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Working Paper — Digitized Version


Discussion Papers, No. 257

Provided in Cooperation with:
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Suggested Citation: Rege, Mari (1999) : Social Norms and Private Provision of Public Goods: Endogenous Peer Groups, Discussion Papers, No. 257, Statistics Norway, Research Department, Oslo

This Version is available at:
http://hdl.handle.net/10419/192239

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Abstract:
The formation of peer groups with social norms for private contributions to a public good is analyzed in an n-player two stage game. First people choose a peer group, then they choose whether to contribute. The first choice is made through a learning process represented by evolutionary dynamics, while the second choice is made by utility maximization. The game has two types of stable states: One in which very few people belong to peer groups with social norms for private contributions, and one in which a large portion of people belong to such peer groups. In the former state nobody contributes, while in the latter everybody contributes. Direct governmental contributions to the public good can move the society to a stable state in which nobody contributes, where as governmental subsidization can move the society to a stable state in which everybody contributes. Indeed, the crowding in caused by subsidization can prevail after policy reversal.

Keywords: crowding in, crowding out, evolution, peer groups, public goods, social norms, social sanctions

JEL classification: D11, H20, H41

Acknowledgement: I thank Geir Asheim, Kjell Amre Brekke, Bård Harstad, Snorre Kverndokk, Karine Nyborg, Aanund Hylland, Jörgen Weibull and the participants at the workshop on social norms (Statistics Norway, January 18, 1999) for helpful discussions and comments. I am also grateful to Christopher Shepard for helping me with the English language.

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"Individual utility functions are simply more complicated than the simple assumptions so far incorporated in neoclassical theory. The task of the social scientist is to broaden the theory to be able to predict when people will act like free riders and when they won't."
Douglass North (1981, p. 46)

1 Introduction

This article will give an evolutionary analysis of the development of peer groups with social norms for private contribution to a public good. People within such peer groups identify themselves as contributors and sanction everyone who is not contributing. The analysis will describe how and when peer groups with social norms for private contribution develop. This is done by analyzing an n-player two stage game. In the first stage each person chooses of which peer group he wants to be a member. This choice is made through a learning process, which is represented by the replicator dynamics from the field of evolutionary game theory. In the second stage each person chooses whether to contribute a given amount to the public good. This choice is made by utility maximization.

In accordance to Coleman (Coleman 1990, p. 242) social norms are defined to be rules of behavior which specify what actions are regarded by a set of persons as proper or correct, or as improper or incorrect. Social norms are enforced by social sanctions. These sanctions normally take the form of approval or disapproval from others (Lindbeck 1997, p. 370). Such a definition of social norms allows them to be analyzed in the framework of neoclassical utility theory by assuming that social approval is an argument in people's utility function. Because of social norms, a person's social approval might be affected by his consumption pattern. This relation is modeled by applying the household production function of Stigler and Becker (1977) to the production of social approval. The production function representing the production of social approval is endogenously determined by the

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1Norms can also be enforced by internal sanctions (Coleman 1990, p. 243). A sanction is internal when a person sanctions himself. This normally results in feelings of self-respect or guilt (Lindbeck 1997, p. 370). A norm enforced by internal sanctions is called an internalized norm. This paper will only analyze the effect of non-internalized norms for private contributions to a public good.
application of evolutionary game theory.

At first sight evolutionary game theory represents a contrast to neoclassical economics: Instead of being a rational choice-maker, the human is seen as a creature doing only that which he has been programmed to do, or as an individual with bounded rationality learning by either imitation or reinforcement\(^2\). Such an approach is taken in several evolutionary analyses of cooperation (e.g. Axelrod 1986, Young and Foster 1991, Sethi and Somanathan 1996). Several evolutionary analysis do, however, show that evolutionary game theory is highly complementary to neoclassical theory (e.g. Frank 1987, Güth 1995, Bester and Güth 1998, Fershtman and Weiss 1998). These analyses retain the assumption that people are guided by self-interested calculation but go on to show how evolutionary game theory can be used to analyze the formation of human preferences that are exogenous in most neoclassical models. An essential assumption is that the preferences associated with highest material payoff succeed. Güth (1995, p. 324) refers to this as the *indirect evolutionary approach*. A drawback of the indirect evolutionary approach emerges in the biological interpretation of the evolutionary dynamics. A learning interpretation is hard to accept because it presupposes that people, in a learning process, choose the preferences associated with highest material payoff\(^3\).

