Hagem, Cathrine; Holtsmark, Bjart; Sterner, Thomas

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Cathrine Hagem, Bjart Holtsmark and Thomas Sterner

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Abstract:
We analyze two mechanism designs for refunding emission payments to polluting firms; Output Based (OB) and Expenditure Based (EB) refunding. In both instruments, emissions fees are returned to the polluting industry, possibly making the policy more easily accepted by policymakers than a standard tax. The crucial difference between OB and EB is that the fees are refunded in proportion to output in the former, but in proportion to the firms’ expenditure on abatement equipment in the latter. We show that to achieve a given abatement target, the fee level in the OB design exceeds the standard tax rate, whereas the fee level in the EB design is lower. Furthermore, the use of OB and EB refunding may lead to large differences in the distribution of costs across firms. Both designs do, strictly speaking, imply a cost-ineffective provision of abatement as firms put relatively too much effort into reducing emissions through abatement technology compared with emission reductions through reduced output. However, this may be seen as an advantage by policymakers if they seek to avoid activity reduction in the regulated sector. We provide some numerical illustrations based on abatement cost information from the Norwegian NOx fund.

Keywords: Refunded charge, Output based, expenditure based; NOx, Tax-subsidy, policy design

JEL classification: Q28, Q25, H2

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Address: Cathrine Hagem, Statistics Norway, Research Department. E-mail: cah@ssb.no
Bjart Holtsmark, Statistics Norway, Research Department. E-mail: bjj@ssb.no
Thomas Sterner, Department of Economics, School of Business, Economics and Law, University of Gothenburg, P.O. Box 640, SE 405 30 Gothenburg, Sweden.
Discussion Papers comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.
Sammendrag

Vi analyserer to ulike mekanismer for tilbakebetaling av miljøavgiftsproveny til forurensende bedrifter; produksjonsbasert (Output Based, OB) og utgiftsbasert (Expenditure Based, EB). I begge systemene blir miljøavgiften tilbakebetalt til den forurensende industriken, noe som kan gjøre at reguleringen blir lettere å akseptere enn et standard avgiftssystem der provenyet blir beholdt av myndighetene. Den vesentlige forskjellen mellom OB og EB er at avgiftene refunderes i forhold til produksjon i den første, og i forhold til investeringer i miljøteknologi i den siste. Vi viser at for å oppnå et bestemt utslippsmål må avgiftsnivået i OB-systemet være høyere enn i et standard avgiftssystem, mens avgiftsnivået i EB er lavere. Bruken av OB og EB kan gi svært ulik fordeling av kostnader mellom bedrifter. Begge systemene vil strengt tatt lede til en ineffektiv reduksjon av utslipp siden bedriftene legger for stor vekt på investeringer i renseteknologi fremfor reduksjoner i produksjonsnivået. Det kan imidlertid sees på som en fordel av politikere som ønsker å unngå produksjonsnedgang i den regulerte sektoren. Vi viser noen numeriske illustrasjoner basert på kostnadsanslag fra det norske NOx fondet.
1. Introduction

It is well known that a uniform tax, levied on all sources of emissions, is a cost effective instrument to reduce emissions. However, environmental taxes tend to be lower than what is required to correct for environmental externalities. One reason for this is concerns that high taxes reduce domestic competitiveness, leading to firm closures and job losses. Furthermore, for transboundary pollutants, such as greenhouse gases, the welfare effect of reduced domestic emissions may be partly offset by increased production and emissions abroad (carbon leakage), see Hoel (1991). Moreover, powerful lobbies may overstate these and other arguments and thus make a cost effective tax regime politically infeasible.

Though not first-best optimal, earmarking of tax revenues may make taxes on emissions more politically feasible. Some people even see the use of the tax revenues as the environmental purpose of the tax (this is discussed extensively in Sterner and Coria, 2011). Kallbekken et al., 2011 show that recycling the revenues to more narrowly targeted groups seems to increase support for taxation. Various ways of refunding the tax revenue ease the economic burdens on the polluters. Refunding emissions turns a tax into a fee, rebated to the polluters according to some specified rules.

This paper compares two methods for refunding emission payments:

- Refund in proportion to output, referred to as output based refunding (OBR).
- Refund in proportion to abatement expenditure, referred to as expenditure based refunding (EBR). (We also use the terms OB or EB designs, schemes or systems respectively).

Sweden has pioneered the use of OBR for NO\textsubscript{x} (sometimes simply referred to as Refunded Emission Payment or REP in the literature\textsuperscript{1}). Among the reasons for the use of OBR rather than a tax, was the difficulty in setting a sufficiently high price on emissions to motivate abatement when faced with a strong industrial lobby against the tax, see further Sterner and Isaksson (2006). In Sweden, fees are refunded to the polluters strictly in proportion to useful output. In the French “Taxe Parafiscale” the fees (for NO\textsubscript{x}, SO\textsubscript{2}, HCl and VOCs) were used generally to subsidize research and abatement, see

\textsuperscript{1} See, e.g. Fischer (2003), Gersbach and Requate (2004), Fredriksson and Sterner (2005), Häglund Isaksson (2005) and Cato (2010). NB also that OBR corresponds to output allocation or “benchmarking” in a tradable permit system, see Fischer (2001). OBR generates an output subsidy and thus gives incentives for excess production. This effect is harmful in a competitive environment, but can increase welfare under imperfect competition, where output is suboptimal (Gersbach and Requate, 2004). Benchmarking in unilateral CO\textsubscript{2} emissions policies is motivated by its potential to reduce carbon leakage and loss of competitiveness (Edwards and Hutton, 2001, and Fischer and Fox, 2007).
Millock et al. (2004). The Norwegian NOx goes a step further and directly ties refunding to actual abatement costs at the firm level.

To our knowledge, EBR has not been previously analyzed\(^2\). The contribution of this paper is the evaluation of the differences in the performance of the EB and OB systems, including a comparison of their qualities and properties with those of a standard environmental tax.\(^3,4\) EBR implies a combination of a fee on pollution and a subsidy on abatement technologies where the latter is financed by the former\(^5\).

