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Suggested Citation: Aasness, Jørgen; Nygård, Odd Erik (2009) : Revenue functions and Dupuit curves for indirect taxes with cross-border shopping, Discussion Papers, No. 573, Statistics Norway, Research Department, Oslo

This Version is available at: https://hdl.handle.net/10419/192555

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## Jørgen Aasness and Odd Erik Nygård

# Revenue functions and Dupuit curves for indirect taxes with cross-border shopping 


#### Abstract

: The partial revenue from each indirect tax and the total revenue from all indirect taxes on consumer goods are derived as functions of all commodity prices, the tax rates of each commodity, total expenditure and demographic variables using a complete demand system. Within this framework we define Dupuit curves, or Laffer curves, and analyze their existence and maximum points theoretically and empirically. The macro demand system is based on exact aggregation across all households in the economy, and on exact aggregation across commodities within a detailed non-homogeneous utility tree. An empirical application for Norway with 55 commodity groups is presented. For beer, wine, spirits and tobacco, consumers can choose among buying at home, cross-border shopping/ tax-free shopping and smuggling. These substitution possibilities increase substantially the price elasticities for these goods. The partial revenue from wine as function of the tax share on wine has a single maximum value close to the actual tax rate in Norway in 1999, conditioned on all the other exogenous variables. The total revenue as a function of the tax share on wine also has a single maximum value, larger than that for the partial revenue. The same results are valid for spirits. For beer and tobacco there is no revenue maximizing tax share.


Keywords: Revenue functions, indirect taxes, complete demand systems, cross-border shopping, tax-free shopping, smuggling, alcohol, tobacco

JEL classification: D12, D6, H2, H31
Acknowledgement: We gratefully acknowledge valuable comments from Vidar Christiansen, Terje Skjerpen and Thor Olav Thoresen, as well as from participants at the 60th Congress of the International Institute of Public Finance, 2004, the 2003 Norwegian Tax Forum, and seminars at Statistics Norway and Norwegian Ministry of Finance. We also thank the Norwegian Research Council's Tax Economics Programme for their financial support.

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## 1. Introduction

In this paper we analyze the effects on consumption patterns and tax revenue of changes in indirect taxes for Norway. In particular we are interested in the following question. Is there, for certain goods, a tax revenue maximizing tax rate, such that the tax revenue increases monotonically with the tax rate, reaches a maximum, and declines thereafter monotonically with the tax rate? This type of relationship between tax revenue and tax rate generates what we call Dupuit curves, after the French economist who described this relationship in 1844 (see Dupuit, 1844). Within our context we define Dupuit curves and seek to establish their existence and maximum point both theoretically and empirically for indirect taxes on goods exposed to cross-border shopping.

We take as point of departure for our analyses an empirically based consumer model, KONSUM, with a system of macro demand functions for all consumption divided into several consumer goods. The model is based on standard economic consumer theory and calibrated using data from various sources and econometric studies, all in accordance with the modeling tradition initiated by Frisch (1959) and Johansen (1960). Their key message was that by assuming an additive utility function, we can by knowing $n$ budget shares, $n$ Engel elasticities and one price elasticity, derive the whole matrix of $n \times n$ price elasticities. This tradition was continued by e.g. Amundsen (1963) and Bjerkholt and Longva (1980), and Aasness and Holtsmark (1993) utilize the same basic idea to develop this further by (i) letting the utility function be based on utility trees, where some goods are substitutes and others complements; (ii) using a specific form on the utility function, making it suitable for global analyses; (iii) letting the macro demand functions be derived be perfect aggregation of the demand functions over all households in Norway; iv) letting the utility function depend on the age of the households' members; (v) taking into account economies of scale in household production; and (vi) calibrating the model based on econometric methods using latent variables, where both random and systematic measurement errors are taken into account.

This model has since been developed further as an independent consumer model and as part of a general equilibrium model (MSG) at Statistics Norway. For our purpose we have incorporated the possibility of cross-border shopping and smuggling, along with a tax revenue model. For beer, wine and spirits, the consumer can choose between buying at home, abroad legally (cross-border - and taxfree shopping) or illegally (smuggling/home produced). Food and non-alcoholic beverages can be bought both at home and abroad legally. This allows us to analyze the effect on consumption patterns
after changing indirect taxation and the ensuing effect on tax revenue. We can also analyze the effects on the Norwegian consumption pattern when foreign prices change.

When calculating revenue effects of changes in indirect taxes, it is often not done within a comprehensive framework consisting of a complete demand system with numerous goods, derived from maximization of a specific utility function. Moreover, cross-border shopping and smuggling are not modelled within such a framework (cf. Walker and Huang, 2003). Often, the only thing taken into account is the direct price elasticity at initial prices, without concern for cross-price effects (NOU, 2003:17, chs. 4 and 9; Crawford and Tanner, 1995). We developed our model in fact for a government appointed commission charged with evaluating the effects of cross-border shopping in Norway and whether tax revenue computation associated with indirect taxes (excise taxes) on cross-border exposed goods could be improved (NOU, 2003:17). ${ }^{1}$

The theoretical literature on commodity tax competition shares some of the features of our study in the sense that it considers cross-border shopping and it often assumes only revenue maximization (Mintz and Tulkens, 1986; Kanbur and Keen; 1993, Wildasin, 1988; Lockwood, 1993, 2001; Nielsen, 2001; Oshawa, 1999; Wang, 1999). However, these studies use game theory to find and characterize different equilibria when nations engage in strategic tax setting. Christiansen (1994) is one instance of a study focusing on cross-border shopping that does not use game theory, but which considers welfare maximization. Turning to empirical studies, they often settle for finding possible reaction curves, without founding this explicitly on e.g. consumer theory (Rork, 2003; Nelson, 2002; Devereux et al., 2007). In our study we find revenue maximizing tax rates conditional on the other countries' tax rates/prices. We will also give examples of how the maximizing tax rates change when foreign prices change, cf. tables 6 and 7. This is the first step in the search for the reaction curve for Norway under revenue maximization. ${ }^{2}$ We differ from the empirical literature mentioned above, in grounding our empirical analyses explicitly on consumer theory and by focusing on changes in consumption patterns within a country, cf. our detailed complete demand system.

In the following we will show that by allowing for cross-border shopping/tax-free shopping, and smuggling, the elasticities for the goods in question increase significantly. Consequently we get revenue maximizing tax shares for wine and spirits. The partial revenue from wine as a function of the tax share on wine has a single maximum value close to the actual tax rate in Norway in 1999,

[^1]conditioned on all the other exogenous variables this year. The total revenue as function of the tax share on wine also has a single maximum value, larger than that of the partial revenue function. These results are valid for spirits as well. We find no revenue maximizing tax share on the other goods included in the analysis.

In Section 2 we go through our model's basic theory and concepts. We introduce the empirical model in Section 3, and present the simulation results in Section 4. We conclude in Section 5.

