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## **Self-delusion in the pursuit of happiness**

**Abstract:**

The paper explores how repeated revisions of consumption plans increase long-run utility. If agents value present anticipations of future consumption, some revisions may be viewed as a benign form of self-delusion. We consider a minimal generalization of the Samuelson discounted utility model to allow for utility linked to next period consumption. Agents are assumed to vary with respect to their sophistication. Different environments likely to facilitate repeated revisions are also considered.

**Keywords:** Intertemporalchoice;selfdelusion;timeinconsistency;naivete;self-control;discountedutilityfunctions;anticipation;memory

**JEL classification:** A12, B49,C70,D11,D60,D74,D91,E21

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You have to begin to lose your memory, if only in bits and pieces, to realize that memory is what makes our lives. Life without memory is no life at all... Our memory is our coherence, our reason, our feeling, even our action. Without it, we are nothing... (I can only wait for the final amnesia, the one that can erase an entire life, as it did my mother's...)  
—Luis Buñuel

## 1 Introduction

We all make plans. Plans involve actions of our future selves. Putting off vacuuming the house may please today's you, but may not go down too well with tomorrow's. Indeed, come tomorrow, you might well look for an excuse to pass the chore on to some future you. But as we know, procrastination, even of seemingly menial tasks, can snowball and result in significant welfare losses.<sup>1</sup> This paper reverses the proposition, however. What if your failure to commit to yesterday's plan may actually produced higher utility over the longer term? If plans for future consumption offset anticipations of present value, revising a plan may increase utility all told. In particular, you may find it better to consume less today than you had planned for or anticipated yesterday. In this scenario the well-known self-control problem of procrastination becomes a self-delusion problem. And the question is not how to improve your self-commitment record, but to find an environment which helps you revise your plans.

My starting point is the much used Samuelson's discounted utility model (Samuelson 1937). As this is a discounted sum of instantaneous utility functions, it involves a premise of total amnesia, as if we plan a sequence of sensations with no value to us except in the heat of the moment. In medical parlance, the economic agent invoked by Samuelson's DU model is suffering from Korsakoff's syndrome.<sup>2</sup> The paper generalizes Samuelson's DU model to allow for utility from memories and anticipations. Two types of agents are considered. The first has preferences as implied by the standard Samuelson utility model (0-korsakoffs); the other has a rudimentary memory (1-korsakoffs). A 1-korsakoff remembers a consumption plan the time period it is made, and may have (positive) anticipations regarding the next period's consumption. But she has no recollection of past consumption or yesterday's plan. A 1-korsakoff can be viewed as the first step towards an agent with full recollection. The question addressed here, is whether this rudimentary form of memory affects intertemporal choice. We find that it does. 1-korsakoffs are very different from

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<sup>1</sup> See O'Donoghue and Rabin (1999) for a discussion and extensive literature review.

<sup>2</sup> Korsakoff's syndrome: Neurological disorder marked by severe amnesia despite clear perception and full consciousness, resulting from chronic alcoholism, head injury, brain illness, or thiamin deficiency. Persons with the syndrome typically cannot remember events in the recent or even immediate past; some retain memories only a few seconds. Longer periods –up to 20 years– may also be forgotten. (Encyclopdia Britannica)

the 0-korsakoffs. They are, as a rule, time inconsistent, and repeated revisions may cut a path to higher long-run utility.

This paper connects with the wider literature on intertemporal choice and time inconsistencies initiated by Strotz (1956) and Pollak (1968). There are essentially two ways to explain time inconsistency in economic decision processes. Either preferences change over time, or agents are myopic (or hyperopic). Time inconsistencies arise in the former case when agents do not take sufficient account of these preference changes (Peleg and Yaari 1973, Becker and Mulligan 1995, Laibson 2001). The latter case is largely about hyperbolic discounting, generating present-biased preferences (Lowenstein and Prelec 1992, Laibson 1997, Dasgupta and Maskin 2005).

Both ways of resolving dynamic inconsistencies raise one important question. How can an agent believe in plans from which her future self will deviate? This question can be addressed in a number of ways. One could look for ways to ensure commitment (Hammond 1976, Laibson 1997). According to Asheim (1997), most commitment schemes change the original decision problem. He introduces revision proof strategies as a refinement of subgame-perfectness. If an agent becomes aware of her self-control problems, she can limit her decision problem by only considering plans that she will actually follow. This contribution ties in with earlier game theoretic approaches (Goldman 1979 1980, Ferreira 1995), for which the search for possible and credible equilibria plays a vital role. We could also explain wrong beliefs about actions of future selves by assuming that agents vary in sophistication. They may, for instance, realize they are present-biased but misjudge its magnitude, or in the case of endogenous preferences the magnitude of future preference changes may escape them (Peleg and Yaari 1973, Becker and Mulligan 1995, Laibson 2001, O'Donoghue and Rabin 1999 2001 2008). This paper ties in with the latter strand of the literature. The agents considered here, i.e. the 0-korsakoffs and 1-korsakoffs, may not take the preferences of their future selves into account. Indeed, they may not even consider the eventuality of their future selves deviating from earlier plans.

If we tend to revise plans, the question is not just how we can believe in them initially. It is also about what happens in the aftermath of the revision. Gul and Pesendorfer (2007) consider the desire for commitment, and Noor (2007) shows that the implications of future temptations are mixed. Agents who are aware of their self-control problems may not take advantage of commitment opportunities. The possibility of indulging temptations in the future is in itself a source of temptation. Halevy (2008) studies diminishing impatience and argues that positive time preference is deeply connected to certainty. Consumption today is certain, planned consumption at a future time is uncertain. This paper's analysis is in some sense transversal to these contributions. Its

main theme is intertemporal choice under full certainty. Learning plays no role in the models presented here. The approach contrasts moreover with recent contributions on intertemporal choice which take an evolutionary approach (Rayo and Becker 2007a, 2007b, Netzer 2009).

The importance of anticipation is not new to economic literature. W. S. Jevons (1888) and H. S. Jevons (1905) explained farsighted behavior as a consequence of utility of anticipation of future consumption. A more recent contribution, but in a similar vein, is Elster and Loewenstein (1992) who consider utility from memory and anticipation. An axiomatic treatment of the role of anticipatory feelings in the case of uncertainty is given in the seminal article of A. Caplin and J. Leahy (2001). They show that time inconsistencies arise naturally in the presence of anticipation. When it comes to intertemporal choice under certainty, Loewenstein (1987) considers a formal model which assumes that a person's instantaneous utility is equal to the utility from consumption plus some function of discounted utility of consumption in future periods.<sup>3</sup> Loewenstein points out that DU anomalies may occur in such a model. In particular, it offers a novel explanation as to why people discount different goods at different rates. More precisely, anticipation creates a varying downward bias on discount rates depending on the nature of the good in question.<sup>4</sup> This paper's contribution to the study of anticipation and intertemporal choice is twofold. First, it extends the Samuelson DU model so as to facilitate the comparison of agents with differing mnemonic capacities. Second, it considers a simple, but critical distinction where anticipation of consumption is connected to plans of consumption, not future consumption per se.

