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Is there a Business Cycle Component in Norwegian Macroeconomic Quarterly Time Series?

Terje Skirpen

Discussion Papers

Statistics Norway Research Department

January 1995
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Abstract:
Some main Norwegian quarterly macroeconomic time series are decomposed into unobserved components within the framework of structural time series models using UCARIMA models. In the most general case we allow for a stationary cyclical component besides a stochastic trend, a stochastic seasonal and an irregular component. The cyclical component is either interpreted as a part of the trend component or as a component which is additive to the trend. For some of the investigated time series it is possible to extract business cycle component, but the the parameters characterizing it are not very precisely estimated and besides the component itself does not seem to be important.

Keywords: Business cycles, UCARIMA

JEL classification: C22, C51, E32

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Introduction

There are several methods available for the study of business cycle components of economic time series. A common property among these is that they try to isolate a cyclical component by removing the trend of the time series after having deseasonalized it (if it is quarterly or monthly).

For instance in the Hodrick-Prescott filter (cf. Hodrick and Prescott (1980)) a residual component are obtained after having removed a non-linear trend by a quadratic cost minimization problem which takes into account both the deviation of the trend from the observed series (or some transformation of it) and the non-smoothness of the trend component. Having obtained the residuals one can ask whether one or more cyclical components are pronounced in these calculated time series. This can for instance be done by investigating whether the log of the spectrum reveals any peaks at the frequencies associated with periods of business cycles length (often claimed to be from about 1.5 to about 8 years). How pronounced are these peaks compared to eventual peaks at the higher frequencies?

Another method is the Beveridge-Nelson procedure (cf. Beveridge and Nelson (1981)) which decomposes the time series into permanent and transitory components. The permanent and transitory components can be viewed as representing the trend and cyclical components respectively if this makes sense from an economic point of view. An important feature of the Beveridge-Nelson procedure is that the trend and cyclical component are generated by the same shocks, but these shocks have only permanent effects on the trend component.

Within structural time series models, which are the subject of this paper, one may also have models sharing this feature. In econometrics the distinction between structural and reduced form models have been fruitful. These models have their relative strength on different areas. A reduced form model is interesting in that it can pick up statistical features of the time series, but the problem is that it can be hard to interpret. For the structural models the situation is the opposite. Instead of viewing the two forms as competitive a natural approach would be to look at the reduced form as a benchmark model which should be encompassed by the structural model. Within the statistical area of univariate time series modelling one has a parallel distinction between reduced form ARIMA models and unobserved component ARIMA models (UCARIMA). The UCARIMA model has the important feature that it imposes an explicit parametric process of all the components making up an observed time series. One implication of this is that all the components can be extracted if the model is identified. Another implication is that the significance of the different components can be formally addressed. For instance one can ask what is the additional explanatory power of a cyclical component if one at the outset already have trend and seasonal components in the structural time series model.

The main result of this paper is that only weak support for a cyclical component is found. At the outset the model framework in this paper was applied to over ten quarterly macroeconomic time series. For the majority of these series it was not possible to extract a cyclical component. In the empirical part of the paper the focus is accordingly on the times series in which we can find some
evidence for a cyclical component. The general failure to establish a significant cyclical component should not be interpreted as if cyclical features are missing. A more appropriate interpretation is that the business cycle is too irregular to be revealed by the modelling framework which is used. This will for instance occur if the length of the business cycle is unstable over the sample period. Significant asymmetries in the different phases of the business cycle will have the same effect.

The rest of the paper is organized in the following way. In section 1 we present the structural time series model. Estimation issues and diagnostic testing are dealt with in section 2. The empirical results are given in section 3 and finally we offer some conclusions. Some technical aspects of the structural time series model, are discussed in two appendixes. Most of the numerical calculations have been made in the software programme STAMP, which builds on the modelling philosophy set out in Harvey (1989).

