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Discussion Paper

Research Department, Central Bureau of Statistics, Norway

No. 89

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Engel Functions, Panel Data, and Latent Variables

– with Detailed Results

by

Jørgen Aasness, Erik Biørn and Terje Skjerpen

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Abstract

A system of consumer expenditure functions is estimated from Norwegian household budget data. Specific features of the approach are: (i) Panel data from individual households are used, which offer far richer opportunities for identification, estimation and testing than cross section data. (ii) Measurement errors are carefully modelled. Total consumption expenditure is modelled as a latent variable, purchase expenditures on different goods and two income measures are used as indicators of this basic latent variable. The usual assumption of no measurement error in total expenditure is clearly rejected. (iii) The distribution of latent total expenditure across households, and its evolution over time, is estimated and important properties tested. (iv) The distribution of individual differences in preferences, represented by individual time invariant latent variables, are modelled, estimated, and tested. (v) We test the hypothesis that preferences are uncorrelated with total consumption expenditure, which is basic to virtually all cross section studies of consumer demand functions.

Keywords: Consumer demand, Engel functions, panel data, preference distributions, latent total expenditure, measurement errors, household expenditure surveys.

* Forthcoming in *Econometrica* except for the detailed results in Appendix B. The paper is a revision, with many new results, of Discussion Paper no 41.

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1. INTRODUCTION

Systems of expenditure functions for consumption commodities, including systems of Engel functions, have been analyzed in a substantial number of scientific papers over the years. (See Deaton (1986) and Blundell (1988) for recent surveys.) Econometric information on expenditure systems is interesting and important for macro-econometric model building, analyses of distributional policies, and several other purposes. The interest often focuses on Engel elasticities and parameters representing the effect on consumption of demographic and socioeconomic characteristics. The vast majority of existing empirical analyses of systems of Engel functions utilizes cross section data from a sample of households with an income variable considered as observed without error. Often no distinction is made between income and total consumption expenditure. However, following the classical articles of Summers (1959) and Liviatan (1961) (see also Friedman (1957) and Cramer (1966)), the problem of measurement error in total expenditure and in income has been recognized as important in analyzing data from household budget surveys. In Liviatan (1961), there is an example from an Israel budget survey indicating that neglecting random measurement errors in total consumption expenditure may bias the estimated marginal budget shares by more than 100 percent. The Norwegian household budget surveys are no exception, see Aasness (1990, p.215). An adequate modeling of measurement errors in total consumption expenditure seems to be important not only in order to avoid large biases in estimated Engel functions but also to assess the variability of preferences and the "true" total consumption expenditure in the population from which the sample is drawn.

The focus of the present paper is on the modeling of measurement errors in consumption in making inference on Engel functions from household budget data. The perspective is, in several respects, wider than in the literature referred to above. First, panel data with two observations from each respondent are used. It is well known that panel data in general offer a far richer opportunity for analyzing individual effects and for controlling for individual 'nuisance' variables than conventional data types (cf Mundlak (1978), Hausman and Taylor (1981), and Griliches and Hausman (1986)). Second, in order to allow for imperfect measurement of income and consumption, they are considered as latent variables. Third, the distribution of latent total consumption expenditure across households, and its evolution over time, is identified and estimated simultaneously with the demand

system. Fourth, individual differences in preferences, represented by individual, time invariant latent variables, are allowed for. An important purpose of the investigation is to quantify the distribution of these differences. Fifth, within this framework, an attempt is made to investigate the possible correlation between latent total expenditure and preferences. The availability of data with more than one replication makes it possible to test for such correlation. As remarked by Griliches and Hausman (1986,p.94), "in the panel data context, a variety of errors-in-variable models may be identifiable and estimable without the use of external instruments". See also Aigner et al (1984, section 3.10).

The paper represents an extension of previous research by Biørn and Jansen (1982) and Aasness (1990,Essay 5). In the former, using panel data, individual differences in consumption are analyzed by means of a complete demand system (including prices) with an error components specification of the disturbance vector, although with errors of measurement in income and consumption disregarded. The latter uses cross section data, thus neglecting the panel aspect, but focuses on the errors of measurement and identifies and estimates a distribution of latent total consumption expenditure across households simultaneously with a system of Engel functions. The present work integrates the two approaches, and extends them by, inter alia, incorporating information on observed incomes from tax records.

The paper is organized as follows. In section 2, we present the basic notation and the general model framework. Four 'dimensions' of the model framework are outlined, specific hypotheses are formulated, and the class of models applied in our empirical investigation is defined. Next, in section 3, the data and the inference procedure, implemented by means of the computer program LISREL 7, are discussed. The main empirical results are presented in section 4, focusing on the structure of measurement errors, the distribution of latent total consumption, the distribution of preferences, the correlation between preferences and total consumption, and the Engel functions. Finally, section 5 concludes and surveys the main empirical findings.

2. MODEL FRAMEWORK AND BASIC NOTATION

Let consumption be divided into I commodity groups and assume that a panel of H households is observed over T years. We specify a system of linear Engel functions

$$(1) \quad \eta_t = a_t + b\xi_t + Cz + \mu, \quad t = 1, \dots, T,$$

where η_t is a $I \times 1$ vector of expenditures, at constant prices, in year t , ξ_t is total expenditure, z is a time invariant $M \times 1$ vector of demographic variables, μ is a time invariant $I \times 1$ vector representing individual preferences attached to the I commodities (and other random effects reflecting unobserved time invariant household characteristics), and a_t , b , and C are matrices of coefficients of dimension $I \times 1$, $I \times 1$, and $I \times M$, respectively. The vectors η_t and μ and the scalar ξ_t are latent, the vector z is observable. Realizations of (η_t, ξ_t, z, μ) for different households are assumed to be independent and, for simplicity, the household subscript is suppressed. The time subscript on the constant vector a_t indicates that shifts in the expenditure functions over time are allowed for. Since, by definition,

$$(2) \quad \iota_1' \eta_t = \xi_t, \quad t = 1, \dots, T,$$

ι_1 denoting the $I \times 1$ vector of ones, the coefficient matrices and the preference vector will be subject to the adding-up restrictions $\iota_1' a_t = 0$, $\iota_1' b = 1$, $\iota_1' C = 0_{1M}$, and $\iota_1' \mu = 0$, 0_{1M} being the $1 \times M$ zero vector.

The $I \times 1$ vector of observed expenditures in year t is

$$(3) \quad y_t = \eta_t + v_t, \quad t = 1, \dots, T,$$

where v_t is a $I \times 1$ vector of measurement errors. (It may also include a vector of disturbances in the Engel functions (1), which cannot be empirically distinguished from the measurement errors.) In household budget surveys, the observed expenditures (y_t) will typically be represented by purchase costs during a relatively short period, while true expenditures (η_t) can be defined precisely with reference to a specific theory of consumer behavior. For a non-durable good, true expenditure could be the value of the consumption flow during the year, v_t representing stock changes in the registration period. For a durable good, true expenditure could, for instance, be the service value of the stock of the good during the period. The difference between the purchase value and the service value

in period t will then be a component of the measurement error.

From (1)-(3) it follows that the observed $I \times 1$ vector of expenditures satisfies

$$(4) \quad y_t = a_t + b\xi_t + Cz + \mu + v_t, \quad t = 1, \dots, T,$$

with observed total expenditure equal to

$$(5) \quad x_t = v_1' y_t = \xi_t + v_t, \quad t = 1, \dots, T,$$

$v_t = v_1' v_t$ being the aggregate measurement error. Formally, (4) says that y_t contains I indicators of the latent total expenditure ξ_t . We also assume that K additional indicators exist, represented by the observed $K \times 1$ vector w_t . Rather than considering w_t as an 'extraneous' vector of instruments for ξ_t relative to (4), we formalize the relationship as

$$(6) \quad w_t = d_t + e\xi_t + Fz + \lambda + \varepsilon_t, \quad t = 1, \dots, T,$$

where d_t , e , and F are coefficient matrices of dimension $K \times 1$, $K \times 1$, and $K \times M$, respectively, λ is a latent time invariant $K \times 1$ vector associated with the indicators (and playing, formally, the same role as μ in (4)), and ε_t is a $K \times 1$ vector of error terms. The coefficients and individual effects (λ) of (6), unlike those of (4), are unrestricted. Otherwise, the two equations are similar, so that formally, the vector (y_t', w_t') contains $I + K$ indicators of ξ_t .

The interpretation of the 'measurement equations' (6), of course, depends on the definition of w_t . The measurement equation of an income variable recorded for tax purposes may represent, on the one hand, the savings behaviour of the household, on the other hand, the definitions of taxable income(s) in the tax code as well as the 'tax paying behavior' of the household. Then λ reflects the thriftiness of the household and its attitude to tax avoidance and tax evasion. If the income variable were properly measured, the corresponding equation in (6) might be interpreted as an 'inverted consumption function', normalized with respect to income, but in general, (6) cannot be given the status as structural relationships with the same degree of autonomy² as (1). Rather (6) represents the reduced or semi-reduced form of a (possibly complex) structural model of the income and wealth distribution mechanism, the statutory tax system, and the spending, saving, and tax paying activity of the individual household. In the following, λ , like μ , will, for brevity, be denoted as a 'preference vector'. Similarly, we will use the term measurement error for both v_t and ε_t .

Let $\xi = (\xi_1 \dots \xi_T)'$, $v = (v_1' \dots v_T')'$, and $\varepsilon = (\varepsilon_1' \dots \varepsilon_T')'$, which have dimensions $T \times 1$, $TI \times 1$, and $TK \times 1$, respectively. We assume that the two composite vectors of 'structural' variables (g) and measurement errors (m),

$$(7) \quad g = (\xi', z', \mu', \lambda')' \quad \text{and} \quad m = (v', \varepsilon')',$$

are uncorrelated, but we allow for correlation within the vectors, specifying their covariance matrices, in partitioned form, as

$$(8) \quad \Sigma_{gg} = \begin{bmatrix} \Sigma_{\xi\xi} & \Sigma_{\xi z} & \Sigma_{\xi\mu} & \Sigma_{\xi\lambda} \\ \Sigma'_{\xi z} & \Sigma_{zz} & 0 & 0 \\ \Sigma'_{\xi\mu} & 0 & \Sigma_{\mu\mu} & 0 \\ \Sigma'_{\xi\lambda} & 0 & 0 & \Sigma_{\lambda\lambda} \end{bmatrix}, \quad \Sigma_{mm} = \begin{bmatrix} \Sigma_{vv} & 0 \\ 0 & \Sigma_{\varepsilon\varepsilon} \end{bmatrix}, \quad \Sigma_{gm} = 0.$$

Owing to the adding-up restriction (2), the columns of $\Sigma_{\mu\mu}$ and $\Sigma_{\xi\mu}$ add to zero. Zero correlation between the preference vectors (μ, λ) and the vector of observed demographic variables is assumed, in order to obtain a framework with completely identifiable models.³ On the other hand, the former are allowed to be correlated with latent total expenditure ($\Sigma_{\xi\mu}, \Sigma_{\xi\lambda} \neq 0$), which is, indeed, an interesting testable hypothesis.

Expressions (4), (6), (7), and (8) define the most general model framework under consideration in this paper. A lot of specific models can be estimated and tested with our data. We focus on specifications along the following dimensions:

E dimension: The marginal distribution of the latent total expenditure vector (ξ).

P dimension: The marginal distribution of the preference vectors (μ, λ) .

C dimension: The joint distribution of the preference and expenditure vectors, represented by their covariances.

M dimension: The contemporaneous covariances of the measurement error vectors (v, ε) .

Each dimension is "parametrized" and specific hypotheses are formulated. Tables I and II give an overview of the specifications in our empirical analysis.

We parametrize the E dimension by assuming that latent total expenditure evolves according to the generalized variance components process

$$\xi_t = q_{0t} + q_t(\chi + u_t), \quad t = 1, \dots, T,$$

where (i) χ is a permanent time invariant component of consumption, $E(\chi) = \Phi_\chi$, $\text{var}(\chi) = \sigma_{\chi\chi}$, (ii) u_t are volatile components representing individual mobility in the distribution, $E(u_t) = 0$, $E(u_t u_s) = \delta_{ts} \sigma_{uu}$ (δ_{ts} being the Kronecker delta), and (iii) q_{0t} and q_t are deterministic trend coefficients (where we, by convention and with no loss of generality, set $q_{01}=0$, $q_1=1$).⁴ The u 's are assumed to be uncorrelated with $(\chi, z, \mu, \lambda, v, \varepsilon)$. If q_{0t} and q_t are independent of t , then ξ_t is (weakly) stationary, otherwise, $\xi_t^* = (\xi_t - q_{0t})/q_t$ has this property. In any case, the process has a constant coefficient of autocorrelation given by $\rho(\xi_t, \xi_s) = \sigma_{\chi\chi} / (\sigma_{\chi\chi} + \sigma_{uu})$ for all t and $s \neq t$, and if $q_{0t} = 0$ for all t , its coefficient of variation is also constant, and equal to $(\sigma_{\chi\chi} + \sigma_{uu})^{1/2} / \Phi_\chi$. These properties seem reasonable, and this parametrization also opens for testing of interesting hypotheses about the consumption growth process. In matrix notation, the process reads

$$(9) \quad \xi = q_0 + Q(\iota_T \chi + u),$$

where $q_0 = (q_{01} \dots q_{0T})'$, $Q = \text{diag}(q_1 \dots q_T)$, and $u = (u_1 \dots u_T)'$. This implies the following restrictions on Σ_{gg} :

$$(10) \quad \begin{aligned} \Sigma_{\xi\xi} &= Q \iota_T \iota_T' Q' \sigma_{\chi\chi} + Q^2 \sigma_{uu}, \\ \Sigma_{\xi z} &= Q \iota_T \Sigma_{\chi z}, \quad \Sigma_{\xi\mu} = Q \iota_T \Sigma_{\chi\mu}, \quad \Sigma_{\xi\lambda} = Q \iota_T \Sigma_{\chi\lambda}. \end{aligned}$$

The P dimension is parametrized by noting that the Engel functions (1) can be interpreted as a complete system of demand equations derived from the linear expenditure system (LES). Assume that (1) is derived from

$$\eta_t = \gamma_t + \beta(\xi_t - \iota' \gamma_t),$$

where γ_t is a stochastic $I \times 1$ vector of 'necessity quantities' in year t , and β is a $I \times 1$ coefficient vector subject to $\iota' \beta = 1$.⁵ Let γ_t be parametrized as

$$\gamma_t = a_t^* + C^* z + \alpha,$$

where C^* is a $I \times M$ matrix representing the effect of the demographic variables on necessity consumption, α is a stochastic $I \times 1$ vector with zero mean and a covariance matrix $\Sigma_{\alpha\alpha}$, representing individual variation in necessity consumption, and a_t^* is a $I \times 1$ vector of constants (representing, inter alia, the effects of the relative price terms in the LES model). Depending on the commodity classification, the covariance matrix $\Sigma_{\alpha\alpha}$ can be restricted in different ways and it is specified as diagonal for our empi-

rical application. It then follows that the coefficients of (1) can be interpreted as

$$a_t = (I - \beta \iota') a_t^*, \quad b = \beta, \quad C = (I - \beta \iota') C^*,$$

and that its preference vector becomes

$$(11) \quad \mu = (I - b \iota') \alpha,$$

so that

$$(12) \quad \Sigma_{\mu\mu} = E(\mu\mu') = (I - b \iota') \Sigma_{\alpha\alpha} (I - \iota b'),$$

$$\Sigma_{\chi\mu} = \Sigma_{\chi\alpha} (I - \iota b').$$

The coefficients a_t^* and C^* are not, however, identifiable, since a_t and C are invariant to replacing a_t^* and C^* by $a_t^* + kb$, $C^* + kb \iota'$, where k is an arbitrary scalar constant, and $\Sigma_{\mu\mu}$ is singular regardless of $\Sigma_{\alpha\alpha}$.