The paper is organized as follows. In section 2 we define k-korsakoff preferences in a T-period consumption scenario of one continuous good. Section 3 considers consumption of one indivisible good that is to be consumed over a given number of time periods. The discussion draws heavily on the agent types, naïfs and sophisticates, introduced by O'Donoghue and Rabin (1999). Whereas naivete tends to yield poor long-run outcomes, the results presented

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<sup>3</sup> Loewenstein (1987) presents a formal model where a person's instantaneous utility function takes the form  $u(c_\tau; c_{\tau+1}, c_{\tau+2}, \dots)$  where the partial derivatives with respect to  $c_{\tau'}$  is positive for all  $\tau' > \tau$ . Loewenstein proposes the following functional form:  $u(c_\tau; c_{\tau+1}, c_{\tau+2}, \dots) = v(c_\tau) + \alpha(\gamma v(c_{\tau+1}) + \gamma^2 v(c_{\tau+2}) + \dots)$  for some  $\gamma < 1$ .

<sup>4</sup> Few empirical studies have considered the extent to which anticipations of future consumption affect intertemporal choice. Loewenstein(1987) reports from a study of undergraduates. They were asked to state the 'most they would pay now' for a kiss from their favorite movie star. They could receive the kiss immediately or later (four possible delays). The students went for the 'three day delayed' kiss. Evidence of self-delusion arising in intertemporal choice, though potentially abundant, is hard to come by empirically. The seminal contributions of Tversky and Kahnemann (1981), Kahnemann and Snell (1992), and Thaler (1999) offer much insight into the complexity of human choice. Their empirical findings highlight the limited nature of agents' ability for self-prediction; decisions also tend to depend on framing. As revisions may be easier to rationalize in situations where new information and new frames of reference are readily at hand, self-delusion may thrive exactly in situations, where it may be hard to unravel. It may also prove hard to identify self-delusion by introspection. This does not rule out that self-delusion is easily observed in others: 'The moment you expose a common man's self-delusion, you take away his happiness' (Ibsen 1884).

here extend the potential downside of sophistication. Sophisticates have less opportunity to revise repeatedly, and as a result may get lower long-run utility in comparison with naïfs. In section 4 we discuss a model, the life span uncertainty model, which provides an example of benign self-delusion among sophisticates. The model is an abstraction of the following scenario. Upon retirement we decide to spend a year in Rio. We have to decide which year to go. We know we will not live forever, but do not know how many years we have left. As a year passes we observe our health, and want to go before it is too late. In a stylized version of this consumption problem, we show that sophisticates can achieve the same long-run utility as naïfs. In other words, a little uncertainty regarding the number of time periods facilitates utility increasing revisions even for sophisticates. Section 5 concludes.

## 2 The DU model with memory and anticipation

In this section we extend Samuelson's discounted utility model include utility of memories of past consumption and anticipations of planned consumption.

**Definition 2.1** *Any consumption vector  $(c_1, \dots, c_T)$  give rise to a vector of memories  $(0, m_1, \dots, m_T)$ , where  $m_i$  is a function of past consumption, that is  $m_i = m_i(c_{<i})$ , where  $c_{<i} = (c_1, \dots, c_{i-1})$  for all  $i > 1$ .*

Anticipation utility is linked to expected consumption, thus in the context of intertemporal choice, *plans* of future consumption. We can formalize anticipations like this:

**Definition 2.2** *Any consumption plan represented by a planned consumption vector  $(c_{1,plan}, \dots, c_{n,plan})$ <sup>5</sup> give rise to a vector of anticipations  $(a_1, \dots, a_T)$  where  $a_i$  is a function of future planned consumption, that is  $a_i = a_i(c_{>i,plan})$  where  $c_{>i,plan} = (c_{i+1,plan}, \dots, c_{T,plan})$  for all  $i > 0$ .*

The anticipations of Definition 2.2 can be viewed as first order anticipations, that is anticipations about consumption. It can be argued that humans do also have anticipations about memories ( $a_i(m_j)$ ), memories of anticipations ( $m_j(a_i)$ ) and anticipations of anticipations ( $a_i(a_j)$ ) et cetera. These can be viewed as second or higher order memories and anticipations. Here, we consider only first order memories and anticipations.

A straightforward generalization of a Samuelson's DU model with anticipation and recollections is:

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<sup>5</sup> The reason for the subscript 'plan' is to make the distinction between consumption and planned consumption explicit.

$$U = \sum_{i=1}^T \delta^{i-1} U_i(c_i, m_i(c_{<i}), a(c_{>i,plan})). \quad (2.1)$$

The utility function ( 2.1) places no limitations on utility from anticipation or recollection. In the following we define preference types that differ in the extent to which they receive utility from memory and anticipation.

Let a person consider an T-period consumption scenario. We will assume that the preferences of all agents can be represented by a DU utility function, but may differ according to their (korsakoff) type. A k-korsakoff has a consumption horizon of k periods into the future and a recollection of the last k periods of consumption including the present time period. This is formalized in the following definition:

**Definition 2.3** *An agent is k-korsakoff, k a positive integer, if the following the following two conditions hold:*

- i.  $m_i(c_1, \dots, c_{i-1}) = m_i(c_{\max(1, i-k-1)}, \dots, c_{i-1})$
- ii.  $a_i(c_{i+1,plan}, \dots, c_{T,plan}) = a_i(c_{i+1,plan}, \dots, c_{\min(T, i+k+1),plan})$

A 0-korsakoff is the economic agent implied by the standard DU function( $DU_0$ ). A 1-korsakoff is the first step with respect to memories and anticipations. She not only enjoys present consumption as the 0-korsakoff, but also looks forward to the next period's planned consumption. In other words, if she plans to eat one hamburger every day, she cherish the thought of tomorrow's burger, while she eats this days hamburger. The next step up, the 2-korsakoff, remembers last periods consumption, enjoys this periods consumption and look forward to the planned consumption in two next time periods. This paper largely concerns 0-korsakoffs and 1-korsakoffs.<sup>6</sup>

**Definition 2.4** *(Discounted utility functions for k-korsakoffs) The intertemporal utility function of a k-korsakoff is given by*

$$DU_k = \sum_{i=1}^T \delta^{i-1} u(m_i(c_{\max(1, i-k-1)}, \dots, c_{i-1}), c_i, a_i(c_{i+1,plan}, \dots, c_{\min(T, i+k+1),plan})) \quad (2.2)$$

where  $\delta < 1$  and  $u, m_i$  and  $a_i$  are continuously differentiable function with positive first order partial derivatives and negative second order partial derivatives with respect to all arguments.

