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Sufficient Welfare Indicators
Allowing Disagreement in
Evaluations of Social Welfare
Abstract:
There is no consensus on how to measure interpersonally comparable, cardinal utility. Despite of this, people repeatedly make welfare evaluations in their everyday lives. However, people do not always agree on such evaluations, and this is one important reason for political disagreements. Thus, to keep in control of the normative premises, decision makers may prefer information which can be used as input in an arbitrary social welfare function to information which is the output from a social welfare function specified by the analyst. In this paper we try to identify sufficient welfare indicators; information which enable decision makers to arrive at welfare evaluations of social states or projects, according to their own ethical beliefs. Our conclusion is that providing factual information about different population groups; their social state, size, and characteristics, may be better for this purpose than the more traditional approach of focusing on ordinal utility information.

Keywords: Disagreement, interpersonal comparison, social welfare.

JEL classification: A11, A13, D6, D7, I31

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1 Introduction

When economists analyze the social desirability of a public project, they rarely discuss ethics. Recognizing that most economists are not better educated for ethical considerations than anyone else, the most common approach is to leave the task of determining the goals of society to the politicians. The role of the economist then becomes to provide factual background information for political decisions.

The exogenous "goals of society" frequently enter economic models in the form of a Samuelson-Bergson welfare function. However, when politicians (or more generally, decision makers) disagree about what the goals ought to be, it is not clear how to define the social welfare function. If a welfare function corresponding to the ethical views of one particular person is used, the output from such an analysis may not be of much interest to another decision maker.

The process of reaching a collective decision can be viewed (very simplified, of course) as consisting of two steps. The first step is that the individual participants in the decision process make up their minds as to which of the alternative social states they prefer. The second step consists of some kind of procedure for aggregating these individual opinions into a collective choice. Majority voting and dictatorship are two examples of such procedures.

The present paper is concerned with the first of these two steps. The scope of the paper is to identify information which any individual participant in a collective choice process can use as input into her subjective social welfare function. The second step, aggregating individual rankings to yield a collective choice, is not discussed here.

Information about a project or about changes in the state of an economy, presented to the decision maker as estimated social net benefits or changes in social welfare, will inevitably involve acceptance of controversial normative judgments if people's ethical beliefs differ. For example, the ratio between benefits and costs resulting from a cost-benefit analysis can be regarded as output from a utilitarian welfare function, given that every individual has the same marginal utility of income; but it cannot be used as input to an arbitrary social welfare function.

The approach of this paper will be to look for reasonable assumptions under which we are able to develop sufficient welfare indicators; that is, aggregated information which
enable individual decision makers to evaluate a project or a state, according to their own ethical beliefs. This is in contrast to the approaches of for example Jorgenson (1990), Mäler (1991) and Hartwick (1990), as well as standard cost-benefit analysis. Common to these is that welfare indicators are derived as output from a social welfare function specified by the analyst.

As early as in 1951 Kenneth Arrow demonstrated that ordinal information on individual preferences is not sufficient to establish a social welfare function (Arrow 1951).\(^1\) Any social welfare function, thus, must be based on cardinal, interpersonally comparable information\(^2\). There is no commonly accepted method of measuring cardinal, interpersonally comparable well-being; in particular, the problem of interpersonal comparisons seems difficult to overcome. However, that people actually make such comparisons in their everyday life is much less controversial. Pollak (1991) draws the following conclusion:

"I am agnostic about the possibility of making interpersonal welfare comparisons. I find the most convincing argument that such comparisons are possible is the frequency with which we make them." Pollak (1991).

Interpersonal comparisons of well-being frequently have a specific purpose; namely, to arrive at a ranking of alternative social states. The comparisons are made informally, and the issues under consideration are frequently of a rather trivial kind: Should I visit my old aunt at the hospital, or should I take my family to the beach? Should Emma or Bill have the last piece of cake? Nevertheless, such everyday decisions are based on subjective opinions about different individuals’ well-being in various social states, and also on how large weight we should attach to each person’s well-being when making the choice. The ability of human beings to make such rankings of social states is the starting point of social choice theory. To quote Arrow (1981): “Each individual is supposed to have a social conscience, an ordering over income distributions. The social welfare function for a given individual determines the answer he or she would give to the following hypothetical question: For all pairs of income distributions, which would I choose if I were a dictator?”

\(^1\)The impossibility theorem states that if the four conditions of 1) Unrestricted domain, 2) The Pareto Principle, 3) Independence of irrelevant alternatives and 4) Non-dictatorship are imposed on a social welfare function, no such function exist. For a discussion of the theorem and its implications see Sen (1970).

\(^2\)Some welfare functions do not require full comparability. A minimax welfare function would only require interpersonal comparison of utility levels, while that same piece of information will be redundant under utilitarianism.
Any democracy is based on the assumption that most people, not only a few experts, are able to make such rankings. The approach of this paper, thus, is based upon the assumption that if given sufficient background information, people are able to compare the well-being of different people and to rank social states.

It may be noted, however, that Arrow's impossibility theorem applies not only to aggregating individual well-beings into a social welfare function, but to aggregating subjective social welfare functions into one function determining collective choice as well. This problem is not solved by our approach. Nevertheless, every society employs mechanisms to reach collective decisions, even though all such mechanisms violate Arrow's axioms. We believe that a reasonable requirement to any collective choice procedure is, at least, that its individual participants have a well-founded opinion about which outcome they regard as socially preferable. Ideally then, every individual ought to have the opportunity to judge the welfare implications of a policy proposal according to his or her own ethical values.

There is no theoretical basis for neutral aggregation of information. Thus, if only completely neutral aggregation is allowed, the only option left for the analyst would be to hand over a huge pile of computer printouts to the politicians. To confront the individual with vast amounts of data may not improve her ability to evaluate a project or a state, because individuals obviously have a limited capacity to digest information. We will therefore have to look for reasonable assumptions under which aggregation can be allowed without too much loss of information. In other words, we are searching for indicators that contain sufficient information for anybody to make his own welfare judgment.

In the next section we present the model of the paper and derive some results. Proofs of the propositions have been relegated to the Appendix. Section 3 provides a discussion of the subjective social welfare functions. Section 4 elaborates on the approach to interpersonal comparisons, whereas section 5 deals with the interpretation of the concept of well-being. In section 6 the role of ordinal utility information is discussed. Section 7 offers some concluding remarks.

2 An approach to individualistic welfare judgments

In this section the model of the paper is presented. Basically, it consists of only two equations. Being very limited in this sense, a discussion of the interpretation of the model is clearly essential. However, to provide an overview, we have chosen to present the formal model and
some mathematical results at the outset. The remains of the paper (sections 3 through 6) is mainly devoted to a discussion of the interpretation of the model; its equations, assumptions and results. Hopefully, many of the reader’s questions which may be left unanswered in section 2 will be dealt with in this part.

