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## Estimating Exchange Rate Dynamics with Diffusion Processes: An Application to Greek EMU Data

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# **HWWA Discussion Paper**

## **ESTIMATING EXCHANGE RATE DYNAMICS WITH DIFFUSION PROCESSES: AN APPLICATION TO GREEK EMU DATA\***

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## ESTIMATING EXCHANGE RATE DYNAMICS WITH DIFFUSION PROCESSES: AN APPLICATION TO GREEK EMU DATA

### Abstract

Recently various exchange rate models capturing the dynamics during the transition from an exchange rate arrangement of floating rates into a currency union have been derived. Technically, these stochastic equilibrium models are diffusion processes which have to be estimated by discretely sampled observations. Using daily exchange rate data prior to the Greek EMU-entrance in January 2001 this paper develops a rigorous estimation procedure. The estimates provide statistical evidence of increased central bank intervention activities in the run-up to the Greek EMU entrance. Thus the modelling and estimation framework establishes an approach for detecting intervention phases in the absence of concrete intervention data.

**Keywords:** Diffusion processes, estimation, exchange rates, EMU, central bank interventions.

**JEL Classification:** C13, C22, F31, F33

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# 1 Introduction

This paper presents a theoretical continuous-time model of exchange rate dynamics prior to a monetary union. Our model explicitly accounts for different central bank intervention intensities in the run-up to the monetary union. The stochastic behavior of the exchange rate implied by the model can be described by a non-stationary diffusion process. We then estimate the structural parameters of the model using discretely sampled, daily exchange rates between the Greek drachma and the euro for a two-year time period preceding the Greek entrance into the European Monetary Union (EMU).

In finance, the continuous-time stochastic modelling of dynamic variables such as interest rates of different maturities on the basis of diffusion processes is in widespread use since the early 1970s. In particular, alternative models for the short-term (instantaneous) interest rate have been put forward, among the most frequently cited being those of Vasicek (1977), Brennan and Schwartz (1979) or Cox et al. (1985).<sup>1</sup> With respect to exchange rate dynamics, continuous-time diffusion models were introduced by Flood and Garber (1983) and later taken up and elaborated in several respects by various authors. Applications range from the derivation of explicit exchange rate equations under alternative currency regimes (Krugman, 1991; Froot and Obstfeld, 1991a, 1991b; Miller and Sutherland, 1994) over the analysis of central bank intervention policies (see Bertola, 1994, and the literature cited there) to the modelling of speculative attacks (Krugman and Rotemberg, 1992; Bertola and Cabellero, 1992).

The theoretical advantages of continuous-time diffusion modeling as opposed to discrete-time models are often characterized by considerable gains in analytical tractability. Being equipped with standard tools of stochastic calculus (such as *Ito*-differentiation and integration techniques) stochastic continuous-time models often yield analytically closed-form solutions thus providing new economic insights and implications which are often cumbersome or even impossible to derive in a discrete-time setting. However, in contrast to theoretical modelling, the empirical validation of continuous-time structural dynamic equations may prove difficult when there are only discretely sampled observations. At the beginning of the 1990s, several authors estimated diffusion-based dynamic equations via Generalized-Methods-of-

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<sup>1</sup>For overviews and empirical applications in discrete time see Chan et al. (1992) or Campbell et al. (1997, Chap. 11).

Moments (GMM) techniques by specifying discrete-time counterparts serving as approximations of the continuous-time diffusion equations.<sup>2</sup> Only recently, refined methods of estimating continuous-time diffusion equations from discretely sampled data have been presented in the literature (e.g. Ait-Sahalia, 1996, 2002; Hansen et al., 1998; Singer, 1998; Darolles and Gourieroux, 2001; Elerian et al., 2001) and it is one of these latter techniques that we will use in our empirical study below.

The paper is organized as follows: Section 2 reviews the continuous-time exchange model. Section 3 describes the data, the econometric estimation technique and the results. Section 4 discusses the estimation results and draws policy conclusions. Section 5 offers some concluding remarks.

## 2 Theoretical models of exchange rate dynamics

In what follows we make use of an exchange rate model in continuous time as presented in Wilfing (2003) or, in a more general form, in Antzoulatos and Wilfing (2003). The path sequences of this model will trace the exchange rate evolution prior to the entrance of two economies into a currency union. In practice, a currency union is typically initiated by a switch in exchange rate regime from an arrangement of more or less flexible rates into a fixed-rate system with an official fixing-parity  $\bar{x}$  (the conversion rate). With respect to timing, we assume that the authorities of both economies agree on the date  $t_S$  as the beginning of the irreversible currency union (the switching date) and that they credibly inform the public at date  $t_A$  (the announcement date) about the conversion rate and the switching date (i.e.  $t_A < t_S$ ). Under this setting agents in financial markets are perfectly informed from  $t_A$  onwards about all modalities of exchange rate fixing at  $t_S$ .