<sup>6</sup> Example A.1 in the appendix is of a 3-korsakoff.



Definition 2.4 states that the time invariant utility function  $u$  satisfies standard assumption of nonsatiation and diminishing marginal utility of consumption. It also states that the same preference structure applies to consumption, memories and anticipations. Note that one time invariant utility function  $u$ , that enters a discounted sum, signifies that a 1-korsakoff takes account of (discounted) future anticipations, that is, has preferences for future anticipations.

We use the following definition of time consistent preferences:

**Definition 2.5** Let  $c_{plan} = (c_{1,plan}, \dots, c_{T,plan})$  and  $c_{plan}^* = (c_{1,plan}^*, \dots, c_{T,plan}^*)$  be two consumption vectors that agree up to time  $j$ , that is  $c_{i,plan} = c_{i,plan}^*$  for  $i \leq j$ . A DU function is said to represent time consistent preferences, if  $U_i(c_i, m_i(c_{<i}), a_i(c_{>i,plan})) > U_i(c_i^*, m_i(c_{<i}^*), a_i(c_{>i,plan}^*))$  for some  $i \leq j$  imply  $U_i(c_i, m_i(c_{<i}), a_i(c_{>i,plan})) > U_i(c_i^*, m_i(c_{<i}^*), a_i(c_{>i,plan}^*))$  for all  $i \leq j$ .

This definition is just a formalized way to say the following. Say you compare two consumption plans, A and B, for a week starting Monday. They agree until Thursday. You find A better than B on Monday. If you have time consistent preferences, you will find A better than B on Tuesday, and Wednesday as well. Preferences that are not time consistent are said to be time inconsistent.

The following theorem states that 1-korsakoffs are time inconsistent.

**Theorem 2.1** A 1-korsakoff agent with preferences given by

$DU_1 = \sum_{i=1}^T \delta^{i-1} u(c_i, a_i(c_{i+1,plan}))$  with  $T \geq 3$ , is time inconsistent if there exist at least one consumption level  $c$ , such that  $\max DU_1(c_1, \dots, c_T)$  given  $c_1 + \dots + c_T = c$ , is an interior solution,  $(c_1^*, \dots, c_T^*)$  ( $c_i^* \neq 0$  for all  $i$ ).<sup>7</sup>

A proof of the theorem is given in the Appendix. The dynamic inconsistency for a 1-korsakoff arises because she picks a plan that involves more consumption in the next period than she actually ends up consuming.<sup>8</sup> Compared to a 0-korsakoff, who is always time consistent, a 1-korsakoff skews consumption towards the future. The time inconsistencies that arise for a 1-korsakoff are structurally similar to those implied by hyperbolic discounting. The following example illustrates this point.

**Example 2.4** An intertemporal utility function for a 1-korsakoff

<sup>7</sup> Not all  $DU_1$  functions have interior solutions. If  $|\frac{du}{da} \frac{da}{dc_{i+1}}| > |\frac{du}{dc_i}|$  for all nonnegative pairs  $(c_i, c_{i+1})$  then  $(c_i, c_{i+1}) = (0, c)$  gives the highest utility for all consumption levels.

<sup>8</sup> Time inconsistent rankings are not merely linked to wrong assessments to next period consumption. Example A.2 in the appendix shows ranking reversals arise also in cases where there are no immediate consumption.

Let  $u(c_i, a(c_{i+1})) = c_i^{\frac{1}{2}} + a(c_{i+1}) = c_i^{\frac{1}{2}} + kc_{i+1}^{\frac{1}{2}}$ , where  $k$  is a (positive) constant. This gives the following  $n$ -period  $DU_1$ -utility function:

$$DU_1(c_{plan}) = \sum_{i=1}^n \delta^{i-1} u(c_i, a_i(c_{i+1,plan})) = c_{1,plan}^{\frac{1}{2}} + (1+k) \sum_{i=2}^T \delta^{i-1} c_{i,plan}^{\frac{1}{2}}$$

The last sum is  $\delta(1+k)$  times the  $DU_1$  at time two (provided)  $c_1 = c_{1,plan}$  in period 1. This relation allows us to make two observations. In this easy case, where utility from anticipation is proportional to utility from consumption, the utility function is structurally equal to an intertemporal utility function with hyperbolic discounting. Present-biased preferences tend to be modeled by an intertemporal utility function of the following type:  $U^t(u_t, \dots, u_T) = \delta^t u_t + \beta \sum_{\tau=t+1}^T \delta^\tau u_\tau$ .<sup>9</sup> In our model  $(1+k)$  plays the role of  $\beta$ . Present-biased preferences and potentially myopic behavior arise when  $\beta < 1$ . In contrast, since we assume  $k > 0$ , are 1-korsakoffs potentially future-biased, and hyperopic.<sup>10</sup>

A valuation of anticipation of next period consumption implies that more weight is given to future consumption, compared to absence of valued anticipation. The effect of the discount factor goes the other way: less weight is put on future consumption compared to present consumption. If  $k+1 = \delta^{-1}$ , then anticipation completely balances the effect of discounting for consumption in time period 2, but  $k$  has no effect on the weight put on consumption in time period 3 compared to consumption in time period 4.

From a welfare perspective there may be the severe implications of time inconsistent behavior. O'Donoghue and Rabin (1999) show that welfare losses may be associated both with procrastination as well as preproperation. An analysis of long-run consequences requires some notion of long-run utility of consumption which allows us to compare different paths of consumption either ex ante or ex post. In the following analysis we rely on the same long-run utility definition as O'Donoghue and Rabin (1999). Long-run utility from consumption is here defined as the sum of utility in all time periods without discounting. This may be viewed as an ex ante utility where all future and past selves are considered equal and this utility is assigned to a time period 0.<sup>11</sup> This utility concept captures the possibility of achieving higher long-run utility by having wrong expectations of future behavior.