For the reader unfamiliar with mathematics, it should be possible to understand the main ideas of the paper without full understanding of the mathematics. The exceptions are equations (1) and (2), which are vital for the following discussion.

2.1 The model

Let $\mathcal{X}$ be a set of social states. Each social state $X \in \mathcal{X}$ is a vector $X = (x_1, \ldots, x_i, y)$, where $x_i \in R$ is the amount of an individual good allocated to individual $i$, and $y \in R$ is a public good. The basic model of the paper is that each individual $j$'s social welfare judgment is based on a Samuelson-Bergsonian welfare function

$$W^j(X) = V^j(\omega_1^j(X), \ldots, \omega_i^j(X))$$

where $\omega_i^j(X)$ is the well-being of individual $i$ in state $X \in \mathcal{X}$ according to $j$'s judgment. Thus the individual social welfare judgment is assumed to conform with the well-known construction from economic welfare theory.

In spite of our desire to leave the ethical considerations to the decision maker, we have already restricted the possible range of social welfare functions considerably: It is assumed that rankings of social states are based solely on the well-being of the individuals in society, i.e. we do not consider the information needed if also, in addition to well-being, rights, freedom or other variables enter the individual social welfare functions. Information of other variables than well-being might be highly relevant in some contexts. However, to bring out the consequences of this restriction would require a separate analysis.

We also need a hypothesis of how individual $j$ makes his judgment on $i$'s well-being. Suppose that a vector of characteristics $\alpha_i$ for individual $i$ is known. The set of possible characteristic configurations is denoted $A$. Let $A = (\alpha_1, \ldots, \alpha_i)$, $A \in A$. The characteristics may, for instance, be age, sex, health, or information of previous choices made by the individual. A characteristic may also be that $i$ belongs to the group who will be affected

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3A value system is "welfaristic" according to Sen (1979) if: "Social welfare is a function of personal utility levels, so that any two social states must be ranked entirely on the basis of personal utility levels in the two states".
by a project, e.g. that i will have to move under the state $X' \in X$. We assume that the characteristics are observables, and hence that there is no element of judgment as to which category a specific individual belongs.

Assumption 1 (Subjective judgement of well-being) For all $j$ there is a function $\nu^j$, defined for all $\alpha_i$ in $A$ and all $(x, y) \in R^I \times R$, such that for all $i$:

$$\omega_i^j(X) = \nu^j(x_i, y; \alpha_i)$$

We will call these functions *evaluation functions*. Existence of evaluation functions implies that if given information about person $i$'s characteristics and the resources allocated to her, any person $j$ is able to make up his mind as to what he believes $i$'s well-being is. Equation 2 further states that person $j$'s evaluation function is assumed to depend only on the social state and the characteristics of individual $i$, not on the identity of $i$. We assume that $\alpha_i$ does not contain full identification if $i$.

The estimates of individual well-being $\omega_i^j$ resulting from the evaluation functions $\nu^j$ are assumed to be cardinal and fully comparable\(^4\) between individuals, from $j$'s point of view. We do not, however, require all individuals to arrive at the same judgement of $i$'s well-being, neither do we assume that $\omega_i^j$ and $\omega_i^k$ are comparable.

It is still possible for person $j$ to disregard person 1's interests, simply by disregarding $\omega_1^j$ in the aggregator $V^j$. To rule out this possibility we add the assumption of anonymous aggregation:

Assumption 2 (Anonymous aggregation) Let $\pi$ be any pivotation of the numbers 1 to $I$, then

$$V^j(\omega_1, ..., \omega_I) = V^j(\omega_{\pi(1)}, ..., \omega_{\pi(I)})$$

Assumption 2 is similar to the assumption on anonymity in Sen (1977). In fact, it rules out not only the possibility of letting people's identity count, but also views like, for example, "the well-being of a criminal should count less than the well-being of an innocent child".\(^5\)

Note that the assumptions require $j$'s social ranking to be derived from universal principles, but we do not require all individuals to arrive at the same social rankings.

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\(^4\)For a definition of full comparability, and an overview of various comparability assumptions, see Sen (1986), pp. 1111-1114.

\(^5\)This claim is modified if one allows for an interpretation of "well-being" as "morally relevant wellbeing". See section 5.3.
2.2 Sufficient welfare indicators

Now we want to consider how much information is needed for any \( j \) to make a welfare judgment. A set of information that is sufficient for anyone to make a welfare judgment is called a sufficient welfare indicator, like the concept 'sufficient statistics' in statistical inference theory:

**Definition 1** A mapping \( T(X, A) \) is a sufficient welfare indicator if for all \( j \), there is a mapping \( F^j \), such that \( W^j(X; A) = F^j(T(X, A)) \), for any \( A \).

Hence, if \( T \) is a sufficient welfare indicator, then \( T \) is all any policy maker needs to know to arrive at his subjective ranking of the social states.

By combining equations (1) and (2) we can easily see that if the vectors \( X \) and \( A \) are known, the decision maker has sufficient information to rank social states. However, this would amount to \( 2I + 1 \) variables,\(^6\) where \( I \) is the number of individuals in society, which would in most practical applications be far too much information to perceive for any decision maker. Thus, we have to turn to the question of aggregating information. The following observation is straightforward:

**Proposition 1** Suppose that \( A \) is finite with \( K \) distinct elements. Suppose furthermore that for all social states and for all \( i \) and \( j \) such that \( \alpha_i = \alpha_j \), we have \( x_i = x_j \). Let \( x_i(k) \) denote the vector of private goods allocated to any member of group \( k \), \( k = 1, \ldots, K \), and \( a(k) \) the group characteristics. Let \( n \in \mathbb{N}^K \) be the vector of natural numbers where component \( n(k) \) is the number of individuals with a combination of characteristics described by the vector \( a(k) \). Then \( y \) and the triples \( (n(k), a(k), \xi(k)) \) for all \( K \) is a sufficient welfare indicator.

The proposition states that if everybody within a group have the same characteristics and the same income, we need to know the size, income and characteristics of the different groups, in addition to information on the collective good \( y \). We require that the amount of the private good should be equal for all with the same characteristics. This will be the case if income is included as a characteristic. Thus, if the population can be divided into homogenous groups, the information required to rank social states is reduced to \( 3K + 1 \) variables, where \( K \) is the number of groups.