In section 3, we show that to achieve a certain abatement target, the fee level under OBR must exceed the standard tax rate, whereas with EBR, it must be set lower. Both OB and EB refunding make technical abatement relatively cheaper than abatement through output reductions, compared to a standard tax. Hence, both designs might appear to lead to a cost-ineffective provision of abatement. However, in the real world, taxes are seldom optimal and we cannot in general tell which of the designs that deviate the most from cost-effectiveness since this also depends on the level of the fee which is attainable with each different design. Furthermore, we show that in cases where output is insensitive to changes in variable costs, both systems are close to cost-effective, but may still lead to very different distribution of costs across firms. In section 4, the potential differences in fee levels and distribution of costs across firms are illustrated by the use of data from the Norwegian NOx fund. Section 5 concludes.

### 2. A model of tax contra refunded fees

Consider a sector consisting of firms \(F = \{1, \ldots, n\}\), producing a commodity in quantities \(q_i\). Production causes emissions, \(e_i\).

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\(^2\) There is a related literature on two-part instruments. There are a number of papers that recognize that the Pigouvian tax level may for various reasons be unattainable, and which therefore, explore alternatives such as various combinations of tax and subsidy, see for instance, Fullerton (1997) and Fullerton and Wolverton (1999) who find that if a tax is infeasible, a satisfactory alternative may combine a subsidy to a clean substitute with a tax on output (see also Fullerton and Mohr 2003 or Walls and Palmer 2001, for similar points). None of these articles however specifically looks at refunding of the tax per se.

\(^3\) We focus on self-financed funds whose income comes exclusively from fees (or designated membership contributions).

\(^4\) An advantage of a standard tax without refunding is that it generates public revenues. In this paper we do not take into consideration the efficiency loss due to the revenue recycling effect (Goulder et al. 1999).

\(^5\) NB this is not the type of subsidy where the policy maker “buys” each unit of reduction from a baseline at a price \(s\), see, e.g., Kolstad (2000), chapter 7. Such subsidies clearly imply a perverse output subsidy. Inspired by the Norwegian and French NOx schemes, we instead model subsidies as the partial payment for the costs of particular abatement equipment.
Let $e_i^0$ and $q_i^0$ denote emissions and production in the absence of environmental policy. Each firm can reduce emissions by abatement technologies or by reduced production. The abatement technologies are denoted $y_i$ and capture various technical measures, including end-of pipe technologies, fuel switching and process optimization.\footnote{We do not address the dynamic effect or irreversibility of investments (see e.g. Coria, 2009, for such models). Furthermore we do not in this paper delve into the incentives for technological progress created by refunding, see Sterner and Turnheim, (2006). See also Jaffe et al. (2002), L"oschel (2002) and Requate (2005) for surveys on technology investment incentives under various policy instruments. Note also that it may be significant in practice to distinguish between measures that are easy to control through simple inspection and those that are either too complex or too subtle to allow for simple monitoring.} We have

\begin{equation}
(1) \quad e_i(r_i, y_i) = e_i^0 - a_i(r_i, y_i),
\end{equation}

where $a(r_i, y_i)$ is the emission reduction (abatement) function and $r_i$ is output reduction ($q_i^0 - q_i$).

Abatement is increasing in output reductions and abatement technologies; $a'_r(\cdot) > 0$, $a'_y(\cdot) > 0$.

Furthermore, we assume that the abatement function is strictly quasi concave, and both $r_i$ and $y_i$ are normal input factors in the production of $a_i$. As a starting point, we consider internal solutions for firm production decisions. However, possible impacts on exit and entry decisions will be discussed in section 3.4.

The negative externality of emissions is assumed to be a linear function of total emissions equal to $\tau E$, where $E$ is global emissions, that is:

\begin{equation}
(2) \quad E = \sum_{i \in F} e_i,
\end{equation}

Furthermore, we define:

\begin{equation}
(3) \quad Q = \sum_{i \in F} q_i,
\end{equation}

And

\begin{equation}
(4) \quad A = \sum_{i \in F} a_i \equiv \sum_{i \in F} e_i^0 - E.
\end{equation}
We assume that all market shares are so small that all firms take all prices, taxes/fees and subsidies as given.

### 2.1 Standard tax

With a standard tax, an individual firm has the following payoff:

\[ \pi_i = p_i q_i - c_i(q_i) - m_i y_i - t e_i(r_i, y_i), \]

where $p_i$ is the product price and $c_i(q_i)$ are production costs of $q_i$, $t$ is the tax on emissions, $m_i$ is the (annuity) cost of abatement technologies $y_i$, and $e_i(r_i, y_i)$ are given by (1). We apply the standard assumptions that marginal cost of production is positive and increasing; $c_i' > 0$ and $c_i'' > 0$. \(^7\)

Maximizing the profit functions yields the following first order conditions\(^8\):

\[ \frac{m_i}{a_i'(r_i, y_i)} = t, \]

\[ \frac{p_i - c_i'}{a_i'(r_i, y_i)} = t. \]

The left hand side of (6) expresses the marginal cost of reducing emissions through abatement equipment, and the left hand side of (7) expresses the marginal cost of emission reductions through output reductions. The intuition behind the first order conditions is well-known: Marginal cost of emissions reductions should equal the marginal benefit of emissions reduction ($t$), regardless of whether the emission reduction occurs through abatement technologies (6), or through output reduction (7).

It is well known that (6)-(7) lead to a cost effective (cost minimizing) combination of output reduction and abatement technology deployment for all emissions levels, and an optimal emission level allocation as long as the level of $t$ corresponds to the true social marginal damage, hereafter named the Pigouvian tax level.

\[^7\] In real life, abatement technologies may also affect the cost of producing $q$. However, our simplification does not affect the main results of the paper.

\[^8\] Second order conditions for an internal solution are given in appendix A.
2.2 Mechanism design for Output Based refunding emissions payments.

With a standard tax, the total tax revenue, $tE$, is collected by the government as public revenue. A fund system implies that the tax becomes a fee and the revenue is collected by the fund. We shall focus on such cases where all the revenue is reimbursed fully to the polluters themselves. We consider two stylized schemes for refunding emission fees: output based refunding (OB) and expenditure based refunding (EB).

In an output based refunding scheme, fees are refunded in proportion to output $q_i$, and the payoff to firm $i$ is:

\[
\pi_i = p_i q_i - c_i(q_i) - m_i y_i - t e_i(q_i, y_i) + q_i t \frac{E}{Q},
\]

where $t$ is the fee (corresponding to a standard tax), and $\frac{E}{Q}$ is the average emission. The refund per unit production is $t \frac{E}{Q}$. The budget constraint is satisfied as $\sum_{i \in F} q_i t \frac{E}{Q} = tE$. This corresponds exactly to refunding the total fee revenue $tE$ in proportion to the firms’ market shares, $\frac{q_i}{Q}$, as in the Swedish NOx scheme. As all firms’ market shares are assumed to be small they do not take into account their influence on the average emission intensity when they make their output and abatement decisions. However, the average emission intensity becomes endogenous and a function of $t$.