To derive an explicit exchange rate path consider first the well-known stochastic version of the monetary exchange rate model with flexible prices in continuous time. In this model, the (logarithmic) spot exchange rate  $x$  at time  $t$ ,  $x(t)$ , can be represented by the sum of a macroeconomic fundamental

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<sup>2</sup>See for example Chan et al. (1992) for estimations of the short-term interest rate. Smith and Spencer (1992) and Lindberg and Söderlind (1992, 1994) estimate exchange rate equations by the so-called *Method-of-Simulated Moments* which can be viewed as a special case of the GMM technique as formalized by Hansen (1982).

$k(t)$  plus a speculative term proportional to the expected rate of change in the exchange rate:

$$x(t) = k(t) + \alpha \cdot \frac{E[dx(t)|\phi(t)]}{dt}, \quad \alpha > 0, \quad (1)$$

where  $E[\cdot|\cdot]$  denotes the expectation operator conditional on the present time- $t$  information set  $\phi(t)$  which includes all information available to rational market participants at time  $t$ . For simplicity, we think of the macroeconomic fundamental  $k$  as a collection of all economic and/or political factors that agents consider to be important for the current valuation of exchange rates.<sup>3</sup>

From a purely mathematical point of view Eq. (1) can be interpreted as follows: the stochastic exchange rate process  $\{x(t)\}$  is the unknown in our problem and endogenous to the monetary exchange rate model behind Eq. (1). On the other hand, the stochastic fundamental process  $\{k(t)\}$  is part of the problem specification and viewed as exogenous to the problem. Whenever  $\{k(t)\}$  is modelled as a diffusion process, Eq. (1) becomes a stochastic differential equation which can be solved by *Ito*-integration techniques and the imposition of adequate constraints reflecting the economic problem at hand.

A further feature of the model is that the specific dynamics chosen for the exogenous fundamental  $k$  characterizes the explicit exchange rate regime before the currency union. We will model two different regimes representing constant and increasing central bank intervention intensity, respectively.

In a first step we assume a managed-float pre-switch regime during which the central banks intervene in foreign exchange markets to stabilize the exchange rate  $x$  near the target value  $\bar{x}$ . The later conversion rate  $\bar{x}$  is assumed to be the same as the target rate pursued by the central banks during the preceding managed-float system. This modelling is in accordance with EMU where the central parities of the exchange rate mechanisms ERM or ERM-II were also applied as the final conversion rates. Such a managed-float pre-switch exchange rate regime can be consistently modelled by letting the exogenous fundamental  $k$  in Eq. (1) follow a mean-reverting Ornstein-Uhlenbeck process with stochastic differential

$$dk(t) = \eta \cdot [\bar{x} - k(t)] \cdot dt + \sigma \cdot dw(t), \quad (2)$$

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<sup>3</sup>For a rationalization of condition (1) stemming from simple monetary-approach models of exchange rate determination see Bertola (1994) and the literature cited there.



where  $\sigma > 0$  denotes the infinitesimal standard deviation and  $dw(t)$  the increment of a Wiener process.

The quantity  $\eta \cdot [\bar{x} - k(t)]$  represents the force that keeps pulling the fundamental  $k$  towards the long-run exchange rate target value  $\bar{x}$  (the later conversion rate), the magnitude being proportional to the current deviation of the process from its target value. The parameter  $\eta \geq 0$  indicates the strength of the target reversion property. Therefore,  $\eta$  can be interpreted as a measure of (the constant) central banks' intervention intensity for stabilizing the exchange rate  $x$  near the target parity  $\bar{x}$ . It should be noted, that strictly speaking the ERM-II (in which Greece took part until its EMU entrance on 1 January 2001) is a target zone regime with a well-defined central parity and an official band width of  $\pm 15\%$ . A justification for approximating such a target zone by a managed-float system is given in Antzoulatos and Wilfling (2003, Footnote 3) and the literature cited there.

The following two special cases are included in the Ornstein-Uhlenbeck specification (2). If  $\eta = 0$ , the fundamental  $k$  evolves along a (driftless) Brownian motion and the pre-switch exchange rate regime is a pure free-float. For  $\eta \rightarrow \infty$  any exchange rate deviation from the target parity  $\bar{x}$  will be instantaneously corrected for. In this case, the pre-switch regime is a fixed-rate system.