1. Model specification

All of the models considered in this paper are nested within the following modelling framework:

\[(1.1) \quad y_t = \mu_t + s_t + J \psi_t + u_t,\]
\[(1.2) \quad \mu_t = \mu_{t-1} + \beta_{t-1} + (1-J) \psi_{t-1} + v_t,\]
\[(1.3) \quad \beta_t = \beta_{t-1} + \omega_t,\]
\[(1.4) \quad s_t = -s_{t-1} - s_{t-2} - s_{t-3} + \varepsilon_t,\]
\[(1.5) \quad \psi_t = \rho \cos (\lambda_c) \psi_{t-1} + \rho \sin (\lambda_c) \psi_{t-1} + \kappa_t, \text{ and}\]
\[(1.6) \quad \psi_t^* = -\rho \sin (\lambda_c) \psi_{t-1} + \rho \cos (\lambda_c) \psi_{t-1} + \kappa_t^*.\]

In equation (1.1) \(y_t\) is an observed quarterly time series (possible transformed). The variable \(J\) is a dummy variable taking on the value 0 or 1. In the \(J=1\) case the observed variable is decomposed in a trend component (\(\mu_t\)), a seasonal component (\(s_t\)), a cyclical component (\(\psi_t\)) and an irregular component (\(u_t\)). The trend component is allowed to follow a random walk with a stochastic trend. The seasonal component (\(s_t\)) is assumed to follow a smooth stochastic process, allowing the seasonal pattern to change over time. Equation (1.5) and (1.6) determine the cyclical component implicitly. The symbol \(\lambda_c\) denotes the frequency, measured in radians, of the cyclical component and corresponds to a periodicity of \(2\pi / \lambda_c\). It is an important aspect of the above model that the frequency is assumed to be
an time invariant parameter. The letter $c$ indicates the a priori belief that the true value of the frequency $\lambda_c$ corresponds to a business cycle. The variable $\psi_t^c$ plays the role of an auxillary variable which makes it easier to represent the cyclical component in the State Space Form. In the $J=0$ case the cyclical component does not occur in equation (1.1). Its effect go through the trend component ($\mu_t$). Let $e_t$ be the vector containing the error terms. It is defined as

\begin{equation}
(1.7) \quad e_t = [u_t, v_t, w_t, e_t, \kappa_t, \kappa_t'].
\end{equation}

The error vectors $e_1, e_2, \ldots$ are assumed to be stochastic independent and normally distributed with expectation zero and with the following diagonal covariance matrix:

\begin{equation}
(1.8) \quad E(e_t e_t') = \text{DIAG} [\sigma_{u u}, \sigma_{v v}, \sigma_{w w}, \sigma_{e e}, \sigma_{\kappa \kappa}, \sigma_{\kappa \kappa}].
\end{equation}

Note that the variance of $\kappa_t^*$ is constrained to be equal to the variance of $\kappa_t$. This assumption is not necessary in order to identify the model, but makes the numerical analysis simpler.

In appendix A we deduce a stationary form representation for the observed variable (after log-transformation). The change in the annual growth rate from one quarter to the next can be written as a lag-distribution over the different error terms, in which $\lambda_c$ and $\varphi$ occur as parameters. It is however possible to move on to yet another representation, which is called the reduced form representation or as in Nerlove, Carvalho and Grether (1979) the canonical representation. The structural time series model implies a certain autocorrelation pattern for the above mentioned stationary variable. A natural question is therefore whether there is an ARMA-model whose autocorrelation pattern exactly matches that of the structural time series model. The answer is affirmative given that certain restrictions are imposed on the coefficients in the ARMA-model. The constrained parameters may be written as functions of the hyperparameters, i.e. the variances of the error terms and the parameters $\lambda_c$ and $\varphi$, of the structural time series model. In principle it is possible to first estimate an unconstrained ARMA model and then test the restrictions implied by the structural time series model.

In model (1.1) - (1.6) we are concerned with estimating the hyperparameters and extraction of the unobserved components. The SSF of the models in (1.1) - (1.6) are given in Appendix B. Since the prediction error decomposition (cf. Schweppe (1965)) is utilized in order to maximize the log-likelihood of the observed variables the SSF turns out to be very useful. After the maximum likelihood estimates of the hyperparameters have been obtained, the unobserved components at each point in time can be estimated using all available information.
2. Estimation issues and diagnostic statistics

In appendix B we have defined the state vector $\eta$, the time invariant transition matrix $T$ and the error vector of the transition equations $\zeta$. We have also defined the vector $A$, which links the observed variable to the state vector.