\(^6\)For each \( i \in I \) information on \( x_i \) and \( \alpha_i \) is needed. In addition information about the public good \( y \) is required. The accounting assumes that the vectors \( \alpha_i \) are counted as only one variable per \( i \), stating the "type" of the individual.
However, if the population cannot be divided into a few homogenous sub-groups, i.e. the number of elements in $A$ is large, this proposition is less useful. In that case, a natural approach would be to make the evaluation of social welfare in two steps. In the first step, the welfare of a subgroup of the population is considered. In the second step we could then aggregate the welfare of a small set of different groups defined by a short list of characteristics. This is a widely used procedure. Marxian class analysis may be seen as an application of this approach. In his seminal "Theory of justice" Rawls (1971) similarly argued to maximize the primary goods of the worst off group. Public debate is also usually framed in terms of the well-being of different interest groups. Would such a procedure be consistent with our model?

The two step procedure requires that the welfare function $V_j$ is separable. Since we have assumed anonymous aggregation, separability must apply to every group. Hence, for the two-step procedure to be feasible, $V_j$ must be strongly separable. The next proposition follows from a result of Debreu (1964), see Barten and Böhm (1987), Th.6.2.

**Proposition 2** If the welfare function is differentiable, the two-step aggregation procedure is possible if and only if the welfare function is utilitarian after a monotone transform, i.e.

$$V(\omega_1, \ldots, \omega_I) = \Phi(\sum \psi(\omega_i))$$

for some monotone transforms $\Phi$ and $\psi$.

For example, quadratic social welfare functions do not meet this requirement. The same is true for rank-dependent social welfare functions.

### 2.3 Locally sufficient welfare indicators

A case of particular interest is the social preferability of a small project. If the project is sufficiently small, differences in individual well-being between social states will be small and the social welfare function can be approximated by a linear function,

$$V^i(\omega) \approx V^i(\bar{\omega}) + \sum \frac{\partial V^j}{\partial \omega_i} (\omega_i - \bar{\omega}_i) = C^i + \sum c^i \omega_i,$$

where $c^i = \frac{\partial V^j}{\partial \omega_i}$. In this case, if $c^i$ is given for all groups, it is possible to make separate judgements for different groups, since

$$V^i(\omega) \approx C^i + \sum G^i_g$$
where $G_i^j = {\sum}_{i \in g} c_i^j \omega_i$.

We can now define a locally sufficient welfare indicator:

**Definition 2** A locally sufficient indicator is an indicator that is sufficient for any $j$ to determine a linearization of her social welfare function.

Note that $c_i^j$ are subjective, and that $j$ may need information to determine these weights. This information is only required for one social state, the status quo. Note also, however, that the weights $c_i^j$ will only be possible to determine through a two-step procedure if $V^j$ satisfies the separability properties stated in Proposition 2.

Hence, a linearization does not really simplify the sufficient welfare indicators. We noted above (Proposition 1) that with $K$ homogenous groups, the sufficient welfare indicator consists of $3K + 1$ variables. The locally sufficient welfare indicators consist in this case of $4K + 2$ variables, since in addition to the state variables, changes in the provision of private and public goods must be known. Once the welfare weights are determined, the additional information required for evaluating another project consists of $K + 1$ variables.

Now, will this amount to more detailed information than any decision maker can reasonably be expected to grasp? If we assume that the separability assumptions of Proposition 2 are fulfilled, it is possible for the decision maker to handle the problem in a two-step (or multi-step) procedure. If the separability assumptions are *not* fulfilled, the decision maker will need full information about the status quo, but once she has used this information to establish her welfare weights, she can evaluate any project using a two-step procedure. Thus, even if $K$ is large, it will in principle be possible to facilitate the problem by dividing it into several partial considerations. Still, if $K$ is large, the evaluation of the project may be too laborious and time-consuming.

A central question then becomes; how can we reduce the number $K$? Fundamentally, any individual is different from every other to some extent. Thus, can we accept *some* differences between individuals and still treat them as equal when looking for sufficient welfare indicators?

Treating heterogenous individuals as if they were homogenous can be done by defining a representative individual for each heterogenous group. Choosing a representative individual is, however, certainly not a task free of value judgments, as will be discussed later. Still, applying such a procedure might be less controversial when only marginal projects are considered.
For every group $g$, let $r(g) \in g$ denote a typical individual, a representative, and $n(g)$, the number of individuals of the group. Now

$$\tilde{G}^j_g = \frac{G^j_g}{n(g)c^j_{r(g)}}$$

normalizes the well-being of the group in terms of the well-being of the representative individual. This normalization becomes especially interesting if we can pick $r(g)$ such that $\omega^j_{r(g)} = \tilde{G}^j_g$ for all $j$ and for all social states. In this case the judgement of the social states can be reduced to a judgement for a hypothetical society with a small number of homogenous groups. We state this formally.

**Proposition 3** If there exists a group structure, and an individual $r(g)$ for each group such that $\omega^j_{r(g)}(x_{r(g)}, y) = \tilde{G}^j_g(X)$, for all social states in a neighborhood, then $y$ and the triples $(r(g), n(g), x_{r(g)})$ for all groups $g$ is a sufficient welfare indicator.

### 3 Subjective social welfare functions

The individual social welfare function $W^j$ is the core of our model. It corresponds to the usual Samuelson-Bergson welfare function, except that its subjective character is emphasized by the superscript $j$, and that the variables are estimates of individual well-being rather than utility. The distinction between well-being and utility, which is discussed in section 5, is not essential to the model.

Several forms of the aggregation function $V$ have been proposed. One type of inequality aversion is represented by the minimax form of $V$. A utilitarian will argue for a linear form of $V$ (see, for instance, Harsanyi (1955). Epstein and Segal (1992), on the other hand, argue that linear aggregators are inappropriate under some contingencies, typically in comparisons of states involving different probabilities of achieving indivisible goods (for instance, life or death) for different individuals. The choice of aggregation function, thus, is a judgment involving important issues such as fairness and equality, on which there seem to be little agreement.

As pointed out by Hume, factual statements can be tested against empirical data, whereas this is impossible for normative claims. This has lead some empiricists to the conclusion that it is of no use to argue about normative statements. In discussing this

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7This is often called Rawlsian, since Rawls (1972) argues for the minimax principle. However, Rawls argues for minimax applied to primary social goods, not to well-being.
point, Sen (1970) makes a distinction between basic and non-basic judgments. A judgment is basic to a person if there are no circumstances in which the person would disagree with the judgment. If a judgment is non-basic, it is an empirical question whether or not the contingencies of the judgment is in fact fulfilled. A claim that a statement is non-basic can be verified simply by finding an instance in which the statement does not apply. However, as Sen points out, the claim that a statement is basic can never be proved, because there always remains the possibility that it is contingent upon some other assumption which one has not thought of yet. It is thus not obvious that there are basic judgments in all value systems.

Elster (1986) discusses the differences in focus between economic social choice theory on the one hand, and Habermas’ theory of “Discourse ethics” on the other. The former is occupied with trying to aggregate individual preferences into social preferences, whereas in the latter approach individual preferences are changed through a process of discourse. Habermas claims that normative statements can be justified, but only through universal acceptance. This may be interpreted as a belief that everyone ultimately shares the same basic judgments, if we just go “deep enough”. If this were the case, disagreements on social welfare judgments could be settled by resolving disagreements regarding the factual premises.