Assuming that the firms under OBR maximize their payoffs specified in (1) and (8), the first order conditions are:

\[
9 \text{ If there is significant diversification of output across firms, refunding could alternatively be based on gross sales values although this would entail incentives to inflate sales figures in ways unrelated to the activities that generate emissions.}
\]

\[
10 \text{ Gersbach and Requate (2004) use this set-up in the competitive version of their model.}
\]

\[
11 \text{ See Fischer (2003) and Sterner and Isaksson (2006) for a model of an OB system where such strategic behavior is discussed. See also Gersbach and Requate (2004) for an analysis of an OB system in the context of imperfect competition in the output market and pre-investment in cleaner technology.}
\]

\[
12 \text{ Second order conditions equals the second order condition for the standard tax system, see Appendix A. However, if we do not assume that the output share of each firm is treated as given, there is an extra factor (1-q_i/Q) in (9) - (10). When all firms’ share of total output is small, this factor is negligible, see further Fischer (2003) and Sterner and Isaksson (2006).}
\]
As in the standard tax system, we find that the marginal cost of emissions reductions through abatement technology should be equated with the level of the fee for all firms, eq. (9). However, we see from (10) that the marginal cost of avoided emissions through output reductions should no longer be equated with the level of the fee \( t \) but multiplied by the factor \( 1 - \frac{E}{Qa'_p(r, y_i)} \).

As \( 1 - \frac{E}{Qa'_p(r, y_i)} < 1 \), the output effect characteristic of a fee is much weaker\(^{13} \), compared with a standard tax system. The intuition here is that the industry does not effectively pay the scarcity rent on the emissions (as this is refunded) and hence there is no reason to expect a significant output effect.

Another way of stating this is that under a tax, a firm would hesitate to produce one more unit of production because they have to pay the costs of the extra pollution incurred. Under an OBR, the extra unit of production also implies more refunds. We see from (1) that, \( a'_p(r, y_i) = c'_i(q_i, y_i) \) and is labeled \textit{marginal emissions intensity} (emissions intensity of the marginal unit produced).

Note that for firms with marginal emissions intensity lower than the average emissions intensity, that is, \( a'_p(r, y_i) < \frac{E}{Q} \), the right hand side of (10) is negative. This implies that that profit maximizing marginal production costs exceed the output price \( p_i < c'_i \). For these firms, the OB system works as a net subsidy on production: The tax reimbursement per unit production more than offset the emissions tax following from one unit production. Hence, firms with marginal emissions intensity lower than the average emissions intensity increase their production under an output based refunding scheme compared to a situation without any environmental policy, where profit maximization yields \( p_i = c'_i \). Firms with higher intensity increase their production.

\(^{13} \) Under simple, but reasonable, assumptions (such as constant emissions intensities) it will be on average zero so there would be no output effect at all.
2.3 Expenditure Based refunding (EB).

With an expenditure based refund scheme, all fee revenues are refunded to the polluters in proportion to their expenditures for abatement and thus the firms have in return a share of their abatement costs. The payoff to a firm $i$ is

\begin{equation}
\pi_i = p_i q_i - c_i(q_i) - (1-s)m_i y_i - te_i(q_i, y_i),
\end{equation}

where $s$ is the subsidy rate for abatement cost expenditure.

As we consider competitive firms, they all take $t$ and $s$ as given when making their decisions. However, it follows from the budget constraint that $s$ must be set at least approximately as a function of $t^{14}$:

\begin{equation}
s = \frac{iE}{MY},
\end{equation}

where

\begin{equation}
MY = \sum_{i \in F} m_i y_i.
\end{equation}

Assuming firms maximize the payoffs specified in (1) and (11), the first order conditions are$^{15}$:

\begin{equation}
\frac{(1-s)m_i}{a'_i(r, y)} = t,
\end{equation}

\begin{equation}
\frac{p_i - c'_i}{a'_i(r, y)} = t.
\end{equation}

By comparing (6) and (14), we see that the expressions for the marginal cost of reducing emissions through abatement technologies differ between systems. As $s$ is a subsidy-rate, not directly addressing any externality, we refer to the left hand side of (6) as the marginal social cost of reducing emissions

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$^{14}$ To ensure an internal solution, we assume that the regulator always sets $t$ such that $s<1$. In practice the budget of the fund might not have to balance exactly every year and $s$ may also be adjusted over time.

$^{15}$ Second order conditions equal the second order conditions for the standard tax system, see Appendix A.
through abatement technologies, whereas the left hand side of (14) is the marginal private cost of reducing emissions through abatement technologies in the EB scheme. It thus follows from (14) and (15) that the social marginal cost of reducing emissions through abatement expenditure exceeds the social marginal cost of reducing emissions through output reductions.

3. A comparison of two mechanisms for refunding

In this section, we evaluate the EB and OB systems regarding cost-effectiveness, and we compare the fee levels and distribution of costs across firms. The standard tax system is used as a benchmark. However, we acknowledge that a standard tax system may, for political or practical reasons, not be achievable. We introduce the subscripts ST, OB and EB to refer to the outcomes of the standard tax system, the OB system, and the EB system, respectively.

3.1 Comparisons with same fee level.

As we saw from the previous section, the output effect of a fee under the OB system is much weaker compared with a tax under a standard tax system. Under the EB system, on the other hand, the fee on pollution is complemented with a subsidy on abatement equipment, and thus makes abatement less costly. Comparing the two systems with the outcome of a tax system, we must decide whether the comparison assumes equal tax rates or equal abatement. We start with the former and derive the following propositions:

**Proposition 1:**

Given any tax level \( t \), we have that \( a_{i}^{EB}(t) > a_{i}^{ST}(t) > a_{i}^{OB}(t) \).