## 2. Basic theory and concepts

We use standard economic consumer theory, see e.g. Deaton and Muellbauer (1980), but with several specifications and interpretations. We take as a starting point the assumption of households maximizing utility subject to a linear budget constraint

$$
\begin{array}{ll}
\mathrm{u}_{\mathrm{ht}}=\mathrm{u}\left(\mathrm{q}_{\mathrm{lh},}, \ldots, \mathrm{q}_{\mathrm{jht}}, \ldots, \mathrm{q}_{\mathrm{nht}} ; \mathrm{a}_{\mathrm{lh}}, \mathrm{a}_{2 \mathrm{hh}}\right), & j \in \mathrm{~J}, \mathrm{~h} \in \mathrm{H}, \\
\mathrm{p}_{\mathrm{lt}} \mathrm{q}_{\mathrm{lht}}+\ldots+\mathrm{p}_{\mathrm{j} t} \mathrm{q}_{\mathrm{jht}}+\ldots+\mathrm{p}_{\mathrm{nt}} \mathrm{q}_{\mathrm{nht}}=\mathrm{y}_{\mathrm{ht}}, & j \in \mathrm{~J}, \mathrm{~h} \in \mathrm{H}, \tag{2}
\end{array}
$$

where $\mathrm{u}($.$) is a utility function, which can be interpreted as an ordinal preference indicator; \mathrm{u}_{\mathrm{lt}}$ is the utility level for household h in time period (or hypothetical situation) $\mathrm{t} ; \mathrm{H}$ is the set of all households in Norway; $q_{j h t}$ is the consumption of commodity group $j$ for household $h$ in time period $t ; J$ is the set of all commodity groups; $n$ is the number of commodity groups in the set $\mathrm{J} ; \mathrm{a}_{1 \mathrm{~h}}$ and $\mathrm{a}_{2 \mathrm{~h}}$ are the number of children and adults respectively in household h ; $\mathrm{p}_{\mathrm{j}}$ is the price of commodity group j in period t ; and $y_{h t}$ is the total consumption expenditure for household $h$ in time period $t$. By maximizing utility (1) subject to the constraint (2) we are able to derive Marshallian demand functions for each household,

$$
\begin{equation*}
\mathrm{q}_{\mathrm{jht}}=\mathrm{g}_{\mathrm{j}}\left(\mathrm{p}_{\mathrm{tt}}, \ldots, \mathrm{p}_{\mathrm{nt}}, \mathrm{y}_{\mathrm{ht}}, \mathrm{a}_{\mathrm{lh}}, \mathrm{a}_{2 \mathrm{~h}}\right), \quad \mathrm{j} \in \mathrm{~J}, \mathrm{~h} \in \mathrm{H}, \tag{3}
\end{equation*}
$$

i.e., the consumption of good j is a function of all prices on all consumer goods, household total consumption expenditure, number of children and number of adults. ${ }^{3}$

Given our particular specification of the utility function, we can add up all households' demand and derive total demand functions for Norway of the type,

[^2]\[

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{jt}}=\mathrm{g}_{\mathrm{j}}\left(\mathrm{p}_{1 \mathrm{t}}, \ldots, \mathrm{p}_{\mathrm{nt}}, \mathrm{Y}_{\mathrm{t}}, \mathrm{~A}_{1 \mathrm{t}}, \mathrm{~A}_{2 \mathrm{t}}, \mathrm{~N}_{\mathrm{t}}\right), \quad \mathrm{j} \in \mathrm{~J}, \tag{4}
\end{equation*}
$$

\]

where $Q_{j t}$ is total private consumption of commodity group $j$ in Norway in period $t ; Y_{t}$ is the sum of total consumption expenditures over all Norwegian households; $\mathrm{A}_{1 t}, \mathrm{~A}_{2 \mathrm{t}}$, and $\mathrm{N}_{\mathrm{t}}$ are the number of children, the number of adults and the number of households respectively, in Norway in period $t$. These variables are defined as simple arithmetical sums of the corresponding household variables and $\mathrm{N}_{\mathrm{t}}$ is the number of elements in the set H of all the households in Norway. The fact that, under certain additional assumptions, the macro demand functions in (4) can be derived from the micro demand functions in (3), is shown for a special case in Aasness and Holtsmark (1993, theorem 2) and analyzed in a more general model in Aasness et al. (1996, pp. 339-341).

From the budget constraints (2) for all households in Norway and the definition of the macro variables we can easily derive a budget constraint for the 'macro consumer':

$$
\begin{equation*}
\mathrm{p}_{1 \mathrm{t}} \mathrm{Q}_{\mathrm{lt}}+\ldots+\mathrm{p}_{\mathrm{jt}} \mathrm{Q}_{\mathrm{jt}}+\ldots+\mathrm{p}_{\mathrm{nt}} \mathrm{Q}_{\mathrm{nt}}=\mathrm{Y}_{\mathrm{t}} \tag{5}
\end{equation*}
$$

i.e., the sum of price times quantity over all the goods is equal to the macro total consumption expenditure. It is also possible to show that there exists a utility function $U($.$) of the type$

$$
\begin{equation*}
\mathrm{U}_{\mathrm{t}}=\mathrm{U}\left(\mathrm{Q}_{1 t}, \ldots, \mathrm{Q}_{\mathrm{jht}}, \ldots, \mathrm{Q}_{\mathrm{nht}} ; \mathrm{A}_{1 \mathrm{t}}, \mathrm{~A}_{2 \mathrm{t} ;} ; \mathrm{N}_{\mathrm{t}}\right), \tag{6}
\end{equation*}
$$

such that if the 'macro consumer' maximizes the utility function (6) subject to the budget constraint (5) we get exactly the same system of demand functions as in (4) above by adding all the demand functions for all the households in Norway.

Each of these theoretical properties have several advantages that could be utilized in the development of practical models for analyzing economic policy. In particular we get macro demand functions in (4) that satisfy exactly the linear budget constraint in (5) for any chosen tax system with subsequent effects on prices and the total expenditure. The system of macro demand functions in (4) satisfies all standard consumer theory properties and relations of an individual consumer, cf. e.g. Deaton and Muellbauer (1980). This gives us a large set of control opportunities, including the restrictions on the sum of columns and rows in Table 4 below, for instance. They could be very useful for e.g. revealing programming errors. By making further restrictions concerning the specification of the utility function, cf. the utility tree in Figure 1, the number of restrictions can grow considerably. This relationship between micro and macro gives us many ways to calibrate and test the model against data from different sources. Of particular interest would be to combine household and macro data, see Aasness and Holtsmark (1993) for an example. Finally, in applied analyses various economic theories such as welfare theory and theory of optimal taxation could be applied, see Aasness et al. (1996) and Schroyen and Aasness (2006) for examples.

In what follows, we refer to prices $\left(\mathrm{p}_{\mathrm{jt}}\right)$ facing consumers as consumer prices. They will be affected by excise taxes, VAT and other indirect taxes and subsidies. The relationship between the consumer price $\left(p_{j}\right)$, price without tax $\left(p_{*}\right)$ and the tax rate $\left(t_{j}\right)$ for good j , is assumed to be

The tax rate $\mathrm{t}_{\mathrm{j} t}$ can be interpreted as a function of all excise taxes, system of VAT, and other indirect taxes and subsidies associated with the consumer good, see (21) below. The tax rate is measured in proportion to the price of the good without taxes $\left(p_{* j}\right)$. We only take into account taxes that raise domestic tax revenue. For commodities bought abroad (cross-border - and tax-free shopping), the tax rate will be zero.

Instead of measuring tax as share of the price without tax $\left(p_{*_{j}}\right)$, it can be measured as a share of the consumer price $\left(p_{j}\right)$. We denote the latter tax share for commodity j as $\tau_{\mathrm{j}}$. The relationship between these two tax concepts is as follows,

$$
\begin{equation*}
\tau_{\mathrm{jt}}=\mathrm{t}_{\mathrm{jt}} /\left(1+\mathrm{t}_{\mathrm{jt}}\right), \quad \mathrm{j} \in \mathrm{~J} \tag{8}
\end{equation*}
$$

From this we realize that when $\mathrm{t}_{\mathrm{j}}$ goes to infinity, $\tau_{\mathrm{j}}$ will approach 1 . Table 4 presents the tax rates $\left(\mathrm{t}_{\mathrm{j}}\right)$ and the tax shares $\left(\tau_{\mathrm{j}}\right)$ for the model base year. We find it useful to exploit both concepts in this paper.
 of good j becomes

$$
\begin{equation*}
\mathrm{T}_{\mathrm{jt}}=\mathrm{t}_{\mathrm{jt}} \mathrm{p}_{\mathrm{jt}} \mathrm{Q}_{\mathrm{jt}} \quad \mathrm{j} \in \mathrm{~J} \tag{9}
\end{equation*}
$$

We will refer to this as the partial revenue from good j . We define the total tax revenue from indirect taxes associated with household's total consumption of all consumer goods as

$$
\begin{equation*}
\mathrm{T}_{\mathrm{t}}=\Sigma_{\mathrm{j} \in \mathrm{~J}} \mathrm{~T}_{\mathrm{jt}}=\Sigma_{\mathrm{j} \in \mathrm{~J}} \mathrm{t}_{\mathrm{j} \mathrm{t}} \mathrm{p}_{\mathrm{j} \mathrm{j} \mathrm{t}} \mathrm{Q}_{\mathrm{it}}, \tag{10}
\end{equation*}
$$

i.e., as the sum of all partial tax revenues.

