-\gamma)}$
$e_1 := \alpha g\left(1 - \frac{\sigma}{r}\left(g + \frac{c(0)}{k(0)}\right)\right)$; $e_2 = \frac{\eta\nu\phi\epsilon\theta^{\beta}r}{1+t_{pw}}\left(\frac{\beta N+\phi\eta\theta}{N^2}\right)$; $e_4 = \frac{\eta\nu\phi\epsilon\theta^{\beta}r}{(1+t_{pw})N}$; $e_5 = \frac{\gamma r}{1+t_z}$
with $N := 1-(1-\phi)h-\phi\eta\theta$.

F: Resource Constraint of the Overall Economy

Divide the resource constraint (37) by k and differentiate the growth rate of capital per worker, $\hat{k} = \dot{k}/k$, with respect to time. This yields

$$\frac{\partial\hat{k}}{\partial t} = -\frac{\dot{k}}{k^2}\left(x - p_z z - \eta\nu w\theta^{\beta} - c\right) + \frac{1}{k}\left(\dot{x} - \dot{p}_z z - \eta\nu\theta^{\beta}\dot{w} - \dot{c}\right) \tag{F1}$$

The growth rate of k is constant in steady state, $\frac{\partial\hat{k}}{\partial t} = 0$, so we have

$$0 = -\hat{k}\left(x - p_z z - \eta\nu w\theta^{\beta} - c\right) + \left(x\hat{x} - p_z z\hat{p}_z - \eta\nu w\theta^{\beta}\hat{w} - c\hat{c}\right) \tag{F2}$$

By observing $\hat{k} = \hat{x} = \hat{p}_z = \hat{w}$, we immediately get $\hat{k} = \hat{c}$.

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