**Proof of proposition 1:**

The outcome of the EB system can be mimicked by a standard tax system with prices on the abatement technology equal to \((1 - s)m_{i} < m_{i}\), and \( t = t^{ST} \). We have that \( \frac{\partial a_{i}}{\partial m_{i}} < 0 \) as

\[
\frac{\partial a_{i}}{\partial m_{i}} = \frac{\partial d_{i}}{\partial t} < 0 \text{ (see appendix A)},
\]

such that \( a_{i}^{EB}(t^{ST}) > a_{i}^{ST}(t^{ST}) \). The outcome of the OB system can be mimicked by a standard tax system with a tax rate of \( t^{ST} \) and \( p^{OB} = p + k^{OB} \), where

\[
k^{OB} = t^{ST} \frac{E^{OB}}{Q^{OB}} > 0.
\]

We have that \( \frac{\partial a_{i}}{\partial p} < 0 \) as \( \frac{\partial a_{i}}{\partial p} = -\frac{\partial q_{i}}{\partial t} < 0 \) (see appendix A), such that

\[
a_{i}^{OB}(t^{ST}) < a_{i}^{ST}(t^{ST}).
\]
3.2 Comparisons with the same target level of abatement.

**Proposition 2:**
Consider a given target for abatement, and let $\bar{T}^{ST}$, $\bar{T}^{OB}$ and $\bar{T}^{EB}$ denote the tax/fee levels which ensure that the target is met in the standard tax system, the OB system, and the EB system, respectively. We then have $\bar{T}^{EB} < \bar{T}^{ST} < \bar{T}^{OB}$.

**Proof of proposition 2:**
The abatement in the standard tax system is an increasing function of the tax rate for all firms

$$\left(\frac{\partial a_i}{\partial t} = a_i \frac{\partial r}{\partial t} + a'_i \frac{\partial y}{\partial t} > 0, \text{ see appendix A}\right).$$

Furthermore, we know from proposition 1 that

$$a_i^{EB}(t) > a_i^{ST}(t) > a_i^{OB}(t), \text{ for all } t.$$ Hence, the $t^{OB}$ which satisfies

$$\sum_i a_i^{ST}(\bar{T}^{ST}) = \sum_i a_i^{OB}(t^{OB})$$

must be larger than $\bar{T}^{ST}$, and the $t^{EB}$ which satisfies

$$\sum_i a_i^{ST}(\bar{T}^{ST}) = \sum_i a_i^{EB}(t^{EB})$$

must be smaller than $\bar{T}^{ST}$.

As the tax rate in the OB system is higher than the tax in the standard tax system, whereas the tax rate in the EB system can be set lower, for identical emissions reductions, there might be fewer objections to the EB system than to the tax system. The OB system does have a higher fee level but thanks to the automatic refund, the average company pays nothing and both theory and experience show that it may be more easily acceptable under some circumstances than a tax (see Fredriksson and Sterner 2005). However it could be costly for firms with emission intensive production and limited abatement options. Furthermore, output based refunding can be controversial to operate when firms produce differentiated products and output itself is hard to measure.

On the other hand, as the EB system redistributes the tax revenue only in terms of subsidies to abatement technologies, firms that have already implemented relevant abatement technologies (or have clean production) receive no refunding of the tax revenue and could complain that they are unfairly treated. In general, the two types of funds may lead to very different distributions of costs across firms. In the numerical section below, we illustrate how the tax rate and distribution of costs across firms may deviate between the two systems.

The magnitude of the differences in tax levels necessary to achieve a specific target depends inter alia on the firms’ sensitivity to changes in variable production costs and on total emissions. For instance, consider a situation where outputs are sensitive to changes in production costs. Abatement induced by
a standard tax system is in that case achieved by significant reductions in output (as a tax on emissions implies higher production costs). As discussed above, the output effect of an emissions fee under the OB system can be absent or very modest. To compensate for this, the fee level may be considerably higher than the standard tax level in order to induce sufficient investment in abatement technology. On the other hand, if output is insensitive to changes in variable production costs, the standard tax system induces abatement mostly through investment in abatement technology, and there is little difference between the fee levels of the OB system and the standard tax system.

We see from (12) that the higher is the tax base (E), the lower \( t \) is necessary to provide a given subsidy rate in the EB system. Hence, the larger the emissions, the larger is the difference in the tax rate between the standard tax system and the EB system.

Note that although the regulator, by appropriate tax/fee levels, can achieve the same total abatement level in all three systems, both EB and OB system will lead to a combination of abatement and output reduction that is cost-ineffective compared to the ideal first best (if and when that is achievable or applicable). In that first best, a standard tax leads to a cost minimizing distribution of output reductions and abatement technology measures for all abatement levels.

**3.3 Further comparisons between EBR and OBR.**

We turn now to an analysis of how emissions are reduced under the different instruments.

**Proposition 3:**

For all levels of emission reductions, the OB and EB systems lead to cost-ineffective combinations of abatement technologies and output reductions. Abatement is achieved by too much investment in abatement technologies and too little output reductions compared with first best.

**Proof of proposition 3:**

We see from (6)-(7) that an optimal combination of \( y_i \) and \( q_i \) implies that

\[
a_{y_i}^*(r_i, y_i) = \frac{m_i}{p_i - c_i'},
\]

whereas from (9) and (10), we find that

\[
a_{y_i}^*(r_i, y_i) = \frac{m_i}{p_i - c_i' + t \frac{E}{Q}}.
\]

From (14) and (15) we find that

\[
\frac{a_{y_i}^*(r_i, y_i)}{a_{y_i}^*(r_i, y_i)} = \frac{(1 - s)m_i}{p_i - c_i'}. \quad \text{Let superscript OB indicate the solution that follows from (9)-(10), and let}
\]
superscript EB indicate the solution that follows from (14)- (15). Due to the quasi concavity assumption of the abatement function, we must have that for any abatement level, \( y_i^{EB} > y_i^{ST} \) and \( r_i^{EB} < r_i^{ST} \), and \( y_i^{OR} > y_i^{ST} \) and \( r_i^{OB} < r_i^{ST} \).

Although the design of the OBR and EBR differ substantially, they both make abatement through technology investments relatively cheaper than abatement through output reduction, compared to a standard tax. With EBR, this follows from the subsidized abatement technology, whereas the OBR subsidizes output.

In practice, OBR and EBR are likely to be used when political or practical conditions make a sufficiently high \( t \) in a standard tax system impossible or undesirable\(^{16}\). Thus, from a political viewpoint, the relevant comparison might be between a standard tax system with a low \( t \), and thus low abatement target, and OB and EB systems with a high(er) abatement target. In this case, production might still be higher under the OB and EB systems, but certainly, there will be higher expenditures on abatement activities. As mentioned, higher production is often considered desirable by politicians. Furthermore they often like the idea of speeding up abatement investments too: these investments also create employment and they may speed up the development of the abatement industry itself through scale effects and learning by doing, possibly even creating export opportunities or other strategic advantages for domestic industry.