From (4), (7), and (9) follows the partial revenue function (PRF) associated with each commodity group,

$$
\begin{equation*}
\mathrm{T}_{\mathrm{jt}}=\mathrm{T}_{\text {PRFj }}\left(\mathrm{t}_{\mathrm{lt}}, \ldots, \mathrm{t}_{\mathrm{nt}}, \mathrm{p}_{* 1 \mathrm{t}}, \ldots, \mathrm{p}_{*_{\mathrm{nt}},} \mathrm{Y}_{\mathrm{t}}, \mathrm{~A}_{\mathrm{t}}, \mathrm{~A}_{2 \mathrm{t}}, \mathrm{~N}_{\mathrm{t}}\right), \quad \mathrm{j} \in \mathrm{~J} \tag{11}
\end{equation*}
$$

Setting all variables constant except for the tax rate of the specific good, we can define the partial revenue from commodity j as a function of the tax share for the same commodity j :

$$
\begin{equation*}
\mathrm{T}_{\mathrm{jt}}=\mathrm{T}_{\text {PRC } \mathrm{j}}\left(\mathrm{t}_{\mathrm{j} \mathrm{t}}\right) \equiv T_{P R F j}\left(\mathrm{t}_{10}, \ldots, \mathrm{t}_{\mathrm{j} \mathrm{t}, \ldots,}, \mathrm{t}_{\mathrm{n} 0}, \mathrm{p}_{* 10}, \ldots, \mathrm{p}_{*_{\mathrm{n} 0}}, \mathrm{Y}_{0}, \mathrm{~A}_{10}, \mathrm{~A}_{20}, \mathrm{~N}_{0}\right), \quad \mathrm{j} \in \mathrm{~J} \tag{12}
\end{equation*}
$$

Since this function can be displayed as a one-dimensional curve, we call it the partial revenue curve (PRC) for commodity j .

We can now formulate a hypothesis:
For some goods $\mathrm{j} \in \mathrm{J}_{\mathrm{PRCM}} \subseteq \mathrm{J}$, the partial revenue curve $\mathrm{T}_{\text {PRCj}}\left(\mathrm{t}_{\mathrm{j}}\right)$ will increase monotonically
(13) to a specific value of the tax rate $\left(\mathrm{t}_{\mathrm{PRCMj}}\right)$, called here the partial tax revenue maximizing tax rate for good j , after which it declines monotonically with the tax rate.

For such goods we call the partial revenue curve a Dupuit curve, in honor of the French economist who described such curves already in Dupuit (1844). We could also call it a Laffer curve, since the American economist Arthur B. Laffer got much publicity in the 1970's by focusing on this possible type of non-monotonic relationship between a tax rate and tax revenue, cf. Fullerton (1981). However, Laffer did not focus on specific indirect taxes, but rather tax on labor and taxes in general.

Given that hypothesis (13) applies for good $j$, and that the partial revenue curve $T_{\text {PRCj }}\left(\mathrm{t}_{\mathrm{j} t}\right)$ is differentiable, the partial tax revenue maximizing tax rate $\left(\mathrm{t}_{\mathrm{PRCj}}\right)$ is determined by the first order condition

$$
\begin{equation*}
\partial \mathrm{T}_{\mathrm{PRCj}}\left(\mathrm{t}_{\mathrm{PRCM}}\right) / \partial \mathrm{t}_{\mathrm{jt}}=0, \tag{14}
\end{equation*}
$$

$$
\mathrm{j} \in \mathrm{~J}_{\mathrm{PRCM}} \subseteq \mathrm{~J} .
$$

This condition can be used to characterize or compute $\mathrm{t}_{\mathrm{PRCmj}}$. From (4), (7), (9), (14) and formulas (5.5) and (5.8) in Sydsæter, Strøm and Berck (2005) it follows that

$$
\begin{equation*}
\tau_{\text {PRCM }}=-1 / \mathrm{e}_{\mathrm{j} j}, \quad \mathrm{j} \in \mathrm{~J}_{\mathrm{PRCM}} \subseteq \mathrm{~J}, \tag{15}
\end{equation*}
$$

where $\mathrm{e}_{\mathrm{ij}}$ is the direct Cournot elasticity for commodity j , from demand function (4). Note that such elasticities are endogenous variables both in our theoretical and empirical model, depending on the partial revenue maximizing tax rate $\left(\mathrm{t}_{\mathrm{PRCMj}}\right)$ and the values of all the conditional variables in (12) above, i.e. $\left(\mathrm{t}_{10}, \ldots, \mathrm{t}_{\mathrm{n} 0}, \mathrm{p}_{* 10}, \ldots, \mathrm{p}_{*_{n} 0}, \mathrm{Y}_{0}, \mathrm{~A}_{10}, \mathrm{~A}_{20}, \mathrm{~N}_{0}\right)$. Relation (15) says that the partial revenue is maximized with respect to the tax on commodity $j$, if the tax share for commodity $j$ is equal to minus the inverse of the direct price elasticity of commodity j . This we call the inverse elasticity rule for revenue maximization, see e.g. Walker and Huang (2003).

In Section 4 below we use simulation techniques to compute the partial revenue curves, within our large empirically based model described in Section 3, for each good in question. We examine whether hypothesis (13) holds, and if it does, we use numerical methods to find the corresponding maximum, given the base year values of the other variables in equation (12).

Note that the set of goods $\mathrm{J}_{\mathrm{PRCM}}$ with proper Dupuit curves as described in (13) can be the empty set $(\phi)$, the set of all goods (J) or any subset of J depending on the form of the utility function in (1), the total expenditure, demographic variables and the price vector in the base year. While we leave a thorough theoretical and empirical analysis of this problem for a later date, we give examples of commodities belonging to $\mathrm{J}_{\mathrm{PRCM}}$, and commodities that do not, in our simulation analysis below.

From (4), (7) and (10) follows the total tax revenue function (TRF) for indirect taxes:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{t}}=\mathrm{T}_{\mathrm{TRF}}\left(\mathrm{t}_{1 \mathrm{t}}, \ldots, \mathrm{t}_{\mathrm{n} t}, \mathrm{p}_{* 1 \mathrm{t}}, \ldots, \mathrm{p}_{*_{\mathrm{n}}}, \mathrm{Y}_{\mathrm{t}}, \mathrm{~A}_{1 \mathrm{t}}, \mathrm{~A}_{2 \mathrm{t}}, \mathrm{~N}_{\mathrm{t}}\right) \tag{16}
\end{equation*}
$$

Setting all variables constant except for the tax rate on some good $j$, we can define the total tax revenue as a function of the tax share for good j :

$$
\begin{equation*}
\mathrm{T}_{\mathrm{t}}=\mathrm{T}_{\mathrm{TRC}}\left(\mathrm{t}_{\mathrm{j} t}\right) \equiv \mathrm{T}_{\mathrm{TRF}}\left(\mathrm{t}_{10}, \ldots, \mathrm{t}_{\mathrm{t} t}, \ldots, \mathrm{t}_{\mathrm{n} 0}, \mathrm{p}_{* 10}, \ldots, \mathrm{p}_{*_{\mathrm{n} 0}}, \mathrm{Y}_{0}, \mathrm{~A}_{10}, \mathrm{~A}_{20}, \mathrm{~N}_{0}\right), \quad \mathrm{j} \in \mathrm{~J} . \tag{17}
\end{equation*}
$$

We call $\mathrm{T}_{\mathrm{TRC}}\left(\mathrm{t}_{\mathrm{j}}\right)$ the total tax revenue curve (TRC) for consumer good j .