### Example 2.5 Long-run utility for a 1-korsakoff

<sup>9</sup> See O'Donoghue and Rabin (2001), for a discussion and an extensive literature review.

<sup>10</sup> If we allow for negative anticipations, that is  $k < 0$ , then this 1-korsakoff model corresponds to a standard hyperbolic DU model with  $\beta = 1+k$  ( $k > -1$ ).

<sup>11</sup> See Goldman (1979) for a brief discussion of different approaches regarding long-run utility

Assume that a 1-korsakoff plans to eat a cake in four days. Her preferences are given by the intertemporal utility function of Example 2.4. We assume that  $k = \delta = \frac{1}{2}$ . In other words, the 1-korsakoff has preferences for anticipations, but at the same time does discount the future. The utility maximizing plan at Day 1 is given in the first row of Table 2.1. The next row gives the best consumption plan at Day 2, given that the 1-korsakoff ate as much cake as planned the first day. Likewise for row 3, given that she consumed on Day 1 and Day 2 did consume according to the best intertemporal plan at Day 1 and Day 2 respectively.

Table 2.1 Consumption plans for 1-korsakoff eating a cake in four days. Consumption is given in percent.

Consumption	Day 1	Day 2	Day 3	Day 4
Best plan Day 1	58	32	8	2
Best plan Day 2	58	25	14	3
Best plan Day 3	58	25	11	6

We see that best plan at Day 1, does involve higher consumption on Day 2, than actually is chosen on Day 2, and likewise for Day 2 and Day 3. Table 2.2 gives the corresponding utility levels for each time period. Some of the utility of 10.4 on the first day, is anticipation utility (2.8). The following day consumption is not 32 but 25. The anticipation utility associated to consumption of 25 the following day is 2.5. In other words the Day 1 1-korsakoff enjoys an excess utility of  $2.8 - 2.5 = 0.3$  due to the difference between actual and planned consumption in time period 2.

Table 2.2 Utility associated with consumption and consumption plans for a 1-korsakoff intending to eat cake in four days.

Consumption	Day 1	Day 2	Day 3	Day 4	Long-run utility
Best plan Day 1	10.4	7.1	3.6	1.5	22.5
Best plan Day 2	10.4	6.9	5.5	1.9	24.7
Best plan Day 3	10.4	6.9	4.6	2.5	24.4

We see also that long-run utility is up from 22.5 to 24.7. The revision at Day 3, does not however give higher long-run utility. This illustrates a potential downside of delayed consumption that affects the last day. The Day 4

1-korsakoff has no cake-eating future. She cannot enjoy consumption anticipation, and she cannot, unlike her previous selves, postpone consumption. In other words, she is at the mercy of the Day 3 1-korsakoff, who can save, and in this case indeed does save, more cake to the last day than is optimal from a long-run perspective. In this numerical example the effect of the second-to-last day revision is small, but can be considerable for small  $\delta$ 's and large  $k$ 's.

### 3 One indivisible good and degrees of sophistication

The dynamic inconsistencies of the 1-korsakoff, notwithstanding her pursuit of higher long-run utility, raise important questions. How can an economic agent believe in a consumption plan and not follow it? Under what circumstances would she actually choose the best plan available to her future selves? Our discussion in this paragraph draws on the framework developed in O'Donoghue and Rabin (1999).

We start out by a motivating example:

**Example 3.1** A 1-korsakoff going to Rio.

Sheila has won a trip to Rio and can choose either to go this year, next year or the year after. We assume her preferences are given by a  $DU_1$ -model with anticipation that is  $U = \sum_i \delta^{i-1} U_i(c_{i,plan}, a_i(c_{i+1,plan}))$ . We also assume for simplicity's sake  $U_i = c_i + a_i(c_{i+1,plan})$ . At the beginning of year 1, she faces three possible consumption plans with the corresponding utilities:

$$U(c, 0, 0) = c$$

$$U(0, c, 0) = a(c) + \delta c$$

$$U(0, 0, c) = \delta a(c) + \delta^2 c,$$

If  $c > a(c) + \delta c$ , then going the first year is preferred. If not, that is, if  $c < a(c) + \delta c$ , postponing the Rio trip to the second year is the better alternative. Since  $\delta < 1$ , going in the second year is always better than waiting to the third year. Moreover, if  $c < a(c) + \delta c$ , and she does not go the first year, she may postpone yet again, and go the third and last year. She may do this even if going the first year is strictly better ( $c > \delta a(c) + \delta^2 c$ ).

This example spurs the following question. What if she does the above com-

putation and realizes that comparing year one with year two is probably irrelevant because she has decided against going in the second year. To address this question we introduce two types of agent, naïfs and sophisticates.

The definitions of naïfs and sophisticates are given in terms of strategies. A strategy is an assignment of an action for every contingency. In the case of consumption of one indivisible good, this is either consume (Y) or not consume (N).

**Definition 3.1** *A perception-perfect strategy for naïfs at time  $t$  is a strategy,  $s^n = (s_1^n, s_2^n, \dots, s_T^n)$  that satisfies  $s_t^n = Y$  if  $U^t(t) = \max(U^t(\tau))$  for all  $\tau \geq t$ .*

**Definition 3.2** *A perception-perfect strategy for sophisticates is a strategy,  $s^s = (s_1^s, s_2^s, \dots, s_T^s)$  that satisfies for all  $t < T$ :  $s_t^s = Y$  if and only if  $U_t(t) \geq U_t(\tau')$  for  $\tau' > t$  such that  $\tau' = \min_{\tau > t} \{\tau | s_\tau^s = Y\}$ .*

Sophisticates only compare the utility of consuming today with later consumption times, where consumption is planned if reached. In other words, they do not consider irrelevant alternatives. Naïfs, on the other hand, compare against all later consumption dates whether consumption then is likely or not. Added to this, in an  $T$  period setting the construction of the game requires that  $s_T^n = Y$ . Consumption must occur in the final period, if not before.<sup>12</sup>

In Example 3.1, if Sheila is a naïf, she will choose to postpone travelling until the third year if  $c < a(c) + \delta c$ . What she fails to realize, though, is that while year two is better for Year-1 Sheila, it is not the best for Year-2 Sheila. Year-2 Sheila will pass on the sandy beaches of Rio to Year-3 Sheila. On the other hand, if Sheila is a sophisticate, she will know that Year-2 Sheila won't be going to Rio. So she will only compare going the first year against going the third year. If  $c > \delta a(c) + \delta^2 c$ , she will go the first year. The results of this example are special cases of the two following theorems.

