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Working Paper

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Working Paper, No. 248

Provided in Cooperation with:

Department of Economics, University of Zurich

Suggested Citation: Lahcen, Mohammed Aït (2017) : Money, inflation, and unemployment in the presence of informality, Working Paper, No. 248, University of Zurich, Department of Economics, Zurich

This Version is available at:

<https://hdl.handle.net/10419/192881>

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**University of
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University of Zurich
Department of Economics

Working Paper Series
ISSN 1664-7041 (print)
ISSN 1664-705X (online)

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Money, Inflation, and Unemployment in the Presence of Informality

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March 2017

Money, Inflation, and Unemployment in the Presence of Informality[☆]

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Abstract

This paper studies the impact of informality on the long-run relationship between inflation and unemployment in developing economies. I present a dynamic general equilibrium model with informality in both labor and goods markets and where money and credit coexist. An increase in inflation affects unemployment through two channels: the matching channel and the hiring channel. On one hand, higher inflation reduces the surplus of monetary trades thus lowering firms entry and increasing unemployment. On the other hand, the lower impact of inflation on formal transactions where credit is partially available shifts firms hiring decision from high separation informal jobs to low separation formal jobs thus reducing unemployment. The model is calibrated to match certain long-run statistics of the Brazilian economy. Numerical results indicate that, in the presence of a sizable informal sector, inflation has a small negative effect on unemployment while producing a significant impact on labor allocation between formal and informal jobs. These results point to the importance of accounting for informality when considering the inflation-unemployment trade-off in the conduct of monetary policy.

Keywords: informality, Phillips curve, money, labor, search and matching

JEL classification: E26, E41, J64, H26, O17

1. Introduction

The long-run relationship between inflation and unemployment in developed economies has been studied extensively. Several authors found compelling evidence against a vertical long-run Philips curve (Karanassou et al., 2003; Beyer and Farmer, 2007; Schreiber and Wolters, 2007; Berentsen et al., 2011) which stands at odds with the assumption of long-run neutrality of money and makes this issue again relevant for welfare and policy analysis. Despite this evidence, the literature on the shape of the long-run Philips curve (LRPC, henceforth) in developing economies is almost non-existent. It is not clear why this issue has been ignored by development macroeconomists especially with regard to the high inflation rates observed in many developing countries. Furthermore, the relationship between inflation and unemployment in the context of a developing economy might be affected by specific institutional

[☆]I am grateful to my adviser Aleksander Berentsen for his useful comments that greatly improved the paper and to my colleagues Lukas Altermatt and Florian Madison for many insightful discussions. I thank Borağan Aruoba, Kenza Benhima, Pierre Cahuc, Gabriele Camera, Mariano Bosch, Pedro Gomis-Porqueras, Fernando Martin, Cyril Monnet, Randall Wright and seminar participants at the University of Basel, the University of Zurich, the Cadi Ayyad University of Marrakech, the Gerzensee alumni conference 2016 and the 10th RGS doctoral conference in economics for valuable comments and suggestions.

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failures not present in developed economies such as strong informality, weak labor institutions and limited financial inclusion. This work aims to help fill this gap in the literature by studying the impact of informality on the shape of the LRPC.

The contribution of this paper consists in developing a theoretical model that captures the relationship between inflation and unemployment in the presence of informality in both labor and goods markets. To this purpose, I introduce a dual labor market into a dynamic general equilibrium model with search frictions similar to Berentsen et al. (2011). I extend the model of Berentsen et al. (2011) mainly in three directions. First, I add stochastic job creation which allows for an endogenously determined size of formal and informal employment. Second, I add credit as a means of payment in formal transactions alongside money. Third, I create a link between the ability of firms to offer credit in trading transactions and their decision to hire workers formally or informally. These three ingredients allow for interactions between inflation, unemployment and informality.

In the model, inflation affects unemployment through two channels: the matching channel and the hiring channel. On one hand, higher inflation reduces the surplus of monetary trades thus lowering firms entry and increasing unemployment. On the other hand, the lower impact of inflation on formal transactions where credit is partially available shifts the hiring decision towards formal jobs with a lower separation rate thus reducing unemployment. The matching technology in the goods market provides an amplification mechanism which can strengthen both channels. The equilibrium effect is ambiguous and depends on the size of the informal sector and the availability of credit in formal transactions. I calibrate the model to match some long-run statistics of the Brazilian economy. Numerical results show that, with a sizable informal sector, inflation has a significant effect on labor reallocation between formal and informal jobs but the net effect on unemployment and output is very small. This results points to the importance for central banks to take into consideration the size of the informal sector in the conduct of monetary policy and in particular with regard to the trade-off between inflation and unemployment.

There is a strong disagreement in the academic literature and among international institutions about the definition of informality. Some of the main ongoing issues concern both the relationship between legal and illegal informal activities and the distinction between informality and the household sector (ILO, 2013). In this paper, I define informality as the market-based production of legal goods and services which is unreported to the government. This definition excludes the household sector as well as criminal activities. The deliberate effort to conceal informal activities from the government is an important aspect in the distinction between the informal sector and the household sector which encompasses all domestically produced goods and services that are exempt from taxation and labor regulation. In the same way, the inherent legal nature of informal activities stands in contrast to the illegal nature of criminal activities even if both are characterized by the active avoidance of government monitoring.

The presence of a sizable informal sector within developing economies has been one of the main issues studied by development macroeconomists. Within the literature on informality, one can distinguish between two main currents. The first one is empirical and is chiefly concerned with the measurement of informality using reduced-form econometric models. This approach faces considerable methodologi-

cal challenges to the extent that agents operating within the informal economy are relentlessly trying to conceal any traces of their activities. To overcome these challenges, economists resort to indirect measurement methods using electricity consumption, money demand and cash transactions, mismatches in national accounts or household surveys. Schneider et al. (2011) provide an extensive survey of this literature.

The second current proposes theoretical models capable of replicating some aspects of agents behavior in order to provide policy recommendations. Early work on this literature focused on the dualistic view of the economy by modeling the presence of an informal sector as the result of barriers to entry into the formal sector facing firms and workers. In more recent models, the informal sector comes to existence as an optimal response to regulatory pressure such as high income tax or stringent labor regulations.

Within the second current, there exists a growing strand studying the effect of different labor institutions and policies on the labor market in the presence of informal employment (Fugazza and Jacques, 2004; Zenou, 2008; Albrecht et al., 2009; Ulyssea, 2010; Bosch and Esteban-Pretel, 2012, 2014; Charlot et al., 2015; Meghir et al., 2015). Another strand focuses on explaining the allocation choices of workers and firms between formal and informal jobs (García, 2013; Almeida and Carneiro, 2012; Boeri and Garibaldi, 2005). This literature also covers other aspects of informal employment such as cyclical flows and transition dynamics between formal and informal labor markets (Bosch and Maloney, 2008, 2010), the wage differentials between formal and informal jobs (El Badaoui et al., 2010) and informal self-employment (Narita, 2014; Albrecht et al., 2009; Satchi and Temple, 2009).

Several authors studied the relationship between taxation and informality (Gomis-Porqueras and Peralta-Alva, 2008; Prado, 2011; Ihrig and Moe, 2004; Aruoba, 2010). These authors usually assume that entering the informal economy is mainly the result of a decision to avoid taxes. The trade-off is often between paying taxes and being part of the formal economy or avoiding taxes and paying a penalty in case of detection by the fiscal authority. The degree of monitoring, legal enforcement and punishment against informality play a strong role in this trade-off. Those parameters can be proxies for the rule of law and the presence of strong institutions which provides broader explanations for the size of the informal sector. One recurrent finding is that the size of the informal sector is negatively affected by an increased level of enforcement or a reduction in the tax rate while weak institutions are often associated with high levels of informality.

This paper stands close to the informality literature dealing with money and inflation. Most of the empirical evidence in this literature points towards a positive relationship between inflation and the size of the informal economy on a cross-country level. Some authors explain this by the use of seigniorage as a source of income for the government in some developing countries due to low tax revenues resulting from a large informal sector. For example, Koreshkova (2006) develops a cash-in-advance model that relates the size of the informal sector to the trade-off between inflation and the ensuing share of seigniorage in the government revenues in one hand and the income tax in the other. She shows that inflation works to smooth the tax burden between the formal and informal parts of the economy. This result offers an interesting explanation for the very high inflation rates observed in some poor countries. Gomis-Porqueras

et al. (2014) introduce a money search model where agents can avoid taxes on part of their income by using cash transactions. They derive a model-based measure of informality and produce country-level estimates which tend to be on the lower range of the more traditional reduced-form estimates such as those reported by Enste and Schneider (2000). This can be explained by the stronger restrictions their theoretical model imposes on data as well as the exclusion of illegal activities from their measurements. Bittencourt et al. (2014) use a monetary overlapping generations model with endogenous tax evasion to study the effect of both financial sector development and inflation on the size of the informal economy. They find that a lower level of financial development provides agents with a higher incentive in participating in tax evasion activities.

The rest of the paper is organized as follows: in the next section I present the theoretical model. Equilibrium is characterized in section 3. Section 4 summarizes some theoretical results derived from the model. The calibration procedure and the numerical results are presented in sections 5 and 6. Section 7 concludes.

2. Model

I consider a setting with discrete time and infinite horizon. In every period, three markets take place sequentially: a decentralized labor market à la Mortensen and Pissarides (1994), called LM, a decentralized goods market following Kiyotaki and Wright (1993) and Lagos and Wright (2005), called DM, and a centralized Walrasian market, called CM. In the CM market, trade is a frictionless process and agents can buy or sell the general good at the equilibrium market price. In the LM and DM, agents must search for matching opportunities and use bargaining to share the match surplus.

Two types of agents live infinitely in this model, firms and households, indexed by f and h respectively and each of measure 1¹. Households work, buy and consume goods and enjoy leisure l . Firms maximize profits by hiring workers to produce goods and then sell the goods to households. Agents discount between periods using the same time preference rate β . There is no discounting between markets within the period.

I consider three employment status for each agent (f or h) depending on the type of work arrangement she is taking part in: formal employment, e , informal employment, i and unemployment, u . I define the value functions U , V and W for the LM, DM and CM respectively. These value functions depend on the agent's type $t \in \{f, h\}$, on her current employment status $j \in \{e, i, u\}$, on the real money balances z she holds as well as on the loans ℓ she extended or received.

Firms and households meet in the LM and try to form bilateral work relationships. I assume random matching based on a matching function, $\mathcal{M} = \mathcal{M}(u, v)$, where the realization of a match is a function of u , the measure of unemployed workers, and v , the measure of posted vacancies. As is standard in the labor search literature, \mathcal{M} is increasing, concave and homogeneous of degree one. On the one hand, a firm with

¹In what follows, the terms *household*, *worker* and *buyer* are used interchangeably. One could also think of a household as being composed of one worker sent to the LM and one buyer sent to the DM. With unit measures of households and firms, the probability that the buyer meets the firm employing his kin worker is 0.

a vacancy finds a worker according to a Poisson process with arrival rate $\alpha_f = \mathcal{M}(u, v)/v = \mathcal{M}(1/\theta, 1)$ where $\theta = v/u$ is the labor market tightness. On the other hand, an unemployed worker finds a job according to a Poisson process with arrival rate $\alpha_h = \mathcal{M}(u, v)/u = \mathcal{M}(1, \theta)$.