We can now formulate a new hypothesis:
For some goods $\mathrm{j} \in \mathrm{J}_{\mathrm{TRCM}} \subseteq \mathrm{J}$, the total revenue curve $\mathrm{T}_{\mathrm{TRC}}\left(\mathrm{t}_{\mathrm{j}}\right)$ will increase
monotonically (18) up to a specific value of the tax rate $\left(\mathrm{t}_{\mathrm{TRCj}}\right)$, here called the total tax revenue maximizing tax
rate for good j , after which it declines monotonically with the tax rate.
Given that hypothesis (18) applies for good j , and that the total revenue curve $\mathrm{T}_{\mathrm{TRCj}}\left(\mathrm{t}_{\mathrm{jt}}\right)$ is differentiable, the total tax revenue maximizing tax rate $\left(\mathrm{t}_{\mathrm{TRC}}\right)$ is determined by the first order condition

$$
\begin{equation*}
\partial \mathrm{T}_{\mathrm{TRCj}}\left(\mathrm{t}_{\mathrm{TRCM}}\right) / \partial \mathrm{t}_{\mathrm{jt}}=0, \quad \mathrm{j} \in \mathrm{~J}_{\mathrm{TRCM}} \subseteq \mathrm{~J} \tag{19}
\end{equation*}
$$

This condition can be used to characterize or compute $\mathrm{t}_{\mathrm{TRCMj}}$. From (4), (7), (9), (10), (17) and formulas (5.5), (5.7), and (5.8) in Sydsæter, Strøm and Berck (2005) it follows that

$$
\begin{equation*}
\tau_{\text {TRCMj }}=-1 /\left(\mathrm{e}_{\mathrm{ij}}+\Sigma_{\mathrm{i} \in \mathrm{~J}-\mathrm{j}} \mathrm{e}_{\mathrm{ij}} \mathrm{~T}_{\mathrm{i}} / \mathrm{T}_{\mathrm{j}}\right) \quad \mathrm{j} \in \mathrm{~J}_{\text {TRCM }} \subseteq \mathrm{J}, \tag{20}
\end{equation*}
$$

where $\mathrm{e}_{1 \mathrm{j}}, \ldots, \mathrm{e}_{\mathrm{nj}}$ are the Cournot elasticities w.r.t. the price of commodity j , from demand function (4), conditioned on the $\mathrm{t}_{\mathrm{TRCMj}}$ as well as on all the other conditional variables in (17) above. Relation (20) says that the total revenue is maximized with respect to tax on commodity $j$, if the tax share for commodity j is equal to minus the inverse of a composite price elasticity, i.e. the direct Cournot elasticity of commodity j plus a weighted average of the cross price Cournot elasticities w.r.t. price $\mathrm{p}_{\mathrm{j}}$. This can be called the inverse composite elasticity rule for revenue maximization, in correspondence with Walker and Huang (2003).

Below we will use simulation techniques to compute the total revenue curves in our large empirically based model, for each tax share in question, and by inspection find if hypothesis (18) holds, and if so by numerical methods find the corresponding maximum, given the base year values of the other variables in equation (17).

Note that the set of goods $\mathrm{J}_{\text {TRCM }}$ as described in (18), can be the empty set ( $\phi$ ), the set of all goods (J) or any subset of J depending on the form of the utility function in (1), the total expenditure, demographic variables and the price vector in the base year. Furthermore, $\mathrm{J}_{\text {TRCM }}$ can be different from $\mathrm{J}_{\text {PRCM }}$ defined in (13) above. A thorough theoretical and empirical analysis of this problem must wait, but we give examples of goods belonging to $\mathrm{J}_{\text {TRCM }}$, and goods that do not, in our simulation analysis below, which is based on our empirical model. We also show how the total tax revenue maximizing tax rate for a commodity $\left(\mathrm{t}_{\mathrm{TRCMj}}\right)$ can depend systematically on other variables held constant in (17), cf. Figure 6 below.

Note that we can define revenue functions and revenue maximizing tax shares when several taxes are allowed to change jointly, cf. Table 5 below for an empirical example.

Let us now assume that the tax system represented by VAT, excise taxes and other tax and subsidies arrangements, can be described by a vector of concrete tax parameters $\left(a_{1 t}, \ldots, a_{m t}\right)$. Further, let us assume that the total tax rate associated with each good is a function of these parameters:

$$
\begin{equation*}
\mathrm{t}_{\mathrm{j}}=t_{j}\left(\mathrm{a}_{1 \mathrm{t}}, \ldots, \mathrm{a}_{\mathrm{mt}}\right), \quad \mathrm{j} \in \mathrm{~J} \tag{21}
\end{equation*}
$$

It follows from (7) and (21) that we can write the consumer prices as a function of the tax parameters $\left(\mathrm{a}_{1 \mathrm{t}}, \ldots, \mathrm{a}_{\mathrm{mt}}\right)$,

$$
\begin{equation*}
\mathrm{p}_{\mathrm{j}}=\mathrm{p}_{* j}\left(1+t_{\mathrm{j}}\left(\mathrm{a}_{1 \mathrm{t}}, \ldots, \mathrm{a}_{\mathrm{mt}}\right)\right) \equiv \mathrm{p}_{\mathrm{j}}\left(\mathrm{a}_{1 \mathrm{t}}, \ldots, \mathrm{a}_{\mathrm{mt}}\right), \quad \mathrm{j} \in \mathrm{~J} \tag{22}
\end{equation*}
$$

There are practical and theoretical advantages to be gained from decomposing the tax model in (7) and (21), instead of considering how the consumer prices $\mathrm{p}_{\mathrm{jt}}$ depend on the concrete tax parameters in (22). In the present version of our model we have not specified (21) and (22). We have only used the tax rates $t_{j}$ in our model base year 1999. A general equilibrium model for Norway (MSG), developed at Statistics Norway, was used to compute these tax rates. ${ }^{4}$

[^3]
## 3. A simulation model of consumer behavior

Our empirical based simulation model (KONSUM-G) is perfectly consistent with the general theory above. ${ }^{5}$ The specification of the utility functions in (1) and (6) is based on the utility tree shown in Figure 1. We show the complete specification with respect to the branch for Food, beverages and tobacco (FBTG). ${ }^{6}$ The branches for the other main groups, Communication (CO), Housing (HO) and Other goods and services (OGS), are also extensive and consist of several levels, though in these cases we confine ourselves to indicating the groups at level two. Within the main group Food, beverages and tobacco (FBTG) we find 18 goods at the lowest level. All together, the model consists of 55 goods.

Figure 1: Utility tree for KONSUM-G


At each node of the utility tree a translated CES subutility function is specified, the "minimum quantities" of which are linear functions of the number of children and adults in the household. The calibration procedure exploits data from various sources and econometric studies. In particular models with latent variables are being used to take care of random and systematic measurement errors in data, cf. Aasness et al. (1993, 2003), Aasness et al. (1994), Aasness and Belsby (1997), Røed Larsen et al. (1997), Wangen and Aasness (2002), and Aasness and Røed Larsen (2003). We have used information

[^4]given in NOU (2003:17) to make estimates on several of the budget shares and elasticities that serve as a basis for our calibration.

Spirits is divided into three commodity groups: bought in Norway (H); bought legally abroad for use in Norway (A); smuggled into the country or distilled illegally in Norway (S). We proceed similarly for wine, beer and tobacco. Two categories of food and non-alcoholic beverages are used: bought in Norway (H) and bought abroad for use in Norway (A). None of the other goods or services are subdivided in this way, but Norwegian consumers' consumption abroad is included as a separate group in another part of the utility tree (OGS).