The following evolutionary analysis draws on the indirect evolutionary approach in assuming that people's actions are guided by self-interested calculation, while evolutionary game theory is used to study the selection of social norms. Applying evolutionary game theory to the selection of norms rather than preferences does, however, allow for a reasonable learning interpretation of the evolutionary dynamics. A social norm for contributing to a public good succeeds if people want to be in peer groups possessing such a norm. Each person will attempt to integrate into a peer group in which his alternative cost of obtaining social approval is as low as possible. The choice of peer group is, however, constrained by incomplete knowledge. People are uncertain of the effect different social norms will have on social approval. Therefore, people choose peer groups in a learning process. Such a process can be represented by the evolutionary dynamics called

\(^{2}\)Weibull (1998) discusses evolutionary game theory's connection with learning models.

\(^{3}\)Bester and Güth (1998) assert that individuals tend to imitate successful attitudes. They do, however, not justify this assertion.
the replicator dynamics which was first developed by Taylor and Jonker (1978).

This paper adds to a series of papers which analyze private provision of public goods, and the effect of various governmental policies on private provision (e.g. Warr 1982 and 1983, Roberts 1984, 1987 and 1992, Bernheim 1986, Bergstrom et al. 1986, Andreoni 1988 and 1990, Andreoni and Bergstrom 1996). In all these papers it is, however, assumed that preferences depend only on private consumption and the total supply of the public good. Andreoni (1990) argues that such an approach implies theoretical results that are counterfactual. It implies, for example, that in large economies virtually no one will contribute to the public good, hence making the Red Cross, the Salvation Army, and the American Public Broadcasting system logical inconsistencies. On this background Andreoni (1990) introduces his theory of warm glow giving. The idea is that one's own contribution to a public good produces a private good - "warm glow" - as a by-product. An approach similar to Andreoni's (1990) is taken by Holländer (1990) who considers social approval as a by-product of contributing to a public good. By in this way including one's own giving explicitly in the utility-function, both Andreoni (1990) and Holländer (1990) show that direct governmental grants will incompletely crowd out private donations to the public good. Furthermore, Andreoni (1990) shows that subsidies will crowd in private donations to the public good. In line with Andreoni (1990) and Holländer (1990), this paper studies private provision of a public good focusing on social approval as a by-product of contributing to the public good. The evolutionary analysis in this paper yields new insight of different governmental policies' crowding out or crowding in of private contributions. The most striking result is that a subsidy might crowd in private provision and that this crowding in might prevail after the policy has been reversed\(^4\). The crowding in might prevail because the social norm for private contribution is conditional: The larger the population share adhering to a norm, the more social approval each person will obtain from adhering to that norm.

The paper is organized as follows: Section 2 presents a public good model with exogenous peer groups. This represents the second stage of the \(n\)-player two stage game. In this stage each person, being in a given type of peer group, chooses whether to contribute a given amount to the public good. This choice is made by utility maximization. Section 3

\(^4\)See the conclusion for a short summary of results.
presents the replicator dynamics and applies this dynamic to endogenize the peer groups. This represents the first stage of the game. In this stage each person chooses, given that he contributes to the public good as found in stage 2, of which peer group he wants to be a member. This choice is made in a learning process. Section 4 discusses how social norms for private contribution are affected by governmental policies. Section 5 concludes the analysis by summarizing the results.

2 Stage Two: Choosing Whether to Contribute

Assume a society consisting of \( n \) people with identical preferences over the two marketable goods: private consumption, \( c \), and a public good, \( G \), in addition to the non-marketable good: social approval, \( q \). These preferences can for each person \( i \) be represented by the utility function

\[
U_i (c_i, q_i, G) = c_i + q_i + w(G)
\]  

which is increasing in all its arguments and concave in \( G \). Additivity in \( c_i, G \) and \( q_i \), in addition to linearity in \( c_i \) and \( q_i \), is assumed in order to simplify the algebra. Normalize such that \( w(0) = 0 \). Let \( c_i \) be the numeraire and let \( p \) be the relative price of good \( G \). Each person \( i \) is spending his income, \( I \), on private consumption, \( c_i \), and his contribution to the public good, \( g_i \). The budget constraint is then given by

\[
I = pg_i + c_i
\]  

Each person’s choice is discrete. He can choose to contribute a fixed amount \( g_i = 1 \) to the public good or not to contribute at all, i.e. \( g_i = 0 \). A person contributing \( g_i = 1 \) will be referred to as a contributor, while a person contributing nothing will be referred to as a non-contributor.

Assume that a share \( \alpha \) of the people in the society has social norms telling people to contribute to a public good\(^5\). These people constitute a peer group in which people identify themselves as contributors. Let \( A \) be the set of people with such social norms for

\(^5\)\( \alpha \) will be endogenously determined by evolutionary dynamics in section 4.
private contribution. Let $B$ be all the other people in the society. People in $A$ sanction non-contributors in both $A$ and $B$, while people in $B$ sanction nobody. People in $A$ will therefore be referred to as sanctioners. Note that a sanctioner is not necessarily a contributor. Talking about doing something is not the same as actually doing it.