The C dimension is parametrized by specifying $\Sigma_{\xi\mu}$ and $\Sigma_{\xi\lambda}$ (or $\Sigma_{\chi\mu}$ and $\Sigma_{\chi\lambda}$, or $\Sigma_{\chi\alpha}$ and $\Sigma_{\chi\lambda}$) as free matrices or a priori restricted to zero. The latter is a basic assumption in virtually all cross section analyses of Engel curves and empirical tests of this hypothesis are thus of considerable interest.

Finally, the M dimension is parametrized by specifying $\tilde{\Sigma}_{vv}$ and $\tilde{\Sigma}_{\epsilon\epsilon}$ as

$$(13) \quad \tilde{\Sigma}_{vv} = I_T \quad \Sigma_{vv}, \quad \tilde{\Sigma}_{\epsilon\epsilon} = I_T \quad \Sigma_{\epsilon\epsilon},$$

where Σ_{vv} and $\Sigma_{\epsilon\epsilon}$ are matrices of contemporaneous covariances, of dimensions $I \times I$ and $K \times K$, respectively. The contemporaneous covariance matrix of measurement errors in expenditures (Σ_{vv}) is specified further. In particular, we can within this framework test the standard assumption of no measurement error in total expenditures ($x_t = \xi_t$, $v_t = 0$, $\iota' \Sigma_{vv} = 0$).⁶

We have investigated the identification of each of the models specified in Tables I and II. The results are as follows: (i) all the models that combine assumption E1 (constant latent total expenditure over time) with C2 (correlation between preferences and latent total expenditure) are not identified, (ii) all the other models are identified. The proofs are somewhat lengthy and tedious, but are available from the authors on request.⁷

3. DATA AND INFERENCE PROCEDURE

The data set is taken from the Norwegian Surveys of Consumer Expenditures for the years 1975-1977, combined with information on incomes from a 'tax file'. Detailed information is given in the Appendix, including the basic data (covariance matrix and mean vector) needed to replicate or extend our econometric calculations. The sample consists of $H=408$ households, each of which is observed $T=2$ times. A five commodity classification comprising the whole budget is used ($I=5$), while the other indicators of total expenditure (the w 's) are two income variables defined for tax purposes ($K=2$). The demographic variables specified in all the $I+K=7$ equations are the number of children and the number of adults in the household ($M=2$).

The 'tax file', giving the two income variables, contains summary information from the individual tax returns for all the personal tax payers in Norway. The income variables — which are (i) net taxable income for central government tax minus total direct taxes and (ii) wage income and net entrepreneurial income used for calculating social security premiums and pension rights in the public social security system — are aggregated across all the individual tax payers in the household to get household income. Since the two income variables have several components, e.g. net wage income, in common, we expect that their measurement errors (ε) are positively correlated, as are also the individual effects (λ), which we take account of in the specification of $\sum_{\varepsilon\varepsilon}$ and $\sum_{\lambda\lambda}$.

Let $s = (y_1' \dots y_T' w_1' \dots w_T' z')$ denote the $(TI+TK+M) \times 1$ vector containing all the values of the observed variables. It is related to g and m (defined in (7)) by a relationship of the form

$$(14) \quad s = \Pi_0 + \Pi_1 g + \Pi_2 m,$$

where $\Pi_0 = (a_1' \dots a_T' d_1' \dots d_T' 0')$ is a $(TI+TK+M) \times 1$ vector of constant terms and Π_1 and Π_2 are matrices of (known or unknown) parameters implicitly defined in section 2. Since g and m are uncorrelated, the covariance matrix of s can be written as

$$(15) \quad \Sigma = \Sigma(\theta) = \Pi_1 \Sigma_{gg} \Pi_1' + \Pi_2 \Sigma_{mm} \Pi_2',$$

where Σ_{gg} and Σ_{mm} are given by (8), whose components are further defined by (10), (12), and (13). The notation $\Sigma(\theta)$ is used to indicate that this matrix is a function of a vector of unknown parameters, θ , in our model.

The realizations of s for the H households in the data set are assumed to be independent.

Let S symbolize the sample covariance matrix of s , with realized values given in Table A1. The estimates of θ are the values that minimize the function

$$(16) \quad F = F(\theta) = \ln|\Sigma(\theta)| + \text{tr}(S\Sigma(\theta)^{-1}) - \ln|S| - (T(I+K)+M).$$

Minimization of F is equivalent to maximization of the likelihood function when assuming that s follows a multivariate normal distribution (cf e.g. Anderson (1958, section 3.2)). This, however, is subject to the qualification that the first order moments of s contain no information which can be used in the estimation of θ . In our case, this is satisfied since the $T(I+K)+M$ first order moments have to be used to estimate the $T(I+K)+M$ independent parameters in $\Pi_0, q_0, \Phi_\chi, \Phi_z$, i.e. the constant terms and expectations of latent total expenditure and demographic variables, which are the parameters that appear in the expressions for the first order moments only. The estimation of these "first order parameters" can be done in a second step after the estimation of θ . (If, however, the "first order parameters" are restricted, e.g. by assuming $q_0=0$, then the maximum likelihood principle and the normality assumption strictly require simultaneous estimation of all parameters from the first and second order sample moments.)

Our model can be formalized as a special case of the LISREL model (cf e.g. Jöreskog (1977)), and the computer program LISREL 7 (cf Jöreskog and Sörbom (1988)) is used to solve the numerical calculations. The function F is minimized by using an algorithm based on the Davidon-Fletcher-Powell method. We got exactly the same estimates using different starting values and different LISREL formulations of the same econometric model. At the minimum of F , the information matrix is computed and used to estimate asymptotic standard errors and t values.⁸

LISREL minimizes the function F without imposing inequality constraints on the admissible values of the parameter vector θ . Thus the LISREL estimate of a parameter interpreted as the variance of a latent variable may well turn out to be negative. This may be regarded as an important drawback of this computer program. However, if our model and its interpretation is correct the LISREL estimates should turn out to have the expected sign, apart from the sampling errors. Thus, if for a given model all the estimated variances are positive, and all the estimates of the

covariance matrices Σ_{gg} and Σ_{mm} are positive semidefinite, we will take this as a confirmation that the model has passed an important test. On the other hand, negative estimates of variances, or negative definite "covariance matrices", indicate either that the model is misspecified or that the sampling errors in its estimates are substantial.

We test a specific model 0 (the null hypothesis) against a more general model 1 (the maintained hypothesis) by a likelihood ratio test. Let F_0 and F_1 be the minimum of F under model 0 and model 1, respectively, and let r be the difference between their number of parameters. It can be shown that minus twice the logarithm of the likelihood ratio is equal to $H(F_0 - F_1)$. This statistic is thus, according to standard normal theory, approximately χ^2 distributed with r degrees of freedom under the null hypothesis. The χ^2 values given in Table II correspond to HF_0 , interpreted as the likelihood ratio test statistic when the alternative hypothesis is an exactly identified model (giving a perfect fit to the sample covariance matrix and accordingly, $F_1 = 0$). The test statistic $H(F_0 - F_1)$ for an arbitrary pair of models can thus be computed by simply taking the difference between the corresponding pair of χ^2 values.

The χ^2 statistic HF_0 can be considered as a measure of the goodness of fit of model 0. As an alternative measure of the goodness of fit of this model we use the Akaike information criterion, which (when disregarding an arbitrary additive constant) can be written as

$$AIC = HF_0 + 2p_0,$$

p_0 denoting the number of parameters estimated under the null. The lower is the value of AIC the better is the fit (see Akaike (1987)).

If one is not willing to assume normality of the data vector s , which in the present context is a rather restrictive assumption, then the estimators derived from minimizing F can be labeled quasi maximum likelihood estimators. These estimators will be consistent, but their efficiency and the properties of the test procedures are not so obvious. There exists a large literature on the robustness of these type of estimators and test procedures for departure from normality, see e.g. Jøreskog and Sørbom (1988) for an extensive list of references, leading to quite different results depending on the assumptions and methods used. We will give three remarks supporting the hypothesis that our results are robust to departures from normality.

A recent and growing literature shows that the estimators and test

statistics derived under normality assumptions within LISREL type of models retain their asymptotic properties for wide departures from normality, exploiting assumptions on independently distributed nonnormal latent variables, see e.g. Anderson and Amemiya (1988), Amemiya and Anderson (1990), Browne (1987) and Browne and Shapiro (1988). Their assumptions are not obviously applicable to all of our models, but for instance the theorem in Browne (1987, p.381) is directly applicable to those not using assumption P2 (LES interpretation with necessity quantities independently distributed).

Another approach, based on an assumption of a multivariate elliptical distribution of the observed variables, shows that the likelihood ratio statistics derived under normality is still applicable, by rescaling the test statistics by a factor equal to the inverse of Mardia's (1970) coefficient of relative multivariate kurtosis, see Browne (1984, section 4) and Shapiro and Browne (1987). In the present data set, this coefficient is 1.306. Dividing the likelihood ratio statistics by this value, will not change the test results in section 4 materially, and all of our qualitative conclusions will remain valid.

A third way of dealing with nonnormality is to use the "asymptotically distribution-free best estimator" suggested by Browne (1984), which utilizes both the second and fourth order empirical moments. We have applied this alternative to some of our models, using the WLS option in LISREL 7, and have compared the results with those obtained within the standard framework. Generally, the estimated standard errors of the parameter estimates are rather invariant to the choice of estimation method, whereas there are some discrepancies regarding the parameter estimates themselves. Although the above distribution-free estimator has optimal properties asymptotically, it may be far from optimal to rely heavily on the fourth order moments using our rather small sample ($H=408$), and we have chosen to use the standard estimator defined by minimization of (16).

4. EMPIRICAL RESULTS

4.1. Hierarchy of models

Numerous models within the general framework described in section 2 can be estimated and tested with our data. A classification of the hypotheses and models along the four dimensions with which we will be concerned is shown in Table I. For each dimension we have picked out 2 or 3 alternative assumptions of particular interest. Combining our assumptions in all possible ways, we obtain $3 \times 3 \times 2 \times 3 = 54$ models, of which 4 are unidentified, 6 are equivalent to other models, and 13 are irrelevant or uninteresting, leaving us with 31 specific models, as shown in Table II. This table presents, for each model, the number of degrees of freedom (df), the chi square statistic (χ^2), and the Akaike information criterion (AIC). Significance probabilities of the likelihood ratio tests of the main hypotheses in Table I are given in Table III, based on all possible pairwise combinations of the models involving these hypotheses. We will use 0.01 as our standard level of significance, unless otherwise stated.

Estimates of the structural parameters are given in Tables IV-VII for a few selected models. A complete record of all the estimated parameters in each of the 31 models is available in Appendix B. The only model for which we give a complete set of estimates in the text, is E3P3C1M1, which we have found a convenient point of reference. Note that this base model implies the standard assumption of no correlation between total expenditures and preferences (C1) and the parsimonious assumption of no correlation between measurement errors of different goods (M1).

We focus on testing hypotheses and on obtaining basic characteristics of structural parameters, including robustness and sensitivity of results with respect to model specification. Our aim is not to select one best model, but rather to get empirical underpinnings of hypotheses on which models are acceptable approximations and which are inappropriate in different settings. The gain obtained by using the more parsimonious parametrizations in Table I may be much larger in other settings than it is in the present exercise. For example, we gain 10 degrees of freedom by going from M3 to M1 in our 5 commodity model, while we would gain 250 degrees of freedom if we split each commodity group into five subgroups and interpret M1 as a 25×25 block diagonal covariance matrix with five nonzero blocks of dimension 5×5 . In our setting, with panel data including both consumption and income registrations, all of our M3 models⁹ are identified, but this

may not be so in other settings, e.g. in a cross section study with only consumption data. Thus our test results can be used to discuss the realism of identification restrictions imposed in other settings.

We now proceed by presenting test and estimation results for each of the four dimensions (M, E, P, and C) in sections 4.2-4.5, and then we present the Engel functions in section 4.6.