Since the model fully satisfies all restrictions known from general consumer theory, the results are easy to interpret and control. Assuming the utility function should satisfy the utility tree in figure 1 produces many more restrictions. Our utility tree implies that every household can make decisions about the composition of expenditures by first considering changes in utility brought about by changes in consumption of goods within the same branch of the utility tree. If the tax on spirits in Norway rises, the consumer can be thought of as choosing his new composition of expenditure in the following way. He would first consider whether to reduce his consumption of spirits bought in Norway and make up for the loss by shopping abroad ( 03 CA ) and smuggling ( 03 CS ). Second he would consider whether to cut back on sprits altogether and buy more wine (03D) and beer (03E). Third he might consider whether to rearrange spending on Food (00), Beverage without alcohol (BO), Beverage with alcohol (BA) and Tobacco (04). He could also adjust the balance of expenditure on the four main groups at the top level of the utility tree. Finally the consumer will run down the utility tree checking whether his budget is optimally distributed between the different groups and subgroups.

Theoretically, increasing the tax on spirits could affect consumption of diet milk which is a part of the Food group ( 00 ). But this can only happen by changing total expenditure on Food (00), i.e., using the same channel as any other price change outside the Food group. Consequently the cross-price elasticity between spirits and diet milk is close to zero, though not exactly. One could extend the utility tree, for instance, and create larger cross-price elasticity between ordinary milk and diet milk compared to the cross-price elasticity between ordinary milk and spirits. The exploitation of such assumptions is essential in our way of modeling.

The model has 55 goods, resulting in a matrix of $55 \times 55=3025$ price elasticities. This matrix satisfies all of the requirements stipulated by consumer theory. Matrices of this size are difficult to use and
present. By assuming an equiproporsional change in all prices within a group, we can exploit Hicks composite commodity theorem and aggregate all the goods within a certain group, and still satisfy the requirements of consumer theory. Tables 1 and 2 present a complete set of elasticities for a grouping of 6 goods and tables 3 and 4 the corresponding elasticities for a grouping of 15 goods. All goods in the utility tree outside the branch for Food, beverages and tobacco are aggregated to form a new group, Other goods (AG). Within this new branch of the utility tree we have made one group, Food and beverages without alcohol (0BO), out of the two groups Food (00) and Beverages without alcohol (BO). The difference between the 6 goods group and the 15 goods group is that the latter distinguishes between goods bought in Norway (H); across the border (A); and smuggled or distilled at home (S), while these are aggregated into one common group for the different types of goods for the 6 good group. The elasticities for the 6 good groups are an exact aggregation of the elasticities for the 15 goods group. In particular it is interesting to study the relationship between the elasticities for the aggregated and disaggregated groups.

Table 1. Price elasticities (Cournot), Engel elasticities (E) and budget shares (w) for food, beverages and tobacco. Main groups

|  | Commodity <br> Croup | $\mathrm{w}(\%)$ | E | ej.0BO | ej.03C | ej.03D | ej.03E | ej.04 | ej.AG | Sum |
| :--- | :--- | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Food/non- |  |  |  |  |  |  |  |  |  |
|  | alcoholic beve- |  |  |  |  |  |  |  |  |  |
| 0BO | rages | 1.386 | 1.306 | -0.188 | -0.001 | 0.000 | -0.001 | -0.008 | -0.110 | 0.000 |
| 03C | Spirits | 0.919 | 1.856 | -0.157 | -0.885 | 0.130 | 0.095 | -0.033 | -0.474 | 0.000 |
| 03D | Wine | 1.164 | 1.139 | -0.135 | 0.188 | -1.247 | 0.133 | -0.047 | -0.664 | 0.000 |
| 03E | Beer | 3.087 | 0.303 | -0.036 | -0.001 | 0.112 | -0.795 | -0.029 | -0.407 | 0.000 |
| 4 | Tobacco | 78.849 | 1.138 | -0.142 | -0.006 | -0.001 | -0.006 | -0.030 | -0.953 | 0.000 |
| AG | Other goods |  | -0.001 | -0.157 | -0.108 | 0.000 |  |  |  |  |
|  | Sum (weighted) | 100.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |  |

Table 2. Price elasticities (Slutsky) for food, beverages and tobacco. Main groups

| Codes | Commodity group | sj.0BO | sj.03C | sj.03D | sj.03E | sj.04 | ej.AG | Sum |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Food/non-alcoholic |  |  |  |  |  |  |  |
| 0BO | beverages | -0.143 | 0.003 | 0.003 | 0.003 | 0.002 | 0.132 | 0.000 |
| 03C | Spirits | 0.037 | -0.867 | 0.142 | 0.110 | 0.008 | 0.570 | 0.000 |
| 03D | Wine | 0.051 | 0.214 | -1.230 | 0.154 | 0.011 | 0.800 | 0.000 |
| 03E | Beer | 0.031 | 0.131 | 0.122 | -0.782 | 0.007 | 0.490 | 0.000 |
| 4 | Tobacco | 0.008 | 0.003 | 0.003 | 0.002 | -0.148 | 0.131 | 0.000 |
| AG | Other goods | 0.024 | 0.010 | 0.009 | 0.007 | 0.005 | -0.056 | 0.000 |
|  | Sum (weighted) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |  |

Table 3.

| Codes | Commodity group | w (\%) | E | ej.0BOH | ej.0BOA | ej. 03 CH | ej.03CA | ej.03CS | ej.03DH | ej.03DA | ej.03DS | ej.03EH | ej.03EA | ej.03ES | ej.04H | ej.04A | ej.04S | ej.AG | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0BOH | Food/ beverages without alcohol; Norway Food/ beverages without alcohol; Cross- | 14.286 | 0.306 | -0.212 | 0.025 | 0.000 | 0.000 | -0.001 | 0.001 | 0.000 | 0.000 | -0.001 | 0.000 | 0.000 | -0.005 | -0.002 | -0.001 | -0.110 | 0.000 |
| 0BOA | border sh. Spirits; Nor- | 0.310 | 0.306 | 1.138 | -1.325 | 0.000 | 0.000 | -0.001 | 0.001 | 0.000 | 0.000 | -0.001 | 0.000 | 0.000 | -0.005 | -0.002 | -0.001 | -0.110 | 0.000 |
| 03CH | way <br> Spirits; Cross- <br> border shop- | 0.675 | 1.563 | -0.181 | -0.004 | -1.412 | 0.213 | 0.154 | 0.118 | 0.018 | 0.017 | 0.108 | 0.003 | 0.001 | -0.028 | -0.009 | -0.003 | -0.559 | 0.000 |
| 03CA | ping <br> Spirits; | 0.287 | 1.563 | -0.181 | -0.004 | 0.501 | -1.699 | 0.154 | 0.118 | 0.018 | 0.017 | 0.108 | 0.003 | 0.001 | -0.028 | -0.009 | -0.003 | -0.559 | 0.000 |
| 03CS | Smuggling | 0.424 | 0.782 | -0.091 | -0.002 | 0.250 | 0.106 | -0.879 | 0.059 | 0.009 | 0.009 | 0.054 | 0.001 | 0.000 | -0.014 | -0.004 | -0.001 | -0.280 | 0.000 |
| 03DH | Wine; Norway Wine; Crossborder shop- | 0.625 | 2.085 | -0.241 | -0.005 | 0.124 | 0.053 | 0.035 | -1.545 | 0.071 | 0.074 | 0.144 | 0.004 | 0.001 | -0.037 | -0.011 | -0.004 | -0.746 | 0.000 |
| 03DA | ping <br> Wine; | 0.093 | 2.085 | -0.241 | -0.005 | 0.124 | 0.053 | 0.035 | 0.476 | -1.950 | 0.074 | 0.144 | 0.004 | 0.001 | -0.037 | -0.011 | -0.004 | -0.746 | 0.000 |
| 03DS | Smuggling | 0.201 | 1.042 | -0.121 | -0.003 | 0.062 | 0.026 | 0.017 | 0.238 | 0.035 | -0.973 | 0.072 | 0.002 | 0.000 | -0.019 | -0.006 | -0.002 | -0.373 | 0.000 |
| 03EH | Beer, Norway Beer; Crossborder shop- | 1.118 | 1.147 | -0.133 | -0.003 | 0.068 | 0.029 | 0.019 | 0.087 | 0.013 | 0.013 | -0.819 | 0.015 | 0.004 | -0.021 | -0.006 | -0.002 | -0.410 | 0.000 |
| 03EA | ping <br> Beer; | 0.030 | 1.147 | -0.133 | -0.003 | 0.068 | 0.029 | 0.019 | 0.087 | 0.013 | 0.013 | 0.540 | -1.344 | 0.004 | -0.021 | -0.006 | -0.002 | -0.410 | 0.000 |
| 03ES | Smuggling <br> Tobacco; | 0.016 | 0.573 | -0.066 | -0.001 | 0.034 | 0.015 | 0.010 | 0.043 | 0.006 | 0.006 | 0.270 | 0.007 | -0.678 | -0.010 | -0.003 | -0.001 | -0.205 | 0.000 |
| 04H | Norway <br> Tobacco; Cross-border | 2.219 | 0.313 | -0.036 | -0.001 | 0.000 | 0.000 | -0.001 | 0.001 | 0.000 | 0.000 | -0.001 | 0.000 | 0.000 | -0.480 | 0.279 | 0.039 | -0.112 | 0.000 |
| 04A | shopping <br> Tobacco; | 0.677 | 0.313 | -0.036 | -0.001 | 0.000 | 0.000 | -0.001 | 0.001 | 0.000 | 0.000 | -0.001 | 0.000 | 0.000 | 0.913 | -1.114 | 0.039 | -0.112 | 0.000 |
| 04S | Smuggling | 0.191 | 0.156 | -0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | -0.001 | 0.000 | 0.000 | 0.457 | 0.139 | -0.677 | -0.056 | 0.000 |
| AG | Other goods | 78.849 | 1.138 | -0.139 | -0.003 | -0.002 | -0.001 | -0.003 | 0.000 | 0.000 | -0.001 | -0.006 | 0.000 | 0.000 | -0.021 | -0.007 | -0.002 | -0.953 | 0.000 |
|  | $\begin{aligned} & \text { Sum (Weigh- } \\ & \text { ted) } \end{aligned}$ | 100.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |  |