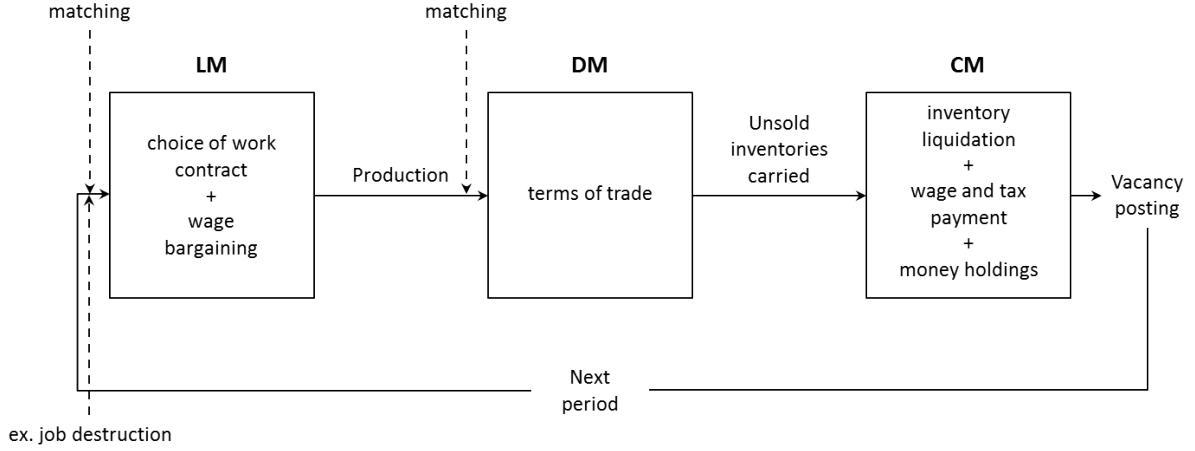


Figure 1: Model structure

At the end of the period, unmatched firms can enter the next period's LM by posting a generic job vacancy. Once an unmatched firm randomly meets a worker it decides to fill the vacancy by hiring him formally or informally. If the worker is not hired the vacancy remains open until the next period.

All workers and firms are ex-ante identical. When a match is formed, the idiosyncratic productivity of the match, ε , is revealed. This productivity is match specific and reflects the quality of the match. It is drawn from a distribution $F(\varepsilon)$ with bounded support $[\underline{\varepsilon}, \bar{\varepsilon}]$. When ε is revealed, the firm and the worker decide whether to engage in a formal or an informal work relationship or to separate and keep searching for a better match. Both formal and informal jobs are destroyed with the same exogenous probability δ .

Ongoing informal labor matches are subject to government monitoring which occurs with probability χ at the beginning of the LM. If detected by the government, the informal firm is charged a fine s_i and the match is terminated causing the firm and the worker to remain unmatched at least until the next period.

Wages are negotiated in the LM and paid in the following CM in terms of the general good. Formal wages are subject to a payroll tax τ collected by the government. Firms employing formal workers are subject to a per-period cost s_e which reflects the administrative costs of formal employment.

Next, firms (sellers) and households (buyers) enter the DM where they can trade the search good q pairwise. Matching in the DM is based on a random matching function, $\mathcal{N} = \mathcal{N}(B, S)$, where B and S are the measures of active buyers and sellers respectively. \mathcal{N} is increasing, concave and homogeneous of degree one. On the one hand, all households take part in the DM as buyers provided they are matched hence $B = 1$. On the other hand, I follow Berentsen et al. (2011) by assuming a firm can take part in the DM market as a seller only if it is able to produce. This is only possible if the firm is part of a formal or informal work relationship in the LM which implies $S = 1 - u$. More specifically, I define S_e as the measure of formal sellers, i.e. firms with a formally filled vacancy in the previous LM, and S_i as the

measure of informal sellers, i.e. firms with informally filled vacancies in the previous LM. Accordingly, the measure of all active firms in the DM is $S_e + S_i = S = 1 - u$.

Buyers can be matched either with a formal or an informal seller. A seller meets a buyer with probability $\sigma_f = \mathcal{N}(B, S)/S$. In the same manner, a buyer meets a formal seller with probability $\sigma_e = \mathcal{N}(B, S_e)/B$ and an informal seller with probability $\sigma_i = \mathcal{N}(B, S_i)/B$. Notice that the LM and DM are related through the DM matching function. This means that an increase in formal or informal employment in the labor market increases the probability for buyers to meet a formal or an informal seller respectively.

The search good q is produced by firms and only households want to consume it during the DM so that there is no double coincidence of wants. Bilateral meetings are anonymous and agents cannot store the general good in the CM market to use it in subsequent periods as a medium of exchange. I assume the existence of a record keeping technology which makes contract enforcement by the government possible in the CM. Acquiring the record keeping technology is costless but its use necessitates that the firm complies with government regulation meaning that it is currently engaged in a formal work contract and is honoring its tax duties. This makes the record keeping technology accessible only for formal firms. As a consequence informal firms resort to money as the only mean of exchange for their transactions in the DM. In contrast, in DM formal matches, formal firms can offer loans to buyers. These loans are a form of deferred payment or supplier credit to be repaid in terms of the general good in the subsequent CM. However, I assume that credit is available only with probability η which is equivalent to imperfect record keeping. This insures the co-existence of credit and money in formal DM transactions. In addition, government punishment is arbitrarily harsh such that default is not an option for buyers.

The CM is a centralized frictionless Walrasian market where goods are traded at the equilibrium price p normalized to 1. Firms liquidate what remains of their production, pay wages and taxes and distribute their profits as dividends to the households. The latter buy and consume the general good, repay their loans and decide how much money to take to the next period.

Finally, I adopt the following convention regarding money. I define real balances z such as $z = m/p = \phi m$ where p is the current price level of the general good in the CM market and ϕ is the price of money in terms of the general good. The aggregate quantity of money in the economy is given by M and grows by the rate $\gamma = M'/M$. In the steady state, I assume that the real value of money supply is constant over time such that $\phi M = \phi' M'$. This implies that inflation is pinned down by the growth rate of the quantity of money available in the economy.

2.1. Households

At the beginning of the LM, the value function for an unemployed worker is:

$$U_u^h(z) = \alpha_h \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max \{V_e^h(\varepsilon, z), V_i^h(\varepsilon, z), V_u^h(z)\} dF(\varepsilon) + (1 - \alpha_h) V_u^h(z)$$

With probability α_h , an unemployed worker entering the LM with real money holdings z is matched with a firm and the match productivity ε is then revealed. Depending on ε the worker is offered a formal job

with value $V_e^h(\varepsilon, z)$, an informal job with value $V_i^h(\varepsilon, z)$, or the match is found to be non-profitable and the firm and worker decide to keep searching for another vacancy with value $V_u^h(z)$ for the worker. With probability $1 - \alpha_h$, the unemployed worker is not matched and keeps searching for at least another period which yields the same value $V_u^h(z)$.

If a worker was employed during the previous period, he enters the LM with his work arrangement still effective. The value functions of formal and informal jobs for a worker with idiosyncratic productivity ε are:

$$\begin{aligned} U_e^h(\varepsilon, z) &= \delta V_u^h(z) + (1 - \delta) V_e^h(\varepsilon, z) \\ U_i^h(\varepsilon, z) &= \chi V_u^h(z) + (1 - \chi) [\delta V_u^h(z) + (1 - \delta) V_i^h(\varepsilon, z)] \end{aligned}$$

The worker might lose his current job with probability δ . In such a case, he becomes unemployed and can be matched with a new firm only starting from the next period. In addition, informal jobs risk termination by the government with probability χ .

In the DM, a buyer (household) is randomly matched with a formal or an informal firm from which he can buy quantity q of the search good which the firm already produced in the previous LM. Both types of firms accept money as a means of payment. However, only formal firms can extend a loan with probability η to be repaid in the next CM. The type of firm a buyer is matched with in the DM is independent of her work status in the LM. The search good provides utility $v(q)$ with $v(0) = 0$, $v' > 0$ and $v'' < 0$. The value function of a buyer entering the DM with employment status $j \in \{e, i, u\}$ is:

$$\begin{aligned} V_j^h(\varepsilon, z) &= \sigma_e \eta (v(q_c) + W_j^h(z - \phi d, \ell)) + \sigma_e (1 - \eta) (v(q_m) + W_j^h(z - \phi d)) \\ &\quad + \sigma_i (v(q_m) + W_j^h(z - \phi d)) + (1 - \sigma_e - \sigma_i) W_j^h(z) \end{aligned}$$

The buyer is matched with a formal firms with probability σ_e . If obtaining credit is possible, which occurs with probability η , the buyer can either use money d , credit ℓ or both to buy quantity q_c . Otherwise the buyer has no choice but to use his previously acquired money in which case he buys quantity q_m . With probability σ_i , the buyer meets an informal seller in which case he is again limited to the use of his money holdings and acquires the same quantity q_m . With probability $1 - \sigma_e - \sigma_i$, the buyer doesn't take part in the DM and so takes her money holdings directly to the CM.

The quantity acquired in formal monetary matches and in informal matches is obviously the same since uncertainty about DM matches resolves long after the buyer had acquired his money holdings and the buyer has no option to adjust his holdings once he learns which type of matches he will face.

When an unemployed household enters the CM, he solves the following maximization problem:

$$W_u^h = \max_{x, z'} \{x + l + \beta U_u^h(z')\}$$

subject to:

$$\begin{aligned} x + \phi m' &= b + D - T + z - \ell \\ z' &= \phi' m' = \frac{\phi m'}{\gamma} \end{aligned}$$

where x is consumption, l is the utility the household gets from leisure, b is the value of unemployment benefits, D are the dividends distributed by the firms and T are lump sum taxes (transfers) collected (distributed) by the government. z are the real balances that the household brings with it to the CM market and z' are the amount of real balances it takes with it to the next period's LM. The loan ℓ that the household eventually obtained in the previous DM is settled in the CM. The first constraint represents the household's budget constraint in the CM while the second defines the real value of the quantity of money m' that the household decides to carry to the next period.

Inserting the budget constraint in the objective function and making use of the linearity of the value function in current money holdings, I get:

$$W_u^h(z, \ell) = b + l + D - T + z - \ell + \max_{z'} \{-\gamma z' + \beta U_u^h(z')\}$$

As we can see, the choice of z' is independent of z . We will see later that it is also independent of the household's employment status.

In the same way, employed households entering the CM face the following maximization problems:

$$W_j^h(\varepsilon, z, \ell) = w_j(\varepsilon) + D - T + z - \ell + \max_{z'} \{-\gamma z' + \beta U_j^h(\varepsilon, z')\} \quad j \in \{e, i\}.$$

Using the linearity of $W_j^h(z, \ell)$ in z and ℓ , V_j^h can be written as:

$$V_j^h(\varepsilon, z) = \sigma_e \eta (v(q_c) - \phi d - \ell) + \sigma_e (1 - \eta) (v(q_m) - \phi d) + \sigma_i (v(q_m) - \phi d) + z + W_j^h(\varepsilon, 0) \quad j \in \{e, i, u\}$$

where $W_j^h(0)$ is the value function of a household entering the CM with no money holdings and no outstanding loans. In order to simplify notation, I drop z and ℓ from the arguments of the value function in the absence of money or loans.