Ruling out arbitrage-opportunities at the moment of transition into the fixed-rate system (i.e. requiring  $\lim_{t \rightarrow t_S} x(t) = \bar{x}$  with probability 1) the (bubble-free) solution of the stochastic differential equation (1) can be derived via means of stochastic calculus. According to Wilfling (2003) the solution is given by

$$x(t) = \bar{x} + \frac{k(t) - \bar{x}}{1 + \alpha\eta} \quad (3)$$

for  $t < t_A$ , while for the interim period  $[t_A, t_S)$  the equilibrium exchange rate path obtains as

$$x(t) = \bar{x} + \frac{k(t) - \bar{x}}{1 + \alpha\eta} \cdot [1 - \exp((1 + \alpha\eta) \cdot (t - t_S)/\alpha)]. \quad (4)$$

These structural exchange rate sequences will serve as a first basis for our estimation framework with Greek EMU data in Section 3.

The second exchange rate regime is characterized by increasing central bank interventions towards the implementation of the monetary union. The fundamental process  $k(t)$  follows a scaled Brownian bridge with stochastic

differential representation

$$dk(t) = \eta \frac{\bar{x} - k(t)}{t_S - t} dt + \sigma dw(t) \quad (5)$$

Comparing (5) with (2) we find that the drift term is increasing proportional to the inverse of the remaining time until the union is implemented; as  $t \rightarrow t_S$  the intervention intensity approaches infinity. Obviously, modelling the fundamental process as a Brownian bridge is only expedient if we restrict attention to the interim period  $[t_A, t_S)$ .

With (5) the (bubble free) solution to (1) is

$$\begin{aligned} x(t) &= \frac{1}{\alpha} \left\{ \int_t^{t_S} e^{(t-r)/\alpha} E[k(r) | \phi(t)] dr + \int_{t_S}^{\infty} e^{(t-r)/\alpha} \bar{x} dr \right\} \\ &= \frac{1}{\alpha} \int_t^{t_S} e^{(t-r)/\alpha} \left( \left( \frac{t_S - r}{t_S - t} \right)^\eta k(t) + \left( 1 - \left( \frac{t_S - r}{t_S - t} \right)^\eta \right) \bar{x} \right) dr \\ &\quad + \bar{x} \cdot e^{(t-t_S)/\alpha}. \end{aligned}$$

Defining

$$I_N(t, t_S) = \int_t^{t_S} e^{(t-r)/\alpha} (t_S - r)^\eta dr$$

we obtain

$$x(t) = \bar{x} + \frac{k(t) - \bar{x}}{\alpha (t_S - t)^\eta} I_N(t, t_S). \quad (6)$$

## 3 Econometric analysis

### 3.1 The data

We analyze exchange rate data of the Greek drachma (GRD) vis-a-vis the euro for the time prior to the Greek EMU entrance on 1 January 2001. The exchange rates are daily spot rates collected for the period between 4 January 1999 until 29 December 2000 (the last trading day). The rates are measured as GRD-prices per euro and were compiled from the historical database provided by the *OANDA-FXTrade*-website.<sup>4</sup> The data provider delivers daily averages of interbank rates recorded at seven days per week. Since trading

<sup>4</sup><http://www.oanda.com/convert/fxhistory>.

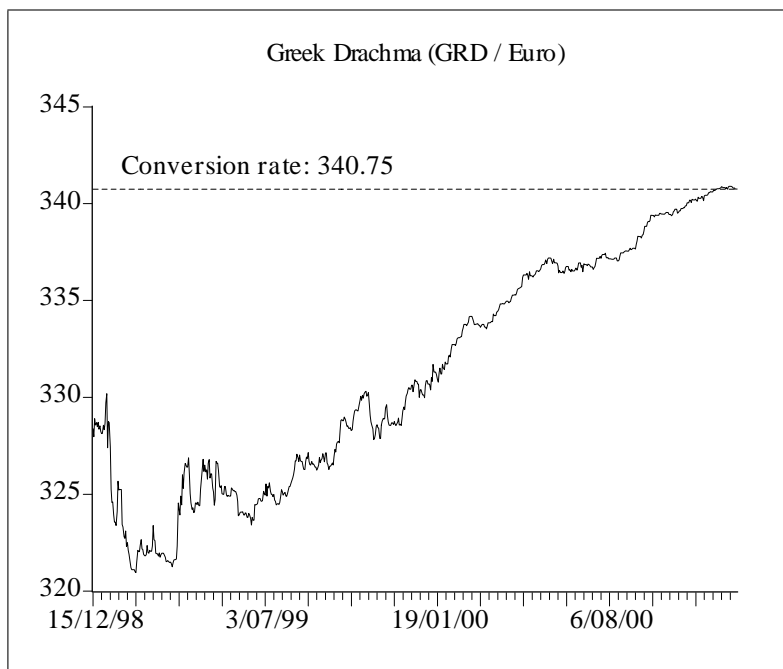


Figure 1: Nominal drachma-euro spot rates

volume and exchange rate changes during weekends are negligible we dropped the weekend quotes.

