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Working Paper

Prices vs. quantities: Technology choice, uncertainty and welfare

Discussion Papers, No. 677

Provided in Cooperation with:
Research Department, Statistics Norway, Oslo

Suggested Citation: Storrøsten, Halvor Briseid (2012) : Prices vs. quantities: Technology choice, uncertainty and welfare, Discussion Papers, No. 677, Statistics Norway, Research Department, Oslo

This Version is available at:
http://hdl.handle.net/10419/192659
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Abstract:
This paper shows that tradable emissions permits and an emissions tax affect the firms' technology choice differently under uncertainty. A tax encourages the most flexible technology if and only if stochastic costs and the equilibrium permit price have sufficiently strong positive covariance, compared with the variance in consumer demand for the good produced. Moreover, the firms' technology choices are socially optimal under tradable emissions permits, but not under an emissions tax. Hence, modeling endogenous technology choice provides an argument in favor of tradable emissions permits as compared with emissions taxes.

Keywords: Regulation; Technology choice; Welfare; Uncertainty; Investment.

JEL classification: H23; Q55; Q58

Acknowledgements: I am grateful to Cathrine Hagem, Michael Hoel and Knut Einar Rosendahl, whose comments improved the paper considerably. Financial support from the Renergi program of the Norwegian Research Council is acknowledged. While carrying out this research I have been associated with CREE - Oslo Centre for Research on Environmentally Friendly Energy. CREE is supported by the Research Council of Norway.

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Discussion Papers comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.
Sammendrag

Artikkelen viser at omsettbare utslippskvoter og en avgift på utslipp påvirker bedrifters teknologivalg ulikt under usikkerhet. Bedriftene velger den samfunnsøkonomisk optimale teknologien under kvotehandel, men ikke under en avgift. Modellering av endogent teknologivalg gir dermed et argument for omsettbare utslippskvoter sammenliknet med en avgift på utslipp.
1 Introduction

Technological improvements have proven essential in mitigating environmental problems such as climate change, depletion of the ozone layer and acid rain. In the longer run, the ability to spur technical innovations and implementation of advanced abatement equipment may be the single most important factor when evaluating public environmental policy.\(^1\) Therefore, it is not surprising that the literature on R&D and firms’ incentives to invest in advanced abatement technology is vast.\(^2\)

However, as pointed out by Krysiak (2008), one aspect of this literature is somewhat surprising: these studies tend to analyze how much is invested,\(^3\) but do not consider the kind of technology that is implemented. This constitutes a shortcoming of the literature. For example, emissions reductions of SO\(_x\) and NO\(_x\) may be achieved either by installing scrubbers\(^4\) or by relying on fuel substitution to, e.g., low-sulfur coal. Similarly, emissions of CO\(_2\) may be reduced by, e.g., a switch from coal to gas or carbon capture and storage (CCS). How this choice is affected by the environmental policy regime is arguably an important consideration in evaluation of public policy. Furthermore, firms’ technology choice will affect the demand for technology and, thereby, the direction of R&D effort (see, e.g., Griliches, 1957 or Ruttan, 2001).

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\(^1\)See, e.g., Kneese and Schultze (1975) or Orr (1976) for an early presentation of this view.

\(^2\)See Jaffe and Stavins (1995) for an empirical approach.

\(^3\)See Jaffe et al. (2002), Löschel (2002), or Requate (2005) for reviews of the literature.

\(^4\)That is, e.g., post-combustion flue-gas desulfurization and selective catalytic reduction, respectively.
This paper examines how environmental regulation affects induced technology choice, and how this influences the optimal choice between regulatory instruments. We consider two types of regulation: tradable emissions permits and an emissions tax. These are presently by far the most important cases where both price- and quantity-based regulatory approaches are suitable. We will not consider how uncertainty concerning the relative slopes of the environmental damage function and the firms' abatement cost functions affects the ranking of price- and quantity-based regulation. That topic is analyzed by, e.g., Weitzman (1974), Hoel and Karp (2001, 2002) and Newell and Pizer (2003).

Under tradable emissions permits, the government sets a cap on aggregate emissions, and the issued licenses to emit (permits) are tradable among firms. Prominent examples of such schemes are found in the EU emissions trading scheme, the US SO$_2$ trading program and various regulatory schemes for NO$_x$ emissions in the US.$^5$ Price-based approaches like harmonized prices, fees, or taxes currently have no international experience (Nordhaus, 2007). However, emissions taxes have considerable national experience. Two examples are the US tax on ozone-depleting chemicals and the Norwegian CO$_2$ tax.

We introduce two sources of uncertainty: demand-side uncertainty represented by random variables in the consumer utility function, and supply-side uncertainty entered through the environmental damage function. The demand-side uncertainty is modeled as a random variable $\sigma_{\text{demand}}$ that affects the consumer's utility function. The supply-side uncertainty is modeled as a random variable $\sigma_{\text{supply}}$ that affects the firms' abatement cost function.

\[ U(c, a, \sigma_{\text{demand}}, \sigma_{\text{supply}}) \]

\[ C(a, \sigma_{\text{supply}}) \]

\[ A(c, a, \sigma_{\text{demand}}, \sigma_{\text{supply}}) \]

\[ P(c, a, \sigma_{\text{demand}}, \sigma_{\text{supply}}) \]

\[ T(c, a, \sigma_{\text{demand}}, \sigma_{\text{supply}}) \]

\[ \text{EU (2003, 2005, 2009)} \] or Convery and Redmond (2007) for more on the EU ETS. Joskow et al. (1998) offer a brief but informative account of the US SO$_2$ trading program. The NO$_x$ programs are the Regional Clean Air Incentives Market (RECLAIM), the Ozone Transport Commission (OTC), NO$_x$ Budget Program, and the NO$_x$ State Implementation Plan (SIP). For details, see Burtraw et al. (2005).

$^5$
uncertainty modeled as random elements in the firms’ abatement cost functions. The model structure and these shocks are outlined in Section 3. In Sections 4 and 5 we demonstrate that either of the two regulatory instruments may induce the most flexible technology; i.e., the technology that best accommodates the firms to respond to new information. Specifically, a tax encourages the most flexibility if and only if the stochastic element in abatement costs and the equilibrium permit price have sufficiently strong positive covariance, compared with the variance in consumer demand for the good produced. Then, we show in section 6 that endogenous technology choice provides a comparative advantage in favor of tradable-quantity regulation compared with Weitzman (1974). Intuitively, the firms’ technology investment decisions affect the fluctuations in aggregate emissions under an emissions tax, and thereby the expected social cost of emissions. This source of externality does not arise under tradable emissions permits where aggregate emissions are fixed.

This paper contributes to the literature by considering regulation, welfare, technology choice and uncertainty in one model setup. Krysiak (2008) does this in the case of production of a public good, but we show that the results in Krysiak (2008) have limited relevance in the important case of pollution abatement. The reason is that the product market for the good of which production cause emissions influences the firms’ investment decisions. As the analysis in Krysiak (2008) does not feature a product market, his results are only valid for comparison of emissions trading and an emissions tax when the demand for the good produced is constant (which would allow us to ignore the product mar-
In the present paper, we explicitly compare price- and quantity-based instruments in the case of environmental pollution. Our results differ from those of Krysiak (2008) in several respects. Most importantly, Krysiak (2008) finds that price-based regulation leads to implementation of a more flexible technology. In contrast, we find that either price- or quantity-based regulation may induce the most flexible technology.