**Theorem 3.1** *Let  $C$  be one indivisible good that is to be consumed in one of the  $T$  time periods. Consider a 1-korsakoff agent with utility function  $DU_1 = \sum_i^T \delta^{i-1} u(c_i, a_i(c_{i+1, plan}))$ . If she is a naïf, she will be time consistent if and only if  $u(C, 0) \geq u(0, a(C)) + \delta u(C, 0)$ .*

**Theorem 3.2** *Let  $C$  be one indivisible good that is to be consumed in one of*

<sup>12</sup> These definitions are the same as given in O'Donoghue and Rabin (1999). However, the implementation in T-period games differ for our sophisticates. An O'Donoghue-Rabin sophisticate does not discount as assumed under the Samuelson DU model. She is present-biased in the sense of only differentiating between now or later, but not between two later periods. Later is just later. In this analysis we do use the standard discounting inferred by the DU model, and in such a case (see Theorem 3.2), is a sophisticate time consistent. In other words, a perception-perfect strategy for sophisticates has more bite in the case of standard discounting.

the  $T$  time periods. Consider a 1-korsakoff agent with utility function  $DU_1 = \sum_i^T \delta^{i-1} u(c_i, a_i(c_{i+1,plan}))$ . If she is a sophisticate, she will be time consistent.

In sum these theorems tell us that although sophisticates may have time inconsistent preferences, do make time consistent plans when they take into account the preferences of their future selves. The case of a 1-korsakoff naïf and one indivisible good allows for an interesting contingency. She can be oblivious to potential future revisions and still be time consistent. This case relies on a weak preference for anticipations compared to the discount rate. On the other hand, if anticipations are not outweighed by the discount rate, naïfs may harvest higher long-run utility from repeated delays of consumption. The following theorem states this formally.

**Theorem 3.3** *Let  $C$  be one indivisible good that is to be consumed in one of the  $T$  time periods. Consider a 1-korsakoff agent with utility function  $DU_1 = \sum_i^T \delta^{i-1} u(c_i, a_i(c_{i+1,plan}))$ . Let  $a = u(0, a(C))$  and  $c = u(C, 0)$ . If she is a naïf, her long-run utility will be:*

i.  $U_{long-run} = c$  if  $a \leq (1 - \delta)c$ .

ii.  $U_{long-run} = c + (T - 1)a$  if  $a > (1 - \delta)c$ .

Sophisticates, on the other hand, due to their sophistication are prevented from having repeated anticipations of next period consumption:

**Theorem 3.4** *Let  $C$  be one indivisible good, that is to be consumed in one of the  $T$  time periods. Consider a 1-korsakoff agent with utility function  $DU_1 = \sum_i^T \delta^{i-1} u(c_i, a_i(c_{i+1,plan}))$ . Let  $a = u(0, a(C))$  and  $c = u(C, 0)$ . If she is a sophisticate, her long-run utility will be*

i.  $U_{long-run} = c$  if  $T$  is odd.

ii.  $U_{long-run} = a + c$  if  $a > (1 - \delta)c$  and  $T$  is even.

According to condition  $a > (1 - \delta)c$ , the utility of anticipation outweighs the discounted utility of next period consumption. If it doesn't, consuming in the first period gives the highest utility. The even and odd condition is driven by the backward induction which follows from a perception-perfect strategy for sophisticates.<sup>13</sup>

<sup>13</sup> This dependence of the parity of time periods is also present in O'Donoghue and Rabin (1999), in the case of time consistent agents, TCs. However, in their Example 1 p. 109, they consider only the case of an even number of time periods ( $T = 4$ ).

Theorems 3.3 and 3.4 are, in sum, a bit discouraging. The 1-korsakoff naïfs can revise consumption plans and enjoy benign self-delusion. They never realize their next period self will make the same calculations as they are doing today. Sophisticates are prevented from utility increasing revisions. At a more technical level, a sophisticate's perception-perfect strategy relies on backward induction, and this prevents unforeseen revisions. In the next section we will briefly discuss an extension to the model that facilitates repeated revision and benign self-delusion also for sophisticates.

#### 4 The life span uncertainty model

In the previous section sophisticates obtained lower long-run utility than naïfs. In this section we consider an extension that facilitates repeated revisions for sophisticates as well as naïfs. The following scenario illustrates the extension. Upon retirement we decide to spend a year in Rio. We know we will not live forever, and need to decide when to go, now or later. As each year passes we observe our health ( $h_t$ ) and know that the probability of enjoying good health for the whole of the following year ( $p_t$ ) is increasing in  $h_t$ . Agents do not know  $p_t$ , but form subjective beliefs  $q(h_t)$  of the probability of a healthy year ahead.

We formalize this with the following definition.

**Definition 4.1** (*Life Span Uncertainty Game (LSU game)*) *An LSU game is a one player game, where the player at every node has to choose between two actions, Y or N (consume or not consume an indivisible good). At any given node  $t$ , there is a positive probability  $(1 - p_t)$  that this node is the last. The probability of continuation  $p_t = p_t(h_t)$  is an increasing function in  $h_t$ . The player observes  $h_t$ , and assigns a subjective probability  $q(h_t)$  for a next time period. The player knows that the true probability  $p_t$  is a function of  $h_t$  and that  $h_t$  is decreasing over time.*

**Definition 4.2** (*naïf in LSU game*) *A perception-perfect decision rule for naïfs in the LSU game is a rule that assign  $s_t^n = Y$  if and only if  $EU^t(t') = \max(EU^t(\tau))$  for  $\tau > t$ , where  $EU^t(t') = q(h_t)U^t(t')$  and  $q(h_t)$  is the subjective probability for reaching  $t'$  given  $h_t$ .*

In discussing the LSU game we assume that both 1-korsakoff naïfs and sophisticates are risk neutral, and maximize expected utility given their observation of the health parameter  $h_t$  and subjective probabilities  $q(h_t)$  for the next time periods.<sup>14</sup>

<sup>14</sup>We do not assume that agents assign lower  $q$ 's for more distant time periods.