In order to utilize the prediction error decomposition we have to introduce some new symbols. Let $\bar{\eta}_{t-1}$ denote the minimum mean square error estimator of $\eta_{t-1}$ at time $t-1$ and let $P_{t-1}$ be the corresponding covariance matrix of the estimator of the state vector at the same time. Let furthermore $\eta_0$ and $P_0$ be the initial state vector and the initial covariance matrix respectively. The optimal estimator of the state vector given past information, i.e. $Y_{t-1} = \{y_t, ..., y_1\}$, is now given by

\begin{equation}
\bar{\eta}_{t-1} = T \bar{\eta}_{t-1},
\end{equation}

the covariance matrix of $\bar{\eta}_{t-1}$ is given by

\begin{equation}
P_{t-1} = TP_{t-1} T^\prime E (\zeta_t \zeta_t').
\end{equation}

When a new observation becomes available $\bar{\eta}_t$ and $P_t$ are changed according to the following updating equations:

\begin{equation}
\bar{\eta}_t = \bar{\eta}_{t-1} + P_{t-1} A_{1}^{-1} f_t \{y_t A \bar{\eta}_{t-1}\} \text{ and}
\end{equation}

\begin{equation}
P_t = P_{t-1} - P_{t-1} A_{1}^{-1} f_t A P_{t-1}
\end{equation}

In (2.3) and (2.4) $f_t$ is defined as

\begin{equation}
f_t = AP_{t-1} A' + \sigma_u.
\end{equation}

It furthermore follows that the optimal predictor of $y_t$ given past information is

\begin{equation}
\bar{y}_{t-1} = E (y_t | Y_{t-1}) = A \bar{\eta}_{t-1}
\end{equation}

and the accompanying one step ahead prediction error is:

\begin{equation}
\psi_t = y_t - \bar{y}_{t-1}.
\end{equation}

Let $d$ be a parameter which denotes the number of non-stationary elements in the state vector. Assuming that the initial state vector has a diffuse prior, we obtain the following conditional loglikelihood function:
In (2.8) $T$ denotes the entire sample size, whereas $(T-d)$ denotes the "effective" sample after having initialized the Kalman filter. The loglikelihood is a function of the vector of hyperparameters. However, by rescaling the variances in a certain way it is possible to concentrate one of the variances out of the loglikelihood function. This facilitates the numerical analysis. The computer programme STAMP supports three different algorithms of which two are applicable in the case with a cyclical component. One of these two algorithms is based on the Fourier transform, whereas the other one is based on maximization in the time domain. For numerical issues the reader should consult Harvey and Peters (1990) and Ng and Young (1990).

In estimating the state vector we have so far only considered past information. Evidently, an estimator with a smaller mean square error can be obtained using the whole sample. These smoothed estimates of the components may be extracted by utilizing the fixed interval smoothing algorithm (cf. Harvey (1990)). In the empirical part of the paper we will be occupied with the smoothed estimates of the components.

The success of the decomposition can be tested by utilizing the estimated standardized innovations:

\begin{equation}
I_t = \frac{\hat{\phi}_t}{\hat{v}_t^{1/2}}
\end{equation}

In this paper we will concentrate on three diagnostic statistics which deals with autocorrelation, heteroscedasticity and nonnormality of the standardized innovations respectively. The autocorrelation at distance $\tau$ of the estimated standardized innovations is given by

\begin{equation}
r_t(\tau) = \frac{\sum_{i = d+1}^{T} (I_t - \bar{I}) (I_{t-\tau} - \bar{I})}{\sum_{i = d+1}^{T} (I_t - \bar{I})^2}, \quad \tau = 1, 2, ...
\end{equation}

A joint test of significance of the first $P$ autocorrelations is then given by the Box-Ljung portemoneau statistic:

\begin{equation}
Q = (T-d)(T-d+2) \sum_{i=1}^{P} (T-d-\tau)^2 r_t^2(\tau)
\end{equation}

It can be shown that, under the absence of autocorrelation, $Q$ is asymptotically $\chi^2 (P-n-1)$, where $n$ is the number of estimated hyperparameters. The number of autocorrelations, $P$, is set to the nearest integer to $(T-d)/3$ from below.
To test for heteroscedasticity we employ the following statistic:

\[
H(h) = \sum_{t = d + 1}^{T - b - 1} \frac{I_t^2}{\sum_{t = d + 1}^{T - b - 1} I_t^2}
\]

(2.12)

Under the absence of heteroscedasticity in the standardised innovations we have, asymptotically, that \( h \) \( H(h) \) is \( \chi^2 \) (\( h \)). In the empirical analysis \( h \) will be set to the nearest integer of \( (T-d)/3 \) from below.

