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Why are banks not recapitalized during crises?

by Matteo Crosignani
Abstract

I develop a model where the sovereign debt capacity depends on the capitalization of domestic banks. Low-capital banks optimally tilt their government bond portfolio toward domestic securities, linking their destiny to that of the sovereign. If the sovereign risk is sufficiently high, low-capital banks reduce private lending to further increase their holdings of domestic government bonds, lowering sovereign yields and supporting the home sovereign debt capacity. The model rationalizes, in the context of the eurozone periphery, the increase in domestic government bond holdings, the reduction of bank credit supply, and the prolonged fragility of the financial sector.

JEL Codes: E44, F33, G21, G28

Keywords: Bank Capital, Sovereign Crises, Risk-Shifting, Government Bonds, Bank Credit
1 Introduction

The recent eurozone debt crisis unveiled the existence of a “diabolic loop” between sovereigns and banks: increased sovereign credit risk impairs the balance sheets of financial institutions that, in turn, rely on government guarantees (Farhi and Tirole (2016), Brunnermeier et al. (2016)). This feedback loop materialized in eurozone peripheral countries where, following the increase in sovereign credit risk, banks reduced their credit supply to firms with negative effects on employment and investment (Acharya et al. (2016)). While the literature has mainly focused on the link running from the banks’ balance sheets to the sovereign’s creditworthiness, little work has been done to understand what drives bank holdings of domestic government bonds during bad times. My goal is to provide a theory to fill this gap.

A clear stylized fact emerges from the eurozone debt crisis. Banks in the eurozone periphery increased their holdings of domestic government bonds as the creditworthiness of the domestic sovereign deteriorated. In Figure 1, I show that the share of total government debt held by domestic banks increased as the sovereign creditworthiness deteriorated from 2009 to 2012. In Figure 2, I show that, during the same period, banks increased their holdings of domestic government bonds and reduced their loans to firms and households.¹

In this paper, I build a tractable model to analyze banks’ holdings of domestic government bonds during sovereign crises. Protected by deposit insurance, low-capital banks hold more domestic than foreign government bonds to link their destiny to that of the home sovereign. Banks risk-shift using domestic government bonds as these assets promise the highest payoff in the good state and limited liability protects banks’ equity holders in case of sovereign default. As sovereign risk becomes sufficiently high, risk-shifting banks reduce

¹While both lower credit demand and lower credit supply are likely at work, a large body of empirical literature documents a contraction of bank credit supply during the eurozone crisis in peripheral countries (Acharya et al. (2016), Bofondi et al. (forthcoming), Bottero et al. (2017), De Marco (2016)).
Figure 1: Share of Sovereign Debt Held by Domestic Banks and CDS Spreads. This figure shows the share of sovereign debt owned by domestic banks and the 5-year USD denominated sovereign CDS spread (dotted line, secondary axis) for Italy, Spain, and Portugal. Plots for Ireland and Greece in the Online Appendix. Source: Bloomberg (CDS spreads) and IMF (government bonds).

private credit to increase even more their holdings of domestic government bonds, lowering sovereign yields and effectively acting as buyers of last resort for the home sovereign.

The model features two countries and two dates. Each country has a financial sector and a government. The financial sector invests in a domestic private lending technology, in domestic government bonds, and in foreign government bonds. The government issues one-period bonds at $t = 0$ and levies taxes on banks’ payoffs from the lending technology at $t = 1$. This payoff is stochastic. In the good state, the payoff is high and the government collects enough taxes to fully repay bondholders. In the bad state, the payoff is low and the government is forced to default. Depositors are protected by a credible deposit insurance.

Banks’ portfolio choice depends on whether the limited liability constraint binds in the bad state. If it does not bind, banks are “well capitalized” and invest in both domestic and foreign government bonds. If it binds, banks are “undercapitalized” and develop a preference, within the government bond portfolio, for domestic bonds. These assets perform well in the good state and poorly in the bad state, exactly when their payoff is entirely used to pay depositors. As shocks are not perfectly correlated across countries, this is not the case for foreign government bonds as their payoff depends on the state of the foreign country.

If the sovereign risk is sufficiently high, government bonds — due to their high yield
Figure 2: Domestic Government Bond Holdings and Credit to the Private Sector. This figure shows bank holdings of domestic government bonds (dotted line) and domestic banks’ credit to firms and households (secondary axis) for Italy, Spain, and Portugal. Plots for Ireland and Greece in the Online Appendix. Source: IMF (debt ownership) and BIS (private credit).

— become an even more attractive asset to risk-shift. Undercapitalized banks reduce their investment in the lending technology to further expand their (domestic) government bond portfolio. This increased demand lowers domestic sovereign yields and increases the home sovereign debt capacity. Governments that maximize welfare and provide a public good face a trade-off when setting capital regulation. On the one hand, well capitalized domestic banks choose a high investment in the lending technology, increasing banks’ profits, capturing the idea that a healthy financial system fosters growth. On the other hand, undercapitalized domestic banks choose a high investment in domestic government bonds expanding the sovereign debt capacity that, in turn, supports the public good provision.

The model relies on three key assumptions. The first is a sufficiently high correlation between the payoff of domestic government bonds and the payoff of the lending technology to ensure that weak banks default when the domestic sovereign defaults. The second is the presence of a credible deposit insurance (or any other guarantee) protecting bank depositors that, in turn, do not require a high return on their savings nor discipline banks by withdrawing their deposits in bad times. The third assumption is a bank balance sheet with a fixed size — an extreme version of funding constraint — that links the purchases of domestic
government bonds to the reduction of private lending.

The analysis in this paper sheds light on the recent experience of peripheral eurozone countries. The eurozone periphery during the crisis — with its high sovereign credit risk and undercapitalized banks — resembles the model environment. In this context, the three key assumptions are realistic. First, eurozone peripheral banks historically have a high exposure to the domestic economy. Second, the European Central Bank (ECB) acted as a de facto supranational deposit insurance during the crisis providing liquidity to banks in peripheral countries against low-quality and illiquid collateral (Drechsler et al. (2016), Crosignani et al. (2017)). Third, external financing was extremely costly for peripheral banks during the crisis: they were unable to access private markets (Garcia-de Andoain et al. (2016)) and were subject to runs (Chernenko and Sunderam (2014), Perignon et al. (forthcoming)).

The model presented in this paper has two clear cross-sectional predictions. During sovereign crises, low-capital banks (i) increase their holdings of domestic government bonds and (ii) reduce their supply of private credit compared with high-capital banks. The empirical literature on the eurozone crisis strongly supports these predictions. Acharya and Steffen (2015) and Drechsler et al. (2016) show that low-capital banks drove the increase in government bond holdings, Bottero et al. (2017) and Bofondi et al. (forthcoming) show that the reduced private credit supply was more pronounced for low-capital banks, and Acharya et al. (2016) and Altavilla et al. (2016) find that low-capital banks purchased domestic government bonds and reduced their credit supply during the crisis.\footnote{The evidence is also consistent with alternative explanations. Low-capital banks might (i) buy domestic government bonds to induce the government to bail them out in case of default (Koetter and Popov (2016)) or (ii) be particularly exposed to government moral suasion (Becker and Ivashina (2016), Ongena et al. (2016), De Marco and Macchiavelli (2015)). The model in this paper can be interpreted as a model of moral suasion where governments are more effective in influencing low-capital banks. For a comprehensive analysis of bank exposure to government debt, see Gennaioli et al. (2014).}

An example clarifies the mechanism. Intesa is the second largest Italian bank with €927...
billion of assets and leverage of 12.5, well above the Italian and European median, in 2010. It is also highly exposed to the domestic economy. Conditional on a domestic sovereign default, Intesa would have suffered losses equivalent to 6.8 times its market value of equity in 2010. While severely hit by the crisis (its stock price fell by 60%), Intesa’s holdings of risky domestic government bonds increased from 8.7% to 11.5% and loans to firms and households decreased from 61% to 59% of total assets from 2009 to 2012. According to the model, Intesa reduced private credit to purchase high-yield domestic government bonds placing a bet on the survival of the home sovereign effectively supporting its debt issuance. Intesa’s equity holders were protected by limited liability and Intesa’s depositors were protected by the ECB that injected €48 billion in its balance sheet at the peak of the crisis.\footnote{The data sources for this examples are publicly available: European Banking Authority stress test data, consolidated bank annual reports, and uptake of ECB liquidity from Bloomberg.}

My contribution is twofold. First, I present a new theory, based on the well-known risk-shifting motive (Jensen and Meckling (1976)), for the accumulation of domestic government bonds on bank balance sheets during sovereign crises. By adding a key role for bank capital, I contribute to the literature on the repatriation of sovereign debt in bad times (Broner et al. (2010), Gennaioli et al. (2016), Broner et al. (2014)), typically based on the role of secondary markets or selective sovereign defaults. I also complement the literature on the banks-sovereign nexus (Brunnermeier et al. (2016), Acharya et al. (2014a), Farhi and Tirole (2016), Leonello (2016), Cooper and Nikolov (2013)) by shifting the focus from the link running from banks’ balance sheets to the government to bank portfolio choice.\footnote{The literature has also attributed the increased holdings of risky government bonds during sovereign crises to their collateral role for interbank loans (Bolton and Jeanne (2011)), their collateral eligibility at the central bank (Uhlig (2013)), and the lack of balance sheet transparency (Ari (2017)).}

Second, I present a new channel for the transmission of sovereign risk to bank credit. The empirical literature on the eurozone crisis documents a negative correlation between
bank holdings of risky sovereign bonds and change in bank credit supply (Bofondi et al. forthcoming), Popov and Van Horen (2015), Bottero et al. (2017), Acharya et al. (2016), De Marco (2016)). This finding is interpreted as the effect of a shock to the value of the government bond portfolio on bank funding costs (Bocola (2016), Perez (2015)).

I provide a new explanation. During sovereign crises, banks might actively reduce their credit to firms to buy more domestic government bonds. While the two channels are not mutually exclusive, only the risk-shifting channel proposed in this paper can explain why banks in peripheral countries increased their holdings of domestic government bonds during the crisis.


My results inform the eurozone policy debate suggesting that a supranational deposit insurance, in presence of weak banks, might exacerbate the banks-sovereign nexus and cause a contraction of bank credit. The model also rationalizes why European authorities adopted liquidity provisions rather than recapitalization programs during the crisis: peripheral countries relied on undercapitalized domestic banks to support their sovereign debt markets.

The remainder of the paper is organized as follows. In Section 2, I illustrate the model setup and define the equilibrium. In Section 3, I present the baseline model. In Section 4, I discuss the role of bank capital. In Section 5, I discuss the model assumptions. In Section 6, I present supportive empirical evidence. Concluding remarks are given in Section 7.

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5 Almeida et al. (2017) document a new “credit ratings” channel by showing that sovereign risk has a negative effect on the real economy following a sovereign rating downgrade.