As we discussed in the introduction, the motivation for refunding is often to appease business and to prevent firm closure, job loss and "carbon leakage". We know that both the EB and OB refunding mechanisms lead to higher output and higher investments in abatement technologies than the standard tax system (proposition 3). The more the outcome deviates from the first best, the higher is the total cost of achieving the emission target. On the other hand, the establishment of a fund is typically motivated by a preference for a different combination of output reductions and abatement technology than what follows from a standard tax system. To determine the preferred fund system it is therefore vital to know how the two different funds perform regarding output reduction versus investments in abatement technology. However, that demands an empirical investigation of the specific regulation.

\(^{16}\) Politicians, who take many of the relevant decisions, rarely want output reduction in any sector. To the opposite, one of the main obstacles for any environmental policy is the threat to jobs. Although a reduction in jobs in some sector might be part of an economically “optimal” strategy, it rarely goes down well with voters. Furthermore the conditions under which the policy is optimal may not be met. This applies inter alia to cases where there is oligopoly or small open economies where there is a threat to competitiveness through foreign competitors who do not face environmental policies.
Proposition 4
Given a specific abatement target, we cannot in general tell whether OBR or EBR leads to the largest output and investment in abatement technologies.

Proof of proposition 4:
For a given emission level, we see from (10) and (15), that \( q_i^{OB} > q_i^{EB} \) (and thus \( y_i^{OB} > y_i^{EB} \) if and only if

\[
(16) \quad t^{OB} \left[ d'_{i}^{*} (r_i^{OB}, y_i^{OB}) - \frac{E}{Q} \right] < t^{EB} d'_{i}^{*} (r_i^{EB}, y_i^{EB})
\]

Consider a situation where all firms are identical and \( a_{p,y}^* = 0 \) (as in the case of end of pipe abatement). If marginal emissions intensity is close to the average, the squared bracket on the left hand side of (16) is close to zero, whereas \( t^{EB} \) must be positive and is increasing in the abatement target. Hence, in this case (16) is satisfied and \( q_i^{OB} > q_i^{EB} \) and \( y_i^{OB} > y_i^{EB} \). Consider a situation where marginal emission intensity is significantly above average. For a sufficiently low \( t^{EB} \) (relative to \( a_{i}^{*} (r_i^{OB}, y_i^{OB}) \)), (16) will not be satisfied. We see from (12), that any given subsidy to abatement technologies (sMY) can be achieved for a very low \( t^{EB} \) if the tax base (E) is sufficiently large. Hence, for a sufficiently large E and sufficiently low demand for abatement, and thus abatement investments, (16) is not satisfied and \( q_i^{OB} < q_i^{EB} \) and \( y_i^{OB} < y_i^{EB} \).

Whether OBR or EBR achieves the abatement target with the highest investment in abatement technologies and thus lowest reduction in output depends on the difference in the costs of output reductions relative to the cost of abatement technology across the two systems.

The difference in relative cost depends inter alia on the ex post emissions and the average emissions intensity relative to marginal emission intensity. Large total emissions and thus a corresponding large tax base induce a large subsidy rate (s) in the EB system, other things being equal (see (12)), and a large subsidy rate favors abatement through abatement technology relative to output reductions. Furthermore, we see from the first order conditions of the OB system that high average emissions intensity relative to marginal emission intensity makes it costly to decrease output and hence favors abatement through abatement technology versus output reductions. Hence, if the total target for abatement is small relative the total emissions, (s is large), and the marginal emissions intensity for
most firm is large relative to the average, \((1 - \frac{E}{Qa'(r_t, y_t)})\) do not deviate too much from 1, we would expect that \(Q^{oB} < Q^{eB}\), and vice versa.

Although both OBR and EBR lead to over investment in abatement technologies and too little output reductions compared with first best, we can derive the following proposition regarding distribution of abatement efforts across firms:

**Proposition 5:**
Both the OB system and the EB system lead to a cost effective distribution of output reductions and abatement technology investments across firms.

**Proof of proposition 5:**
From (10) we see that marginal social costs of production \((p_t - c'_t - ta'_t(r_t, y_t))\) are equalized across all firms \(= \frac{E}{Q}\). From (9) we see that that marginal social costs of abatement technology \(m - ta'_t(q_t, y_t)\) are equalized across firms \(=0\). As \(\frac{1}{1-s}\) is identical for all firms, we see from (14) that social marginal cost of emissions reductions are equalized across all firms. Furthermore, it follows from (15) that marginal cost of emission reductions through reduced output are equalized across all firms.

Proposition 5 follows from the fact that within each of the fund systems, marginal costs of abatement through output reductions are equalized across all firms and marginal cost of abatement through investment in abatement technology is equalized across all firms.

Although both EBR and OBR lead to a cost-ineffective provision of abatement (proposition 3), the cost-ineffectiveness of the system may not always be very prominent. A fund system might be applied to industries where a standard tax system mainly would induce abatement through only one of the sources; output reductions or abatement technology investments. The first situation occurs if firms’ decision regarding abatement technologies is insensitive to changes in the price of the technologies, and the latter situation occurs if output is insensitive to changes in variable production costs. These situations render both fund systems close to cost-effective in the short run (see section 3.4 for a discussion on long run implications).
Corollary 1:

If the OB or EB systems are used to regulate an industry where either output is insensitive to changes in variable costs or the level of investment in abatement technologies is insensitive to changes in their prices, both systems can be designed to yield outcomes that are close to first best.

If for instance production is capital intensive, with large sunk costs, and a given production capacity, it may be profitable for the firm to operate close to the production capacity under all of the emissions policy systems. In that case, almost all emissions reductions occur through investment in abatement technologies. As both OB and EB systems distribute abatement cost effectively across firms (proposition 5), any distribution of abatement technologies following from a standard tax system can (almost) be achieved by the OB and EB systems.

Note that it follows from proposition 2 that although the outcome of both OBR and EBR can be close to first best, the tax rate and the distribution of cost across firms may differ substantially between the two systems. This is exemplified in the numerical illustration in section 0.