Each person $i$ meets a given number of people after he has chosen $g_i \in \{0, 1\}$. These people will all observe person $i$'s choice. The people from $A$ observing $i$'s choice will sanction him if $g_i = 0$. If people met each other independently of peer group, then both a person in $A$ and a person in $B$ would have expected a share $\alpha$ of the people he meet to be sanctioners. However, it is a sociological and empirical fact that people socialize more often with people similar to themselves (Fischer, 1982. p.7). Thus, a person meets more often people from his own kind of peer group. Assume that, among the people a person meets, a share $k > 0$ are from his own peer group, and that a share $(1 - k)$ he meets independently of peer group. Then, the share of sanctioners each person $i$ meets is given by

$$z_i = \begin{cases} k + (1 - k) \alpha & \forall i \in A \\ (1 - k) \alpha & \forall i \in B \end{cases}$$

Thus, the difference between a person in $A$ and $B$ is that a person in $A$ sanctions while a person in $B$ does not sanction, and that a person in $A$ meets more sanctioners than a person in $B$.

Note that there is no cost associated with sanctioning in this model. A person identifying himself as a contributor will not start yelling at a person who does not contribute. He will instead quietly disapprove the non-contributor and consider him as irresponsible. The fact that the non-contributor is conscience about this disapproval imposes a cost on him. He experiences the feeling of diminished social approval. The so called sanctioner has, however, intentionally not made any effort to sanction him.\(^6\)

Each person $i$'s social approval can be represented by the household production function

$$q_i = z_i \cdot (g_i - \bar{g})$$

where $\bar{g}$ denotes the average contribution of people in the society and $l$ is a measure of how much each person potentially can benefit from private contributions to the public.

\(^6\)See Brennan and Petitt (1993) for further discussion on costs of sanctioning.
good. More precisely $l$ denotes the differences in individual utility, in terms of private consumption, between a society in which everybody contributes and a society in which nobody contributes. Hence,

$$l = w(n) - p$$

(5)

The first factor, $z_i$, in (4) reflects that each person $i$ feels the social norm more strongly the more sanctioners he meets. This is similar to Lindbeck (1997) and Lindbeck et al. (1999) in which it is presupposed that a social norm is felt more strongly the greater the number of people who obey it. The second factor, $l$, reflects that the social norms for private contribution is felt more strongly the more each person benefits from these private contributions. The more people benefit from private contributions, the more important sanctioners perceive the social norms enforcing these private contributions, and the more they sanction. This captures the idea that social norms can arise because of market failure which has been argued by a number of authors (e.g. Arrow 1971, Ullmann-Margalit 1977, North 1981, Coleman 1990). Market failures for which there are no corresponding norms show, however, that market failure alone is not a sufficient condition for a norm to arise (Elster 1989).

The third factor, $(g_i - \bar{g})$, reflects that the average behavior in the society determines the norm. A person will be rewarded if he contributes more than the average contribution in society, and punished if he contributes less. When some people improve their social approval by contributing more to the provision of the public good, then the social approval of other people in society will worsen.

Equation (1), (2) and (4) imply that each person’s maximization problem is given by

$$\max_{g_i \in \{0,1\}} I - pg_i + z_i (g_i - \bar{g}) + w(G)$$

(6)

Assume that there is a large number of people in the society and that each person $i$’s contribution to the public good has a negligible effect on welfare derived from the public good, i.e. $w(G) - w(G - 1) \approx 0$. Thus, people’s only motivation to contribute to the public good is social approval. Then, each person $i$ will contribute if and only if

$$z_i l > p$$

(7)
i.e. a person will contribute if and only if the benefit of the increase in social approval by contributing is higher than the cost of the decrease in private consumption. Equation (3) and (7) imply that all people in A will contribute if

$$\alpha > \frac{p}{l(1-k)} - \frac{k}{(1-k)} \equiv \tilde{\alpha}$$

(8)

and that both people in A and B will contribute if

$$\alpha > \frac{p}{l(1-k)} \equiv \tilde{\alpha}$$

(9)

Note that there exist $\alpha$ and $\tilde{\alpha}$ such that $0 \leq \alpha < \tilde{\alpha} \leq 1$ if and only if $kl \leq p \leq l(1-k)$. Then (5) implies that there exist $\alpha$ and $\tilde{\alpha}$ such that $0 \leq \alpha < \tilde{\alpha} \leq 1$ if and only if $w(n) \frac{k}{k+1} \leq p \leq w(n) \frac{1-k}{2-k}$. It is a Pareto improvement that everybody contributes if and only if $p < w(n)$. Hence, people will contribute voluntarily only if it is a Pareto improvement that everybody contributes. Note for later purposes that there exist $\alpha$ and $\tilde{\alpha}$ such that $0 < \alpha < \tilde{\alpha} < 1$ if and only if $w(n) \frac{k}{k+1} < p < w(n) \frac{1-k}{2-k}$.