6 Moreover, a bank lending channel triggered by a shock to the value government bonds is at odds with the observation that 85% of sovereign bonds were not marked-to-market during the crisis (De Marco (2016)).
2 Model

In this section, I set up the baseline model and define the equilibrium concept. I make some assumptions that I discuss in Section 5.

2.1 Setup

The economy starts at $t = 0$ and terminates at $t = 1$. There are two symmetric countries $i \in I$, where $I = \{A, B\}$. Each country has a government and a banking sector. There is universal risk neutrality and no discounting. I describe the model setup for one country, omitting for simplicity the country superscripts. In Figure 3, I summarize the timeline of the economy for one country.

Banks The financial sector is made of a representative bank with a balance sheet of size one, debt $L \in (0, 1)$ maturing at $t = 1$, and equity $1 - L$. It maximizes profits by investing in domestic government bonds, foreign government bonds, and a domestic private lending technology.

Assumption 1: Banks cannot invest in the foreign private lending technology.

The lending technology is risky. It can be hit by a negative shock between $t = 0$ and $t = 1$. An investment of $k$ at $t = 0$ yields $\epsilon_H f(k)$ with probability $\theta$ and $\epsilon_L f(k)$ with probability $1 - \theta$ at $t = 1$, where $\theta \in (0, 1)$ and $\epsilon_H > \epsilon_L$.

Assumption 2: Lending technology shocks are uncorrelated across countries.

I assume that $f(\cdot)$ is continuous, strictly increasing, strictly concave, and satisfies Inada conditions. Banks can also invest in the two government bond markets. In particular, they invest $\alpha(1 - k)$ in the domestic market and $(1 - \alpha)(1 - k)$ in the foreign market. Domestic government bonds pay an (endogenous) gross interest rate $R$. Similarly, foreign government
bonds pay an (endogenous) gross interest rate $R^*$. The choice variable $\alpha \in [0, 1]$ captures the “home bias” of the financial sector in the government bond portfolio. If $\alpha = 1$, banks invest only in domestic bonds (“perfect” home bias). If $\alpha = 0$, banks invest only in foreign bonds. Banks maximize profits and are subject to limited liability. The left panel of Figure 4 illustrates the investment opportunities for banks in country $i \in I$.

**Depositors** Depositors hold bank debt $L$ and are protected by a supranational deposit insurance. Depositors and the deposit insurance are unmodeled. As opposed to a nationally funded deposit insurance, the supranational deposit insurance is able to credibly protect depositors in case of domestic sovereign default.\(^7\) Because depositors do not suffer losses caused by an eventual bank default, the return on bank deposits is one.

\(^7\)One example of a supranational deposit insurance is the European Deposit Insurance Scheme (EDIS), scheduled to start in July 2017. In Section 5.1, I discuss the role that the deposit insurance plays in the model in greater detail.
Figure 4: Investment Opportunities. The left panel illustrates the investment opportunity set of banks in country $i \in \mathcal{I}$. They can invest, at $t = 0$, in (i) the (domestic) lending technology, (ii) domestic government bonds, and (iii) foreign government bonds. The right panel shows the payoff from (i) and (ii), following the realization of the payoffs from the lending technology at $t = 1$.

**Government**  The government issues one-period bonds at $t = 0$ and collects taxes and repays bondholders at $t = 1$. The government applies an exogenous tax rate $\tau$ to the payoff of the private lending technology.\(^8\) Tax collection is subject to a negative (sovereign) shock: part of it disappears and therefore cannot be used to repay bondholders at $t = 1$. Motivated by the need to roll over a sizable portion of existing public debt, the government exhaust its debt capacity at $t = 0$ issuing one-period government bonds.

### 2.2 Sovereign Default and Debt Capacity

In this subsection, I illustrate how the sovereign might default and derive its debt capacity.

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\(^8\)The taxation of the lending technology captures the idea that loans to firms generate economic growth, therefore increasing taxable income and tax revenues.
Sovereign Default  The sovereign defaults when tax collection is too low to fully repay bondholders. There is no strategic default, as the government, conditional on having sufficient funds, always repays its debt. In case of default, the government applies an (endogenous) haircut $1 - \lambda$ to its payments to bondholders. If the recovery value $\lambda < 1$, the government defaults, being able to repay only a fraction $\lambda$ of the payments due. If the recovery value $\lambda = 0$, the government defaults on the entire debt.

The right panel of Figure 4 shows that the realized payoff $f(k)$ from the lending technology is split between banks (after-tax revenues) and the government (tax collection). This uncertain tax collection, after being hit by the sovereign shock $y$, is then used to repay (domestic and foreign) banks. The sovereign might default for two (not mutually exclusive) reasons. First, tax collection might be low because the country is in the bad state and the payoff from the lending technology — the tax base — is low. Second, tax collection might be insufficient to fully repay bondholders because it is eroded by a sufficiently large sovereign shock $y$.

Debt Capacity  Banks anticipate that the government might default and constrain its public debt issuance accordingly. In particular, I assume, as in Acharya and Rajan (2013), that banks are willing to invest in public debt if the payments due to bondholders are less than or equal to the expected tax collection minus the sovereign shock $y$:

$$ DR \leq E(\tau) f(k) - y $$

To gain tractability, I rewrite $y$ as a fraction $\gamma$ of tax collection in the bad state of the world. I refer to $\gamma$ as the "sovereign risk" in the economy. Formally,

$$ y = \gamma \tau f(k) $$
Rearranging the last two expressions, I obtain the government debt capacity:

$$D \leq \frac{\tau \Delta \epsilon_f(k)}{R}$$  \hspace{1cm} (1)

where $\Delta \epsilon = \theta \epsilon_H + (1 - \theta - \gamma) \epsilon_L < \mathbb{E}(\epsilon)$ is the expected portion of tax collection that is used to repay bondholders.\(^9\) Note that $\gamma$ captures the fraction of tax collection lost because of the sovereign shock $y$ in the bad state.

### 2.3 Agents’ Problem and Equilibrium Definition

Having derived the government debt capacity, I now illustrate the banks’ optimization problem. At $t = 0$, banks maximize profits by investing in the private lending technology, domestic government bonds, and foreign government bonds, subject to limited liability:

$$\max_{\alpha, k} \mathbb{E} \left( [\Pi - L]^+ \right)$$  \hspace{1cm} (2)

where

$$\Pi_{s,s^*} = \underbrace{(1 - \tau) \epsilon_s f(k)}_{\text{revenues from lending technology}} + \underbrace{\alpha (1 - \kappa) \lambda_s R}_{\text{revenues from domestic bonds}} + \underbrace{(1 - \alpha)(1 - k) \lambda_s R^*}_{\text{revenues from foreign bonds}}$$

where $s \in S$ is a state of the world and the asterisk indicates a foreign variable. The productivity parameter $\epsilon$ and the recovery value $\lambda$ depend on the domestic state of the world. In particular, the uncertainty about the productivity parameter $\epsilon$ spreads to the sovereign bonds as governments repay bondholders with an uncertain tax collection at $t = 1$.

In equilibrium, governments exhaust their debt capacity, banks maximize profits, and the two bond markets clear. Hereafter, I use the following equilibrium definition:

\(^9\)As $\gamma$ is a constant, $y$ depends on the equilibrium investment $k$ in the lending technology.
**Definition 1.** Given initial debt levels $L^i$, tax rates $\tau^i$, lending technologies $f^i$, probabilities $\theta^i$, productivity parameters $\epsilon^i$, and sovereign risks $\gamma^i$, where $i \in I$, an equilibrium is

- gross return on government bonds $R^i$
- public debt issuance $D^i$
- recovery values on public debt $\lambda^s$, for $s \in S$
- financial sectors’ investment decisions $\alpha^i, k^i$

such that

- bond markets clear
- financial sectors maximize profits
- governments exhaust their debt capacity at $t = 0$, collect taxes on the payoff from the domestic lending technology, and repay bondholders

According to market clearing conditions, the sum of domestic and foreign demand for government bonds for each country must be equal to the supply of public debt by the sovereign.

The two bond market clearing conditions are:

$$\alpha^A(1 - k^A) + (1 - \alpha^B)(1 - k^B) = D^A$$
$$\alpha^B(1 - k^B) + (1 - \alpha^A)(1 - k^A) = D^B$$

In each of the two equations above, the first term on the left-hand side is the domestic demand for sovereign bonds and the second term is the foreign demand for sovereign bonds.

Governments exhaust their debt capacity and therefore choose the highest level of debt $D$ subject to (1):

$$D^i = \frac{\tau^i \Delta f(k^i)}{R^i}$$  \hspace{1cm} (3)
While in the good state tax collection minus $y$ is sufficiently high to repay bondholders, the government is forced, in the bad state, to write-down part of its debt, applying a haircut $1 - \lambda < 1$. The following lemma formalizes the intuition.

**Lemma 1.** Governments only default in the bad domestic state ($\lambda_H^i = 1$, $\forall i \in \mathcal{I}$). The sovereign debt recovery value in the bad state is $\lambda_L^i = \epsilon^i_L(1 - \gamma^i)(\Delta^i)^{-1} \in (0, 1)$, $\forall i \in \mathcal{I}$.

On the one hand, the recovery value $\lambda$ is increasing in firm productivity $\epsilon$, as higher productivity leads to a higher payoff from investing in the lending technology and, therefore, a higher tax collection. On the other hand, the recovery value $\lambda$ is decreasing in the sovereign risk $\gamma$, as a higher sovereign risk erodes tax collection.

Having obtained the supply of government bonds, I now turn to derive the demand for government bonds solving banks’ portfolio problem. Given Inada conditions, banks always invest $k > 0$ in the lending technology. Depending on whether the limited liability constraint binds in the bad state, there are two relevant cases. If the initial debt $L$ is sufficiently low, the limited liability constraint never binds and banks are “well capitalized” (W banks). If the initial debt $L$ is sufficiently high, the limited liability constraint binds in the bad state and banks are “undercapitalized” (U banks).\textsuperscript{10}

Suppose that banks in country $A$ choose a high home bias $\alpha$ and therefore default when the domestic economy is in the bad state. In this case, their problem can be rewritten as

\textsuperscript{10}I formally characterize the relation between the initial debt $L$ and banks’ capitalization in Section 4.1.
the following simplified version of (2):

\[
\max_{\alpha, k} \theta \left[ \Pi_H - L \right] \\
\text{s.t.} \\
\Pi_H = (1 - \tau) e_H f(k) + \alpha (1 - k) R + (1 - \alpha) (1 - k) E^*(\lambda^*) R^*
\]

where the subscript \( H \) indicates the good state of the world and the asterisk indicates a foreign variable. The above maximization problem captures the incentive of undercapitalized banks to risk-shift on their insured depositors. These banks only care about the payoff in the good state as the payoff in the bad state is entirely used to repay bondholders.