Table 4. Price elasticities (Slutsky), Engel elasticities (E), tax shares (t,t). Detailed groups

| Codes | Commodity group | t | $\tau$ | sj.0BOH | sj.0BOA | sj.03CH | sj.03CA | sj.03CS | sj.03DH | sj.03DA | sj.03DS | sj.03EH | sj.03EA | sj.03ES | sj.04H | sj.04A | sj.04S | sj.AG | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0BOH | Food/nonalcoholic beverages; Norway Food/nonalcoholic beverages; Cross-border | 0.260 | 0.206 | -0.168 | 0.026 | 0.002 | 0.001 | 0.001 | 0.002 | 0.000 | 0.000 | 0.002 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.132 | 0.000 |
| 0BOA | sh. Spirits; Nor- | 0.000 | 0.000 | 1.182 | -1.324 | 0.002 | 0.001 | 0.001 | 0.002 | 0.000 | 0.000 | 0.002 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.132 | 0.000 |
| 03CH | way <br> Spirits; Crossborder shop- | 3.349 | 0.770 | 0.042 | 0.001 | -1.401 | 0.217 | 0.161 | 0.128 | 0.019 | 0.021 | 0.126 | 0.003 | 0.001 | 0.007 | 0.002 | 0.000 | 0.673 | 0.000 |
| 03CA |  | 0.000 | 0.000 | 0.042 | 0.001 | 0.511 | -1.695 | 0.161 | 0.128 | 0.019 | 0.021 | 0.126 | 0.003 | 0.001 | 0.007 | 0.002 | 0.000 | 0.673 | 0.000 |
| 03CS | Smuggling | 0.000 | 0.000 | 0.021 | 0.000 | 0.256 | 0.109 | -0.876 | 0.064 | 0.010 | 0.010 | 0.063 | 0.002 | 0.000 | 0.003 | 0.001 | 0.000 | 0.337 | 0.000 |
| 03DH | Wine; Norway Wine; Crossborder shop- | 1.841 | 0.648 | 0.056 | 0.001 | 0.138 | 0.059 | 0.043 | -1.532 | 0.073 | 0.079 | 0.168 | 0.005 | 0.001 | 0.009 | 0.003 | 0.000 | 0.898 | 0.000 |
| 03DA | ping <br> Wine; | 0.000 | 0.000 | 0.056 | 0.001 | 0.138 | 0.059 | 0.043 | 0.489 | -1.948 | 0.079 | 0.168 | 0.005 | 0.001 | 0.009 | 0.003 | 0.000 | 0.898 | 0.000 |
| 03DS | Smuggling | 0.000 | 0.000 | 0.028 | 0.001 | 0.069 | 0.029 | 0.022 | 0.244 | 0.036 | -0.971 | 0.084 | 0.002 | 0.001 | 0.004 | 0.001 | 0.000 | 0.449 | 0.000 |
| 03EH | Beer; Norway Beer; Crossborder shop- | 1.661 | 0.624 | 0.031 | 0.001 | 0.076 | 0.032 | 0.024 | 0.094 | 0.014 | 0.015 | -0.806 | 0.015 | 0.004 | 0.005 | 0.001 | 0.000 | 0.494 | 0.000 |
| 03EA | ping <br> Beer; | 0.000 | 0.000 | 0.031 | 0.001 | 0.076 | 0.032 | 0.024 | 0.094 | 0.014 | 0.015 | 0.553 | -1.344 | 0.004 | 0.005 | 0.001 | 0.000 | 0.494 | 0.000 |
| 03ES | Smuggling <br> Tobacco; | 0.000 | 0.000 | 0.015 | 0.000 | 0.038 | 0.016 | 0.012 | 0.047 | 0.007 | 0.008 | 0.277 | 0.007 | -0.678 | 0.002 | 0.001 | 0.000 | 0.247 | 0.000 |
| 04H | Norway <br> Tobacco; Cross-border | 2.414 | 0.707 | 0.008 | 0.000 | 0.002 | 0.001 | 0.001 | 0.003 | 0.000 | 0.000 | 0.002 | 0.000 | 0.000 | -0.473 | 0.281 | 0.040 | 0.135 | 0.000 |
| 04A | shopping <br> Tobacco; | 0.000 | 0.000 | 0.008 | 0.000 | 0.002 | 0.001 | 0.001 | 0.003 | 0.000 | 0.000 | 0.002 | 0.000 | 0.000 | 0.920 | -1.112 | 0.040 | 0.135 | 0.000 |
| 04S | Smuggling | 0.000 | 0.000 | 0.004 | 0.000 | 0.001 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.460 | 0.140 | -0.677 | 0.067 | 0.000 |
| AG | Other goods | 0.168 | 0.144 | 0.024 | 0.001 | 0.006 | 0.002 | 0.002 | 0.007 | 0.001 | 0.001 | 0.007 | 0.000 | 0.000 | 0.004 | 0.001 | 0.000 | -0.056 | 0.000 |
|  | Sum (Weighted) | 0.211 | 0.174 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |  |

Note that the column sums in tables $1-4$ are zero. They are weighted sums of the elasticities which follow from the consumer's budget constraint. Similarly, the row sums are all zero as a consequence of the demand functions being homogeneous of degree zero in prices and total expenditure. Note also that the cross-price elasticities are all positive in tables 2 and 4. They are Slutsky elasticities, i.e., we imagine the consumer receiving income compensation such that the utility level stays the same after a price increase. Since the consumer goods are all substitutes, every Slutsky cross-price elasticity will be positive. In tables 1 and 3 we have Cournot elasticities, i.e., the consumer receives no income compensation and several of the cross-price elasticities become negative due to negative income effects. Nevertheless, some goods are sufficiently strong substitutes to make the substitution effect dominate and the cross-price elasticities end up as positive. In particular we can observe that we get positive Cournot elasticities between every beverage with alcohol both in tables 1 and 3 . We also get positive Cournot elasticities in Table 3 between goods of the same type, but depending on whether they are bought at home $(\mathrm{H})$, abroad (A) or smuggled or produced illegally at home (S).