Plugging this in the expression for U_u^h and inserting its next period expression into W_u^h I get the following recursive formulation:

$$W_u^h(z, \ell) = b + l + D - T + z - \ell + Z + \beta \left(\alpha_h \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max \{W_e^h(\varepsilon'), W_i^h(\varepsilon'), W_u^h\} dF(\varepsilon') + (1 - \alpha_h) W_u^h \right)$$

where

$$Z \equiv \max_{z'} \{(\beta - \gamma)z' + \beta (\sigma_e \eta (v(q'_c) - \phi' d' - \ell') + \sigma_e (1 - \eta) (v(q'_m) - \phi' d') + \sigma_i (v(q'_m) - \phi' d'))\}$$

In the same fashion, I get the following recursive formulations for W_e^h and W_i^h :

$$\begin{aligned} W_e^h(\varepsilon, z, \ell) &= w_e(\varepsilon) + D - T + z - \ell + Z + \beta\delta W_u^h + \beta(1 - \delta)W_e^h(\varepsilon) \\ W_i^h(\varepsilon, z, \ell) &= w_i(\varepsilon) + D - T + z - \ell + Z + \beta(1 - (1 - \chi)(1 - \delta))W_u^h + \beta(1 - \chi)(1 - \delta)W_i^h(\varepsilon) \end{aligned}$$

From the expression of Z we clearly see that the choice of real money holdings z' is completely independent of the employment status of the household and hence independent from the productivity of the match ε . This is because, for firms, all households are the same in the DM.

2.2. Firms

The value for a firm of entering the LM with a vacancy is:

$$U_u^f = \alpha_f \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max \left\{ V_e^f(\varepsilon) - s_e, V_i^f(\varepsilon), V_u^f \right\} dF(\varepsilon) + (1 - \alpha_f)V_u^f$$

with probability α_f the firm is matched with a worker. The match productivity level ε is then revealed and depending on it the firm will decide whether to enter a formal work contract which gives the value $V_e^f(\varepsilon) - s_e$, where s_e represents administrative fixed costs related to formal employment, an informal work arrangement with value $V_i^f(\varepsilon)$ or else leave the match and keep searching with value V_u^f . Finally, with probability $1 - \alpha_f$ the firm is not matched and is left with continuation value V_u^f .

The value of a firm entering the LM with a formal job with idiosyncratic productivity ε is:

$$U_e^f(\varepsilon) = (1 - \delta)(V_e^f(\varepsilon) - s_e) + \delta V_u^f$$

where δ is the probability of job destruction. Similarly, the value of firm entering with an informal job is:

$$U_i^f(\varepsilon) = (1 - \chi) \left((1 - \delta)V_i^f(\varepsilon) + \delta V_u^f \right) + \chi(V_u^f - s_i)$$

where χ is the probability of match termination by the government and s_i is the fine incurred by the firm if the match is terminated.

Matched firms produce the general good in the LM. The output of the match is the sum of the general productivity y and the match specific productivity ε . Firms take their output to the DM and CM for sale. At the beginning of the DM, a firm which had produced in the LM is matched with probability σ_f to a household, supplies him with quantity q at cost $c(q)$ with $c' > 0$ and $c'' \geq 0$ against a payment of money ϕd or a loan ℓ if the firm is formal. Formal firms can only offer credit with probability η whereas with probability $1 - \eta$ they can only sell against money. The remaining output $y + \varepsilon - c(q)$ is carried to the CM and sold at price p normalized to 1. With probability $1 - \sigma_f$, the firm is not matched and carries all of its output $y + \varepsilon$ to be sold at the CM. This leaves us with the following value functions at

the beginning of the DM:

$$\begin{aligned} V_e^f(\varepsilon) &= \sigma_f \eta W_e^f(y + \varepsilon - c(q_c), \phi d, \ell) + \sigma_f (1 - \eta) W_e^f(y + \varepsilon - c(q_m), \phi d) + (1 - \sigma_f) W_e^f(y + \varepsilon) \\ V_i^f(\varepsilon) &= \sigma_f W_i^f(y + \varepsilon - c(q_m), \phi d) + (1 - \sigma_f) W_i^f(y + \varepsilon) \end{aligned}$$

In the CM, firms liquidate their unsold inventories and pay wages to their workers. Since holding money is costly and firms have no use for it they do not need to carry any balances to the next period. As a consequence, the value of a firm with productivity ε which carries inventory x , real balances z and loans ℓ into the CM simplifies to:

$$\begin{aligned} W_e^f(\varepsilon, x, z, \ell) &= x - (1 + \tau)w_e(\varepsilon) + z + \ell + \beta U_e^f(\varepsilon) \\ W_i^f(\varepsilon, x, z) &= x - w_i(\varepsilon) + z + \beta U_i^f(\varepsilon) \end{aligned}$$

As opposed to firms employing informal workers, formal firms face a payroll tax on their wage payments.

As in the DM, an unfilled firm has nothing to sell and thus cannot take part in the CM. However, this firm can decide to post a vacancy at the end of the CM with the value:

$$W_u^f = \max \{0, -k + \beta U_u^f\}$$

where k is the cost paid in the CM for posting a vacancy. Assuming free entry, firms keep entering until the benefits of entry are exhausted: $V_u^f = W_u^f = 0$.

The structure of the problem of the firm is slightly different from the household's since firm's payoff at the DM depends on its employment status in the previous LM. Hence I write a recursive formulation in terms of beginning of DM value function V . Using the linearity of W^f , I rewrite V_e^f as:

$$V_e^f(\varepsilon) = \sigma_f \eta (\phi d + \ell - c(q_c)) + \sigma_f (1 - \eta) (\phi d - c(q_m)) + y + \varepsilon - (1 + \tau)w_e(\varepsilon) + \beta U_e^f(\varepsilon)$$

Inserting the expression for U_e^f and making use of the free entry condition I get:

$$V_e^f(\varepsilon) = R_e(\varepsilon) - (1 + \tau)w_e(\varepsilon) + \beta(1 - \delta)(V_e^f(\varepsilon) - s_e)$$

where $R_e(\varepsilon) = y + \varepsilon + \sigma_f \eta (\phi d + \ell - c(q_c)) + \sigma_f (1 - \eta) (\phi d - c(q_m))$ is the current period expected revenue of a formal firm. In the same way, I write the value of an informal firm entering the CM as:

$$V_i^f(\varepsilon) = R_i(\varepsilon) - w_i(\varepsilon) + \beta(1 - \chi)(1 - \delta)V_i^f(\varepsilon) - \beta \chi s_i$$

where $R_i(\varepsilon) = y + \varepsilon + \sigma_f (\phi d - c(q_m))$ represents the expected current period revenues for an informal firm.

2.3. Government

Government prints money at a rate γ and collects lump sum tax T and payroll tax τ which are used to finance unemployment benefits b and public spending G incurred in the CM market. The government budget constraint can be written as follows:

$$G + ub = T + \tau \int w_e(\varepsilon) d\hat{F}(\varepsilon) + n_i \chi s_i + (\gamma - 1)\phi M$$

where τ is levied on all formal wages paid in the CM market and \hat{F} is the distribution of productivity levels of formal employees. The last right hand side term represents seigniorage income.

3. Equilibrium

The model reduces to the following set of functional equations:

$$V_e^f(\varepsilon) = R_e(\varepsilon) - (1 + \tau)w_e(\varepsilon) + \beta(1 - \delta)(V_e^f(\varepsilon) - s_e) \quad (1)$$

$$V_i^f(\varepsilon) = R_i(\varepsilon) - w_i(\varepsilon) - \beta\chi s_i + \beta(1 - \chi)(1 - \delta)V_i^f(\varepsilon) \quad (2)$$

$$\frac{k}{\beta\alpha_f} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max \{V_e^f(\varepsilon) - s_e, V_i^f(\varepsilon)\} dF(\varepsilon) \quad (3)$$

$$W_e^h(\varepsilon) = w_e(\varepsilon) + D - T + Z + \beta\delta W_u^h + \beta(1 - \delta)W_e^h(\varepsilon)$$

$$W_i^h(\varepsilon) = w_i(\varepsilon) + D - T + Z + \beta(1 - (1 - \chi)(1 - \delta))W_u^h + \beta(1 - \chi)(1 - \delta)W_i^h(\varepsilon)$$

$$W_u^h = b + l + D - T + Z + \beta \left[\alpha_h \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max \{W_e^h(\varepsilon'), W_i^h(\varepsilon'), W_u^h\} dF(\varepsilon') + (1 - \alpha_h)W_u^h \right] \quad (4)$$

In what follows, I solve for the steady state equilibrium of the model. The steady state definition used here is in terms of both constant real value of money supply, $\phi M = \phi' M'$, and equal in and out-flows in the labor market, $\Delta u = \Delta n_i = \Delta n_e = 0$.

3.1. Optimal decisions in the DM

The terms of trade in the DM are determined using the generalized Nash bargaining solution. I assume that the bargaining power of buyers φ is the same for all types of DM matches.

When a buyer is part of a pure monetary match, the bargaining problem can be formulated as follows:

$$\max_{q_m, d} [v(q_m) - \phi d]^\varphi [\phi d - c(q_m)]^{1-\varphi}$$

subject to:

$$\phi d \leq z$$

$$c(q_m) \leq y + \varepsilon$$

Since money is costly, households don't have an incentive to carry more money than they intend to spend in the DM market which makes the first constraint binding. In addition, I assume that the

second constraint is never binding which allows us to write the problem as an unconstrained optimization problem. This results in the following first order condition:

$$z = \frac{\varphi v'(q_m)c(q_m) + (1 - \varphi)v(q_m)c'(q_m)}{\varphi v'(q_m) + (1 - \varphi)c'(q_m)} \equiv g(q_m)$$

As a consequence, the Nash bargaining solution is a pair (q_m, d) that satisfies $q_m = g^{-1}(z)$ and $\phi d = z$. Notice that $\partial q_m / \partial z = \partial g^{-1}(z) / \partial z = 1 / g'(q_m) \geq 0$ meaning that more money holdings increases q .

Next, I solve for the terms of trade in credit formal matches:

$$\max_{q_c, d, \ell} [v(q_c) - \phi d - \ell]^\varphi [\phi d + \ell - c(q_c)]^{1-\varphi}$$

subject to:

$$\begin{aligned} \phi d &\leq z \\ c(q_c) &\leq y + \varepsilon \end{aligned}$$

Using the same assumptions as before, the bargaining problem results in the following first order conditions:

$$\begin{aligned} z + \ell &= \frac{\varphi v'(q_c)c(q_c) + (1 - \varphi)v(q_c)c'(q_c)}{\varphi v'(q_c) + (1 - \varphi)c'(q_c)} \equiv g(q_c) \\ z + \ell &= (1 - \varphi)v(q_c) + \varphi c(q_c) \end{aligned}$$

Combining both first order conditions we get the following optimality condition:

$$[v'(q_c) - c'(q_c)] [v(q_c) - c(q_c)] = 0$$

Hence the optimal solution is $q_c = q^*$ the efficient quantity which solves $v'(q) = c'(q)$.

The efficient quantity q^* can be consumed either using credit or money. So without loss of generality I assume that in matches where credit is possible buyers will use exclusively credit. As a consequence, the Nash bargaining solution is a pair (q_c, ℓ) that satisfies $q_c = q^*$ and $\ell = g(q^*)$.

It is worth noting that since ℓ is essentially an intra-period transfer what really matters is that buyers matched with formal firms providing credit will always consume the efficient quantity q^* . Because $q_m \leq q^*$ always holds, this offers formal firms a higher trading surplus in the DM compared to informal firms.