For practical purposes, however, it seems quite unrealistic to reach such a state of universal agreement, even if disagreements were in fact caused only by diverging beliefs about factual circumstances. The reason is, of course, that even if people do in fact share the same basic judgments, the contingencies of non-basic disagreements might be too complex to be resolved without prohibitive costs. Hence, whether or not we believe that people ultimately share the same basic judgments, we will for most practical purposes have to face the fact that decision makers disagree on the social desirability of a proposed policy; and further, this disagreement will not necessarily be resolved by providing additional factual information.

We assume that the utility or well-being functions entering the welfare functions are cardinal and interpersonally comparable. Our model does not require that cardinal interpersonally comparable well-being is measurable, only that individuals possess evaluation functions, which will be further discussed in the next section.

In spite of the wish to leave ethical considerations for the desicion makers, we have restricted the possibility of such considerations in an important way. For simplicity we
assume that only individual well-beings enter the welfare functions. Many people would maintain that people's well-being cannot be regarded as the only information needed to rank social states. Respect of human rights, religious considerations, the extent of individual freedom; these are variables which may be important to many people per se, not only because they may contribute to individual well-being. We do not claim that variables other than well-being are not important; but inclusion of such variables would require a separate analysis. Our analysis supports a claim that information aimed at facilitating welfare judgments should be more descriptive, and our crude guess is that the inclusion of additional variables in the welfare functions will reinforce this argument.

4 Evaluating individual well-being

Data about an individual's choice among a set of alternatives will only reveal information about his ordinal preferences. Arrow's (1951) impossibility theorem has confirmed that ordinal information is not sufficient to determine a social welfare function, and the theorem's robustness to changes of the assumptions has demonstrated that it is difficult to avoid interpersonal comparisons of utility in welfare judgments. (See e.g. Sen (1979), Kelsey (1987) and Starret (1988)). Cardinality alone is not sufficient, as shown in Sen (1970, Ch.8*, Th.2). There are some alternative routes to circumventing the Arrow dilemma, such as relaxing the axioms of unrestricted domain or universal scope, but these seem unattractive in many applications.

A central element of the model above is the judgments of individual well-being. In this section we take a closer look at the assumption about individual evaluation functions; its motivation, interpretation, and implications.

4.1 Comparisons of well-being

Even though interpersonal comparisons of well-being has been recognized to be essential to welfare measurements, the possibility of making such comparisons is disputed. That we actually do make such comparisons is much less controversial, as illustrated by the quotation from Pollak in the introduction of this paper.

The problems of measuring interpersonally comparable well-being arises at least partly from the fact that there is a central element of subjective judgement in measurement of well-being. Since we do not know how do describe this variable in an objective manner,
our approach is to recognize the subjective character of such judgements, and concentrate on providing the information needed to actually make subjective judgments. We simply assume that people do make interpersonal comparisons of well-being. Assumption 1 implies that given information on an individual i’s characteristics, and the amount of private and public goods available to her, any individual j is able to make up his mind as to what he thinks i’s well-being is.

The functional form of the evaluation functions is similar to the utility functions used in the theory of “extended preferences” (see, for example, Arrow (1977) or Harsanyi (1987)). However, the evaluation functions treat $\alpha_i$ as causes of preference and not as objects of preference (see Broome (1993)). In other words, if $\nu_j^i(X; \alpha_1) > \nu_j^i(X; \alpha_2)$, then person j thinks that person 1 is better off in social state X than person 2, but it does not necessarily mean that j would prefer to be 1 rather than being 2. We do not require that people have preferences over causes of preferences.8

How people form their evaluation functions is a psychological issue and we will not pursue it at any length here. We will simply assume that the functions $\nu_j^i$ are defined for all relevant alternatives; people are able to make up their minds as to what they think other people’s well-beings are or will be, even though they may sometimes feel that their opinion might not be well-founded.

Such functions may not be constant over time. Rather, it seems reasonable to assume that as individuals gather more information about psychological issues, and as they gain more experience in life and meet more people, the evaluation functions will change. It also seems likely that the evaluation functions will change as society changes and different attitudes become fashionable. A quote from Harsanyi (1955) may illustrate this: ”We tend to assume that the average man finds greater satisfaction in solving mathematical puzzles because, allegedly, men in general have greater abstractive ability than women.” We believe this tendency has changed quite a lot during the period since 1955.

Our guess is, nevertheless, that this process is likely to be fairly slow, so that taking the

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8For instance, consider a society where income is equally distributed. Imagine also that older people have a higher marginal utility of income than young people do. We might believe that in this society, getting older increases an individual’s utility, but this is not the same as having a preference for being old to being young, ceteris paribus. Note, however, that some variables might be both causes and objects of preference; like, for example, the characteristic of being beautiful. In that case the variable should be included in both $X$ and $\alpha_i$. In this paper $X$ is defined rather narrowly, but individual income can be included both in $X$ and in $\alpha_i$.  

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functions as given at the specific point of time when the welfare evaluation is to be made is appropriate.\footnote{Still, one should perhaps not disregard the possibility that in instances where the policy decision to be made involves highly unfamiliar events, psychological information ought to be provided to help decision makers adjust their evaluation functions.}

4.2 Anonymous well-being judgement

A crucial point in Kant's moral philosophy is that a valid moral claim must apply universally. Any moral claim must be derived from a principle that is defendable as a universal principle. Violation of assumption 1 is clearly at odds with Kantian moral philosophy, but our assumption is considerably weaker than the Kantian requirement. The assumption rules out that \( j \) can exaggerate the intensity or level of a specific person \( i \) (e.g., \( i = j \)) without doing the same to all persons with the same characteristics. In other words, the egoist is not allowed to explicitly exaggerate the effects of a project on his own well-being; however, he is still perfectly free to give special treatment to people who have two freckles on their left elbow (like himself), are born in 1955 (like himself), and have two small children (like himself).

This use of evaluation functions requires anonymity in the sense that it rules out the possibility that individual \( i \) is given special treatment, \( \omega_i(X) = \nu_i(X; \alpha_i, i) \). With no extra assumptions on \( \alpha_i \) such a requirement is empty, since a complete identification of \( i \) may be included in \( \alpha_i \).