2 Review of related literature

This paper relates to two important strands of the literature; i.e., that of price-induced innovation and that of prices versus quantities. We present only a brief overview here. Beginning with the literature on price-induced innovation, Morton and Schwartz (1968) show that optimal technology choice depends on the initial technology, the relative factor prices and the relative costs of acquiring different types of innovations. Magat (1978) introduces regulation and finds that effluent taxes and effluent standards lead to a distinctively different allocation of R&D funds between improvements in abatement technology and production technology. Kon (1983) looks at the role of output price uncertainty and shows that it can lead to investment in more labor-intensive technologies. Mendelsohn (1984) examines investment under price- and quantity-based regulation. He finds that quantity-based instruments have an advantage, because price-based regulation induce excessive variation in output. Mills (1984) shows that an unregulated competitive firm will invest more in flexibility if demand uncertainty

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6Krysiak (2008) addresses this point on page 1282.
increases. Lund (1994) allows R&D growth to take more than one direction, and shows that this may create the need for interplay between R&D subsidies and a carbon tax. Zhao (2003) finds that abatement cost uncertainties reduce firms’ investment incentive under both tradable emissions permits and emission taxes if the investment is irreversible, and more so under taxes. Kaboski (2005) shows that relative input price uncertainty can cause investment inaction as the firms wait to get more information about what type of technology is most profitable to implement. Fowlie (2010) examines the US NO\textsubscript{x} Budget Program and finds that deregulated plants were less likely to implement more capital intensive environmental compliance options compared with regulated or publicly owned plants.

The literature on implications of uncertainty on optimal choice of policy instruments (without technology investment) is extensive. In a seminal article, Weitzman (1974) shows that a higher ratio of the slope of marginal damages relative to the slope of marginal abatement costs favors quotas. Hoel and Karp (2002) and Newell and Pizer (2003) extend this result to stock pollutants with additive uncertainty.\textsuperscript{7} They also find that an increase in the discount rate or the stock decay rate favors tax usage, and obtain numerical results that suggest that taxes dominate quotas for the control of greenhouse gases. Hoel and Karp (2001) examine the case with stock pollutants and multiplicative uncertainty. Their analytical results are ambiguous, but, using a numerical model, they find

\textsuperscript{7}Additive and multiplicative uncertainty applies to the intercept and the slope of marginal abatement costs, respectively.
that taxes dominate quotas for a wide range of parameter values under both additive and multiplicative uncertainty in the case of climate change mitigation policies. Stavins (1996) shows that positive correlation between marginal costs and marginal benefits works in favor of quantity-based instruments with flow pollutants. Hybrid policies that combine price- and quantity-based policies have been examined by, e.g., Roberts and Spence (1976), Weitzman (1978), Pizer (2002), Jacoby and Ellerman (2004), and Krysiak and Oberauner (2010). These studies suggest that hybrid policies generally dominate a single instrument approach.

3 The modeling framework

Consider a sector featuring \( n \) risk-neutral firms that supply a homogeneous good \( q \). One unit of production causes one unit of emissions that is subject to either an emissions tax or tradable emissions permits regulation. This could, for example, be the power sector located in a country (or group of countries like the EU) that mitigates carbon emissions in order to meet its Kyoto requirements, or the electricity sector covered by the NO\(_x\) Budget Program in the US. We assume divisibility between the costs of abatement and other production costs. This is reasonable in the case of end-of-pipe abatement technology like, e.g., carbon capture and storage.\(^8\) In order to focus on the abatement technology

\[^{8}\text{Without this assumption, we would have additional spillover effects under both regulatory approaches (featuring cross derivatives between the elements } a \text{ and } q \text{ in the cost function). We argue in Section 7 that our main results do not depend qualitatively on the functional}\]
choice, we let the cost of producing the good (without abatement) be given by $q_i^2/2$ for any firm $i \in N = \{1, 2, \ldots, n\}$. Finally, perfect competition is assumed in all markets.9

We extend the model of Weitzman (1974) by analyzing the long-run regulation problem where any firm $i \in N$ can choose the technology parameters $\alpha_i$ and $\beta_i$ in the following abatement cost function:

$$c_i(q_i, a_i) = \frac{1}{2} q_i^2 + (\alpha_i + \eta_i) a_i + \frac{\beta_i}{2} a_i^2. \quad (1)$$

Here $a_i$ is firm $i$’s abatement and $\eta_i \sim (0, \sigma^2_{\eta})$ is a firm-specific stochastic variable.10 The chosen set of technology parameters $\{\alpha_i, \beta_i\}$ incurs investment costs $k(\alpha_i, \beta_i)$, with $\alpha_i, \beta_i > 0$; $k(\alpha_i, \beta_i) \geq 0$; $k_{\alpha}, k_{\beta} < 0$; and $k_{\alpha\alpha}, k_{\beta\beta} \geq 0$;11 These assumptions imply that reducing the short-run costs always increases capital costs, and that the marginal costs of reducing $\alpha_i$ and $\beta_i$ increase for lower values of these technology parameters (i.e., more advanced technology). This is in accordance with the standard assumption of decreasing marginal productivity of capital. Because a lower value on $\beta_i$ reduces the slope on the marginal abatement cost function, it may be interpreted as a higher level of flexibility.12 That is, if a firm reduces $\beta_i$, it increases its ability to respond to new information.

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9Results by Joskow et al. (1998) and Convery and Redmond (2007) indicate, respectively, that the US market for sulfur dioxide emissions and the EU emissions trading scheme are competitive.

10As usual, $\eta_i \sim (0, \sigma^2_{\eta})$ means that $\eta_i$ is randomly distributed with expected value 0 and variance $\sigma^2_{\eta}$.

11This specification is equal to the production technology specification in Krysiak (2008) if we omit the term $q_i^2/2$.

12Stigler (1939) and Marschak and Nelson (1962) early referred to the firms’ ability to
For example, abatement of NO\textsubscript{x} from electricity production is possible through, e.g., installation of Selective Catalytic Reduction (SCR), which incur high capital costs and can reduce emissions by up to 90 percent, or Selective Non-Catalytic reduction (SNCR), which have lower investments costs but only reduce emissions rates with up to 35 percent. In terms of our stylized functional form, SCR technology will be characterized by a lower value on \( \beta_i \) than that of the SNCR technology. Similarly, emissions reduction of CO\textsubscript{2} is possible by use of, e.g., CCS or by fuel substitution. While CCS is capital intensive and allows for large emissions reductions with relatively small increases in marginal abatement costs (low \( \beta_i \)), fuel substitution is less capital intensive but cannot achieve high emissions reductions without increasing marginal costs substantially (high \( \beta_i \)).