In Figure 5, I illustrate banks’ payoffs at \( t = 1 \). The left panel shows the payoffs of well capitalized banks (W banks), and the right panel shows the payoffs of undercapitalized banks (U banks). The payoff from investing in domestic bonds depends on the domestic state. If well capitalized, banks obtain the full payoff in the good state and the post-haircut payoff in the bad state. If undercapitalized, banks obtain the full payoff in the good state and zero in the bad state, where the entire government bond payoff is used to repay bondholders.
The payoff from investing in foreign bonds depends only on the foreign state and, in the bad domestic state, banks might use it to repay their depositors. Caring only about the good state, undercapitalized banks prefer domestic over foreign government bonds.

To get some intuition, consider banks’ maximization problem and solve for the optimal home bias, taking partial derivatives in (2) with respect to $\alpha$:

$$W_{\text{banks}}:\begin{cases}
\alpha = 1 & \text{if } E(\lambda)R > E^*(\lambda^*)R^* \\
\alpha = 0 & \text{if } E(\lambda)R < E^*(\lambda^*)R^* \\
\alpha \in [0, 1] & \text{if } E(\lambda)R = E^*(\lambda^*)R^*
\end{cases} \tag{5a}$$

$$U_{\text{banks}}:\begin{cases}
\alpha = 1 & \text{if } R > E^*(\lambda^*)R^* \\
\alpha = 0 & \text{if } R < E^*(\lambda^*)R^* \\
\alpha \in [0, 1] & \text{if } R = E^*(\lambda^*)R^*
\end{cases} \tag{5b}$$

where, using Lemma 1, $E(\lambda) = \theta + \lambda L(1 - \theta) \in (0, 1)$ and $E^*(\lambda^*) = \theta^* + \lambda^* L(1 - \theta^*) \in (0, 1)$. Given risk neutrality, well capitalized banks invest only in the government bonds with the highest expected return $E(\lambda)R^*$. However, undercapitalized banks prefer to invest in domestic bonds, as the payoff from foreign bonds is entirely used to repay depositors in the bad state.

### 3 Bank Capital and Portfolio Choice

In this section, I solve a baseline version of the model. To isolate the role of bank capital in this economy, I assume that the two countries are identical except for the level of bank debt $L'$. In Section 3.1, I show that financial sectors, if both well capitalized, invest in both
domestic and foreign government bonds. In Section 3.2, I show that the undercapitalization of (at least) one financial sector causes home bias in the entire economy. In Section 3.3, I show that, if the sovereign risk is sufficiently high, a government with undercapitalized domestic banks pays lower yields and has a larger debt capacity compared with a government with well capitalized domestic banks.

3.1 Well Capitalized Banks

I assume that the two countries are identical and differ only in the level of private debt $L^i$. 

Assumption 3: The two countries have identical $\theta \in (0,1)$, $\tau \in (0,1)$, $f(\cdot)$, $\epsilon$, and $\gamma$.

Depending on the two financial sectors’ levels of bank debt $L$, the economy could be in four states: WW, UW, UU, or UU. The first (second) letter refers to whether banks in country A (B) are well capitalized or undercapitalized. For example, the UW case corresponds to the case where country A’s financial sector is undercapitalized and country B’s financial sector is well capitalized.

In this subsection, I analyze the case where both financial sectors are well capitalized. In this benchmark case, the two financial sectors invest in both government bonds and have the same home bias in equilibrium.

Proposition 1. Financial sectors, if both are well capitalized, have the same home bias $\alpha^i = \alpha$ for all $i \in I$.

By symmetry, both banks choose the same investment $k^i = k$ in the lending technology and allocate the same share $\alpha^i = \alpha$ (home bias) of the remaining unit balance sheet capacity to domestic government bonds.

The economy presents a continuum of equilibria characterized by different levels of banks’ home bias. Figure 6 shows two equilibria from this continuum. The left panel shows a
Figure 6: WW Case. This figure illustrates two equilibria from the continuum of equilibria in the WW case. The left (right) panel illustrates a high (low) home bias equilibrium. The high home bias equilibrium where both financial sectors allocate the largest share of their government bond portfolio domestically. This equilibrium is that observed in the data where, for the majority of countries, domestic banks own the largest share of public debt. The right panel shows a low home bias equilibrium, where both governments face sizable foreign demand for their bonds. Crucially, quantities and prices do not depend on the home bias, which is indeterminate in equilibrium.

I obtain closed-form solutions using a simple square root function for the lending technology ($f(k) = \sqrt{k}$). With this functional form, the model yields intuitive expressions for private lending, government yields, and sovereign debt:

\[
\begin{align*}
    \k_{WW} & = \frac{(1-\tau)E(\epsilon)}{(1-\tau)E(\epsilon) + 2E(\lambda)\Delta_0} \\
    \R_{WW} & = \frac{1}{2E(\lambda)^{1/2}} \left( E(\epsilon)(1-\tau)E(\epsilon)(1-\tau) + 2E(\lambda)\tau\Delta_0 \right)^{1/2} \\
    \D_{WW} & = \frac{2\tau E(\lambda)\Delta_0}{E(\epsilon)(1-\tau) + 2E(\lambda)\Delta_0}
\end{align*}
\]

for all $i \in I$, where the WW subscripts refer to the case where both financial sectors are well capitalized. On the one hand, private lending is decreasing in the tax rate. As the tax base is made exclusively by the payoff from the lending technology, a higher $\tau$ reduces the after-tax
revenues from private lending, causing banks to tilt their portfolio toward government bonds. On the other hand, private lending is increasing in the sovereign risk $\gamma$. As $\gamma$ increases, the government is forced to write-down a larger share of its debt in the bad domestic state. Government bonds become riskier and banks invest more in private lending. A higher $\gamma$ also lowers the government debt capacity as banks anticipate that the sovereign default might be particularly harsh.

3.2 Undercapitalized Banks

I now analyze the economy where at least one country has undercapitalized banks. As shown in (5a) and (5b), undercapitalized banks develop a preference within the government bond portfolio for domestic bonds. These assets perform well in the good state and poorly in the bad state, exactly when all revenues are used to pay depositors. As shocks are uncorrelated across countries, this is not the case for foreign government bonds, because their payoff only depends on the state of the foreign country. In equilibrium, undercapitalized banks invest only domestically ($\alpha = 1$), regardless of the capitalization of foreign banks.

**Proposition 2. (Home Bias)** An undercapitalized financial sector has perfect home bias ($\alpha = 1$), regardless of the capitalization of the foreign financial sector.

Suppose that country A has undercapitalized banks and country B has well capitalized banks (UW case). The undercapitalization of one financial sector causes perfect home bias ($\alpha = 1$) in the entire economy. The transmission operates through prices: banks in country A risk-shift investing in domestic government bonds, lowering their yield. Well capitalized

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11In Section 5.2, I analyze an economy where the shocks hitting the lending technology are correlated across countries.
banks in country B then also tilt their government bond portfolio domestically as foreign sovereign bonds, because of their low yield, are no longer attractive.

Similar to the WW case discussed in the previous subsection, I assume a square root production function to obtain closed-form solutions. I also maintain this assumption going forward, as exact expressions for quantities and prices provide intuition on the channels at work.

Assumption 4. The lending technology has a square-root functional form \( f(k) = \sqrt{k} \).

To understand the origin of the perfect home bias, I first solve the case where both financial sectors are undercapitalized (UU case). The closed-form solutions are:

- \( k_i^{UU} = \frac{(1 - \tau)\epsilon_H}{(1 - \tau)\epsilon_H + 2\tau \Delta_s} \)
- \( R_i^{UU} = \frac{1}{2} \left( (1 - \tau)\epsilon_H((1 - \tau)\epsilon_H + 2\tau \Delta_s) \right)^{1/2} \)
- \( D_i^{UU} = \frac{2\tau \Delta_s}{(1 - \tau)\epsilon_H + 2\tau \Delta_s} \)

for all \( i \in \mathcal{I} \), where the UU subscripts indicate an economy where both countries have undercapitalized domestic banks. Similar to the WW case, private lending is decreasing in the tax rate and increasing in the sovereign risk \( \gamma \). A higher probability of the bad state (lower \( \theta \)) reduces the supply of sovereign bonds, as banks fear that the government is more likely to default in the bad state. Crucially, there is no effect of \( \theta \) on demand for sovereign bonds, as undercapitalized banks, seeking high payoff volatility, only care about the good state, regardless of its likelihood. Hence, in equilibrium, sovereign yields are decreasing in \( \theta \).

### 3.3 Undercapitalized Banks and High Sovereign Risk

I now compare the UU and WW cases and show that the comparison of equilibrium quantities and prices crucially depends on the level of sovereign risk \( \gamma \) in the economy. Intuitively,
undercapitalized banks want to risk-shift and can do so using either private lending or domestic government bonds.\textsuperscript{12} The choice between these two assets depends on the respective payoffs. The asset best suited to risk-shifting yields the highest payoff in the good state and the lowest payoff in the bad state.

**Proposition 3. (Crowding-Out)** If $\gamma > 1 - \theta$, in an economy with undercapitalized domestic banks (UU), governments have a higher debt capacity, pay lower rates, and banks reduce private lending compared with an economy with well capitalized banks (WW).

In an economy with high sovereign risk ($\gamma > 1 - \theta$), undercapitalized banks risk-shift using domestic government bonds as they offer the highest payoff in the good state to compensate investors for holding high sovereign risk. The condition $\gamma > 1 - \theta$ can be rewritten as $\lambda < \frac{\epsilon}{\epsilon_H}$, namely the bond recovery value in the bad state has to be sufficiently low to ensure that banks choose domestic government bonds to risk-shift.\textsuperscript{13} By investing a high fraction of their un-allocated balance sheet capacity in government bonds, banks crowd-out private lending. However, in an economy with low sovereign risk ($\gamma \leq 1 - \theta$), discussed in Section 5.2, undercapitalized banks reduce their investment in government bonds to invest more in the private lending technology.

To get some intuition, I rearrange the condition in Proposition 3 as follows:

$\gamma > 1 - \theta \iff \lambda < \frac{\epsilon_L}{\epsilon_H} \iff \left( \frac{\mathbb{E}(DomGovtBond_{UU})}{\mathbb{E}(PrivLending_{UU})} \right) < \left( \frac{\mathbb{E}(DomGovtBond_{WW})}{\mathbb{E}(PrivLending_{WW})} \right)$

\textsuperscript{12}In Proposition 2, I illustrate why undercapitalized banks prefer domestic government bonds over foreign government bonds to risk-shift.

\textsuperscript{13}This inequality corresponds to government bonds performing worse than the private lending technology in the bad state. According to Moody’s (2014), the average value-weighted recovery rate on defaulted government bonds is 26% compared to 38% of senior unsecured corporate issuers during the period from 1983 to 2013.
where, in the last inequality, the left-hand side is the ratio of the expected payoff from domestic government bonds to the expected payoff from the private lending technology in the WW case and the right-hand side is the same ratio in the UU case.

