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Voluntary Disclosure in Unfair Contests

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Voluntary Disclosure in Unfair Contests*

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Abstract This paper studies incentives for the interim voluntary disclosure of verifiable information in probabilistic all-pay contests. Considered are *unfair* contests, i.e., contests in which, subject to activity conditions, one player (the favorite) is interim always more likely to win than the other player (the underdog). A condition is identified that ensures that a given contest is unfair regardless of disclosure decisions. Under this condition, full revelation is the unique perfect Bayesian equilibrium outcome of the contest with pre-play communication. This is so because the weakest type of the underdog will try to moderate the favorite, while the strongest type of the favorite will try to discourage the underdog—so that the contest unravels. We also show that self-disclosure may, with positive probability, provoke unintended reactions, i.e., “dominant” or “defiant” behavior. Moreover, while individually rational for the marginal type, the unraveling may be strictly Pareto inferior from an ex-ante perspective. Our main conclusion is just the opposite of the corresponding finding for the deterministic all-pay auction. The proofs employ lattice-theoretic methods and an improved version of Jensen’s inequality.

Keywords Unfair contests · Incomplete information · Self-disclosure · Unraveling · Strategic complements and substitutes · Dominance and defiance

JEL Classification C72 Non-cooperative Games · D74 Conflict, Conflict Resolution, Alliances, Revolutions · D82 Asymmetric & Private Information · J71 Discrimination

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1. Introduction

The economics literature has a long tradition of studying incentives for the voluntary disclosure of private information. Seminal contributions by Grossman (1981) and Milgrom (1981) considered markets with quality uncertainty in which buyers derive their beliefs from verifiable information. In this type of environment, a product that is not certified will be perceived to be of average quality only. Therefore, in the absence of countervailing forces or alternative channels of communication, a seller would find it optimal to disclose any verifiable information that provides evidence of above-average quality. But since this is anticipated by the buyers, the perceived quality of uncertified products will decline further. As a result, there is an unraveling process that may ultimately force all sellers to disclose their private information. Since its inception, this *disclosure principle* has been gradually refined and extended through a large number of contributions.¹

However, all-pay contests do not satisfy existing conditions sufficient for the disclosure principle in the *strong form*, according to which any perfect Bayesian (or sequential) equilibrium entails full revelation.² The purpose of the present paper is it to extend the strong-form disclosure principle to a large class of probabilistic contests. Clearly, there are forces that favor disclosure in contests. After all, if a player is relatively strong, then there should be, at least in some cases, a strict benefit from letting the weaker opponent know about this. Indeed, with the information revealed, the contest should become more lopsided, which would make it easier for the stronger player to win. Conversely, a relatively weak player might want to inform the oppo-

¹See, for instance, Verrecchia (1983), Dye (1985), Okuno-Fujiwara et al. (1990), Seidman and Winter (1997), Benoît and Dubra (2006), Giovannoni and Seidman (2007), Van Zandt and Vives (2007), and Hagenbach et al. (2014). Surveys of the literature can be found in Milgrom (2008) and Dranove and Jin (2010).

²The main complication is that best-response mappings are not monotone in a contest. Cf. Denter et al. (2014) and Kovenock et al. (2015). For an introduction to the burgeoning literature on all-pay contests, see Konrad (2009).

ment that the battle will be excessively unequal, thereby inducing the stronger player to choose a more moderate strategy.³ While these effects (especially the first) might look familiar, their significance for the scope of the disclosure principle has apparently been overlooked so far.

In this paper, we extend the standard model of a probabilistic contest (Rosen, 1986; Dixit, 1987) by allowing for pre-play communication of verifiable information (Okuno-Fujiwara et al., 1990; Van Zandt and Vives, 2007; Hagenbach et al., 2014). The contestants are assumed to differ in their marginal cost of effort, which is private information to them. However, at a stage preceding the contest, any player may interim, i.e., subsequent to having observed her type, choose to disclose that information to her opponent. The focus of the present paper lies on contests that are *unfair* in the sense that there is one player that, subject to activity, is interim always strictly more likely to win than the other player.⁴ We identify a condition that guarantees that the contest stage ensuing the pre-play exchange of information is unfair regardless of players' disclosure decisions. As will be discussed, this condition is consistent with both asymmetric technologies (e.g., O'Keeffe et al., 1984; Meyer, 1992; Feess et al., 2008; Epstein et al., 2013; Franke et al., 2014) and heterogeneous type distributions (e.g., Amann and Leininger, 1996; Maskin and Riley, 2000).

In this type of framework, we evaluate the incentives of individual types of each player to voluntarily disclose their private information. Moreover, we characterize the unique perfect Bayesian equilibrium outcome of the resulting two-stage game. Our

³Real-world examples of such disclosure include, for instance, weapon tests (Beardsley and Asal, 2009), witness intimidation (Maynard, 1994), bragging (Alfano and Robinson, 2014), and acts of supplication (Pedrick, 1982; van Kleef et al., 2006). It is conceded that in some of these examples, misrepresentation may be possible to some degree, and that verifiability may be costly to some extent. However, since the incentives that we identify are all strict, this does not generally invalidate our analysis.

⁴The focus on unfair contests is motivated by the fact, established below, that the strong-form disclosure principle may fail in ex-ante symmetric contests.

main result says that, provided that the contest stage is unfair in the sense discussed above, the *only* outcome of the revelation game consistent with the assumption of perfect Bayesian rationality is the one in which all the privately held information is unfolded prior to the contest. Thus, we find general conditions under which the strong-form disclosure principle applies to a standard contest setting.

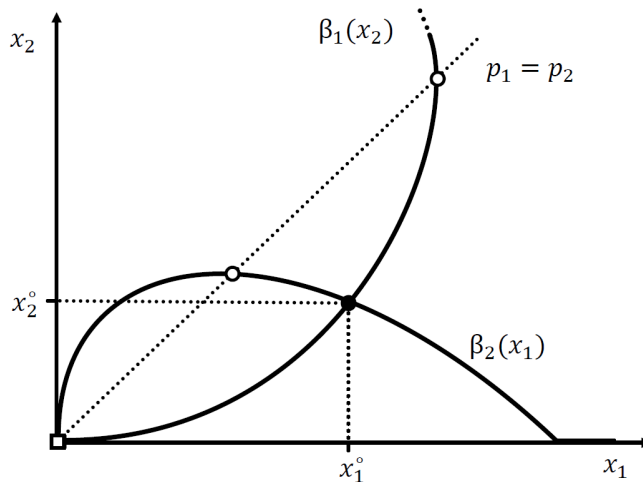


Figure 1. Best-response curves in an asymmetric contest of complete information.

The analysis revisits Dixit's (1987, p. 893) observation that, in an asymmetric contest of complete information, the player more likely to win, the so-called favorite, has a best-response function that is locally strictly increasing at the equilibrium, whereas the player less likely to win, the so-called underdog, has a best-response function that is locally strictly declining at the equilibrium. For example, in Figure 1, at the equilibrium (x_1^o, x_2^o) , player 1's best-response function $\beta_1 \equiv \beta_1(x_2)$ is strictly increasing, while player 2's best-response function $\beta_2 \equiv \beta_2(x_1)$ is strictly declining. We extend Dixit's (1987) observation to a setting with incomplete information. Thus, also in our setting, there will be a favorite whose best-response mapping is locally strictly increasing at the equilibrium, and an underdog whose best-response mapping is locally strictly declining at the equilibrium. There is, however, an important difference.

Specifically, with incomplete information, strategy spaces are multi-dimensional lattices, reflecting the fact that each type of a given player may choose a different effort level.

While the proof of the unraveling result is not entirely straightforward, there is a simple story. Specifically, in view of the high effort level to be expected from an uninformed favorite, the weakest type of the underdog will have a strict incentive to self-disclose, so as to moderate the favorite. But given that this is anticipated, any silent types of the underdog will be confronted with an even higher effort of the favorite. The weakest of those remaining types will therefore choose to disclose her type, and so on. Thus, there is an unraveling of the underdog’s side. However, in the resulting contest with one-sided incomplete information, the unraveling continues on the side of the favorite. Indeed, the respective strongest type of the favorite has a strict incentive to self-disclose, so as to discourage the underdog. In the end, there is necessarily full revelation of all private information.

A central part of our analysis examines the monotone comparative statics of the contest stage with respect to changes in the information structure. For this, we draw on intuitions suggested by recent work on parameterized games of strategic heterogeneity (Monaco and Sabarwal, 2016). Quite notably, however, existing conditions do not apply to our model.⁵ To clarify this point, we construct two examples in which a player’s self-disclosure may trigger, respectively, a “dominant” or “defiant” reaction that runs squarely against the player’s intention to moderate or discourage the opponent.⁶ These examples show that the relevant comparative statics of the Bayesian equilibrium is, in general, monotone *for one player only*. In contrast, Monaco and

⁵Similarly, standard methods such as total differentiation or variational inequalities fail to yield any useful results.

⁶A conceptual discussion of dominance and defiance, with numerous examples from politics and history, can be found in Caygill (2013).

Sabarwal’s (2016, Th. 5) conditions, like any of the conditions in the literature that we are aware of, imply the monotone comparative statics of the entire equilibrium profile. Thus, we indeed cannot make use of existing methods. Instead, we develop a novel argument that will be outlined in the body of the paper.⁷

Finally, we compare full revelation with a benchmark outcome in which players do not have the option to disclose their private information. This benchmark outcome will be referred to as *mandatory concealment*. For a somewhat more structured environment with one-sided incomplete information and an unbiased lottery technology, we show that the unraveling is ex-ante strictly undesirable for a privately informed underdog. In other words, full revelation obtains in this case just because the underdog has ex ante no means of committing herself to not revealing her type. The proof of this result relies on an improved version of Jensen’s inequality, which in turn is derived using the theory of moment spaces (Dresher, 1953). We then go on and show that, depending on parameters, the unraveling may even lead to a strictly Pareto inferior outcome. We call this outcome the “disclosure trap.” However, such possibilities are clearly not universal, i.e., there are examples in which a privately informed contestant will appreciate disclosure not only interim, i.e., when being of the marginal type, but also from an ex-ante perspective.

While noise and private information are essential in applications, probabilistic contests of incomplete information have been studied since the early 90’s only.⁸ The general framework with one-sided and two-sided private valuations was introduced by Hurley and Shogren (1998a, 1998b). Wärneryd (2003) made the intriguing observa-

⁷Even though the comparative statics is a central element of our analysis, additional arguments are needed to put a definite sign on the type-specific incentives for disclosure. These additional arguments will likewise be outlined in the body of the paper.

⁸Indeed, Rosen (1986, fn. 7) still complained that “few analytical results” were available. Early papers considering probabilistic contests with incomplete information include Linster (1993) and Baik and Shogren (1995), among others.

tion that, in a set-up with one-sided incomplete information about a common valuation, the uninformed player is more likely to win than the informed player. Malueg and Yates (2004) analyzed a symmetric two-player Tullock contest with two equally likely, but possibly correlated types. Schoonbeek and Winkel (2006) pointed out that, in a contest of one-sided incomplete information, individual types may remain inactive. For a large class of probabilistic incomplete-information contests, including those considered in the present paper, Einy et al. (2015) established existence of a Bayesian equilibrium, while Ewerhart and Quartieri (under review) proved existence of a unique Bayesian equilibrium.

The present paper falls into the recent and quickly expanding literature concerned with the disclosure of *verifiable* information in contests.⁹ Research in this literature has tended to focus on either ex-ante voluntary disclosure, optimal disclosure policies, or interim voluntary disclosure.¹⁰ Ex-ante voluntary disclosure in probabilistic contests has been studied by Denter et al. (2014), in particular. Assuming a probabilistic contest technology with one-sided incomplete information, they showed that a “laissez-faire” policy regarding the informed player’s ex-ante disclosure decision leads to lower expected lobbying expenditures than a policy of mandatory disclosure.¹¹ The second topic, optimal disclosure policies in contests, has recently seen a strong development. In particular, effort-maximizing disclosure policies have been characterized

⁹Another form of pre-play communication, not considered in the present paper, is the signaling of unverifiable information. See, e.g., Katsenos (2010), Slantchev (2010), Fu et al. (2013), Heijnen and Schoonbeek (2017), and Yildirim (2017).

¹⁰Numerous additional research questions, related to learning, feedback, and motivation, for example, arise in the analysis of dynamic contests of incomplete information. Such research questions have been dealt with in papers by Clark (1997), Yildirim (2005), Krämer (2007), Münster (2009), Zhang and Wang (2009), Aoyagi (2010), Ederer (2010), and Goltsman and Mukherjee (2011), for instance.

¹¹Relatedly, Wu and Zheng (2017) considered a symmetric two-player lottery contest with two equally likely, independently drawn types for each player. In this framework, they showed that ex-ante disclosure decisions are fully revealing if and only if the two possible type realizations are sufficiently close to each other.

by Zhang and Zhou (2016) and Serena (2017) for probabilistic technologies, and by Fu et al. (2014) and Chen et al. (2017) for deterministic technologies.¹² The present analysis is concerned, however, with the third topic, i.e., the interim voluntary disclosure in contests. As far as we know, there is only one paper that has dealt with this issue on a comparable level of generality.¹³ Specifically, Kovenock et al. (2015) showed that, regardless of whether valuations are private or common, the interim information sharing game followed by an all-pay auction admits a perfect Bayesian equilibrium in which no player ever shares her private information. The present analysis is complementary to that of Kovenock et al. (2015) in the sense that, instead of the all-pay auction, we consider a probabilistic contest. Overall, the review of the literature suggests that the specific research question pursued in the present paper, viz. the analysis of incentives for the interim voluntary disclosure of hard information in ex-ante asymmetric contests with probabilistic technologies and two-sided incomplete information, has not been addressed in prior work.

The remainder of this paper is structured as follows. Section 2 introduces the setup. The main result is stated in Section 3. Section 4 discusses contestants' incentives for interim voluntary disclosure. In Section 5, we provide examples for nonmonotone reactions to self-disclosure. A commitment problem is discussed in Section 6. Section 7 concludes. Appendix A contains auxiliary results, while all proofs of the results of this paper have been relegated to Appendix B.

¹²In a similar vein, Einy et al. (2017) studied the value of public information in Tullock contests with nonlinear costs. Optimal disclosure policies have been extensively analyzed also in models of population uncertainty. See Münster (2006), Myerson and Wärneryd (2006), Lim and Matros (2009), Fu et al. (2011), Feng and Lu (2016), and Fu et al. (2016), among others.

¹³However, Epstein and Mealem (2013) considered a lottery contest with one-sided incomplete information, and characterized the perfect Bayesian equilibrium outcome in the case of two possible type realizations. In fact, Epstein and Mealem (2013) considered also an extension with more than two types, yet they did not characterize the perfect Bayesian equilibrium outcome in that case.

2. Set-up

Following the general framework of Okuno-Fujiwara et al. (1990),¹⁴ the dynamic game to be considered is composed of two consecutive stages, referred to in the sequel as revelation stage and contest stage. These stages will be separately described below, following the backward order of the subsequent analysis.

2.1 The contest stage

Two players (or teams), referred to as contestants $i = 1, 2$, exert costly efforts so as to increase their respective probability of winning a given prize that is commonly valued at $V > 0$. Contestant i 's effort (or bid) is denoted by $x_i \geq 0$. It is assumed that player i 's payoff may be written as

$$\Pi_i(x_1, x_2; c_i) = p_i(x_1, x_2)V - c_i x_i, \quad (1)$$

where $p_i(x_1, x_2)$ denotes i 's probability of winning, and $c_i > 0$ contestant i 's marginal cost of effort. Without loss of generality, the value of the prize will be normalized to $V = 1$. Following Rosen (1986), we will assume that

$$p_i(x_1, x_2) = \begin{cases} \frac{\gamma_i h(x_i)}{\gamma_1 h(x_1) + \gamma_2 h(x_2)} & \text{if } x_1 + x_2 > 0 \\ \gamma_i / (\gamma_1 + \gamma_2) & \text{if } x_1 + x_2 = 0, \end{cases} \quad (2)$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$ are parameters, while $h : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a continuous function that is twice continuously differentiable at positive bid levels, with $h(0) = 0$, $h' > 0$, and $h'' \leq 0$.¹⁵ It will also be assumed that the *curvature* of the production function

¹⁴See also the extensions by Van Zandt and Vives (2007) and Hagenbach et al. (2014).