For the moment we assume that the information to be included in \( \alpha_i \) is restricted and does not include complete identification. There is then still the question of whether anonymous well-being judgment is a plausible description of how people would think. The anonymity assumption may, however, be interpreted normatively. The purpose of the paper is to identify the information required to make a subjective welfare evaluation. We may then choose to proceed as if people were making anonymous well-being judgments, thus not taking into account the information requirements of those who wish, for instance, to put special emphasis on their own situation without giving the same treatment to another person in a similar position. This is, of course, somewhat in conflict with the spirit of this paper, since we would like to leave as much of the ethical considerations as possible to the decision maker. The normative restriction we are talking about here is, however, probably not so controversial.
Note also that we do allow the possibility that people put special emphasis on characteristics they possess themselves, as long as they apply the same principles to everybody else having the same characteristics.

### 4.3 Restricting characteristics information

In practice, $\alpha_i$ may consist of a great many elements. To arrive at operational sufficient welfare indicators, we will therefore have to restrict the amount of information included in $\alpha_i$. How should this restriction be made? Are there elements of $\alpha_i$ that should be ignored?

If the population consisted of a few homogenous groups, this question would be easy to answer; we could then include only those characteristics which distinguished the groups from each other, and proposition 1 would apply, yielding simple and straightforward sufficient welfare indicators.

Propositions 2 and 3 approach the problems of a more complex world, but, not surprisingly, the results are somewhat negative. Proposition 2 asserts that we can facilitate the evaluation by using a two-step procedure. This does not necessarily help us much, though; either the groups are large and inhomogenous, and the welfare evaluation for each group becomes difficult, or we would have to work with very small homogenous groups. Even if we divide the population into very small groups, *some* restrictions will nevertheless have to be made, since all individuals differ to some extent. Moreover, Proposition 2 states that the two-step procedure is not possible for all functional forms of $V^j$. Thus, even if we disregard the latter problem, we somehow have to cope with the question of how much detailed information a person can perceive, or is willing to pay attention to, before the information works more to confuse him than to enlighten him.

Proposition 3 states that if there is a representative individual for each group, then the task of identifying locally sufficient welfare indicators can be approached as if the population consisted of homogenous groups. The question of identifying the representative individual of each group would probably be controversial, however. For example, a person who has a Rawlsian welfare function would presumably choose the worst-off individual within each group as the representative, whereas the utilitarian would choose some kind of "average" person. The ethical problems of choosing a representative individual is exactly the starting point of this paper; if one accepts the idea of choosing such a representative, why not just choose a representative for the whole population and use his or her well-being as the measure
of social welfare? We believe, however, that even if the procedure of aggregating individuals into broad groups implies that some ethical considerations must be made, implicitly or explicitly, this is better than not giving any information at all about the differences between people.

One possible approach to arrive at criteria for aggregating people into groups, thus restricting characteristics information, is the view taken by Scanlon (1991); ‘...to construct a more concrete conception of welfare in terms of particular goods and conditions that are recognized as important to a good life even by people with divergent values’ (emphasis added). Scanlon proceeds to refer to John Rawls’ "A Theory of Justice" (Rawls 1971) as the best known attempt to carry out such a strategy. He concludes: ‘Giving up the idea that value judgments can be avoided altogether allows us to make, within moral argument, the kind of interpersonal comparison mentioned earlier. We can still pursue the aim of Neutrality by basing our moral arguments on a conception of the elements that are important in making a life good that is at least widely shared.’

The challenge will then be to determine whether there is a wide consensus in society of what is the ‘elements that are important in making a life good’. Or, to translate the idea into our context, which characteristics are commonly perceived to be most important for the evaluation of someone’s well-being. If there is such a consensus, the task of choosing the representative individual, or choosing which characteristics to include in the description of \( \alpha \), becomes much easier.

Note that there is a similar problem on how detailed information to provide about the social states. Note moreover that restrictions on \( \alpha_i \) may be important to the description of the social states \( X \). The difference between two social states \( X \) and \( Y \) as perceived by \( i \) may depend upon the set of characteristics \( \alpha_i \). Avoiding value judgments altogether is at this point impossible.

The issue of restricting the amount of information on characteristics will have to be subject to further considerations. Our preliminary answer is that the analyst will have to select the most important features to be reported in each case. This may be done by preparatory research, trying empirically to identify the most important causes of preference relevant to the project under consideration. The idea of Scanlon cited above could be applied by reporting the elements of \( \alpha_i \) that are widely held to be important in making a life good.
5 Perception of the concept of well-being

5.1 Utility and well-being

The classical utilitarians took the term "utility" to represent the individual's happiness or fulfillment of desires. Today the most popular interpretation of 'utility' among economists is as a numerical representation of choice behaviour.\(^{10}\) According to this approach, if the individual chooses \(A\) over \(B\), then by definition a higher utility is attached to \(A\) than to \(B\).

In this paper, the concept of utility is taken to mean such a numerical representation of choice behaviour. As have probably been noted, however, we have not used the term "utility", but instead "well-being" of the variables entering the social welfare functions. The distinction between utility and well-being is not essential to our model, but provides one additional reason why the subjectivistic approach we have chosen may be defended. We will return to this below.

The well-being of an individual \(i\) is meant to describe the extent to which individual \(i\)'s personal interests are fulfilled. If maximization of well-being is the only motive underlying choice behaviour, then utility and well-being will coincide. There are, however, reasons to claim that well-being should not be seen as utility. This is due to what Sen (1985b, 1987) calls the *agency aspect* of utility. Sen argues that individuals act not only to pursue their own well-being, but sometimes also to pursue other goals, for instance duty to some other person or cause. The following quote illustrates the point.

"Once upon a time two boys found a cake. One of them said, 'Splendid! I will eat the cake. 'The other one said, 'No, that is not fair! We found the cake together, and we should share and share alike, half for you and half for me. 'The first boy said, 'No, I should have the whole cake!' Along came an adult who said, 'Gentlemen, you shouldn't fight about this: you should *compromise*. Give him three quarters of the cake.'\(^{11}\)

Among the different sharing options, the first boy chooses all for him and nothing for the other, while the other boy chooses an equal sharing. Thus, when utility is defined the way we did here, the first boy has the highest utility when he gets the whole cake, whereas the second boy has the highest utility if he only gets half the cake. A natural interpretation

\(^{10}\)For a discussion of the different interpretations of utility and their use, see Sen (1985a).

\(^{11}\)Smullyan (1980, p.56), as cited in Elster (1986).
of the situation described (though not the only possible one) is that the first boy expresses his purely egotistic preferences, the well-being he derives from eating one whole cake is higher than the well-being derived from eating half a cake. A reasonable interpretation of the second boy's statement, however, is that even though he too would perhaps like to have the whole cake himself, he is concerned about fairness. In fact, the second boy may be expressing his individual social welfare function, or his social preferences, where both boys' well-being enter, whereas the first boy expresses only his own well-being. In that case it is not obvious that their statements should be treated equally when used as inputs in a social welfare judgment.