Banks’ portfolio choice affects prices and the domestic government debt capacity. Undercapitalized banks hold more domestic government bonds and invest less in the private lending technology \((k_{UU} < k_{WW})\) compared with well capitalized banks. The resulting lower tax collection reduces the government debt capacity as banks fear that the sovereign might be unable to repay them at \(t = 1\). However, in equilibrium, the high demand for government bonds lowers sovereign yields, offsetting the negative effect of the lower tax collection. Hence, a government with undercapitalized domestic banks has a higher debt capacity \((D_{UU} > D_{WW})\) and pays lower sovereign yields \((R_{UU} < R_{WW})\) compared with a government with well capitalized domestic banks.

Given that the effect on the government debt capacity operates through prices, I isolate the risk-shifting component \(\eta\) in sovereign yields:

\[
R_{WW} = \frac{1}{2} \left( \frac{E(\epsilon)}{E(\lambda)} (1 - \tau) \left( \frac{E(\epsilon)}{E(\lambda)} (1 - \tau) + 2\tau \Delta_\epsilon \right) \right)^{1/2}
\]

\[
R_{UU} = \frac{1}{2} \left( \frac{E(\epsilon)}{E(\lambda)} (1 - \tau) \left( \frac{E(\epsilon)}{E(\lambda)} (1 - \tau) + 2\tau \Delta_\epsilon \right) \right)^{1/2}
\]

where

\[
\eta = \frac{\epsilon H E(\lambda)}{E(\epsilon)} > 0
\]

The term \(\eta\) captures the equilibrium effect of banks’ risk-shifting behavior. If \(\eta = 1\) (if and only if \(\gamma = 1 - \theta\)), the portfolio choice of undercapitalized banks does not affect government bond yields. This is the particular case where government bonds and the private lending technology are equally desirable assets for a risk-shifting bank. If \(\eta \neq 1\), the portfolio choice of undercapitalized banks does affect government bond yields. In particular, if \(\eta < 1\) (high sovereign risk), government bonds are the best assets to risk-shift and the increased
demand for domestic government bonds by undercapitalized banks lowers domestic sovereign yields.

Suppose now that, in an economy with high sovereign risk, country A has undercapitalized banks and country B has well capitalized domestic banks (UW case). Both countries have, again, perfect home bias \( \alpha^i = \alpha \), but country A faces a higher demand for its government bonds compared with country B. The undercapitalized banks in A invest more, risk-shifting on their insured depositors, in domestic government bonds and invest less in the private lending technology compared with well capitalized banks in B. Equilibrium quantities and prices are given by:

\[
\begin{align*}
R^A_{UW} &= R_{UU} & R^B_{UW} &= R_{WW} \\
k^A_{UW} &= k_{UU} & k^B_{UW} &= k_{WW} \\
D^A_{UW} &= D_{UU} & D^B_{UW} &= D_{WW}
\end{align*}
\]

Similar to the UU and WW cases, there is perfect home bias in the economy and equilibrium quantities and prices only depend on the capitalization of domestic banks.

4 Equilibrium Bank Capital

This section focuses on bank capitalization. In Section 4.1, I characterize the level of bank debt that makes a bank well capitalized or undercapitalized. In Section 4.2, I analyze the preference of the government — in an environment where the government is in charge of capital regulation and maximizes welfare — over the capitalization of domestic banks.

\[\text{14} \text{The WU case trivially follows by symmetry.}\]
4.1 Characterizing Bank Capital

In the previous sections, I label banks as undercapitalized or well capitalized based on whether the limited liability constraint binds in the bad state. But, of course, the level of bank debt that triggers the limited liability constraint to bind depends on the bank’s investment decision. In particular, with perfect home bias, the payoff of the financial sector at \( t = 1 \) consists of the payoff from the lending technology and the payoff from holding domestic government bonds. Hence, in equilibrium, the bank payoff in the bad state is:

\[
\Pi_{L, HB} = [(1 - \tau) \epsilon_L f(k^i) + \lambda^i_L (1 - k^i) R^i - L^i]^+ = [(1 - \tau \gamma^i) f(k^i) - L^i]^+ +
\]

where the subscript refers to the bad state of the world in an economy with perfect home bias. This expression suggests that there is a bank debt threshold \( L \) such that a bank is well capitalized if \( L \leq \bar{L} \) and undercapitalized if \( L > \bar{L} \).

**Lemma 2.** There exists a threshold \( \bar{L} \) such that banks in \( i \in \mathcal{I} \) are undercapitalized if \( L^i > \bar{L} \) and well capitalized if \( L^i \leq \bar{L} \).

The left panel of Figure 7 shows the threshold level \( \bar{L} \) of bank debt for the financial sector in country \( i \). If the initial level of bank debt is greater than the threshold, the limited liability constraint binds in the bad state and (undercapitalized) banks solve (4). If the initial level of debt is lower than the threshold, the limited liability constraint does not bind in the bad state and (well capitalized) banks solve (2). The right panel of the figure maps the initial debt level \( L^A \) of banks in country A and the initial debt level \( L^B \) of banks in country B to the four cases in the economy: UU, UW, WU, and WW.

I now compare the expected payoffs of agents in the UU and WW cases in an economy with high sovereign risk (\( \gamma > 1 - \theta \)). In Table 1, I summarize the expected payoffs at \( t = 1 \) for
Figure 7: Bank Capitalization. This figure shows how the initial bank debt $L$ delimits four cases in the economy: UU, UW, WU, and WW. The left panel shows the two areas W and U for country $i \in I$. The right panel combines the level of bank debt $L$ in country A ($x$-axis) and country B ($y$-axis).

Bank depositors, bank equity holders, and the supranational deposit insurance. Depositors are clearly indifferent between the two cases as they are protected by the deposit insurance in the bad state. Deposit insurance is never required to make payments in the WW case, but is required to pay depositors in the UU case with probability $1 - \theta$. Bank equity holders obtain the expected profits in both cases. They prefer the WW case if the gains from having well capitalized banks are higher than the losses shifted to the deposit insurance in the bad state in the UU case. More formally, equity holders prefer well capitalized if and only if

$$
\mathbb{E}(\Pi(k_{WW})) - \theta \Pi_H(k_{UU}) \geq L(1 - \theta)
$$

where the right-hand side is the value of expected losses shifted to the deposit insurance.


Table 1: Expected Payoffs at \( t = 1 \). This table shows expected payoffs of bank depositors, bank equity holders, and the deposit insurance at \( t = 1 \) in the UU and WW cases.

### 4.2 Governments and Domestic Bank Capital

In the baseline model, governments exhaust their debt capacity and collect taxes to repay bondholders. I now assign a more active role to governments and analyze their preferences toward the capitalization of domestic banks in an environment with high sovereign credit risk (\( \gamma > 1 - \theta \)). In particular, governments provide a public good and act as capital regulators deciding — before banks make their investment decision at \( t = 0 \) — the level of initial bank debt \( L \) to maximize domestic welfare. More formally, their problem is:

\[
\max L f(k(L)) + g(D(L)) \quad \text{s.t. } D(L) \leq \frac{\tau \Delta f(k(L))}{R(L)} \tag{7}
\]

where I omit the country superscripts for simplicity. The objective function is made of two terms. The first term is the profits of bank equity holders. The second term is the benefit from the provision of the public good funded by the sovereign debt \( D \), where \( g \) is a continuous, strictly increasing, and strictly concave function. Note that depositors do not appear in the government objective function, as, protected by the deposit insurance, they never incur losses on their deposits.

In an economy with high sovereign risk (\( \gamma > 1 - \theta \)), the government faces a trade-off. On the one hand, well capitalized banks choose a high investment in the lending technology, increasing the payoff for equity holders. On the other hand, undercapitalized banks — driven by the risk-shifting motive — choose a high investment in domestic government bonds, lowering sovereign yields and therefore expanding the sovereign debt capacity that, in turn,
supports the public good provision.\footnote{The government objective function in (7) can easily accommodate an extra term to account for the eventual national contribution to the supranational deposit insurance. This term would capture an extra element in the trade-off faced by the government as having undercapitalized banks is costly for the government.}

**Proposition 4.** If $\gamma > 1 - \theta$, a government that solves (7) chooses $L > \bar{L}$ if and only if

$$\frac{\partial g}{\partial D} \frac{\partial D}{\partial L} > \frac{\partial f}{\partial k} \frac{\partial k}{\partial L}$$

If the inequality above holds, the government prefers undercapitalized domestic banks. The left-hand side captures the positive effect of high bank debt on the sovereign debt capacity and, therefore, the provision of public good. The right-hand side captures the negative (crowding-out) effect of high bank debt on investment in the lending technology. In sum, undercapitalized banks act as buyers of last resort for the domestic sovereign, lowering sovereign yields and funding the public good. Moreover, this role of domestic banks in sustaining the sovereign debt capacity is particularly important during sovereign crises, when the marginal benefit of public spending is likely high (Auerbach and Gorodnichenko (2012)).\footnote{The literature on fiscal multipliers shows that the effect public spending is state dependent. Multipliers are larger in crises and recessions than in expansions.}

## 5 Discussion

In Section 5.1, I discuss the model assumptions, focusing on the three key ones (high correlation between domestic payoffs, supranational deposit insurance, and fixed-size bank balance sheets). In Section 5.2, I extend the baseline model in several directions. In Section 6.1, I show that, in the context of the eurozone crisis, the model assumptions are realistic.
5.1 Assumptions

High Correlation Between Domestic Payoffs  The first key assumption is a sufficiently high correlation between the payoff of the (domestic) lending technology and the payoff of domestic government bonds. Because the private sector cannot invest in the foreign private lending technology, the sovereign mechanically defaults only if the domestic economy is in the bad state as the low payoff of the domestic lending technology reduces tax collection, therefore triggering the sovereign default. While this assumption simplifies the solution, the risk-shifting mechanism survives if banks can also invest in the foreign private lending technology, as long as banks allocate a sufficiently large share of the investment in the lending technology domestically.\textsuperscript{17} In Section 5.2, I discuss a model extension where banks can invest in the foreign lending technology.

Supranational Deposit Insurance  The second key assumption is the presence of a credible deposit insurance that protects depositors in the bad state. This guarantee must be realistically funded by a supranational body, as a nationally funded guarantee is unlikely to have sufficient funds to protect bank depositors in case of a sovereign default. Thanks to this guarantee, depositors do not require a high return on their savings nor do they discipline banks by withdrawing their deposits in bad times.\textsuperscript{18} This assumption represents a key departure with respect to the “doom loop” literature where the bank-sovereign nexus is typically the result of bank holdings of sovereign bonds paired with government guarantees. In this paper, I decide to not model guarantees and instead focus on what drives bank holdings of domestic government bonds during bad times.