To test for normality we use the statistic:

\[
N = \left( \frac{T - d}{6} \right) b_1^* \left( \frac{T - d}{24} \right) (b_2 - 3)^2 \text{, where}
\]

(2.13)

\[
\sqrt{b_1} = (\hat{\sigma}^2)^{-\frac{3}{2}} \sum_{t = 1}^{T - d} (I_t - \bar{I})^3 / (T - d) \text{ and}
\]

(2.14)

\[
b_2 = (\hat{\sigma}^2)^{-2} \sum_{t = 1}^{T - d} (I_t - \bar{I})^4 / (T - d)
\]

(2.15)

In (2.14) and (2.15) \( \hat{\sigma}^2 \) denotes the estimate of the parameter concentrated out of the loglikelihood. The first term on the right hand side of (2.13) takes account of skewness, whereas the second one takes account of excess kurtosis. Under normality \( N \) is asymptotically \( \chi^2 \) (2).
3. Empirical illustrations

In this section we give some empirical illustrations of the structural time series approach to the business cycle modelling. Three quarterly time series from the National Accounts are decomposed. The time series are: the GDP mainland (Q6), total private consumption (C) and the price deflator of private consumption (PC). The decomposition is made after taking the natural logarithm. Thus the component model of the untransformed variables are multiplicative.

Table 1 displays the estimates of the hyperparameters, together with the value of the loglikelihood kernel of different models of the three variables. For all the three variables we have picked out three models. For each variable the models are given a consecutive number. Model 1 is a reference model without a cyclical component. With regard to ln (Q6) and ln (PC) model 2 and 3 are the models with additive and trend cyclical component respectively. Since we were unable to obtain convergent estimates of the model with an additive cyclical component for the ln (C) variable, we present two models based on the trend cycle specification. The difference between the second and third model for this variable is that we have assumed a fixed slope for the trend component in the latter. Table 2 contains the results from the diagnostic checking of the standardized innovations. The smoothed estimates of the unobserved components of the time series of some of the models are depicted in figures 1-4. Since we model the log of the time series, we have transformed the smoothed components by applying the antilog operator. For each model we operate with three graphs. In graph a) we have the actual series together with the smoothed trend. In graph b) we depict the cyclical component, and finally the seasonal and irregular components are displayed in graph c.
Table 1. Estimates of hyperparameters. Standard error in paranthesis. The variances and their standard errors are multiplied by 1000.

<table>
<thead>
<tr>
<th>Model variable</th>
<th>Cycle</th>
<th>Irregular</th>
<th>Loglikelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma^2_m$</td>
<td>$\sigma^2_m$</td>
<td>$\lambda_c$</td>
</tr>
<tr>
<td>Production ln (Q6)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0995</td>
<td>0.006</td>
<td>0.0017</td>
</tr>
<tr>
<td></td>
<td>(0.0596)</td>
<td>(0.0007)</td>
<td>(0.0021)</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0004</td>
<td>0.0367</td>
</tr>
<tr>
<td></td>
<td>(0.1698)</td>
<td>(0.0005)</td>
<td>(0.1173)</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>0.0005</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Consumption ln (C)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.1780</td>
<td>0.0003</td>
<td>0.0062</td>
</tr>
<tr>
<td></td>
<td>(0.0610)</td>
<td>(0.0003)</td>
<td>(0.0039)</td>
</tr>
<tr>
<td>2</td>
<td>0.0364</td>
<td>0.0002</td>
<td>0.0063</td>
</tr>
<tr>
<td></td>
<td>(0.1013)</td>
<td>(0.0003)</td>
<td>(0.0040)</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>-</td>
<td>0.0065</td>
</tr>
<tr>
<td></td>
<td>(0.0727)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Consumption price ln (PC)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0277</td>
<td>0.0081</td>
<td>0.0008</td>
</tr>
<tr>
<td></td>
<td>(1.6496)</td>
<td>(2.1836)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0023</td>
<td>0.0008</td>
</tr>
<tr>
<td></td>
<td>(0.0014)</td>
<td>(0.0005)</td>
<td>(0.0097)</td>
</tr>
<tr>
<td>3</td>
<td>0.0232</td>
<td>0.0019</td>
<td>0.0008</td>
</tr>
<tr>
<td></td>
<td>(0.0196)</td>
<td>(0.0016)</td>
<td>(0.0005)</td>
</tr>
</tbody>
</table>

The general result seems to be that it does not give very much additional explanatory power to augment the reference model with a cyclical component. Incorporating a cyclical term means that three new parameters have to be estimated ($\sigma^2_m$, $\lambda_c$ and $\rho$). The log-likelihood kernel value only increases moderately. However, it is not straightforward to utilize for instance the LR-test to formally test the reference model against the cyclical model. Since we are on the boundary of the admissible parameter space (because of the impossibility of negative variances) under the null hypothesis, the LR-statistic will not be asymptotically $\chi^2$-distributed. In the same way care must be taken in
interpreting the implied t-values of the parameters characterising the cyclical component.