4.2. Structure of measurement errors

Three hypotheses on the structure of the covariance matrix of measurement errors on commodity groups (Σ_{VV}) are presented in Table I.4, test results are given in Table III.4, and estimates of this covariance matrix are presented in Table IV.

The standard hypothesis in applied consumer econometrics is M2, i.e. no measurement errors in total expenditure, with the implied singularity of the covariance matrix due to the adding-up condition, but no specific restrictions on the measurement error vector otherwise. This hypothesis is clearly rejected against M3 (no restrictions) irrespective of the maintained assumptions chosen within our class of models. Given the standard assumption C1 of no correlation between preferences and total expenditure, we can even reject the M2 hypothesis with a significance level as low as 10^{-6} , based on our moderate sample size of 408 households.

Hypothesis M1, with a diagonal covariance matrix of measurement errors, is much more restrictive in terms of number of free parameters than M2, but gives a substantially better fit, both according to χ^2 and AIC, irrespective of maintained assumptions otherwise, cf Table II. This diagonality hypothesis is not rejected against M3, given adequate assumptions in the E and P dimension, i.e. E3 and P3 or P2. Imposing P1 (no individual differences in preferences), M1 is rejected against M3, which is not surprising because M3 can pick up correlations between the suppressed preference variables while M1 cannot. Somewhat more surprisingly, the M1 hypothesis is also rejected against M3 if we impose the restrictive assumption E1 or E2 with respect to the distribution of latent total expenditure.

From Table IV we see that the estimated variances of measurement errors in the M1 and M3 models are quite close, and none of the covariances in the latter are significantly different from zero, which strengthens the conjecture that M1 is an appropriate approximation to M3. (Table IV has E3, C1, and P3 as maintained assumptions, but similar results are obtained for all models containing E3 and P3 or P2.)

TABLE I

Classification of hypotheses and models

A specific model is labeled $E_i P_j C_h M_k$, which means that the model is based on hypothesis E_i w.r.t. the distribution of latent total expenditure (ξ), hypothesis P_j w.r.t. the distribution of preference variables (μ, λ), hypothesis C_h w.r.t. the covariation between latent total expenditure and preference variables, and hypothesis M_k w.r.t. the contemporaneous covariances of the measurement errors (v, ϵ).

1. Hypotheses w.r.t. the distribution of latent total expenditure

Label	Parameter restrictions			Interpretation
	$\sigma_{\xi\xi}$	q_2	σ_{uu}	
E3	free	free	free	No restrictions
E2	free	free	0	Equal growth factor (q_2) for all consumers
E1	free	1	0	Constant latent total expenditure over time for each consumer

2. Hypotheses w.r.t. the distribution of preference variables

Label	Parameter restrictions		Interpretation
	$\sum_{\mu\mu}$	$\sum_{\lambda\lambda}$	
P3	free ¹	free	No restrictions on covariances between preference variables (μ) ¹
P2	$\sum_{\alpha\alpha} = \text{free}^2$	free	LES interpretation with independently distributed necessity quantities
P1	0	0	No individual differences in preferences

¹ Except for the restrictions from the adding-up condition
² $\sum_{\mu\mu} = (I - b'l') \sum_{\alpha\alpha} (I - l'b')$, $\sum_{\alpha\alpha} = \text{diagonal}$

3. Hypotheses w.r.t. covariation between latent total expenditure and preference variables

Label	Parameter restrictions		Interpretation
	$\sum_{\chi\mu}$	$\sum_{\chi\lambda}$	
C2	free	free	Preference variables are correlated with latent total expenditure
C1	0	0	Preference variables are uncorrelated with latent total expenditure

4. Hypotheses w.r.t. contemporaneous covariances of the measurement errors

Label	Parameter restrictions		Interpretation
	\sum_{vv}	$\sum_{\epsilon\epsilon}$	
M3	free	free	No restrictions
M2	$l' \sum_{vv} = 0$	free	No measurement error in total expenditure
M1	$\sum_{vv} = \text{diag}$	free	.. Measurement errors are uncorrelated across goods

TABLE II
Overview of fitted models^{ab}

Covariance structure of measurement errors	Total expenditure distribution	Covariation between preferences and total expenditure					
		C2		C1			
		Preference distribution		Preference distribution			
		P3	P2	P3	P2	P1	
M3	E3	df ₂	73	78	79	84	92
		χ^2	130.47	135.48	155.81	163.74	765.35
		AIC	0.963	0.962	0.957	0.954	0.842
	E2	df ₂	74	79	80	85	93
		χ^2	140.92	153.83	160.99	169.18	766.69
		AIC	0.961	0.957	0.956	0.953	0.841
E1	df ₂	-a	-a	81	86	94	
	χ^2			174.21	182.23	780.45	
	AIC			0.951	0.949	0.838	
M2	E3	df ₂	78	83	84	89	97
		χ^2	150.76	163.86	277.62	284.94	1348.62
	AIC	0.959	0.956	0.928	0.926	0.795	
M1	E3	df ₂	83	88	89	94	102
		χ^2	147.99	157.55	175.17	183.47	1005.84
		AIC	0.957	0.955	0.950	0.947	0.740
	E2	df ₂	84	89	90	95	103
		χ^2	167.71	181.30	190.53	199.14	1007.83
		AIC	0.951	0.947	0.945	0.943	0.737
E1	df ₂	-a	-a	91	96	104	
	χ^2			207.14	215.64	1021.52	
	AIC			0.939	0.936	0.734	

^a The models are generated from all possible combinations of assumptions in the dimensions E, P, C, and M, see Table I for definitions. However, note that: (i) Models combining assumptions E1 and C2 are not identified and thus not fitted. (ii) Models combining C2 and P1 are equivalent to models combining C1 and P1, and only the latter are tabulated. (iii) Models combining M2 with E2 or E1 are immediately rejected by looking at the individual data, e.g. M2 and E1 implies that $x_1=x_2$ for each household while this is not true for any household in the sample. These models are thus not interesting and are left out.

^b For each model are presented the number of degrees of freedom (df), the chi square statistics (χ^2), and the Akaike information criterion (AIC) in comparison to a model with no restrictions on the covariance matrix, cf section 3 for definitions.

TABLE III

Significance probabilities in likelihood ratio tests^a

1. Tests of E-hypotheses			3. Tests of C-hypotheses		
Maintained assumptions	Null and alternative hypotheses		Maintained assumptions	Null and alternative hypotheses	
	E1 vs E2	E2 vs E3		C1 vs C2	
P3, C2, M3	b)	0.001227	E3, P3, M3	0.000295	
P2, C2, M3	b)	0.000018	E2, P3, M3	0.002691	
P3, C1, M3	0.000277	0.022848	E3, P2, M3	0.000084	
P2, C1, M3	0.000303	0.019681	E2, P2, M3	0.017702	
P1, C1, M3	0.000208	0.247034	E3, P3, M2	0.000000	
P3, C2, M1	b)	0.000009	E3, P2, M2	0.000000	
P2, C2, M1	b)	0.000001	E3, P3, M1	0.000134	
P3, C1, M1	0.000046	0.000089	E2, P3, M1	0.000859	
P2, C1, M1	0.000049	0.000075	E3, P2, M1	0.000230	
P1, C1, M1	0.000216	0.158341	E2, P2, M1	0.006645	
2. Test of P-hypotheses			4. Tests of M-hypotheses		
Maintained assumptions	Null and alternative hypotheses		Maintained assumptions	Null and alternative hypotheses	
	P1 vs P2	P2 vs P3		M1 vs M3	M2 vs M3
E3, C2, M3	0.000000	0.414661	E3, P3, C2	0.063621	0.001102
E2, C2, M3	0.000000	0.024237	E2, P3, C2	0.002811	c)
E3, C1, M3	0.000000	0.160136	E3, P2, C2	0.014752	0.000031
E2, C1, M3	0.000000	0.146071	E2, P2, C2	0.002193	c)
E1, C1, M3	0.000000	0.155137	E3, P3, C1	0.035921	0.000000
E3, C2, M2	0.000000	0.022460	E2, P3, C1	0.001018	c)
E3, C1, M2	0.000000	0.197909	E1, P3, C1	0.000280	c)
E3, C2, M1	0.000000	0.088707	E3, P2, C1	0.031912	0.000000
E2, C2, M1	0.000000	0.018435	E2, P2, C1	0.000870	c)
E3, C1, M1	0.000000	0.140459	E1, P2, C1	0.000232	c)
E2, C1, M1	0.000000	0.125668	E3, P1, C1	0.000000	0.000000
E1, C1, M1	0.000000	0.130748	E2, P1, C1	0.000000	c)
			E1, P1, C1	0.000000	c)

^a See Tables I and II for detailed definitions of hypotheses and models.^b Since models with both E1 and C2 are not identified, and thus not estimated, the test can not be performed.^c Since models with both M2 and E2 or E1 are not estimated (see Table II, footnote a) the test can not be performed.

TABLE IV

Covariance matrix of measurement errors (Σ_{vv}) based on hypothesis M1, M2 or M3^a

	Food, beverages and tobacco			Clothing and footwear			Housing, fuel and furniture			Travel and recreation			Other goods and services		
	M3	M2	M1	M3	M2	M1	M3	M2	M1	M3	M2	M1	M3	M2	M1
Food, beverages and tobacco	10.13 (0.73)	9.06 (0.64)	9.82 (0.72)												
Clothing and footwear	0.89 (0.59)	-0.91 (0.50)	0 ^b	13.38 (0.94)	10.96 (0.77)	13.15 (0.93)									
Housing, fuel and furniture	0.32 (0.86)	-1.50 (0.74)	0 ^b	0.78 (0.97)	-2.29 (0.81)	0 ^b	27.24 (1.98)	24.12 (1.69)	26.92 (1.96)						
Travel and recreation	2.52 (1.53)	-6.86 (0.99)	0 ^b	3.72 (1.72)	-7.79 (1.10)	0 ^b	-3.47 (2.53)	-19.66 (1.81)	0 ^b	90.61 (6.18)	39.04 (2.74)	89.02 (6.16)			
Other goods and services	0.74 (0.38)	0.21 (0.34)	0 ^b	1.01 (0.43)	0.03 (0.37)	0 ^b	0.25 (0.63)	-0.67 (0.55)	0 ^b	0.63 (1.11)	-4.72 (0.74)	0 ^b	5.42 (0.39)	5.16 (0.36)	5.32 (0.39)

^a Maintained assumptions: E3, P3 and C1, cf Table I. Standard deviations in parentheses.^b A priori restriction.

Imposing the hypothesis of no measurement error in total expenditure (M2) leads to smaller estimates of the variances of measurement errors for each of the five commodity groups, and in particular so for the group which has the absolutely and relatively largest variance of measurement errors, namely Travel and recreation. Imposing M2 also strongly changes the pattern of covariances, implying, inter alia, significantly negative correlation between the measurement error of Travel and recreation and that of other commodity groups. Thus, imposing the false hypothesis M2 not only gives a significantly worse fit than M3 and the parsimonious model M1, but also strongly distorts other characteristics of the covariance matrix, such as relative size of the variances and sign and relative size of the covariances.

The estimates of the covariance matrix of the error terms of the income relations ($\Sigma_{\varepsilon\varepsilon}$) are robust to model specification within the class of models not including P1. In the base model (E3P3C1M1), the estimate of $(\sigma_{\varepsilon\varepsilon}^{11}, \sigma_{\varepsilon\varepsilon}^{12}, \sigma_{\varepsilon\varepsilon}^{22})$ is (57, 54, 93) with standard deviations (4, 6, 9).¹⁰ The errors in the two income measures are thus strongly positively correlated, as expected, cf section 3. In P1 models (no individual differences in preferences), the ε variables pick up variation in the suppressed λ variables, and the estimated covariance and variances are substantially larger than in the corresponding P2 or P3 models. In e.g. model E3P1C1M1, the estimate of $(\sigma_{\varepsilon\varepsilon}^{11}, \sigma_{\varepsilon\varepsilon}^{12}, \sigma_{\varepsilon\varepsilon}^{22})$ is (181, 115, 168), with standard deviations (10, 11, 23).

In order to compare the relative size of the variance of measurement errors across goods and income measures, we define a parameter ρ^i to be equal to $1 - \sigma_{vv}^{ii} / \sigma_{yy}^{ii}$ for commodity i , $i=1, \dots, I$, and correspondingly for the income measures. Observe that ρ^i is analogous to the squared coefficient of multiple correlation in classical linear regression analysis. In our model, we have that $\sigma_{yy}^{ii} = \sigma_{\eta\eta}^{ii} + \sigma_{vv}^{ii}$, since, by assumption, the v 's are uncorrelated with all the elements of the η 's. Thus ρ^i can also be written as $\sigma_{\eta\eta}^{ii} / \sigma_{yy}^{ii}$, i.e. the ratio between the variance of the latent 'structural' component of the equation and the variance of its observed left hand side variable. It gives a measure of the signal/noise ratio for our observed consumption and income variables, all of which can be considered as indicators of latent total expenditure. The ranking of these variables according to the estimated¹¹ value of ρ^i in the base model (E3P3C1M1), with estimated values of ρ^i in period 1 in parenthesis, is: 1) Income measure 2 (0.94), 2) Income measure 1 (0.89), 3) Food, beverages and tobacco (0.70), 4) Housing, fuel and furniture (0.51), 5) Other goods and

services (0.48), 6) Clothing and footwear (0.41) and Travel and recreation (0.41). This ranking list is robust to model specification if we exclude models with M2 and P1 assumptions. The corresponding ratio for observed total expenditure ($\sigma_{\xi\xi}/\sigma_{xx}$) is 0.73.

4.3. Distribution of latent total expenditure

Three hypotheses on the evolution of the distribution of latent total expenditure across households are presented in Table I.1, significance probabilities of likelihood ratio tests are given in Table III.1, and estimates of parameters of the distribution are presented, for selected models, in Table V.