Figure 1 depicts the nominal drachma-euro spot rates for the sampling period along with a marker on the vertical axis for the ERM-II central parity of 340.75 GRD per euro. As mentioned before, this parity also served as the final conversion rate  $\bar{x}$  on 1 January 2001. All in all, the exchange rate shows a clear tendency to converge towards the conversion parity at the end of the interim period. The convergence may be due to two alternative reasons, both providing important sources of interpretation in the next section:

1. On the one hand the no-arbitrage condition used to derive Eq. (4) requires that  $\lim_{t \rightarrow t_S} x(t) = \bar{x}$  with probability 1. Observe that this type of convergence towards  $\bar{x}$  is independent of the specific value of the intervention parameter  $\eta$ . Hence, one reason for the convergence is merely the agents' removal of riskless profit opportunities at the moment of transition into the currency union.
2. On the other hand it is evident from Eq. (4) that for any  $t \in [t_A, t_S)$

the present exchange rate  $x(t)$  can be pushed arbitrarily close to the conversion rate  $\bar{x}$  by choosing  $\eta$  sufficiently high or, in other words, by sufficiently high central bank intervention activities.<sup>5</sup> In this case convergence towards the central parity is due to the central banks' intervention policy.

Clearly, the distinction between these two sources of exchange rate convergence is of high interest.

The last issue to be tackled before estimating the model concerns the following question: What is the empirical equivalent of the announcement date  $t_A$  from the theoretical model above? Obviously, in practice agents in foreign exchange markets are (more or less) rational and usually well informed so that in general they must be supposed to anticipate the prospective switch in the exchange rate regime long before any official announcement by the political or the institutional authorities in charge. Consequently, relying solely on speeches of central bank governors or on other official announcements is likely to be misleading in identifying the empirical counterpart of the theoretical announcement date  $t_A$ . For our analysis below we therefore utilize the results of a Markov-switching-GARCH analysis provided by Antzoulatos and Wilfing (2003). The main idea of their approach is that the exchange rate Eqs. (3) and (4) imply a clear-cut volatility structure in daily exchange rate returns. This volatility structure is characterized by a switch from a high volatility regime to a regime of low volatility at that moment when agents in foreign exchange markets incorporate the prospective currency union with the announced conversion rate  $\bar{x}$  in their currency valuation schemes. Inferential techniques associated with Markov-switching-GARCH models can identify the empirical interim period  $[t_A, t_S)$  in a statistically significant manner. Making use of the results in Antzoulatos and Wilfing (2003), we subsequently consider the time between 1 March and 31 December 2000 as the empirical interim period. In technical terms this implies that we consider 1 March 2000 as the break point  $t_A$  between the successive structural exchange rate Eqs. (3) and (4) in our estimation framework.

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<sup>5</sup>To see this latter claim observe that for arbitrary  $t \in [t_A, t_S)$  the first factor of the product on the right-hand side of Eq. (4) converges to zero with probability 1 as  $\eta \rightarrow \infty$  while the second factor in squared brackets is bounded from above by the value 1.

### 3.2 Estimation technique and results

There is a considerable literature on the estimation of diffusion processes using discretely sampled data. These estimation methods are, however, predominantly designed for stationary ergodic processes and therefore not directly applicable for our model where the stochastic process is non-stationary between  $t_A$  and  $t_S$ .

We estimate the parameters  $\alpha$ ,  $\eta$  and  $\sigma$  of the exchange rate diffusion processes by maximum likelihood as suggested in Singer (1998). Our observations are the timings (our time unit is one year) and the exchange rates

$$(t_1, x(t_1)), \dots, (t_n, x(t_n)).$$

Due to the Markov property of the process  $x(t)$  the likelihood function can be calculated from the one-step transition densities. Let  $p(t_{i-1}, x, t_i, y; \theta)$  denote the transition density from  $x$  to  $y$  between time  $t_{i-1}$  and time  $t_i$ . For ease of notation we write  $x_i = x(t_i)$  from now on. The likelihood function of all observations (conditional on  $x_1$ ) is

$$L_n(\alpha, \eta, \sigma) = \prod_{i=2}^n p(t_{i-1}, x_{i-1}, t_i, x_i; \alpha, \eta, \sigma).$$