We add \( \eta_i \sim (0, \sigma^2_{\eta}) \) to firm \( i \)'s abatement costs. For example, this reflects fluctuations in factor prices or factor productivity, or a breakdown of abatement equipment. As argued by Weitzman (1974), the determination of \( \eta_i \) could involve elements of genuine randomness, but might also stem from lack of information. The abatement cost shock \( \eta_i \) enters our functional form linearly, which is similar to, e.g., Weitzman (1974), Hoel and Karp (2002), and Karp and Zhang (2006). Note that all firms share the same uncertainty and menu of possible abatement cost structures. Therefore, they choose equal abatement technologies (because they are identical in period 2). We henceforth suppress the firm-specific subscript \( i \) except where necessary (i.e., on variables that differ change production levels in response to new information as their “flexibility”. Mills (1984) and Krysiak (2008) carry on this terminology.
across firms) to streamline notation.

Let the utility of consumption of $q$ be given by the strictly concave function:

$$u(q_j) = bq_j - \frac{d}{2}q_j^2 + \varepsilon_j q_j - pq_j,$$

for consumer $j \in M = \{1, 2, \ldots, m\}$, with $b, d > 0$, $\varepsilon_j \sim (0, \sigma^2_\varepsilon)$ and $p$ being the product price on $q$. We may interpret the stochastic element $\varepsilon_j$ as random variation in preferences, or in the supply of other goods that are compliments or substitutes to $q$. Alternatively, $\varepsilon_j$ may model private information consumer $j$ has on its own preferences; i.e. only the distribution $\varepsilon_j \sim (0, \sigma^2_\varepsilon)$ is common knowledge until $\varepsilon_j$ is revealed in period 3. We assume that the demand-side shocks $\varepsilon_j$ and the abatement cost shocks $\eta_i$ are independently distributed random variables, i.e., the expected value $E(\varepsilon_j \eta_i) = 0$ for all $i \in N$ and $j \in M$, and that $\varepsilon_j$ and $\eta_i$ are symmetrically correlated across consumers and firms, respectively. We state the following lemma on the properties of the correlation coefficients:

**Lemma 1** Let the correlation coefficients be given by $\rho_\eta = E(\eta_i \eta_{i'}) / \sigma^2_\eta$ for all firms $i, i' \in N$ ($i \neq i'$) and $\rho_\varepsilon = E(\varepsilon_j \varepsilon_{j'}) / \sigma^2_\varepsilon$ for all consumers $j, j' \in M$ ($j \neq j'$). Then we have $\rho_\eta \in [-1/(n-1), 1]$ and $\rho_\varepsilon \in [-1/(m-1), 1]$.

**Proof.** See the Appendix. \[\square\]

Note in particular that the lower bounds on the correlation coefficients $\rho_\eta$ and $\rho_\varepsilon$ become arbitrarily close to zero as the number of firms $n$ or consumers $m$ increases, respectively.\[13\]

\[13\]Our assumption that $\rho_\eta = E(\eta_i \eta_{i'}) / \sigma^2_\eta$ is similar to Krysiak (2008), who assumes $\rho_\eta \in$
The model is organized in three periods. First, in period 1, the regulator sets the emissions tax or a binding cap on aggregate emissions. The firms react to the regulation and invest in abatement technology in period 2. Finally, the firms choose their abatement and production levels in period 3. We assume that the outcomes of the stochastic variables are determined between periods 2 and 3. That is, decisions in periods 1 and 2 are made under uncertainty, while firms have full information in period 3. The firms’ production and abatement decisions in period 3 are made contingent on the firms’ abatement technology decisions in period 2. So, the firms’ investment decisions are formulated as a two-stage game: the payoffs in period 3 determine the technology investment decisions in period 2. The model is solved by backwards induction and our equilibrium concept is that of a subgame perfect equilibrium.

4 Consumption, production and abatement

Let $\sigma$ refer to the permit price and $\tau$ denote the emissions tax. The profit function in period 3 of any firm $i \in N$ is given by:

$$
\pi_i = \max_{q_i, a_i} \left[ pq_i - \frac{1}{2} q_i^2 - (\alpha + \eta_i) a_i - \frac{\beta}{2} a_i^2 - w (q_i - a_i) \right],
$$

where $w \in \{ \sigma, \tau \}$, $q_i > 0$, $a_i \in (0, q_i]$. Both the permit price $\sigma$ and the emissions tax $\tau$ remain to be determined. Because technology may differ across the regulatory regimes, we have $\alpha \in \{ \alpha_\sigma, \alpha_\tau \}$ and $\beta \in \{ \beta_\sigma, \beta_\tau \}$.\footnote{As a notational convention, “$x$” may refer to variable/parameter $x$ under either regulatory regime. If confusion is possible, we use “$x_\sigma$” and “$x_\tau$” to refer to $x$ under tradable emissions [0,1].} We get the following...
first-order conditions for any firm \(i \in N\):

\[
q_i = q = p - w, \quad (4)
\]

\[
a_i = \frac{1}{\beta} (w - \alpha - \eta_i), \quad (5)
\]

under the assumptions of interior solutions for production \(q_i\) and abatement \(a_i\). Equation (4) implies that the industry supply function is given by \(n(p - w)\).

We also observe that the second-order conditions to the maximization problem (3) are fulfilled. Note that each firm’s production and abatement levels are random variables before the outcomes of the stochastic events are known (i.e., in periods 1 and 2).

The consumers maximize utility as given by equation (2). The first order condition for any consumer \(j \in M\) is:

\[
q_j = \frac{1}{d} (b + \varepsilon_j - p). \quad (6)
\]

Hence, the market demand is given by \(\frac{1}{d} \sum_{j \in M} (b + \varepsilon_j - p)\). Because aggregate supply must equal aggregate demand,\(^\text{15}\) the product market equilibrium condition is given by:

\[
n (p - w) = \frac{1}{d} \sum_{j \in M} (b + \varepsilon_j - p). \quad (7)
\]

This equation implicitly yields the equilibrium product price \(p\). Under tradable quantity regulation, the regulator sets a binding cap on aggregate emissions denoted \(S_{\sigma}\). The emissions trading market-clearing condition then becomes

\(^{15}\text{Open economy considerations are briefly discussed in Section 7.}\)
(remember that one unit of production causes one unit of emissions):

\[ S_{\sigma} = nq_{\sigma} - \sum_{i \in N} a_{i\sigma}, \]  \hspace{1cm} (8)

where \( q_{\sigma} \) and \( a_{i\sigma} \) refer to the optimal levels of production and abatement under tradable emissions permits, respectively. Equation (8) implicitly yields the equilibrium permit price \( \sigma \).

Under price-based regulation, the regulator sets an emissions tax \( \tau \). In order to simplify comparison of the regulatory regimes, we let \( \tau \) be determined implicitly as the tax that realizes the expected emissions level \( S_{\tau} \).\(^\text{16}\) Hence, the emissions tax solves:

\[ S_{\tau} = E \left( nq_{\tau} - \sum_{i \in N} a_{i\tau} \right). \]  \hspace{1cm} (9)

Because the expectations operator is present in equation (9), but not in equation (8), the two regulatory instruments differ with respect to the risk imposed upon the regulated firms. In particular, regulation ensures that actual aggregate emissions (\( Z \)) are equal to the emissions target \( S \) under tradable-quantity regulation, while \( Z \) is endogenous under price-based regulation.