3.2. Optimal decisions in the CM

Given the terms of trade they expect to face in the next DM, households decide on the optimal amount of money holdings they will carry from the CM to the next period:

$$\max_z \left\{ (\beta - \gamma)z + \beta [\sigma_e \eta (v(q^*) - g(q^*)) + (\sigma_e(1 - \eta) + \sigma_i) (v(g^{-1}(z)) - z)] \right\}$$

Using the independence of q^* from z , the above problem can be simplified to:

$$\max_z \{(\beta - \gamma - \beta\sigma_e(1 - \eta) - \beta\sigma_i)z + \beta(\sigma_e(1 - \eta) + \sigma_i)v(g^{-1}(z))\}$$

Assuming an interior solution, the first order condition is:

$$\frac{v'(q_m)}{g'(q_m)} = \frac{\gamma - \beta}{\beta(\sigma_e(1 - \eta) + \sigma_i)} + 1$$

Using the Fisher equation $\frac{\gamma}{\beta} = 1 + i$, I get the following expression:

$$\frac{v'(q_m)}{g'(q_m)} = \frac{i}{\sigma_e(1 - \eta) + \sigma_i} + 1 \quad (5)$$

where i is the nominal interest rate and σ_e and σ_i depend on the measure of formal and informal firms in the LM. For a given fraction of credit matches η and interest rate i , equation (5) works as a transmission channel which determines DM output in monetary matches q_m as a function of LM conditions which are consistent with the buyer's maximization problems in the DM and CM. If I set $\eta = 0$, equation (5) collapses to the LW curve in Berentsen et al. (2011). In this case, the DM transmission channel depends only on the level of unemployment.

An increase in either σ_i or σ_e or a decrease in η or i pushes q_m towards q^* . Intuitively buyers will carry just enough money to trade in informal transactions and monetary formal transactions such that the higher the probability of monetary matches the higher the marginal return on money and the more interesting it is to carry it across periods.

3.3. Optimal decisions in the LM

The choice of the optimal work arrangement, i.e. the arrangement offering the highest present discounted value, depends on the idiosyncratic productivity of the match ε . When ε is revealed at the beginning of the LM, the matched firm and worker are each faced with two decisions: first, which type of work arrangement to choose given that keeping the match is worthwhile and second, what should the wage be. The first decision determines the total match surplus and the second determines the way the total surplus will be split. The two decisions combined determine the surplus of each party. I start by solving the wage bargaining problem for each job type and then, given the wage bargaining solution, I solve for the optimal work arrangement as a function of ε .

The surplus of a LM match, after choosing the work arrangement, is defined as the total gain from

the match by both the firm and the worker:

$$\begin{aligned} S_e(\varepsilon) &= V_e^f(\varepsilon) - s_e - V_u^f + V_e^h(\varepsilon, z, \ell) - V_u^h(z, \ell) \\ &= V_e^f(\varepsilon) - s_e + W_e^h(\varepsilon) - W_u^h \end{aligned}$$

$$\begin{aligned} S_i(\varepsilon) &= V_i^f(\varepsilon) - V_u^f + V_i^h(\varepsilon, z) - V_u^h(z) \\ &= V_i^f(\varepsilon) + W_i^h(\varepsilon) - W_u^h \end{aligned}$$

The total surplus in both types of jobs is independent of money holdings and outstanding loan obligations which simplifies the problem greatly.

In what follows, I assume Nash bargaining with termination threat points as a way to decide the wage when a firm and a worker are matched. This leads to the sharing rules:

$$\omega_e (V_e^f(\varepsilon) - s_e) = (1 + \tau)(1 - \omega_e) (W_e^h(\varepsilon) - W_u^h) \quad (6)$$

$$\omega_i V_i^f(\varepsilon) = (1 - \omega_i) (W_i^h(\varepsilon) - W_u^h) \quad (7)$$

where ω_e and ω_i are the bargaining power of formal and informal workers respectively. Notice that in the case of a strictly positive proportional wage tax τ , identical Nash bargaining power for workers in both types of jobs is not enough to guarantee identical surplus sharing rules. This follows directly from the surplus sharing rules (6) and (7). The marginal payroll tax reduces the match surplus and creates a wedge which distorts the way the surplus is split between the worker and the firm. However, since τ is proportional to the wage, reducing the wage increases the total surplus of the match even if it reduces the surplus going to the worker.

Using the model's equations and the bargaining solutions (6) and (7), I derive the following wage equations:

$$\begin{aligned} w_e(\varepsilon) &= \frac{\omega_e}{1 + \tau} (R_e(\varepsilon) - s_e) - (1 - \omega_e)(D - T + Z) + (1 - \omega_e)(1 - \beta)W_u^h \\ w_i(\varepsilon) &= \omega_i (R_i(\varepsilon) - \beta\chi s_i) - (1 - \omega_i)(D - T + Z) + (1 - \omega_i)(1 - \beta)W_u^h \end{aligned}$$

which implies that the value functions of both firms and households are linear and increasing in ε . This implies the following proposition:

Proposition 1. *Under appropriate parameter values there exists a productivity threshold ε_e above which matched firms will always choose a formal work arrangement, another threshold ε_i below which matched firms will prefer to separate and keep searching. If ε falls between these two thresholds matched firms will choose an informal work arrangement. Workers decision follows the same pattern but not necessarily the same threshold values.*

This proposition follows from the linearity of the model's solution in ε and in particular the fact that

the household's optimal money holdings are independent of his LM status. Proposition 1 is illustrated in figure 2.

It remains to be seen whether a matched pair will always agree on those thresholds. In what follows I focus on parameter values for which these two thresholds exist and satisfy the condition $0 \leq \varepsilon_i \leq \varepsilon_e$.

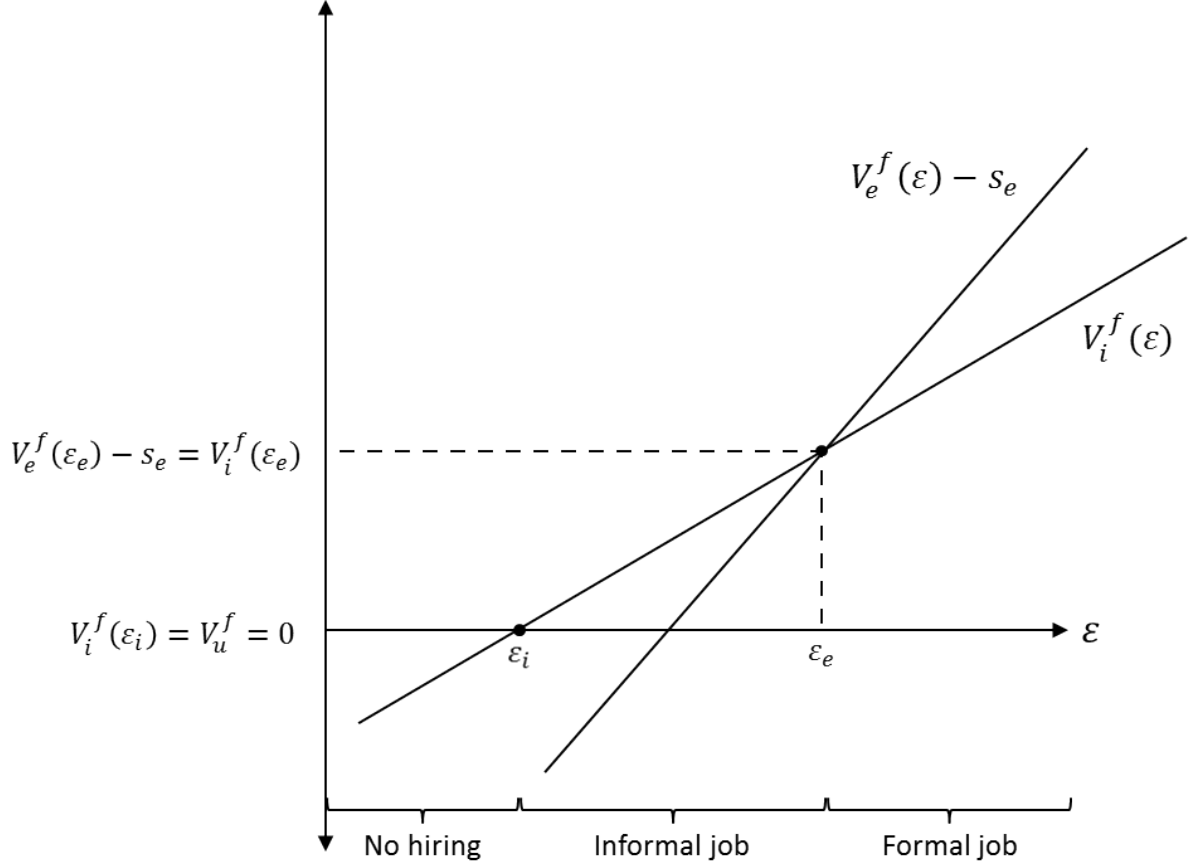


Figure 2: Hiring decision thresholds

The two productivity thresholds result from two equilibrium conditions. More formally, ε_i is the productivity level that makes a matched firm indifferent between hiring a worker informally and keeping the vacancy unfilled:

$$V_i^f(\varepsilon_i) = V_u^f = 0 \quad (8)$$

whereas ε_e is defined as the level of productivity that makes the firm indifferent between an informal and a formal work contract:

$$V_e^f(\varepsilon_e) - s_e = V_i^f(\varepsilon_e) \quad (9)$$

Now, do the indifference equations of the worker imply the same decision thresholds as the firm? The answer is no, or at least not all the time.

Proposition 2. *A matched firm and worker always agree on the separation threshold ε_i but might disagree on the threshold ε_e which determines the choice of the optimal work arrangement for the match.*

Proposition 2 is a direct result of the difference in sharing rules under formal and informal jobs which creates a discontinuity in the way the surplus is shared rising the possibility that for the same productivity level the firm and worker disagree on what is the optimal choice of contract. Intuitively, going from an informal to a formal contract for the same productivity level might simultaneously increase the total surplus and decrease the share of one of the parties. The proof of proposition 2 is written in appendix B.2.

In order to rule out any possible disagreement, I follow Bosch and Esteban-Pretel (2012) and make the simplifying assumption

$$\omega_i = \frac{\omega_e}{1 + \tau(1 - \omega_e)} \quad (10)$$

resulting in the surplus being shared in the same way independently of the choice of the work contract. This implies $\omega_i \leq \omega_e$, in line with the empirical fact that formal workers are usually more organized and enjoy more legal protection compared to informal workers.