If the number of observations is large (as in our application) the unknown marginal distribution of  $x_1$  has only negligible impact on the unconditional likelihood and may be ignored. Hence, the unconditional loglikelihood may be approximated by

$$l_n(\alpha, \eta, \sigma) = \sum_{i=2}^n \ln(p(t_{i-1}, x_{i-1}, t_i, x_i; \alpha, \eta, \sigma)).$$

Since at each point of time  $t$  the exchange rate  $x(t)$  is a linear function of the Gaussian fundamental process  $k(t)$  the transition densities are Gaussian. Hence, the mean and variance of  $x(t + \Delta)$  conditional on the information set  $\phi(t)$  can be derived from the conditional mean and variance of the fundamental process  $k(t + \Delta)$ .

If the fundamental process is modelled as an Ornstein-Uhlenbeck process the conditional moments of  $k(t)$  are well-known. Writing  $x_i = x(t + \Delta)$  and  $x_{i-1} = x(t)$  we obtain the following conditional moments for the exchange

rate:

$$E(x_i|\phi(t_{i-1})) = \begin{cases} x_{i-1} + (\bar{x} - x_{i-1})(1 - e^{-\eta\Delta_i}) & \text{for } t < t_A \\ x_{i-1} + (\bar{x} - x_{i-1})\left(1 - e^{-\eta\Delta_i}\frac{f_i}{f_{i-1}}\right) & \text{for } t_A \leq t < t_S \end{cases},$$

$$Var(x_i|\phi(t_{i-1})) = \begin{cases} \frac{1}{2\eta}\left(\frac{\sigma}{1+\alpha\eta}\right)^2(1 - e^{-2\eta\Delta_i}) & \text{for } t < t_A \\ \frac{1}{2\eta}\left(\frac{\sigma}{1+\alpha\eta}\right)^2(1 - e^{-2\eta\Delta_i})f_i^2 & \text{for } t_A \leq t < t_S \end{cases},$$

for  $i = 2, \dots, n$  where  $\Delta_i = t_i - t_{i-1}$  and  $f_i = 1 - e^{(1+\alpha\eta)(t_i-t_S)/\alpha}$ . Since weekends are not included in our sample, the time step  $\Delta_i$  is either one day or three days (i.e.  $\Delta_i = \frac{1}{365}$  or  $\Delta_i = \frac{3}{365}$ ). The transition density is

$$p(t_{i-1}, x_{i-1}, t_i, x_i; \alpha, \eta, \sigma) = \frac{1}{\sqrt{Var(x_i|\phi(t_{i-1}))}}\varphi\left(\frac{x_i - E(x_i|\phi(t_{i-1}))}{\sqrt{Var(x_i|\phi(t_{i-1}))}}\right)$$

where  $\varphi(u)$  is the probability density function of the standard normal distribution. The loglikelihood is

$$\begin{aligned} l_n(\alpha, \varepsilon, \sigma) &= -\frac{1}{2}\sum_{i=2}^n \frac{(x_i - E(x_i|\phi(t_{i-1})))^2}{Var(x_i|\phi(t_{i-1}))} & (7) \\ &\quad -\frac{1}{2}\sum_{i=1}^n \ln(Var(x_i|\phi(t_{i-1}))) - \frac{n}{2}\ln(2\pi) \\ &= -\frac{1}{2}\sum_{i=2}^n \left[ \frac{(x_i - E_i)^2}{V_i} + \ln(V_i) + \ln(2\pi) \right] \\ &= -\frac{1}{2}\sum_{i=2}^n A_i \end{aligned}$$

with  $E_i = E(x_i|\phi(t_{i-1}); \alpha, \eta, \sigma)$  and  $V_i = Var(x_i|\phi(t_{i-1}); \alpha, \eta, \sigma)$  and  $A_i = \frac{(x_i - E_i)^2}{V_i} + \ln(V_i) + \ln(2\pi)$ . Numerical maximization of the loglikelihood yields the estimates of the parameters. The maximization may be facilitated by using the partial derivatives of  $l_n$  with respect to its parameters (see appendix). As a by-product of the numerical maximization procedure we use the inverse

of the final information matrix as the estimated covariance matrix of the parameter estimators.