A perception-perfect strategy for sophisticates in this case, requires a refinement of Definition 3.2. A refinement may be achieved in the following way:

Let  $a = u(0, a(C))$  and  $c = u(C, 0)$ . If  $a + q(h_t, t)c > c$ , then it is better to go the next year. The sophisticate realizes that this reasoning is conditional on going the next year. Moreover, if  $a + q(h_{t+1})c > c$  the next-year sophisticate will not go. In this model there is a critical health level  $h_c$  such that  $a + q(h_c)c = c$ . When  $h_c$  is reached, there is no point in postponing consumption. In other words, if the sophisticate believes that  $h_{t+1} \leq h_c$ , she may enjoy anticipation at year  $t$ . As  $h_t$  is decreasing, an  $h_t$  close to  $h_c$  may be read as a high probability of  $h_{t+1} \leq h_c$ . We formalize this into the following termination criterion:

**Definition 4.3** ( *$\epsilon$ -criterion*)

*If the player observes that  $h_t - h_c < \epsilon$ , then  $q(h_{t+1} < h_c) = 1$ , else  $q(h_{t+1} < h_c) = 0$ .*

This termination criterion can be used to formulate perception-perfect strategies for 1-korsakoff sophisticates in the LSU game.

**Definition 4.4** ( *$\epsilon$ -sophisticate in a LSU game*)

*A perception-perfect decision rule for an  $\epsilon$ -sophisticate in an LSU game is a rule that assigns*

- 1.  $(s_t^s, s_{t+1}^s) = (N, Y)$  if  $q(h_t)(a + c) > c$  and  $h_t - h_c < \epsilon$  ( $q(h_{t+1} < h_c) = 1$  by the  $\epsilon$ -criterion).*
- 2.  $(s_t^s, s_{t+1}^s) = (Y, Y)$  if  $q(h_t)(a + c) \leq c$  or  $h_t - h_c \geq \epsilon$  ( $q(h_{t+1} < h_c) = 0$  by the  $\epsilon$ -criterion).*

*for every  $t$ .*

**Theorem 4.1** *1-korsakoff naïfs and 1-korsakoff  $\epsilon$ -sophisticates have the same expected long-run utility in the LSU game provided  $h_t - h_c < \epsilon$  for all  $t$ . The expected long-run utility is in this case:  $E(U) = a \sum_{i=1}^{\infty} (\pi_{i=0}^{t-1} p_k) q_t$ .*

The theorem tells us that the LSU game allows for utility increasing revisions for sophisticates as well as for naïfs. In this case they would both also achieve the same long-run utility. The conditions for this to happen are restrictive. The health parameter needs to remain low but high enough all the same to exceed  $h_c$  for all time periods. Sophisticates as well as naïfs end up consuming nothing. Their long-run utility is pure anticipation.



## 5 Conclusion

Under the assumption that people value present anticipations of future consumption, this paper considers long-run utility benefits from repeated revisions. As yesterday's anticipation resides safely in the past, you can consume less today than you planned yesterday. This paper explores an extension to Samuelson's discounted utility model that allows agents to remember their plans and value future anticipation. These agents, called 1-korsakoffs, are very different from the economic agent inferred by the standard Samuelson's discounted utility model (0-korsakoffs). 1-korsakoffs may achieve higher long-run utility from repeated revisions of consumption plans.

When we assume that agents vary with respect to sophistication, we get structurally similar results as O'Donoghue and Rabin (1999). A naïf fails to realize that her future self may deviate from her original plan. Repeated revisions provide an opportunity for higher long-run utility. A sophisticate, on the other hand, doesn't have as many opportunities to engage in repeated revisions, as she considers only consumption plans that are likely to be followed. At a technical level, backward induction in consumption scenarios with a fixed number of time periods prevents a sophisticate from anticipating higher consumption levels of future selves than actually occur.

In sum, the results presented here illustrate the difficulties of uniting sophistication and self-delusion even in the pursuit of a higher long-run utility. This somewhat discouraging insight may be sweetened by a conjecture. Benign self-delusion may be viable for sophisticates in consumption scenarios with a higher degree of complexity and uncertainty than the stylized consumption scenarios considered here. The final model extension where uncertainty regarding the number of time periods facilitates repeated revisions also for sophisticated agents can be read as modest evidence in favor of this conjecture.

## Appendix

### *Proof of Theorem 2.1*

It is enough to establish time inconsistency in the three-period case. By assumption there exist a total consumption level  $c^*$ , such that the highest utility level as perceived at time period 1, gives a plan of non zero consumption  $(c_{1,plan}^*, c_{2,plan}^*, c_{3,plan}^*)$  in all time periods ( $c_{i,plan}^* \neq 0$  for  $i \in \{1, 2, 3\}$ ). We will write  $c_i^*$  for  $c_{i,plan}^*$ , for notational convenience. We have the following expression for the maximal utility of C at time 1:

$$U_{\star}^{(1)} = u(c_1^*, a(c_2^*)) + \delta u(c_2^*, a(c_3^*)) + \delta^2 u(c_3^*, 0)$$

Consider the following small change of the optimal consumption bundle,  $\epsilon$  less in time 2 and  $\epsilon$  more in time 3:

$$U_\epsilon^{(1)} = u(c_1^*, a(c_2^* - \epsilon)) + \delta u(c_2^* - \epsilon, a(c_3^* + \epsilon)) + \delta^2 u(c_3^* + 2, 0)$$

By construction is  $U_\star^{(1)} > U_\epsilon^{(1)}$ , since  $u(c_1^*, a(c_2^*)) > u(c_1^*, a(c_2^* - \epsilon))$  by non-satiation of  $a()$  and  $u()$ , it follows that

$$\delta u(c_2^* - \epsilon, a(c_3^* + \epsilon)) + \delta^2 u(c_3^* + 2, 0) - \delta u(c_2^*, a(c_3^*)) - \delta^2 u(c_3^*, 0) > 0$$

This inequality is  $\delta U_\epsilon^{(2)} - \delta U_\star^{(2)} > 0$ , where  $\delta U_\epsilon^{(2)}$  and  $\delta U_\star^{(2)}$  are the utility of the consumption plans as perceived at time 2. In other words  $(c_{2,plan}^* - \epsilon, c_{3,plan}^* + \epsilon)$  gives higher utility at time 2 than  $(c_{2,plan}^*, c_{3,plan}^*)$ , and we have a reversal of rankings of the two consumption plans  $(c_{1,plan}^*, c_{2,plan}^*, c_{3,plan}^*)$  and  $(c_{1,plan}^*, c_{2,plan}^* - \epsilon, c_{3,plan}^* + \epsilon)$ .