3.4 Effects on exit and entry of firms.

In the modeling of the performance of the two types of funds in the previous sections, we ignored potential impacts on exit and entry. A big difference between a standard tax and a refunded fee is that the former generates public revenue, whereas the fund systems are revenue neutral, as the emission tax (fee) collected by the fund are redistributed to the firms. A standard tax may lead to a situation where some firms are no longer able to cover their average cost for any output level, and it is profitable for them to close down production. Compared with a standard tax system, the fund systems may prevent closures as the firms receive an income from the reimbursement of emission taxes. Hence, refunding could in principle lead to a lower degree of exit compared with a standard tax. We cannot in general tell which of the fund systems (EB or OB) would have a stronger effect in preventing closures, as the two types of funds may lead to very different distributions of costs across firms. For instance, as the EB system redistributed the tax revenue only in terms of subsidies to abatement technologies, it benefits firms with large current options for abatement investments (which may often be coupled with other investments decided by plant vintage). Firms that have low emission levels compared to the average would, on the other hand, tend to prefer OBR to EBR, as they receive a higher than average refund of their tax payment.
We argued in section 5 that firms with marginal emission intensity lower than average emission intensity would increase their production under an OB scheme compared to a situation without any environmental policy. Thus, an OB system may induce entry of new firms with low emissions per unit output, as output based refunding in that case works as a net subsidy on production. On the other hand an EB system might also help avoid the exit of a plant that has to undergo relatively expensive reparations or refurbishment if the firm is able to get the investment classified as an improvement from the viewpoint of abatement and thus get a subsidy reducing the expenditure incurred. It is worth noticing that the issue of monitoring and enforcement connected to emissions that may serve dual purposes and verification of actual (as opposed to claimed) abatement costs could be quite tricky in some cases.

4. Numerical illustration based on the Norwegian NOx Fund.17

In January 2007 the Norwegian government imposed a tax of 15 NOK/kg NOx (2 Euros/kg NOx) on emissions from the main industrial sources including large vessels. In May 2008 the Norwegian NOx Fund was proposed by industry and then established as an alternative to the tax and replaced the tax as the main NOx policy instrument in Norway. Firms that choose to be a member of the Fund are exempted from paying the NOx tax, and pay instead a membership fee to the Fund that is proportional to their respective NOx emissions. The Fund is a private foundation owned and managed by the most relevant business organizations, and the fact that a private foundation in this way has been given the mandate to replace tax collection is quite unique. It illustrates the obvious fact that if an industry is going to be forced to pay abatement through a tax, it is in their collective interest, to try to arrange the same outcome voluntarily to avoid the payment on those emissions that are allowed. In return the Fund has therefore committed to carry out reductions in NOx emissions within Norwegian jurisdiction such that total NOx emissions from the relevant industries drop to below 98 000 tons NOx/year, 20 000 tons below the 2007 level.

Firms that are engaged in the offshore oil- and gas extraction pay a member fee of 11 NOK/kg NOx emitted. Other firms’ fee is 4 NOK/kg NOx. The firms in the offshore oil- and gas extraction are responsible for 43 per cent of the fund members’ NOx emissions, but provide 68 per cent of the Fund’s revenue.

17 This section builds on data received from the Norwegian NOx Fund, the collaboration with which is gratefully acknowledged.
As mentioned, the Norwegian NOx Fund represents an example of an expenditure based (EB) policy for regulation of NOx emissions. The Fund operates on a non profit basis and its revenue has so far almost completely been refunded to the firms as support for emission reducing investments.

4.1 Illustrative difference in fee levels.

The following numerical example is based on the Norwegian policy but simplified to illustrate primarily Proposition 2, by showing to what extent a certain emission reduction in an EB system could be achieved by a fee that is lower than the corresponding tax in a standard tax system\textsuperscript{18}.

Figure 1.

---

\textsuperscript{18} The data available from the NOx Fund comprises only the cost of abatement through investments in abatement technology. Hence, we have no information about the potential for and the cost of activity reductions as a mean for emission reductions. We therefore assume throughout the numerical illustration that the level of production activity is given. As shown in Corollary 1, this assumption renders both systems close to cost-effective, thus the illustrations eliminate potential differences in the two systems’ properties regarding cost-effectiveness. However, this may not be a very strict assumption for the industries covered by the Norwegian NOx Fund, at least in the short run. A major share of the Fund’s members operates within the fishing industry or produce supply services for the offshore oil and gas industries. Hence, we are to a large extent dealing with firms that base their activities on boats and other relatively capital intensive production concepts that are likely to operate close to the production capacity, whether they are supposed to pay a NOx tax or not. Hence, it appears likely that emission reductions, in fact, by a large share of the firms mainly will be carried out through installation and use of abatement technologies as well as adjusting or renewing the engines.
Based on data received from the Norwegian NOx Fund, we have estimated the involved industries’ total costs related to emission reductions up to 20 000 tons NOx, which was the Fund’s commitment. We show how thanks to the subsidy, significant abatement was achieved in spite of a very low fee level.\(^\text{19}\)

The marginal cost curve in Figure 1 (single solid line) provides estimated marginal abatement costs, based on data from the Norwegian NOx Fund.\(^\text{20}\) Thus, this curve also shows the tax level of a tax or an OB system (given our assumption of fixed outputs) for the corresponding achieved emission reduction levels given along the horizontal axes. Annual emission reductions of 20 000 tons NOx would have required a tax level slightly below 20 NOK/kg NOx. This suggests that the Norwegian tax of 15 might not have been quite high enough. A higher tax would no doubt have met with some resistance from the polluters. One solution to this would be, as in Sweden to have a high fee level but refund the fee in proportion to output to make the fee more politically acceptable.

The dashed curve in Figure 1 shows what fee level would have been sufficient if abatement costs would have been subsidised by 50%. Note that there could be many such lines showing different combinations of subsidy rates and fee levels. If for instance abatement costs would have been subsidised by 90%, then a fee of just 2 NOK/kg NOx would have been sufficient to cover abatement costs up to 20 NOK/kg NOx and would thus have made it possible to attain the aggregate abatement target of 20 000 tons NOx. Naturally, we must also check for each particular level whether or not the combination of subsidy and fee selected is consistent with the Fund’s budget constraint, see equation (12).\(^\text{21}\) Interestingly enough, it appears from the data we have, that even a fee of 2 NOK/kg NOx would actually have been enough to finance a subsidy rate at 90 per cent thus balancing the budget of the

\(^{19}\) As mentioned the Norwegian NOx Fund has two different fee levels; both are low compared to the original tax of 15 NOK/kg NOx or to the Swedish output based fee of 50 SEK or roughly 45 NOK. Our numerical example abstracts from this assuming all members pay the same fee, and we show how this fee level could be reduced and still meet the same level of ambition.