Using the entire observation span ( $n = 518$  observations) we obtain the following point estimates

$$\begin{aligned}\hat{\alpha} &= 0.54444 \\ \hat{\eta} &= 0.64941 \\ \hat{\sigma} &= 0.02837\end{aligned}$$

with the estimated covariance matrix

$$\widehat{Cov} \left( \begin{bmatrix} \hat{\alpha} \\ \hat{\eta} \\ \hat{\sigma} \end{bmatrix} \right) = \begin{bmatrix} 0.010743 & 0.024160 & 0.000457 \\ 0.024160 & 0.094656 & 0.001377 \\ 0.000457 & 0.001377 & 0.000023 \end{bmatrix};$$

all three parameters are significant at level  $\alpha = 0.05$ .

The ML point estimates do change significantly when the estimation period is shortened. Figure 2 displays the point estimates for  $\eta$  as a function of the date of the first observation included in the estimation procedure; the dashed lines are pointwise 0.95-confidence intervals. While  $\sigma$  and  $\alpha$  decline,  $\eta$  is moving strongly upwards. Even though the confidence intervals are fairly wide, the dependence on the time period underlying the estimation suggests that our model might be misspecified. In particular, the increase in  $\eta$  could indicate an upward drift in the intervention intensity not accounted for by the Ornstein-Uhlenbeck specification of the fundamental process  $k(t)$ .

The second model specification is consistent with increasing central bank interventions. If the fundamental process is assumed to be the Brownian bridge (5) its conditional moments are

$$\begin{aligned}E[k(t + \Delta) | \phi(t)] &= \left( \frac{t_S - t - \Delta}{t_S - t} \right)^\eta k(t) + \left( 1 - \left( \frac{t_S - t - \Delta}{t_S - t} \right)^\eta \right) \bar{x}, \\ Var[k(t + \Delta) | \phi(t)] &= \frac{\sigma^2 (t_S - t)}{2\eta - 1} \left( \frac{t_S - t - \Delta}{t_S - t} - \left( \frac{t_S - t - \Delta}{t_S - t} \right)^{2\eta} \right).\end{aligned}$$

Writing  $x_i = x(t + \Delta)$  and  $x_{i-1} = x(t)$  and using (6) we find

$$E(x_i | \phi(t_{i-1})) = x_{i-1} + (x_{i-1} - \bar{x}) \left( \frac{I_N(t_i, t_S)}{I_N(t_{i-1}, t_S)} - 1 \right)$$

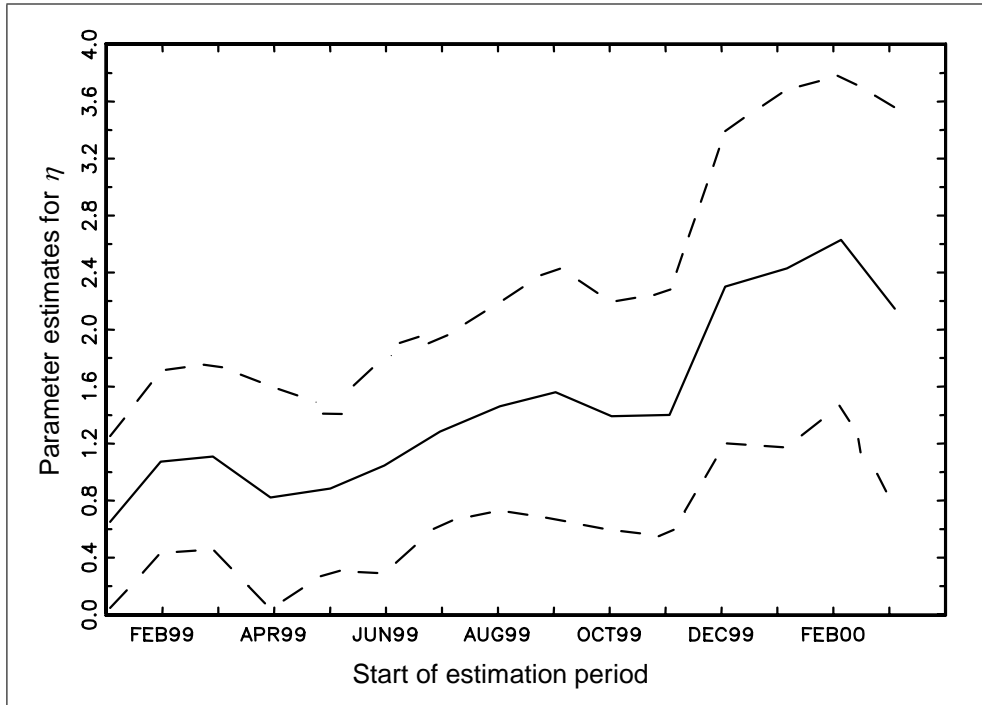


Figure 2: Estimates for  $\eta$  under shortenings of the estimation period

and

$$\text{Var}(x_i | \phi(t_{i-1})) = \frac{\sigma^2 I_N^2(t_i, t_S)}{\alpha^2 (2\eta - 1)} \cdot ((t_S - t_i)^{1-2\eta} - (t_S - t_{i-1})^{1-2\eta}).$$