Hypothesis E1, with constant latent total expenditure for each household over the two observation periods, is strongly rejected. It is rejected against E2 (equal growth factor) regardless of which maintained assumptions are chosen. Likelihood ratio tests of E1 against E3 (no restrictions), which can be performed from the χ^2 values in Table II, would generally make the significance probabilities even lower, for the base model less than 10^{-6} . An interpretation of this rejection of E1 is that many Norwegian households had a substantial change in their latent total consumption over the period 1975-1977. Due to this fact, our sample, although rather small and covering only two periods, can enable us to investigate covariation between preferences and total expenditure, which could not be identified under E1, see section 4.5.

Hypothesis E2, with equal growth factor and no change in the ranking of households according to total expenditure ($\sigma_{uu}=0$), is also rejected in most of the tests, but the results are not so robust. E2 is not rejected when using P1 as a maintained assumption, not even if we choose a significance level as high as 0.1. Since P1 itself is strongly rejected, see section 4.4, a reasonable interpretation of this result is that in order to perform appropriate tests of specific properties of the distribution of total expenditure one has to model the distribution of preferences (at least to some degree). The significance probabilities in testing E2 against E3 are larger when assuming M3 than when using M1, and if combining C1 and M3, then E2 is not rejected against E3 at a significance level of 0.01, but it is rejected at a 0.05 level. Thus we get a similar but somewhat bleaker picture of the E dimension when using the flexible assumption M3 than when using the parsimonious, but appropriate, assumption M1.

Table V presents, for eight models, estimates of parameters in the distribution of latent total expenditure. The models were selected by starting

TABLE V

Distribution of latent total expenditure in selected models^a

Parameter	Base model	Change in M-dimension		Change in P-dimension		Change in C-dimension	Change in E-dimension	
	E3P3C1M1	E3P3C1M3	E3P3C1M2	E3P2C1M1	E3P1C1M1	E3P3C2M1	E2P3C1M1	E1P3C1M1
σ_{XX}	380.02 (33.68)	376.32 (33.90)	348.05 (32.91)	380.56 (33.72)	266.98 (25.05)	341.96 (34.19)	381.12 (33.16)	420.96 (34.93)
ρ_{02}	-1.16 (1.53)	-0.77 (1.50)	-2.86 (1.92)	-1.16 (1.53)	0.90 (1.18)	-3.70 (2.18)	-1.24 (1.41)	2.98 (0.86)
ρ_2	1.104 (0.030)	1.094 (0.029)	1.147 (0.041)	1.104 (0.030)	1.052 (0.014)	1.168 (0.048)	1.106 (0.027)	1 ^b
σ_{uu}	15.15 (4.60)	10.38 (4.64)	158.56 (12.48)	15.39 (4.63)	-3.30 (2.01)	35.20 (9.63)	0 ^b	0 ^b
σ_{vv}	144.23 (6.62)	161.56 (12.45)	0 ^b	143.57 (6.58)	207.91 (6.30)	141.83 (7.26)	150.33 (6.68)	151.46 (6.74)
CV^c	0.500 (0.026)	0.495 (0.026)	0.566 (0.025)	0.501 (0.026)	0.409 (0.022)	0.489 (0.026)	0.491 (0.026)	0.516 (0.026)
CA^d	0.962 (0.012)	0.973 (0.012)	0.687 (0.026)	0.961 (0.012)	1.013 (0.008)	0.907 (0.025)	1 ^b	1 ^b
σ_{Xz}^1	8.80 (1.40)	8.85 (1.41)	8.52 (1.36)	8.80 (1.40)	9.07 (1.27)	8.52 (1.36)	8.82 (1.40)	9.19 (1.46)
σ_{Xz}^2	10.00 (1.09)	10.05 (1.09)	9.83 (1.07)	10.00 (1.09)	10.23 (1.00)	9.70 (1.07)	9.99 (1.09)	10.52 (1.13)

^a See Table I for model descriptions. Standard deviations in parentheses.

^b A priori restriction.

^c Coefficient of variation in period 1: $CV = (\sigma_{XX} + \sigma_{uu})^{1/2} / \Phi_X$

^d Coefficient of autocorrelation: $CA = \sigma_{XX} / (\sigma_{XX} + \sigma_{uu})$.

out with the base model (E3P3C1M1) and then making change of assumptions along the different dimensions. A sensitivity analysis showed that the conclusions below are robust to the choice of base model (see Appendix B).

The estimates of the variance of the permanent component of latent total expenditure ($\sigma_{\chi\chi}$) are quite similar in most of the models. In the E1 models, the estimated values of $\sigma_{\chi\chi}$ are larger than in the other models, which is not surprising since $\sigma_{\chi\chi}$ here will be a compromise between the variances of latent total expenditure in period 1 and period 2. (Recall that the other models have a q_2 estimate larger than 1.) The estimates of $\sigma_{\chi\chi}$ are smallest in the P1 models, again a reasonable result since suppressing the preference variables (μ) increases the variance of the estimated measurement errors (v), and thus also the variance of the measurement errors in total expenditure ($v = \sum v_i$), which again decreases the variance in latent total expenditure (cf equation (5)).

For the time specific constant term (q_{0t}), which is normalized to zero in period 1, the estimate for period 2 is not significantly different from zero in any model, except when inappropriately assuming E1. This implies that the hypothesis of a constant coefficient of variation in the distribution of latent total expenditure is not rejected (cf section 2). The estimates of the coefficient of variation of latent total expenditure are about 0.50 in the most relevant models. The estimated growth factor q_2 significantly exceeds unity in all the models which has this as a free parameter, indicating a pronounced growth in latent total expenditure among Norwegian households in the period 1975-1977.

The estimates of the variance (σ_{uu}) of the volatile component in latent total expenditure (u_t) are significantly positive, again supporting the E3 hypothesis, but the estimates are rather small. The only exception are M2 models where the estimate of σ_{uu} is large, since it to some extent captures the effect of the suppressed measurement errors in total expenditure, the sum $\sigma_{uu} + \sigma_{vv}$ being of the same order of magnitude in the M2 and in the relevant M3 and M1 models. Accordingly, the estimated coefficient of autocorrelation of the latent total expenditure is larger than 0.90 in the relevant M3 and M1 models, while it is as low as 0.69 in the M2 models.

The estimates of the covariances of the permanent component of total expenditure and the demographic variables ($\sigma_{\chi z}$) are almost the same in all models. Our model of the consumption growth process, (9), implies that $\sigma_{\xi z}^2 = q_2 \sigma_{\xi z}^1$, $i=1,2$, which is a testable hypothesis. We have tested it by the likelihood ratio method, for a few model specifications, and it was not rejected. (These test results are available from the authors on request.)

4.4. Distribution of preferences

The distribution of preferences across the population of consumers is, in one interpretation of our model, represented by the distribution of the individual effects in the Engel functions (μ) and in the income measurement equations (λ). Three hypotheses regarding this distribution are presented in Table I.2, significance probabilities of likelihood ratio tests are given in Table III.2, and estimates of $\sum_{\mu\mu}$ in two selected models are given in Table VI.

The hypothesis of no preference variation (P1) is strongly rejected, the significance probability is less than 10^{-6} for each of the twelve possible likelihood ratio tests. Furthermore, (i) the estimators of many of the other parameters seem to be substantially biased when using P1 (cf e.g. section 4.3), (ii) our estimates of the parameters of the preference distributions seem reasonable (cf below), and (iii) we have not experienced any serious practical problem in estimating these preference distributions (individual effects). Thus we conclude with a strong recommendation of modeling these preference distributions.

The hypothesis of independently distributed necessity quantities in the linear expenditure system, which generates the parsimonious covariance structure P2 of the preference variables (μ), is not rejected against the most general specification P3 in any of the twelve possible likelihood ratio tests. Furthermore, at least for C1 models, (i) the estimates of $\sum_{\mu\mu}$ are very close in P2 and P3 models (see Table VI), (ii) the estimates of the other parameters are almost identical in P2 and P3 models (see Table V for an example), (iii) the estimates are easily interpreted and no unreasonable results have been found, and (iv) the P2 models give a better fit than P3 according to the Akaike information criterion. Thus we conclude that P2 is an appropriate approximation in our setting. It seems reasonable that similar approximations can be found appropriate also in other settings, where the gain from using the parsimonious parameterization can be much larger. For example, we gain 5 degrees of freedom when going from P3 to P2 in our 5 commodity model, while we would gain 250 degrees of freedom if we split each commodity group into five subgroups and interpret P2 as a 25x25 block diagonal covariance matrix ($\sum_{\alpha\alpha}$) with five nonzero blocks of dimension 5x5.

TABLE VI

Covariance matrix of preference variables ($\sum_{\mu\mu}$) based on hypothesis P2 or P3^a

	Food, beverages and tobacco		Clothing and footwear		Housing, fuel and furniture		Travel and recreation		Other goods and services	
	P3	P2	P3	P2	P3	P2	P3	P2	P3	P2
Food, beverages and tobacco	6.32 (0.84)	5.94 (0.79)								
Clothing and footwear	-0.20 (0.51)	-0.59 (0.22)	3.01 (0.73)	3.05 (0.72)						
Housing, fuel and furniture	-0.80 (0.76)	-1.96 (0.49)	-2.19 (0.69)	-0.90 (0.37)	7.74 (1.46)	8.10 (1.41)				
Travel and recreation	-4.94 (1.11)	-3.16 (0.75)	-0.76 (0.93)	-1.61 (0.56)	-4.10 (1.52)	-4.91 (1.21)	10.32 (2.57)	10.49 (2.54)		
Other goods and services	-0.29 (0.35)	-0.23 (0.16)	0.14 (0.31)	0.04 (0.13)	-0.77 (0.48)	-0.33 (0.27)	-0.52 (0.63)	-0.81 (0.42)	1.32 (0.33)	1.34 (0.33)

^a Maintained assumptions: E3, C1 and M1, cf Table I. Standard deviations in parentheses.

Since the μ 's add to zero they will tend to be negatively correlated. Indeed, there are no significantly positively correlated pairs of μ 's in Table VI. There are three pairs of goods for which the correlation in the μ 's is significantly negative in the P3 model: Food, beverages and tobacco vs. Travel and recreation, Clothing and footwear vs. Housing, fuel and furniture, and Housing, fuel and furniture vs. Travel and recreation. These covariances are also significantly negative in the P2 model, assuming independent necessity quantities (α), and can thus be explained as an effect of the budget constraint. Note also that the standard errors of the estimators of the nondiagonal elements in $\sum_{\mu\mu}$ are substantially smaller in the parsimonious P2 model than in the flexible P3 model.

The estimates of the variances of the necessity expenditures (with standard errors in parenthesis) in the E3P2C1M1 model is 7.2 (1.1) for Food, beverages and tobacco, 3.2 (0.9) for Clothing and footwear, 11.1 (2.5) for Housing fuel and furniture, 18.1 (6.1) for Travel and recreation, and 1.1 (0.4) for Other goods and services. One possible relative measure of the preference variation is the standard deviation of α divided by the mean expenditure (η). The ranking list of the commodities according to this measure is: Clothing and footwear (0.407), Other goods and services (0.338), Travel and recreation (0.330), Housing, fuel and furniture (0.326), and Food, beverages and tobacco (0.254).

4.5. Correlation between preferences and latent total expenditure

A maintained hypothesis in most empirical work on consumer behavior is independence of preferences and income (or total expenditure). This can be tested by means of our panel data. The likelihood ratio tests in Table III lead to rejection of the hypothesis C1 against C2, for nine out of ten possible combinations of maintained assumptions.

The estimates of the covariances of the preference variables (μ, λ) and the permanent component of latent total expenditure (χ) are, in model E3P3C2M1, with standard errors in square brackets:

$$\sum_{\chi\mu} = \begin{pmatrix} 20.8, & 4.7, & 6.6, & -36.4, & 4.3 \\ [6.0, & 4.9, & 7.0, & 10.2, & 3.3] \end{pmatrix}, \quad \sum_{\chi\lambda} = \begin{pmatrix} 67.1, & 164.0 \\ [19.8, & 36.9] \end{pmatrix}.$$

We see that that the preference variable for Food, beverages and tobacco is significantly positively correlated with latent total expenditure, the correlation is significantly negative for Travel and recreation, while the estimated covariances are not significantly different from zero for the other three commodity groups. Also, the λ variables are significantly posi-

tively correlated with latent total expenditure for both income measures. One possible interpretation of this result is that the preferences for savings are positively correlated with total expenditure and income. However, this correlation could for example also be explained by a negative correlation between the degree of tax evasion and measured income, cf section 2.

The results are, to some extent, similar in the other C2 models. In particular, the estimated covariance between the preference variable (μ_i) and the latent total expenditure is positive for Food, beverages and tobacco and negative for Travel and recreation for all the ten C2 models. However, the numerical values of the estimates of these covariances and their standard errors show substantial variation between the C2 models. Furthermore, the estimates of the other parameters vary substantially between different C2 models and between C2 and C1 models (cf section 4.6 below). In addition, some of the estimates for the C2 models seem unreasonable. This leads us to consider the C2 estimates more as an empirical illustration of a model and an approach than as convincing substantial results. For future empirical research on this important issue we would recommend using models with more elaborated dynamics, data with longer time span, and information on the stocks of durables (cf footnote 6).