\(^{20}\) The abatement costs per unit of NOx depend on the lifetime of the projects and the discount rate. We assume a discount rate of 7 percent. The lifetime of the projects vary. In order to make the numerical example transparent and illustrative we have here assumed that all projects have lifetimes of 10 years, although a share of the projects probably has longer lifetimes.

\(^{21}\) It follows from equation (14) that if the subsidy rate is s, then abatement projects with unit costs below \(t/(1-s)\) are commercially profitable. Hence, with the subsidy rate set to 90 per cent, abatement projects with unit costs ten times the fee level will be commercially profitable. Hence, a fee equal of 2 NOK/kg NOx would mean that projects up to 20 NOK/kg NOx are profitable. Annual emissions are then reduced to 98 000 kg NOx. Hence, the Fund’s annual revenue would be 184 million NOK. The costs of all the projects are 204 million NOK (the area below the grey curve). 90% of this abatement cost is 183 Million NOK and thus the Fund’s budget constraint is (almost exactly) met.
This illustrates an important difference between the two refund systems considered – in our example, the OB system requires a fee that is about an order of magnitude larger than the required fee in the EB system. Note, however, that an important factor to explain this difference in the fee level is that the emission reduction considered is relatively modest. Hence, the Fund’s income basis is to a large extent preserved. If the emission reductions were much more ambitious, the result would be different.

4.2 Differences in distributional outcomes.

This subsection presents a numerical example to illustrate the different distributional effects of the OB and EB refunding. The numerical example shows that the choice of abatement policy system potentially has large inter-firm distributional effects. Firstly, the example shows that firms could collect significant gains when a refund system is introduced, compared to the standard tax system. Secondly, the numerical examples will illustrate that different firms will prefer different systems. Firms that have few or no options for cheap emission reductions will collect small or no refunds in the EB system, but might collect a significant refund in the OB system. Firms with considerable potential for low cost emission reductions prefer the EB system.

In the Norwegian NOx Fund a typical abatement project is investment in a less emission intensive vessel engine. Consider, for example, two firms, A and B, both with an identical cargo vessel. Both produce 0.02 per cent of total output from the regulated industry. Corresponding to the previous numerical example, we assume that output is not changed by the NOx policies considered, neither in the firms considered nor in total. From the NOx Fund’s data we know that a typical cargo vessel could reduce its emission by approximately 60 tons NOx annually (from 80 to 20), with an average abatement cost of approximately 5 NOK/kg NOx. Hence, we construct the following example: The firms are identical in all respects except one: Firm A has before the NOx policy was introduced already replaced its engine with a new one leading to lower NOx emissions. This type of investment could in certain cases be profitable to the firms as a new engine not only causes lower NOx emissions, but also uses the fuel more efficiently. For one reason or the other, (maybe because of a younger vintage) firm B still uses the ship’s original engine with higher emissions. While the vessel of firm A emits only 20 tons NOx, firm B’s vessel emits 80 tons NOx/year, see Table 1. Furthermore, we apply the fee levels

---

22 The levels imposed by the Norwegian NOx Fund (11 and 4 NOK/kg NOx for oil and gas industries and other industries, respectively) are higher than in our example. We have assumed that the fee is maintained throughout the life time of the investment projects, assumed to be 15 years. If the Norwegian NOx Fund sustains their high fee levels, it would appear that the Fund could make a significant surplus.

23 As the firm specific data from the Norwegian NOx Fund are confidential, we construct a hypothetical example.
derived in the previous subsection \( t_{OB} = 20 \text{ NOK/kg NOx} \), and \( t_{EB}=2 \text{ NOK/kg NOx} \) and the subsidy rate of 90%. Hence, in both systems it is profitable to reduce emissions to the level where marginal abatement costs are 20 NOK/kg NOx.

Table 1. A numerical example with two firms

<table>
<thead>
<tr>
<th></th>
<th>Standard tax</th>
<th>OB</th>
<th>EB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Firm A</td>
<td>Firm B</td>
<td>Firm A</td>
</tr>
<tr>
<td>Emissions ex ante (tons NOx)</td>
<td>20</td>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>Emissions ex post (tons NOx)</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Abatement costs (Thousand NOK)</td>
<td>-</td>
<td>300</td>
<td>-</td>
</tr>
<tr>
<td>Fee/tax rate (NOK/kg NOx)</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Refund (Thousand NOK)</td>
<td>-</td>
<td>-</td>
<td>400</td>
</tr>
<tr>
<td>Fee/tax paid (thousand NOK)</td>
<td>400</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Net costs (Thousand NOK)</td>
<td>400</td>
<td>700</td>
<td>-</td>
</tr>
</tbody>
</table>

Assume that the technical conditions are such that firm B could reduce its emissions by 60 tons NOx at a cost of 300 KNOK. Hence, firm B experiences average abatement costs of 5 NOK/kg NOx, while the costs of emission reductions in firm A would exceed 20 NOK/kg NOx and are therefore not profitable in any of the considered systems.

Table 1 presents the outcome of the standard tax system, and the OB and EB systems. It follows from our assumptions that the outcome for total abatement and the distribution of abatement across firms are identical across all systems. Furthermore, both firms’ ex post emissions intensities equal the ex post average emissions intensity. Table 1 shows payments in the first year (thus including the abatement payment as well as taxes and refunds). Firm B always pays its 300 in abatement costs. Emission payments are either 400 with tax or OBR, or 40 under EBR. Refunds are also 400 under OBR since these firms are average firms (with respect to emissions intensity). Under EBR only firm B gets any refund, in our example 90% of 300. We see that both firms are better off with the OB or EB systems compared with a standard tax regime. The firm with the large potential for abatement (firm B) prefers the EB system, whereas firm A, with no profitable abatement options, prefers the OB system.

For firm A, there is no cost associated with the OB system. As its emissions intensity equals the ex post average emission intensity, all of its emissions payment is refunded in the OB system. Even though the fee is much lower in the EB system, the firm prefers the OB system as it does not receive any refunding in the EB system.
As firm B’s ex post emission intensity also equals the average emissions intensity, all of its fee payment is reimbursed in the OB system. However, with the OB system the firm has to cover the full costs of its investments in abatement technologies. In total, firm B’s costs of the OB system is significantly larger than under the EB system, where 90% of the abatement costs are reimbursed and the level of the fee very low.