From table 1 it is seen that the estimate of the variance of the error term, $\sigma^2_v$, is very low. This suggests that the model may be simplified. Instead of restricting $\sigma^2_v$ to zero it seems more appropriate to pursue the idea of a fixed slope in the trend component, i.e. restricting $\sigma^2_{wv}$ to zero. However following this route of action for the production and the price variable resulted in convergence problems. In the preliminary estimates $\sigma^2_v$ was still close to zero. Besides the value of the dampening factor, $\varphi$, was very close to 1, which implies a non-stationary cycle. Furthermore the value of the frequency was very low and could accordingly not be associated with the typical periodicity of a business cycle. These results emphasize once again the problem of distinguishing between a deterministic and a stochastic trend.

The point estimates of the frequencies of the cyclical components do not indicate a length of the cycle which is inconsistent with a business cycle. For the ln (Q6) variable the implied period is about 10 years in the additive cycle interpretation and about 7 years in the level trend case. With regard to the consumption variable, ln (C), the estimated frequency corresponds to a period of about 7 years in both models. The shortest length of the cycle is obtained for the price variable where the implied estimate is between 5 and 6 years. However, because of substantial uncertainty in the estimates of the frequencies the cyclical periods are not very well determined.

Table 2 gives no evidence of autocorrelation or heteroscedasticity in the standardized innovations. Non normality seems however to be a problem for the consumption variables. This may be due to outliers and could be taken care of by introducing an appropriate dummy variable in the measurement equation.
Table 2. Diagnostic tests of different models

<table>
<thead>
<tr>
<th>Model</th>
<th>Diagnostic statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q</td>
</tr>
<tr>
<td>Production</td>
<td></td>
</tr>
<tr>
<td>(ln(Q6))</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11.69</td>
</tr>
<tr>
<td>2</td>
<td>11.75</td>
</tr>
<tr>
<td>3</td>
<td>11.16</td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
</tr>
<tr>
<td>(ln(C))</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>9.54</td>
</tr>
<tr>
<td>2</td>
<td>9.56</td>
</tr>
<tr>
<td>3</td>
<td>9.18</td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
</tr>
<tr>
<td>price</td>
<td></td>
</tr>
<tr>
<td>(ln(PC))</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11.32</td>
</tr>
<tr>
<td>2</td>
<td>8.46</td>
</tr>
<tr>
<td>3</td>
<td>8.59</td>
</tr>
</tbody>
</table>

1) Significant at the 5 percentage significance level.

Conclusions and discussion

In this paper we have tried to decompose some quarterly economic time series using unobserved component models. Our main concern was with the cyclical component. The estimate of the frequency of the cyclical component implied a periodicity which was not inconsistent with the typical periodicity of a business cycle. However the cyclical component did not possess much additional information compared with the reference model in which no cyclical component was allowed. Besides for many other macroeconomic time series (not reported) it was not possible to extract a cyclical component at all within this modelling framework. It is not obvious how this result should be interpreted. In business cycle analysis it is often claimed that it is necessary to have a long span of time in order to obtain precise results with regard to the cyclical component and many researchers have in agreement with this worked with data covering the whole century. In this empirical analysis the sample length is 28 years. However having a large sample period accentuates problems connected to structural breaks. This has often implied that researchers have been forced to estimate different models for different subperiods. Since the hyperparameters in the structural time series model are assumed to be
timeinvariant, it will be hard to extract a business cycle component unless it show a high degree of regularity. In Brasil and Souza (1993) the timeinvariance assumption has been somewhat relaxed. Both the frequency of the cyclical component and the dampening factor are allowed to develop according to a random-walk scheme. However, this implies a much more complicated model since the transition equation in the state space model now becomes nonlinear. In the influential paper by Hamilton (1989) the assumption of symmetric cyclical behaviour is abandoned. A parameter interpreted as the drift in the log of US real GNP depends on which of two states the economy is in, and the state itself is governed by a first-order Markov process. Another approach to the possible asymmetry in business cycles is the one taken by Brännäs and De Gooijer (1994). Working within the class of the so-called autoregressive-asymmetric moving average (ARasMA) models positive and negative shocks are, at the outset, allowed to have different effects and the restriction of symmetry can be tested. If it is hard to extract a business cycle component within the class of symmetric models, as it is in this paper, it seems appropriate to search for asymmetrical features in the business cycle.
References