Solving the systems of equations (4), (5), (7) and (8) under tradable-quantity regulation, and (4), (5), (7) and (9) under price-based regulation, we get the regulatory regime contingent reduced form solutions to the endogenous variables in period 3. These are given in Table 2 in the appendix. The first- and second-order moments in the probability distributions of selected variables are summarized in Table 1.

\(^{16}\)It does not affect our results whether the regulator chooses \( \tau \) directly or via \( S_{\tau} \) in equation (9), because the regulator correctly foresee the firm’s actions.
We observe from Tables 1 and 2 that the firm’s production level is independent of the stochastic element to abatement costs \( \eta_i \) under an emissions tax. This reflects that the marginal cost of emissions is constant and equal to the tax in equilibrium. Together with our assumption of separability between abatement costs and other production costs, this leaves the total costs of production independent of the abatement cost shock \( \eta_i \) under an emissions tax. Under tradable quantity regulation, in contrast, the production of any firm \( i \in N \) decreases in the stochastic shocks to the abatement cost \( \eta_i \) of all the \( i = 1, 2, \ldots, n \) firms. This occurs because the total cost of production depends on the permit price, which is strictly increasing in abatement costs. We also observe that the stochastic elements in the consumer’s utility functions \( \varepsilon_j \) affect the optimal production level stronger under an emissions tax than under tradable emissions.
permits. The reason is that the cap on aggregate emissions forces the firms to increase the aggregate abatement level when production increases due to the demand side shock $\sum_{j\in M} \varepsilon_j > 0$. The associated increase in the permit price increases total production costs, and thereby dampens the firms’ response to $\sum_{j\in M} \varepsilon_j > 0$. This mechanism is absent under price-based regulation, because the emissions tax is constant.

Proceeding to abatement, optimal abatement levels are independent of the demand-side shocks $\varepsilon_j$ under price-based regulation. This occurs because the firms simply abate until marginal abatement costs are equal to the emissions tax, leaving aggregate emissions endogenous. Under tradable emissions permits $\sum_{j\in M} \varepsilon_j > 0$ increases abatement, because fluctuations in aggregate emissions (caused by fluctuations in production) must be mirrored by aggregate abatement in order to satisfy the emissions cap. We last note that the stochastic element to abatement cost $\eta_i$ affects optimal abatement levels stronger under an emissions tax than under tradable emissions permits. The reason is that $\text{Cov}(\sigma, \eta_i) = \frac{V}{n} (1 + (n - 1)\rho_\eta) \sigma_i^2 > 0$, given $\rho_\eta \in (\frac{-1}{n-1}, 1)$ (cf. Table 2). So a high (low) equilibrium permit price tends to occur together with high (low) realized abatement costs. This reduces the firms’ responses to the abatement costs shocks. Again, this mechanism is absent under price-based regulation, because the emissions tax is constant.

Finally, expected aggregate emissions are equal across the two regulatory regimes if $S_\sigma = S_\tau$, cf. Table 1. Equal expected production and abatement levels across the regimes requires both $S_\sigma = S_\tau$ and equal technology, however.
The reason is that the regulator has only one instrument available for each regulatory regime (i.e., the emissions cap \( S \) or the tax \( \tau \)), while the firms have three decision variables. That is, if the regulatory instruments are used to impose equal expected aggregate emissions across the regimes, the regulator cannot ensure equal expected production levels. Therefore, although tradable emissions permits and an emissions tax may be equivalent with respect to expected aggregate emissions, the regulatory regimes will in general have different effects on the product market when the abatement cost structure is endogenous. Note that the regulator could alternatively calibrate its instruments in order to induce equal expected aggregate production instead of emissions across the regulatory regimes.

The following lemma formalizes important parts of the above discussion:

**Lemma 2** Assume \( S_\sigma = S_\tau \) and let the firms’ profit maximization problem be given by equation (3). Then we have the following:

(i) \( E(q_\sigma) = E(q_\tau) \) if \( \alpha_\sigma = \alpha_\tau \) and \( \beta_\sigma = \beta_\tau \).

(ii) \( \text{Var}(q_\sigma) \geq (\leq) \text{Var}(q_\tau) \Leftrightarrow \frac{\sigma^2}{\tau^2} \geq (\leq) \left( \frac{1}{V_{\sigma}} - 1 \right) \frac{V_{\tau}}{V_{\sigma}}. \)

(iii) \( \text{Var}(a_\sigma) \geq (\leq) \text{Var}(a_\tau) \Leftrightarrow \frac{\sigma^2}{\tau^2} \geq (\leq) \frac{1}{V_{\sigma}} \left[ \frac{1}{V_{\sigma} V_{\tau} + 1} + \frac{1}{\tau^2} \right]. \)

**Proof.** The Lemma is obtained from Table 1. ■

Remember that the \( V \)'s are constants defined in Table 1. Part (i) in Lemma 2 implies that \( E(a_\sigma) = E(a_\tau) \) if \( S_\sigma = S_\tau \) and the firms choose identical technology under tradable emissions permits and an emissions tax.
5 The firms’ investment decisions

The impetus of our analysis of the firms’ investment decisions in period 2 is that their abatement technology choice depends on the expected abatement levels and the extent of anticipated future fluctuations in abatement. For a given expected abatement level, intuition suggests that the firms are willing to pay higher investment costs in order to increase flexibility (reduce $\beta$) if the variance in the abatement level is large. For example, if the equilibrium permit price turns out to be unexpectedly high in period 3, a firm may reduce its costs with a higher level of abatement. The firm can increase its adaptability to such future events by investing in a more flexible technology in period 2.

In period 2, any firm $i \in N$ maximizes expected profits with respect to abatement cost structure as determined by $\alpha$ and $\beta$:

$$\Pi = \max_{\alpha,\beta} E [\pi - k (\alpha, \beta)] , \quad (10)$$

with $\pi$ given by equation (3). The interior solution to the maximization problem (10) is characterized by the following first-order conditions (see the appendix):

$$-k_\alpha = E (a) , \quad (11)$$

$$-k_\beta = \frac{1}{2} \left( Var (a) + (E (a))^2 \right) , \quad (12)$$

with expectations $E(a)$ and variances $Var(a)$ as given by Table 1 for each regulatory regime. Not surprisingly, a large expected abatement level increases capital costs and decreases both $\alpha$ and $\beta$. Moreover, we show in the proof of Proposition 1 below that a larger variance increases flexibility (reduce $\beta$), as conjectured above.
In the introduction to this paper, we outlined the following research question: how does environmental regulation influence firms’ technology choices through the disparate risk environments that is imposed upon the firms? The following proposition compares the induced technology choices under the benchmark criterion of equal expected aggregate emissions across the two regulatory regimes:

**Proposition 1** Let the firms’ profit maximization problems be given by equation (10) and assume \( S_\sigma = S_\tau \). Then we have \( \beta_\sigma \leq (\geq) \beta_\tau \) if and only if \( \text{Var}(a_\sigma) \geq (\leq) \text{Var}(a_\tau) \). In addition, if \( \Pi_{\alpha\beta} \leq 0 \), \( \beta_\sigma \leq (\geq) \beta_\tau \) implies \( \alpha_\sigma \leq (\geq) \alpha_\tau \). If \( \Pi_{\alpha\beta} \geq 0 \), \( \beta_\sigma \leq (\geq) \beta_\tau \) implies \( \alpha_\sigma \geq (\leq) \alpha_\tau \).