As a result of assumption (10), I can combine the surplus sharing rules and the free entry condition (3) to rewrite equation (4) as:

$$W_u^h = \frac{b + l + D - T + Z + \frac{\omega_e}{(1+\tau)(1-\omega_e)} \frac{\alpha_h}{\alpha_f} k}{1 - \beta}$$

which I use to simplify the wage equations above to:

$$\begin{aligned} w_e(\varepsilon) &= \frac{\omega_e}{1 + \tau} \left(R_e(\varepsilon) + \frac{\alpha_h}{\alpha_f} k - s_e \right) + (1 - \omega_e)(b + l) \\ w_i(\varepsilon) &= \omega_i \left(R_i(\varepsilon) + \frac{\alpha_h}{\alpha_f} k - \beta \chi s_i \right) + (1 - \omega_i)(b + l) \end{aligned}$$

The last equilibrium condition results from restating the free entry condition (3) in terms of the two thresholds defined above:

$$\frac{k}{\beta \alpha_f} = \int_{\varepsilon_e}^{\bar{\varepsilon}} V_e^f(\varepsilon) - s_e \, dF(\varepsilon) + \int_{\varepsilon_i}^{\varepsilon_e} V_i^f(\varepsilon) \, dF(\varepsilon)$$

Replacing with the appropriate expressions (cf. appendix A) and using integration by part yields

$$\frac{k}{\beta \alpha_f} = \frac{(1 - \omega_e)}{1 - \beta(1 - \delta)} \left(\bar{\varepsilon} - \varepsilon_e - \int_{\varepsilon_e}^{\bar{\varepsilon}} F(\varepsilon) \, d\varepsilon \right) + \frac{(1 - \omega_i)}{1 - \beta(1 - \chi)(1 - \delta)} \left(\varepsilon_e - \varepsilon_i - \int_{\varepsilon_i}^{\varepsilon_e} F(\varepsilon) \, d\varepsilon \right) \quad (11)$$

Equation (11) is the job creation condition which determines how many firms enter by posting vacancies v and given u determines the LM tightness θ . The left hand side represents the expected cost of entry while the right hand side represents its expected benefit. Firms enter the LM up to the point where the two are equalized.

Proposition 3. *The cost of entry is increasing in θ since posted vacancies take a longer time to be filled when θ increases. The profits from entry are decreasing in θ since a tighter market increases the outside opportunity for workers which increases wages and reduces the net expected profit for firms. This means*

that there is a unique θ that satisfies equation (11).

This proposition is a standard result in the labor search models. Its proof is shown in appendix B.3.

3.4. Steady-state equilibrium

From the optimal decisions in the LM, I get the following job filling and separation rates:

$$JFR_e = \alpha_h(1 - F(\varepsilon_e))$$

$$JFR_i = \alpha_h(F(\varepsilon_e) - F(\varepsilon_i))$$

$$JSR_e = \delta$$

$$JSR_i = (1 - \chi)\delta + \chi$$

Employment in the formal and informal sectors are denoted n_e and n_i and the economy-wide unemployment rate is denoted u . The rates above imply that employment measures evolve according to the following laws of motion:

$$\Delta u = \delta n_e + ((1 - \chi)\delta + \chi)n_i - \alpha_h(1 - F(\varepsilon_i))u$$

$$\Delta n_e = \alpha_h(1 - F(\varepsilon_e))u - \delta n_e$$

$$\Delta n_i = \alpha_h(F(\varepsilon_e) - F(\varepsilon_i))u - ((1 - \chi)\delta + \chi)n_i$$

which depend on θ , ε_e , and ε_i and where $u + n_e + n_i = 1$ holds for every period.

The steady state distribution of the labor force is:

$$\bar{u} = \frac{\delta \bar{n}_e + ((1 - \chi)\delta + \chi)\bar{n}_i}{\alpha_h(1 - F(\varepsilon_i))} \quad (12)$$

$$\bar{n}_e = \frac{\alpha_h(1 - F(\varepsilon_e))\bar{u}}{\delta} \quad (13)$$

$$\bar{n}_i = \frac{\alpha_h(F(\varepsilon_e) - F(\varepsilon_i))\bar{u}}{(1 - \chi)\delta + \chi} \quad (14)$$

Under the assumption that each household holds a share of a market portfolio composed of all active firms in the economy, the equilibrium dividend income D is equal to aggregate profits Π :

$$\Pi = n_e \int_{\varepsilon_e}^{\bar{\varepsilon}} R_e(\varepsilon) - (1 + \tau)w_e(\varepsilon) - s_e \, dF(\varepsilon) + n_i \int_{\varepsilon_i}^{\varepsilon_e} R_i(\varepsilon) - w_i(\varepsilon) - \chi s_i \, dF(\varepsilon) - u\theta k$$

Next, I define the steady state equilibrium of this economy.

Definition 1. A steady-state equilibrium in this economy is: (i) a set of productivity thresholds $\{\varepsilon_e, \varepsilon_i\}$, (ii) a level of LM tightness θ , (iii) and a quantity q_m consumed in monetary DM matches, which together satisfy

- Optimal decisions of firms and households in the LM;
- Free entry (job creation) condition;

- *Optimal decisions of firms and households in the DM;*
- *Steady-state equations of employment and unemployment.*

As usual in monetary models, equilibrium is not unique. In addition, there could be multiple monetary equilibria.

4. Theoretical results

In what follows, I discuss several results from the model related to the relation between inflation, unemployment and informality.

In the model, the equilibrium level of unemployment depends on both firms entry through the matching probability α_h and on the hiring standards of firms through the two productivity thresholds ε_i and ε_e . I label the first mechanism *the matching channel* and the second *the hiring channel*. To see how these two mechanisms affect unemployment, I plug equations (13) and (14) in (12) to get

$$\bar{u} = \frac{1}{\frac{JFR_e}{JSR_e} + \frac{JFR_i}{JSR_i} + 1} = \frac{1}{\frac{\alpha_h(1 - F(\varepsilon_e))}{\delta} + \frac{\alpha_h(F(\varepsilon_e) - F(\varepsilon_i))}{(1 - \chi)\delta + \chi} + 1} \quad (15)$$

where the steady state unemployment depends on the level of labor market tightness, both productivity thresholds, government monitoring and the job destruction rate. Because of the difference in job separation rates between formal and informal jobs, the labor steady state allocation between the two types of jobs matters for the level of unemployment. For a given initial level of unemployment, shifting labor from formal jobs to informal jobs, characterized by a higher separation, will increase \bar{u} .

Proposition 4. *We have the following partial equilibrium results:*

$$\frac{\partial \bar{u}}{\partial \theta} < 0 \quad ; \quad \frac{\partial \bar{u}}{\partial \varepsilon_i} > 0 \quad ; \quad \frac{\partial \bar{u}}{\partial \varepsilon_e} > 0$$

Proposition 4 follows directly from equation (15). The first partial derivative simply means that an increase in θ increases the matching probability for workers which in turn increases both job finding rates and reduces unemployment. This is the matching channel referred to above. For the second result, an increase in ε_i keeping the rest constant increases firms hiring standards which reduces the informal jobs finding rate and increases the steady state unemployment. The third result means that an increase in ε_e increases the job finding rate in the informal sector and reduces the job finding rate in the formal sector. Since informal jobs have a higher separation rate, having a higher share of these jobs increases the steady state unemployment. The last two results illustrate the hiring channel.

Inflation and the nominal interest rate interact with unemployment and informality mainly through the matching and hiring channels described above. An increase in inflation and hence i reduces q_m through equation (5). The resulting decrease in DM surplus for monetary matches affects both the entry of firms and their labor allocation between formal and informal jobs.

Proposition 5. *An increase in inflation reduces the DM surplus in monetary matches which transmits to employment through two channels:*

1. *Change in the entry of firms (matching channel);*
2. *Change in the hiring standards of firms ε_i and ε_e (hiring channel).*

The equilibrium effect on unemployment depends on the interaction of these two channels.

In order to disentangle the two effects, I look at the impact of i on the productivity thresholds and LM tightness θ each separately. For a given level of θ , an increase in the productivity thresholds will lead to a decrease in job finding rates and hence an increase in unemployment.

Proposition 6. *An increase in inflation and i keeping θ constant*

- *Unambiguously increases ε_i ;*
- *Has an ambiguous effect on ε_e .*

The proof of this proposition is given in appendix B.4. One can see clearly that ε_i is directly affected by i through the DM match surplus. An increase in i reduces the trade surplus and lowers expected profits from entry. This in turn increases ε_i because firms require a higher match productivity in order to break even as can be seen from equation (8).

Proposition 7. *In equilibrium, an increase in inflation lowers the entry of firms and hence the LM tightness θ .*

The proof of this proposition is presented in appendix B.5. Intuitively, an increase in i reduces the expected gain from vacancy posting which results in less firms entering the LM and lowers market tightness θ . Lower market tightness reduces the cost of posting vacancies which pushes downward the hiring standards of firms and increases the expected profits mitigating the first effect. In equilibrium, entry falls but less than if the hiring decision was exogenous.

Now I turn to the effect of inflation on informality. For a given θ , informal employment is determined by the difference between the two productivity thresholds ε_e and ε_i :

$$\begin{aligned} \varepsilon_e - \varepsilon_i = & \Gamma_1(1 - \omega_e) \left[\sigma_f \eta [(g(q_m) - c(q_m)) - (g(q^*) - c(q^*))] + s_e - \beta \chi s_i + \tau(b + l) \right. \\ & \left. + \frac{\tau}{(1 + \tau)} \frac{\omega_e}{(1 - \omega_e)} \theta k \right] \end{aligned} \quad (16)$$

Equation (16) simply tells us that for an informal sector to exist, the benefits of informality in both the labor and goods markets should be higher than its costs. Put differently, for n_i to be positive the condition $\varepsilon_e - \varepsilon_i > 0$ needs to hold which implies:

$$s_e + \tau(b + l) + \frac{\tau}{(1 + \tau)} \frac{\omega_e}{(1 - \omega_e)} \frac{\alpha_h}{\alpha_f} k > \beta \chi s_i + \sigma_f \eta [(g(q^*) - c(q^*)) - (g(q_m) - c(q_m))]$$

where the LHS sums the tax avoiding benefits of informality while the RHS is the sum of the government monitoring and inflation costs. The inflation tax on informality is amplified by an increase in the availability of credit in DM transactions. Thus financial development increases the opportunity cost of holding money lowering the surplus of informal transactions and encouraging newly created firms to allocate labor to formal jobs.

Proposition 8. *An increase in inflation, keeping θ constant, reduces the size of informal employment.*

Proposition 8 results directly from proposition 6. Keeping θ constant, the effect of i on the size of informal employment as measured by (16) is negative. It follows then that, since inflation is a tax on monetary transactions, the higher the initial size of informal employment, the higher the share of monetary transactions in the DM and hence the stronger the effect of inflation on the economy.

5. Calibration

The calibration exercise focuses on the long-run properties of the data by matching certain model-based statistics with their equivalent long-run empirical averages. The model is calibrated using Brazilian data. Each period in the model corresponds to a quarter. Limited data availability restricts the sample to the period from 1996q1 to 2014q4. The reason I focus my numerical analysis on Brazil is that it has a sizable informal sector, close to the average level in Latin American countries (Perry et al., 2007), and relatively good statistics about it. This is because the distinction in Brazil between formal and informal work is quite straightforward (Gerard and Gonzaga, 2016).

In order to conduct numerical exercises, I choose the following functional forms: the utility function in the DM is $v(q) = Aq^{1-a}/(1-a)$. Utility in the CM is linear. Firms' cost function in the DM has the form: $c(q) = Cq^\zeta$. The matching functions in the LM and DM are described by $\mathcal{M}(u, v) = \xi u^{1-\sigma} v^\sigma$ and $\mathcal{N}(B, S_j) = BS_j/(B + S)$ respectively.

I separate the model's parameters into two groups: exogenous parameters and jointly calibrated parameters. The first group consists of parameters which I directly set to a specific value while the second group is jointly calibrated with the equilibrium solution such that the model matches certain targets from the data.