Obviously, it is not an easy task to define exactly if someone is acting out of personal or social interests (or both). Some economists would state that it does not matter what the motives are, since regardless of the motives, respecting consumer sovereignty requires us to accept that the consumer himself is the best judge as to what is good for him.\textsuperscript{12} However, including our concern for others in a social welfare function may lead to a kind of double counting of the beneficiary's benefits (see Milgrom (1992) and Johansson (1992)). The reasoning of the adult in the example above is an example of such double counting.

One may argue that the discrepancy between utility and well-being disappears if the model is sufficiently extended, and that people's concern for each other is ultimately caused by self-interest. One example of this idea is the prisoner's dilemma: If it is assumed that the game is played only once, cooperative behaviour would seem to imply an agency aspect; but when the game is to be repeated an infinite number of times, cooperation may be a perfectly rational way to maximize own well-being. Thus, if we observe that people cooperate in a one-shot prisoner's dilemma game and conclude that people act as agents, our model may actually be misspecified (or the participants have misinterpreted the game) and the participants may in fact be acting only out of self-interest, regarding the game as a repeated one.

Even if all behaviour might in a similar way be ultimately motivated by maximization of well-being alone, we will for practical purposes have to restrict the model. Hence, it might still be necessary to separate utility and well-being within the context of a particular\textsuperscript{12}Hausman and McPherson (1993) argue that paternalism is to induce someone to act differently from what he otherwise would, with the justification that the alternative action is to his own benefit. Making judgments about well-being, on the other hand, is not paternalism, since such judgments can be made without inducing anybody to act in a specific way.
tractable model. Sen (1987) provides an extensive discussion of the relevance of a separation of the two concepts.

5.2 Integrated judgment of well-being and social welfare

While the possible distinction between utility and well-being is discussed above, several other interpretations of well-being are conceivable. If people have different opinions of what a good life is, they will probably also have different opinions about what the variables to be included in the social welfare function should be. The approach of leaving the judgment of individual well-being to the decision makers themselves leaves open the possibility that people actually use quite different notions of well-being.

We are not certain, however, that this is merely a disadvantage of the approach. If people use different interpretations of well-being, this will of course contribute to disagreement in the judgments \( \omega_j^i \). But then this difference in interpretation might possibly be taken account of in the welfare function \( V^j \). Such correction is not possible if the decision maker is only presented with data on well-being (or utility) as judged by someone else.

Actually, it can frequently be very difficult to separate one’s judgment on someone’s well-being from one’s judgment on social welfare. For example, if I believe that the poorer people’s interests should count more in a cost-benefit analysis, this may be explained in two ways: 1) I believe in classical utilitarianism, but think that poor people get larger marginal increases in well-being when their income change than rich people do, or 2) I believe that everybody gets the same marginal change in well-being when income changes, but I think poor people’s interest should count more in a judgment of social welfare.\(^\text{13}\) It is not obvious that decision makers are able to distinguish their judgment on individual well-being from judgments on social welfare. This question is particularly relevant if it is not obvious to the decision maker how to interpret the concept of well-being.

The interpretation of \( \omega_j^i \) as the result of a subjective judgment made by \( j \), rather than some ”objective” representation of \( i \)’s well-being, actually relieves our approach from one serious problem faced by utilitarian philosophy. The problem concerns the interpersonal comparison of well-being evaluated behind a ”veil of ignorance”, a position which is frequently referred to: If the decision maker does not know whom she will become, and is thus

\(^{13}\)The assumption of anonymous aggregation does actually not allow the view that poor people’s interests should count more. It allows, however, that the interests of people who have relatively less well-being in status quo should count more that the interests of other people.
stripped of her own personality, what basis does she have to make any judgments at all?\textsuperscript{14} The theory of extended preferences encounters this problem, since it is assumed that people have preferences over which person they would like to be. Even though the functional form of $\nu^j$ in our approach is quite similar to the extended utility function employed in the theory of extended preferences, the difference in interpretation is therefore important.

One could actually regard the $\nu$ functions as representations of the decision maker's view on the morally relevant aspect of well-being. For instance, we may take the view that if a person is a sadist we should not attach any weight to this in estimating his well-being in a state where he tortures somebody (this example is taken from Sen (1979)). What the morally relevant aspects of well-being are, is then a moral philosophic question. If this choice is left to the analyst, it might not be possible for the individual decision maker to translate the information given into the concepts he himself find most relevant.

Harsanyi (1955) takes another view, stating that “interpersonal comparisons of utility are not value judgments based on some ethical or political postulates but rather are factual propositions based on certain principles of inductive logic.” (pp. 319-320.) We believe our approach have room for both an interpretation of the evaluation functions according to Harsanyi's view and as morally relevant well-being, as long as every one individual $j$ applies only one $\nu^j$ function when evaluating different people's well-being.

5.3 Anonymous judgment and ethical considerations

Some people have a view on justice which implies that if someone has caused his bad fortune himself, then his loss of well-being because of the self-induced event should not count in a social welfare function. A common attitude thus seems to be that personal characteristics which are caused by the individual's own choices should be treated differently from characteristics of which he does not have - nor has previously had - any control. This observation may suggest that only characteristics which the individual does not control should be included in $\alpha_i$, thus helping us to restrict characteristics information.

A problem with this approach is of course that it implies a moral view which will not be shared by everyone, and so is at odds with the intentions of sufficient welfare indicators. We think, therefore, that information on self-induced characteristics should not be excluded a priori. Moreover, the assumption of anonymous aggregation makes it difficult to fit such

\textsuperscript{14}For a discussion of this problem, see Rawls (1971), pp 173-175, and Griffin (1986), pp 108-113.
opinions of justice into our framework. Anonymous welfare judgment does not allow us to use the welfare function to "punish" someone for a previous act. Identification of the person could be made by adding as a characteristic "having done this or that". However, characteristics do not enter the individual social welfare functions directly, only the evaluation functions. The only possibility of applying our framework in the case of such views on justice is, therefore, to apply the notion of "morally relevant well-being" as discussed above.

The problems of applying the framework to specific ethical views can probably be ascribed to the failure to allow for other variables than well-being in the social welfare functions. If information on rights and obligations were allowed, a broader range of ethical views would be permitted within our framework. An interesting theme for further investigation, though beyond the scope of this paper, is to explore the implications for the propositions of including such information in the welfare functions.

6 The role of ordinal utility information

So far in the paper, we have not made any restrictions on whether j's judgment of i's well-being \( \omega_i^j \) should conform with i's ordinal preferences. We believe that there are several reasons for not requiring this.

The first reason is that there might be a discrepancy between utility and well-being, so that data on i's actual choices may not represent ordinal information on i's well-being. The second reason is that the evaluation functions are supposed to describe the kind of well-being evaluations people make in everyday life, and it is easy to find examples (for instance, when it comes to the issues of drugs and alcohol) where j's evaluation of i's well-being does not correspond to the observed choices made by i. Finally, even if people tried to respect ordinal utility information when evaluating other people's well-being, they would frequently be faced with situations where information about i's ordinal preferences was not available. In that case they would have to guess, and their guesses might not be correct.