The loglikelihood function (7) can be calculated and numerically maximized as before. Using only the observations from the interim period the point estimates are

$$\begin{aligned} \hat{\alpha} &= 0.02190 \\ \hat{\eta} &= 1.23910 \\ \hat{\sigma} &= 0.00719 \end{aligned}$$

and the covariance matrix estimate is

$$\widehat{\text{Cov}} \left( \begin{bmatrix} \hat{\alpha} \\ \hat{\eta} \\ \hat{\sigma} \end{bmatrix} \right) = \begin{bmatrix} 7.4610 \times 10^{-5} & -1.6156 \times 10^{-3} & 2.1288 \times 10^{-6} \\ -1.6156 \times 10^{-3} & 1.0290 \times 10^{-1} & -6.9862 \times 10^{-8} \\ 2.1288 \times 10^{-6} & -6.9862 \times 10^{-8} & 2.1120 \times 10^{-7} \end{bmatrix}.$$



Again, all three parameter estimates are significant at the 5% level.

## 4 Interpretation and policy issues

The econometric results of the preceding section are suggestive of increasing central bank intervention activities towards the Greek EMU entrance on 1 January 2001 for two reasons: (a) The estimates of the (constant) mean-reversion parameter  $\eta$  in the Ornstein-Uhlenbeck process from Eq. (2) increase in response to shortenings of the estimation period (see Figure 2). And (b), the parameter estimate of the parameter  $\eta$  in the scaled Brownian-bridge process (5) is statistically significant at the 5%-level. Nevertheless, it should be taken into account that—for both fundamental processes—interpreting  $\eta$  solely as the intervention intensity on foreign exchange markets can be misleading since other mean-reverting factors, potentially included in  $\eta$ , may have been effective. For example, under realistic assumptions of well-functioning international capital markets the increased degree of mean-reversion can also be caused by an active interest rate policy.<sup>6</sup>

However, the exchange rate evolution of the Greek drachma in Figure 1 apparently exhibits a strict exchange rate convergence towards the EMU conversion parity during the second half of the year 2000 arousing further suspicion of an active intervention policy stance. The potential need to increase intervention activities prior to a currency union was discussed in general terms on the road to Stage III of EMU under the phrasing *institutional frontloading*.<sup>7</sup> The idea behind this interim policy design was to coordinate exchange rate and monetary policy of the accepted EMU countries prior to the eventual exchange rate fixing in order to achieve two objectives: (a) to prevent inherent speculative attacks, and (b), to credibly steer the exchange rates towards their final conversion rates in the case of severe misalignments prior to the fixed-rate system. With respect to this latter point, Figure 1 clearly reveals that the Greek exchange rates were permanently overvalued before 1 January 2001 thus indicating the potential need for an active strategy.

It is also apparent from Figure 1 that exchange rate volatility decreased at the end of the interim period. Currently, the impacts of foreign exchange

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<sup>6</sup>See Antzoulatos and Wilfing (2003) for an analysis of interest rate spreads between Greece and Euroland prior the Greek EMU-entrance.

<sup>7</sup>See among others De Grauwe (1996, p. 24).

market interventions on exchange rate volatility are discussed controversially in the literature.<sup>8</sup> Today the theoretical literature emphasizes the signalling effect as the prevailing channel of transmission of central bank interventions on exchange rates. The idea is that (preferably coordinated) interventions may credibly signal to market participants the authorities' intentions about future macroeconomic policies thus inducing stabilizing effects on foreign exchange markets. Clearly, concerted exchange rate interventions between the European and the Greek central bank may have provided such an unambiguous signal with stabilizing volatility effects on the Drachma-Euro exchange rate prior to the Greek EMU entrance.

## 5 Concluding remarks

This paper estimates two versions of a continuous-time model of exchange rate dynamics prior to a currency union. In statistical terms the structural exchange rate equations are diffusion processes (i.e. stochastic processes whose trajectories are continuous with probability 1) and the parameter estimation is performed on the basis of discretely sampled, daily exchange rates between the Greek drachma and the euro for an adequately chosen time period preceding the Greek EMU entrance on 1 January 2001.