Let us first consider the price elasticities for tobacco. From table 1 we see that the direct price elasticity for the main group tobacco is -0.16 . Compared with results produced by econometric studies, in absolute value terms, this is rather low. On the other hand, if we observe the direct price elasticity for tobacco bought in Norway, it is -0.48 , i.e., between $0.4-0.5$ in absolute value, an interval considered reasonable by the commission set up by the government to report on cross-border trade (NOU 2003:17). And as Table 3 shows, the price elasticity for tobacco bought outside the country is -1.11. The fact that this elasticity is much larger in absolute value makes sense since it has a smaller share of total tobacco consumption than tobacco bought in Norway, as the first column of Table 3 shows. The cross-price elasticities between the different tobacco groups are clearly positive, which is in line with the utility tree in Figure 1.

The direct cross-price elasticities for spirits, wine and beer are $-0.9,-1.3$ and -0.8 , respectively. This is largely in line with the considerations of the commission, but the price elasticities for spirits are somewhat lower. From Table 3 we see that the corresponding elasticities for spirits, wine and beer bought in Norway are $-1.4,-1.6$ and -0.8 , respectively. They have all increased in absolute value, but only slightly for beer, some for wine, and more for spirits. This is because a large share of the total consumption of spirits derives from cross-border shopping/tax-free purchases, and smuggling/illegal distilling, as the budget shares in Table 3 show. These price elasticities are therefore somewhat higher than what the commission considers to be reasonable, although they refer to price and income elasticities when measuring consumption in pure alcohol, while the price elasticities in tables 1-4 are based on
consumption in 1999 prices. The latter will probably be higher because decreasing prices or increasing total expenditure tends to favor the more expensive wine and spirits brands (Aasness (1990), Essay 2). The logic in our model requires that we measure the consumption of different commodity groups in Norwegian kroner, but we could in a consistent way add some relations for the consumption measured in different ways, e.g., as the amount of pure alcohol. Nevertheless, in the present model this is not done.

The mechanism of the model could be partly illustrated by increasing the excise tax on spirits in Norway, affecting the tax share for spirits. These are some of the effects : i) the tax revenue from spirits will increase for a given level of demand for spirits; ii) the tax revenue will decrease because of falling demand caused by the higher tax rate and retail price; iii) in particular it generates a tax leakage consisting of revenue from excise taxes and value added tax caused by increased cross-border and taxfree shopping and smuggling/illegal distilling; (iv) the increase in tax on spirits causes a substitution from spirits to wine and beer bought in Norway which increases the tax revenue; (v) the increase in tax on spirits causes demand to fall for other goods bought in Norway because the real value of the total expenditure decreases as the price on spirits increases. This decreases the total tax revenue. All of these effects affect the outcome simultaneously, and our model gives us the sum of the effects. It will, of course, depend on the properties of the model and assumptions regarding crucial quantities entering our model. Which of the effects that dominates will change when the tax on spirits changes, and in a systematic way, as shown in the forthcoming figures.

## 4. Simulation results

## Effects on revenue of changes in taxes on alcohol

Figure 2 shows how the revenue from tax on wine increases when the tax share increases until it reaches a maximum, after which the partial tax revenue decreases when the tax share increases. The same applies to spirits. The partial tax revenue maximizing tax share (as the share of consumer price, including VAT) is smallest for wine (0.648) and largest for spirits $(0.730)$. This is caused by the higher price elasticity for wine, cf. the inverse elasticity rule in (15) and the direct price elasticity for wine bought at home in Table 3 ( $\mathrm{e}_{03 \mathrm{DH} .03 \mathrm{DH}}=-1.545$ ).

Strictly speaking, our simulations refer to the year 1999, the base year for our model, but similar qualitative results are likely to apply for other years as well. The actual tax shares for 1999 are also depicted in the figure. The actual tax share in 1999 for wine ( 0.648 ) is approximately equal to the tax share value maximizing the partial tax revenue for wine. For spirits the actual tax share (0.770) is larger than the tax share maximizing the partial revenue from spirits.

Figure 2. Partial tax revenue from spirits, wine and beer as a function of their respective tax share. A vertical line indicates maximum tax revenue. A cross indicates tax share in 1999


Source: Statistics Norway, KONSUM-G-march-2003
Figure 3. Total tax revenue from indirect taxes as a function of tax share for spirits, wine and beer respectively. A vertical line indicates maximum total indirect tax revenue. A cross indicates tax share in 1999


Source: Statistics Norway, KONSUM-G-march-2003

From Figure 2 we observe that the partial tax revenue from beer increases monotonically with the tax share. In the case of beer we are not able to find any maximizing value for the partial tax revenue. This is caused by the fact that the price elasticity for beer is much lower than the price elasticity for wine and spirits, cf. Table 3. Higher rates of cross-border shopping in the future and higher price elasticity for beer could entail a maximum also for the partial tax revenue curve for beer. The slope of the partial tax revenue curve for beer is rather steep, possibly because we measure the tax as the share of the consumer price. Increasing the tax measured as share of the consumer price from 0.6 to 0.85 in Figure 2 is equivalent to increasing the tax measured as ratio of the price without tax from 1.5 to 5.67 (i.e., from $150 \%$ to $567 \%$ ), cf. (8).

The results in Figure 2 just contain the partial tax revenue from the tax share we change. For the fiscal policy the total tax revenue is more relevant. Figure 3 shows the total tax revenue as a function of the tax shares for the different commodity groups. The total tax revenue curves for wine and spirits have a single maximum. These maximum points are located to the right of the corresponding points for the partial tax revenue curves. This is due to the fact that when the tax on wine increases, the demand for spirits and beers, which also have high tax shares, increases. See the inverse composite elasticity rule in (20) and the (cross-)price elasticities in Table 3. The same reasoning applies to spirits. If the tax share grows sufficiently, the increased tax leakage associated with cross-border shopping and smuggling causes the total tax revenue to decline when the tax shares increase. The total tax revenue maximizing tax shares (including VAT) are smallest for wine (0.733) and largest for spirits (0.785). The actual tax share in 1999 for wine $(0.648)$ is considerably lower than the total tax revenue maximizing tax share ( 0.733 ). The actual tax share for spirits in $1999(0.770)$ is also lower than the total tax revenue maximizing tax share ( 0.785 ), according to our model.

The tax revenue curves in figures 2 and 3 show how the tax revenue changes after changing one tax share at a time, keeping all other variables constant. We can also compute how these tax revenue curves change in response to shifts in other exogenous variables in the model. We could, for instance, explore how changing tax shares for other commodities would affect the tax revenue maximizing tax shares.

Table 5 shows total tax revenue from different combinations of tax shares for spirits and wine. Each row and column in the table can be interpreted as a total tax revenue curve for wine and spirits, respectively. This allows us to interpret the table as giving us different total tax revenue curves for each commodity depending on the tax share value of the other commodity.