**Proof.** See the appendix. ■

Remember that the condition for \( \text{Var}(a_\sigma) \geq (\leq) \text{Var}(a_\tau) \) is given in Lemma 2. Strict inequalities in the condition on the variances yield strict inequalities between technology parameters \( \alpha \) and \( \beta \) across the two regulatory regimes.

Proposition 1 has two important consequences. First, the two regulatory instruments typically induce implementation of different technologies. The unequal choices of technology when \( S_\sigma = S_\tau \) follow from the different economic environments with regard to risk caused by the two regulatory regimes (the regimes are equal when \( \sigma_\eta = \sigma_\varepsilon = 0 \)). This implication corroborates a point emphasized by Krysiak (2008): the choice of environmental policy instrument can have a lock-in effect. That is, a switch between price- and quantity-based regulations could render existing technology suboptimal and, therefore, devalue the installed equipment and the acquired technological knowledge. If the resultant loss of sunk technology investment costs is substantial, it may deter a
change of regulatory instrument once it has been implemented.

Second, Proposition 2 states that both types of regulation may induce stronger incentives to choose the most flexible technology (lower $\beta$). This result differs from Krysiak (2008), who finds that price-based regulation always leads to implementation of a more flexible technology. The explanation for this difference is that stochastic demand for the good (of which production causes emissions) increases the variance in abatement under tradable emissions permits, and, hence, the incentives to invest in flexibility. This contrasts with an emissions tax, where the optimal abatement level is independent of the product price. Because the model in Krysiak (2008) does not feature a product market, this effect does not occur in his model.

We last observe that the regimes yield the same technology in the particular case of a continuum of firms and independent stochastic variables when $S_\sigma = S_\tau$. This is true because the probability $P (|\sigma - \tau| > \epsilon) \to 0$ as $n \to \infty$ when $\rho_\eta = \rho_\epsilon = 0$ for some (infinitely) small constant $\epsilon$; i.e. the probability distribution of the market clearing permit price collapses around its expected value (by the law of large numbers), which becomes equal to the emissions tax. So, Proposition 1 implies that the characteristics of tradable emissions permits converge toward those of price-based regulation as the number of firms increases when the random variables are independently distributed.
6 Prices versus quantities with endogenous technology choice

Weitzman (1974) shows that a higher ratio of the slope of marginal damages relative to the slope of marginal abatement costs favors quotas. In this paper, we have shown that price- and quantity-based regulation influences the long-run technology choice of firms in different ways. How then is the ranking of price-versus quantity-based instruments provided in Weitzman (1974) affected by this technology choice effect?

Because our focus of interest is endogenous technology choice, we isolate the regulation-dependent effects on social welfare imposed by the firms’ choice of technology. We characterize the technology chosen by a benevolent social planner, given that firms implement the profit maximizing production and abatement levels given in Table 2. In terms of our model, the firms’ decisions in period 3 remain unaltered, but we let the social planner choose the technology that maximizes social welfare in period 2. We then compare the social planners’ choice with the firms’ technology choice derived in Section 5. We will not consider how uncertainty concerning the relative slopes of the environmental damage function and the firms’ abatement cost functions affects the ranking of price- and quantity-based regulation.\(^{17}\)

\(^{17}\)That topic is analyzed by, e.g., Weitzman (1974), Hoel and Karp (2001, 2002) and Newell and Pizer (2003).
aggregate emission levels under tradable emissions permits and an emissions tax. The reason is that the regimes generally induce different technologies and, hence, different marginal abatement costs (cf. Proposition 1). Moreover, as in, e.g., Weitzman (1974), the regulator cannot implement an optimal policy that ensures the aggregate marginal abatement cost to equal marginal environmental damage, because the demand side shocks $\varepsilon_j$ and the abatement cost shocks $\eta_i$ are stochastic and unknown to the regulator. Therefore, the regulator can only achieve a second-best outcome.

Let $G(Z)$ depict the social cost of aggregate emissions $Z$, with $G_Z > 0$ and $G_{ZZ} \geq 0$. Expected social welfare can be calculated as:

$$W = \max_{\alpha, \beta} E \left[ \sum_{j \in M} \left( b q_j - \frac{d}{2} q_j^2 + \varepsilon_j q_j \right) - \sum_{i \in N} \left( c_i(q_i, a_i) + k(\alpha, \beta) \right) - G(Z) \right],$$

where $c_i(\cdot)$ is given by equation (1). Moreover, $q_i = q_j$ and $a_i$ denote the equilibrium levels of production and abatement under tradable emissions permits or an emissions tax, as given by Table 2.

We now characterize the technology that is socially optimal, given the expected aggregate emissions levels and the firms’ profit-maximizing output and abatement decisions under the two regulatory approaches. Under tradable emission permits, realized aggregate emissions $Z$ are equal to the binding emissions cap $S$. Hence, the social cost of emissions is a constant given by $G(S)$. Maximization of $W$ with respect to the technology parameters $\alpha$ and $\beta$ then yields
the following first order conditions (see the appendix):

\[-k_\alpha = E(a), \quad (14)\]
\[-k_\beta = \frac{1}{2} \left( \text{Var}(a) + (E(a))^2 \right). \quad (15)\]

This is identical to the profit maximizing firms’ technology choice under tradable emissions permits (cf. equations 11 and 12). It follows that firms will implement the socially optimal technology under tradable emissions permits.

Under tax-based regulation, the level of aggregate emissions is endogenous and the social cost of emissions is given by \( G(Z) \) with \( Z = S + \frac{1}{2} \sum_{i \in N} \eta_i + V_2 \sum_{j \in M} \varepsilon_j \) (cf. Table 2). Maximization of \( W \) with respect to the technology parameters \( \alpha \) and \( \beta \) then yields the following first order conditions (see the appendix):

\[-k_\alpha = E(a), \quad (16)\]
\[-k_\beta = \frac{1}{2} \left( \text{Var}(a) + (E(a))^2 \right) - \frac{1}{n\beta} \text{Cov} \left( G_Z, \sum_{i \in N} \eta_i \right). \quad (17)\]

These conditions differ from the profit maximizing firms’ choice under tradable emissions permits given by equations (11) and (12).

We observe that the covariance on the right-hand side of equation (17) is zero when \( G(Z) \) is linear in aggregate emissions, because \( G_Z \) is then a constant. In this particular case the technology implemented by the firms coincides with that of the social planner. Conversely, \( \text{Cov} \left( G_Z, \sum_{i \in N} \eta_i \right) > 0 \) if the social cost of emissions is a strictly convex function. The reason is that \( G_Z \) obtains high values induced by high aggregate emissions when \( \sum_{i \in N} \eta_i \) is large. This yields a positive covariance on the right-hand side of equation (17). Therefore,
the social planner will choose a less flexible technology (higher $\beta$) than the technology chosen by the profit-maximizing firms.