4.6. Engel functions

The estimated system of Engel functions in the base model (E3P3C1M1) is presented in Table VII.1. The results are almost identical for all the six models combining assumptions C1, M1, P2 or P3, and E3 or E2 or E1, see Appendix B. The estimates do not differ materially if M1 is replaced by M3. On the other hand, if we allow for correlation between preferences and latent total expenditure, i.e. replace C1 by C2, the estimated system of Engel functions differs not only from that of the base model but also between different versions of C2 models (cf below). The class of models with no preference variation (P1) also has Engel functions which differ somewhat from the base model, but since these differences are moderate and are not very interesting they will not be further discussed. The Engel functions in the models disregarding measurement error in total expenditure (M2) also differ from those in the corresponding M3 and M1 models. The difference between the vector of Engel elasticity estimates for model E3P3C1M2 and the corresponding vector for the base model (E3P3C1M1),

TABLE VII

Engel functions and income-consumption relations^a

1. Engel functions (equation (4))							
Commodity group	Engel elasticities ^{c)} E_i	Budget share		Demographic effects		Constant term	
		Average ^{b)}	Marginal	Child	Adult	year 1	year 2
		ω_i	b_i	C_{i1}	C_{i2}	a_{i1}	a_{i2}
Food, beverages and tobacco	0.63 (0.05)	0.256 (0.005)	0.162 (0.012)	0.906 (0.148)	0.569 (0.236)	2.125 (0.475)	1.650 (0.484)
Clothing and footwear	1.14 (0.11)	0.107 (0.004)	0.122 (0.011)	0.109 (0.132)	0.048 (0.210)	-0.675 (0.428)	-0.991 (0.436)
Housing, fuel and furniture	1.08 (0.07)	0.248 (0.005)	0.268 (0.016)	-0.330 (0.192)	-1.526 (0.307)	2.908 (0.620)	2.795 (0.632)
Travel and recreation	1.10 (0.07)	0.313 (0.007)	0.343 (0.021)	-0.492 (0.253)	1.122 (0.409)	-3.785 (0.814)	-2.891 (0.834)
Other goods and services	1.38 (0.10)	0.076 (0.002)	0.105 (0.007)	-0.193 (0.090)	-0.213 (0.143)	-0.573 (0.292)	-0.563 (0.297)

2. Income-consumption relations (equation (6))

Income measure	Income-consumption ratio		Demographic effects		Constant term	
	Average ^{b)}	Marginal	Child	Adult	year 1	year 2
		e_k	f_{k1}	f_{k2}	d_{k1}	d_{k2}
Income measure 1	0.971 (0.023)	0.514 (0.053)	-1.384 (0.673)	0.474 (1.066)	-2.291 (2.138)	0.078 (2.176)
Income measure 2	1.380 (0.036)	1.110 (0.100)	-0.122 (1.270)	11.111 (2.011)	-13.566 (4.025)	-13.483 (4.097)

^a Estimates for model E3P3C1M1 with standard deviations in parentheses.

^b Ratio of means in the full sample including both periods.

^c $E_i = b_i/\omega_i$.

given in first column of Table VII, is $(-0.16, -0.03, -0.17, 0.36, -0.34)$, indicating a positive bias for Travel and recreation and a negative bias for the other four commodity groups. More dramatic biases when neglecting measurement errors in total expenditure have been estimated by Liviatan (1961), Cramer (1966), and Aasness (1990, Essay 5), for several detailed commodity groups. The main contribution of the present study in this respect is not estimates of such biases, but the firm rejection it gives of the M2 hypothesis in formal tests (cf section 4.2). For this purpose, the specification with only five commodity groups is fully appropriate.

The estimated Engel elasticity for Food, beverages and tobacco is positive and significantly less than one for all the 31 models, confirming once again Engel's law, now in a framework with errors in variables and with preferences allowed to be correlated with latent total expenditure. The estimates are lower in the C2 models than in the C1 models. This agrees with the result that the preference variable for this commodity group is positively correlated with latent total expenditure (cf section 4.5), since these parameters compete in explaining the positive covariance between the consumption of Food, beverages and tobacco and latent total expenditure. On the other hand, Travel and recreation is classified as a luxury (i.e. its estimated Engel elasticity exceeds unity) by all the 31 models. As noted in section 4.5, the preference variable for Travel and recreation is estimated to be negatively correlated with total expenditure, and thus its estimated Engel elasticity should be expected to be larger in the C2 models than in the C1 models, which is in fact the case.

It should be noted, however, that the standard deviations of the estimates and the sensitivity of the results increase substantially when replacing C1 by C2. For example, the estimates of the marginal budget share for Food, beverages and tobacco vary in the interval $(0.157, 0.169)$, with estimated standard deviations in $(0.012, 0.013)$, in the C1 class containing 12 models (without M2 and P1), while they vary in the interval $(0.007, 0.098)$, with estimated standard deviations in $(0.022, 0.068)$, in the C2 class containing 8 models (without M2 and P1). For Travel and recreation, the estimates of the marginal budget shares vary in the interval $(0.328, 0.348)$ for the same C1 class with 12 models, with estimated standard deviations in $(0.021, 0.023)$, while the estimates vary in the interval $(0.374, 0.800)$, with standard deviations in $(0.035, 0.110)$, for the same C2 class with 8 models. Thus, relaxing the standard maintained assumption of independence between preferences and latent total expenditure has its costs in terms of decreased precision in our knowledge about the values of the

parameters of the Engel functions. The loss of information on Engel curves when replacing C1 by C2 will, of course, depend on the type of data used. Our panel, with only 408 households and two replications, may be too small to get precise information on C2 models. On the other hand, good panel data on household expenditures are quite rare.

Estimates of e_1 and e_2 , which can be interpreted as reduced form parameters reflecting saving behavior and other effects, cf section 2, are presented in Table VII.2, for the base model. The estimates are significantly positive, confirming our hypothesis that both income measures are good indicators of latent total expenditure. Viewed as measures of "true income", both these measures probably contain not only random, but also systematic measurement errors, which makes it difficult to give clearcut interpretations about saving behavior from the values of the parameters. The estimates of e_k vary considerably between the main groups of models, the largest estimates are obtained in the P1 models (no preference variables) and the smallest estimates are obtained in the C2 models (preference variables allowed to be correlated with latent total expenditure).

The estimated effects on consumption of household size and composition are also presented in Table VII for the base model. The estimates are quite close in all models not containing P1, M2, or C2, and all of them have the following characteristics. The effect on food consumption of an additional child or of an additional adult, given the latent total expenditure, is significantly positive, in agreement with Engel's law. The estimated effects on Clothing and footwear are also positive, both for children and adults, but these effects are small and not significantly different from zero. The effect of the demographic variables on the expenditure on Housing, fuel and furniture is negative, and significantly so for adults. The estimated effect on expenditure on Travel and recreation of an additional adult is significantly positive, while the estimated effect of an additional child is negative. The number of adults and children affect expenditure on Other goods and services negatively, but not significantly, and the magnitudes of the coefficients are small.

5. CONCLUSION

In this paper, a new framework for econometric modeling of a system of consumer expenditures has been presented and applied successfully on Norwegian panel data from 1975-1977. Thirty-one systematically selected model specifications have been estimated and tested, and the results have been compared across models. Many of the substantial results are robust with respect to model specification, while other results are distorted when choosing inappropriate specifications.

The hypothesis of no measurement error in total expenditure, which still is a standard assumption in analyses of household expenditure surveys, is neatly nested within our modeling framework and is strongly rejected in all of our tests. The relative variance of the measurement error in total expenditure is also of considerable magnitude. Thus there may be substantial biases in the estimators of e.g. Engel elasticities when no measurement error in total expenditure is erroneously assumed, but the sign and size of the biases will of course vary between commodity groups. The parsimonious specification assuming independently distributed measurement errors across commodity groups gives a much better fit to our data and seems to be an appropriate approximation with our five broad commodity groups. It would be of considerable interest to investigate whether similar results could be obtained from household surveys from other countries. However, the possibilities for testing these hypotheses are much smaller with cross section data than when using panel data.

The estimated variances of all the latent variables (total expenditure, preference variables, measurement errors) are positive in all of our models (with one unimportant exception), and this was not imposed as constraints on the estimation procedure. Thus our econometric interpretation of the statistical models of the observed consumption and income variables has passed an important test. The estimated parameters are of reasonable magnitude and confirm our view that this is a fruitful approach to consumer econometrics.

There has been a significant general growth in latent total expenditure in Norway in 1975-1977. The coefficient of autocorrelation of this variable is high, but there has been a significant change in the ranking of the households by latent total expenditure in this rather short period. We found no significant change in the inequality in the distribution of latent total expenditure, as measured by its coefficient of variation.

There is a substantial variation in preferences, as represented by the

vectors μ and λ , across households. Our model class can be interpreted as relating to a population of households, each having a Stone-Geary utility function. An assumption of independently distributed "necessity quantities" across commodities places strong restrictions on the covariance structure of the "preference variables" in our econometric model, but these restrictions are not rejected. This independence assumption seems to be a good approximation in our setting with five broad commodity aggregates.

A fundamental assumption in virtually all cross section analyses of consumer demand functions is that preferences and total expenditure are uncorrelated. This hypothesis is tested and rejected within our linear framework. The preference variable for Food, beverages and tobacco is positively correlated with latent total expenditure, indicating a positive bias in the estimator of the Engel elasticity when assuming zero correlation. The preference variable for Travel and recreation is negatively correlated with total expenditure, indicating a negative bias in the estimator of the Engel elasticity when assuming zero correlation. These results should not be taken too far, but they indicate a fruitful starting point for future research on an important issue.

The empirical study has confirmed our conjecture that this type of latent variable approach is fruitful for econometric analysis of surveys of household expenditures. The models and the statistical procedures may, of course, be refined and extended in various directions, but probably more important gains would be obtained by using more and better data, e.g. panel data with more than two replications, data on stocks of durables, and data with a more detailed commodity classification.

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APPENDIX A: DATA

The data base for this study is taken from the Norwegian Surveys of Consumer Expenditure for the years 1975-1977, which give data in the incomplete cross-section/time series format. The sampling method is a three-stage stratified design, giving a selfweighted random sample of all the private households in Norway. The rate of nonresponse averages about 30%. It is lower for households asked to give their second report than for those asked the first time. The data base is described in Biørn and Jansen (1980), and we have used data from a subsample of H=408 households for which two reports exist in the years 1975-1977, one half observed in 1975 and 1976 and the other half in 1976 and 1977. This is the same data set as used in Biørn and Jansen (1982), with a few modifications as noted below, and with the addition of income data taken from 'tax files'.

Purchase expenditures on consumption goods are recorded by a combination of bookkeeping and interviews. Each household is asked to keep detailed accounts of its expenses over a period of two weeks. For commodities with a low purchase frequency, expenses during the last 12 months are registered in a concluding interview at the end of the accounting period. Housing expenses are measured by rent (including maintenance and repairs), whereas other durable goods are represented by the value of last year's purchases.

The expenditure data are collected evenly throughout the year, 1/26 of the households participating in a particular year are observed between 1st and 14th of January, another 1/26 between 15th and 28th of January, and so on. The nominal expenditures are deflated by price indexes constructed from the basic data used in calculating the official Norwegian Consumer Price Index. Each series of monthly Laspeyres indexes is converted to a periodicity of 14 days. The households report with an interval of exactly one year. By constructing annual aggregates, we get two annual reports from the 408 households, which we formally treat as if it were a two period balanced panel, although the two time periods are not exactly the same for all households.

The two income measures, obtained from tax files, are:

- w_1 : Taxable income for the central government tax assessment minus taxes.
- w_2 : Income base used for calculating social security premiums and pension rights in the public social security system. It includes wages and net entrepreneurial income, but excludes capital income (positive and negative, e.g. interests received and paid) and pensions.

The demographic variables used to characterize the household size and composition are:

z_1 : The number of children, i.e. persons with age ≤ 15 years.

z_2 : The number of adults, i.e. persons with age ≥ 16 years.

We have used the observations on the latter variables from the first reporting period and analyze the data as if they were the same in both periods, i.e. individual specific. (The effects of the one year aging of each household member will then be captured by the period specific constant term.) For ten households, the number of household members (z_1+z_2) changed by more than one between the two periods. These were deleted from our sample, since they could hardly be considered as representing the same household in both periods. (This was not done by Biørn and Jansen (1982) which explains why they used 418 households as against our 408 households.)

An inspection of this data set showed that some of the second order sample moments were very sensitive to a few extremely large observations. Some sort of robust procedure [cf Huber (1981) or Hampel et al (1986)] seemed to be needed. We found the idea of winsorizing the upper tail of the distribution promising in our setting. We chose to winsorize moderately, by setting the eight largest values of each variable in the original data set equal to its ninth largest observation (which is an estimate of the 98 per cent quantile). This procedure was followed for all the basic expenditure data by applying the above rule on each of the commodities in a detailed grouping (28 goods) before aggregation to the five commodity grouping. The resulting covariance matrix, on which our econometric analysis in this paper is based, is given in Table A1.