¹⁵Our assumption of a concave production function is motivated by the fact that the existence and uniqueness of Bayesian equilibrium in probabilistic contests with incomplete information has been studied, up to this point, predominantly for this case, so that relaxing that assumption would take us away from the main focus of this paper. In contrast, the extension to player-specific production functions is easily feasible, yet does not yield additional insights.

h , i.e., $\underline{\rho} \equiv \underline{\rho}(h) = \inf\{\rho \geq 1 \mid h^\rho \text{ convex}\}$ is well-defined.¹⁶

This set-up includes, as an important special case, the example of the biased *Tullock contest* (Tullock, 1975; Leininger, 1993; Clark and Riis, 1998), where the production function is given by $h(z) = h^{\text{TUL}}(z; r) \equiv z^r$ for an arbitrary parameter $r \in (0, 1]$. In the Tullock case, $\underline{\rho}(h^{\text{TUL}}) = 1/r$. The *lottery contest* corresponds to the case $r = 1$, and hence $\underline{\rho} = 1$.

Each player $i \in \{1, 2\}$ is privately informed about her marginal cost c_i . It is commonly known, however, that player i 's type is drawn ex-ante, independently across players, from a given probability distribution over the finite set $C_i = \{c_i^1, \dots, c_i^{K_i}\}$, where $K_i \geq 1$. The ex-ante probability of type c_i^k is denoted by $q_i^k \equiv q_i(c_i^k)$, for $k = 1, \dots, K_i$, where probabilities sum up to one, i.e., $q_i^1 + \dots + q_i^{K_i} = 1$. Without loss of generality, we assume that all possible type realizations have a positive probability, i.e., $q_i^k > 0$ for any $i \in \{1, 2\}$ and any $k \in \{1, \dots, K_i\}$. Moreover, types will be ordered such that

$$\underline{c}_i \equiv c_i^1 < \dots < c_i^{K_i} \equiv \bar{c}_i \quad (i \in \{1, 2\}). \quad (3)$$

Thus, \underline{c}_i denotes the most efficient, or strongest type of player i , whereas \bar{c}_i denotes the least efficient, or weakest type of player i .¹⁷

A *bid schedule* for player $i \in \{1, 2\}$ is an arbitrary function $\xi_i : C_i \rightarrow \mathbb{R}_+$. Denote by X_i the set of i 's bid schedules. A pair of bid schedules $\xi^* = (\xi_1^*, \xi_2^*) \in X_1 \times X_2$ is a *Bayesian equilibrium* if, for any type $c_i \in C_i$ of any player $i \in \{1, 2\}$, the effort

¹⁶The curvature $\underline{\rho}(h)$ corresponds to the smallest ρ for which the production function h is ρ -convex (cf., e.g., Anderson and Renault, 2003). For general background on generalized concavity, see Caplin and Nalebuff (1991a, 1991b).

¹⁷Our set-up is isomorphic to a model in which costs are commonly known but valuations are private information for the contestants (e.g., Hurley and Shogren, 1998b). This can be seen by normalizing payoff functions in the agent-normal form of the Bayesian contest game. Further, we conjecture that our results extend to the case of continuous type spaces (Fey, 2008; Ryvkin, 2010; Wasser, 2013a, 2013b; Ewerhart, 2014), yet we also suspect that the technical complications necessary would not be rewarded by additional insights.

level $x_i = \xi_i^*(c_i)$ maximizes type c_i 's expected payoff $E_{c_j}[\Pi_i(x_i, \xi_j^*(c_j); c_i)]$, where $E_{c_j}[\cdot]$ denotes the expectation over the realizations of $c_j \in C_j$, with $j \neq i$. Following Schoonbeek and Winkel (2006), a type $c_i \in C_i$ that chooses an equilibrium effort $\xi_i^*(c_i) > 0$ ($\xi_i^*(c_i) = 0$) will be called *active* (*inactive*). The discontinuity of the payoff functions at the origin implies that, at any Bayesian equilibrium, both players are necessarily active with positive probability.¹⁸ By the same token, at least one player will have to be active with probability one. We will make use of the following existing result.

Lemma 1. *The contest stage has a unique Bayesian Nash equilibrium.*¹⁹

Proof. See Appendix B. \square

In the special cases of complete and one-sided incomplete information, the following notation will be used. When it is commonly known that, for $i \in \{1, 2\}$, player i 's type is $c_i = c_i^\circ$ for some $c_i^\circ \in C_i$, then i 's equilibrium strategy will be written as $x_i^\circ = x_i^\circ(c_1^\circ, c_2^\circ)$. Further, when it is commonly known that player i 's type is $c_i = c_i^\#$ for some $c_i^\# \in C_i$, while player j 's type, with $j \neq i$, is uncertain, then equilibrium strategies will be written as $x_i^\# = x_i^\#(c_i^\#)$ for player i and as $\xi_j^\# = \xi_j^\#(\cdot; c_i^\#)$ for player j , so that $\xi_j^\#(c_j; c_i^\#)$ is type c_j 's equilibrium effort.

2.2 The revelation stage

At a stage preceding the contest, players are given the opportunity to simultaneously and independently disclose their marginal costs of effort to their respective

¹⁸To see this, suppose that one player is always inactive. Then, any sufficiently small positive bid is a better response than the zero bid, but any positive bid is suboptimal. Hence, there is no best response if one player is always inactive.

¹⁹Lemma 1 extends to mixed strategies. Indeed, since each player is active with positive probability, and payoffs functions are own-bid l.s.c. at the origin, expected payoffs against the opponent's equilibrium strategy are strictly concave over \mathbb{R}_+ , so that it is suboptimal to randomize strictly.

opponent.²⁰ Following the Bayesian persuasion approach, it is assumed that private information cannot be misrepresented (in the sense of providing evidence for being of another type). Further, the decision to self-disclose does not lead to any direct costs.²¹ Let $S_i \subseteq C_i$ denote the set of types of player $i \in \{1, 2\}$ that choose to disclose their private information.

A *belief* about player i is a mapping $\mu_i : C_i \rightarrow [0, 1]$ such that $\sum_{k=1}^{K_i} \mu_i(c_i^k) = 1$. We denote by $\Delta(C_i)$ the set of all beliefs about i . Beliefs are updated in response to the observation of verifiable information. Consequently, there are three basic scenarios for each contestant $i \in \{1, 2\}$.

(i) Suppose first that player i discloses $c_i \in C_i$. Then, player i is believed to be of type c_i with probability one, i.e., $\mu_i(c_i) = 1$.

(ii) Next, suppose that player i does not disclose her type, and that player i 's decision to not disclose is a possibility on the equilibrium path, i.e., $C_i \setminus S_i \neq \emptyset$. Then, c_i is expected to be in the set-theoretic complement of S_i . Hence, by Bayes' rule,

$$\mu_i(c_i) = \begin{cases} q_i(c_i) / \sum_{c'_i \in C_i \setminus S_i} q_i(c'_i) & \text{if } c_i \in C_i \setminus S_i \\ 0 & \text{if } c_i \in S_i. \end{cases} \quad (4)$$

(iii) Finally, suppose that player i does not disclose her type, and that i 's decision to not disclose is an off-equilibrium event, i.e., $C_i \setminus S_i = \emptyset$. Then, the belief about player i may be specified as an arbitrary probability distribution $\mu_i = \mu_i^0$ over C_i .²²

²⁰Thus, the revelation stage offers a binary decision for each type. However, our main result continues to hold provided that contestants' message correspondences each contain an *evidence base* (Hagenbach et al., 2014). For example, the disclosure decision may alternatively establish an upper bound for the favorite's cost parameter and a lower bound for the underdog's cost parameter, respectively.

²¹Introducing costs for disclosing information would not change our conclusions, provided those are not too large compared to the benefits of self-disclosure identified below.

²²Off-equilibrium beliefs in the fully revealing perfect Bayesian equilibrium will be specified below by giving full weight to the respective *worst-case type* (Seidman and Winter, 1997; Hagenbach et

Since the Bayesian equilibrium at the contest stage is unique, the expected continuation payoff for any type $c_i \in C_i$ with $i \in \{1, 2\}$ from the contest stage is well-defined for any given pair of beliefs $(\mu_1, \mu_2) \in \Delta(C_1) \times \Delta(C_2)$ such that $\mu_i(c_i) > 0$. A *perfect Bayesian equilibrium*, in reduced form, of the contest with pre-play communication is therefore composed of sets $S_1 \subseteq C_1$ and $S_2 \subseteq C_2$ of revealing types, and off-equilibrium beliefs $\mu_i^0 \in \Delta(C_i)$ for any $i \in \{1, 2\}$ with $S_i = C_i$, such that (i) for each type $c_i \in S_i$, the expected continuation payoff from self-disclosure weakly exceeds any expected payoff that c_i could realize from not disclosing and subsequently choosing an arbitrary bid $x_i \geq 0$, and such that (ii) for each type $c_i \in C_i \setminus S_i$, the expected continuation payoff from non-disclosure weakly exceeds the expected continuation payoff from self-disclosure.²³

3. The unraveling theorem

3.1 Unfair contests

A probabilistic contest of either complete or incomplete information will be called *unfair* when there is one player that is active with probability one and that, provided that the other player is also active with probability one, is interim always more likely to win than the other player.

To formulate conditions on the primitives of the model that ensure that a contest is unfair *regardless of disclosure decisions*, we introduce the following parameters. To start with, let the ratio $\sigma = \underline{c}_2/\bar{c}_1$ denote player 1's *lowest relative resolve*, where the

al., 2014). In the present setting, the worst-case type of the favorite is the least efficient type, \bar{c}_1 , whereas the worst-case type of the underdog is the most efficient type, \underline{c}_2 . See also the discussion following Theorem 1.

²³Type-dependent signal spaces and continuous strategy sets preclude a direct reference to the standard definition of a perfect Bayesian equilibrium in a multi-stage game with observable actions (Fudenberg and Tirole, 1991, p. 331). Otherwise, however, the definition is standard. Note also that we restrict attention to pure strategies at the revelation stage. This is for expositional reasons only. Our main result (Theorem 1 below) holds likewise when players may use randomized strategies at the revelation stage.

terminology is adapted from Hurley and Shogren (1998a, 1998b). E.g., if player 1 is interim always more efficient than player 2, then σ strictly exceeds one and corresponds to the worst-case cost advantage of player 1 compared to player 2. Second, we will denote by $\pi_i = \sqrt{\underline{c}_i/\bar{c}_i}$ the *predictability* of player i 's marginal cost, where $i \in \{1, 2\}$. Thus, a predictability equal to one (strictly lower than one) for a player corresponds to complete information (incomplete information) about her type. Finally, the parameter $\gamma = \gamma_2/\gamma_1$ will be referred to as the *net bias* of the contest technology. For example, a net bias equal to one (strictly below one, strictly above one) corresponds to a contest technology that is unbiased (biased against player 2, biased against player 1).

The following assumption will be imposed throughout the analysis.

Assumption 1. $\gamma < \gamma^*(\pi_1, \pi_2, \sigma, \underline{\rho}) \equiv \frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1} \cdot \widehat{\sigma}(\sigma, \underline{\rho})$, where

$$\widehat{\sigma}(\sigma, \underline{\rho}) = \begin{cases} \sigma & \text{if } \sigma \leq 1 \\ \sigma^{1/\underline{\rho}} & \text{if } \sigma > 1. \end{cases} \quad (5)$$

Intuitively, with Assumption 1 in place, player 1 is in a quite strong position relative to player 2. As will be shown below, this implies that the contest, even though of incomplete information, is structurally similar to the complete-information contest considered by Dixit (1987). We also remark that the specific form of the inequality has been derived from the proof of Lemma 2 below and thus constitutes a sufficient but not necessary condition for the contest to be unfair.²⁴

The comparative statics of γ^* is straightforward. Indeed, it can be readily verified that, when positive, $\gamma^*(\pi_1, \pi_2, \sigma, \underline{\rho})$ is strictly increasing in each of the three parameters π_1 , π_2 , and σ , as well as monotone declining in $\underline{\rho}$. Thus, the assumption

²⁴For a discussion of what happens when Assumption 1 is dropped, see the end of this section.

of unfairness is more likely to hold when the net bias discriminates more strongly against player 2, when marginal costs are more predictable, when player 1's lowest relative resolve is larger, or when the production function has a lower curvature. In particular, the comparative statics with respect to π_1 , π_2 , and σ should convince the reader that, if Assumption 1 holds for a given contest, changes to the information structure caused by pre-play disclosure decisions can never invalidate Assumption 1.²⁵

One can also check that the case of a biased contest with ex-ante symmetric type distributions (i.e., $\underline{c}_1 = \underline{c}_2 \leq \bar{c}_1 = \bar{c}_2$), as discussed, e.g., by Drugov and Ryvkin (2017), is consistent with Assumption 1.²⁶ In particular, in the limit case of complete information and symmetric costs (i.e., $\underline{c}_1 = \bar{c}_1 = \underline{c}_2 = \bar{c}_2$), Assumption 1 just says that the contest technology is biased against player 2 (i.e., $\gamma_2 < \gamma_1$).

Returning to the general case, we make the following observation that is crucial for our analysis.

Lemma 2. (Underdog and favorite)

Suppose that Assumption 1 holds. Then, (i) all types of player 1 are active; and (ii), provided that all types of player 2 are active as well, $p_1(\xi_1^(c_1), \xi_2^*(c_2)) > \frac{1}{2} > p_2(\xi_1^*(c_1), \xi_2^*(c_2))$ for any $c_1 \in C_1$ and $c_2 \in C_2$.*

Proof. See Appendix B. \square

Thus, Assumption 1 implies that player 1 is active with probability one. Moreover, provided that player 2 is likewise active with probability one, player 1 is interim, i.e., for any realization of types c_1 and c_2 , more likely to win than player 2.²⁷ In

²⁵This point will be discussed more formally in the proof of Lemma 2 below.

²⁶Indeed, in this case, $\gamma^* = \frac{(3\pi-2)\pi^2}{2-\pi}$, with $\pi \equiv \pi_1 = \pi_2 = \sqrt{\sigma}$. For example, for $\pi = 0.8$, we get $\gamma^* = 0.21$.

²⁷The activity of all types of player 2 is actually needed. Intuitively, an inactive type of player 2 may lower the marginal incentives of player 1, and may thereby invalidate the conclusion of Lemma 2(ii).

other words, Assumption 1 ensures that the contest stage is unfair regardless of the disclosure decisions taken by the contestants at the revelation stage. Lemma 2 is proven by the combination of several inequalities, all of which are derived from the first-order conditions necessary for players' bid schedules to be in equilibrium. Given this result, we henceforth will refer to player 1 as the *favorite* and to player 2 as the *underdog*.

3.2 Main result

We will use the term *full revelation* to characterize the perfect Bayesian equilibrium, or the perfect Bayesian equilibrium outcome, in which all types disclose their private information. The main result of the present paper is the following.

Theorem 1. (Disclosure principle)

Suppose that Assumption 1 holds. Then, full revelation is the unique perfect Bayesian equilibrium outcome in the incomplete-information contest with pre-play communication.

Proof. See Appendix B. \square

Theorem 1 states that the strong-form disclosure principle applies to any contest that satisfies Assumption 1. Note that no activity conditions have been imposed. This is noteworthy because, in general, corner solutions are known to be consistent with the existence of a perfect Bayesian equilibrium with no revelation of private information (Okuno-Fujiwara et al., 1990, Ex. 4). In our framework, however, this problem cannot occur.

An overview discussion of the proof of Theorem 1 will be provided in the next section. In fact, the proof may be of some interest because it examines the before-mentioned incentives for interim voluntary disclosure, identifying also quite intuitive

effects such as a beneficial self-disclosure for the weakest type of the underdog, as well as strategies of intimidation (or discouragement) for the strongest type of the favorite. We also check that full revelation is indeed a perfect Bayesian equilibrium.²⁸

It is instructive to compare Theorem 1 with the case of auctions with interdependent valuations, where the incentives to reveal a private signal are typically strongest at the bottom of the signal support (Benoît and Dubra, 2006). In both cases, voluntary disclosure aims at reducing the opponent’s incentives for bidding too aggressively. However, unlike the auction setting, the contest may induce also the most optimistic types to reveal their private information. Indeed, as Proposition 2 below will show, this is definitely so for the type at the top of the signal support, corresponding to the strongest type of the favorite.

Hagenbach et al. (2014) identified necessary and sufficient conditions for the existence of a fully revealing sequential equilibrium with “extremal” off-equilibrium beliefs that implements a given Nash equilibrium action profile on and off the equilibrium path. They assumed that a player that surprises her opponent by not revealing her type is deemed to be the *worst-case type*, i.e., the type that no other type would like to masquerade as. In our setting, the worst-case type is either the most efficient type of the underdog, or the least efficient type of the favorite. Thus, the off-equilibrium beliefs they constructed correspond precisely to those that will be used below to establish the equilibrium property.²⁹ Theorem 1 complements the analysis of Hagenbach et al. (2014) by providing conditions sufficient for the uniqueness of the perfect Bayesian equilibrium outcome in the special case of probabilistic all-pay contests.