\textsuperscript{17} The home bias in the investment in the private lending technology is backed by data and can be motivated, for example, by the need to monitor investors.

\textsuperscript{18} See, for example, Black et al. (1978) for a discussion of deposit insurance and bank risk taking.
Supranational deposit insurance is not the only assumption that can sustain the proposed risk-shifting mechanism. Alternatively, I can assume that depositors can withdraw their savings and invest at a cost in foreign assets. In this environment, banks and governments can take advantage of depositors — who have a “hold-up” problem — up to the cost of investing abroad.

**Fixed Bank Balance Sheet Size** The third key assumption is a fixed bank balance sheet size that links the purchases of domestic government bonds to the crowding-out of private lending. Note that, in an environment with frictionless private credit markets, the purchases of sovereign debt do not necessarily imply a negative effect on private credit as government bond holdings can be financed by new borrowing. As in Gennaioli et al. (2014), financial frictions that constrain private borrowing during crises motivate this assumption.

**Other Assumptions** The country-specific shocks are symmetric and uncorrelated in the baseline model. Probabilities $\theta^i = \bar{\theta}$, for all $i \in \mathcal{I}$, simplify the algebra but come with a loss of generality. This assumption is not crucial as the risk-shifting mechanism is robust to asymmetric country-level shocks, as long as these shocks are non-perfectly correlated. However, in case the two shocks are perfectly correlated, the risk-shifting incentive ceases to generate home bias in the government bond market.

Finally, I assume a square-root functional form for the lending technology to obtain closed-form solutions for prices and isolate the equilibrium effect $\eta$ of banks’ risk-shifting behavior. All the results derived in the previous section still hold for definite parameter regions with a generic functional form as long as this function is continuous, strictly increasing, strictly concave and satisfies Inada conditions.
5.2 Extensions of the Baseline Model

Relaxing Assumption 1: Foreign Private Lending  I now allow banks to invest in domestic and foreign private lending technologies, in addition to the global sovereign bond market. The left panel of Figure 8 shows the investment opportunity set of a representative bank in country $i \in I$. Similar to the baseline model, banks choose how to allocate the unit balance sheet capacity between the private lending technology ($k > 0$) and the government bond market ($1 - k$). Moreover, banks now choose, within each asset class, the share that is invested domestically, namely, $\mu$ and $\alpha$ for the lending technology and the bond market, respectively. The right panel of the figure shows the payoffs at $t = 1$ once uncertainty in both countries is resolved.

The proceeds from investing in private lending are taxed at the exogenous rate $\tau$ and domestic sovereigns use tax collection to repay bondholders. Should tax collection be lower than payments due to bondholders, sovereigns default. Sovereign default can happen for two reasons, depending on the share $\mu$ of private lending that is allocated domestically. If $\mu$ is high, default happens because the home economy is in the bad state and domestic banks have invested a lot in the domestic lending technology. A low realization of domestic private lending has a sizable effect on the tax base and, consequently, on the ability of the sovereign to repay bondholders: the government is forced to default exactly when banks realize low profits from the lending technology. This mechanism is at work in the baseline model as sovereign default is positively correlated with the home state of the world. If $\mu$ is low, default happens because the foreign economy is in the bad state and domestic banks have invested a lot in the foreign lending technology. Regardless of the domestic state, a low realization of foreign private lending might cause a domestic sovereign default. In other words, a sufficiently low $\mu$ breaks the positive correlation between banks’ revenues from private lending and the payoff of domestic government bonds. With $\mu$ sufficiently low, the incentive of undercapitalized banks to risk-shift vanishes.
Formally, banks in $i \in I$ invest domestically if and only if:

$$R_i' \geq R_j' \left(1 - 1_U \left(1 - \frac{\mathbb{E}(\lambda_{i,H,s}^j)}{\mathbb{E}(\lambda_{i,H,s}^j)}\right)\right) \quad i \neq j \quad i, j \in I$$

where $1_U$ is an indicator variable equal to one if banks are undercapitalized, and $\lambda_{i,H,s}^j$ is the recovery value of government bonds of country $i$ when country $i$ is in $s_i$ state and country $j$ is in $s_j$ state. One the one hand, well capitalized banks ($1_U = 0$) invest domestically if and only if the home sovereign yield is greater than the foreign sovereign yield. On the other hand, undercapitalized banks ($1_U = 1$) have an incentive to tilt their government bond portfolio toward the security that pays the most in the good state. Domestic government bonds serve this purpose if, in the good home state, their recovery value is higher than the
recovery value of foreign government bonds:

$$\mathbb{E}(\lambda_{i,H,s}^j) < \mathbb{E}(\lambda_{i,H,s}^i)$$

where expectations are taken with respect to country $j$ probability. If the inequality holds, undercapitalized banks need to be compensated to hold foreign bonds and the mechanism of the baseline model still holds. Perhaps not surprisingly, the condition holds if the correlation between revenues from domestic government bonds and private lending technology is sufficiently high. This correlation depends, as discussed, on the choice variable $\mu$.\(^{19}\)

Relaxing Assumption 2: Correlated Shocks I now assume that the shocks hitting the two private technologies are correlated, with $\text{corr}(\epsilon^A, \epsilon^B) = \rho$. Maintaining the marginal probabilities unchanged in both countries, I compute the following joint probabilities:

- $\text{Prob}(s^A = H \cap s^B = H) = \theta(1 - (1 - \rho)(1 - \theta))$
- $\text{Prob}(s^A = H \cap s^B = L) = (1 - \rho)\theta(1 - \theta)$
- $\text{Prob}(s^A = L \cap s^B = L) = (1 - \theta)(\rho + (1 - \rho)(1 - \theta))$

where $s^i \in S$ is the state of country $i \in \mathcal{I}$ and $\text{Prob}(s^A = H \cap s^B = L) = \text{Prob}(s^A = L \cap s^B = H)$ by symmetry. Lemma 1 still holds, as governments default in the bad state only, when the tax base is not sufficient to repay bondholders.

Note that in the WW case, only marginal probabilities matter and the equilibrium is therefore identical to the one obtained in the baseline model. In particular, financial sectors

\(^{19}\)In the Online Appendix, I derive, in this more general model environment, the conditions needed for banks to maintain the risk-shifting incentive.
invest in domestic government bonds if $R \geq R^*$. Nevertheless, the non-zero correlation affects the portfolio choice of undercapitalized banks. Because of the binding limited liability constraint, these banks make investment decisions to maximize the payoff in the good state. In this state, the domestic government never defaults and bondholders get the high payoff $R$. In the good domestic state, the payoff of foreign government bonds crucially depends on the correlation coefficient. If $\rho < 1$, the foreign government might default in the domestic good state, introducing the home bias discussed in the baseline model. If $\rho = 1$, the two governments always default at the same time and the banks’ problem is therefore unchanged from the WW case. For a generic $\rho \in [-1, 1)$, undercapitalized banks invest domestically if and only if:

$$R \geq (1 - (1 - \rho)(1 - \lambda)(1 - \theta) \frac{R^*}{\text{required compensation to hold foreign bonds}}$$

where the term in parentheses captures the required compensation needed by domestic undercapitalized banks to hold foreign government bonds.

### Relaxing Assumption 3: One Safe Country, One Risky Country

I now assume that the lending technology is risky in country A ($\theta^A \in (0, 1)$) and riskless in country B ($\theta^B = 1$). On the one hand, the problem faced by banks in A is unchanged from the baseline model: Lemma 1 holds and the government is forced to default on part of its debt in the bad state. On the other hand, as there is no uncertainty, the government in B always has sufficient tax collection at $t = 1$ to repay bondholders. Country B never defaults. In order to isolate the effect of the different shock probabilities on the government bond market, I normalize the country B production function so that banks in the two countries obtain, in
expectation, the same payoff from private lending.\footnote{Formally, $\Delta^A = E^A(\bar{\epsilon}) - \gamma\bar{\epsilon}^A$ for $i \in I$. In other words, (i) an investment of $k^B$ in country B’s private lending technology yields $(1 - \tau)\Delta_1f(k^B)$ with certainty at $t = 1$ and (ii) an investment of $k^A$ in country A’s private lending technology yields $(1 - \tau)\Delta_1f(k^A)$ in expectation at $t = 1$.}

Note that there are only two cases: the case where both financial sectors are well capitalized (WW) and the case where banks in A are undercapitalized and banks in B are well capitalized (UW).\footnote{Banks in B can only be well capitalized, as their limited liability constraint never binds.} As in the baseline model, if both countries have well capitalized banks, banks invest in both sovereign bonds. Equilibrium gross interest rates are such that:

$$E^A(\lambda^A)R^A = R^B$$

where superscripts refer to countries. The risk-adjusted return on government bonds in the two countries must be equal so that the risky country A needs to pay its bondholders more than country B to remunerate them for the higher sovereign risk. Higher government yields in A reduce the government debt capacity. In equilibrium, the riskless country has higher debt capacity and attracts foreign banks, as shown in the left panel of Figure 9.

Suppose now that banks in A are undercapitalized. The unique equilibrium is illustrated
in the right panel of the figure. As in the baseline model, undercapitalization of one financial sector generates perfect home bias in the entire economy, with both sovereigns facing only domestic demand for their bonds. The riskless country still enjoys a higher debt capacity and pays lower yields compared with the risky country.

Relaxing Assumption 4: Lending Technology with a General Functional Form

Bank portfolio choice is unaffected if I replace the square-root lending technology with a generic functional form. However, closed-form solutions cannot be obtained. For example, in a perfect home bias equilibrium, the two market clearing conditions are:

$$1 - k'(R, L') = \tau f(k'(R, L')) \Delta(N'(R'))$$

The results based on closed-form solutions (e.g., crowding-out) would still hold in certain parameter regions.

Analysis of the Low Sovereign Risk Case ($\gamma \leq 1 - \theta$)

I now analyze the economy with low sovereign risk ($\gamma \leq 1 - \theta$) or, in other words, with a sufficiently high recovery value of government bonds in the bad state ($\lambda \geq \epsilon_L/\epsilon_H$). Compared with the baseline model, in the WW case, both governments face domestic and foreign demand for their bonds at the same interest rate. As the recovery value on government bonds $\lambda$ increases (lower $\gamma$), banks invest more in the relatively safer government bonds reducing private lending. Similarly, the UU case is unchanged from the baseline model.

However, in the case where one financial sector is well capitalized and one financial sector is undercapitalized, the economy has two equilibria, as shown in Figure 10 for the UW case. The left panel illustrates the standard perfect home bias equilibrium where the binding limited liability induces undercapitalized banks in A to tilt their government bond portfolio domestically. However, banks now invest more in the lending technology and less in
Figure 10: Two UW Equilibria ($\gamma \leq 1 - \theta$ case). This figure illustrates the two equilibria in the UW case when $\gamma \leq 1 - \theta$. The left panel shows the perfect home bias equilibrium and the right panel shows the asymmetric equilibrium. Orange banks are undercapitalized and yellow banks are well capitalized.

the government bond market compared with the WW case. Seeking higher payoff volatility, banks choose private lending as a tool to risk-shift, as domestic government bonds are too safe for this purpose. The government tax collection therefore increases, driven by a higher tax base. In equilibrium, the lower demand for bonds causes sovereign yields to increase. As a result, the government has a lower debt capacity and faces high sovereign yields.