The chosen values for the first group of parameters are listed in table 1. β is set such that the real interest in the model matches the average of the difference between the risk-free nominal interest rate and the rate of inflation in the data. There are several possibilities when it comes to the calibration of the flow value of unemployment $b + l$. The standard one is to target a given percentage of the average productivity which corresponds in this model to the average productivity in both formal and informal matches. Shimer (2005) sets the value to 40%, Hall and Milgrom (2008) choose 0.71 while Hagedorn and Manovskii (2008) argue for a value of 95%. I set the target at 40% and use the unconditional average productivity such that:

$$\frac{b + l}{y + \mathbb{E}[\varepsilon]} = 0.4$$

Table 1: Exogenously set parameters

Parameter	Description	Value
β	Discount factor	0.99
i	Nominal interest rate	0.02
l	Value of leisure	0.13
b	UI benefits	0.47
y	General productivity level	1.00
C	Parameter of DM cost function	1.00
τ	Payroll tax	0.37
η	Credit availability in formal DM matches	0.20
s_e	Cost of formalization	0.01
δ	Job destruction rate	0.03
χ	Probability of government monitoring	0.07
ξ	Efficiency of LM matching function	1.00
σ	Elasticity of LM matching function	0.50
ω_e	LM formal workers bargaining power	0.50
ω_i	LM informal workers bargaining power	0.42
$F(\varepsilon)$	Distribution of match productivity	Uniform $[0,1]$

Unemployment insurance b is usually calibrated in the labor economics literature such that the average replacement ratio in the model matches the empirical replacement rate. In Brazil, the benefit level ranges between 100% to 187% of the minimum wage which means that the replacement rate is very high for formal workers at the bottom of the wage distribution (Gerard and Gonzaga, 2016). These benefits are only received by unemployed workers and informal workers who were formally employed. Trying to take this into account will complicate the calibration without much added value. To keep things relatively simple I assume that all informal workers can draw on unemployment benefits which I set at a replacement ratio of 50% and follow Pissarides (2000) by choosing the simpler expression:

$$\frac{b}{(1 - \tau)(y + \mathbb{E}[\varepsilon])} = 50\%$$

The elasticity of the LM matching σ is set to 0.5 as is standard in the literature. The matching function efficiency ξ is set to 1. The LM bargaining power of formal workers ω_e is set equal to σ in order to satisfy the Hosios efficiency condition. The bargaining power of informal workers is set to $\omega_i = \frac{\omega_e}{1 + \tau(1 - \omega_e)}$ such that the surplus of the match is split in the same proportion in formal and informal matches as in Bosch and Esteban-Pretel (2012). The parameter of the cost function C is set to 1. The payroll tax τ is set at 37.5% of formal wages as reported by the World Bank's Doing Business for non-wage labor costs in Brazil². The parameter s_e corresponds to labor related administrative costs incurred by formal firms. In Brazil, 1 day is spent on average to formalize a job (Bosch and Esteban-Pretel, 2012) which corresponds to 1/90 of quarter. The share of credit formal matches η is set at 20% which corresponds roughly to the average share of credit card transactions in total consumer spending transactions in Brazil³. From the

²These correspond to a 9% direct wage tax, a 20% employer's social security contribution and an 8.5% severance contribution.

³The figures are readily available at the website of the Associação Brasileira das Empresas de Cartões de Crédito e Serviços.

model the job separation rates in formal and informal matches are defined as follows:

$$JSR_e = \delta$$

$$JSR_i = \delta + \chi - \delta\chi$$

I set the rate of job destruction δ to match an observed quarterly job separation rate in formal labor of 3% (Bosch and Esteban-Pretel, 2012). For informal labor, there is very little evidence on the intensity of government monitoring χ . Therefore I use it to match the observed quarterly job separation rate in informal job matches such that $\chi = \frac{JSR_i - \delta}{1 - \delta}$. For Brazil, the observed JSR_i is 10% which implies a value for χ of around 7%.

Once all the exogenous parameters are set I proceed to jointly calibrate the second group of parameters. This group comprises utility function parameters A and a , cost function parameter ζ , DM bargaining power φ , informality punishment s_i and the cost of posting vacancies k . The joint calibration procedure consists in solving for the vector of calibrated parameters \mathcal{P} and the equilibrium solution $\{\theta, \varepsilon_i, \varepsilon_e\}$ which together reduce the distance (squared percentage difference) between the targeted statistics \mathcal{S}_{data} and the corresponding model statistics \mathcal{S}_{model} while taking the system of steady state equilibrium equations (11), (A.1) and (A.2) as equality constraints and imposing interval bounds \mathcal{I} on the value of some parameters:

$$\begin{aligned} \min_{\mathcal{P}, \theta, \varepsilon_i, \varepsilon_e} & (\mathcal{S}_{model}(\theta, \varepsilon_i, \varepsilon_e; \mathcal{P}) - \mathcal{S}_{data})^2 \\ \text{s. t.} & \quad \mathbf{EC}(\theta, \varepsilon_i, \varepsilon_e; \mathcal{P}) = 0 \quad ; \quad \mathcal{P} \in \mathcal{I} \end{aligned}$$

In order to specify the targets of the calibration procedure, I need to define how the model maps into the data. For that, I make the following definitions:

$$\begin{aligned} Y_{e_{DM}} &= n_e \sigma_f [\eta(g(q_e^c) - c(q_e^c)) + (1 - \eta)(g(q_e^m) - c(q_e^m))] \\ Y_{i_{DM}} &= n_i \sigma_f (g(q_i) - c(q_i)) \\ Y_{e_{CM}} &= n_e \left(y + \int_{\varepsilon_e}^{\bar{\varepsilon}} \varepsilon \, d \frac{F(\varepsilon)}{F(\bar{\varepsilon}) - F(\varepsilon_e)} \right) \\ Y_{i_{CM}} &= n_i \left(y + \int_{\varepsilon_i}^{\varepsilon_e} \varepsilon \, d \frac{F(\varepsilon)}{F(\varepsilon_e) - F(\varepsilon_i)} \right) \end{aligned}$$

where $Y_{e_{DM}}$ and $Y_{i_{DM}}$ are the net aggregate output of formal and informal firms sold in the DM and $Y_{e_{CM}}$ and $Y_{i_{CM}}$ are their net aggregate output sold in the CM respectively.

Depending on the country, the real GDP in the data might or might not take into account informal activities (Andrews et al., 2011). In some countries, GDP accounts only for formal activities which is the case for example in the US. In some others, GDP includes formal and some of the informal activities. This is increasingly the case in EU countries. For the purposes of calibration I assume that a share of informal activities is included in Brazil's GDP data. This allows me to define recorded activities in the model as the total of formal firms sales in both DM and CM and informal firms sales in the CM. M is

defined in the model as the amount of cash carried by households to be spent in informal and monetary formal transactions in the DM. In the data M corresponds to either M1 or sweep-adjusted M1 (Aruoba et al., 2011). As a consequence, I define the size of the informal sector as a share of observed GDP as:

$$\frac{Y_i}{Y} = \frac{Y_{i_{DM}} + Y_{i_{CM}}}{Y_{e_{DM}} + Y_{e_{CM}} + Y_{i_{CM}}}$$

and the model based money demand equation as:

$$L(i) = \frac{g(q_i)}{4 * (Y_{e_{DM}} + Y_{e_{CM}} + Y_{i_{CM}})}$$

where $L(i)$ depends on i through q_i and the LM measures and on the utility parameters A and a through $g(q)$.

Following Lucas (2000) and Lagos and Wright (2005), A and a are included in the joint-calibration in order to fit the model-based money demand function $L(i) \equiv \frac{M}{pY}$ to the data. The idea is to match two moments: the average real money balances at the average nominal interest rate and the elasticity of money demand to the nominal interest rate. To estimate the interest elasticity of money demand that I will target in the calibration, I use a log-log specification:

$$\log M/PY_t = \beta_1 + \beta_2 \log i_t + \nu_t$$

The OLS estimate of β_2 is used as a point estimate for the interest elasticity of money demand ϵ :

$$\epsilon = \frac{\partial L(i)}{\partial i} \frac{i}{L(i)}$$

while β_1 is used to pin down average money demand at the average i .

Following Aruoba et al. (2011) I use φ , the bargaining power of buyers in the DM, to target an average markup in the DM trades of 30%. Since the CM is a competitive market, the markup there is 0. The markup in informal transactions is:

$$\mu_i = \frac{g(q_i)/q_i}{c'(q_i)} - 1$$

and in formal transactions it is

$$\mu_e^c = \frac{g(q_e^c)/q_e^c}{c'(q_e^c)} - 1$$

$$\mu_e^m = \frac{g(q_e^m)/q_e^m}{c'(q_e^m)} - 1$$

The average markup in DM trades can be written as

$$\mu_{DM} = \frac{n_e(\eta\mu_e^c + (1-\eta)\mu_e^m) + n_i\mu_i}{n_e + n_i}$$

Taking the economy wide average, I get:

$$\bar{\mu} = \left(\frac{n_e(\eta\mu_e^c + (1-\eta)\mu_e^m) + n_i\mu_i}{n_e + n_i} \right) \frac{Y_{e_{DM}} + Y_{i_{DM}}}{Y}$$

The parameter k is used to target the average unemployment rate in Brazil over the period 1996-2014. The informality punishment s_i is included to target an informal employment of 30% of the total labor force. ζ is used to target an average size of the retail sector in Brazil of 20%.

Table 2: Calibration results

		Data	Model
Calibrated parameters			
k	Cost of vacancy posting	-	1.448
s_i	Informality punishment	-	4.411
φ	Bargaining power in DM	-	0.258
A	Parameter of DM utility	-	2.124
a	Parameter of DM utility	-	0.083
ζ	Parameter of DM cost	-	1.447
Calibration targets			
u	Unemployment	0.095	0.095
n_i	Informal employment	0.300	0.300
Y_{DM}/Y	Size of retail sector	0.200	0.200
μ_{DM}	Average markup in retail sector	0.300	0.300
$L(\bar{i})$	Average Money Demand	0.215	0.215
ϵ	Elasticity of MD to i	-0.418	-0.418
Equilibrium solution			
θ	LM tightness	-	0.767
ε_e	Formality threshold	-	0.781
ε_i	Informality threshold	-	0.420
Sum of squared residuals		-	0.000

Table 4 shows the result of the joint calibration. The model is able to match all of the targets based on reasonable values for the calibrated parameters. For example, the model is able to match the Brazilian money demand data as can be seen in figure 3.

6. Numerical results

In order to test the validity of the model I compare the model results with some empirical statistics which I did not specifically include in the calibration procedure. In table 3, I report the resulting values for different measures of the size of the informal sector using the model-based GDP measures introduced in the previous section. The different informality measures are based on different assumptions regarding the inclusion of informal sector in official GDP figures and are within reasonable range of the literature estimates of the size of the informal sector in Brazil, notably Schneider et al. (2011) who report an estimated average size of the informal sector in Brazil of 39% over the period 1999-2007 using the MIMIC (multiple indicators-multiple causes) model, a linear latent variables econometric approach.

Formal wages are on average 40% higher than informal wages which is in line with the literature on informality. The labor share of total income in the model economy is 71.7% whereas this share amounts to around 60% in Brazil.