²⁸In principle, to reveal all private information, it suffices that, for each player, all but one type disclose their private information. The residual type is then indifferent between concealing or revealing her information. As a result, the uniqueness claim to be made concerns only the outcome, rather than the equilibrium.

²⁹In particular, given our assumption of finite type spaces, full revelation can be seen to satisfy the consistency property of a sequential equilibrium (Kreps and Wilson, 1982) in our framework.

We conclude this section by showing that the conclusion of Theorem 1 need not hold when Assumption 1 is dropped.

Example 1.³⁰ **(Failure of the strong-form disclosure principle)** In the ex-ante symmetric set-up specified in Table I, neither type benefits from unilateral disclosure. In this particular case, this is so even though the self-disclosure by the strong type $c_1 = c_1^1$ marginalizes the weak type $\bar{c}_2 = c_2^2$ of the opponent.

Player 1		Player 2	
$q_1(c_1^1) = 0.5$	$q_1(c_1^2) = 0.5$	$q_2(c_2^1) = 0.5$	$q_2(c_2^2) = 0.5$
$c_1^1 = 1$	$c_1^2 = 13$	$c_2^1 = 1$	$c_2^2 = 13$
Assumption 1 does not hold: $\gamma = 1 > \gamma^* = -0.0522$			
$\xi_1^*(c_1^1) = 0.1582$	$\xi_1^*(c_1^2) = 0.0122$	$\xi_2^*(c_2^1) = 0.1582$	$\xi_2^*(c_2^2) = 0.0122$
$x_1^\#(c_1^1) = 0.1111$	–	$\xi_2^\#(c_2^1, c_1^1) = 0.2222$	$\xi_2^\#(c_2^2, c_1^1) = 0.0000$
–	$x_1^\#(c_1^2) = 0.0133$	$\xi_2^\#(c_2^1, c_1^2) = 0.1019$	$\xi_2^\#(c_2^2, c_1^2) = 0.0187$
$E_{c_2}[\Pi_1(\xi_1^*(c_1^1), \xi_2^*(c_2); c_1^1)] = 0.5561 > 0.5556 = E_{c_2}[\Pi_1(x_1^\#(c_1^1), \xi_2^\#(c_2, c_1^1); c_1^1)]$ $E_{c_2}[\Pi_1(\xi_1^*(c_1^2), \xi_2^*(c_2); c_1^2)] = 0.1276 > 0.0928 = E_{c_2}[\Pi_1(x_1^\#(c_1^2), \xi_2^\#(c_2, c_1^2); c_1^2)]$			

Table I. Failure of the strong disclosure principle.

Thus, in a contest that is not unfair, all types concealing their private information may well be a perfect Bayesian equilibrium.

4. Understanding the unraveling result

This section discusses the mechanics underlying Theorem 1. We start by deriving basic monotonicity properties of players' best-response mappings. Then, the disclosure decision of the weakest type of the underdog is dealt with. Finally, we consider the disclosure decision of the strongest type of the favorite.

³⁰All the numerical examples in this paper are based on the unbiased lottery contest. Moreover, to exclude the possibility of rounding errors in the first-order conditions, we have double-checked any close case using a working precision of as much as 256 decimal digits.

4.1 Lattice-theoretic framework

Given two bid schedules $\xi_i, \widehat{\xi}_i \in X_i$, we write $\xi_i \succeq_i \widehat{\xi}_i$ when $\xi_i(c_i) \geq \widehat{\xi}_i(c_i)$ holds for any $c_i \in C_i$. Thus, (X_i, \succeq_i) is the set of bid schedules equipped with the product order.³¹ As usual, we will write $\xi_i \succ_i \widehat{\xi}_i$ if $\xi_i \succeq_i \widehat{\xi}_i$ and there is $c_i \in C_i$ such that $\xi_i(c_i) > \widehat{\xi}_i(c_i)$. Moreover, the subscript i in \succeq_i and \succ_i will be dropped whenever there is no risk of ambiguity.

Denote by $X_j^* \subseteq X_j$ the set of bid schedules ξ_j for player $j \in \{1, 2\}$ that admit, for any type $c_i \in C_i$ of player $i \neq j$, a unique maximizer $x_i \equiv \widetilde{\beta}_i(\xi_j; c_i) \in \mathbb{R}_+$ of the expected payoff function $x_i \mapsto E_{c_j}[\Pi_i(x_i, \xi_j(c_j); c_i)]$. Given $\xi_j \in X_j^*$, the bid schedule $\beta_i(\xi_j) = \widetilde{\beta}_i(\xi_j; \cdot) : C_i \rightarrow \mathbb{R}_+$ will be called the *best-response bid schedule* against ξ_j . In Appendix A, it is shown that, for any $\xi_j \in X_j^*$, the best-response bid schedule $\beta_i(\xi_j)$ is weakly declining in the type, and strictly so at positive bid levels.

The mapping $\beta_i : X_j^* \rightarrow X_i$ that maps a given bid schedule ξ_j^* of player j to player i 's best-response bid schedule against ξ_j^* will be referred to as player i 's *best-response mapping*. In the case of complete information, the best-response mapping satisfies monotonicity properties only on a strict subset of the opponent's strategy space.³² This is likewise so in the case of incomplete information. We will say that *player 1's domain condition* holds at $(\xi_2; c_1) \in X_2^* \times C_1$ if (i) $\widetilde{\beta}_1(\xi_2; c_1) > 0$, and (ii) $p_1(\widetilde{\beta}_1(\xi_2; c_1), \xi_2(c_2)) > \frac{1}{2}$ for any $c_2 \in C_2$. Thus, player 1's domain condition at $(\xi_2; c_1)$ requires that type c_1 's best-response bid against ξ_2 is interior, and wins with a probability strictly exceeding one half against any of player 2's types. Similarly, we will say that *player 2's domain condition* holds at $(\widehat{\xi}_1, c_2) \in X_1^* \times C_2$ if (i) $\widetilde{\beta}_2(\widehat{\xi}_1; c_2) > 0$, and (ii) $p_2(\widehat{\xi}_1(c_1), \widetilde{\beta}_2(\widehat{\xi}_1; c_2)) < \frac{1}{2}$ for any $c_1 \in C_1$. Thus, player 2's domain condition at $(\widehat{\xi}_1; c_1)$ requires that type c_2 's best-response bid against $\widehat{\xi}_1$ is interior, and wins

³¹In fact, (X_i, \succeq_i) is a lattice, yet this property will not be used below.

³²See Dixit (1987). See also Gama and Rietzke (2017), who offer a lattice-theoretic discussion of the complete-information set-up.

with a probability strictly below one half against any of player 1's types. Using these definitions, we obtain the following useful result.

Lemma 3. (Strict monotonicity of best-response mappings)

(i) Let $\xi_2, \hat{\xi}_2 \in X_2^*$ with $\xi_2 \succ \hat{\xi}_2$, and let $c_1 \in C_1$. If player 1's domain condition holds at $(\xi_2; c_1)$, then $\tilde{\beta}_1(\xi_2; c_1) > \tilde{\beta}_1(\hat{\xi}_2; c_1)$. In particular, if player 1's domain condition holds at $(\xi_2; c_1)$ for every $c_1 \in C_1$, then $\beta_1(\xi_2) \succ \beta_1(\hat{\xi}_2)$.

(ii) Let $\xi_1, \hat{\xi}_1 \in X_1^*$ with $\xi_1 \succ \hat{\xi}_1$, and let $c_2 \in C_2$. If player 2's domain condition holds at $(\hat{\xi}_1; c_2)$, then $\tilde{\beta}_2(\xi_1; c_2) < \tilde{\beta}_2(\hat{\xi}_1; c_2)$. In particular, if player 2's domain condition holds at $(\hat{\xi}_1; c_2)$ for every $c_2 \in C_2$, then $\beta_2(\xi_1) \prec \beta_2(\hat{\xi}_1)$.

Proof. See Appendix B. \square

This lemma shows that the domain conditions are sufficient to ensure that a type's best-response bid and a player's best-response bid schedule, respectively, move in a strictly monotone way to changes in the opponent's bid schedule. For example, in the case of player 1, the best-response bid of type c_1 will strictly rise in response to an increase of player 2's bid schedule. If player 1's domain condition holds at all of her types, then we get a strict order relation even between the best-response bid schedules. Similar comparative statics properties hold for player 2, whose best-response mapping is, however, strictly declining under the assumptions of Lemma 3. In sum, the contest with two-sided incomplete information exhibits, subject to domain conditions, comparative statics properties analogous to those of the complete-information contest.

4.2 Benefits of self-disclosure for the underdog

In this subsection, we study the incentives of the weakest type of the underdog to disclose her type, given a candidate equilibrium in which all types conceal their informa-

tion. Consider, consequently, a contest with incomplete information in which player 2 has at least two possible type realizations. Let $\xi^* = (\xi_1^*, \xi_2^*)$ denote the Bayesian equilibrium at the contest stage. For the weakest type of the underdog \bar{c}_2 , the probability of winning and the expected payoff are given by $p_2^* = E_{c_1}[p_2(\xi_1^*(c_1), \xi_2^*(\bar{c}_2))]$ and $\Pi_2^* = E_{c_1}[\Pi_2(\xi_1^*(c_1), \xi_2^*(\bar{c}_2); \bar{c}_2)]$, respectively. Consider, next, the Bayesian equilibrium $(\xi_1^\#, x_2^\#)$ in the contest with one-sided incomplete information that results when the weakest type of the underdog reveals her type. Then, type \bar{c}_2 's probability of winning and expected payoff are given by $p_2^\# = E_{c_1}[p_2(\xi_1^\#(c_1), x_2^\#)]$ and $\Pi_2^\# = E_{c_1}[\Pi_2(\xi_1^\#(c_1), x_2^\#; \bar{c}_2)]$, respectively. The following result summarizes the comparative statics of the equilibrium at the contest stage with respect to the disclosure decision by the weakest type of the underdog.

Proposition 1. (Self-disclosure by the weakest type of the underdog)

Suppose that Assumption 1 holds, and that the underdog has at least two possible type realizations. Then, a unilateral disclosure by the weakest type of the underdog, \bar{c}_2 ,

- (i) induces type \bar{c}_2 to strictly raise her effort, i.e., $x_2^\# > \xi_2^*(\bar{c}_2)$;*
- (ii) strictly raises type \bar{c}_2 's interim probability of winning, i.e., $p_2^\# > p_2^*$ (even against any given type of player 1); and*
- (iii) strictly raises type \bar{c}_2 's expected payoff, i.e., $\Pi_2^\# > \Pi_2^*$.*

Proof. See Appendix B. \square

Thus, after revealing her relative weakness, the weakest type of the underdog behaves as if gaining confidence. She bids more aggressively and wins with a strictly higher probability. Moreover, the disclosure is always strictly beneficial for her. In the proof of the unraveling result, we will actually need only part (iii) of Proposition 1.

However, as will become clear, parts (i) and (ii) are crucial steps that need to be made in order to derive part (iii).

Note that the conclusions of Proposition 1 are immediate for any type of the underdog that is inactive in ξ^* . Indeed, disclosure is the only way for such types to ensure an active participation, a positive probability of winning, and a positive expected payoff. Thus, Proposition 1 shows that the weakest type of the underdog has an incentive to disclose her type even when she foresees herself being active after concealment.

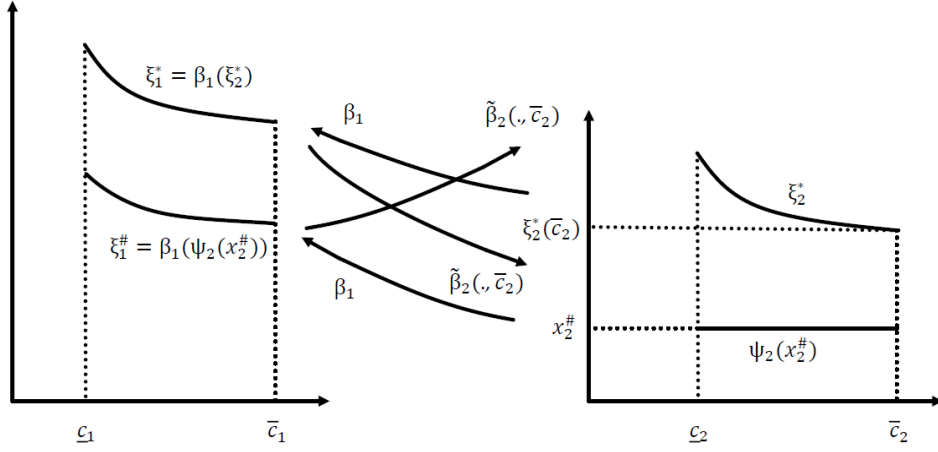


Figure 2. A lattice-theoretic argument.

The fact that the weakest type of the underdog raises her effort after self-disclosure may be unexpected. To understand this point, suppose that, instead of strictly raising her effort, the weakest type of the underdog were to weakly lower her effort after disclosure, i.e., $x_2^\# \leq \xi_2^*(\bar{c}_2)$, as shown in the diagram on the right-hand side of Figure 2. Consider now the *flat* bid schedule $\psi_2(x_2^\#) \in X_2$ that prescribes an effort of $x_2^\#$ for each type $c_2 \in C_2$ of the underdog. Then, since there are at least two types for the underdog, and since the equilibrium bid schedule ξ_2^* is strictly declining, we get $\xi_2^* \succ \psi_2(x_2^\#)$. From the strict monotonicity of player 1's best-response mapping, after

checking domain conditions, we therefore obtain $\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(x_2^\#)) = \xi_1^\#$, as shown in the diagram on the left-hand side of Figure 2. Applying now the strictly declining best-response mapping of the weakest type of the underdog, checking also here the domain condition, we arrive at $\xi_2^*(\bar{c}_2) = \tilde{\beta}_2(\xi_1^*; \bar{c}_2) < \tilde{\beta}_2(\xi_1^\#; \bar{c}_2) = x_2^\#$, which yields the desired contradiction. Thus, the weakest type of the underdog indeed gains in confidence after self-disclosure.

Based upon this fact, it can be shown that self-disclosure strictly raises also the probability of winning for the weakest type of the underdog. Ultimately, this is a consequence of what we call the *Stackelberg monotonicity* of the complete-information model. By this, we mean the fact that an increase of player i 's bid, subject to an optimal response by the opponent j , always raises player i 's winning probability (and strictly so in the interior). Intuitively, a higher effort is rewarded in terms of higher winning probabilities.³³ Applied to the present situation, this says that a Stackelberg-leading player 2 that raises her bid from $\xi_2^*(\bar{c}_2)$ to $x_2^\#$ strictly increases her probability of winning. But type \bar{c}_2 's probability of winning with her bid $\xi_2^*(\bar{c}_2)$ in the Stackelberg setting is already strictly higher than in the Bayesian equilibrium under two-sided incomplete information, because player 1's best-response bid schedule against the leader's bid $\xi_2^*(\bar{c}_2)$ is strictly lower than ξ_1^* in the product order. Combining these two insights, it follows that indeed, the probability of winning for the weakest type of the underdog rises strictly subsequent to self-disclosure. In fact, this is so even for any given type of the favorite.

Finally, we check that the weakest type of the underdog has a strict incentive to disclose her private information. The proof we managed to come up with exploits, in the spirit of the envelope theorem, type \bar{c}_2 's first-order condition in order to rewrite

³³This property, for which we did not find a suitable reference, may be seen as an analogue of Dixit's (1987, Eq. 8) precommitment result. However, in contrast to that result, the Stackelberg monotonicity property holds regardless of contestants' relative strengths.

her expected payoff from the contest as a monotone function of ex-post winning probabilities and bids. Given parts (i) and (ii) of Proposition 1, this suffices to prove the claim. Unfortunately, however, developing a simple intuition for part (iii) seems to be more intricate.

Through repeated application of Proposition 1, the underdog's side of the contest equilibrium is seen to unravel. Indeed, all types of the underdog except the worst-case type will, when in the position of the weakest type that is foreseen to conceal, find it strictly optimal to voluntarily disclose their private information. Thus, incomplete information is effectively one-sided in any perfect Bayesian equilibrium.

4.3 Benefits of self-disclosure for the favorite

From the previous section, we know that, in any perfect Bayesian equilibrium, the type of the underdog is public information at the contest stage. Let $c_2^\#$ denote the commonly known cost type of the underdog. Given this setting with one-sided incomplete information, we will study the incentive of the strongest type of the favorite to disclose her private information to the underdog.