Most of the extreme observations are due to large purchase of some particular goods during the short registration period. For example, the largest outlier reflects a large purchase of an item containing jewelry etc. during the two week accounting period. Thus the recorded extreme purchases are poor indicators of the true consumption variable we are interested in (cf section 2), which increases the uncertainty of the parameter estimates. Such problems are common in the analysis of family budgets. Below we give some arguments supporting the above winsorization procedure. First, by reducing the influence of a few outliers, the skewness and kurtosis of some of the variables decreased dramatically, making our test procedures and standard errors, relying on asymptotic theory, more appropriate (cf section 3). Second, the variance of the observed variables were also considerably reduced, which might be expected to bias the esti-

Table A1
Covariance matrix of the observed variables^{a)}

	Food, beverages and tobacco		Clothing and footwear		Housing, fuel and furniture		Travel and recreation		Other goods and services	
	y11	y12	y21	y22	y31	y32	y41	y42	y51	y52
y11	33.866399									
y12	23.931881	34.277795								
y21	9.771840	9.894488	22.692229							
y22	10.362220	12.385392	9.556499	23.523477						
y31	16.573568	17.280461	7.069731	11.628316	54.839144					
y32	17.656239	19.497959	6.757210	13.738555	30.243543	61.781544				
y41	21.432868	21.064339	14.633581	14.944365	24.033823	22.290424	131.351104			
y42	26.029033	32.328722	15.823002	26.467252	34.579921	31.770744	55.026045	179.365061		
y51	6.995257	6.929788	5.202998	5.008590	7.746768	8.136831	14.012352	14.429180	9.590906	
y52	7.414759	9.006666	4.324339	6.445988	9.069422	10.505322	11.609091	15.243447	5.218842	11.862711
w11	51.588551	48.147343	32.549316	31.866398	59.113226	60.929099	127.655839	116.312647	29.988223	28.188578
w12	57.612568	58.135318	35.658711	40.761958	63.376137	64.759473	133.288826	149.462801	32.373895	30.914929
w21	107.289966	97.009444	64.431758	67.096449	140.906382	133.627689	238.857808	234.022386	57.959994	50.511329
w22	113.944568	108.692940	69.767003	78.773507	141.262465	136.863114	235.569617	278.668461	62.880572	55.233889
z1	2.912353	3.029978	1.318979	1.289102	1.469587	2.181751	1.699158	3.172564	0.540539	0.760104
z2	2.197420	2.294813	1.334488	1.339847	1.310812	1.752556	4.525261	4.460283	0.836724	0.995516

a) Measurement unit: 1000 Norwegian 1974 kroner

Table A1 (cont.)
Covariance matrix of the observed variables

	Income measure 1		Income measure 2		Number of children	Number of adults
	w11	w12	w21	w22	z1	z2
w11	487.977561					
w12	467.259940	573.080053				
w21	766.034038	774.003609	1626.371358			
w22	752.773693	890.587960	1624.143500	1851.203952		
z1	2.281886	4.351582	9.495276	12.433148	1.579200	
z2	12.660054	13.595467	20.156360	21.558213	0.078527	0.826605

mated structural parameters. However, our hypothesis that this will only substantially reduce the estimated variances of the measurement errors, while not changing systematically the estimates of the other structural parameters, was supported by a sensitivity analysis in which we varied the degree of winsorization between 0 and 2 percent [cf Appendix B, tables 16-17].¹² Third, by winsorizing the data at a fairly detailed level of aggregation, we will not, to a large extent, censor rich households having large purchases of many goods. They will be represented by their proper large expenditure even if one (or several) of the recorded purchases at the disaggregated level may have been influenced by the chosen procedure. Fourth, by winsorizing at a disaggregated level, we obtain a consistent system of accounts for all aggregated commodity groupings which can be constructed from it. Fifth, in our context, winsorizing observations seems preferable to deleting them, since the latter might substantially reduce the effective sample size and also throw away valuable information on large purchases. Sixth, by following the above simple rule of winsorization, other researchers may replicate our results and analyze the properties of our procedure theoretically and empirically.

The sample means of our observed variables are

y_1 : (10.5581, 4.3886, 9.8937, 11.9299, 2.9802), w_1 : (38.0961, 55.1871),
 y_2 : (10.5655, 4.4365, 10.5787, 13.8448, 3.3036), w_2 : (41.9946, 58.5761),
 and z : (0.7672, 2.2377).

APPENDIX B: DETAILED RESULTS

B.1. Introduction

The main text above contains selected results from 31 models, with comments on the robustness of the results across all models. This appendix presents a full record of the estimation results without comments. In a preliminary version of the paper having the same title (issued as Discussion paper no 41, Central Bureau of Statistics, Oslo, Norway, 1988), we gave a full account of all the 13 models in the M1 dimension. However, since this preliminary version was written, we have been able to take full account of a restriction associated with the P2 models. As a consequence, the estimation results differ for 5 out of the 13 models. In the present paper, the number of models has been increased to 31, which reflects that we now give a more elaborate treatment of the distribution of measurement errors in expenditures.

Before the list of tables is displayed we present the model in scalar form and all the scalar symbols which appear in the tables.

B.2 Model and symbols in scalar notation

Engel function for commodity i :

$$\eta_{it} = a_{it} + b_i \xi_t + c_{i1} z_1 + c_{i2} z_2 + \mu_i.$$

Adding-up condition of expenditures:

$$\xi_t = \sum_{i=1}^5 \eta_{it}.$$

Measured consumption of commodity i :

$$y_{it} = \eta_{it} + v_{it}.$$

Measured total consumption:

$$x_t = \sum_{i=1}^5 y_{it} = \xi_t + v_t.$$

Measurement error in total consumption:

$$v_t = \sum_{i=1}^5 v_{it}.$$

Income-consumption equation, income type k:

$$w_{kt} = d_{kt} + e_k \xi_t + f_{k1} z_1 + f_{k2} z_2 + \lambda_k + \varepsilon_{kt}.$$

Total consumption process:

$$\xi_t = q_{0t} + q_t (\chi + u_t).$$

Modeling of preference variables (in P2 models):

$$\mu_i = \alpha_i - \beta_i \sum_{j=1}^5 \alpha_j.$$

Variances and covariances:

$$\sigma_{vv}^{ij} = E(v_{it} v_{jt}), \quad \sigma_{\mu\mu}^{ij} = E(\mu_i \mu_j), \quad i, j = 1, \dots, 5,$$

$$\sigma_{\alpha\alpha}^{ii} = E(\alpha_i \alpha_i), \quad i = 1, \dots, 5,$$

$$\sigma_{\varepsilon\varepsilon}^{kr} = E(\varepsilon_{kt} \varepsilon_{rt}), \quad \sigma_{\lambda\lambda}^{kr} = E(\lambda_k \lambda_r), \quad k, r = 1, 2,$$

$$\sigma_{\chi\chi} = \text{var}(\chi), \quad \sigma_{uu} = E(u_t^2), \quad \sigma_{vv} = E(v_t^2),$$

$$\sigma_{\chi z}^m = \text{cov}(\chi, z_m), \quad m = 1, 2,$$

$$\sigma_{\chi\mu}^i = \text{cov}(\chi, \mu_i), \quad i = 1, \dots, 5,$$

$$\sigma_{\chi\lambda}^k = \text{cov}(\chi, \lambda_k), \quad k = 1, 2.$$

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Table 1
Classification of hypotheses and models

A specific model is labeled $E_i P_j C_h M_k$, which means that the model is based on hypothesis E_i w.r.t. the distribution of latent total expenditure (ξ), hypothesis P_j w.r.t. the distribution of preference variables (μ, λ), hypothesis C_h w.r.t. the covariation between latent total expenditure and preference variables, and hypothesis M_k w.r.t. the contemporaneous covariances of the measurement errors (v, ϵ).

1. Hypotheses w.r.t. the distribution of latent total expenditure

Label	Parameter restrictions			Interpretation
	$\sigma_{\chi\chi}$	q_2	σ_{uu}	
E3	free	free	free	No restrictions
E2	free	free	0	Equal growth factor (q_2) for all consumers
E1	free	1	0	Constant latent total expenditure over time for each consumer

2. Hypotheses w.r.t. the distribution of preference variables

Label	Parameter restrictions		Interpretation
	$\sum_{\mu\mu}$	$\sum_{\lambda\lambda}$	
P3	free ¹	free	No restrictions on covariances between preference variables (μ) ¹
P2	$\sum_{\alpha\alpha} = \text{free}^2$	free	LES interpretation with independently distributed necessity quantities
P1	0	0	No individual differences in preferences

¹ Except for the restrictions from the adding-up condition
² $\sum_{\mu\mu} = (I - b1') \sum_{\alpha\alpha} (I - 1b')$, $\sum_{\alpha\alpha} = \text{diagonal}$

3. Hypotheses w.r.t. covariation between latent total expenditure and preference variables

Label	Parameter restrictions		Interpretation
	$\sum_{\chi\mu}$	$\sum_{\chi\lambda}$	
C2	free	free	Preference variables are correlated with latent total expenditure
C1	0	0	Preference variables are uncorrelated with latent total expenditure

4. Hypotheses w.r.t. contemporaneous covariances of the measurement errors

Label	Parameter restrictions		Interpretation
	\sum_{vV}	$\sum_{\epsilon\epsilon}$	
M3	free	free	No restrictions
M2	$1' \sum_{vV} = 0$	free	No measurement error in total expenditure
M1	$\sum_{vV} = \text{diag}$	free	Measurement errors are uncorrelated across goods

Table 2
Overview of fitted models^{ab}

Covariance structure of measurement errors	Total expenditure distribution	Covariation between preferences and total expenditure					
		C2		C1			
		Preference distribution		Preference distribution			
		P3	P2	P3	P2	P1	
M3	E3	df	73	78	79	84	92
		χ^2	130.47	135.48	155.81	163.74	765.35
		AIC	0.963	0.962	0.957	0.954	0.842
	E2	df	74	79	80	85	93
		χ^2	140.92	153.83	160.99	169.18	766.69
		AIC	0.961	0.957	0.956	0.953	0.841
	E1	df	-a	-a	81	86	94
		χ^2			174.21	182.23	780.45
		AIC			0.951	0.949	0.838
M2	E3	df	78	83	84	89	97
		χ^2	150.76	163.86	277.62	284.94	1348.62
		AIC	0.959	0.956	0.928	0.926	0.795
M1	E3	df	83	88	89	94	102
		χ^2	147.99	157.55	175.17	183.47	1005.84
		AIC	0.957	0.955	0.950	0.947	0.740
	E2	df	84	89	90	95	103
		χ^2	167.71	181.30	190.53	199.14	1007.83
		AIC	0.951	0.947	0.945	0.943	0.737
	E1	df	-a	-a	91	96	104
		χ^2			207.14	215.64	1021.52
		AIC			0.939	0.936	0.734

^a The models are generated from all possible combinations of assumptions in the dimensions E, P, C, and M, see Table I for definitions. However, note that: (i) Models combining assumptions E1 and C2 are not identified and thus not fitted. (ii) Models combining C2 and P1 are equivalent to models combining C1 and P1, and only the latter are tabulated. (iii) Models combining M2 with E2 or E1 are immediately rejected by looking at the individual data, e.g. M2 and E1 implies that $x_1=x_2$ for each household while this is not true for any household in the sample. These models are thus not interesting and are left out.

^b For each model are presented the number of degrees of freedom (df), the chi square statistics (χ^2), and the Akaike information criterion (AIC) in comparison to a model with no restrictions on the covariance matrix, cf section 3 for definitions.

Table 3

Significance probabilities in likelihood ratio tests^a

1. Tests of E-hypotheses

Maintained assumptions	Null and alternative hypotheses	
	E1 vs E2	E2 vs E3
P3, C2, M3	b)	0.001227
P2, C2, M3	b)	0.000018
P3, C1, M3	0.000277	0.022848
P2, C1, M3	0.000303	0.019681
P1, C1, M3	0.000208	0.247034
P3, C2, M1	b)	0.000009
P2, C2, M1	b)	0.000001
P3, C1, M1	0.000046	0.000089
P2, C1, M1	0.000049	0.000075
P1, C1, M1	0.000216	0.158341

3. Tests of C-hypotheses

Maintained assumptions	Null and alternative hypotheses	
	C1 vs C2	
E3, P3, M3	0.000295	
E2, P3, M3	0.002691	
E3, P2, M3	0.000084	
E2, P2, M3	0.017702	
E3, P3, M2	0.000000	
E3, P2, M2	0.000000	
E3, P3, M1	0.000134	
E2, P3, M1	0.000859	
E3, P2, M1	0.000230	
E2, P2, M1	0.006645	

2. Test of P-hypotheses

Maintained assumptions	Null and alternative hypotheses	
	P1 vs P2	P2 vs P3
E3, C2, M3	0.000000	0.414661
E2, C2, M3	0.000000	0.024237
E3, C1, M3	0.000000	0.160136
E2, C1, M3	0.000000	0.146071
E1, C1, M3	0.000000	0.155137
E3, C2, M2	0.000000	0.022460
E3, C1, M2	0.000000	0.197909
E3, C2, M1	0.000000	0.088707
E2, C2, M1	0.000000	0.018435
E3, C1, M1	0.000000	0.140459
E2, C1, M1	0.000000	0.125668
E1, C1, M1	0.000000	0.130748

4. Tests of M-hypotheses

Maintained assumptions	Null and alternative hypotheses	
	M1 vs M3	M2 vs M3
E3, P3, C2	0.063621	0.001102
E2, P3, C2	0.002811	c)
E3, P2, C2	0.014752	0.000031
E2, P2, C2	0.002193	c)
E3, P3, C1	0.035921	0.000000
E2, P3, C1	0.001018	c)
E1, P3, C1	0.000280	c)
E3, P2, C1	0.031912	0.000000
E2, P2, C1	0.000870	c)
E1, P2, C1	0.000232	c)
E3, P1, C1	0.000000	0.000000
E2, P1, C1	0.000000	c)
E1, P1, C1	0.000000	c)

^a See Tables I and II for detailed definitions of hypotheses and models.

^b Since models with both E1 and C2 are not identified, and thus not estimated, the test can not be performed.

^c Since models with both M2 and E2 or E1 are not estimated (see Table II, footnote a) the test can not be performed.