**Proposition 1**

- If $\alpha < \alpha$, then neither people in A nor people in B will contribute voluntarily. The total provision is $G = 0$.

- If $\alpha < \alpha < \tilde{\alpha}$, then people in A will contribute voluntarily, while people in B will free-ride on these contributions. The total provision of the public good is $G = \alpha n$ and the average contribution is $g = \alpha$.

- If $\alpha > \tilde{\alpha}$, then both people in A and people in B will contribute voluntarily. The total provision of the public good is $G = n$ and the average contribution is $g = 1$.

- People will contribute voluntarily only if it is a Pareto improvement that everybody contributes.

Note that equation (5), (8) and (9) imply that people get stronger incentives to contribute if $\alpha$ or $w(n)$ increases, or if $p$ decreases. The intuition behind this is that an increase in $\alpha$ or $w(n)$, or an decrease in $p$ will reduce the alternative cost of obtaining social approval and hence, give people stronger incentives to contribute.
3 Stage One: Choosing Peer Group

The previous section analyzed how each person, being in a given type of peer group, will choose to contribute to the public good. This section analyzes how each person chooses, given that he contributes to the public good as described in the previous section, the peer group he wants to be a member of.

3.1 Evolutionary Dynamics

Social norms have an impact on individual welfare because people have preferences for social approval which is influenced by social norms. The possibility to obtain social approval varies between peer groups due to different social norms within different peer groups. A person in a peer group with social norms for private contribution to a public good will obtain higher social approval by contributing to the public good than a person in a peer group with no such norms. The former receives higher social approval because he meets more people who appreciate his behavior. Conversely, a person in a peer group with social norms for private contribution, will obtain lower social approval by not contributing to the public good than a person in a peer group with no such norms. The former receives lower social approval because he meets more people who do not appreciate his behavior.

In the long run people seek to integrate into those peer groups possessing the social norms which will maximize their individual welfare. In his well known analysis of social relations Fischer (1982, p. 4) says:

The initial relations are given us — parents and close kin — and often other relations are imposed on us — workmates, in-laws, and so on. But over time we become responsible; we decide whose company to pursue, whom to ignore or to leave as casual acquaintances, whom to neglect and break away from. (...) We each build a network — which is one part of building a life.

And in all this activity, we make choices as best we can to attain the values we hold dear.

The choice of social relations is, however, constrained. Fischer (1982, p. 4) emphasizes how this choice is constrained by available information, personality, available pool of
people, society, and social contexts. Particularly relevant to this analysis is how the choice of social relations is constrained by incomplete knowledge of different social norms' impact on social approval.

Because a person's choice of social relations is constrained by available information, he will learn by trial and error or by imitation when choosing his peer group. Many people end up integrating into a peer group with social norms which will reduce their individual welfare. This is especially true for people who only have experience with a few kinds of peer groups. They do not understand the impact different peer groups have on individual welfare or how social norms varies between peer groups. For this reason they often integrate into peer groups in which the alternative cost of obtaining social approval is higher than in their earlier peer group. However, every time a person obtains experience with new peer groups his information set improves. Eventually, he will be able to understand the links between individual welfare and different social norms. In addition, he will associate certain norms with certain peer groups. If he then once more has to change peer group, he will seek a peer group in which his alternative cost of obtaining social approval is as low as possible. Hence, in a learning process people will, in the long run, integrate into peer groups with social norms which make them better off.

Weibull (1998) discusses how the replicator dynamics (Taylor and Jonker 1978) can be viewed as an approximation for different learning processes. The replicator dynamics was originally developed in order to study the selection of strategies. It says: those subpopulations that are associated with better-than-average strategies grow, while those associated with worse-than-average strategies decline, or more precisely: the growth rate of the population share using a certain strategy equals the difference between the strategy’s current payoff and the current average payoff in the population (Weibull 1995, p. 73). This evolutionary dynamic can easily be applied to the selection of social norms instead of strategies. The replicator dynamics for selection of social norms says: those subpopulations that are associated with better-than-average social norms grow, while those associated with worse-than-average social norms decline.