If type \underline{c}_1 decides to conceal her private information, then the ensuing contest is of one-sided incomplete information, with equilibrium efforts $\xi_1^\#(\underline{c}_1) \equiv \xi_1^\#(\underline{c}_1; c_2^\#)$ and $x_2^\# \equiv x_2^\#(c_2^\#)$. Type \underline{c}_1 's probability of winning and expected payoff are consequently given by $p_1^\# = p_1(\xi_1^\#(\underline{c}_1), x_2^\#)$ and $\Pi_1^\# = \Pi_1(\xi_1^\#(\underline{c}_1), x_2^\#; \underline{c}_1)$, respectively. If, however, type \underline{c}_1 decides to disclose her private information, then the ensuing contest is of complete information, with equilibrium efforts $x_i^\circ \equiv x_i^\circ(\underline{c}_1, c_2^\#)$, for $i = 1, 2$. In that case, type \underline{c}_1 's probability of winning and expected payoff are given by $p_1^\circ = p_1(x_1^\circ, x_2^\circ)$ and $\Pi_1^\circ = \Pi_1(x_1^\circ, x_2^\circ; \underline{c}_1)$, respectively. The following result summarizes the comparative statics of the one-sided incomplete-information contest with respect to a revelation by the strongest type of the favorite.

Proposition 2. (Self-disclosure by the strongest type of the favorite)

Suppose that Assumption 1 holds and that the type of the underdog is public information, whereas the favorite has at least two possible type realizations. Then, a unilateral disclosure by the strongest type of the favorite, \underline{c}_1 ,

- (i) induces the underdog to strictly lower her effort, i.e., $x_2^\circ < x_2^\#$;*
- (ii) allows type \underline{c}_1 to strictly lower her effort, i.e., $x_1^\circ < \xi_1^\#(\underline{c}_1)$;*
- (iii) strictly raises type \underline{c}_1 's probability of winning, i.e., $p_1^\circ > p_1^\#$; and*
- (iv) strictly raises type \underline{c}_1 's expected payoff, i.e., $\Pi_1^\circ > \Pi_1^\#$.*

Proof. See Appendix B. \square

Thus, if the type of the underdog is public, then the self-revelation by the strongest type of the favorite discourages the underdog. As a result, the strongest type of the favorite exerts a lower effort, but still wins with higher probability. Clearly then, she finds it strictly optimal to reveal her private information to the underdog. The proof of Proposition 2 employs the same methods that have been used before. However, given that informational incompleteness is one-sided, the argument is of course much simpler in this case.³⁴

As shown in the Appendix, an iterated application of Proposition 2 implies that also the favorite's side unravels. Thus, in the presence of Assumption 1, full revelation is the only outcome consistent with the assumption of perfect Bayesian rationality. But, as already discussed, disclosure by all types of both players is indeed a perfect Bayesian equilibrium of the contest with pre-play communication, which then completes the proof of Theorem 1.

³⁴Part (ii) of Proposition 2 can actually be shown to hold also in the case of two-sided incomplete information, using an argument similar to the one used for Proposition 1. Beyond this observation, however, the analogy is incomplete. In fact, we conjecture that parts (iii) and (iv) of Proposition 2 do not generalize to a setting with two-sided incomplete information.

5. Nonmonotone reactions to disclosure

This section documents two additional effects, intuitively corresponding to “dominant” and “defiant” reactions, that naturally arise in the study of the comparative statics of the Bayesian equilibrium at the contest stage with respect to changes in the information structure. On a more technical level, this will also allow us to clarify the relationship between the present analysis and the recent contribution of Monaco and Sabarwal (2016).

5.1 Equilibrium responses to the underdog’s self-disclosure

While self-disclosure by the weakest type of the underdog tends to have an overall moderating effect on the favorite, some types of the favorite may actually respond by becoming more aggressive.

Example 2. (“Dominant reaction”) Consider the set-up specified in Table II. As can be seen, after the self-disclosure by type $\bar{c}_2 = c_2^2$ of the underdog, the weak type $\bar{c}_1 = c_1^2$ of the favorite raises her effort.

Player 1		Player 2	
$q_1(c_1^1) = 0.5$	$q_1(c_1^2) = 0.5$	$q_2(c_2^1) = 0.05$	$q_2(c_2^2) = 0.95$
$c_1^1 = 0.1$	$c_1^2 = 0.2$	$c_2^1 = 3.7$	$c_2^2 = 7.8$
Assumption 1 holds: $\gamma = 1 < \gamma^* = 1.2103$			
$\xi_1^*(c_1^1) = 0.1592$	$\xi_1^*(c_1^2) = 0.1042$	$\xi_2^*(c_2^1) = 0.0572$	$\xi_2^*(c_2^2) = 0.0011$
\vee	\wedge		\wedge
$\xi_1^\#(c_1^1) = 0.1495$	$\xi_1^\#(c_1^2) = 0.1051$	–	$x_2^\# = 0.0023$

Table II. Equilibrium bids before and after the underdog’s self-disclosure.

Example 2 shows that the self-disclosure by the weakest type of the underdog need not cause a generally soothing shift in the favorite’s bid schedule. Indeed, in response to

learning that the underdog is weak, only the strong type of the favorite decreases her bid, whereas the weak type of the favorite raises her bid, as if being challenged. For intuition, note that there are two countervailing effects. On the one hand, following the self-disclosure by the weakest type of the underdog, the favorite's belief regarding the underdog's type collapses and henceforth assigns probability one to the weakest type of the underdog. Clearly, this induces all of the favorite's types to *lower* their respective bids. On the other hand, the weakest type of the underdog will raise her bid after having disclosed her type, which induces all of the favorite's types to likewise *raise* their respective bids. Since the two effects have opposite signs, the overall effect of the underdog's self-disclosure on the bid of a given type of the favorite is, in general, ambiguous.

Despite this flexibility, the model does impose some structure of the favorite's reaction. First, not all types of the favorite may simultaneously raise their bids in response to the self-disclosure by the weakest type of the underdog. Indeed, this would be incompatible with our earlier conclusion that the weakest type of the underdog necessarily raises her bid. Second, even a dominant reaction of the favorite will never be strong enough to press the probability of winning for the weakest type of the underdog weakly below her probability of winning under concealment.

5.2 *Equilibrium responses to the favorite's self-disclosure*

The following example demonstrates that, in analogy to the case just considered, a type of the underdog may actually raise her effort after the favorite's attempt to discourage her.

Example 3. (“Defiant reaction”) Consider the set-up specified in Table III. It can be seen that, in response to the favorite's attempt to discourage the underdog, only the two weaker types of the underdog lower their respective efforts, whereas the

strongest type of the underdog actually raises her effort.

Player 1		Player 2		
$q_1(c_1^1) = 0.5$	$q_1(c_1^2) = 0.5$	$q_2(c_2^1) = 0.1$	$q_2(c_2^2) = 0.1$	$q_2(c_2^3) = 0.8$
$c_1^1 = 0.2$	$c_1^2 = 0.6$	$c_2^1 = 7.5$	$c_2^2 = 11$	$c_2^3 = 11.5$
Assumption 1 holds: $\gamma = 1 < \gamma^* = 1.6914$				
$\xi_1^*(c_1^1) = 0.1195$	$\xi_1^*(c_1^2) = 0.0647$	$\xi_2^*(c_2^1) = 0.0205$	$\xi_2^*(c_2^2) = 0.0031$	$\xi_2^*(c_2^3) = 0.0014$
\vee		\wedge	\vee	\vee
$x_1^\# = 0.0872$	—	$\xi_2^\#(c_2^1) = 0.0206$	$\xi_2^\#(c_2^2) = 0.0018$	$\xi_2^\#(c_2^3) = 0.0000$

Table III. Equilibrium bids before and after the favorite’s self-disclosure.

In fact, the example illustrates again another possibility, seen before in a symmetric environment, viz. that a type of the underdog may become so discouraged that she decides to exert zero effort.³⁵

5.3 Relationship to Monaco and Sabarwal (2016)

Monaco and Sabarwal (2016) introduced an interesting new class of games that they referred to as parameterized games of strategic heterogeneity. For the precise definition, we refer the reader to the original paper. Very roughly, however, in such games, strategy spaces are lattices and payoff functions allow for strategic complements and substitutes at the same time. One can check that, under suitable constraints on bids, the incomplete-information contests considered in the present paper are indeed parameterized games of strategic heterogeneity. Rather unexpectedly, however, we found that the contraction-mapping approach of Monaco and Sabarwal (2016, Th. 5) need not go through—when the contest is too unbalanced. The problem is that, as

³⁵This possibility is reminiscent of the *drop-out* identified by Parreiras and Rubinchik (2010). However, in their setting, intimidation is caused by the presence of additional players, whereas in our setting, intimidation is caused by disclosed information.

recently noted by Wärneryd (2016) in a different context, the best-response iteration in a sufficiently asymmetric contest need not be a contraction. Figure 1 above illustrates this fact for the case of complete information, but the situation is quite similar under incomplete information.³⁶

6. A commitment problem

While self-disclosure is always individually rational for some type of some player, other types of the same player might subsequently suffer from an increased level of competition. It turns out that this is indeed feasible. More specifically, it will be shown in this section that the unraveling may lead to higher ex-ante levels of rent dissipation for both players and to a lower ex-ante probability of winning for the underdog. In fact, the unraveling may be strictly undesirable for both contestants.

To illustrate this point, we will compare the equilibrium scenario of full revelation (FR) with the hypothetical benchmark of mandatory concealment (MC). Let $\mathbf{C}^{\text{FR}} = E[c_1 x_1^\circ(c_1, c_2) + c_2 x_2^\circ(c_1, c_2)]$ and $\mathbf{C}^{\text{MC}} = E[c_1 \xi_1^*(c_1) + c_2 \xi_2^*(c_2)]$, respectively, denote total expected costs under full revelation and under mandatory concealment.³⁷ Further, for $i \in \{1, 2\}$, let $p_i^{\text{FR}} = E[p_i(x_1^\circ(c_1, c_2), x_2^\circ(c_1, c_2))]$ and $p_i^{\text{MC}} = E[p_i(\xi_1^*(c_1), \xi_2^*(c_2))]$ denote player i 's ex-ante probability of winning under full revelation and under mandatory concealment. Finally, likewise for $i \in \{1, 2\}$, let $\Pi_i^{\text{FR}} = p_i^{\text{FR}} - E[c_i x_i^\circ(c_1, c_2)]$ and $\Pi_i^{\text{MC}} = p_i^{\text{MC}} - E[c_i \xi_i^*(c_i)]$ denote player i 's ex-ante expected payoff under full revelation and mandatory concealment. A specific setting that allows to draw some clear-cut conclusions is assumed in the following result.

³⁶To see this for Example 2, let $\bar{\beta}_1(\xi_2) = \beta_1(\psi_2(\xi_2(\bar{c}_2)))$ denote player 1's best-response bid schedule against $\psi_2(\xi_2(\bar{c}_2))$, where $\xi_2 \in X_2^*$. Monaco and Sabarwal (2016, Th. 5) required that $\bar{\beta}_1(\hat{\xi}_2) \preceq \xi_1^*$, where $\hat{\xi}_2 = \beta_2(\hat{\xi}_1)$ and $\hat{\xi}_1 = \bar{\beta}_1(\xi_2^*)$. A numerical computation shows that $\hat{\xi}_1(\underline{c}_1) = 0.1016$, $\hat{\xi}_1(\bar{c}_1) = 0.0715$, and $\hat{\xi}_2(\bar{c}_2) = 0.0194$. As a result, $\bar{\beta}_1(\hat{\xi}_2)(\underline{c}_1) = 0.4208 > 0.1592 = \xi_1^*(\underline{c}_1)$ and $\bar{\beta}_1(\hat{\xi}_2)(\bar{c}_1) = 0.2919 > 0.1042 = \xi_1^*(\bar{c}_1)$. Thus, $\bar{\beta}_1(\hat{\xi}_2) \succ \xi_1^*$, in conflict with the required condition.

³⁷ $E[\cdot] = E_{c_1, c_2}[\cdot]$ denotes the ex-ante expectation.

Proposition 3. (“Commitment problem”)

Consider an unbiased lottery contest satisfying Assumption 1. Suppose that the type of the favorite is public information, whereas the underdog has at least two possible type realizations. Assume also that, under mandatory concealment, all types are active. Then,

- (i) $\mathbf{C}^{\text{FR}} > \mathbf{C}^{\text{MC}}$ (in both cases, expected costs split evenly between the players),³⁸*
- (ii) the underdog’s (the favorite’s) ex-ante probability of winning is strictly lower (strictly higher) under full revelation than under mandatory concealment, i.e., $p_2^{\text{FR}} < p_2^{\text{MC}}$ ($p_1^{\text{FR}} > p_1^{\text{MC}}$); and*
- (iii) the ex-ante payoff for the underdog is strictly lower under full revelation than under mandatory concealment, i.e., $\Pi_2^{\text{FR}} < \Pi_2^{\text{MC}}$.³⁹*

Proof. See Appendix B. \square

The result above shows that the option to disclose private information may be undesirable for a contestant. Intuitively, there is an externality that the disclosing marginal type imposes upon the silent submarginal types. The externality is a virtual one only, because two type realizations of the same contestant never coexist. Notwithstanding, the inability to commit leads to a situation in which the privately informed player loses in expected terms by the unraveling.

The following example illustrates the possibility that the unraveling may actually be ex-ante undesirable for both contestants.

³⁸Thus, the effort of the favorite is strictly higher under full revelation than under mandatory concealment. The expected effort of the underdog, however, may either rise or fall, depending on parameters.

³⁹The payoff comparison for the favorite is ambiguous, i.e., depending on parameters, it may be that $\Pi_1^{\text{FR}} \geq \Pi_1^{\text{MC}}$, or as in Example 4 below, that $\Pi_1^{\text{FR}} < \Pi_1^{\text{MC}}$.

Example 4. (“**Disclosure trap**”) The setting specified in Table IV satisfies the assumptions of Proposition 3, and hence, illustrates the conclusions of the proposition. More importantly, it can be seen that the unraveling leads the contestants into a strictly Pareto inferior outcome.

Player 1		Player 2	
$q_1(c_1^1) = 1$		$q_2(c_2^1) = 0.5$	$q_2(c_2^2) = 0.5$
$c_1^1 = 1$		$c_2^1 = 2$	$c_2^2 = 3$
Assumption 1 holds: $\gamma = 1 < \gamma^* = 1.2660$			
$x_1^\circ(c_1^1, c_2^1) = 0.2222$		$x_2^\circ(c_1^1, c_2^1) = 0.1111$	–
$x_1^\circ(c_1^1, c_2^2) = 0.1875$		–	$x_2^\circ(c_1^1, c_2^2) = 0.0625$
$x_1^\# = 0.2020$		$\xi_2^\#(c_2^1) = 0.1158$	$\xi_2^\#(c_2^2) = 0.0575$
$\mathbf{c}^{\text{FR}} = 0.4097$		$>$	$\mathbf{c}^{\text{MC}} = 0.4040$
$p_1^{\text{FR}} = 0.7083 > p_1^{\text{MC}} = 0.7071$			$p_2^{\text{FR}} = 0.2917 < p_2^{\text{MC}} = 0.2929$
$\Pi_1^{\text{FR}} = 0.5035 < \Pi_1^{\text{MC}} = 0.5050$			$\Pi_2^{\text{FR}} = 0.0868 < \Pi_2^{\text{MC}} = 0.0909$

Table IV. Equilibrium bids under full revelation and mandatory concealment.

Thus, in contrast to the more common situation in which the receiver in a persuasion game, such as an employer, a consumer, or a health insurer, tends to benefit from the unraveling, sometimes even unduly so, this need not be the case in a contest.

7. Conclusion

In this paper, we have identified general conditions under which a probabilistic contest with pre-play communication admits full disclosure as the unique perfect Bayesian equilibrium outcome. Interestingly, our main result is just the opposite of the corresponding finding for the all-pay auction (Kovenock et al., 2015). Moreover, given that the usual assumptions for the uniqueness of the fully revealing equilibrium outcome (Milgrom, 1981; Okuno-Fujiwara et al., 1990; Seidman and Winter, 1997; Van

Zandt and Vives, 2007) fail to hold for contests, our results mean an extension of existing theory. In particular, the strong-form disclosure principle is more general than previously perceived.⁴⁰

Regarding methods, we have extended the existing lattice-theoretic approach for the analysis of the comparative statics of equilibria in games of strategic heterogeneity (Monaco and Sabarwal, 2016). Furthermore, we have developed an improved version of Jensen’s inequality, which might be of independent interest.