□

### *Proof of Theorem 3.1*

A perception-perfect strategy for naïfs at time  $t$  is a strategy,  $s^n = (s_1^n, s_2^n, \dots, s_T^n)$  that satisfies  $s_t^n = Y$  if  $U^t(t) = \max(U^t(\tau))$  for  $\tau > t$  or  $t' > \min_{\tau > t} \{\tau | U^t(\tau) = \max(U^t(\tau') | \tau' \geq t)\}$ . That is, she will delay consumption if and only if there exist a  $\tau$ ,  $\tau > t$ , that gives higher utility. Since she is a 1-korsakoff, and the instantaneous utility function is equal for all periods, at any given time  $t$ , the present utility of time periods  $t + 2, \dots, T$  is strictly less than  $U^t(t)$  (Note that  $U^{t+1+i}(t) < \max\{U^t(t), U^{t+1}(t)\}$  due to  $\delta < 1$ ). In other words, consumption at time  $t$ ,  $s_t^n = Y$ , if and only if  $U^t(t) = u(C, 0) \geq u(0, a(C)) + \delta u(C, 0)$ .

### *Proof of Theorem 3.2*

A perception-perfect strategy for sophisticates, is one where  $s^s = (s_1^s, s_2^s, \dots, s_T^s)$  that satisfies for all  $t < T$  :  $s_t^s = Y$  if and only if  $U_t(t) \geq U_t(\tau')$  for  $\tau' > t$  such that  $\tau' = \min_{\tau > t} (\tau | s_\tau^s = Y)$ . That is she will only choose action  $Y$  if the present utility of  $Y$  gives a higher utility than the present utility of the next (planned)  $Y$ . Since she is a 1-korsakoff, and the instantaneous utility function is equal for all periods, at any given time  $t$ , the present utility of time periods  $t + 2, \dots, T$  is strictly less than  $U^t(t)$  (Note that  $U^{t+1+i}(t) < \max\{U^t(t), U^{t+1}(t)\}$  due to  $\delta < 1$ ). In other words, consumption at time  $t$ ,  $s_t^s = Y$ , if and only if one the two conditions hold:

1.  $U^t(t) = u(C, 0) \geq u(0, a(C)) + \delta u(C, 0)$  and  $s_{t+1}^s = Y$
2.  $s_{t+1}^s = N$ .

As the action at time  $t$  is uniquely determined by the time invariant condition  $u(C, 0) \geq u(0, a(C)) + \delta u(C, 0)$  and the action at time  $t + 1$ , the one unique perception-perfect strategy will be determined by backward induction. In other words, she will be time consistent.

□

*Proof of Theorem 3.3*

In the first time period she compares  $c$  versus  $a + \delta c$ . If  $a + \delta c \leq c$ ,  $s_1^n = Y$ , (since all later consumption times will be discounted ( $c > \delta^i(a + \delta c)$  for all  $i > 0$ )).  $s^n = (s_1^n, s_2^n, \dots, s_T^n) = (Y, Y, \dots, Y)$ . This gives a long-run utility of  $c$ . If  $a + \delta c > c$ , then  $s_1^n = N$ , since consuming the next period is better. Then  $s^n = (s_1^n, s_2^n, \dots, s_T^n) = (N, Y, Y, \dots, Y)$  defines a perception-perfect strategy. The same reasoning also applies to a naïf who has reached time period  $t$ . That is  $s_t^n = (s_t^n, s_{t+1}^n, \dots, s_T^n) = (N, Y, Y, \dots, Y)$  defines a perception-perfect strategy at time  $t$ . The long-run utility is  $c + (T - 1)a$ , since at every time period consumption in the next time period gives the highest utility except when the final period  $T$  is reached.

□

*Proof of Theorem 3.4*

Using backward induction we get  $s_{T-1}^s = Y$  if and only if  $c \geq a + \delta c$ . That is  $a \leq (1 - \delta)c$ . In this case (by induction again)  $s^s = (s_1^s, s_2^s, \dots, s_T^s) = (Y, Y, \dots, Y)$  is a perception-perfect strategy. This gives long-run utility  $c$ . If  $a < (1 - \delta)c$  then  $s_{T-1}^s = N$ . In this case  $s_{T-2}^s$  must be equal to  $Y$ , since  $c > \delta a + \delta^2 c$  (Note:  $a < (1 - \delta)c$  implies  $\delta a + \delta^2 c < \delta(1 - \delta)c + \delta^2 c = \delta c < c$  (only next period anticipation for 1-korsakoffs, and  $s_{T-1}^s = N$  implies no planned consumption in time period  $(T - 1)$ ). By induction we get that  $s^s = (s_1^s, s_2^s, \dots, s_T^s) = (Y, N, Y, \dots, N, Y)$  if  $T$  is odd, and  $s^s = (s_1^s, s_2^s, \dots, s_T^s) = (N, Y, N, \dots, N, Y)$  if  $N$  is even. By construction these strategies are perception-perfect strategies for the odd and even cases respectively. The long-run utility associated with these strategies are  $c$  if  $T$  odd, and  $a + c$  if  $T$  even.

□

*Proof of Theorem 4.1*

If  $h_t - h_c < \epsilon$  for all  $t$ , the perception-perfect decision rule for  $\epsilon$ -sophisticates and naïfs agrees at every achieved decision node (time interval). Furthermore their beliefs regarding future behavior are the same in that both believe in next period consumption. The long-run utility is given by  $q_1 a + p_1 q_2 a + p_2 p_1 q_3 a + \dots = a \sum_{i=1}^{\infty} (\pi_{i=0}^{t-1} p_k) q_t$ .

□

*Example A.1*

Going to Rio, the case of a 3-korsakoff.

Sheila has won a trip to Rio. She has three options. Go this year, next year or the year after. We assume for expository purposes that her preferences are given by an additive  $DU_3$ -model with memory and anticipation,  $U_i = c_i + m_i(c_{<i}) + a_i(c_{>i}, \text{plan})$ , with a slight notational abuse measuring all three in utility directly.

Sheila has three possible consumption plans  $(1, 0, 0)$ ,  $(0, 1, 0)$ , and  $(0, 0, 1)$ .<sup>15</sup> Their corresponding utility evaluated at the start of year one is:

$$U(1, 0, 0) = 1 + \delta m^1 + \delta^2 m^2$$

$$U(0, 1, 0) = a^1 + \delta + \delta^2 m^1$$

$$U(0, 0, 1) = a^2 + \delta a^1 + \delta^2,$$

where  $m^1 = m_2(c_{<2}) = m_2((1, 0, 0))$ , the superscript tells us that its memories of consumption in the previous period and  $m^2 = m_3(c_{<3}) = m_3((1, 0, 0))$ , the superscript tells us that is memories of consumption two periods back. We adopt the same notation for anticipation; that is  $a^1$  is anticipation of next period anticipation, and  $a^2$  is anticipation of consumption two periods later.