Appendix A. Implications of the structural time series model

Let the lag-operator be defined as

\[
L^i Z_t = Z_{t-i}
\]

Under the assumption that the dampening factor \( p \) lies in the interval \((0,1)\), the solution of \( \psi_t \) is

\[
\psi_t = \frac{[(1- p \cos (\lambda p) ) L \kappa + p \sin (\lambda p) L \kappa]}{[1- 2 p \cos (\lambda p) L + p^2 L^2]}.
\]

Using the lag-operator notation equations (1.2) to (1.4) can be rewritten as

\[
(1- L) \mu_t = L \beta_t + (1- L) L \psi_t + v_t,
\]

\[
(1- L) \beta_t = w_t, \text{ and}
\]

\[
(1 + L+ L^2 + L^3) s_t = e_t.
\]

Let the filter \( G(L) \) be defined as:

\[
G(L) = (1- L)^2 (1 + L^2+ L^3).
\]

Applying this filter on (1.1) in the J=1 case yields:

\[
G(L) y_t = (1+ L+ L^2+ L^3) (1- L) [(1- L) \mu_t] + (1- L)^2 [(1+ L+ L^2+ L^3) s_t] + G(L) \psi_t + G(L) u_t.
\]

Inserting from the equations (A.3) and (A.5) yields:

\[
G(L) y_t = (1+ L+ L^2+ L^3) (1- L) [L \beta_t + v_t] + (1- L)^2 e_t + G(L) \psi_t + G(L) u_t.
\]

Inserting from (A.4) yields furthermore:

\[
G(L) y_t = (1+ L+ L^2+ L^3) [L w_t + (1- L) v_t] + (1- L)^2 e_t + G(L) \psi_t + G(L) u_t.
\]

The filter \( G(L) \) may also be written as:
Equation (A.10) shows that the implication of running the filter $G(L)$ on a time series is to construct the change in the four quarter change from one period to the next. Rewriting equation (A.9) in this new notation yields:

$$\Delta \Delta_4 y_t = (1 + L + L^2 + L^3) [L w_t + \Delta v_t] + \Delta^2 \varepsilon_t + \Delta \Delta_4 \psi_t + \Delta \Delta_4 u_t.$$  \hfill (A.13)

Let now

$$x_t = \Delta \Delta_4 y_t,$$  \hfill (A.14)

and let the three filters implicit in equation (A.2) be defined as:

$$A(L) = (1 - 2 \rho \cos \lambda \ L + \rho^2 \ L^2),$$  \hfill (A.15)

$$B(L) = (1 - \rho \cos \lambda \ L),$$  \hfill (A.16)

$$C(L) = \rho \sin \lambda \ L.$$  \hfill (A.17)

If we multiply equation (A.13) with the filter in (A.15) we obtain

$$A(L) x_t = A(L) (1 + L + L^2 + L^3) [L w_t + \Delta v_t] + A(L) \Delta^2 \varepsilon_t + \Delta \Delta_4 B(L) \kappa_t + \Delta \Delta_4 C(L) \kappa_t + A(L) \Delta \Delta_4 u_t.$$  \hfill (A.18)

As can be seen from the right hand side of equation (A.18) the maximum power of $L$ is 7, meaning that the variable $x_t$ is following a constrained ARMA (2,7) process. In the $J=0$ case equation (1.24) is slightly altered and is given as:

$$A(L) x_t = A(L) (1 + L + L^2 + L^3) [L w_t + \Delta v_t] + A(L) \Delta^2 \varepsilon_t + \Delta_4 \ V(L) \kappa_t + \Delta \Delta_4 \ L, C(L) \kappa_t + A(L) \Delta \Delta_4 u_t.$$  \hfill (A.19)