5. Concluding remarks

Our point of departure was that policy makers and indeed the public at large usually emphasize a number of aspects of environmental policy in addition to efficiency: activity reductions in local industries, possible job losses, leakage effects (polluting industries relocate to other countries), and distributional concerns loom large. Hence, the environmental policy instrument preferred by most economists, the Pigouvian tax, is seldom as popular as economists think it deserves to be. We have analyzed two alternative mechanisms that imply refunding emission payments to polluting firms; Output Based (OB) and Expenditure Based refunding (EB). Our main findings are as follows:

Given a certain emission target, the tax/fee level that ensures that the target is met is lower in the standard tax system than in the OB system, but even lower in the EB system than in the standard tax system. These differences, in turn, lead to differences in the distribution of cost across firms. Both refunding mechanisms lead to cost-ineffective combinations of abatement and output reduction, with higher output and more abatement than efficient from the viewpoint of a simple economic model. However, this effect may be seen as an advantage by policymakers if they seek to avoid activity reduction in the regulated sector.

Intuitively one might perhaps expect that an EB system would lead to higher investment in abatement technologies than the OB systems. However, we found that this is not a general result. Both the considered mechanisms minimize output reduction.

Although both refund mechanisms lead to the types of inefficiencies mentioned above, both systems lead to an efficient distribution of production and abatement technologies across firms. If output is insensitive to changes in variable costs, both OB and EB refunding can be designed to yield outcomes that are relatively close to first best.
Finally we use numerical examples to show that the choice of refund mechanism might have important consequences with regard to distributional effects. Firms that already have implemented abatement technologies before the introduction of a refund mechanism, are likely to prefer a an OB system, while a firm that still has profitable investment options in abatement technologies, would prefer the EB system.

Note also that an important difference between the two systems is the regulator’s needs for information to regulate the systems effectively. All systems (including a regular tax) require information about firms’ emissions. In addition, the OB system requires information about output, which is usually available but in many cases may be open to manipulation (through transfer pricing within concerns for instance or through vertical integration). In the Swedish NOx policy case, the physical heat and energy output of boilers was chosen as a readily verifiable and relevant measure. It is not unlikely that the lack of applications in other areas is due to the difficulty of using for instance gross sales values as a measure.

The EB system, on the other hand, demands knowledge about the firms’ costs of purchasing and utilizing the abatement technology. Due to asymmetric information between the Fund’s manager and the firms, the firms may gain large informational rents by overstating the cost of the abatement technology. Recall here that emission reductions often is a consequence of different types of commercially profitable projects, as rebuilding engines for higher efficiency, and so forth. Furthermore, in order to gain informational rent, the firms may not choose the most cost-effective abatement technology project, but instead implement technologies where their private cost information advantage is the largest.

In this paper, we mention various reasons why policy makers may prefer fund systems to a standard tax system. An area for future research is to analyze more specifically how the optimal choice of emission policy mechanisms depends on how the policy makers’ emphasize the various policy targets (emissions reductions, industries’ activity level, income distribution, diffusion of abatement technologies, etc.).

We believe that economists have put insufficient interest on the use of revenues collected by environmental fees. For business, NGOs and politicians alike, this is often the main issue. Our model has shown how refunding, either in proportion to output or to pay partially for abatement, can improve the incentives under some conditions. Experience from Sweden, Norway and other countries have shown that this type of refunding can make the policies more politically acceptable at least in the case of such cases as NOx from large industrial sources. Whether or not there may be a case for applications in the climate area is an interesting issue left for future research.
References


Appendix A

In this appendix, we derive second order conditions for global profit maximum, and show how the equilibrium outcomes changes with $t$, $m$ and $p$ in the standard tax system.

We write the second order derivatives of the profit function as follows:

\[
\frac{\partial \pi_i}{\partial q_i} = \pi_{q_i}, \frac{\partial^2 \pi_i}{\partial y_i \partial q_i} = \pi_{qy}, \frac{\partial^2 \pi_i}{\partial y_i \partial y_i} = \pi_{yy}, \frac{\partial^2 \pi_i}{\partial q_i \partial y_i} = \pi_{qy}.
\]

Given that the second order conditions for global profit maximum are satisfied we must have; $\pi_{yy} = ta_{yy}^* < 0,$ $\pi_{qq} = -c_r^t + ta_{yr}^* < 0,$ and $\pi_{yy} \cdot \pi_{qq} - (\pi_{yq}^* )^2 > 0.$ $\pi_{qy}^* = -ta_{yr}^*.$

To find how the equilibrium outcomes of the standard tax system changes with $t$, $m$ and $p$ we totally differentiate the two equations (6) and (7). Given the second order conditions and our assumption about the abatement functions, ($r_i$ and $y_i$ are normal input factors), we are able to sign some of the effects:

\[
\frac{\partial y_i}{\partial t} = -\frac{a_{r_i} \cdot \pi_{qy}^* + a_{r_i} \cdot \pi_{qy}^*}{D} > 0,
\]

\[
\frac{\partial q_i}{\partial t} = -\frac{a_{r_i} \cdot \pi_{qy}^* + a_{r_i} \cdot \pi_{qy}^*}{D} < 0,
\]

\[
\frac{\partial y_i}{\partial m_i} = \frac{\pi_{yq}^*}{D} < 0,
\]

\[
\frac{\partial q_i}{\partial m_i} = -\frac{\pi_{yq}^*}{D},
\]

\[
\frac{\partial y_i}{\partial p} = \frac{\pi_{yq}^*}{D},
\]

\[
\frac{\partial q_i}{\partial p} = -\frac{a_r \cdot \pi_{yq}^*}{D} > 0.
\]

Where

\[
D = \pi_{yy} \cdot \pi_{qq} - (\pi_{yq}^* )^2.
\]
Statistics Norway

Oslo:
PO Box 8131 Dept
NO-0033 Oslo
Telephone: + 47 21 09 00 00
Telefax: + 47 21 09 00 40

Kongsvinger:
NO-2225 Kongsvinger
Telephone: + 47 62 88 50 00
Telefax: + 47 62 88 50 30
E-mail: ssb@ssb.no
Internet: www.ssb.no

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