Our analysis took us naturally to a formalization of several intuitive concepts for which, to our knowledge, a flexible and all-encompassing framework in the realm of contest theory has been lacking so far. These concepts include strategic attempts of individual types to either moderate or discourage an opponent through pre-play communication of verifiable information, as well as the possibility of seemingly irrational, “dominant” or “defiant”, reactions to such attempts. Clearly, these latter findings are unexpected given the absence of behavioral elements in our framework. Therefore, the further analysis of such effects appears to us as a valuable route for future research.⁴¹

⁴⁰Theorem 1 continues to hold when the revelation stage is replaced by a sequential-move model in which the disclosure decision is made first by the favorite. But also in the case where the underdog moves first, we have found (by a definite result for the lottery contest, and an extensive numerical search for more general technologies) that full revelation remains the unique perfect Bayesian equilibrium outcome. Thus, even if disclosure decisions are made sequentially, it does not seem possible for the players to escape the logic of the unraveling result.

⁴¹For instance, our numerical exercises suggest that only the weakest types of the favorite may exhibit dominant reactions, and that only the strongest types of the underdog may exhibit defiant reactions.

Appendix A: Auxiliary results

In this Appendix, we state and prove a number of auxiliary results. Lemma A.1 collects some properties of a transformation introduced by Wärneryd (2003).⁴² Lemma A.2 establishes a basic monotonicity property of the best-response bid schedule. Lemma A.3 provides bounds on the bid distributions. Lemma A.4 establishes the Stackelberg monotonicity property of the complete-information contest. Finally, Lemma A.5 offers an extension of Jensen's inequality.

Lemma A.1 (Wärneryd's transformation)

Let $\Phi(z) = h(z)/h'(z)$, for any $z > 0$. Then,

$$(i) \lim_{z \searrow 0} \Phi(z) = 0,$$

$$(ii) 1 \leq \Phi' \leq \underline{\rho}, \text{ and}$$

$$(iii) (d \ln h)/(d \ln \Phi) = 1/\Phi'.$$

Proof. (i) By assumption, h is differentiable in the interior of the strategy space, with h' positive and declining. Hence, $\lim_{z \rightarrow 0} h'(z) \in (0, \infty]$. Moreover, by continuity, $\lim_{z \rightarrow 0} h(z) = 0$. The claim follows. (ii) Note first that $\Phi' = 1 - (hh''/(h')^2) \geq 1$ by the concavity of h . To see that $\Phi' \leq \underline{\rho}$, take some $\rho > \underline{\rho}$ such that h^ρ is convex. Then, in the interior of the strategy space,

$$\rho(\rho - 1)h^{\rho-2}(h')^2 + \rho h^{\rho-1}h'' \geq 0. \quad (6)$$

Recall that $\underline{\rho} \geq 1$. Hence, $\rho > 1$. Dividing (6) by $\rho h^{\rho-2}(h')^2 > 0$, and rearranging, one obtains $\Phi' \leq \rho$. Taking the limit $\rho \rightarrow \underline{\rho}$, the claim follows. (iii) A straightforward

⁴²See also Inderst et al. (2007).

calculation shows that

$$\frac{d \ln h(z)}{d \ln \Phi(z)} = \left(\frac{dh(z)}{h(z)} \right) / \left(\frac{d\Phi(z)}{\Phi(z)} \right) = \frac{h'(z)dz}{h(z)} \cdot \frac{\Phi(z)}{\Phi'(z)dz} = \frac{1}{\Phi'(z)} \quad (z > 0), \quad (7)$$

as claimed. This proves the lemma. \square

Lemma A.2 (Best-response bid schedules)

Let $\xi_j \in X_j^*$ and $c_i, \hat{c}_i \in C_i$ for $i \neq j$ such that $c_i > \hat{c}_i$. Then, $\tilde{\beta}_i(\xi_j; c_i) \leq \tilde{\beta}_i(\xi_j; \hat{c}_i)$, where the inequality is strict if $\tilde{\beta}_i(\xi_j; \hat{c}_i) > 0$.

Proof. Take an arbitrary bid schedule $\xi_j \in X_j^*$ of player j . The assertion is obvious for $\tilde{\beta}_i(\xi_j; c_i) = 0$. Suppose instead that $x_i \equiv \tilde{\beta}_i(\xi_j; c_i) > 0$. Then, the necessary first-order condition for type c_i implies

$$\frac{\partial E_{c_j}[p_i(x_i, \xi_j(c_j))]}{\partial x_i} = c_i. \quad (8)$$

We will show first that player i 's marginal probability of winning, i.e., the left-hand side of equation (8), is strictly declining in i 's bid. Indeed, because the best-response bid $\tilde{\beta}_i(\xi_j; c_i)$ exists, there is a type $c_j \in C_j$ such that $\xi_j(c_j) > 0$. A straightforward calculation shows, therefore, that

$$\begin{aligned} & \frac{\partial^2 E_{c_j}[p_i(x_i, \xi_j(c_j))]}{\partial x_i^2} \\ &= \frac{\partial}{\partial x_i} E_{c_j} \left[\frac{\gamma_i \gamma_j h'(x_i) h(\xi_j(c_j))}{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))^2} \right] \end{aligned} \quad (9)$$

$$= E_{c_j} \left[\frac{\gamma_i \gamma_j h(\xi_j(c_j)) \{ (\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j))) h''(x_i) - 2\gamma_i (h'(x_i))^2 \}}{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))^3} \right] \quad (10)$$

$$< 0, \quad (11)$$

which proves the claim. There are now two cases. Assume first that $\hat{x}_i > 0$. For this

case, it is claimed that $\hat{x}_i > x_i$. To provoke a contradiction, suppose that $\hat{x}_i \leq x_i$. Then, since the marginal probability of winning for player i is strictly declining in i 's bid,

$$\hat{c}_i = \frac{\partial E_{c_j}[p_i(\hat{x}_i, \xi_j(c_j))]}{\partial x_i} \geq \frac{\partial E_{c_j}[p_i(x_i, \xi_j(c_j))]}{\partial x_i} = c_i, \quad (12)$$

in conflict with $\hat{c}_i < c_i$. Hence, $\hat{x}_i > x_i$, as claimed. Assume next that $\hat{x}_i = 0$, i.e., type \hat{c}_i finds it optimal to respond to ξ_j with a zero effort. But then, clearly, strictly higher marginal costs induce type c_i to do the same, i.e., $x_i = 0$. The lemma follows. \square

Lemma A.3 (Bounds on the bid distributions)

Let $\xi^* = (\xi_1^*, \xi_2^*)$ be a Bayesian equilibrium in a contest such that both players are active with probability one. Then, the following two inequalities hold:

$$\gamma_i h(\xi_i^*(\underline{c}_i)) \leq \frac{1}{\pi_i} \cdot \gamma_i h(\xi_i^*(\bar{c}_i)) + \frac{1 - \pi_i}{\pi_i} \cdot \gamma_j h(\xi_j^*(\underline{c}_j)) \quad (j \neq i) \quad (13)$$

$$h(\xi_2^*(\bar{c}_2)) \leq \frac{1}{\bar{\sigma}} \cdot h(\xi_1^*(\underline{c}_1)) \quad (14)$$

Proof. Take an arbitrary type $c_i \in C_i$ of player i . Since, by assumption, $\xi_i^*(c_i) > 0$, the necessary first-order condition for type c_i holds, i.e.,

$$E_{c_j} \left[\frac{\gamma_i \gamma_j h'(\xi_i^*(c_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(c_i)) + \gamma_j h(\xi_j^*(c_j)))^2} \right] - c_i = 0, \quad (15)$$

where $j \neq i$. To prove the first claim, evaluate (15) at $c_i = \bar{c}_i$. Then, making use of

Lemma A.2 and the concavity of h , we get

$$\bar{c}_i = E_{c_j} \left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\bar{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2} \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}{\gamma_i h(\xi_i^*(\bar{c}_i)) + \gamma_j h(\xi_j^*(c_j))} \right)^2 \right] \quad (16)$$

$$= E_{c_j} \left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\bar{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2} \cdot \underbrace{\left(1 + \frac{\gamma_i h(\xi_i^*(\underline{c}_i)) - \gamma_j h(\xi_i^*(\bar{c}_i))}{\gamma_i h(\xi_i^*(\bar{c}_i)) + \gamma_j h(\xi_j^*(c_j))} \right)^2}_{\text{monotone increasing in } c_j} \right] \quad (17)$$

$$\geq E_{c_j} \left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\bar{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2} \right] \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\bar{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))} \right)^2 \quad (18)$$

$$= E_{c_j} \left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\underline{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2} \right] \times \underbrace{\left(\frac{h'(\xi_i^*(\bar{c}_i))}{h'(\xi_i^*(\underline{c}_i))} \right)}_{\geq 1} \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\bar{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))} \right)^2 \quad (19)$$

$$\geq \underline{c}_i \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\bar{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))} \right)^2. \quad (20)$$

Dividing by $\underline{c}_i > 0$, and using $\pi_i = \sqrt{\underline{c}_i / \bar{c}_i}$, we obtain

$$\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\bar{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))} \leq \frac{1}{\pi_i}. \quad (21)$$

Inequality (13) follows. To prove the second claim, one multiplies type c_i 's first-order condition (15) by $\Phi(\xi_i^*(c_i))$, and subsequently takes expectations. This yields

$$E_{c_i} [c_i \Phi(\xi_i^*(c_i))] = E_{c_1, c_2} \left[\frac{\gamma_1 \gamma_2 h(\xi_1^*(c_1)) h(\xi_2^*(c_2))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2} \right] \quad (i = 1, 2), \quad (22)$$

where $E_{c_1, c_2}[\cdot]$ denotes the ex-ante expectation. Exploiting the fact that equilibrium bid schedules are monotone declining (by Lemma A.2), and that $\Phi' > 0$, this implies

$$\underline{c}_2 \Phi(\xi_2^*(\bar{c}_2)) \leq E_{c_2} [c_2 \Phi(\xi_2^*(c_2))] = E_{c_1} [c_1 \Phi(\xi_1^*(c_1))] \leq \bar{c}_1 \Phi(\xi_1^*(\underline{c}_1)), \quad (23)$$

or, using that $\Phi(\xi_2^*(\bar{c}_2)) > 0$,

$$\frac{\Phi(\xi_1^*(\underline{c}_1))}{\Phi(\xi_2^*(\bar{c}_2))} \geq \frac{c_2}{\bar{c}_1} = \sigma. \quad (24)$$

There are two cases. Assume first that $\xi_1^*(\underline{c}_1) \geq \xi_2^*(\bar{c}_2)$. Then, using $\Phi' \leq \underline{\rho}$ (see Lemma A.1), we obtain

$$\ln \left(\frac{h(\xi_1^*(\underline{c}_1))}{h(\xi_2^*(\bar{c}_2))} \right) = \int_{\xi_2^*(\bar{c}_2)}^{\xi_1^*(\underline{c}_1)} d \ln h(z) \quad (25)$$

$$= \int_{\xi_2^*(\bar{c}_2)}^{\xi_1^*(\underline{c}_1)} \frac{d \ln h(z)}{d \ln \Phi(z)} d \ln \Phi(z) \quad (26)$$

$$= \int_{\xi_2^*(\bar{c}_2)}^{\xi_1^*(\underline{c}_1)} \frac{1}{\Phi'(z)} d \ln \Phi(z) \quad (27)$$

$$\geq \frac{1}{\underline{\rho}} \int_{\xi_2^*(\bar{c}_2)}^{\xi_1^*(\underline{c}_1)} d \ln \Phi(z) \quad (28)$$

$$= \frac{1}{\underline{\rho}} \ln \left(\frac{\Phi(\xi_1^*(\underline{c}_1))}{\Phi(\xi_2^*(\bar{c}_2))} \right). \quad (29)$$

Using (24), this implies

$$h(\xi_2^*(\bar{c}_2)) \leq \frac{1}{\sigma^{1/\underline{\rho}}} \cdot h(\xi_1^*(\underline{c}_1)). \quad (30)$$

Assume next that $\xi_1^*(\underline{c}_1) < \xi_2^*(\bar{c}_2)$. Then using $\Phi' \geq 1$ (taken likewise from Lemma A.1) delivers

$$\ln \left(\frac{h(\xi_2^*(\bar{c}_2))}{h(\xi_1^*(\underline{c}_1))} \right) = \int_{\xi_1^*(\underline{c}_1)}^{\xi_2^*(\bar{c}_2)} \frac{d \ln \Phi(z)}{\Phi'(z)} \leq \int_{\xi_1^*(\underline{c}_1)}^{\xi_2^*(\bar{c}_2)} d \ln \Phi(z) = \ln \left(\frac{\Phi(\xi_2^*(\bar{c}_2))}{\Phi(\xi_1^*(\underline{c}_1))} \right). \quad (31)$$

Hence, in that case,

$$h(\xi_2^*(\bar{c}_2)) \leq \frac{1}{\sigma} \cdot h(\xi_1^*(\underline{c}_1)). \quad (32)$$

Thus, exploiting that $\underline{\rho} \geq 1$,

$$h(\xi_2^*(\bar{c}_2)) \leq h(\xi_1^*(\underline{c}_1)) \cdot \max \left\{ \frac{1}{\sigma}, \frac{1}{\sigma^{1/\underline{\rho}}} \right\} \quad (33)$$

$$= h(\xi_1^*(\underline{c}_1)) \cdot \begin{cases} 1/\sigma & \text{if } \sigma \leq 1 \\ 1/\sigma^{1/\underline{\rho}} & \text{if } \sigma > 1. \end{cases} \quad (34)$$

Clearly, this proves (14). \square

Lemma A.4 (Stackelberg monotonicity)

Let $x_2 > \hat{x}_2 \geq 0$ and $c_1 \in C_1$ such that $x_1 = \tilde{\beta}_1(\psi_2(x_2), c_1)$ and $\hat{x}_1 = \tilde{\beta}_1(\psi_2(\hat{x}_2), c_1)$.

If $\hat{x}_1 > 0$ then,

(i) $p_2(x_1, x_2) > p_2(\hat{x}_1, \hat{x}_2)$, and

(ii) $\Pi_1(x_1, x_2; c_1) < \Pi_1(\hat{x}_1, \hat{x}_2; c_1)$.

Proof. (i) By assumption, $\hat{x}_1 = \tilde{\beta}_1(\psi_2(\hat{x}_2), c_1) > 0$. Therefore, $x_2 > \hat{x}_2$ implies $p_2(\hat{x}_1, x_2) > p_2(\hat{x}_1, \hat{x}_2)$. Assume first that $x_1 \leq \hat{x}_1$. Then, clearly, $p_2(x_1, x_2) \geq p_2(\hat{x}_1, x_2)$. Combining the last two inequalities, we arrive at $p_2(x_1, x_2) > p_2(\hat{x}_1, \hat{x}_2)$, as claimed. Assume, next, that $x_1 > \hat{x}_1$. Then, the necessary first-order conditions associated with the respective optimality of \hat{x}_1 and x_1 hold. As for \hat{x}_1 , we find

$$\frac{\gamma_1 h'(\hat{x}_1) \gamma_2 h(\hat{x}_2)}{(\gamma_1 h(\hat{x}_1) + \gamma_2 h(\hat{x}_2))^2} = c_1. \quad (35)$$

Multiplying by $\gamma h(\hat{x}_2)/h'(\hat{x}_1)$, where $\gamma = \gamma_2/\gamma_1$ as before, yields

$$(p_2(\hat{x}_1, \hat{x}_2))^2 = \frac{c_1 \gamma h(\hat{x}_2)}{h'(\hat{x}_1)}. \quad (36)$$

Similarly, one shows that the optimality of x_1 implies

$$(p_2(x_1, x_2))^2 = \frac{c_1 \gamma h(x_2)}{h'(x_1)}. \quad (37)$$

Recalling that h is strictly increasing and that h' is weakly declining, we see that $(p_2(x_1, x_2))^2 > (p_2(\hat{x}_1, \hat{x}_2))^2$. The claim follows.