If she does not travel the first year, she will compare,  $a^1 + \delta$  to  $1 + \delta m^1$  the second year. Assume that  $a^1 + \delta > 1 + \delta m^1$ , then she will rate traveling the second year as better than the first year, provided that  $m^1 > m^2$ , i.e. she values first order memories higher than second order. She will be time inconsistent if  $a^1 + \delta + \delta^2 m^1 > a^2 + \delta a^1 + \delta^2$ . In other words, she is time inconsistent if  $a^1 > \max(1 - \delta + \delta m^1, (a^2 - \delta + (1 - \delta^2)m^1)/(1 - \delta))$  and  $m^1 > m^2$ . In this case time inconsistency relies on high valuation of next period consumption compared to anticipation during later periods and next period memories. The question of time inconsistency under full recollection is shown to be nontrivial by this example, and depends on the relative strength of utility of anticipation and memories as well as the discount factor.

*Example A.2*

<sup>15</sup> The consumption utility of the Rio trip is normalized to 1

## Existence of ranking reversals in the case of no immediate consumption

Let  $DU_1 = \sum_i^T \delta^{i-1} u(c_i, a_i(c_{i+1}, \text{plan}))$  be a  $DU_1$ -function, and assume there exist at least one consumption level,  $c$ , such that  $\max DU_1$  given  $c$ , has an interior solution. We will prove the existence of two consumption plans C, and D, with no immediate consumption, where C and D agree up to time period 2; C is ranked strictly better than D in time period 1; and D is ranked strictly better than C in time period 2.

Assume that  $(c_2^*, c_3^*)$  is an interior solution of the  $\max DU_1 \sum_i^2 \delta^{i-1} u(c_i, a_i(c_{i+1}, \text{plan}))$  given  $c_2 + c_3 = c$ .

We consider a 5 period consumption scenario. Let consumption plan A be  $(0, 0, c_2^*, c_3^*, 0)$  and consumption plan B be  $(0, 0, 0, c_2^*, c_3^*)$ . Table A.1 gives the two consumption plans:

Table A.1

Consumption plan (Time period)	1	2	3	4	5
A	0	0	$c_2^*$	$c_3^*$	0
B	0	0	0	$c_2^*$	$c_3^*$

At time period 1, A is strictly preferred to B, since  $\delta < 1$ . (Note that at time period 1  $U_B = \delta U_A$ .)

Next we will construct two consumption plans, C and D, which give the same utility level evaluated at time 1. First let consumption plan C be given by  $(0, c_2^* + e, c_3^* - e, 0)$  ( $e < c_3^*$ ). Then let consumption plan D be given by  $(0, f_4 + f_5, c_2^* + e - f_4, c_3^* + e - f_5)$  where  $f_5 \leq c_3^*$  and  $f_4 \leq c_2^*$ . The following table summarizes these consumption plans.

Consumption plan (Time period)	1	2	3	4	5
C	0	0	$c_2^* + e$	$c_3^* - e$	0
D	0	0	$f_4 + f_5$	$c_2^* + e - f_4$	$c_3^* + e - f_5$

For every  $e$  there exist  $f_4$  and  $f_5$  such that D is ranked higher than C, at time 1. One way to achieve this is to choose  $f_4$  and  $f_5$  in such a way that D is equal to the consumption plan A  $(0, c_2^*, c_3^*, 0)$ , that is  $f_5 = c_3^* + e$  and  $f_4 + f_5 = c_2^*$  ( $f_4 = c_2^* - c_3^* + e$ ).

Now if  $(f_4, f_5) = (0, 0)$  C is better than D. And if  $(f_4, f_5) = (c_2^* - c_3^* + e, c_3^* + e)$  D is better than C. By continuity there of  $U$  there exist a lowest  $1 > s > 0$  such that  $(f_4, f_5) = (s(c_2^* - c_3^* + e), s(c_3^* + e))$  gives D and C ranked equal.

In the following we choose  $f_4$  and  $f_5$  such that C and D are ranked equal *at time period 1*.

We have the following expression for the utility of C at time 1,  $U_C^{(1)}$ :

$$U_C^{(1)} = \delta u(0, a(c_2^* + e)) + \delta^2 u(c_2^* + e, a(c_3^* - e)) + \delta^3 u(c_3^* - e, 0)$$

$$U_D^{(1)} = \delta u(0, a(f_4 + f_5)) + \delta^2 u(f_4 + f_5, a(c_2^* + e - f_4)) + \delta^3 u(c_2^* + e - f_4, a(c_3^* - e - f_5)) + \delta^4 u(c_3^* - e - f_5, 0)$$

By construction  $U_C^{(1)} = U_D^{(1)}$ . By taking the difference between these two expressions and writing the terms  $\delta$  on the one side, and higher orders  $\delta$  on the other we arrive at the following equality:

$$u(0, a(c_2^* + e)) - u(0, a(f_4 + f_5)) + \delta u(f_4 + f_5, a(c_2^* + e - f_4)) + \delta^2 u(c_2^* + e - f_4, a(c_3^* - e - f_5)) + \delta^3 u(c_3^* - e - f_5, 0) - \delta u(c_2^* + e, a(c_3^* - e)) - \delta^2 u(c_3^* - e, 0)$$

Note that  $u(0, a(c_2^* + e)) - u(0, a(f_4 + f_5)) > 0$  since  $a$  is a monotonly increasing function and  $c_2^* + e > f_4 + f_5$  (recall by construction  $\max f_4 + f_5 = c_2^* < c_2^* + e$ ).

The right hand side of the above equality divided by  $\delta$  is the difference  $U_D^{(2)} - U_C^{(2)}$ , that is, the difference between the utility of the consumption plan as perceived at time 2.

By continuity of  $U$ , D is strictly preferred to C in a neighborhood N of  $(f_4, f_5)$  in  $R^2$ . Recall that  $(f_4, f_5) = (s(c_2^* - c_3^* + e), s(c_3^* + e))$ , where  $s$  was the lowest such that  $U_C^{(1)} = U_D^{(1)}$ . In particular, by construction any smaller  $s' < s$ , imply  $U_C^{(1)} > U_D^{(1)}$ . Since the neighborhood N contains a smaller  $s' = s - \epsilon$ , this  $s'$  gives an  $(f'_4, f'_5)$  such that  $U_C^{(1)} > U_D^{(1)}$  and  $U_C^{(2)} < U_D^{(2)}$ . In other words, C is strictly preferred at time 1 and D is strictly preferred at time 2.

□

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