In the model presented in this paper the replicator dynamics can be represented by

7Analyzing α as being continuous is a good approximation in a large population.
\[ \dot{\alpha} = \alpha \cdot (V_A(\alpha) - \bar{V}(\alpha)) \]  

(10)

where \( \dot{\alpha} \) denotes the time derivative. Because \( \bar{V}(\alpha) = \alpha V_A(\alpha) + (1 - \alpha)V_B(\alpha) \), (10) can be rewritten as

\[ \dot{\alpha} = \alpha (1 - \alpha)(V_A(\alpha) - V_B(\alpha)) \]  

(11)

The following definitions will be useful\(^8\):

**Definition 1** A state \( \dot{\alpha} \in [0, 1] \) is *stationary* if \( \dot{\alpha}(\dot{\alpha}) = 0 \).

**Definition 2** A state \( \dot{\alpha} \in [0, 1] \) is *Lyapunov stable* if \( \dot{\alpha} \) is stationary and if no small perturbation of \( \dot{\alpha} \) induces a movement away from \( \dot{\alpha} \).

**Definition 3** A state \( \dot{\alpha} \in [0, 1] \) is *asymptotically stable* if \( \dot{\alpha} \) is Lyapunov stable and all small perturbations of \( \dot{\alpha} \) induces a movement back to \( \dot{\alpha} \).

### 3.2 Stable States

This section applies the replicator dynamics in order to analyze the development of social norms for private contribution to a public good within the model presented in section 2. Equation (11) shows that a welfare comparison between people in A and B for different levels of \( \alpha \) is necessary in order to do this analysis. Such a welfare comparison is possible because people in A and B have identical preferences. The only difference between people in A and B is that people in A meet more sanctioners than people in B\(^9\). Equation (6), (3)  

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\(^8\)Because the evolutionary dynamics in this model is rather simple, it is sufficient to work from these informal and intuitive definitions of stability taken from Weibull (1995, p. 243). For a formal definition, see Weibull (1995, p. 243).

\(^9\)In addition, people in A are sanctioners. Section 1 argued, however, that there is no cost associated with sanctioning.
Figure 1:

and Proposition 1 imply that the indirect utility level for different levels of $\alpha$ for people in $A$ is given by

$$V_A(\alpha) = \begin{cases} 
I & \forall \alpha \leq \bar{\alpha} \\
I - p + w(n) + l(k + (1 - k)\alpha)(1 - \alpha) & \forall \bar{\alpha} < \alpha \leq \alpha \\
I - p + w(n) & \forall \alpha > \alpha 
\end{cases}$$

and for people in $B$

$$V_B(\alpha) = \begin{cases} 
I & \forall \alpha \leq \bar{\alpha} \\
I + w(n) - l(1 - k)\alpha^2 & \forall \bar{\alpha} < \alpha \leq \alpha \\
I - p + w(n) & \forall \alpha > \alpha 
\end{cases}$$

The difference in indirect utility level between people in $A$ and $B$, $\Delta V(\alpha) = V_A - V_B$, is then given by

$$\Delta V(\alpha) = \begin{cases} 
0 & \forall \alpha \leq \bar{\alpha} \\
-p + l(k + (1 - 2k)\alpha) & \forall \bar{\alpha} < \alpha \leq \alpha \\
0 & \forall \alpha > \alpha 
\end{cases} \quad (12)$$

The last section found that $0 < \bar{\alpha} < \alpha < 1$ if and only if $w(n)\frac{k}{k+1} < p < w(n)\frac{1-k}{2-k}$. Thus, if $w(n)\frac{k}{k+1} < p < w(n)\frac{1-k}{2-k}$, then $\Delta V(\alpha)$ has the shape as depicted in figure 1. $\alpha_0$ is defined to be the $\alpha$ solving $\Delta V(\alpha) = 0$. Thus,
\[ \alpha_0 = \frac{p}{l(1 - 2k)} - \frac{k}{1 - 2k} \] (13)

The intuition behind the shape of \( \Delta V (\alpha) \) in figure 1 is as follows: For low \( \alpha \), a person’s increase in social approval, \( q_i \), from contributing is small because he meets few sanctioners. For \( \alpha < \tilde{\alpha} \) the increase in \( q_i \) is so small that nobody wants to contribute. Then everybody has the same private consumption and social approval, and hence, individual welfare is the same in \( A \) and \( B \). For higher \( \alpha \), a person’s increase in social approval, \( q_i \), from contributing is larger because he meets more sanctioners. For \( \alpha < \alpha < \tilde{\alpha} \) the increase in \( q_i, i \in A \), is sufficient to make people in \( A \) contribute. A contribution from everybody in \( A \) imposes an externality on everybody in society because it increases the average contribution level \( \bar{g} \). This externality is larger for people in \( A \) than for people in \( B \) because people in \( A \) meet more sanctioners. In equilibrium the people in \( A \) will have higher social approval than people in \( B \) because they are also increasing their social approval by contributing. Despite this increase in social approval, for \( \alpha < \alpha < \alpha_0 \) the people in \( B \) are doing better than the people in \( A \) because the higher social approval does not compensate for the loss in private consumption. However, the difference in social approval between contributors and non-contributors will increase when \( \alpha \) increases because the number of sanctioners increases. For \( \alpha_0 < \alpha < \tilde{\alpha} \) people in \( A \) are doing better than people in \( B \) because the higher social approval for people in \( A \) more than compensates for their loss in private consumption. People in \( B \) will still not contribute because they meet too few sanctioners to make it optimal. For \( \alpha > \tilde{\alpha} \) people in \( B \) meet the sufficient amount of sanctioners to make it optimal for them to contribute. Then everybody has the same private consumption and social approval, and hence, individual welfare is the same in \( A \) and \( B \).