(ii) As a consequence of the envelope theorem,

$$\frac{d\Pi_1(\tilde{\beta}_1(\psi_2(x_2); c_1), x_2; c_1)}{dx_2} = \left. \frac{\partial \Pi_1(x_1, x_2; c_1)}{\partial x_2} \right|_{x_1 = \tilde{\beta}_1(\psi_2(x_2); c_1)} \quad (38)$$

$$= - \frac{\gamma_1 h(\tilde{\beta}_1(\psi_2(x_2); c_1)) \gamma_2 h'(x_2)}{(\gamma_1 h(\tilde{\beta}_1(\psi_2(x_2); c_1)) + \gamma_2 h(x_2))^2} \quad (39)$$

$$< 0. \quad (40)$$

Thus, player 1 benefits from the lowered effort of player 2. This proves the second claim, and hence, the lemma. \square

Lemma A.5 (Improved Jensen's inequality)⁴³

Let $g : (1, \infty) \times (1, \infty) \rightarrow \mathbb{R}$ be a twice continuously differentiable function with Hessian matrix

$$H_g \equiv H_g(x, y) = \begin{pmatrix} \frac{\partial^2 g(x, y)}{\partial x^2} & \frac{\partial^2 g(x, y)}{\partial x \partial y} \\ \frac{\partial^2 g(x, y)}{\partial y \partial x} & \frac{\partial^2 g(x, y)}{\partial y^2} \end{pmatrix}, \quad (41)$$

and let Y be a nondegenerate random variable with finite support in $(1, \infty)$. If

$$\left\{ x > 1, y \geq x^2, d_x > 0, d_y > 0, \frac{d_y}{d_x} > \frac{y-1}{x-1} \right\} \Rightarrow (d_x \ d_y) (H_g(x, y)) \begin{pmatrix} d_x \\ d_y \end{pmatrix} > 0, \quad (42)$$

⁴³This auxiliary result is used in the proof of Proposition 3. It also helped us to see through the analysis of sequentially taken disclosure decisions (see fn. 40). Alternative extensions of Jensen's inequality have been proposed by Pittenger (1990), Guljaš et al. (1998), and Liao and Berg (2017), in particular. However, those results do not render the payoff comparisons made in the proof of Proposition 3.

then

$$E[g(Y, Y^2)] > g(E[Y], E[Y^2]). \quad (43)$$

Proof. By induction. Assume first that the random variable Y has precisely two possible realizations $y_1, y_2 \in (1, \infty)$. Without loss of generality, $y_1 < y_2$. Consider the auxiliary mapping $f : [0, 1] \rightarrow \mathbb{R}^2$ defined through

$$f(t) = (1-t) \begin{pmatrix} y_1 \\ y_1^2 \end{pmatrix} + t \begin{pmatrix} y_2 \\ y_2^2 \end{pmatrix} \quad (t \in [0, 1]). \quad (44)$$

By assumption, g is strictly convex along the straight line described by f .⁴⁴ In particular, the composed mapping $g \circ f$ is strictly convex. Therefore, when t is considered a random variable that assumes the value $t = 0$ with probability $q_1 = \text{pr}(Y = y_1) > 0$ and the value $t = 1$ with probability $q_2 = 1 - q_1 = \text{pr}(Y = y_2) > 0$, then

$$E[g(Y, Y^2)] = E[g(f(t))] \quad (45)$$

$$> g(f(E[t])) \quad (46)$$

$$= g(q_1 y_1 + (1 - q_1) y_2, q_1 y_1^2 + (1 - q_1) y_2^2) \quad (47)$$

$$= g([E[Y], E[Y^2]]). \quad (48)$$

This proves the claim in the case that Y has two realizations only. Suppose that the claim has been shown for $K \geq 2$ realizations, and assume that Y has $K + 1$ realizations $y_1 < \dots < y_{K+1}$, with respective probabilities $q_k = \text{pr}(Y = y_k) > 0$, where $k = 1, \dots, K + 1$. Consider the random variable Y' that assumes value y_k , for

⁴⁴To see this, let $x = (1-t)y_1 + ty_2 > 1$, $y = (1-t)y_1^2 + ty_2^2 \geq x^2$, $d_x = y_2 - y_1 > 0$, and $d_y = y_2^2 - y_1^2 > 0$. Then, $d_y/d_x = y_2 + y_1 > 1 + y_1 = (y-1)/(x-1)$, so that the precondition in (42) indeed holds true.

$k = 2, \dots, K + 1$, with probability

$$q'_k = \frac{q_k}{1 - q_1} = \frac{q_k}{\sum_{\kappa=2}^{K+1} q_\kappa}. \quad (49)$$

Thus, Y' follows a conditional distribution after learning $Y \neq y_1$. In particular,

$$E[Y] = q_1 y_1 + (1 - q_1) E[Y'], \quad (50)$$

$$E[Y^2] = q_1 y_1^2 + (1 - q_1) E[(Y')^2]. \quad (51)$$

Moreover, by the induction hypothesis, inequality (43) holds for Y' , i.e.,

$$E[g(Y', (Y')^2)] > g(E[Y'], E[(Y')^2]). \quad (52)$$

As above, we define an auxiliary mapping

$$\tilde{f}(t) = (1 - t) \begin{pmatrix} y_1 \\ y_1^2 \end{pmatrix} + t \begin{pmatrix} E[Y'] \\ E[(Y')^2] \end{pmatrix} \quad (t \in [0, 1]). \quad (53)$$

Clearly, $E[(Y')^2] > E[Y']^2$. Therefore, as illustrated in Figure 3, the vector that directs from $\begin{pmatrix} y_1 \\ y_1^2 \end{pmatrix}$ to $\begin{pmatrix} E[Y'] \\ E[(Y')^2] \end{pmatrix}$ is steeper than the vector that directs from $\begin{pmatrix} y_1 \\ y_1^2 \end{pmatrix}$ to $\begin{pmatrix} E[Y'] \\ E[Y']^2 \end{pmatrix}$. Hence, g is strictly convex also along the linear path described by \tilde{f} .⁴⁵ Thus, $g \circ \tilde{f}$ is strictly convex.

⁴⁵Indeed, letting $x = (1 - t)y_1 + tE[Y'] > 1$, $y = (1 - t)y_1^2 + tE[(Y')^2] > x^2$, $d_x = E[Y'] - y_1 > 0$, and $d_y = E[(Y')^2] - y_1^2 > 0$, we see that $d_y/d_x = (E[(Y')^2] - y_1^2)/(E[Y'] - y_1) > (E[(Y')^2] - y_1^2)/(E[Y'] - y_1) = E[Y'] + y_1 > 1 + y_1 = (y - 1)/(x - 1)$, so that the precondition in (42) holds true also in this case.

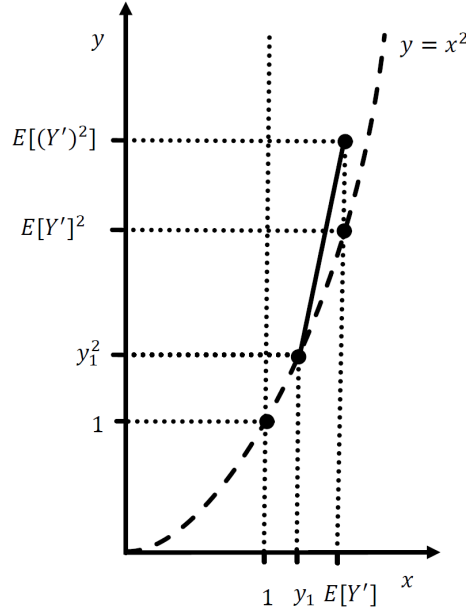


Figure 3. Improved Jensen's inequality.

Therefore, considering t as a random variable that assumes the value $t = 0$ with probability $q_1 = \text{pr}(Y = y_1) > 0$ and the value $t = 1$ with probability $1 - q_1 > 0$, relationships (50-53) imply

$$E[g(Y, Y^2)] = q_1 g(y_1, y_1^2) + (1 - q_1) E[g(Y', (Y')^2)] \quad (54)$$

$$> q_1 g(y_1, y_1^2) + (1 - q_1) g(E[Y'], E[(Y')^2]) \quad (55)$$

$$= E[g(\tilde{f}(t))] \quad (56)$$

$$> g(\tilde{f}(E[t])) \quad (57)$$

$$= g(q_1 y_1 + (1 - q_1) E[Y'], q_1 y_1^2 + (1 - q_1) E[(Y')^2]) \quad (58)$$

$$= g(E[Y], E[Y^2]). \quad (59)$$

Thus, the claim holds for $K + 1$ realizations. This completes the induction, and thereby, the proof of the lemma. \square

Appendix B: Proofs

This Appendix contains formal proofs of the results stated in the body of this paper.

Proof of Lemma 1. This is a special case of a result in Ewerhart and Quartieri (2013). The details are omitted. \square

Proof of Lemma 2. Part (ii) will be proved first. So assume that all types of both players are active in the contest. There are two cases.

Case A. All types of both players conceal their private information. We make use of Lemma A.3. Letting $i = 2$ in (13) yields

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \frac{1}{\pi_2} \cdot \gamma_2 h(\xi_2^*(\bar{c}_2)) + \frac{1 - \pi_2}{\pi_2} \cdot \gamma_1 h(\xi_1^*(\underline{c}_1)). \quad (60)$$

Combining this with (14) delivers

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \underbrace{\left\{ \frac{1}{\pi_2} \cdot \frac{\gamma}{\bar{\sigma}} + \frac{1 - \pi_2}{\pi_2} \right\}}_{\equiv \alpha} \cdot \gamma_1 h(\xi_1^*(\underline{c}_1)), \quad (61)$$

where $\gamma = \gamma_2/\gamma_1$, as before. Letting $i = 1$ in (13), and plugging the result into (61) yields

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \alpha \cdot \left\{ \frac{1}{\pi_1} \cdot \gamma_1 h(\xi_1^*(\bar{c}_1)) + \frac{1 - \pi_1}{\pi_1} \cdot \gamma_2 h(\xi_2^*(\underline{c}_2)) \right\}. \quad (62)$$

To be able to solve for $\gamma_2 h(\xi_2^*(\underline{c}_2))$, we assume for the moment that

$$1 - \alpha \frac{1 - \pi_1}{\pi_1} > 0. \quad (63)$$

Then, rewriting (62), we obtain

$$\gamma_2 h(\xi_2^*(c_2)) \leq \underbrace{\left\{ \frac{\alpha \cdot \frac{1}{\pi_1}}{1 - \alpha \cdot \frac{1 - \pi_1}{\pi_1}} \right\}}_{\equiv \lambda} \cdot \gamma_1 h(\xi_1^*(\bar{c}_1)). \quad (64)$$

Thus, $\gamma_2 h(\xi_2^*(c_2)) \leq \lambda \cdot \gamma_1 h(\xi_1^*(\bar{c}_1))$. We claim that inequality (63) holds. Indeed, starting with Assumption 1, we find that

$$\gamma < \frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1} \cdot \hat{\sigma} \quad (65)$$

$$\Leftrightarrow \frac{\gamma}{\hat{\sigma}} + 1 < \frac{2\pi_2}{2 - \pi_1} \quad (66)$$

$$\Leftrightarrow \underbrace{\frac{(\gamma/\hat{\sigma}) + 1}{\pi_2}}_{=\alpha+1} < \underbrace{\frac{2}{2 - \pi_1}}_{=\frac{\pi_1}{2 - \pi_1} + 1} \quad (67)$$

$$\Leftrightarrow \alpha < \frac{\pi_1}{2 - \pi_1} \quad (68)$$

$$\Leftrightarrow 1 - \frac{\alpha(1 - \pi_1)}{\pi_1} > \frac{\alpha}{\pi_1}. \quad (69)$$

Clearly, this implies (63). Moreover, it can be readily verified that (69) implies $\lambda < 1$.

Therefore, $\gamma_2 h(\xi_2^*(c_2)) \leq \gamma_1 h(\xi_1^*(\bar{c}_1))$. Using the monotonicity of equilibrium bid schedules (Lemma A.2 above), this proves

$$\gamma_2 h(\xi_2^*(c_2)) \leq \gamma_1 h(\xi_1^*(c_1)) \quad (c_1 \in C_1, c_2 \in C_2). \quad (70)$$

Clearly this proves part (ii) for the case that all types of both players conceal their private information.

Case B. Some type of some player discloses her private information. The conclusion remains valid even if not all types conceal. To understand why, note that disclosure by some types means that, in the relevant information set at the contest stage, the

sets C_1 and C_2 are replaced by nonempty subsets, respectively. Therefore, player 1's lowest relative resolve $\sigma = \underline{c}_2/\bar{c}_1$ rises weakly. Given that the curvature $\underline{\rho} \geq 1$ stays unchanged, this implies that $\hat{\sigma}(\sigma, \underline{\rho})$ rises weakly as well. Further, player 1 and 2's predictabilities π_1 and π_2 fall weakly, while the net bias γ stays the same. Therefore, Assumption 1 continues to hold, and the argument detailed under case A goes through as before.

This concludes the proof of part (ii) of the lemma.

It remains to verify part (i) of the lemma, i.e., that all types of player 1 are active. Suppose not. Then, all types of player 2 are active. Denote by $\emptyset \neq C_1^* \subsetneq C_1$ the set of active types of player 1, and by $q_1^* = \sum_{c_1 \in C_1^*} q_1(c_1)$ the ex-ante probability that player 1 is active. Then, since any positive bid wins against an inactive type with probability one, the corresponding terms in player 2's first-order condition vanish, so that

$$\sum_{c_1 \in C_1^*} q_1(c_1) \frac{\gamma_2 h'(\xi_2^*(c_2)) \gamma_1 h(\xi_1^*(c_1))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2} = c_2 \quad (c_2 \in C_2). \quad (71)$$

In the *modified contest*, player 1's type set C_1 is replaced by the subset C_1^* , the probability distribution $q_1(\cdot)$ is replaced by $q_1^*(c_1) = q_1(c_1)/q_1^*$, and player 2's type set C_2 is replaced by

$$\frac{C_2}{q_1^*} = \left\{ \frac{c_2}{q_1^*} \mid c_2 \in C_2 \right\}. \quad (72)$$

Denote by $\xi_1^*|_{C_1^*}$ the restriction of the mapping $\xi_1^* : C_1 \rightarrow \mathbb{R}_+$ to C_1^* , and by $\xi_2^*|_{q_1^*} : \frac{C_2}{q_1^*} \rightarrow \mathbb{R}_+$ the bid schedule for player 2 in the modified contest that satisfies $\xi_2^*|_{q_1^*}(\frac{c_2}{q_1^*}) = \xi_2^*(c_2)$ for any $c_2 \in C_2$. We claim that $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ is a Bayesian equilibrium in the modified contest. Indeed, quite obviously, the first-order condition of any active type

of player 1 holds in the modified contest. Moreover, dividing (71) by $q_1^* > 0$, we get

$$\sum_{c_1 \in C_1^*} \frac{q_1(c_1)}{q_1^*} \frac{\gamma_2 h'(\xi_2^*(c_2)) \gamma_1 h(\xi_1^*(c_1))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2} = \frac{c_2}{q_1^*} \quad (c_2 \in C_2), \quad (73)$$

i.e., also the first-order condition of any type of player 2 holds in the modified contest. Since all types of both players are active in $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ and since, in addition, the expected payoff against a player that is always active is strictly concave in the own bid, this proves the claim, i.e., $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ is indeed a Bayesian equilibrium in the modified contest. Next, one notes that, since Assumption 1 holds for the original contest, Assumption 1 holds also for the modified contest (because π_1 and σ rise weakly, while γ , $\underline{\rho}$, and π_2 stay the same). From the first part of the proof, applied to the modified contest, it therefore follows that

$$\gamma_2 h(\xi_2^*(c_2)) \leq \gamma_1 h(\xi_1^*(c_1)) \quad (c_1 \in C_1^*, c_2 \in C_2). \quad (74)$$

Now, by assumption, some types of player 1 remain inactive in the original contest. Since, by Lemma A.2, ξ_1^* is monotone declining, this clearly implies $\xi_1^*(\bar{c}_1) = 0$. Consequently, the marginal productivity at the zero bid level $h'(0) = \lim_{\varepsilon \searrow 0} \frac{h(\varepsilon)}{\varepsilon}$ is finite. Moreover, type \bar{c}_1 's marginal payoff at the zero bid level is weakly negative, i.e.,

$$E_{c_2} \left[\frac{\gamma_1 h'(0)}{\gamma_2 h(\xi_2^*(c_2))} \right] \leq \bar{c}_1. \quad (75)$$

Plugging (74) into (75), we see that

$$\frac{h'(0)}{h(\xi_1^*(c_1))} \leq \bar{c}_1 \quad (c_1 \in C_1^*). \quad (76)$$

Moreover, Assumption 1 implies

$$\frac{\gamma_2}{\gamma_1} = \gamma < \underbrace{\frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1}}_{\leq 1} \cdot \underbrace{\widehat{\sigma}(\sigma, \underline{\rho})}_{\leq \sigma} \leq \sigma = \frac{c_2}{c_1}. \quad (77)$$

Multiplying inequality (76) by $(\gamma/q_1^*) > 0$, exploiting (77), and taking expectations over all $c_1 \in C_1^*$, we get

$$\sum_{c_1 \in C_1^*} \frac{q_1(c_1)}{q_1^*} \frac{\gamma_2 h'(0)}{\gamma_1 h(\xi_1^*(c_1))} < \frac{c_2}{q_1^*}. \quad (78)$$

Thus, in the modified contest, the marginal expected payoff of type (c_2/q_1^*) at the zero bid level is strictly negative. But this is impossible given that she is active and her expected payoff against $\xi_1^*|_{C_1^*}$ is strictly concave. The contradiction shows that, indeed, all types of player 1 are active in the original contest. \square

Proof of Theorem 1. It is shown first that incomplete revelation is incompatible with the assumption of a perfect Bayesian equilibrium. The proof is by contradiction. Suppose that there is a perfect Bayesian equilibrium in which not all private information is disclosed. Then, there is at least one information set on the equilibrium path in which at least one of the players has at least two types that may realize with positive probability. By suitably redefining C_1 and C_2 , we may assume without loss of generality that all types conceal their types in that scenario. Suppose first that $|C_2| \geq 2$. Then, Proposition 1 implies that the weakest type of the underdog has a strict incentive to unilaterally deviate at the revelation stage, in conflict to the equilibrium assumption. Suppose next that $|C_2| = 1$. Then, since there is incomplete information, $|C_1| \geq 2$. But, again, this cannot be part of a perfect Bayesian equilibrium by Proposition 2. Thus, either way, we obtain a contradiction, and the claim

follows.