The second main research question outlined in the introduction asked how the technology choice effect matters for the choice between regulatory instruments. We state the following result:

**Proposition 2** Let social welfare be given by equation (13) and Table 2 give the firms’ actions. Then, the firms’ technology choices are socially optimal under tradable emissions permits. Under an emissions tax, the firms choose a too flexible and, hence, socially suboptimal technology, unless $\text{Cov}(G_Z, \sum_{i \in N} \eta_i) = 0$ (e.g., if social damage from emissions is linear).  

**Proof.** The proposition follows from equations (11), (12) and (14) to (17).

Proposition 2 implies that endogenous technology choice provides a comparative advantage in favor of tradable-quantity regulation as compared with Weitzman (1974). What is the economic intuition underlying this result? In contrast with tradable emissions permits, aggregate emissions may differ from its expected value under an emissions tax. This will increase the expected social cost of emissions if the damage function is strictly convex, which is a well-known result. The novel aspect here is that the firms can influence the size of this difference by their choice of technology. We see this from the variance in aggregate emissions under an emissions tax, which is given by $\frac{\mu}{2} V_\eta \sigma^2_\eta + n V_\epsilon^2 V_\sigma^2$ (cf. Table 1). This expression obviously decreases in the technology parameter $\beta$,  

\[ \text{Cov}(G_Z, \sum_{i \in N} \eta_i) = 0 \text{ in equation (17), e.g., when } G(Z) \text{ is quadratic and } \rho_\eta = -1/(n-1). \]
given \( \rho_\eta > -1/(n - 1) \). Therefore, if the social planner could choose the technology, they would reduce the fluctuations of aggregate emissions around its expected value by investing in a less flexible technology. This decreases the expected social cost of emissions. In contrast, the firms face a given price per unit of emissions under an emissions tax and, consequently, have no incentive to internalize the convexity of environmental damage cost in their technology investment decisions. Therefore, the firms implement a too flexible abatement cost structure under the emissions tax. Intuitively, the social planner would, if given the opportunity, choose a less flexible technology in period 2 in order to limit the firms’ ability to deviate from the expected aggregate emissions levels in period 3. Importantly, this externality source does not arise under tradable emissions permits, because aggregate emissions are fixed.

Proposition 1 states that either an emissions tax or tradable emissions permits could induce the most flexible technology. Proposition 2 entails that endogenous technology choice provides a bias in favor of tradable-quantity regulation as compared with Weitzman (1974), because an emissions tax induces a too flexible and, hence, socially suboptimal technology. These two propositions do not contradict each other. The reason is that the social planner would choose the socially optimal technology, given the firms’ behavior as induced by the regulatory regime. Therefore, it is feasible that the firms choose the most inflexible technology under an emissions tax, and that this is still too flexible when compared with the socially optimal technology under price-based regulation.

We also observe that the variance in realized emissions under an emissions tax
converge towards zero as \( n \to \infty \) if \( \rho_\eta = \rho_\varepsilon = 0 \). Hence, \( \text{Cov} \left( G_Z, \sum_{i \in N} \eta_i \right) \) approaches zero and price-based regulation yields the socially optimal technology choice. This is consistent with our previous observation that the characteristics of tradable emissions permits converge toward those of price-based regulation as the number of firms increases when the random variables are independently distributed.

How important is the technology choice effects given in Propositions 2 and 3? If we compare the firms’ technology choice under an emissions tax (cf. equations 11, 12) with the socially optimal technology (cf. equations 14 to 17), we see that the comparative advantage of emissions trading increases in \( \text{Cov} \left( G_Z, \sum_{i \in N} \eta_i \right) \) when \( t \leq 0 \). Because this covariance can be expressed as \( E \left( G_Z \sum_{i \in N} \eta_i \right) \), and \( G_Z \) increases in aggregate emissions \( Z = S + \frac{1}{\beta} \sum_{i \in N} \eta_i + V_2 \sum_{j \in M} \epsilon_j \), \( \text{Cov} \left( G_Z, \sum_{i \in N} \eta_i \right) \) must be an increasing function of \( E \left( (\sum_{i \in N} \eta_i)^2 \right) = n(1 + (n-1)\rho_\eta)\sigma_\eta^2 \), given that \( G(Z) \) is strictly convex. Thus, the difference between the firms’ technology choice and the socially optimal technology increases in \( \sigma_\eta^2 \) and \( \rho_\eta \); i.e., in the variance and correlation of the shocks that affect the firms’ realized abatement costs (\( \eta_i \)). The reason is that higher values of \( \sigma_\eta^2 \) and \( \rho_\eta \) cause flexibility to induce a larger variation in aggregate emissions under price-based regulation, and, thereby, a larger social cost of a too flexible technology. Therefore, the bias in favor of tradable-quantity regulation implied by Proposition 2 increases with the level of abatement cost uncertainty and the correlation

\[ \text{Cov} \left( G_Z, \sum_{i \in N} \eta_i \right) = \frac{n\gamma}{\beta} \left( 1 + (n-1)\rho_\eta \right) \sigma_\eta^2 \]

\[ 19 \text{ A particularly simple example is } G = \frac{3}{2} Z^2, \text{ which yields } \text{Cov} \left( G_Z, \sum_{i \in N} \eta_i \right) = \frac{n\gamma}{\beta} \left( 1 + (n-1)\rho_\eta \right) \sigma_\eta^2 \]
across firms’ abatement costs.

It follows that we could expect tradable emissions permits to have a stronger comparative advantage if large shocks that originate from the same sources and affect many firms similarly influence the firms’ abatement costs.\textsuperscript{20} If such incidents play an important role in the overall risk the regulated firms face, a very flexible abatement technology, together with an emissions tax, could possibly greatly increase the variance in total emissions and, thereby, substantially increase social damage. It is also important to note that the comparative advantage of tradable-quantity regulation increases in the convexity of the social costs of aggregate emissions. This convexity varies across pollutants. For example, the NO\textsubscript{x} Budget Program controlling smog-causing pollution in the US may face quite strongly convex damage costs,\textsuperscript{21} while regulation that mitigates greenhouse gas emissions from a limited number of countries may operate with an approximately linear environmental damage function.

\textsuperscript{20}For example, Parsons et al. (2009) states that a disruption in delivery of low-sulfur coal because of track failures in October 2005 created a bottleneck that reduced deliveries significantly. In addition, a pair of coalmines had extended outages. The price of low-sulfur coal trading in the Midwest peaked in December 2005 at a level triple the price a year earlier. The shortage in low-sulfur coal forced 11 power companies to shift to higher-sulfur coal with corresponding higher SO\textsubscript{2} emissions.

\textsuperscript{21}See, e.g., Mauzerall et al. (2005).
7 Conclusion

This paper demonstrates that environmental regulation has a risk-related technology choice effect. That is, the choice of policy instrument affects which type of technology to be implemented. We show that the firms' technology choices are socially optimal under tradable emissions permits, but not under an emissions tax. The reason is that the firms' technology investment decisions affect variation in aggregate emissions under an emissions tax, and thereby the expected social cost of emissions. This source of externality does not arise under tradable emissions permits, where aggregate emissions are fixed. Therefore, we conclude that endogenous technology choice provides a comparative advantage in favor of tradable-quantity regulation as compared with the well-known criterion in Weitzman (1974).