On the background of figure 1, (11) implies that the replicator dynamics can be illustrated as in figure 2. Observe the following from this figure: If \( \alpha \in [\alpha_0, \tilde{\alpha}] \), then \( \dot{\alpha} > 0 \). People will seek to be in the peer group with social norms for private contribution to a public good because they can obtain higher individual welfare by contributing and meeting many people who appreciate such behavior. \( \alpha \) will increase to the stable state, \( \tilde{\alpha} \), in which everybody voluntarily contributes to the public good. If \( \alpha \in [\underline{\alpha}, \alpha_0] \), then \( \dot{\alpha} < 0 \). People will seek to move away from the peer group with social norms for private contribution because they can obtain higher individual welfare by not contributing and meet...
as few people as possible disliking such behavior. \( \alpha \) will decrease to the stable state, \( \alpha \), in which nobody voluntarily contributes to the public good. If \( \alpha \in [0, \alpha] \cup [\beta, 1] \), then \( \dot{\alpha} = 0 \). People are indifferent between the two types of peer-groups and have no incentive to change peer group. Thus, the Lyapunov stable states are given by\(^{10} \):

\[
[0, \alpha] \cup [\beta, 1]
\]

because a small perturbation of a state \( \dot{\alpha} \in [0, \alpha] \cup [\beta, 1] \) will not induce a movement away from \( \dot{\alpha} \). None of the Lyapunov stable states are asymptotically stable. The sets \([0, \alpha]\) and \([\beta, 1]\) are, however, asymptotically stable sets. A small perturbation of for example a state \( \dot{\alpha} \in [0, \alpha] \) will, if the perturbation changes \( \alpha \) to a \( \dot{\alpha} \in (\alpha, \alpha_0) \), induce a movement back towards a \( \alpha \in [0, \alpha] \). The state \( \alpha_0 \) is stationary but not stable because a small perturbation of the state will induce a movement away from \( \alpha_0 \).

The stable states found above imply that there exist two types of stable states: One state, \( \alpha \in [\beta, 1] \), in which everybody contributes to the public good, and another state, \( \alpha \in [0, \alpha] \), in which nobody contributes. Recall that the condition \( w(n) \frac{k}{k+1} < p < w(n) \frac{1-k}{2-k} \) was assumed fulfilled when drawing the graph in figure 1 and 2. Note that this condition is a necessary and sufficient condition for the existence of the two types of stable states. This implies that \( w(n) - p > 0 \) is a necessary condition for the existence

\(^{10}\text{Indeed, these are the asymptotically stable states of any payoff monotonic or weakly payoff-positive dynamics. See Weibull (1995) for definitions.} \)
of both types of stable states. Furthermore, note that the difference in individual welfare between a state in which everybody contributes and a state in which nobody contributes is given by $l = w(n) - p$. Hence, if there exist two types of stable states, one state in which everybody contributes and another state in which nobody contributes, then the former state will always Pareto dominate the latter.

To summarize this section:

**Proposition 2** Assume that $w(n) \frac{k}{k+1} < p < w(n) \frac{1-k}{2-k}$. Then, there exist two types of stable states: One stable state, $\alpha \in [\bar{\alpha}, 1]$, in which everybody contributes to the provision of the public good, and another stable state, $\alpha \in [0, \underline{\alpha}]$, in which nobody contributes. The former type of stable state Pareto dominates the latter.

**Proposition 3** If the share of people with social norms for private contribution to the public good gets above the unstable stationary state, $\alpha_0$, then more and more people will integrate into networks with social norms for private contribution. This process will continue until the society reaches the evolutionary stable state, $\bar{\alpha}$, in which everybody contributes to the provision of the public good.

4  Governmental Policies

This section shows that governmental subsidization of the public good is likely to increase voluntary contributions, while a governmental provision of the public good can crowd out voluntary contributions. This is because a subsidy will decrease the alternative cost of obtaining social approval, while a governmental provision of the public good will increases the alternative cost of obtaining social approval.