Table 5. Total revenue from indirect taxes as a function of tax shares for spirits and wine. Measured in mill. 1999 kroner

|  | 0.85 | 86742.84 | 86764.09 | 86783.40 | 86800.65 | 86815.74 | 86828.55 | 86838.93 | 86846.76 | 86851.88 | 86854.14 | 86853.36 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.84 | 86779.16 | 86800.38 | 86819.67 | 86836.91 | 86852.01 | 86864.83 | 86875.24 | 86883.12 | 86888.30 | 86890.64 | 86889.96 |
|  | 0.83 | 86807.56 | 86828.73 | 86847.98 | 86865.20 | 86880.28 | 86893.10 | 86903.52 | 86911.43 | 86916.65 | 86919.05 | 86918.45 |
|  | 0.82 | 86828.97 | 86850.08 | 86869.28 | 86886.46 | 86901.51 | 86914.31 | 86924.74 | 86932.65 | 86937.90 | 86940.33 | 86939.79 |
|  | 0.81 | 86844.21 | 86865.25 | 86884.38 | 86901.51 | 86916.51 | 86929.28 | 86939.69 | 86947.59 | 86952.85 | 86955.30 | 86954.79 |
|  | 0.80 | 86853.92 | 86874.88 | 86893.94 | 86911.00 | 86925.95 | 86938.67 | 86949.04 | 86956.92 | 86962.17 | 86964.63 | 86964.13 |
|  | 0.79 | 86858.69 | 86879.55 | 86898.53 | 86915.51 | 86930.39 | 86943.06 | 86953.38 | 86961.22 | 86966.45 | 86968.89 | 86968.40 |
|  | 0.78 | 86858.99 | 86879.75 | 86898.63 | 86915.53 | 86930.33 | 86942.93 | 86953.19 | 86960.99 | 86966.17 | 86968.59 | 86968.09 |
|  | 0.77 | 86855.25 | 86875.90 | 86894.67 | 86911.48 | 86926.19 | 86938.71 | 86948.90 | 86956.64 | 86961.77 | 86964.15 | 86963.62 |
|  | 0.76 | 86847.83 | 86868.36 | 86887.02 | 86903.72 | 86918.34 | 86930.77 | 86940.88 | 86948.54 | 86953.61 | 86955.94 | 86955.37 |
|  | 0.75 | 86837.05 | 86857.45 | 86876.00 | 86892.58 | 86907.09 | 86919.42 | 86929.44 | 86937.02 | 86942.03 | 86944.30 | 86943.67 |
|  |  | 0.64 | 0.65 | 0.66 | 0.67 | 0.68 | 0.69 | 0.7 | 0.71 | 0.72 | 0.73 | 0.74 |
| Tax on wine as share of the consumer price |  |  |  |  |  |  |  |  |  |  |  |  |

The italicized cells in the table indicate the tax shares maximizing the total tax revenue. As we see, the maximizing tax share for wine ( 0.73 ) is completely independent of the tax share value for spirits. The maximizing tax share for spirits is also not very substantially affected by the tax share value for wine (from 0.78 to 0.79 ).

When the other tax share change, total tax revenue changes substantially. Whether the shift is upward or downward depends on the size of the other tax share. When we increase the tax share for wine, the total tax revenue curve shifts upward as long as the tax share for wine remains below 0.73 .

There is one combination of tax shares for the two commodities where we simultaneously are at the maximum of each total tax revenue curve, namely 0.79 for spirits and 0.73 for wine. If we increase the tax share for one or both commodities beyond this the total tax revenue will decrease.

Effects on alcohol consumption, cross-border shopping and smuggling of changes in taxes on alcohol Figure 4 shows that when the tax shares for spirits increase, the consumption of spirits bought in Norway decreases, while it increases for spirits bought abroad and consumed in Norway. Smuggling increases too with an increase in the tax share. Intuitively this sounds very reasonable; however the quantitative effects are uncertain.

Total consumption of spirits falls when the taxes on spirits bought in Norway increase, which is in line with all conventional knowledge. But this will only apply if the tax share is less than the tax share minimizing total consumption. If the tax share increases beyond this level the total consumption of spirits will increase according to our model. The tax share minimizing total consumption of spirits is 0.92 . This is way beyond the actual tax share in 1999 ( 0.77 ), and since then the tax share for spirits has fallen in Norway. The existence of this consumption minimizing tax share in our model is an interesting result, and deserves further study.

Figure 4. Consumption of spirits as a function of the tax share for spirits. A cross indicates tax share in 1999


[^5]Figure 5. Consumption of beverages with alcohol as a function of price abroad for beverages with alcohol. Relative prices on alcohol are assumed constant. 1999 price is normalized to 1


Source: Statistics Norway, KONSUM-G-march-2003
Figure 6. Total tax revenue from indirect taxes as a function of the tax share for spirits, evaluated for three different price levels abroad for beverages with alcohol. A vertical line indicates maximum total tax revenue. A cross indicates tax share in 1999


[^6]Figure 7. Total tax revenue from indirect taxes as a function of the tax share for wine, evaluated for three different price levels abroad for beverages with alcohol. A vertical line indicates maximum total tax revenue. A cross indicates tax share in 1999


Source: Statistics Norway, KONSUM-G-march-2003

## Effects of changes in prices on alcohol abroad

Figure 5 shows that lowering the price of alcoholic beverages ${ }^{7}$ sold in neighboring countries increases the purchase of such beverages by cross-border shopping Norwegians, while lowering consumption of smuggled wares and sale of alcoholic beverages in Norway. The effect of increased cross-border shopping will dominate and the total consumption of beverages with alcohol will increase.

Figure 6 shows that the total tax revenue curve for spirits will shift sharply downward in response to falling prices abroad. The total revenue maximizing tax shares in Norway will also decrease if prices abroad decline, although the effects are not very dramatic. We see a similar result for wine in Figure 7. Here the maximizing tax share is largely independent of the price abroad, according to our simulation model.

[^7]
## Other simulation results

We shall briefly mention some other simulation results. We were unable to find a maximum value for either the partial or total tax revenue curve for tobacco and for food and non-alcoholic beverages, cf. the result for beer. This is caused by the low price elasticities for these commodity groups.

## 5. Conclusions

In this paper we derived the partial revenue from each indirect tax and the total revenue from all indirect taxes on consumer goods as functions of all commodity prices, the tax rates on each commodity, total expenditure and demographic variables using a complete demand system. Within this framework we analyzed the existence and maximum point of Dupuit curves associated with certain goods exposed to cross-border shopping /tax-free shopping and smuggling.

We showed that allowing for substitution to cross-border/tax-free shopping, along with smuggling, increases the elasticities substantially for the goods in question. As a consequence we find revenue maximizing tax shares for wine and spirits. The partial revenue from wine as a function of the tax share on wine has a single maximum value close to the actual tax rate in Norway anno 1999, conditioned on all the other exogenous variables in 1999. The total revenue as function of the tax share on wine also has a single maximum value, which is larger than that for the partial revenue. This emphasizes the importance of including the cross-price effects in computations of changes in tax revenue resulting from changes in tax share. The partial revenue from spirits as a function of the tax share for spirits also has a single maximum, though it is lower than the actual tax rate in 1999. On the other hand, the total revenue as a function of the tax share on spirits has a single maximum that is higher than the actual 1999 tax rate.

No such maximum exists in the case of beer, nor Food/non-alcoholic beverages and tobacco as the elasticity on these goods is too small.

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[^1]:    ${ }^{1}$ One of the authors, J. Aasness, was a member of this commission.

[^2]:    ${ }^{2}$ Elaborating on this, Nygård (2009) seeks equilibrium tax rates for spirits within a game theoretical framework with participation of the Scandinavian countries.
    ${ }^{3}$ By minimizing the costs (2) subject to a given utility level (1), we get Hicksian demand functions, see e.g. Deaton and Muellbauer (1980) for basic concepts and Indahl et al. (2001) for an application of this type of model. All the results below, such as aggregation over all consumers and the derivation of different revenue functions, can be shown in an analogous way by taking the Hicksian demand functions as a starting point. This enables analyses of policy changes based an empirical model of this type. Nevertheless, in the following analyses, we rely entirely on the Marshallian demand functions.

[^3]:    ${ }^{4}$ Thanks to Birger Strøm for providing this.

[^4]:    ${ }^{5}$ Earlier versions of the model are documented in Aasness and Holtsmark (1993), Wold (1998) and Indahl et al. (2001). See Sommervoll and Aasness (2001) for a version used for global simulations analyses with climate gas minimizing consumer prices.
    ${ }^{6}$ For a general discussion of utility trees, see Deaton and Muellbauer (1980, ch. 5)

[^5]:    Source: Statistics Norway, KONSUM-G-march-2003

[^6]:    Source: Statistics Norway, KONSUM-G-march-2003

[^7]:    ${ }^{7}$ We assume that the prices on beverages with alcohol are changed in the same proportions. Every price is normalized to 1 in the year 1999.