In addition to the standard perfect home bias equilibrium, the economy can fall in an asymmetric equilibrium where country A faces both domestic and foreign demand for its bonds, illustrated in the right panel of the figure. Country A attracts foreign banks by offering them a high interest rate, which can be sustained by the high domestic tax base. Because of poorly capitalized foreign banks, country B pays a high interest rate and has lower debt capacity compared with the WW case.

6 Empirical Evidence

The model sheds light on the experience of peripheral eurozone countries during the recent crisis. In this section, I show that in this empirical setting the model assumptions are realistic and the evidence consistent with the model predictions.
6.1 Mapping the Model to the Eurozone Crisis

Peripheral eurozone countries, with their high sovereign risk and low-capital banks, resemble the model environment in many ways. Their sovereign credit risk started increasing in 2009 and reached record highs at the end of 2011, inducing the ECB to adopt extraordinary measures to preserve the euro. In addition to high sovereign risk, banking sectors in these countries were, and many still are, notoriously undercapitalized (Greenlaw et al. (2013), Acharya et al. (2014b)). Finally, any euro-denominated government bond has zero regulatory risk weight, a preferential regulatory treatment that mirrors the model environment where banks can only invest in government bonds.\(^{22}\)

Moreover, the three key model assumptions are realistic in the eurozone periphery. First, highly intertwined with the domestic sovereign, banks are likely to default in case of domestic sovereign default. In Table 2, I use data on the credit risk exposures of major peripheral banks in December 2010 and show the exposures at default $EAD_{ij}$ of bank $i$ vis-à-vis country $j.^{23}$ I then rank banks according to their $EAD_{ij}/E_i$ ratio, where $E_i$ is the market value of equity of bank $i$. Of the hundreds bank-country pairs, the table only reports the 26 pairs where the ratio is greater than one, suggesting that bank $i$ is very vulnerable to an eventual default of country $j$. Half of these high exposures (highlighted) are domestic, namely, where the country of incorporation of the bank is the same country the bank is exposed to.

Second, the ECB acted as a de facto supranational deposit insurance for banks. Starting

\(^{22}\)Thanks to the zero risk weight, euro-denominated government bonds are a particularly attractive asset for eurozone banks (Korte and Steffen (2016)). Under the Capital Requirement Directive (CRD), “exposures to Member States’ central governments and central banks denominated and funded in the domestic currency of that central government and central bank shall be assigned a risk weight of 0%” (Directive 2006/48/EC, Annex VI, Part 1(4)).

\(^{23}\)For example, the first line illustrates the exposure at default of EFG Eurobank Ergasias (Greek bank) vis-à-vis Greece. The last line illustrates the exposure at default of Unicredit (Italian bank) vis-à-vis Austria. The data are publicly available on the website of the European Banking Authority.

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in October 2008, banks could in fact obtain unlimited liquidity from the ECB provided they pledged sufficient eligible collateral. Crucially, eligible collateral included low-quality and illiquid assets (peripheral government bonds, bank covered bonds, mortgage-backed securities, asset-backed securities, and selected bank loans) allowing banks to effectively shift the
risk of some of the potential sovereign default losses to the central bank (Uhlig (2013)). The empirical literature confirms this interpretation and shows that the ECB generous collateral rules allowed banks to maintain stable funding sources, tapping the central bank liquidity facilities at more attractive terms (collateral eligibility and haircut) compared with private markets. Drechsler et al. (2016) and Crosignani et al. (2017) find that, during the crisis, peripheral banks purchased risky eligible collateral securities and pledged them at the ECB to secure stable funding. In practice, peripheral banks relied progressively more on the central banks as a funding source (Garcia-de Andoain et al. (2016)).

Third, at the peak of the crisis, peripheral banks were not able to obtain external financing. If anything, these banks were having a hard time maintaining their funding sources and were subject to sudden funding dry-ups. Chernenko and Sunderam (2014) show that U.S. money market funds significantly cut back their funding to eurozone banks as the crisis deteriorated in summer 2011 and Carpinelli and Crosignani (2017) document that the wholesale funding of Italian banks collapsed by 5 percentage points in the second half of 2011, forcing banks to access the ECB facilities to replace their funding sources.

6.2 Supporting Empirical Evidence

The model has two clear predictions. During sovereign crises, low-capital banks increase their holdings of domestic government bonds and reduce their private credit supply.

The recent empirical literature on the eurozone crisis strongly supports these cross-sectional predictions. Acharya et al. (2016) and Altavilla et al. (2016) examine the joint evolution of private credit supply and government bond holdings in the run-up to the crisis. They show that weakly capitalized banks drove the reduction of credit supply to firms and the increase in holdings of domestic government bonds. Their findings are consistent with papers focusing — in isolation — on holdings of government bonds (Acharya and Steffen (2015), Drechsler et al. (2016)) and credit supply to firms (Bottero et al. (2017), Bofondi et al.
The former find that undercapitalized banks purchased more domestic bonds compared with better capitalized banks. The latter find that the increased sovereign credit risk caused a more severe credit tightening for banks with low regulatory capital.

While the empirical evidence supports the mechanism proposed in the model, the different evolution of government bond holdings and private credit of undercapitalized and well capitalized banks might, of course, also be driven by other factors. For example, governments might force weak domestic banks to buy more domestic bonds in bad times promising, in exchange, more tolerant supervision (Becker and Ivashina (2016), Ongena et al. (2016), De Marco and Macchiavelli (2015)). Furthermore, low-capital banks might optimally buy more domestic government bonds to incentivize the government to bail them out to prevent a costly fire sale of government bonds (Koetter and Popov (2016)).

7 Conclusion

During the eurozone crisis, banks in the eurozone periphery increased their holdings of domestic government bonds and reduced private lending. Motivated by this evidence, I build a tractable model to explain the cause, rationale, and consequences of bank holdings of domestic government bonds during sovereign crises. The cause is low bank capital. Undercapitalized banks purchase domestic government bonds linking their destiny to that of the sovereign. The rationale is risk-shifting. While, in case of domestic sovereign default, banks are protected by limited liability, home sovereign debt guarantees a high payoff in the good state of the world. The consequences are a crowding-out effect of private credit

\[\text{Note that the model presented in this paper can be interpreted as a model of moral suasion where the government is more successful in repressing those banks that, for a risk-shifting motive, already have an incentive to buy more domestic debt.}\]
and increased government debt capacity. When the sovereign risk is high, undercapitalized banks reduce the supply of private lending to further increase domestic government bond holdings. This increased demand for domestic government bonds reduces sovereign yields therefore supporting the sovereign debt capacity.

The model suggests that national regulators face a trade-off when setting capital regulation in presence of high sovereign risk. On the one hand, well capitalized banks lend to firms and households fostering growth. On the other hand, low-capital banks optimally act as buyers of last resort for the home sovereign, exactly when it needs to borrow the most. During periods of high sovereign risk, sovereigns with undercapitalized domestic banks have a higher debt capacity and pay lower sovereign yields compared with sovereigns with well capitalized domestic banks.

The model rationalizes the prolonged fragility of eurozone peripheral banks and the reluctance to recapitalize them during the crisis. Greenlaw et al. (2013) show that these banks were undercapitalized before and throughout the crisis, and Acharya et al. (2014b) show that regulators failed to both assess the extent of their capital need and improve their soundness. Moreover, European policy makers have been reluctant to (i) implement the Basel III capital requirements, (ii) apply non-zero risk weights to risky eurozone sovereign bonds, and (iii) force banks to mark-to-market their government bond holdings.25

25First, capital requirements in the eurozone follow the Capital Requirements Directive that implemented the Basel II and Basel III capital standards. Member states were expected to implement the directive into national law by the end of 2012. Even if the European Banking Authority warned in October 2011 that banks had to raise $146 billion to meet new capital requirements, the deadline was not respected, and the directive was put in place in January 2014. Second, as Danièle Nouy — Chair of the Single Supervisory Mechanism — admitted in a statement (“Sovereigns are not risk-free assets. That has been demonstrated, so now we have to react. What I would admit is that maybe it’s not the best moment in the middle of the crisis to change the rules [...]”), regulators were concerned that applying a non-zero risk weight to euro-denominated government bonds would have encouraged a sell-off. Third, in May 2010 the Bank of Italy allowed Italian banks not to mark-to-market their government bond holdings.
The analysis in this paper warns against establishing a supranational deposit insurance in the presence of weak and geographically undiversified banks. In this environment, in periods of high sovereign risk, low-capital banks might risk-shift on their insured deposit holders by buying domestic sovereign bonds and reducing their private credit supply. This behavior might help governments access public debt markets at a time when they likely need to borrow, but it also exacerbates the sovereign-bank nexus. When in place, an international safety net should be paired with careful bank supervision to preserve international financial stability. I believe these are promising avenues for future research.

References


Appendix A  Derivations and Proofs

A.1 Baseline Model

In this section, I present model derivations and proofs. Superscripts indicate countries. In Figure A.1, I illustrate the nine possible banks-sovereign “arrangements” in the economy.

Proof of Lemma 1. Using (3) the payment due to bondholders at $t = 1$ is

$$DR = \Delta \tau f(k)$$

In the good state, tax collection minus $y$ is greater than payments due to bondholders if:

$$\tau f(k)(\epsilon_H - \gamma \epsilon_L) > \Delta \tau f(k)$$

$$\epsilon_H > \mathbb{E}(\epsilon)$$

In the good state, the government is always able to fully repay bondholders. In the bad state, tax collection (minus the sovereign shock $y$) is greater than payments due to bondholders if:

$$\tau f(k)\epsilon_L(1 - \gamma) > \Delta \tau f(k)$$

$$0 > \theta(\epsilon_H - \epsilon_L)$$

In the bad state the government always defaults on part of its debt. The haircut $\lambda$ is the parameter such that tax collection equals the post-haircut payments due to bondholders:

$$\tau\epsilon_L f(k) - g = \lambda DR$$

$$\lambda_L = \epsilon_L(1 - \gamma)\Delta^{-1}$$
Figure A.1: Nine Candidate Arrangements. This figure illustrates the nine possible arrangements between banks and governments in the economy. Note that the degenerate arrangements where a bank does not invest in the bond markets are not included.