4.1  Governmental Subsidies

Assume the government subsidizes private contributions at a rate $s$ and pays for this subsidy by levying a lump sum tax $\frac{sG}{n}$ on each person in society. Then the relative price of the public good is given by $p = \hat{p} - s$ and people's income is given by $I = \hat{I} - \frac{sG}{n}$. Independent of the subsidy rate $s$, the differences in individual utility, in terms of private consumption, between a society in which everybody contributes and a society in which
nobody contributes is still given by \( l = w(n) - p \). Hence, differentiating (8), (9), (12) and (13) with respect to \( s \) yields

\[
\frac{\partial \Delta V (\alpha)}{\partial s} = 1 \quad \forall \quad \alpha \in \langle \bar{\alpha}, \hat{\alpha} \rangle
\]

\[
\frac{\partial \bar{\alpha}}{\partial s} = \frac{\partial \hat{\alpha}}{\partial s} = -\frac{1}{l(1-k)} < 0
\]

\[
\frac{\partial \alpha_0}{\partial s} = -\frac{p}{l(1-2k)} < 0
\]

Then, (11) and (12) imply that imposing a subsidy, \( s \), will shift the evolutionary path, \( \hat{\alpha} (\alpha) \), upwards to the left. The evolutionary path shifts upwards because the alternative cost of contributing to the public good or obtaining social approval has decreased.

Possible impacts of a governmental subsidy on voluntary contributions can now easily be illustrated in figure 3. In a society with no subsidies the evolutionary path, \( \hat{\alpha} (\alpha) \), is given by the solid line. Assume that a society with no subsidy is in a stable state \( \bar{\alpha} \in \langle \alpha_0', \bar{\alpha} \rangle \) with no private contributions to the public good. Then the government decides to subsidize the public good. The evolutionary path, \( \hat{\alpha} (\alpha) \), shifts upwards to the left and is now represented by the dashed line. \( \bar{\alpha} \) is no longer a stable state. Since \( \bar{\alpha} > \alpha_0' \), evolutionary forces will increase \( \alpha \) to the stable state, \( \bar{\alpha}' \), in which everybody contributes to the public good. The subsidy will crowd in voluntary contributions. Note that such a crowding in of voluntary contributions will prevail after the policy change is reversed if \( \alpha \) has increased such that \( \alpha > \alpha_0 \).

Whether subsidization of the public good will move the society to the stable state in which everybody contributes to the public good is dependent on the size of the subsidy and the initial stable state \( \bar{\alpha} \). The subsidy must shift the evolutionary path sufficiently such that \( \bar{\alpha} > \alpha_0' \). If not, the society will remain in the stable state with no private contributions. To summarize:

**Proposition 4** Assume a society is in a stable state, \( \bar{\alpha} \in (0, \bar{\alpha}] \), in which nobody voluntarily contributes to the public good. Then the government begins subsidizing the public good such that the evolutionary path shifts upwards to the left, and \( \alpha, \alpha_0 \) and \( \bar{\alpha} \) decreases to \( \bar{\alpha}', \alpha_0' \) and \( \bar{\alpha}' \). If the change in the evolutionary path is such that
• $\alpha' < \hat{\alpha}$, then $\alpha$ increases to the stable state, $\hat{\alpha}'$, in which everybody voluntarily contributes to the public good. The governmental subsidy crowds in voluntary contribution. This crowding in of voluntary contributions will prevail after the policy change is reversed if $\alpha$ has increased such that $\alpha > \alpha_0$.

• $\alpha' < \hat{\alpha} < \alpha'_0$, then $\alpha$ decreases to the stable state, $\alpha'$, in which nobody voluntarily contributes to the public good. The governmental subsidy will, however, temporarily crowd in some voluntary contributions.

• $\hat{\alpha} < \alpha'$, then the society remains in the stable state, $\hat{\alpha}$, in which nobody voluntarily contributes to the public good. The governmental subsidy has no impact on voluntary contributions.

4.2 Governmental Provision of Public Goods

Assume the government contributes an amount $\hat{G}$ to the provision of the public good and pays for this contribution by levying a lump sum tax $\frac{\hat{G}}{n}$ on each person in society. Then people’s income is given by $I = \hat{I} - \frac{\hat{G}}{n}$, and the difference in individual utility between a society in which everybody contributes voluntarily and a society in which nobody contributes voluntarily is given by $l = w \left( n + \hat{G} \right) - w \left( \hat{G} \right) - p$. Hence, differentiating (8), (9),
Figure 4:

(12) and (13) with respect to \( \hat{G} \) yields

\[
\frac{\partial \Delta V (\alpha)}{\partial \hat{G}} = (k + (1 - 2k) \alpha) \left( w' (n + \hat{G}) - w' (\hat{G}) \right) < 0 \quad \forall \alpha \in (\underline{\alpha}, \bar{\alpha})
\]

\[
\frac{\partial \alpha}{\partial \hat{G}} = \frac{\partial \bar{\alpha}}{\partial \hat{G}} = -\frac{p}{l^2 (1 - k)} \left( w' (n + \hat{G}) - w' (\hat{G}) \right) > 0
\]

\[
\frac{\partial \alpha_0}{\partial \hat{G}} = -\frac{p}{l^2 (1 - 2k)} \left( w' (n + \hat{G}) - w' (\hat{G}) \right) > 0
\]

since \( w'' < 0 \). Then, (11) and (12) imply that a government contribution, \( \hat{G} \), will shift the evolutionary path, \( \hat{\alpha} (\alpha) \), downwards to the right. The evolutionary path shifts downwards because the alternative cost of contributing to the public good or obtaining social approval has increased.

Possible impacts of governmental contributions on voluntary contributions are now easily illustrated in figure 4. In a society with no governmental contribution, \( \hat{\alpha} (\alpha) \), is given by the solid line. Assume that such a society is in the stable state \( \bar{\alpha} \) in which everybody contributes to the public good. Then the government wants an even higher provision of the public good and decides to contribute an amount \( \hat{G} \). The evolutionary path, \( \hat{\alpha} (\alpha) \), shifts downwards to the left and is now represented by the dashed line. Now all people in \( B \) will stop contributing because contributing yields less social approval than before. However, since people in \( A \) are doing better than people in \( B \) evolutionary forces will make \( \alpha \) increase to the new stable state \( \alpha = \bar{\alpha}' \) in which everybody contributes. The governmental policy did only temporarily crowd out some voluntary contributions.
If the government contribution is very large, however, then the crowding out of voluntary contribution is complete and permanent. Assume again that a society with no governmental intervention is in the stable state $\bar{\alpha}$ in which everybody contributes to the public good. Then the government decides to contribute a large amount $\hat{G}$ such that the evolutionary path shifts downwards to the left and is represented by the dotted line. Then all people in $B$ will stop contributing. Furthermore, since people in $A$ are doing worse than people in $B$ evolutionary forces will make $\alpha$ decrease to the new stable state $\alpha = \alpha''$ in which nobody contributes voluntarily $\alpha$. The governmental policy has crowded out all voluntary contribution. To summarize:

**Proposition 5** Assume a society is in the stable state $\bar{\alpha} \geq \alpha$ in which everybody voluntarily contributes to the public good. Then the government decides to provide an amount $\hat{G}$ of the public good such that the evolutionary path shifts downwards to the right, and $\alpha, \alpha_0$ and $\bar{\alpha}$ decreases to $\alpha', \alpha'_0$ and $\bar{\alpha}'$. If the change in the evolutionary path is such that

- $\hat{\alpha} > \alpha'$, then the society remains in the stable state, $\hat{\alpha}$, in which everybody voluntarily contributes to the public good. The governmental contribution has no impact on voluntary contributions.

- $\bar{\alpha}_0 < \hat{\alpha} < \alpha'$, then $\alpha$ increases to the stable state, $\alpha'$, in which everybody voluntarily contributes to the public good. The governmental contribution will, however, temporarily crowd out some voluntary contributions.

- $\bar{\alpha} < \hat{\alpha} < \bar{\alpha}_0$, then $\alpha$ decreases to the stable state, $\bar{\alpha}$, in which nobody voluntarily contributes to the public good. The governmental contribution crowds out all voluntary contribution.

- $\hat{\alpha} < \bar{\alpha}'$, then the society remains in the state $\hat{\alpha}$. This state has, however, changed to a stable state in which nobody voluntarily contributes to the public good. The governmental contribution crowds out all voluntary contribution.
5 Conclusion

This paper has provided an evolutionary analysis of the development of peer groups with social norms for private contribution to a public good. The analysis found the necessary and sufficient conditions for the existence of two types of stable states: One in which very few people belong to peer groups with social norms for private contribution, and one in which a large portion of people belong to such peer groups. In the former state nobody contributes, while in the latter everybody contributes. This finding is consistent with empirical observations that collective action will sometime succeed, but will also sometimes fail (see Ostrom, 1990).

This existence of several stable states yields important policy implications. Governmental policies can influence to which stable state the society will converge. Governmental subsidization of a public good is likely to move a society to a stable state in which everybody contributes voluntarily, while a governmental policy contributing directly to the provision of a public good is likely to move a society to a stable state in which nobody contributes voluntarily. This is because a subsidy will decrease the alternative cost of obtaining social approval, where as a governmental provision of the public good can increase the alternative cost of obtaining social approval. The analysis does not only show that subsidies will crowd in private contributions. It shows, indeed, that it may even be possible that this crowding in prevails after the policy change is reversed.
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