The estimation results for both variants of the exchange rate model provide a fair amount of econometric evidence of an active monetary and/or exchange rate policy in the run-up to the currency union. Both structural exchange rate equations exhibit a tendency of increasing mean-reversion in exchange rates prior to the Greek EMU entrance. Besides several economic (and in particular monetary) policy components this property of mean reversion is very likely to reflect (at least partly) the intervention activities of the Greek and/or the European central bank in the foreign exchange market.

In general, central banks often tend to be rather restrictive in providing intervention data. Therefore, the modelling and estimation framework presented in this paper establishes a convenient approach of detecting potential intervention phases in the absence of concrete intervention data.

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<sup>8</sup>For a recent survey along with an empirical investigation see Beine et al. (2003) and the literature cited there.

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## Appendix

The partial derivative of the loglikelihood function with respect to  $\alpha$  is

$$\frac{\partial l_n}{\partial \alpha} = -\frac{1}{2} \sum_{i=2}^n \frac{\partial A_i}{\partial \alpha}$$

where

$$\begin{aligned} \frac{\partial A_i}{\partial \alpha} &= \frac{-2(x_i - E_i) \frac{\partial E_i}{\partial \alpha} V_i - (x_i - E_i)^2 \frac{\partial V_i}{\partial \alpha}}{V_i^2} + \frac{1}{V_i} \frac{\partial V_i}{\partial \alpha} \\ &= \frac{-2(x_i - E_i) \frac{\partial E_i}{\partial \alpha} - \frac{(x_i - E_i)^2}{V_i} \frac{\partial V_i}{\partial \alpha} + \frac{\partial V_i}{\partial \alpha}}{V_i} \end{aligned}$$

and similarly for  $\eta$  and  $\sigma$ . The required partial derivatives are

$$\begin{aligned} \frac{\partial E_i}{\partial \alpha} &= \begin{cases} 0 \\ (\bar{x} - x_{i-1}) (1 - f_{i-1}) \frac{(1-f_{i-1})e^{\Delta_i/\alpha} \Delta_i + e^{-\eta \Delta_i} (t_{i-1} - t_S) - e^{\Delta_i/\alpha} (t_i - t_S)}{f_{i-1}^2 \alpha^2} \end{cases} \\ \frac{\partial V_i}{\partial \alpha} &= \begin{cases} \frac{e^{-2\eta \Delta_i} - 1}{(1+\alpha\eta)^3} \sigma^2 \\ \sigma^2 (1 - e^{-2\eta \Delta_i}) f_i \frac{(1-f_i)(t_i - t_S)(1+\alpha\eta)/\alpha - \alpha\eta f_i}{\alpha\eta(1+\alpha\eta)^3} \end{cases} \end{aligned}$$

with  $f_i = 1 - \exp((1 + \alpha\eta)(t_i - t_S)/\alpha)$ ;

$$\begin{aligned} \frac{\partial E_i}{\partial \eta} &= \begin{cases} (\bar{x} - x_{i-1}) \Delta_i e^{-\eta \Delta_i} \\ (\bar{x} - x_{i-1}) \frac{\Delta_i e^{-\eta \Delta_i} - (1-f_{i-1})e^{-\eta \Delta_i} (t_i + \Delta_i - t_S) + (1-f_{i-1})e^{\Delta_i/\alpha} (t_i - t_S)}{f_{i-1}^2} \end{cases} \\ \frac{\partial V_i}{\partial \eta} &= \begin{cases} \sigma^2 \frac{(e^{-2\eta \Delta_i} - 1) + 3\alpha\eta(e^{-2\eta \Delta_i} - 1) + 2\Delta_i e^{-2\eta \Delta_i} \eta(1+\alpha\eta)}{2\eta^2(1+\alpha\eta)^3} \\ \sigma^2 f_i \frac{2\eta(1+\alpha\eta)\Delta_i e^{-2\eta \Delta_i} f_i - (1+3\alpha\eta)f_i(1 - e^{-2\eta \Delta_i}) - 2\eta(1+\alpha\eta)(1-f_i)(t_i - t_S)(1 - e^{-2\eta \Delta_i})}{2\eta^2(1+\alpha\eta)^3} \end{cases} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial E_i}{\partial \sigma} &= 0 \\ \frac{\partial V_i}{\partial \sigma} &= \begin{cases} \sigma \frac{1 - e^{-2\eta \Delta_i}}{\eta(1+\alpha\eta)^2} \\ \sigma (1 - e^{-2\eta \Delta_i}) \frac{f_i^2}{\eta(1+\alpha\eta)^2}. \end{cases} \end{aligned}$$