Table 4.1

Variances of measurement errors in expenditures in M1 models

Commodity/ income measure	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expen- diture				No individual differences in preferences				
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	σ_{w}^{11}	9.77 (0.71)	9.90 (0.71)	10.15 (0.71)	10.16 (0.71)	9.82 (0.72)	9.82 (0.72)	10.31 (0.72)	10.33 (0.72)	10.32 (0.72)	10.34 (0.72)	21.58 (1.07)	21.70 (1.08)	21.68 (1.08)
Clothing and footwear	σ_{w}^{22}	12.81 (0.93)	13.25 (0.93)	13.49 (0.95)	13.50 (0.95)	13.15 (0.93)	13.14 (0.93)	13.57 (0.95)	13.56 (0.94)	13.70 (0.95)	13.70 (0.95)	19.01 (0.94)	19.05 (0.95)	19.05 (0.95)
Housing, fuel and furniture	σ_{w}^{33}	26.54 (2.05)	27.24 (1.94)	28.00 (1.96)	28.09 (1.97)	26.92 (1.96)	26.98 (1.97)	27.66 (1.92)	27.71 (1.92)	27.65 (1.92)	27.68 (1.92)	44.55 (2.23)	44.95 (2.25)	44.90 (2.25)
Travel and recreation	σ_{w}^{44}	87.51 (6.78)	82.46 (7.00)	98.59 (6.88)	96.67 (6.72)	89.02 (6.16)	88.32 (6.13)	93.32 (6.24)	92.60 (6.20)	94.29 (6.30)	93.63 (6.27)	114.29 (5.70)	113.75 (5.67)	113.86 (5.68)
Other goods and services	σ_{w}^{55}	5.19 (0.39)	5.37 (0.38)	5.48 (0.38)	5.47 (0.38)	5.32 (0.39)	5.31 (0.38)	5.47 (0.38)	5.47 (0.38)	5.51 (0.39)	5.50 (0.38)	8.48 (0.42)	8.55 (0.43)	8.55 (0.43)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 4.2

Variances and covariances of measurement errors in expenditures in M2 models^{a)}

Commodity/ income measure	Parameter	MODEL				
		Preference variables correlated with total expenditure		Preference variables uncorrelated with total expenditure		No individual differences in preferences
		E3P3C2	E3P2C2	E3P2C1	E3P2C1	E3P1C1
Food, beverages and tobacco	σ_{VV}^{11}	8.80 (0.62)	8.91 (0.62)	9.06 (0.64)	9.22 (0.64)	15.72 (0.78)
Clothing and footwear	σ_{VV}^{22}	10.96 (0.77)	11.08 (0.77)	10.96 (0.77)	10.99 (0.77)	14.49 (0.72)
Housing, fuel and furniture	σ_{VV}^{33}	23.51 (1.65)	23.70 (1.66)	24.12 (1.69)	24.19 (1.68)	31.41 (1.56)
Travel and recreation	σ_{VV}^{44}	36.09 (2.53)	36.03 (2.52)	39.04 (2.74)	38.93 (2.72)	51.36 (2.55)
Other goods and services	σ_{VV}^{55}	4.64 (0.35)	5.06 (0.35)	5.16 (0.36)	5.17 (0.36)	6.74 (1.02)
Food etc. vs. clothing etc.	σ_{VV}^{12}	-0.91 (0.49)	-0.82 (0.45)	-0.91 (0.50)	-0.73 (0.45)	-1.05 (0.53)
Food etc. vs. Housing etc.	σ_{VV}^{13}	-1.96 (0.72)	-1.40 (0.67)	-1.50 (0.74)	-1.03 (0.68)	-1.92 (0.78)
Food etc. vs. Travel etc.	σ_{VV}^{14}	-5.95 (0.93)	-6.84 (0.91)	-6.86 (0.99)	-7.72 (0.96)	-12.74 (1.09)
Food etc. vs. Others	σ_{VV}^{15}	0.02 (0.33)	0.15 (0.31)	0.21 (0.34)	0.26 (0.31)	-0.01 (0.36)
Clothing etc. vs. Housing etc.	σ_{VV}^{23}	-2.33 (0.80)	-3.37 (0.75)	-2.29 (0.81)	-3.11 (0.74)	-4.48 (0.76)
Clothing etc. vs. Travel etc.	σ_{VV}^{24}	-7.73 (1.06)	-6.86 (1.00)	-7.79 (1.10)	-7.18 (1.00)	-8.95 (1.01)
Clothing etc. vs. Others	σ_{VV}^{25}	0.21 (0.37)	-0.03 (0.33)	0.03 (0.37)	0.03 (0.33)	-0.01 (0.35)
Housing etc. vs. Travel etc.	σ_{VV}^{34}	-18.28 (1.70)	-18.04 (1.66)	-19.66 (1.81)	-19.31 (1.71)	-23.98 (1.64)
Housing etc. vs. Others	σ_{VV}^{35}	-0.74 (0.54)	-0.89 (0.49)	-0.67 (0.55)	-0.74 (0.49)	-1.03 (0.51)
Travel etc. vs. Others	σ_{VV}^{45}	-4.13 (0.70)	-4.29 (0.66)	-4.73 (0.74)	-4.72 (0.68)	-5.69 (0.95)

^{a)} See table 1 and 2 for model description. Standard deviations in parentheses.

Table 4.3
Variances and covariances of measurement errors in expenditures in M3 models

a)

Commodity/ income measure	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expen- diture						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	σ_{W}^{11}	10.24 (0.72)	10.28 (0.72)	10.16 (0.71)	10.22 (0.72)	10.13 (0.73)	10.16 (0.74)	10.40 (0.72)	10.44 (0.73)	10.41 (0.72)	10.44 (0.73)	22.06 (1.10)	22.00 (1.09)	21.99 (1.09)
Clothing and footwear	σ_{W}^{22}	13.40 (0.94)	13.57 (0.94)	13.49 (0.94)	13.59 (0.95)	13.38 (0.94)	13.51 (0.95)	13.54 (0.93)	13.66 (0.94)	13.66 (0.94)	13.80 (0.95)	19.25 (0.96)	19.23 (0.96)	19.24 (0.96)
Housing, fuel and furniture	σ_{W}^{33}	27.25 (1.98)	27.58 (1.94)	27.98 (1.96)	28.10 (1.97)	27.24 (1.98)	27.25 (1.98)	27.92 (1.94)	27.93 (1.94)	27.90 (1.94)	27.87 (1.93)	46.19 (2.30)	45.76 (2.29)	45.75 (2.29)
Travel and recreation	σ_{W}^{44}	87.22 (7.02)	83.89 (7.15)	99.08 (6.91)	97.17 (6.76)	90.61 (6.18)	89.45 (6.11)	93.62 (6.20)	92.48 (6.30)	94.45 (6.25)	93.37 (6.18)	114.65 (5.71)	115.28 (5.74)	115.45 (5.75)
Other goods and services	σ_{W}^{55}	5.43 (0.39)	5.47 (0.38)	5.48 (0.38)	5.50 (0.38)	5.42 (0.39)	5.43 (0.39)	5.48 (0.38)	5.49 (0.38)	5.52 (0.38)	5.53 (0.38)	8.77 (0.44)	8.71 (0.43)	8.72 (0.43)
Food etc. vs. Clothing etc.	σ_{W}^{12}	0.99 (0.59)	1.12 (0.56)	0.95 (0.58)	1.23 (0.55)	0.89 (0.59)	1.13 (0.56)	1.09 (0.59)	1.34 (0.56)	1.17 (0.59)	1.42 (0.56)	4.45 (0.74)	4.41 (0.74)	4.41 (0.74)
Food etc. vs. Housing etc.	σ_{W}^{13}	0.47 (0.85)	0.68 (0.83)	0.54 (0.84)	0.91 (0.81)	0.23 (0.86)	0.64 (0.84)	0.75 (0.84)	1.09 (0.82)	0.74 (0.84)	1.06 (0.82)	8.09 (1.16)	7.93 (1.15)	7.92 (1.15)
Food etc. vs. Travel etc.	σ_{W}^{14}	3.24 (1.64)	2.81 (1.59)	3.32 (1.58)	1.74 (1.49)	2.52 (1.53)	1.41 (1.46)	3.48 (1.52)	2.37 (1.44)	3.66 (1.53)	2.58 (1.45)	7.31 (1.79)	7.30 (1.79)	7.31 (1.79)
Food etc. vs. Others	σ_{W}^{15}	0.79 (0.38)	0.73 (0.37)	0.81 (0.37)	0.88 (0.36)	0.74 (0.38)	0.73 (0.37)	0.88 (0.37)	0.87 (0.36)	0.91 (0.38)	0.91 (0.36)	3.63 (0.51)	3.57 (0.50)	3.57 (0.50)
Clothing etc. vs. Housing etc.	σ_{W}^{23}	0.79 (0.97)	0.50 (0.91)	1.07 (0.96)	0.66 (0.92)	0.78 (0.97)	0.20 (0.91)	1.10 (0.96)	0.54 (0.90)	1.22 (0.96)	0.65 (0.90)	4.20 (1.06)	4.10 (1.06)	4.11 (1.06)
Clothing etc. vs. Travel etc.	σ_{W}^{24}	3.65 (1.81)	4.08 (1.77)	4.84 (1.82)	4.82 (1.70)	3.72 (1.72)	3.95 (1.56)	4.45 (1.72)	4.65 (1.56)	4.79 (1.74)	4.98 (1.57)	8.44 (1.68)	8.47 (1.68)	8.51 (1.69)
Clothing etc. vs. Others	σ_{W}^{25}	1.02 (0.43)	1.22 (0.40)	1.10 (0.43)	1.22 (0.40)	1.01 (0.43)	1.13 (0.40)	1.12 (0.43)	1.24 (0.40)	1.19 (0.43)	1.32 (0.40)	3.15 (0.47)	3.10 (0.47)	3.11 (0.47)
Housing etc. vs. Travel etc.	σ_{W}^{34}	-3.77 (2.68)	-2.85 (2.60)	-0.97 (2.61)	-0.34 (2.54)	-3.47 (2.53)	-2.83 (2.39)	-1.92 (2.50)	-1.23 (2.35)	-1.66 (2.51)	-0.95 (2.37)	7.30 (2.58)	7.25 (2.58)	7.31 (2.58)
Housing etc. vs. Others	σ_{W}^{35}	0.27 (0.63)	0.39 (0.60)	0.47 (0.61)	0.27 (0.60)	0.25 (0.63)	0.03 (0.60)	0.46 (0.61)	0.23 (0.59)	0.51 (0.61)	0.27 (0.59)	4.47 (0.73)	4.31 (0.72)	4.32 (0.72)
Travel etc. vs. Others	σ_{W}^{45}	0.62 (1.17)	0.91 (1.14)	1.39 (1.15)	1.35 (1.10)	0.63 (1.11)	0.94 (1.02)	1.15 (1.10)	1.47 (1.02)	1.38 (1.11)	1.70 (1.03)	5.74 (1.14)	5.69 (1.13)	5.72 (1.14)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 5

Variances and covariance of measurement errors in incomes^{a)}

		MODEL													
Income measure	Parameter	M ₁	Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure						No individual differences in preferences		
			E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Income measure 1	$\sigma_{\epsilon\epsilon}^{11}$	M1	58.83 (4.49)	55.95 (4.46)	62.05 (4.35)	62.15 (4.36)	57.44 (4.44)	57.43 (4.44)	61.85 (4.34)	61.86 (4.34)	63.27 (4.44)	63.27 (4.44)	181.03 (9.77)	175.45 (9.31)	178.58 (9.47)
Income measure 2	$\sigma_{\epsilon\epsilon}^{22}$	M1	101.09 (8.72)	87.78 (9.17)	111.84 (7.85)	112.40 (7.89)	92.82 (9.13)	92.81 (9.11)	111.71 (7.85)	111.73 (7.86)	114.64 (8.04)	114.64 (8.04)	167.81 (22.79)	128.98 (8.90)	135.55 (9.33)
Income measure 1 vs. income measure 2	$\sigma_{\epsilon\epsilon}^{12}$	M1	57.14 (5.54)	50.82 (5.62)	62.96 (5.18)	63.31 (5.20)	53.57 (5.59)	53.56 (5.58)	62.70 (5.17)	62.70 (5.17)	64.92 (5.31)	64.92 (5.31)	114.61 (11.41)	99.89 (8.27)	105.32 (8.57)
Income measure 1	$\sigma_{\epsilon\epsilon}^{11}$	M2	61.44 (4.31)	61.43 (4.31)			62.36 (4.37)	62.36 (4.37)					268.33 (13.30)		
Income measure 2	$\sigma_{\epsilon\epsilon}^{22}$	M2	61.88 (5.09)	61.86 (5.09)			63.59 (5.22)	63.59 (5.22)					371.83 (21.67)		
Income measure 1 vs. income measure 2	$\sigma_{\epsilon\epsilon}^{12}$	M2	109.59 (7.68)	109.57 (7.68)			112.77 (7.91)	112.77 (7.91)					908.94 (45.05)		
Income measure 1	$\sigma_{\epsilon\epsilon}^{11}$	M3	56.92 (4.48)	55.86 (4.46)	62.04 (4.35)	62.18 (4.36)	58.22 (4.50)	58.15 (4.50)	61.88 (4.34)	61.88 (4.34)	63.27 (4.44)	63.27 (4.44)	180.50 (10.60)	187.88 (9.85)	191.77 (10.05)
Income measure 2	$\sigma_{\epsilon\epsilon}^{22}$	M3	52.00 (5.66)	49.68 (5.68)	62.91 (5.18)	63.31 (5.20)	54.84 (5.82)	54.69 (5.80)	62.71 (5.17)	62.71 (5.17)	64.92 (5.31)	64.92 (5.31)	98.06 (13.95)	118.09 (8.84)	124.94 (9.13)
Income measure 1 vs. income measure 2	$\sigma_{\epsilon\epsilon}^{12}$	M3	88.53 (9.36)	83.51 (9.50)	111.77 (7.85)	112.35 (7.88)	94.71 (9.84)	94.42 (9.80)	111.66 (7.85)	111.67 (7.85)	114.64 (8.04)	114.64 (8.04)	81.29 (31.27)	135.58 (9.24)	143.38 (9.73)

^{a)} See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 6.1
Squared coefficient of multiple correlation in M1 models^{a)}

Commodity/ income measure	MODEL																									
	Preference variables correlated with total expenditure								Preference variables uncorrelated with total expenditure								No individual differences in preferences									
	E3P3C2		E3P2C2		E3P3C2		E2P2C2		E3P3C1		E3P2C1		E2P3C1		E2P2C1		E1P3C1		E1P2C1		E3P1C1		E2P1C1		E1P1C1	
	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period	Period
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	
Food, beverages and tobacco	0.70	0.72	0.70	0.72	0.70	0.71	0.69	0.71	0.70	0.72	0.70	0.72	0.68	0.71	0.68	0.71	0.70	0.70	0.70	0.70	0.36	0.37	0.36	0.37	0.36	0.36
Clothing and footwear	0.42	0.47	0.42	0.44	0.40	0.43	0.39	0.44	0.41	0.45	0.41	0.45	0.40	0.43	0.40	0.43	0.41	0.41	0.41	0.41	0.17	0.18	0.17	0.18	0.18	0.18
Housing, fuel and furniture	0.51	0.57	0.52	0.55	0.49	0.55	0.48	0.55	0.51	0.56	0.51	0.55	0.50	0.55	0.50	0.55	0.53	0.53	0.53	0.53	0.22	0.25	0.22	0.24	0.23	0.23
Travel and recreation	0.39	0.48	0.41	0.51	0.32	0.40	0.35	0.41	0.41	0.45	0.42	0.46	0.38	0.42	0.39	0.43	0.40	0.40	0.40	0.40	0.25	0.27	0.26	0.28	0.27	0.27
Other goods and services	0.49	0.54	0.48	0.51	0.46	0.51	0.46	0.52	0.48	0.52	0.48	0.52	0.47	0.51	0.47	0.51	0.49	0.49	0.49	0.49	0.20	0.22	0.19	0.21	0.20	0.20
Income measure 1	0.89	0.89	0.89	0.90	0.88	0.89	0.88	0.89	0.89	0.89	0.89	0.89	0.88	0.89	0.88	0.89	0.88	0.88	0.88	0.88	0.65	0.67	0.66	0.68	0.66	0.66
Income measure 2	0.94	0.94	0.95	0.95	0.93	0.94	0.93	0.94	0.94	0.95	0.94	0.95	0.93	0.94	0.93	0.94	0.93	0.93	0.93	0.93	0.90	0.91	0.92	0.93	0.92	0.92

^{a)} See table 1 and 2 for model descriptions. See section 4.2 for definition of the squared coefficient of multiple correlation.