**Proof of Proposition 1.** From the maximization problem of the banking sector I get:

\[
k = f^{t-1} \left( \frac{E(\lambda)(\alpha R + (1 - \alpha)R^*)}{E(\epsilon)(1 - \tau)} \right)
\]

(A1a)

\[
k_{LL} = f^{t-1} \left( \frac{\alpha R + E(\lambda)(1 - \alpha)R^*)}{\epsilon R(1 - \tau)} \right)
\]

(A1b)

where the subscript \( LL \) indicates that the limited liability constraint binds in the bad state, i.e., the banking sector solves (4).

First, I show that arrangements (h) and (i) are not candidate equilibria. Consider arrangement (h) where both banks invest in B. Market clearing for country A is violated as \( k^A > 0 \)
(Inada condition). Second, I show that arrangements (e) and (f) are not candidate equilibria. Consider arrangement (e) where A banks invest only abroad and B banks invest in both sovereign bonds. It is easy to show that $R_B = R_A$ and $k_B = k_A$. I then reach a contradiction since governments face different demand for bonds, having the same debt capacity in equilibrium. Third, I show that arrangements (c) and (d) are not candidate equilibria. Consider arrangement (c). Since, A’s banks invest in both countries, it must be that $R_B = R_A$ and $k_B = k_A$. I then reach a contradiction since governments have the same debt capacity, but face different demands, in equilibrium. Fourth, I need to rule out the degenerate arrangements where one banking sector does not hold any government bonds, hence investing $k = 1$ in the lending technology. Suppose $k_A = 1$. If financial sector B invests in both types of government bonds, it must be that $R_A = R_B$. Hence, $k_B = 1$ reaching a contradiction, as both governments have a strictly positive debt capacity. Suppose country A faces zero demand for its bonds. In equilibrium, it must be that $R_A = \infty$ and $R_B = \infty$ since $R_B \geq R_A$. In that case, country B has zero debt capacity too. Finally, suppose that country B faces no demand for its bonds. Similar to the case where A faces no demand for its bonds, I reach a contradiction as both interest rates are infinite. I now show that in equilibrium the two financial sectors must have the same home bias ($\alpha^i = \alpha$, for $i = A, B$). In each of the three candidate arrangements (a), (b), and (g), it must be that $R_B = R_A$ and $k_B = k_A$. I need to show that countries have the same home bias in arrangement (a). Market clearing conditions can be written as $(\alpha^A + 1 - \alpha^B)(1 - k) = D^A$ and $(\alpha^B + 1 - \alpha^A)(1 - k) = D^B$. I reach a contradiction unless $\alpha^A = \alpha^B$. 

**Closed-Form Solutions.** Having shown that the candidate arrangements (a), (b), and (g) have the same home bias $\alpha^i = \alpha$, for $i \in \mathcal{I}$, I now use a square root production function to get closed-form solutions. The two (symmetric) market clearing conditions are therefore:

$$1 - k = \frac{\tau \Delta \sqrt{k}}{R}$$
Plugging in (A1a),

\[ R_{WW} = \frac{1}{2E(\lambda)} \left( E(\epsilon)(1 - \tau)(E(\epsilon)(1 - \tau) + 2E(\lambda)\tau \Delta) \right)^{1/2} \]  
(A2a)

\[ k_{WW} = \frac{E(\epsilon)(1 - \tau)}{E(\epsilon)(1 - \tau) + 2E(\lambda)\tau \Delta} \]  
(A2b)

\[ D_{WW} = \frac{2E(\lambda)\tau \Delta}{E(\epsilon)(1 - \tau) + 2E(\lambda)\tau \Delta} \]  
(A2c)

where the subscript WW indicates the capitalization level of the financial sector. Note that \( k_{WW} \in (0, 1) \). It is also easy to show that \( \frac{\partial k_{WW}}{\partial \tau} < 0 \) and \( \frac{\partial k_{WW}}{\partial \gamma} > 0 \).

Proof of Proposition 2. Note first that \( E(\lambda) < 1 \). First, I show that arrangements (a), (f), (e), and (g) are not candidate equilibria when at least one banking sector risk-shifts. Arrangement (a): In the UU case, in equilibrium it must be that \( R^A > R^B \) and \( R^B > R^A \) to have both financial sector investing abroad. In the UW case, similarly, in equilibrium it must be that \( R^B > R^A \) and \( R^B = R^A \). Case WU is symmetric. In each of these three cases, I reached a contradiction. Arrangement (g): In the UU case, in equilibrium, it must be that \( R^A \leq E(\lambda)R^B \leq E(\lambda)^2R^A \). In the UW case in equilibrium, it must be that \( R^A \leq E(\lambda)R^B \leq E(\lambda)^2R^B \). In the WU case, in equilibrium it must be that \( R^A = R^B \) and \( R^B < E(\lambda)R^A \). Case WU is symmetric. In each of these three cases we reached a contradiction. Arrangement (f): In the UU case, in equilibrium, it must be that \( R^A = \theta R^B \). In the UW case in equilibrium it must be that \( R^B \leq E(\lambda)R^A \leq E(\lambda)^2R^B \). In the WU case in equilibrium it must be that \( R^A = R^B \) and \( R^B < E(\lambda)R^A \). Arrangement (e) follows by symmetry. The proof used in Proposition 1 can be used again to show that arrangements (h) and (i) are not candidate equilibria. Finally, arrangement (c) in UU and UW case is not an equilibrium as markets do not clear (\( R^B > R^4 \) and \( k^B < k^A \)). In the WU case, arrangement (c) is a viable equilibria only if \( \gamma < 1 - \theta \). Finally, arrangement (b) is always an equilibrium as long as one
financial sector is undercapitalized. Equilibrium prices solve, in each country,

\[ 1 - k^l = \frac{\tau \Delta_i f(k^l)}{R^l} \]

□

**Proof of Proposition 3.** From (A2a)-(A2c) and (A3a)-(A3c) the claim trivially follows.

**Closed-Form Solutions.** I get closed-form solutions using a square root production function. From Proposition 2, when both financial sectors are undercapitalized, the economy has a unique perfect home bias equilibrium where both financial sectors invest only domestically.

\[
\begin{align*}
R^l_{UU} &= R^H_{UU} = \frac{1}{2} \left( (1 - \tau) \epsilon_H \left( (1 - \tau) \epsilon_H + 2\tau \Delta_i \right) \right)^{1/2} \quad (A3a) \\
k^l_{UU} &= k^H_{UU} = \frac{(1 - \tau) \epsilon_H}{(1 - \tau) \epsilon_H + 2\tau \Delta_i} \quad (A3b) \\
D^l_{UU} &= D^H_{UU} = \frac{2\tau \Delta_i}{(1 - \tau) \epsilon_H + 2\tau \Delta_i} \quad (A3c)
\end{align*}
\]

Note that \( k_{UU} \in (0,1) \) iff \( \gamma < (1 - \theta) + (1 - \tau + 2\tau \theta) \epsilon_H (2\tau \epsilon_L)^{-1} \). In the case where one financial sector is undercapitalized and one financial sector is well capitalized, the unique equilibrium is:

\[
\begin{align*}
R^l_{UW} &= R^H_{UW} = R^H_{WW} \quad (A4a) \\
k^l_{UW} &= k^H_{UW} = k^H_{WW} \quad (A4b) \\
D^l_{UW} &= D^H_{UW} = D^H_{WW} \quad (A4c)
\end{align*}
\]

□

**Proof of Lemma 2.** Define the banks’ payoff in the good state and in the bad state, with
perfect home bias, as follows:

\[ \Pi^{\text{high}}(k) = (1 - \tau)\epsilon_H f(k) + R(1 - k) \]
\[ = ((1 - \tau)\epsilon_H + \tau\Delta) f(k) \]
\[ \Pi^{\text{low}}(k) = (1 - \tau)\epsilon_L f(k) + R\lambda(1 - k) \]
\[ = (1 - \tau\gamma) f(k) \]

It is easy to show that \( \Pi^{\text{low}}(\tilde{k}) < \Pi^{\text{high}}(\tilde{k}) \), for every \( \tilde{k} \). Define the unconstrained problem

\[ \max_k E(\Pi(k)) - L \]

with solution \( k^* \in (0, 1) \) and the (limited liability) constrained problem:

\[ \max_k \theta \Pi^{\text{high}}(k) - L\theta \]

with solution \( k^{**} \). Finally, let \( \tilde{k} \) be such that \( \Pi^{\text{low}}(\tilde{k}) = L \). If the limited liability constraint does not bind, banks solve the unconstrained problem. If the limited liability constraint binds, banks solve the constrained problem. There are four cases: (i) if \( k^* \geq \tilde{k} \) and \( k^{**} \geq \tilde{k} \), the solution is \( k^* \) as:

\[ E(\Pi(k^*)) - L = \theta \Pi^{\text{high}}(k^*) - L\theta + (1 - \theta)\Pi^{\text{low}}(k^*) - L(1 - \theta) \]
\[ \geq \theta \Pi^{\text{high}}(k^{**}) - L\theta + (1 - \theta)\Pi^{\text{low}}(k^{**}) - L(1 - \theta) \]
\[ \geq \theta \Pi^{\text{high}}(k^{**}) - L\theta \]
(ii) if \(k^* \leq \overline{k}\) and \(k^{**} \leq \overline{k}\), the solution is \(k^{**}\) as:
\[
\theta \Pi^{high}(k^{**}) - L \theta = \theta \Pi^{high}(k^*) - L \theta \\
\geq \theta \Pi^{high}(k^*) - L \theta + (1 - \theta) \Pi^{low}(k^*) - L(1 - \theta) \\
\geq E(\Pi(k^*)) - L
\]

(iii) if \(k^* \geq \overline{k}\) and \(k^{**} \leq \overline{k}\), the solution is \(k^*\) as:
\[
E(\Pi(k^*)) - L \geq \theta \Pi^{high}(k^*) - L \theta + (1 - \theta) \Pi^{low}(k^*) - L(1 - \theta) \\
\geq \theta \Pi^{high}(k^{**}) - L \theta \\
\geq \theta \Pi^{high}(k^{**}) - L \theta
\]

(iv) if \(k^* \leq \overline{k}\) and \(k^{**} \geq \overline{k}\), the solution is \(k^{**}\) as:
\[
\theta \Pi^{high}(k^{**}) - L \theta + (1 - \theta) \Pi^{high}(k^{**}) - L(1 - \theta) \\
\geq \theta \Pi^{high}(k^{**}) - L \theta + (1 - \theta) \Pi^{high}(k^*) - L(1 - \theta) \\
\geq \theta \Pi^{high}(k^{**}) - L \theta \\
\geq \theta \Pi^{high}(k^{**}) - L \theta
\]

as \(k^{**} \geq k^*\). Hence, the solution to the banks’ portfolio problem is \(k^*\) if \(k^* \geq \overline{k}\) and \(k^{**}\) if \(k^* < \overline{k}\).

I can then obtain the threshold debt level \(\overline{L}\) such that \(\overline{L} = (1 - \tau \gamma) (k_{HW})^{1/2}\). If \(L \leq \overline{L}\), the banks are unconstrained and if \(L > \overline{L}\), banks are constrained.