This paper contributes to the literature by considering regulation, welfare, technology choice and uncertainty in one model setup. Krysiak (2008) does the same in the case of production of a public good. He finds that price-based regulation leads to implementation of the most flexible technology, and that this is socially suboptimal. In contrast, we find that price-based regulation may induce the least flexible technology. This occurs if the variance in consumer demand is sufficiently strong compared with the covariance between the stochastic element in abatement costs and the permit price. The reason for this difference is that the product market influences the firms' investment decisions in the case of pollution abatement, and the analysis in Krysiak (2008) does not feature a product market.
Our representation of technology is very stylized and adopted to get tractable analytical results. This does not affect the qualitative results in Section 5, because ambiguity under our restrictive functional forms implies ambiguity in the general case too. Regarding the welfare analysis in Section 6, our main results are arguably likely to be robust against changes in the model setup: firstly, the technology choice effect is likely to be more pronounced if the firms could influence its technology configuration even more. Secondly, the source for inefficient technology choice under an emissions tax, caused by the covariance between the stochastic elements to firms’ abatement costs and marginal environmental damage, cannot arise under tradable emissions permits, because the cap on aggregate emissions is given. On the other hand, the theoretical model does not feature possibly important elements like, e.g., market power, R&D externalities, distortionary taxes or non-uniform pollutants, which are likely to cause inefficient technology investment under tradable emissions permits.\footnote{A previous version of the paper shows that the firms’ technology choice is socially suboptimal under both price- and quantity-based regulation if there is a distortionary tax in the product market (and that the ranking become ambiguous).} Further, the model is static and does not feature dynamic aspects like, e.g., gradual disclosure of information, consumer savings or accumulation of stock pollutants. Moreover, we assumed an exogenous number of firms, although Spulber (1985) shows that the exit and entry of firms influence the ranking of regulatory instruments. Further, Mills (1984) demonstrates that competitive equilibrium with free entry and exit may sustain a higher number of firms if demand fluctuates than if demand is stationary at its expected value. Finally, we have only consid-
ered a closed economy, but it can be shown that our qualitative results remain valid if the firms in the regulated area sell the produced good in a world market with an exogenous stochastic product price.\textsuperscript{23}

Most importantly, this paper indicates that it is insufficient to consider only static properties and induced investment levels when evaluating a potential regulatory instrument; it is also important to assess the characteristics of the induced technology. In particular, the possible comparative advantage of tradable emissions permits over an emissions tax, induced by a higher ratio of the slope of marginal damages relative to the slope of marginal abatement costs, tends to be even stronger than shown by Weitzman (1974).

\textsuperscript{23}However, if the social planner can affect the world market product price (in the derivation of the socially optimal technology), it is reasonable to conjecture that they will invest more (less) in technology than the competitive firms if the regulated area is a net importer (exporter), in order to decrease (increase) the product price.
Appendix

Proof of Lemma 1. We first prove that $\rho_\eta \in [-1/(n - 1), 1]$. A matrix is a valid covariance matrix if and only if it is positive semi-definite. With $n'$ identical firms and $\rho_\eta = E(\eta_i\eta_\ell)/\sigma_\eta^2$ the covariance matrix is given by the following $n' \times n'$ matrix:

$$
\begin{pmatrix}
1 & \rho_\eta & \cdots & \rho_\eta \\
\rho_\eta & 1 & \cdots & \rho_\eta \\
\vdots & \vdots & \ddots & \vdots \\
\rho_\eta & \rho_\eta & \cdots & 1
\end{pmatrix}
$$

The determinant of this matrix is given by $(1 - \rho_\eta)^{n' - 1}(1 + \rho_\eta(n' - 1))$. It can then be shown that the principal minors of our $n \times n$ covariance matrix satisfy the criteria necessary for positive semi-definiteness if and only if $\rho_\eta \in [-1/(n - 1), 1]$ (use the determinant criteria for positive semi-definiteness with the given formula for $n' = 1, n' = 2, \ldots, n' = n$). The proof that $\rho_\varepsilon \in [-1/(m - 1), 1]$ is similar.

Derivation of equations (11) and (12): Firm $i$’s first order condition wrt. $\alpha$ is:

$$
\frac{d\Pi_i}{d\alpha} = E\left[(p - w - q_i) \frac{dq_i}{d\alpha} + (w - \alpha - \eta_i - \beta a_i) \frac{da_i}{d\alpha} - k_\alpha - a_i\right] = 0
$$

$$
\iff -k_\alpha = E(a_i)
$$

while its first order condition wrt. $\beta$ is:
\[
\frac{d\Pi_i}{d\beta} = E \left[ (p - w - q_i) \frac{dq_i}{d\alpha} + (w - \alpha - \eta_i - \beta a_i) \frac{da_i}{d\alpha} - k_\beta - \frac{1}{2} a_i^2 \right] = 0
\]
\[
\Leftrightarrow -k_\beta = \frac{1}{2} E \left( a_i^2 \right) = \frac{1}{2} \left( \text{var}(a) + (E(a))^2 \right).
\]

We used the first order conditions (4) and (5) (the envelope theorem) in the derivations.

**Proof of Proposition 1.** We first derive the second order conditions to the maximization problem (10). The Hessian is given by:

\[
\begin{pmatrix}
\Pi_{\alpha\alpha} & \Pi_{\alpha\beta} \\
\Pi_{\alpha\beta} & \Pi_{\beta\beta}
\end{pmatrix} =
\begin{pmatrix}
-k_{\alpha\alpha} - \frac{dE(a)}{d\alpha} & -k_{\alpha\beta} - \frac{dE(a)}{d\beta} \\
-k_{\alpha\beta} - \frac{1}{2} \frac{dE(a^2)}{d\alpha} & -k_{\beta\beta} - \frac{1}{2} \frac{dE(a^2)}{d\beta}
\end{pmatrix},
\]

where \(\frac{1}{2} \frac{dE(a^2)}{d\alpha} = \frac{dE(a)}{d\beta} = -E(a)\sigma^3\), cf. Table 1. The conditions for the Hessian to be negative semi-definite are \(\Pi_{\alpha\alpha} \leq 0\) and \(\Pi_{\alpha\alpha} \Pi_{\beta\beta} - \Pi_{\alpha\beta}^2 \geq 0\). We assume the last equation holds with strict inequality.