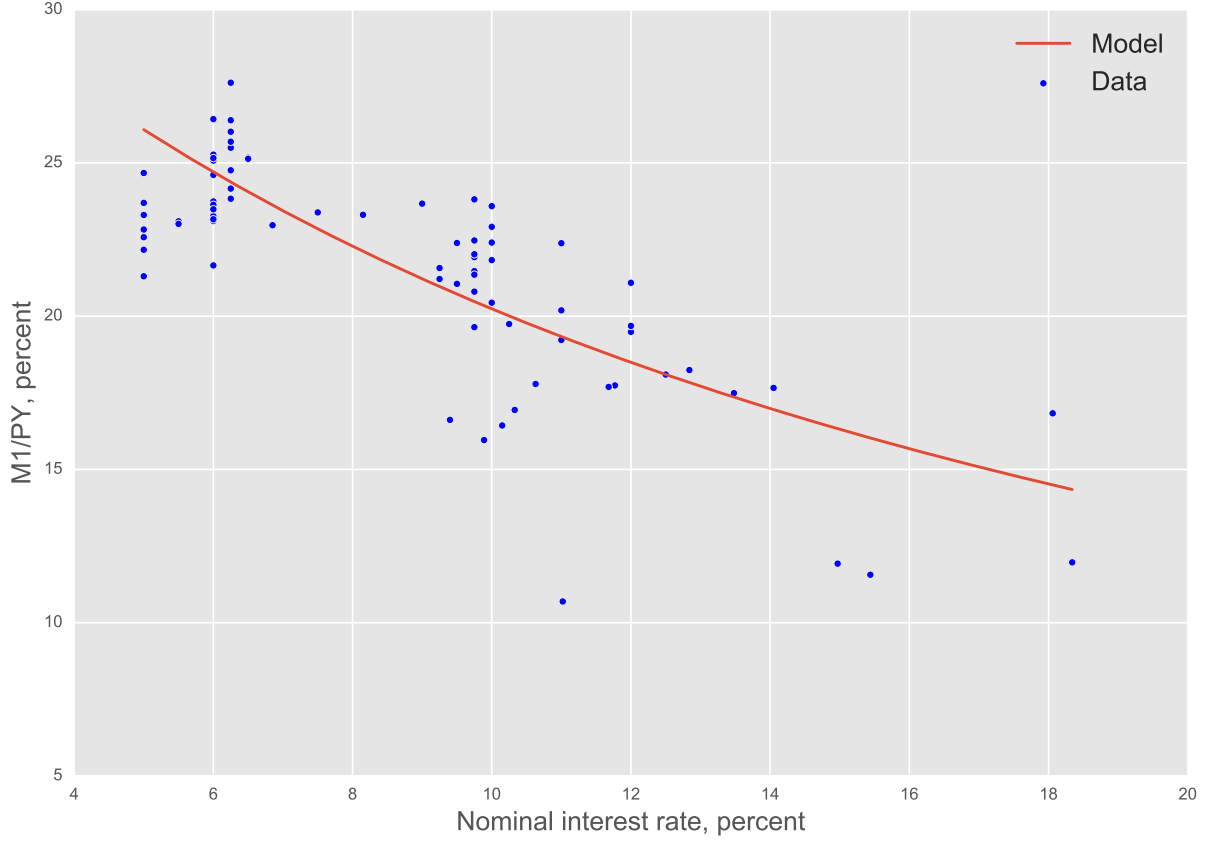


Figure 3: Money demand: model vs. data

Table 3: Different possible measures of the informal sector

Measure	$\frac{Y_i}{Y_e}$	$\frac{Y_i}{Y_e + Y_{iCM}}$	$\frac{Y_i}{Y_e + Y_i}$
Value	41.8%	31.0%	29.5%

Table 4: Model validation

		Data	Model
JFR_e	Formal job finding rate	0.150	0.191
JFR_i	Informal job finding rate	-	0.316
q_i/q^*	Ratio of monetary to credit consumption	-	0.429
$\ell\%$	Share of credit in retail sales (value)	0.150	0.217
μ	Economy average markup	-	0.060
$\Sigma w(\varepsilon)/Y$	Share of labor income	0.600	0.717
$(1 + \tau)\bar{w}_e(\varepsilon)/\bar{w}_i(\varepsilon)$	Average wage difference	-	1.400
$b/\bar{w}_e(\varepsilon)$	Average UI replacement rate	-	0.291

In table 5, I report elasticities of various measures to changes in some policy parameters. A 1% increase in inflation around the steady state equilibrium reduces both informal employment and unemployment by 0.42% and 0.03% respectively. This corresponds to the interpretation that an increase in inflation affects the labor allocation from high separation rate informal jobs to low separation rate formal jobs which simultaneously reduces the size of the informal sector and reduces unemployment. To see that, figure 4 shows the relationship between the nominal interest rate and hence inflation with the composition of

the labor force. An increase in nominal interest rates from 5% to 18% drives informality down from 35% to around 20% with a slightly negative impact on unemployment. However, increasing inflation by 1% reduces the aggregate output by 0.04%. This is because inflation taxes also formal monetary transactions which limits its effectiveness as a tool against informality.

Increasing the availability of credit for formal firms by 1% reduces unemployment and informal employment by 0.1% and 0.33% respectively. Interestingly enough, financial development is effective at reducing unemployment while inflation is better at fighting informality. This implies that a combination of the two instruments might be a better policy.

In line with the literature, an increase of 1% in the payroll tax increases unemployment and informal employment by 0.48% and 1.34% respectively. Better government monitoring and lower red tape costs, two proxies for the quality of a country institutions such as the rule of law and the strength of the state, are the best way to reduce the size of the informal sector. Increasing χ , s_i or reducing s_e reduces both unemployment and informality without hurting output. However, it is known that institutions are very sticky and any positive impact of improving them is not likely to show up over the short or medium run.

Table 5: Elasticities w.r.t. changes in different policy parameters

	i	η	τ	s_e	χ	s_i
u	-0.03	-0.10	0.48	0.02	-1.14	-0.63
n_i	-0.42	-0.33	1.34	0.07	-4.83	-2.62
JFR_e	0.24	0.28	-1.22	-0.06	3.60	2.02
JFR_i	-0.38	-0.23	0.85	0.04	-2.86	-1.99
Y	-0.04	0.02	-0.06	-0.01	0.14	0.09
Y_i/Y	-0.45	-0.37	1.45	0.07	-5.28	-2.80

7. Conclusion

In this paper, I introduced a dynamic general equilibrium model of inflation, unemployment and informality. This model provides a flexible framework to understand interactions between the informal sector and the rest of the economy. In the model, informality results from the optimal choices of firms and households given the frictions they face when deciding how to allocate labor, how much to consume and how to pay for their consumption. A main innovation of this model is to connect informality in both labor and goods markets by constraining buyers matched with firms employing informal workers to the use of money as a means of payment in retail transactions. Inflation affects unemployment and informality by taxing money holdings agents carry from one period to the next. The higher cash intensity of informal transactions makes informal firms and jobs more vulnerable to inflation compared to the formal sector.

One would think that a bigger informal sector will make the positive long run relation between inflation and unemployment stronger. However, if we consider the differences in separation rates between formal and informal jobs then the long-run Philips curve might become vertical or slightly negatively sloped. This is because the composition of the labor force in terms of formal and informal jobs is affected by monetary conditions prevailing in the economy. An increase in inflation reduces the trade surplus for both formal and informal firms. This lowers the benefits of vacancy posting reducing in turn entry of firms. Lower

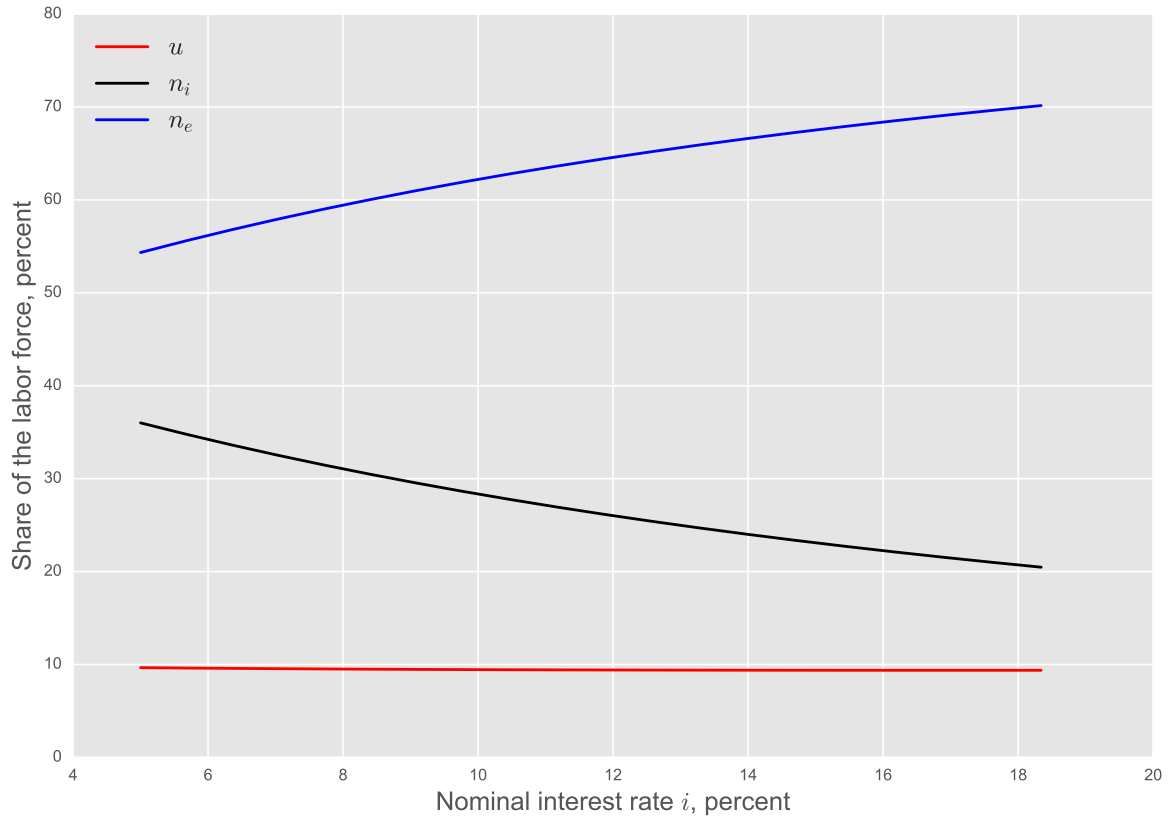


Figure 4: Effect of a change in i on steady state labor force composition

firm entry pushes unemployment upwards which reduces the matching probability in the goods market further amplifying the fall in job creation and the rise of unemployment. This positive effect of inflation on unemployment is referred to as the matching channel. An opposite effect works through the hiring channel where an increase in inflation pushes new entrants towards formal jobs. Since formal jobs exhibit a lower job separation rate, this shift from informal to formal employment reduces unemployment. Since buyers are randomly matched to firms in the retail sector, the higher ratio of formal to informal firms increases the buyers' matching probability with formal firms which further reduces the attractiveness of informality. The existence of this two channels renders the effect of inflation on unemployment ambiguous.

I presented the results of a calibration exercise to match certain long-run statistics of the Brazilian economy. The model is able to replicate most of the stylized facts found in the empirical literature on informality. Numerical results show that inflation has only a slightly positive effect on unemployment. However, the impact of inflation on labor allocation between formal and informal jobs is significant. These results indicate that the composition of the labor force between formal and informal employment matters for the inflation-unemployment trade-off which points to the importance of understanding and measuring informality for the conduit of monetary policy.

Appendix A Derivation of the productivity thresholds

To solve for the two equilibrium thresholds ε_e and ε_i , one way would be to rewrite the model's set of functional equations in terms of the thresholds and the model's parameters as detailed below.