This means that, quite in contradiction to the more traditional economic approaches to the ranking of social states, ordinal utility information plays no essential role in our model. In the next section, however, we will discuss how ordinal utility information could in some instances be used to determine unobservable parts of \( \alpha_i \). In section 6.2 we will discuss the

\[ ^{15} \text{For a discussion of this restriction, see Hylland (1991).} \]
role of ordinal utility information if we, as a special case, assume that \( j \)'s judgment of \( i \)'s well-being is made in accordance with \( i \)'s ordinal preferences.

6.1 Identifying interest groups

When some of the variables or parameters in the subjective social welfare functions are unknown, additional information will be valuable. It is likely that maximization of well-being is an important motivation in many occasions, and in some cases probably the most important. The observed choices of individual \( i \) may thus be important additional information.

To determine the functional value of the evaluation functions \( \nu^j \), data on individual characteristics is needed. In principle, we need information on all characteristics which can be regarded as causes of preference. In practice some of this information will be unknown, some of it may even be unobservable.

We will now assume that the population can be divided into a finite number of groups, such that members of each group \( k \) possess similar characteristics \( \alpha_k \). Call these groups "interest groups". Assume further that we know the characteristics associated with each group, and the total number of individuals in society, but we do not know which person belongs to which group, and hence not the size of the groups.

For simplicity, suppose there are only two interest groups, and as an example, let us name them "nature lovers" and "materialists". We do not have data on personal characteristics, only on the social state, which we will in this case limit to a description of the income distribution and the provision of a common good, environmental quality. We want to consider whether or not to undertake a project which will improve environmental quality. Suppose the costs of the project are equally distributed among individuals.

We might then obtain ordinal information by conducting a contingent valuation study, measuring people's willingness to pay for the project.\(^{16}\) Then information about each individual's willingness to pay, combined with data on his or her income, could be used to infer which interest group the individual belongs to: If two persons have equal income and have

\(^{16}\)Alternatively, one could measure "willingness to accept" as opposed to "willingness to pay". These two measures will in general not coincide and need not be of the same magnitude (Hanemann 1991). This difference might well be important if one interprets the sum of individual WTP or WTA as a measure of the change in social welfare due to the project, as is frequently done. However, we are not considering aggregation of individual WTP here; rather, we are concerned about relative differences between persons. In this context the difference may be less important.
differing marginal willingness to pay, they belong to different interest groups.

Note, however, that this does not imply any defense for using aggregate willingness to pay as a welfare indicator. The assumptions needed for that is discussed below. Further, using ordinal information to identify which group an individual belongs to will not be possible under all alternative assumptions about \( \nu \) and all possible projects. Other information than willingness to pay may also provide an equally useful tool for group identification, for instance a survey where people are asked to state their opinions rather than amounts of money.

6.2 Willingness to pay as a welfare measure

The conclusion above does not rest on the assumption that judgments of well-being should correspond to the ordinal preferences of the individual in question. Having determined that some individuals belong to the group "materialists", the decision maker is perfectly free to think that an improvement in environmental quality is particularly important to those people. In this subsection, however, we will assume that the judgments \( \omega_i \) are made in accordance with \( i \)'s ordinal preferences.

Most practical applications of cost-benefit analyses ignore the welfare weights and simply compare total willingness to pay with total costs. Recognizing the lack of a theoretical foundation for such a procedure, it has been suggested to report total willingness to pay and total costs for subgroups of the population. Under which conditions will this procedure be consistent with the model suggested in this paper? Under what conditions can aggregate willingness to pay for a public project, reported separately for several subgroups of the population, be used as a locally sufficient welfare indicator?

Proposition 3 only required a linearization of the aggregator \( V^j \). We now consider linearizing the individual well-being too. For simplicity, suppose the private good \( x_i \) is money. Then the marginal willingness to pay \( p_i \) for the public good \( y \) is

\[
p_i = \frac{\partial \omega_i}{\partial y} \frac{\partial \omega_i}{\partial x_i}
\]

If well-being is interpreted as utility, the marginal willingness to pay can in principle be determined from the ordinal preferences over pairs \((x_i, y)\), and \( p_i \) will be independent of \( j \). Linearizing the welfare function gives

\[
dW^j = \sum_{i=1}^{I} \beta^j_i (p_i dy + dx_i)
\]
where the welfare weights are \( \beta_i^j = \frac{\partial V_j}{\partial \omega_i^j} \). This is the well-known cost-benefit rule with welfare weights, see Dreze and Stern (1986) or Starret (1988).

Note first that if \( \omega_i^j(x, y) \) is completely determined by the characteristics \( \alpha_i \), and if all individuals in the same group has the same income, then the marginal willingness to pay would be equal for all individuals in the group. In this case, proposition 1 applies. For practical purposes the interesting case is when willingness to pay is different within the group. In this case willingness to pay either reveals information about income or about unobservable characteristics.

Except under very strong conditions, this information would be important to determine the welfare weights. Thus welfare weights has to be a function of willingness to pay, which is inconsistent with the procedure of unweighted aggregation within each group. Proposition 4 states that such unweighed summation can only yield a sufficient welfare indicator under very strict and probably unrealistic assumptions about the functions \( V^j \) and \( \omega^j_i \).

**Proposition 4** Suppose the characteristics of \( i \) consist of an observable part \( \gamma_i \) and an unobservable \( \zeta_i \), i.e. \( \alpha_i = (\gamma_i, \zeta_i) \). Suppose furthermore that for all \( i \) and \( j \), the welfare weights \( \beta_i^j \) are independent of the distribution of income \( x_i \) and unobservable characteristics \( \zeta_i \), within the group. If \( \frac{\partial V_j}{\partial \omega_i} > 0 \), \( \frac{\partial^2 V_j}{\partial x_i^2} \leq 0 \), and \( \frac{\partial^2 \omega}{\partial x_i^2} \leq 0 \), then for all \( i \) and \( j \), the welfare function must be purely utilitarian

\[
V^j = \sum \omega_i^j \text{ for all } j,
\]

and the well-being must be quasi linear of the form

\[
\nu^j((x, y); \alpha_i) = c^j(y; \gamma_i)x_i + \phi^j(y; \alpha_i).
\]

The requirements that \( V \) is increasing and that \( V \) and \( \omega \) are concave seem reasonable. A violation of \( \frac{\partial^2 V_j}{\partial \omega_i^2} \leq 0 \) would mean that an extra unit of well-being for the better off individuals are given higher weight than the changes for the worst off. The proposition proves that the rule that the worst off should be given higher weights is not consistent with constant welfare weights. Similarly \( \frac{\partial^2 \omega}{\partial x_i^2} \leq 0 \) rules out that the rich ones are more effective in making utility out of a dollar. The proposition then states that the opposite judgement, that the poorer individual would get most utility out of an extra dollar, is inconsistent with the assumption on constant welfare weights.