Proof of Proposition 4. Taking first order conditions with respect to \(L\) in (7) and applying Lemma 2, the claim trivially follows. 

\qed
Online Appendix†

Additional Derivations

In this Online Appendix, we present derivations related to the discussion of model extensions presented in Section 5.2.

Relaxing Assumption 1: Foreign Private Lending Let \( \mu \in (0, 1) \) be the fraction of investment in the private lending technology that is allocated domestically. Since the two lending technologies are identical, banks are indifferent whether investing in domestic or foreign private lending and tax collection in state \( s \in S \) is \( \tau f(k)(\epsilon_s(\mu)^{1/2} + \mathbb{E}(\epsilon)(1 - \mu)^{1/2}) \).

Let \( \lambda_{i,s}^{i',s'} \) be the recovery value on government bonds of country \( i \) when it is in state \( s_i \in S \) and country \( j \) is \( s_j \in S \) \( (i \neq j) \). By construction, \( \lambda_{i,s}^{i',s'} \) is such that:

\[
D R \lambda_{i,s}^{i',s'} = \left( \epsilon_s(\mu)^{1/2} + \epsilon_s(1 - \mu)^{1/2} \right) \tau f(k) - g
\]

\[
\lambda_{i,s}^{i',s'} = \frac{\epsilon_s(\mu)^{1/2} + \epsilon_s(1 - \mu)^{1/2} - \gamma \epsilon_L}{\mathbb{E}(\epsilon)(\mu)^{1/2} + (1 - \mu)(1/2)} \quad \forall s_i, s_j \in S
\]

It is then trivial to show that (i) if \( s_i = s_j = H \) there is never default, (ii) if \( s_i = H \) and \( s_j = L \) there is no default if and only if \( \mu \geq \overline{\mu} \), (iii) if \( s_i = L \) and \( s_j = H \) there is no default if and only if \( \mu \leq \underline{\mu} \), (iv) if \( s_i = s_j = L \) there is always default, where \( \underline{\mu} \) is such that \( (1 - \theta)(1 - \mu)^{1/2} = \theta(1 - \mu)^{1/2} \) and \( \overline{\mu} \) is such that \( \theta(1 - \overline{\mu})^{1/2} = (1 - \theta)(1 - \overline{\mu})^{1/2} \). I can then solve

†Date: July 2017. Not for publication. The opinions expressed are those of the author and do not necessarily reflect the views of the Board of Governors of the Federal Reserve System, or anyone associated with this institution. All results have been reviewed to ensure that no confidential information is disclosed. All errors are my own. Email: matteo.crosignani@frb.gov.
for the expected recovery value of domestic and foreign government bonds at $t = 0$.

\[
\begin{align*}
\mathbb{E}(\lambda_{H,s}^i) &= \theta + (1 - \theta)(1 - \mu_i \sigma(1 - \lambda_{H,L}^i)) \\
\mathbb{E}(\lambda_{L,s}^i) &= \theta(1 - 1 - \mu_i \sigma(1 - \lambda_{L,H}^i)) + (1 - \theta) \lambda_{L,H}^i \\
\mathbb{E}(\lambda_{H,L}^i) &= \theta + (1 - \theta)(1 - \mu_i \sigma(1 - \lambda_{H,L}^i)) \\
\mathbb{E}(\lambda_{L,L}^i) &= \theta(1 - 1 - \mu_i \sigma(1 - \lambda_{L,H}^i)) + (1 - \theta) \lambda_{L,L}^i
\end{align*}
\]

I can show that $\mathbb{E}(\lambda_{H,s}^i) > \mathbb{E}(\lambda_{L,s}^i)$ and $\mathbb{E}(\lambda_{H,L}^i) > \mathbb{E}(\lambda_{L,L}^i)$, implying that the payoff in the domestic bad state of the world is still dominated by the payoff in the good state. When well capitalized banks invest domestically if:

$$R \geq R^*$$

Undercapitalized banks invest domestically if and only if

$$R \geq \left( \frac{\mathbb{E}(\lambda_{H,s}^i)}{\mathbb{E}(\lambda_{H,L}^i)} \right) R^*$$

where $\frac{\mathbb{E}(\lambda_{H,s}^i)}{\mathbb{E}(\lambda_{H,L}^i)} < 1$ if and only if $\mu^i \geq \overline{\theta}$ and $\mu^j \geq \overline{\theta}$.

Relaxing Assumption 2: Correlated Shocks Assume that $\text{corr}(\epsilon^A, \epsilon^B) = \rho$. Lemma 1 holds ($\lambda$ is unchanged from the baseline model). Hence, the probabilities in the four states of the world are:

\[
\begin{align*}
\text{Prob}(s^A = H \cap s^B = H) &= \theta(1 - (1 - \rho)(1 - \theta)) \\
\text{Prob}(s^A = H \cap s^B = L) &= \text{Prob}(s^A = L \cap s^B = H) = (1 - \rho)\theta(1 - \theta) \\
\text{Prob}(s^A = L \cap s^B = L) &= (1 - \theta)(\rho + (1 - \rho)(1 - \theta))
\end{align*}
\]
Consider country A (B follows by symmetry). Domestic government bonds pay $R$ in the good state and $\lambda R$ in the bad state. Foreign government bonds pay $(1 - (1 - \rho)(1 - \theta)) + \lambda(1 - \rho)(1 - \theta)$ in the good state and $\theta(1 - \rho) + \lambda(\rho + (1 - \rho)(1 - \theta))$ in the bad state where I used the following conditional probabilities

\[
\begin{align*}
Prob(s^B = L \mid s^A = H) &= (1 - \rho)(1 - \theta) \\
Prob(s^B = H \mid s^A = H) &= 1 - (1 - \rho)(1 - \theta) \\
Prob(s^B = L \mid s^A = L) &= \rho + (1 - \rho)(1 - \theta) \\
Prob(s^B = H \mid s^A = L) &= \theta(1 - \rho)
\end{align*}
\]

W banks invest domestically if and only if $R \geq R^*$. U banks invest domestically if and only if:

\[
R \geq R^*(1 - (1 - \lambda)(1 - \rho)(1 - \theta))
\]

Relaxing Assumption 3: One Safe Country, One Risky Country Assume now that $\theta^A < 1$ and $\theta^B = 1$. Moreover, assume that $\Delta_i^t = \theta^A + (1 - \theta^A - \gamma)$ for $i \in I$. In other words, an investment of $y$ in country A lending technology yields, in expectation, the same in country B’s lending technology. The two government debt capacities are:

\[
D_i^t R_i^t = \tau f(k^i) \Delta_i
\]

Lemma 1 holds for country A as in the bad state it partially defaults (haircut $\lambda^A$). Country B is riskless and never defaults: its banks are always well capitalized as there is no uncertainty. The banking sector optimal investment $k^B$ is given by:

\[
k^B = \left( \frac{(1 - \tau)E^A(\epsilon)}{2(\beta R^A + (1 - \beta)R^A E^A(\lambda^A))} \right)^2
\]
In the WW case, countries invest in A’s debt if and only if $R_A^A(\lambda^A) \geq R_B^B$. The only possible arrangement is therefore (a) where both countries face a strictly positive demand for their own debt. Hence, $R_A^A(\lambda^A) = R_B^B$ and $k^A = k^B = k$. From the two market clearing conditions, in equilibrium, it must be that $\alpha^A < \alpha^B$, $D^B > D^A$, and $R^B < R^A$.

In the UW case, banks in A invest domestically if and only if $R^A \geq R^B$, while banks in B invest domestically if and only if $R^B \geq E_A(\lambda^A)R^A$. I can discard arrangements (h) and (i) because they would, as proved before, give a contradiction. Similarly arrangements (a), (e), (f), and (g) are not candidate equilibria as there are no prices such that banks optimize in equilibrium. In arrangement (c), $R^A = R^B$ and $k^A > k^B$, and markets do not clear. If $1 - \theta > \gamma$, arrangement (d) is a candidate equilibrium with $k^A \geq k^B$ and $R^A \geq R^B$. Finally, the financial autarky equilibrium (b) is characterized as usual.

Relaxing Assumption 4: Lending Technology with General Functional Form No results in the paper rely on the square-root functional form, which is assumed without loss of generality.

Analysis of the Low Sovereign Risk Case ($\gamma \leq 1 - \theta$) Suppose the economy is in the UW state. The only candidate arrangements are type (b) and (c). Type (b) is an autarkic

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1The arrangements where one country faces zero demand for its debt ((h) and (i)) are not candidate equilibria as the interest rate on such country debt would go to infinite in equilibrium, attracting domestic and foreign banks.
equilibrium with the following quantities and prices.

\[ R_{UW,aut}^A = R_{UU} \]
\[ R_{UW,aut}^B = R_{WW} \]
\[ k_{UW,aut}^A = k_{UU} \]
\[ k_{UW,aut}^B = k_{WW} \]
\[ D_{UW,aut}^A = D_{UU} \]
\[ D_{UW,aut}^B = D_{WW} \]

where \textit{aut} indicates “autarky.” In the asymmetric equilibrium or type (c) arrangement, I have

\[ R_{UW,asy} = R_{UW,asy} = \left( (R_{WW})^2 + (R_{UU})^2 \right)^{1/2} \]
\[ k_{UW,asy}^A = \left( \frac{1 - \tau}{2R_{UW,asy}} \right)^2 \]
\[ k_{UW,asy}^B = \left( \frac{1 - \tau}{2E(\lambda)R_{UW,asy}} \right)^2 \]
\[ \alpha_{UW,asy}^B = \frac{2\tau \Delta_a (1 - \tau) E(\epsilon) - \Delta_a (1 - \tau) E(\epsilon) + 2\tau \Delta_a (1 - \tau) E(\epsilon)}{\epsilon H (1 - \tau) E(\epsilon)} \]
\[ D_{UW,asy}^A = (1 - k_{UW,asy}^A) (1 - \alpha_{UW,asy}^B) \]
\[ D_{UW,asy}^B = \alpha_{UW,asy}^B (1 - k_{UW,asy}^B) \]

where \textit{asy} indicates that the equilibrium is asymmetric (the well capitalized financial sector invests domestically and non-domestically).
Additional Figures

Figure OA.1: Fact 1 and Fact 2 (Greece and Ireland). The top figure shows the share of sovereign debt owned by domestic banks (solid orange line, primary axis, (%)) and the 5-year USD-denominated sovereign CDS spread (dotted blue line, secondary axis, (%) for Greece and Ireland. The bottom figures show holdings of domestic government bonds by domestic banks (dotted blue line, primary axis, tn EUR) and domestic banks’ credit to the non-financial private sector (solid orange line, secondary axis, tn EUR) for Greece and Ireland. Source: CDS spreads from Bloomberg, credit to non-financial entities from the Bank for International Settlements, government debt ownership data from the IMF.
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