Next, it will be shown that there are off-equilibrium beliefs such that self-disclosure by all types of both players constitutes a perfect Bayesian equilibrium. To this end, we specify beliefs μ_1^0 and μ_2^0 as follows. The underdog expects a favorite that does not disclose her private information to be of type $c_1 = \bar{c}_1$ with probability one. Thus, $\mu_1^0(c_1) = 1$ if $c_1 = \bar{c}_1$, and $\mu_1^0(c_1) = 0$ otherwise. Similarly, the favorite expects an underdog that does not disclose her private information to be of type $c_2 = \underline{c}_2$ with probability one. Thus, $\mu_2^0(c_2) = 1$ if $c_2 = \underline{c}_2$, and $\mu_2^0(c_2) = 0$ otherwise.⁴⁶ To check the equilibrium property, consider first an arbitrary type $c_1 \in C_1$ of the favorite. If c_1 complies with equilibrium self-disclosure, and is matched with some type $c_2 \in C_2$ of the underdog, then c_1 receives a complete-information equilibrium payoff of

$$\Pi_1^\circ(c_1, c_2) = \Pi_1(x_1^\circ(c_1, c_2), x_2^\circ(c_1, c_2); c_1) \quad (79)$$

$$= \Pi_1(\tilde{\beta}_1(x_2^\circ(c_1, c_2); c_1), x_2^\circ(c_1, c_2); c_1). \quad (80)$$

If, however, the favorite chooses to not disclose her type, then, given the off-equilibrium beliefs specified above, an underdog of type c_2 expects the favorite to be of the worst-case type \bar{c}_1 and, having revealed her own type c_2 to the favorite, chooses an effort of $x_2^\circ(\bar{c}_1, c_2)$. Responding optimally to type c_2 's bid, the deviating favorite of type c_1 chooses an effort of $\tilde{\beta}_1(x_2^\circ(\bar{c}_1, c_2); c_1)$ at the contest stage, and consequently receives a payoff of

$$\Pi_1^{\text{dev}}(c_1, c_2) = \Pi_1(\tilde{\beta}_1(x_2^\circ(\bar{c}_1, c_2); c_1), x_2^\circ(\bar{c}_1, c_2); c_1). \quad (81)$$

A straightforward application of Monaco and Sabarwal (2016, Th. 3) shows that,

⁴⁶ Alternatively, one could argue that the least efficient type of the favorite and the most efficient type of the underdog do not disclose their types, in which case the consideration of off-equilibrium beliefs would not be necessary in the first place.

given Assumption 1, $x_2^\circ(c_1, c_2) \leq x_2^\circ(\bar{c}_1, c_2)$.⁴⁷ We claim that $\Pi_1^\circ(c_1, c_2) \geq \Pi_1^{\text{dev}}(c_1, c_2)$. Indeed, if $x_2^\circ(c_1, c_2) < x_2^\circ(\bar{c}_1, c_2)$ then, by Lemma A.4(ii), $\Pi_1^\circ(c_1, c_2) > \Pi_1^{\text{dev}}(c_1, c_2)$. Moreover, if $x_2^\circ(c_1, c_2) = x_2^\circ(\bar{c}_1, c_2)$ then $\Pi_1^\circ(c_1, c_2) = \Pi_1^{\text{dev}}(c_1, c_2)$, which proves the claim. Taking expectations over all $c_2 \in C_2$ yields

$$E_{c_2} [\Pi_1^\circ(c_1, c_2)] \geq E_{c_2} [\Pi_1^{\text{dev}}(c_1, c_2)] \quad (c_1 \in C_1). \quad (82)$$

Hence, a deviation is not profitable for any type $c_1 \in C_1$. On the other hand, if any type of the underdog deviates, and the favorite interprets this as a tactic of the strongest type of the underdog, then one can show in complete analogy that the equilibrium condition holds.⁴⁸ It follows that self-disclosure by all types of both players is indeed a perfect Bayesian equilibrium. This proves the theorem. \square

Proof of Lemma 3. (i) Let $\xi_2, \hat{\xi}_2 \in X_2^*$ with $\xi_2 \succ \hat{\xi}_2$, and $c_1 \in C_1$. By assumption, player 1's domain condition holds at $(\xi_2; c_1)$. We wish to show that $x_1 \equiv \tilde{\beta}_1(\xi_2; c_1) > \tilde{\beta}_1(\hat{\xi}_2; c_1) \equiv \hat{x}_1$. To provoke a contradiction, suppose that $\hat{x}_1 \geq x_1$. From the domain condition, we have $x_1 > 0$. Therefore, both x_1 and \hat{x}_1 are positive, so that the corresponding first-order conditions imply

$$E_{c_2} \left[\frac{\gamma_1 h'(x_1) \gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2} \right] = E_{c_2} \left[\frac{\gamma_1 h'(\hat{x}_1) \gamma_2 h(\hat{\xi}_2(c_2))}{(\gamma_1 h(\hat{x}_1) + \gamma_2 h(\hat{\xi}_2(c_2)))^2} \right] = c_1. \quad (83)$$

Fix some $c_2 \in C_2$ for the moment. Letting $x = \gamma_1 h(\tilde{\beta}_1(\xi_2; c_1))$ and $y = \gamma_2 h(\xi_2(c_2))$, the domain condition implies $x > y$. Clearly, the mapping $y \mapsto y/(x+y)^2$ is strictly

⁴⁷For a self-contained argument, it suffices to replicate earlier arguments. Indeed, suppose that $x_2^\circ(c_1, c_2) > x_2^\circ(\bar{c}_1, c_2)$. Clearly, all equilibrium efforts are positive under complete information. Therefore, using Lemma 2(ii), player 1's domain condition holds at $(x_2^\circ(c_1, c_2); c_1)$, so that, by Lemma 3(i), $x_1^\circ(c_1, c_2) > x_1^\circ(\bar{c}_1, c_2)$. Moreover, using Lemma 2(ii) another time, player 2's domain condition is seen to hold at $(x_1^\circ(\bar{c}_1, c_2); c_2)$, so that by Lemma 3(ii), $x_2^\circ(c_1, c_2) < x_2^\circ(\bar{c}_1, c_2)$, which yields the desired contradiction. The claim follows.

⁴⁸The details are omitted.

increasing over the interval $[0, x]$. Therefore, noting that $\xi_2 \succ \widehat{\xi}_2$ implies $y \geq \widehat{y} \equiv \gamma_2 h(\widehat{\xi}_2(c_2))$, we see that

$$\frac{\gamma_1 h'(x_1) \gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2} \geq \frac{\gamma_1 h'(x_1) \gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \quad (c_2 \in C_2), \quad (84)$$

with strict inequality for at least one $c_2 \in C_2$. Moreover, from $\widehat{x}_1 \geq x_1$,

$$\frac{\gamma_1 h'(x_1) \gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \geq \frac{\gamma_1 h'(\widehat{x}_1) \gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \quad (c_2 \in C_2). \quad (85)$$

Combining (84) and (85), and subsequently taking expectations, we arrive at

$$E_{c_2} \left[\frac{\gamma_1 h'(x_1) \gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2} \right] > E_{c_2} \left[\frac{\gamma_1 h'(\widehat{x}_1) \gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \right], \quad (86)$$

in conflict with (83). The contradiction shows that $x_1 > \widehat{x}_1$, as claimed. Moreover, if player 1's domain condition holds for any $c_1 \in C_1$, then $\widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1)$ for any $c_1 \in C_1$, which indeed implies $\beta_1(\xi_2) \succ \beta_1(\widehat{\xi}_2)$.

(ii) The proof is similar. Let $\xi_1, \widehat{\xi}_1 \in X_1^*$ with $\xi_1 \succ \widehat{\xi}_1$, and $c_2 \in C_2$. By assumption, player 2's domain condition holds at $(\widehat{\xi}_1; c_2)$. Suppose that $x_2 \equiv \widetilde{\beta}_2(\xi_1; c_2) \geq \widetilde{\beta}_2(\widehat{\xi}_1; c_2) \equiv \widehat{x}_2$. Then, from the domain condition, $\widehat{x}_2 > 0$. Hence,

$$E_{c_1} \left[\frac{\gamma_2 h'(\widehat{x}_2) \gamma_1 h(\widehat{\xi}_1(c_1))}{(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2))^2} \right] = E_{c_1} \left[\frac{\gamma_2 h'(x_2) \gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(x_2))^2} \right] = c_2. \quad (87)$$

Fix some $c_1 \in C_1$, and let $\widehat{x} = \gamma_2 h(\widetilde{\beta}_2(\widehat{\xi}_1; c_2))$ and $\widehat{y} = \gamma_1 h(\widehat{\xi}_1(c_1))$. By the domain condition, $\widehat{x} < \widehat{y}$. Moreover, the mapping $\widehat{y} \mapsto \widehat{y}/(\widehat{x} + \widehat{y})^2$ is strictly declining for

$\hat{y} \geq \hat{x}$. Hence, given that $\hat{\xi}_1 \prec \xi_1$ implies $\hat{y} \leq y \equiv \gamma_1 h(\xi_1(c_1))$, we see that

$$\frac{\gamma_2 h'(\hat{x}_2) \gamma_1 h(\hat{\xi}_1(c_1))}{\left(\gamma_1 h(\hat{\xi}_1(c_1)) + \gamma_2 h(\hat{x}_2)\right)^2} \geq \frac{\gamma_2 h'(\hat{x}_2) \gamma_1 h(\xi_1(c_1))}{\left(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(\hat{x}_2)\right)^2} \quad (c_1 \in C_1), \quad (88)$$

with strict inequality for some $c_1 \in C_1$. Moreover, from $\hat{x}_2 \leq x_2$,

$$\frac{\gamma_2 h'(\hat{x}_2) \gamma_1 h(\xi_1(c_1))}{\left(\gamma_1 h(\xi_1(c_2)) + \gamma_2 h(\hat{x}_2)\right)^2} \geq \frac{\gamma_2 h'(x_2) \gamma_1 h(\xi_1(c_1))}{\left(\gamma_2 h(\xi_1(c_1)) + \gamma_2 h(x_1)\right)^2} \quad (c_1 \in C_1). \quad (89)$$

Combining (88) and (89), and taking expectations, we arrive at

$$E_{c_1} \left[\frac{\gamma_2 h'(\hat{x}_2) \gamma_1 h(\hat{\xi}_1(c_1))}{\left(\gamma_1 h(\hat{\xi}_1(c_1)) + \gamma_2 h(\hat{x}_2)\right)^2} \right] > E_{c_1} \left[\frac{\gamma_2 h'(x_2) \gamma_1 h(\xi_1(c_1))}{\left(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(x_2)\right)^2} \right], \quad (90)$$

in contradiction to (87). It follows that, indeed, $\hat{x}_2 > x_2$. In particular, provided that player 2's domain condition holds for any $c_2 \in C_2$, it follows that $\beta_2(\xi_1) \prec \beta_2(\hat{\xi}_1)$. This concludes the proof. \square

Proof of Proposition 1. As noted in the body of the paper, the conclusions of Proposition 1 are immediate if $\xi_2^*(\bar{c}_2) = 0$. We may, therefore, assume without loss of generality that $\xi_2^*(\bar{c}_2) > 0$. Since, by Lemma A.2, the equilibrium bid schedule ξ_2^* is weakly declining, actually all types of player 2 are active in ξ_2^* . Using Lemma A.2 another time, one sees that ξ_2^* is even strictly declining. These observations will be tacitly used below. We now prove the three assertions made in the statement of the proposition.

(i) First, it is shown that self-disclosure induces the weakest type of the underdog to strictly raise her bid, i.e., $\xi_2^*(\bar{c}_2) < x_2^\#$. To provoke a contradiction, suppose that $\xi_2^*(\bar{c}_2) \geq x_2^\#$. Then, because ξ_2^* is strictly declining and there are at least two possible

type realizations for player 2, we get $\xi_2^* \succ \psi_2(x_2^\#)$. We claim that player 1's domain condition holds at $(\xi_2^*; c_1)$, for any $c_1 \in C_1$. To see this, take some $c_1 \in C_1$. Then, from Lemma 2(i), $\tilde{\beta}_1(\xi_2^*; c_1) = \xi_1^*(c_1) > 0$. Further, since all types of player 2 are active in ξ_2^* , Lemma 2(ii) implies that $p_1(\xi_1^*(c_1), \xi_2^*(c_2)) > \frac{1}{2}$ for any $c_2 \in C_2$, which proves the claim. We may, therefore, apply Lemma 3(i) so as to obtain

$$\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(x_2^\#)) = \xi_1^\#. \quad (91)$$

Next, it is claimed that player 2's domain condition holds at $(\xi_1^\#; \bar{c}_2)$. Since $(\xi_1^\#(\cdot), x_2^\#)$ is an equilibrium in the contest with one-sided incomplete information, we have $x_2^\# > 0$, i.e., player 2 is active with probability one. Applying Lemma 2(ii) shows, therefore, that $p_2(\xi_1^\#(c_1), x_2^\#) < \frac{1}{2}$, for any $c_1 \in C_1$. Since $\tilde{\beta}_2(\xi_1^\#; \bar{c}_2) = x_2^\#$, this means that $p_2(\xi_1^\#(c_1), \tilde{\beta}_2(\xi_1^\#; \bar{c}_2)) < \frac{1}{2}$, for any $c_1 \in C_1$. I.e., player 2's domain condition at $(\xi_1^\#; \bar{c}_2)$ is indeed satisfied. Therefore, using relationship (91) and Lemma 3(ii), we see that

$$\xi_2^*(\bar{c}_2) = \tilde{\beta}_2(\xi_1^*; \bar{c}_2) < \tilde{\beta}_2(\xi_1^\#; \bar{c}_2) = x_2^\#, \quad (92)$$

in contradiction to $\xi_2^*(\bar{c}_2) \geq x_2^\#$. Thus, $\xi_2^*(\bar{c}_2) < x_2^\#$, as claimed.