If well-being and utility do not coincide, then not even ordinal information on well-being is available from market data. In that case, an approach based on ordinal utility information
would require not only assumptions on cardinality and interpersonal comparison, but also about how the ordinal utility information should be interpreted in terms of well-being.

While the assumption of congruence of the two concepts of utility and well-being may be a fair approximation in the case of ordinary market behavior, our guess is that divergence between them may be more important in the context of valuation of public goods. When asked to value a public good, it may not be clear to the respondent whether he is supposed to apply his individual social welfare function ("what would I choose if I was a dictator") or whether he is only supposed to value the public good according to his own private interests. This may be a problem in the interpretation of results from contingent valuation studies.\(^{17}\)

### 7 Concluding remarks

Most economic approaches to social welfare evaluations are based on ordinal utility information, applying varying assumptions about how interpersonal comparisons should be made and how utilities should be aggregated into a social welfare function. In this paper, we recognize that there are essential elements of subjectivity in evaluations of social welfare, and that people do not always agree. Thus, our approach has been to incorporate these subjective elements into the model. Our aim has been to identify sufficient welfare indicators; i.e. aggregated information which enable decision makers to evaluate a project or social state according to their own ethical beliefs.

The welfare indicators we have developed focus on information about different groups of the population; the size of these groups and their characteristics. Ordinal utility information, which is central to most welfare theoretic works, takes a much less prominent place in our model. It may be used as an indicator of which group an individual belongs to, but is not used directly in the evaluation of social welfare.

Not surprisingly, the results are negative in the sense that if the heterogeneity of the population is to be taken fully into account, the sufficient welfare indicators become exceedingly complicated. If the population can be divided into homogenous groups, however, then the number of individuals in each group becomes a sufficient welfare indicator. The same holds if one is allowed to divide the population into a limited number of groups, and choose a representative individual from each group. The choice of representative individual...\(^{17}\)

\(^{17}\)This was discussed in an interesting lecture by Amartya Sen at the Economy and Ecology Conference in Oslo, June 1993, but unfortunately no written proceedings of the conference have been published.
may depend on ethical views; hence our model does not yield indicators which are ethically completely neutral. This would nevertheless be a too demanding goal; no aggregation is absolutely neutral. For a decision maker who wants to keep track of the moral basis of the welfare evaluation, descriptions of the characteristics of a broad partition of the population may still be better than a number telling her whether a project is socially desirable or not.

References


A Proof of proposition 1

Let \((\alpha_1, \ldots, \alpha_I)\) be the true vector of characteristics. For any \(n\) let \((\hat{\alpha}_1, \ldots, \hat{\alpha}_I)\) be an arbitrary vector of characteristics consistent with \(n\), and let \((\hat{x}_1, \ldots, \hat{x}_I)\) be the corresponding vector.
of private goods. By the assumption about \( n \), there exist a pivotation such that \( \hat{\alpha}_i = \alpha_{\pi(i)} \), and \( \hat{x}_i = x_{\pi(i)} \). Thus by anonymous welfare judgments, \( \nu^i((\hat{x}_i, y), \hat{\alpha}_i) = \nu^i((x_{\pi(i)}, y), \alpha_{\pi(i)}) = \omega^j_{\pi(i)}((x_{\pi(i)}, y)) \), where \( \omega^j_{\pi(i)}(x_i, y) \) is the ‘true’ judgment that \( j \) would make about \( i \) in state \( X \), with complete information. Hence by anonymous aggregation

\[
V_j(\nu^j((\hat{x}_1, y), \hat{\alpha}_1), \ldots, \nu^j((\hat{x}_I, y), \hat{\alpha}_I)) = V_j(\omega^j_1(x_1, y), \ldots, \omega^j_I(x_I, y)) = W^j_j(X).
\]

Let \( F^j_j(X, n) = V^j_j(\nu^j_j((\hat{x}_1, y), \hat{\alpha}_1), \ldots, \nu^j_j((\hat{x}_I, y), \hat{\alpha}_I)) \). \( F^j_j \) is well defined since it is independent of the particular choice of \( (\hat{\alpha}_1, \ldots, \hat{\alpha}_I) \) by anonymous aggregation. Since \( F^j_j(X, n) = W^j_j(X) \), this completes the proof.

**B Proof of proposition 4**

By assumption

\[
\frac{\partial V}{\partial \omega_i} \frac{\partial \omega_i}{\partial x_i} = \beta
\]

where \( \beta \) is a constant independent of \( x_i \) and \( \zeta_i \). We have dropped the superscript \( j \) to simplify the notation. Differentiating (5) with respect to \( x_i \) we get

\[
\frac{\partial^2 V}{\partial \omega_i^2} \left[ \frac{\partial \omega_i}{\partial x_i} \right]^2 + \frac{\partial V}{\partial \omega_i} \frac{\partial^2 \omega_i}{\partial x_i^2} = 0.
\]

By assumption both terms are non-positive, and hence both must be zero. Furthermore, since marginal utility of income is strictly positive, \( \frac{\partial^2 V}{\partial \omega_i^2} = 0 \), and we concludes that the welfare function is linear in each individual well-being. Since the welfare weights should be independent of income distribution within the group, differentiating with respect to \( x_k \) for \( k \neq i \), gives \( \frac{\partial^2 V}{\partial \omega_i \partial x_k} = 0 \). This proves that \( V = \sum_i a \omega_i \) for some positive scalar \( a \). Normalizing to \( a = 1 \) proves the first part of the proposition.

Since \( \frac{\partial V}{\partial \omega_i} > 0 \), we also concludes that \( \frac{\partial^2 \omega_i}{\partial x_i^2} = 0 \), and hence individual well-being is linear in \( x_i \). Differentiating (5) with respect to \( \zeta_i \) we get

\[
\frac{\partial^2 V}{\partial \omega_i^2} \frac{\partial \omega_i}{\partial x_i} \frac{\partial \omega_i}{\partial \zeta_i} + \frac{\partial V}{\partial \omega_i} \frac{\partial^2 \omega_i}{\partial x_i \partial \zeta_i} = 0.
\]

Since \( \frac{\partial^2 V}{\partial \omega_i^2} = 0 \) we conclude that \( \frac{\partial^2 \omega_i}{\partial x_i \partial \zeta_i} = 0 \). Thus \( \omega \) and hence \( \nu \) must be of the required form

\[
v((x_i, y); \gamma_i, \zeta_i) = c(y, \gamma_i)x_i + \phi(y; \gamma_i, \zeta_i).
\]

This concludes the proof.
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