Again the variable $x_t$ follows a constrained ARMA (2,7) process, and $y_t$ can similarly be viewed as a restricted ARIMA (2, 5, 7) process. The implication of the two ways of implementing the cycle should be sought in the different autocorrelation pattern of the $x_t$ variable.
Appendix B. The SSF

In this section we show how the structural time series models may be written out in the SSF, by use of Kalman filter techniques. Both ways of treating the trend component are considered. In the SSF one distinguishes between the measurement equation (in the univariate case) and the transition equations. The measurement equation states how the observed variable is linked to the unobserved state vector

\[ y_t = A \eta_t + u_t \]

In (B.1) \( \eta_t \) is a vector containing the state variables. A is a line vector which link the observed variable to the state vector, whereas \( u_t \) is the measurement error. The state vector is generated according to the following 1. order vector process:

\[ \eta_t = T \eta_{t-1} + \zeta_t \]

It should be emphasized that the SSF in (B.1) and (B.2) is not on its most general form. Especially it should be noted that the matrices A and T are both timeinvariant.

i) The J=1 case

The state vector and the transition matrix are given respectively as:

\[ \eta_t = [\mu_t, \beta_t, s_t, s_{t-1}, s_{t-2}, \psi_t, \psi_{t-1}]' \]

and

\[
T = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho \cos (\lambda) & \rho \sin (\lambda) & 0 \\
0 & 0 & 0 & 0 & 0 & -\rho \sin (\lambda) & \rho \cos (\lambda)
\end{bmatrix}
\]

Note that when the cyclical component is not considered in the analysis, the dimension of the \( \eta_t \) vector and the T matrix are 5x1 and 5x5 respectively. Furthermore the matrix T is completely known and does not have to be estimated from the data. The disturbance vector \( \zeta_t \) is defined as

\[ \zeta_t = [v_t, w_t, e_t, 0, 0, \kappa_t, \kappa_t] \]

According to the earlier stated stochastic assumptions, we have the following diagonal (and singular)
covariance matrix of the $\zeta$-vector:

$$(B.6) \quad \mathbb{E}(\zeta, \zeta') = \text{DIAG} [\sigma^2_{wv}, \sigma^2_{vw}, \sigma^2_{ev}, 0, 0, \sigma^2_{ew}, \sigma^2_{ee}].$$

The vector $A$ is given by

$$(B.7) \quad A = [1, 0, 1, 0, 1, 0].$$

The measurement error $u_i$ is the same as in equation (1.1). In the outlined model we have seven hyperparameters $[\sigma^2_{wv}, \sigma^2_{vw}, \sigma^2_{ew}, \sigma^2_{ee}, \rho, \lambda_c]$ which must be obtained before the final unobserved component can be extracted. Before we consider estimation and decomposition of the model, we have to clarify how the state-space form of the structural time series model is altered when applying the alternative interpretation of the cyclical component. The transition vector and disturbance vector is unaltered, but the transition matrix and the $A$-matrix are slightly changed:

ii) The $J=0$ case

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho \cos (\lambda_c) & \rho \sin (\lambda_c) \\
0 & 0 & 0 & 0 & 0 & -\rho \sin (\lambda_c) & \rho \cos (\lambda_c)
\end{bmatrix}
\]

and

$$(B.9) \quad A = [1, 0, 1, 0, 0, 0].$$
Figure 1a. Actual and trend values of production (Q6). Mrd. (1991) Norwegian kroner. The additive cycle case

Figure 2a. Actual and trend values of production (Q6). Mrd. (1991) Norwegian kroner. The trend cycle case

Figure 1b. Cyclical component of production (Q6). The additive cycle case

Figure 2b. Cyclical component of production (Q6). The trend cycle case

Figure 1c. Seasonal and irregular components of production (Q6). The additive cycle case

Figure 2c. Seasonal and irregular components of production (Q6). The trend cycle case
Figure 3a. Actual and trend values of consumption \( (C) \). Mrd. (1991) Norwegian kroner. The trend cycle case

Figure 4a. Actual and trend values of consumer prices \( (PC) \). The additive cycle case

Figure 3b. Cyclical component of consumption \( (C) \). The trend cycle case

Figure 4b. Cyclical component of consumer price \( (PC) \). The additive cycle case

Figure 3c. Seasonal and irregular components of consumption \( (C) \). The trend cycle case

Figure 4c. Seasonal and irregular components of consumer price \( (PC) \). The additive cycle case
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