(ii) Next, it is shown that, after disclosure, the probability of winning for the weakest type of the underdog rises strictly, i.e.,

$$p_2^\# = E_{c_1}[p_2(\xi_1^\#(c_1), x_2^\#)] > E_{c_1}[p_2(\xi_1^*(c_1), \xi_2^*(\bar{c}_2))] = p_2^*. \quad (93)$$

In fact, we will prove the somewhat stronger statement

$$p_2(\xi_1^\#(c_1), x_2^\#) > p_2(\xi_1^*(c_1), \xi_2^*(\bar{c}_2)) \quad (c_1 \in C_1). \quad (94)$$

Take some type $c_1 \in C_1$. It is claimed first that $\tilde{\beta}_1(\psi_2(\xi_2^*(\bar{c}_2)); c_1) > 0$, as shown in the left diagram of Figure 4. Indeed, because player 2 is always active in ξ_2^* , the mapping $x_1 \mapsto E_{c_2}[\Pi_1(x_1, \xi_2^*(c_2); c_1)]$ is strictly concave on \mathbb{R}_+ , and vanishes at $x_1 = 0$. Therefore, the optimality of $\xi_1^*(c_1) > 0$ implies $E_{c_2}[\Pi_1(\xi_1^*(c_1), \xi_2^*(c_2); c_1)] > 0$. But the flat bid schedule $\psi_2(\xi_2^*(\bar{c}_2))$ is everywhere weakly lower than ξ_2^* . Therefore, $E_{c_2}[\Pi_1(\xi_1^*(c_1), \psi_2(\xi_2^*(\bar{c}_2)); c_1)] > 0$, i.e., type c_1 is able to realize a positive payoff against the flat bid schedule $\psi_2(\xi_2^*(\bar{c}_2))$. Since $\xi_2^*(\bar{c}_2) > 0$, it follows that type c_1 's best-response bid against $\psi_2(\xi_2^*(\bar{c}_2))$ is positive, as claimed. Next, from the previous step, we know that $x_2^\# > \xi_2^*(\bar{c}_2)$. Invoking Lemma A.4(i), and noting that $\xi_1^\# = \beta_1(\psi_2(x_2^\#))$, it follows that

$$p_2(\xi_1^\#(c_1), x_2^\#) > p_2(\tilde{\beta}_1(\psi_2(\xi_2^*(\bar{c}_2)); c_1), \xi_2^*(\bar{c}_2)) \quad (c_1 \in C_1). \quad (95)$$

Next, comparing the strictly declining equilibrium bid schedule $\xi_2^* = \beta_2(\xi_1^*)$ with the flat bid schedule $\psi_2(\xi_2^*(\bar{c}_2))$, and recalling that there are at least two types, we obtain $\xi_2^* \succ \psi_2(\xi_2^*(\bar{c}_2))$. Moreover, as seen above, all types of player 2 are active. Hence, by Lemma 2(ii), $p_1(\xi_1^*(c_1), \xi_2^*(c_2)) > \frac{1}{2}$ for any $c_1 \in C_1$ and any $c_2 \in C_2$, so that via $\tilde{\beta}_1(\xi_2^*; c_1) = \xi_1^*(c_1)$, player 1's domain condition is seen to hold at $(\xi_2^*; c_1)$, for any $c_1 \in C_1$. Therefore, by Lemma 3(i), $\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(\xi_2^*(\bar{c}_2)))$, as illustrated in Figure 4.⁴⁹ In particular,

$$\xi_1^*(c_1) \geq \tilde{\beta}_1(\psi_2(\xi_2^*(\bar{c}_2)); c_1) \quad (c_1 \in C_1). \quad (96)$$

Therefore,

$$p_2(\tilde{\beta}_1(\psi_2(\xi_2^*(\bar{c}_2)); c_1), \xi_2^*(\bar{c}_2)) \geq p_2(\xi_1^*(c_1), \xi_2^*(\bar{c}_2)) \quad (c_1 \in C_1). \quad (97)$$

⁴⁹The figure shows an example where $x_2^\# < \xi_2^*(\bar{c}_2)$. In general, we may also have that $x_2^\# \geq \xi_2^*(\bar{c}_2)$.

Combining (95) and (97) yields (94). In particular, this proves $p_2^\# > p_2^*$, as claimed.

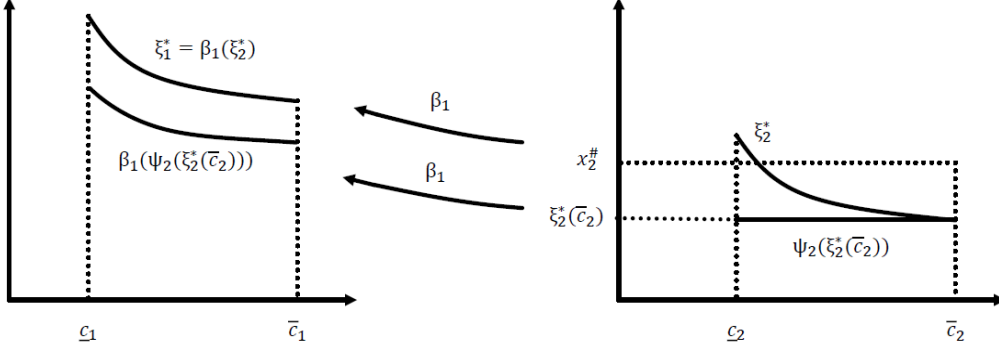


Figure 4. Proof of Proposition 1(ii).

(iii) Finally, we show that the weakest type of the underdog has a strict incentive to disclose her type. Clearly, the equilibrium effort $x_2^\#$ is positive. One can check that type \bar{c}_2 's first-order condition is equivalent to

$$E_{c_1} \left[p_2(\xi_1^\#(c_1), x_2^\#) - \left(p_2(\xi_1^\#(c_1), x_2^\#) \right)^2 \right] = \bar{c}_2 \Phi(x_2^\#). \quad (98)$$

Exploiting (98), we obtain for type \bar{c}_2 's expected payoff from self-disclosure,

$$\Pi_2^\#(\bar{c}_2) = E_{c_1} \left[\left(p_2(\xi_1^\#(c_1), x_2^\#) \right)^2 \right] + \bar{c}_2 \left(\Phi(x_2^\#) - x_2^\# \right). \quad (99)$$

In a completely analogous fashion, we can convince ourselves that concealment grants type \bar{c}_2 a payoff of

$$\Pi_2^*(\bar{c}_2) = E_{c_1} \left[\left(p_2(\xi_1^*(c_1), \xi_2^*(\bar{c}_2)) \right)^2 \right] + \bar{c}_2 \left(\Phi(\xi_2^*(\bar{c}_2)) - \xi_2^*(\bar{c}_2) \right). \quad (100)$$

Now, from (94), we see that

$$E_{c_1} \left[\left(p_2(\xi_1^\#(c_1), x_2^\#) \right)^2 \right] > E_{c_1} \left[\left(p_2(\xi_1^*(c_1), \xi_2^*(\bar{c}_2)) \right)^2 \right]. \quad (101)$$

Moreover, from Lemma A.1, $\Phi' \geq 1$, so that the mapping $x_2 \mapsto \Phi(x_2) - x_2$ is monotone increasing in x_2 . But, as shown above, $\xi_2^*(\bar{c}_2) < x_2^\#$. It follows that the weakest type of the underdog has indeed a strict incentive to reveal her type. This proves the final claim, and concludes the proof of the proposition. \square

Proof of Proposition 2. Since x_1° and x_2° are equilibrium efforts under complete information, we have $x_1^\circ > 0$ and $x_2^\circ > 0$. Similarly, one notes that $x_2^\# > 0$. Moreover, by Lemma 2(i), all types of player 1 are active in $\xi_1^\#$, so that by Lemma A.2, the bid schedule $\xi_1^\#$ is strictly declining. We now prove the four assertions made in the statement of Proposition 2.

(i) It is claimed that $x_2^\circ < x_2^\#$. To provoke a contradiction, suppose that $x_2^\circ \geq x_2^\#$. Lemma 2(ii) implies $p_1(x_1^\circ, x_2^\circ) > \frac{1}{2}$, so that in view of $x_1^\circ = \tilde{\beta}_1(x_2^\circ; \underline{c}_1)$, player 1's domain condition holds at $(x_2^\circ; \underline{c}_1)$. Hence, by Lemma 3(i), if even $x_2^\circ > x_2^\#$, then

$$x_1^\circ = \tilde{\beta}_1(x_2^\circ; \underline{c}_1) > \tilde{\beta}_1(x_2^\#; \underline{c}_1) = \xi_1^\#(\underline{c}_1). \quad (102)$$

If, however, $x_2^\circ = x_2^\#$, then it is immediate that $x_1^\circ = \xi_1^\#(\underline{c}_1)$. Thus, either way, we arrive at $x_1^\circ \geq \xi_1^\#(\underline{c}_1)$, so that $\psi_1(x_1^\circ) \succeq \psi_1(\xi_1^\#(\underline{c}_1))$. Moreover, given that player 1 has at least two types, and that $\xi_1^\#$ is strictly declining, $\psi_1(\xi_1^\#(\underline{c}_1)) \succ \xi_1^\#$. Hence, $\psi_1(x_1^\circ) \succ \xi_1^\#$. Lemma 2(ii) implies that $p_2(\xi_1^\#(c_1), x_2^\#) < \frac{1}{2}$ for any $c_1 \in C_1$. Thus, recalling that $x_2^\# = \tilde{\beta}_2(\xi_1^\#; c_2^\#)$, player 2's domain condition holds at $(\xi_1^\#; c_2^\#)$. Therefore, using Lemma 3(ii), we arrive at

$$x_2^\# = \tilde{\beta}_2(\xi_1^\#; c_2^\#) > \tilde{\beta}_2(\psi_1(x_1^\circ); c_2^\#) = x_2^\circ, \quad (103)$$

a contradiction. It follows that $x_2^\circ < x_2^\#$, as claimed.

(ii) Next, it is shown that $x_1^\circ < \xi_1^\#(\underline{c}_1)$. From the previous step, we know that

$x_2^\# > x_2^\circ$. Via Lemma 2(ii), we see that $p_1(\xi_1^\#(\underline{c}_1), x_2^\#) > \frac{1}{2}$. Thus, the domain condition for player 1 holds at $(x_2^\#; \underline{c}_1)$. Lemma 3(i) implies, therefore, that

$$\xi_1^\#(\underline{c}_1) = \tilde{\beta}_1(x_2^\#; \underline{c}_1) > \tilde{\beta}_1(x_2^\circ; \underline{c}_1) = x_1^\circ. \quad (104)$$

Thus, the effort of the strongest type of the favorite will indeed be strictly lower after self-disclosure.

(iii) Given part (i) above, we have $x_2^\circ < x_2^\#$. Recalling that $x_1^\circ > 0$, Lemma A.4(i) implies $p_2(x_1^\circ, x_2^\circ) < p_2(\xi_1^\#(\underline{c}_1), x_2^\#)$, so that $p_1(x_1^\circ, x_2^\circ) > p_1(\xi_1^\#(\underline{c}_1), x_2^\#)$. Thus, type \underline{c}_1 indeed wins with a strictly higher probability after self-disclosure.

(iv) The claim that $\Pi_1^\circ > \Pi_1^\#$ follows now directly from Lemma A.4(ii). This completes the proof. \square

Proof of Proposition 3. (i) Let $c_1^\# \in C_1$ denote the public type of the favorite. For the unbiased lottery contest, an interior equilibrium may be easily derived from the corresponding first-order conditions (Hurley and Shogren, 1998a; Epstein and Mealem, 2013; Zhang and Zhou, 2016). In our set-up, this yields equilibrium bids

$$x_1^\# = \left(\frac{E[\sqrt{c_2}]}{c_1^\# + E[c_2]} \right)^2, \text{ and} \quad (105)$$

$$\xi_2^\#(c_2) = \sqrt{\frac{x_1^\#}{c_2}} - x_1^\# \quad (c_2 \in C_2), \quad (106)$$

where we dropped, for convenience, the subscript c_2 from the expectation operator. Using these expressions, total expected costs under mandatory concealment are easily

derived as

$$\mathbf{C}^{\text{MC}} = c_1^\# x_1^\# + E[c_2 \xi_2^\#(c_2)] \quad (107)$$

$$= (c_1^\# - E[c_2])x_1^\# + E[\sqrt{c_2}] \sqrt{x_1^\#} \quad (108)$$

$$= \frac{(c_1^\# - E[c_2])E[\sqrt{c_2}]^2}{(c_1^\# + E[c_2])^2} + \frac{E[\sqrt{c_2}]^2}{c_1^\# + E[c_2]} \quad (109)$$

$$= \frac{2c_1^\# E[\sqrt{c_2}]^2}{(c_1^\# + E[c_2])^2}. \quad (110)$$

Note that this formula entails, in particular, the complete-information case where c_2 is public as well. Therefore, being an expectation over such complete-information scenarios, total expected costs under full revelation amount to

$$\mathbf{C}^{\text{FR}} = E \left[\frac{2c_1^\# c_2}{(c_1^\# + c_2)^2} \right]. \quad (111)$$

To compare the two expressions, we apply Lemma A.5 with $Y = \sqrt{c_2/c_1^\#}$ and $g(x, y) = g_1(x, y) \equiv \frac{2x^2}{(1+y)^2}$. The Hessian of the mapping g_1 is given by

$$H_{g_1}(x, y) = \begin{pmatrix} \frac{4}{(1+y)^2} & -\frac{8x}{(1+y)^3} \\ -\frac{8x}{(1+y)^3} & \frac{12x^2}{(1+y)^4} \end{pmatrix}. \quad (112)$$

It suffices to show that, for any $x > 1$, $y \geq x^2$, $d_x > 0$, $d_y > 0$ such that $\frac{d_y}{d_x} > \frac{y-1}{x-1}$, the quadratic form

$$(d_x \ d_y) (H_{g_1}(x, y)) \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{4}{(1+y)^2} d_x^2 - \frac{16x}{(1+y)^3} d_x d_y + \frac{12x^2}{(1+y)^4} d_y^2 \quad (113)$$

$$= \frac{4d_x^2}{(1+y)^2} \left(1 - \frac{x}{1+y} \frac{d_y}{d_x} \right) \left(1 - \frac{3x}{1+y} \frac{d_y}{d_x} \right) \quad (114)$$

attains a positive value. To see this, one checks that

$$\frac{x}{y+1} \cdot \frac{d_y}{d_x} > \underbrace{\frac{x}{y+1} \cdot \frac{y-1}{x-1}}_{\text{increasing in } y} \geq \frac{x}{x^2+1} \cdot \frac{x^2-1}{x-1} = \frac{x^2+x}{x^2+1} > 1. \quad (115)$$

Clearly then, the right-hand side of (114) is positive. This proves the claim. It follows that

$$\mathbf{C}^{\text{FR}} = E \left[\frac{2(c_2/c_1^\#)}{(1 + (c_2/c_1^\#))^2} \right] > \frac{2E \left[\sqrt{c_2/c_1^\#} \right]^2}{(1 + E[c_2/c_1^\#])^2} = \mathbf{C}^{\text{MC}}, \quad (116)$$

i.e., total expected costs are indeed strictly higher under full revelation than under mandatory concealment. In particular, given that, by equation (22), expected costs in the lottery contest are the same across contestants, and given that the favorite's type is public, the favorite exerts a higher effort under full revelation than under mandatory concealment.

(ii) From (105) and (106), player 1's probability of winning is easily determined as

$$p_1^{\text{MC}} = E \left[\frac{x_1^\#}{x_1^\# + \xi_2^\#(c_2)} \right] = E \left[\sqrt{x_1^\# c_2} \right] = \frac{E \left[\sqrt{c_2} \right]^2}{c_1^\# + E[c_2]}, \quad (117)$$

under mandatory concealment, and by

$$p_1^{\text{FR}} = E \left[\frac{c_2}{c_1^\# + c_2} \right] \quad (118)$$

under full revelation. We again apply Lemma A.5 for $Y = \sqrt{c_2/c_1^\#}$, but using this time the mapping $g(x, y) = g_2(x, y) \equiv \frac{x^2}{1+y}$. The corresponding Hessian reads

$$H_{g_2}(x, y) = \begin{pmatrix} \frac{2}{1+y} & -\frac{2x}{(1+y)^2} \\ -\frac{2x}{(1+y)^2} & \frac{2x^2}{(1+y)^3} \end{pmatrix}. \quad (119)$$

Suppose that $x > 1$, $y \geq x^2$, $d_x > 0$, and $d_y > 0$. Then, clearly,

$$(d_x \ d_y) (H_{g_2}(x, y)) \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{2d_x^2}{1+y} \left(1 - \frac{x}{1+y} \frac{d_y}{d_x}\right)^2 \geq 0. \quad (120)$$

Moreover, from relationship (115), inequality (120) is even strict, which implies strict convexity of g_2 along the relevant linear path segment. Thus, we have

$$p_1^{\text{FR}} = E \left[\frac{(c_2/c_1^\#)}{1 + (c_2/c_1^\#)} \right] > \frac{E \left[\sqrt{c_2/c_1^\#} \right]^2}{1 + E[c_2/c_1^\#]} = p_1^{\text{MD}}, \quad (121)$$

and, consequently, also $p_2^{\text{FR}} < p_2^{\text{MD}}$.

(iii) Since expected costs are equal across players in the lottery contest, ex-ante expected payoffs for the underdog are given by $\Pi_2^{\text{FR}} = p_2^{\text{FR}} - \frac{\mathbf{C}^{\text{FR}}}{2}$ under full revelation, and by $\Pi_2^{\text{MD}} = p_2^{\text{MD}} - \frac{\mathbf{C}^{\text{MD}}}{2}$ under mandatory concealment. As seen above, $p_2^{\text{FR}} < p_2^{\text{MD}}$ and $\mathbf{C}^{\text{FR}} > \mathbf{C}^{\text{MD}}$. Hence, $\Pi_2^{\text{FR}} < \Pi_2^{\text{MD}}$, as claimed. \square

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