I start from the firm's side. First, I subtract from (2) the same expression evaluated at ε_i to get

$$V_i^f(\varepsilon) - V_i^f(\varepsilon_i) = \frac{R_i(\varepsilon) - w_i(\varepsilon) - \beta\chi s_i}{1 - \beta(1 - \chi)(1 - \delta)} - \frac{R_i(\varepsilon_i) - w_i(\varepsilon_i) - \beta\chi s_i}{1 - \beta(1 - \chi)(1 - \delta)}$$

Using the first threshold condition this simplifies to:

$$V_i^f(\varepsilon) = \frac{(1 - \omega_i)(\varepsilon - \varepsilon_i)}{1 - \beta(1 - \chi)(1 - \delta)}$$

Applying the same steps to (1) I get:

$$V_e^f(\varepsilon) - s_e = \frac{(1 - \omega_e)(\varepsilon - \varepsilon_e)}{1 - \beta(1 - \delta)} + \frac{(1 - \omega_i)(\varepsilon_e - \varepsilon_i)}{1 - \beta(1 - \chi)(1 - \delta)}$$

Using the threshold conditions, the wage equations and the corresponding value functions, I solve for the two productivity thresholds to get:

$$\varepsilon_i = -y - \sigma_f(g(q_m) - c(q_m)) + \beta\chi s_i + \frac{\omega_i}{1 - \omega_i}\theta k + b + l \quad (\text{A.1})$$

$$\begin{aligned} \varepsilon_e = \Gamma_1(1 - \omega_e) \left[-y - \sigma_f\eta(g(q^*) - c(q^*)) - \sigma_f(1 - \eta)(g(q_m) - c(q_m)) + s_e + (1 + \tau)(b + l) \right. \\ \left. + \frac{\omega_e}{1 - \omega_e}\theta k \right] - \Gamma_2(1 - \omega_i)\varepsilon_i \end{aligned} \quad (\text{A.2})$$

where

$$\begin{aligned} \Gamma_1 &= \frac{(1 - \beta(1 - \chi)(1 - \delta))}{(1 - \beta(1 - \chi)(1 - \delta))(1 - \omega_e) - (1 - \beta(1 - \delta))(1 - \omega_i)} \\ \Gamma_2 &= \frac{(1 - \beta(1 - \delta))}{(1 - \beta(1 - \chi)(1 - \delta))(1 - \omega_e) - (1 - \beta(1 - \delta))(1 - \omega_i)} \end{aligned}$$

The two thresholds along with the job creation condition

$$\frac{k}{\beta\alpha_f} = \frac{(1 - \omega_e)}{1 - \beta(1 - \delta)} \left(\bar{\varepsilon} - \varepsilon_e - \int_{\varepsilon_e}^{\bar{\varepsilon}} F(\varepsilon) \, d\varepsilon \right) + \frac{(1 - \omega_i)}{1 - \beta(1 - \chi)(1 - \delta)} \left(\varepsilon_e - \varepsilon_i - \int_{\varepsilon_i}^{\varepsilon_e} F(\varepsilon) \, d\varepsilon \right)$$

form the system of equilibrium equations.

Appendix B Proofs

B.1 Effect of an increase in i on q_m

To measure the effect of an increase in i on q_m I proceed by totally differentiating equation (5):

$$\begin{aligned} \frac{v''(q_m)g'(q_m) - v'(q_m)g''(q_m)}{g'(q_m)^2} dq_m &= \frac{1}{\sigma_e(1-\eta) + \sigma_i} di - \frac{i}{(\sigma_e(1-\eta) + \sigma_i)^2} (1-\eta) d\sigma_e - \frac{i}{(\sigma_e(1-\eta) + \sigma_i)^2} d\sigma_i \\ &+ \frac{i}{(\sigma_e(1-\eta) + \sigma_i)^2} d\eta \end{aligned}$$

Evaluating the above expression at the Friedman rule ($i = 0$) and rearranging the terms, I get:

$$\frac{dq_m}{di} = \frac{1}{\sigma_e(1-\eta) + \sigma_i} \left(\frac{g'(q_m)^2}{v''(q_m)g'(q_m) - v'(q_m)g''(q_m)} \right) < 0$$

The effect of i away from the Friedman rule depends on the effect of inflation on LM conditions which affects the matching probabilities in the DM.

B.2 Proof of Proposition 2

To prove proposition 2 we compare the worker's decision thresholds resulting from his indifference equations to those resulting from the firm's indifference equations.

The worker's decision thresholds should be satisfying the following two indifference equations:

$$\begin{aligned} V_i^h(\varepsilon_i) &= V_u^h \\ V_e^h(\varepsilon_e) &= V_i^h(\varepsilon_e) \end{aligned}$$

Starting from the firms conditions (8) and (9) and applying the sharing rules (6) and (7) to derive the implied indifference levels for the worker I get

$$\begin{aligned} V_i^h(\varepsilon_i) &= V_u^h \\ \frac{(1+\tau)(1-\omega_e)}{\omega_e} (V_e^h(\varepsilon_e) - V_u^h) &= \frac{1-\omega_i}{\omega_i} (V_i^h(\varepsilon_e) - V_u^h) \end{aligned}$$

The two equation sets above result in the same indifference equation for ε_i which means that the firm and worker always agree on the separation decision. However, depending on parameter values, the resulting decision thresholds ε_e which determine the choice of the work arrangement are not necessarily the same.

B.3 Proof of Proposition 3

Differentiating the job creation condition (11) w.r.t. θ yields:

$$-\frac{k}{\beta\alpha_f^2} \frac{\partial\alpha_f}{\partial\theta} = \left(\frac{1-\omega_i}{1-\beta(1-\chi)(1-\delta)} - \frac{1-\omega_e}{1-\beta(1-\delta)} \right) (1-F(\varepsilon_e)) \frac{\partial\varepsilon_e}{\partial\theta} - \frac{1-\omega_i}{1-\beta(1-\chi)(1-\delta)} (1-F(\varepsilon_i)) \frac{\partial\varepsilon_i}{\partial\theta}$$

From equations (A.1) and (A.2), I can show that:

$$\begin{aligned}\frac{\partial \varepsilon_i}{\partial \theta} &= \frac{\omega_i}{1 - \omega_i} k > 0 \\ \frac{\partial \varepsilon_e}{\partial \theta} &= (\Gamma_1 \omega_e - \Gamma_2 \omega_i) k > 0\end{aligned}$$

for the parameter values used in the calibration. From the definition of the matching function, we also have $\frac{\partial \alpha_f}{\partial \theta} < 0$. Hence, it can be easily shown that the LHS of the equation above is positive while the RHS is negative. This implies that the LHS of equation (11) is increasing in θ while the RHS is decreasing in θ . Hence θ is unique.

B.4 Proof of Proposition 6

Differentiating equation (A.1) with respect to i keeping θ constant gets:

$$\left. \frac{\partial \varepsilon_i}{\partial i} \right|_{\partial \theta = 0} = -\sigma_f \left(\frac{\partial g(q_i)}{\partial q_i} - \frac{\partial c(q_i)}{\partial q_i} \right) \left. \frac{\partial q_i}{\partial i} \right|_{\partial \theta = 0} > 0$$

This result is valid for $q_i \ll q^*$.

Differentiating equation (A.2) with respect to i keeping θ constant gets:

$$\begin{aligned}\left. \frac{\partial \varepsilon_e}{\partial i} \right|_{\partial \theta = 0} &= \Gamma_1 (1 - \omega_e) \left[-\sigma_f (1 - \eta) \left(\frac{\partial g(q_e^m)}{\partial q_e^m} - \frac{\partial c(q_e^m)}{\partial q_e^m} \right) \left. \frac{\partial q_e^m}{\partial i} \right|_{\partial \theta = 0} \right] - \Gamma_2 (1 - \omega_i) \left. \frac{\partial \varepsilon_i}{\partial i} \right|_{\partial \theta = 0} \\ &= \left[1 - \eta \Gamma_1 (1 - \omega_e) \right] \left. \frac{\partial \varepsilon_i}{\partial i} \right|_{\partial \theta = 0} < 0\end{aligned}$$

where the sign depends on parameter values and in particular on η , the share of credit transactions in formal DM matches. For the values specified in the calibration $\left. \frac{\partial \varepsilon_e}{\partial i} \right|_{\partial \theta = 0} > 0$.

B.5 Proof of Proposition 7

Differentiating the job creation condition (11) w.r.t. i yields:

$$\begin{aligned}-\frac{k}{\beta \alpha_f^2} \frac{\partial \alpha_f}{\partial \theta} \frac{d\theta}{di} &= \left(\frac{1 - \omega_i}{1 - \beta(1 - \chi)(1 - \delta)} - \frac{1 - \omega_e}{1 - \beta(1 - \delta)} \right) (1 - F(\varepsilon_e)) \left(\frac{\partial \varepsilon_e}{\partial i} + \frac{\partial \varepsilon_e}{\partial \theta} \frac{d\theta}{di} \right) \\ &\quad - \frac{1 - \omega_i}{1 - \beta(1 - \chi)(1 - \delta)} (1 - F(\varepsilon_i)) \left(\frac{\partial \varepsilon_i}{\partial i} + \frac{\partial \varepsilon_i}{\partial \theta} \frac{d\theta}{di} \right)\end{aligned}$$

Rearranging the terms, I get:

$$\frac{d\theta}{di} = \frac{\left(\frac{1 - \omega_i}{1 - \beta(1 - \chi)(1 - \delta)} - \frac{1 - \omega_e}{1 - \beta(1 - \delta)} \right) (1 - F(\varepsilon_e)) \frac{\partial \varepsilon_e}{\partial i} - \frac{1 - \omega_i}{1 - \beta(1 - \chi)(1 - \delta)} (1 - F(\varepsilon_i)) \frac{\partial \varepsilon_i}{\partial i}}{\left(\frac{1 - \omega_e}{1 - \beta(1 - \delta)} - \frac{1 - \omega_i}{1 - \beta(1 - \chi)(1 - \delta)} \right) (1 - F(\varepsilon_e)) \frac{\partial \varepsilon_e}{\partial \theta} + \frac{1 - \omega_i}{1 - \beta(1 - \chi)(1 - \delta)} (1 - F(\varepsilon_i)) \frac{\partial \varepsilon_i}{\partial \theta} - \frac{k}{\beta \alpha_f^2} \frac{\partial \alpha_f}{\partial \theta}}$$

From the results above, it can easily be shown that the numerator is negative. The denominator

depends on the signs of $\frac{\partial \varepsilon_i}{\partial \theta}$ and $\frac{\partial \varepsilon_e}{\partial \theta}$. From equations (A.1) and (A.2) I get:

$$\begin{aligned}\frac{\partial \varepsilon_i}{\partial \theta} &= \frac{\omega_i}{1 - \omega_i} k > 0 \\ \frac{\partial \varepsilon_e}{\partial \theta} &= (\Gamma_1 \omega_e - \Gamma_2 \omega_i) k > 0\end{aligned}$$

for the parameter values used in the calibration. Since $\frac{\partial \alpha_f}{\partial \theta} < 0$, I conclude that $\frac{d\theta}{di} < 0$.

This result shows that the total effect of an increase in inflation and i on the hiring standards (i.e. $\frac{d\varepsilon_i}{di}$ and $\frac{d\varepsilon_e}{di}$) is ambiguous.

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