We observe from equations (11), (12) and Table 1 that the regulatory regimes induce equal technology if and only if \(\text{Var}(a_\sigma) = \text{Var}(a_\tau)\) when \(S_\sigma = S_\tau\). Differentiating equations (11) and (12) wrt. \(z \in \{\sigma, \eta, \rho, \epsilon\}\), i.e., increasing \(\text{Var}(a)\) using exogenous parameters, we get:

\[
-k_{\alpha\alpha} \frac{d\alpha}{dz} - \frac{dE(a)}{d\alpha} \frac{d\alpha}{dz} - k_{\alpha\beta} \frac{d\beta}{dz} - \frac{dE(a)}{d\alpha} \frac{d\beta}{dz} = 0,
\]
\[
-k_{\alpha\beta} \frac{d\alpha}{dz} - \frac{1}{2} \frac{dE(a^2)}{d\alpha} \frac{d\alpha}{dz} - k_{\beta\beta} \frac{d\beta}{dz} - \frac{1}{2} \frac{dE(a^2)}{d\beta} \frac{d\beta}{dz} = \frac{1}{2} \frac{dE(a^2)}{dz}.
\]

Or, equivalently, using matrix notation and the definitions from the Hessian:
\[
\begin{pmatrix}
\Pi_{\alpha\alpha} & \Pi_{\alpha\beta} \\
\Pi_{\alpha\beta} & \Pi_{\beta\beta}
\end{pmatrix}
\begin{pmatrix}
d\alpha \\
d\beta
\end{pmatrix}
= \begin{pmatrix}
0 \\
1 \frac{d\text{Var}(a)}{dz}
\end{pmatrix},
\]
where we also used \(\frac{dE(a^2)}{dz} = \frac{d\text{Var}(a)}{dz}\) (cf. Table 1). Solving for the changes in the technology parameters, we get:

\[
\begin{pmatrix}
d\alpha \\
d\beta
\end{pmatrix}
= \begin{pmatrix}
\frac{1}{\Pi_{\alpha\alpha} \Pi_{\beta\beta} - \Pi_{\alpha\beta}^2} \left(-\frac{1}{2} \frac{d\text{Var}(a)}{dz} \Pi_{\alpha\beta} - \frac{1}{2} \frac{d\text{Var}(a)}{dz} \Pi_{\alpha\alpha}\right)
\end{pmatrix}.
\]

It then follows from the second order conditions that \(\frac{d\alpha}{dz} \leq (\geq) 0 \Leftrightarrow \Pi_{\alpha\beta} \geq (\leq) 0\) and that \(\frac{d\beta}{dz} < 0\). Hence, \(\beta_\sigma \leq (\geq) \beta_\tau\) if and only if \(\text{Var}(a_\sigma) \geq (\leq) \text{Var}(a_\tau)\).

This proves the proposition.

**Derivation of equations (14) and (16):** Equation (13) is equivalent with:

\[
W = \max_{\alpha, \beta} E \left[ \sum_{j \in M} \left( b q_j - \frac{d}{2} q_j^2 + \varepsilon_j q_j \right) - \sum_{i \in N} \left( \frac{1}{2} q_i^2 + (\alpha + \eta_i) a_i + \frac{\beta}{2} a_i^2 + k(\alpha, \beta) \right) - G(Z) + X \right],
\]

with:

\[
X = p \left( \sum_{i \in N} q_i - \sum_{j \in M} q_j \right) + w \sum_{i \in N} (q_i - q_i + a_i - a_i) = 0.
\]

Differentiating with respect to \(\alpha\) and rearranging we get:

\[
\frac{dW}{d\alpha} = E \left[ \sum_{j \in M} \left( (b - dq_j + \varepsilon_j - p) \frac{dq_j}{d\alpha} \right) + \sum_{i \in N} \left( (p - w - q_i) \frac{dq_i}{d\alpha} + (w - \alpha - \eta_i - \beta a_i) \frac{da_i}{d\alpha} - k_\alpha - a_i \right) \right],
\]

\[
+ E \left[ \left( \sum_{i \in N} q_i - \sum_{j \in M} q_j \right) \frac{dp}{d\alpha} + \sum_{i \in N} (a_i - a_i + q_i - q_i) \frac{dw}{d\alpha} + w \sum_{i \in N} \left( \frac{dq_i}{d\alpha} - \frac{da_i}{d\alpha} \right) - G_Z \frac{dZ}{d\alpha} \right],
\]

\[
= E \left[ \sum_{i \in N} (k_\alpha + a_i) + (w - G_Z) \frac{dZ}{d\alpha} \right],
\]

\[
= -nk_\alpha - nE(a_i),
\]
where we used the first order conditions (4), (5), (6) and $\frac{dZ}{d\alpha} = 0$ (cf. Table 2).

Setting $\frac{dW}{d\alpha} = 0$ and dividing by $n$ yields equations (14) and (16).

**Derivation of equations (15) and (17):** Following the steps from the derivation of equations (14) and (16) above, but differentiating with respect to $\beta$, the first order conditions become:

$$
\frac{dW}{d\beta} = \mathbb{E} \left[ \sum_{i \in N} \left( k_\beta + \frac{1}{2} (a_i)^2 \right) + (w - G Z) \frac{dZ}{d\beta} \right] = 0,
$$

$$
\Leftrightarrow -nk_\beta - \frac{n}{2} \mathbb{E} \left( \sum_{i \in N} (a_i)^2 \right) + \mathbb{E} \left[ (w - G Z) \frac{dZ}{d\beta} \right] = 0,
$$

$$
\Leftrightarrow -k_\beta = \frac{1}{2} \left( \text{var} \left( a \right) + \left( \mathbb{E} \left( a \right) \right)^2 \right) + \frac{1}{n} \mathbb{E} \left[ (w - G Z) \frac{dZ}{d\beta} \right].
$$

Under emissions trading we have $\frac{dZ}{d\beta} = \frac{dS}{d\beta} = 0$ (cf. Table 2). Insertion yields equation (15). Under a tax we have $\frac{dZ}{d\beta} = \frac{1}{\beta^2} \sum_{i} \eta_i$ (cf. Table 2). Hence,

$$
\mathbb{E} \left[ (\tau - G Z) \frac{dZ}{d\beta} \right] = \mathbb{E} \left[ (\tau - G Z) \frac{1}{\beta^2} \sum_{i} \eta_i \right] = \frac{1}{\beta^2} \mathbb{E} \left[ G Z \sum_{i} \eta_i \right] = \frac{1}{\beta^2} \text{cov} \left[ G Z, \sum_{i} \eta_i \right].
$$

Insertion yields equation (17).
Table 2: Reduced form solutions for endogenous variables in period 3

<table>
<thead>
<tr>
<th>Tradable emissions permits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma, \tau = \beta_{V_1} \left( V_2 \left( \frac{1}{m} \sum_{j \in M} \varepsilon_j + b \right) - \frac{\delta}{n} \tau + \frac{1}{m} \left( \frac{1}{n} \sum_{i \in N} \eta_i + \alpha_n \right) \right)$</td>
</tr>
<tr>
<td>$q = V_{3\sigma} \left( b + \frac{\delta}{n} \beta_{V_3} - \alpha_n - \frac{1}{m} \sum_{i \in N} \eta_i + \frac{1}{m} \sum_{j \in M} \varepsilon_j \right)$</td>
</tr>
<tr>
<td>$a = V_{3\sigma} \left( b - \alpha_n - \frac{1}{n} \frac{a}{3} S_n + \frac{1}{m} \sum_{j \in M} \varepsilon_j + \left( \frac{dn}{m \beta_{V_3}} + \frac{1}{m} \right) \frac{1}{n} \sum_{i \in N} \eta_i \right) - \frac{1}{\beta_{V_3}} \eta_i$</td>
</tr>
<tr>
<td>$Z = S_{\sigma}$</td>
</tr>
</tbody>
</table>

$V_1 = \frac{m + \alpha n}{m + \beta n}, \ V_2 = \frac{m}{m + \beta n}$ and $V_3 = \frac{m}{m + \beta n}$.

$V_1, V_2, V_3 \in (0, 1)$
References


Krysiak, F. C. (2008): Prices vs. quantities: The effects on technology