Table 6.2
Squared coefficient of multiple correlation in M2 models^{a)}

Commodity/ income measure	MODEL									
	Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure				No individual differences in preferences	
	E3P3C2 Period		E3P2C2 Period		E3P3C1 Period		E3P2C1 Period		E3P1C1 Period	
	1	2	1	2	1	2	1	2	1	2
Food, beverages and tobacco	0.73	0.75	0.72	0.74	0.71	0.74	0.71	0.73	0.51	0.56
Clothing and footwear	0.50	0.55	0.50	0.54	0.50	0.55	0.50	0.55	0.34	0.40
Housing, fuel and furniture	0.56	0.61	0.56	0.51	0.55	0.60	0.55	0.60	0.42	0.50
Travel and recreation	0.73	0.79	0.74	0.79	0.74	0.78	0.74	0.78	0.64	0.70
Other goods and services	0.50	0.54	0.50	0.54	0.49	0.53	0.48	0.53	0.34	0.40
Income measure 1	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.48	0.51
Income measure 2	0.94	0.94	0.94	0.94	0.93	0.93	0.93	0.93	0.46	0.50

^{a)} See table 1 and 2 for model descriptions. See section 4.2 for definition of the squared coefficient of multiple correlation.

Table 6.3
Squared coefficient of multiple correlation in M3 models^{a)}

Commodity/ income measure	MODEL																									
	Preference variables correlated with total expenditure								Preference variables uncorrelated with total expenditure								No individual differences in preferences									
	E3P3C2 Period		E3P2C2 Period		E3P3C2 Period		E2P2C2 Period		E3P3C1 Period		E3P2C1 Period		E2P3C1 Period		E2P2C1 Period		E1P3C1 Period		E1P2C1 Period		E3P1C1 Period		E2P1C1 Period		E1P1C1 Period	
	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
Food, beverages and tobacco	0.70	0.70	0.69	0.70	0.70	0.70	0.69	0.71	0.69	0.71	0.69	0.71	0.68	0.70	0.68	0.70	0.69	0.69	0.69	0.69	0.35	0.36	0.35	0.36	0.35	0.35
Clothing and footwear	0.41	0.43	0.41	0.42	0.40	0.43	0.39	0.43	0.41	0.43	0.41	0.43	0.40	0.43	0.39	0.42	0.41	0.41	0.40	0.40	0.16	0.17	0.16	0.17	0.17	0.17
Housing, fuel and furniture	0.52	0.55	0.52	0.53	0.49	0.54	0.48	0.55	0.51	0.55	0.51	0.55	0.50	0.54	0.50	0.54	0.52	0.52	0.52	0.52	0.20	0.22	0.21	0.23	0.22	0.22
Travel and recreation	0.39	0.48	0.41	0.50	0.32	0.40	0.34	0.41	0.41	0.44	0.41	0.45	0.39	0.42	0.39	0.43	0.40	0.40	0.40	0.40	0.25	0.27	0.25	0.27	0.26	0.26
Other goods and services	0.48	0.50	0.48	0.50	0.47	0.51	0.46	0.51	0.48	0.51	0.47	0.51	0.47	0.51	0.47	0.50	0.49	0.49	0.48	0.48	0.18	0.19	0.18	0.20	0.19	0.19
Income measure 1	0.89	0.90	0.89	0.90	0.88	0.89	0.88	0.89	0.89	0.89	0.89	0.89	0.88	0.89	0.88	0.89	0.88	0.88	0.88	0.88	0.65	0.67	0.63	0.66	0.64	0.64
Income measure 2	0.95	0.95	0.95	0.95	0.93	0.94	0.93	0.94	0.94	0.95	0.94	0.95	0.93	0.94	0.93	0.94	0.93	0.93	0.94	0.94	0.95	0.96	0.92	0.93	0.92	0.92

^{a)} See table 1 and 2 for model descriptions. See section 4.2 for definition of the squared coefficient of multiple correlation.

Table 7.1
Distribution of latent total expenditure in M1 models^{a)}

Parameter	MODEL												
	Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure						No individual differences in preferences		
	E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
σ_{XX}	341.96 (34.19)	356.04 (33.91)	359.41 (34.19)	360.57 (33.96)	380.02 (33.68)	380.56 (33.72)	381.12 (33.16)	381.90 (33.20)	420.96 (34.93)	421.74 (34.95)	266.98 (25.05)	262.14 (24.70)	275.98 (25.72)
q_2	1.168 (0.048)	1.139 (0.040)	1.152 (0.044)	1.159 (0.043)	1.104 (0.030)	1.104 (0.030)	1.106 (0.027)	1.106 (0.027)	1 ^{b)}	1 ^{b)}	1.052 (0.014)	1.051 (0.014)	1 ^{b)}
σ_{uu}	35.20 (9.63)	29.85 (7.53)	0 ^{b)}	0 ^{b)}	15.15 (4.60)	15.39 (4.63)	0 ^{b)}	0 ^{b)}	0 ^{b)}	0 ^{b)}	-3.30 (2.01)	0 ^{b)}	0 ^{b)}
q_0	-3.696 (2.184)	-2.556 (1.882)	-3.071 (1.986)	-3.341 (1.967)	-1.165 (1.527)	-1.163 (1.530)	-1.244 (1.413)	-1.242 (1.414)	2.979 (0.863)	2.979 (0.861)	0.904 (1.178)	0.956 (1.178)	2.979 (1.011)
σ_{VV}	141.83 (7.26)	138.22 (7.34)	155.70 (7.27)	153.89 (7.12)	144.21 (6.62)	143.57 (6.58)	150.33 (6.68)	149.67 (6.64)	151.46 (6.74)	150.85 (6.71)	207.91 (6.30)	208.01 (6.28)	208.02 (6.29)
cV	0.489 (0.026)	0.494 (0.026)	0.477 (0.026)	0.478 (0.026)	0.500 (0.026)	0.501 (0.026)	0.491 (0.026)	0.492 (0.026)	0.516 (0.026)	0.517 (0.026)	0.409 (0.022)	0.407 (0.022)	0.418 (0.023)
cA	0.907 (0.025)	0.923 (0.020)	1 ^{b)}	1 ^{b)}	0.962 (0.012)	0.961 (0.012)	1 ^{b)}	1 ^{b)}	1 ^{b)}	1 ^{b)}	1.013 (0.008)	1 ^{b)}	1 ^{b)}
σ_{Xz}^1	8.52 (1.36)	8.64 (1.37)	8.64 (1.37)	8.59 (1.37)	8.80 (1.40)	8.80 (1.40)	8.82 (1.40)	8.82 (1.40)	9.19 (1.46)	9.19 (1.46)	9.07 (1.27)	9.06 (1.27)	9.19 (1.30)
σ_{Xz}^2	9.70 (1.07)	9.83 (1.07)	9.77 (1.07)	9.74 (1.07)	10.00 (1.09)	10.00 (1.09)	9.99 (1.09)	9.99 (1.13)	10.52 (1.13)	10.52 (1.13)	10.23 (1.00)	10.23 (1.00)	10.52 (1.02)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

b) A priori restrictions.

Table 7.2
Distribution of latent total expenditure in M2 models^{a)}

Parameter	MODEL				
	Preference variables correlated with total expenditure		Preference variables uncorrelated with total expenditure		No individual differences in preferences
	E3P3C2	E3P2C2	E3P3C1	E3P2C1	E3P1C1
σ_{XX}	330.13 (31.24)	329.38 (31.22)	348.05 (32.91)	348.05 (32.91)	348.05 (32.91)
q_2	1.158 (0.041)	1.158 (0.042)	1.147 (0.041)	1.147 (0.041)	1.147 (0.041)
σ_{uu}	157.05 (12.36)	157.08 (12.37)	158.56 (12.48)	158.56 (12.48)	158.56 (12.48)
q_0	-3.311 (1.938)	-3.302 (1.939)	-2.864 (1.920)	-2.864 (1.920)	-2.864 (1.920)
σ_{VV}	0 ^{b)}	0 ^{b)}	0 ^{b)}	0 ^{b)}	0 ^{b)}
cV	0.555 (0.025)	0.555 (0.025)	0.566 (0.025)	0.566 (0.025)	0.566 (0.025)
cA	0.678 (0.027)	0.677 (0.027)	0.687 (0.026)	0.687 (0.026)	0.687 (0.026)
σ_{Xz}^1	8.47 (1.34)	8.48 (1.34)	8.52 (1.36)	8.52 (1.36)	8.52 (1.36)
σ_{Xz}^2	9.78 (1.05)	9.78 (1.05)	9.83 (1.07)	9.83 (1.07)	9.83 (1.07)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

b) A priori restrictions.

Table 7.3

Distribution of latent total expenditure in M3 models^{a)}

Parameter	MODEL												
	Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure						No individual differences in preferences		
	E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
σ_{XX}	357.993 (34.250)	363.971 (34.078)	352.443 (35.200)	347.997 (35.052)	376.315 (33.895)	377.508 (33.974)	372.864 (33.619)	374.288 (33.706)	406.712 (35.250)	408.170 (35.312)	246.341 (26.377)	251.127 (26.579)	262.702 (27.634)
q_2	1.118 (0.037)	1.104 (0.033)	1.136 (0.047)	1.161 (0.050)	1.094 (0.029)	1.094 (0.029)	1.096 (0.028)	1.096 (0.028)	1 ^{b)}	1 ^{b)}	1.045 (0.013)	1.047 (0.013)	1 ^{b)}
σ_{uu}	21.309 (9.355)	20.694 (6.273)	0 ^{b)}	0 ^{b)}	10.384 (4.636)	10.734 (4.692)	0 ^{b)}	0 ^{b)}	0 ^{b)}	0 ^{b)}	3.995 (2.222)	0 ^{b)}	0 ^{b)}
q_0	-1.693 (1.793)	-1.148 (1.656)	-2.432 (2.114)	-3.430 (2.240)	-0.769 (1.500)	-0.763 (1.503)	-0.839 (1.467)	-0.834 (1.469)	2.979 (0.940)	2.979 (0.938)	1.172 (1.394)	1.110 (1.391)	2.979 (1.261)
σ_{VV}	159.68 (13.82)	159.95 (13.26)	183.20 (12.80)	180.05 (12.60)	161.56 (12.45)	160.47 (12.44)	176.04 (12.00)	175.16 (11.94)	179.73 (12.24)	178.90 (12.19)	324.47 (16.28)	323.23 (16.22)	323.74 (16.25)
c_V	0.490 (0.026)	0.493 (0.026)	0.472 (0.027)	0.469 (0.027)	0.495 (0.026)	0.496 (0.027)	0.486 (0.026)	0.487 (0.026)	0.507 (0.027)	0.508 (0.027)	0.398 (0.024)	0.399 (0.024)	0.408 (0.025)
c_A	0.944 (0.019)	0.946 (0.016)	1 ^{b)}	1 ^{b)}	0.973 (0.012)	0.972 (0.011)	1 ^{b)}	1 ^{b)}	1 ^{b)}	1 ^{b)}	0.984 (0.009)	1 ^{b)}	1 ^{b)}
σ_{Xz}^1	8.750 (1.386)	8.803 (1.395)	8.720 (1.381)	8.601 (1.370)	8.855 (1.406)	8.854 (1.407)	8.872 (1.403)	8.871 (1.404)	9.187 (1.461)	9.187 (1.462)	9.109 (1.330)	9.117 (1.333)	9.187 (1.362)
σ_{Xz}^2	9.931 (1.082)	9.994 (1.087)	9.845 (1.082)	9.733 (1.075)	10.055 (1.094)	10.056 (1.095)	10.045 (1.091)	10.046 (1.092)	10.524 (1.132)	10.524 (1.133)	10.228 (1.041)	10.222 (1.043)	10.524 (1.065)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

b) A priori restrictions.

Table 8.1
 Variances and covariances of preference variables (μ) in M1 models^{a)}

Commodity/ income measure	Parameter	MODEL									
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure					
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1
Food, beverages and tobacco	σ_{μ}^{11}	7.90 (1.41)	7.52 (1.15)	9.57 (3.42)	7.08 (1.44)	6.23 (0.84)	5.94 (0.79)	6.00 (0.84)	5.68 (0.79)	5.96 (0.84)	5.63 (0.79)
Clothing and footwear	σ_{μ}^{22}	3.20 (0.76)	3.82 (0.88)	3.36 (1.14)	2.87 (0.72)	3.01 (0.73)	3.05 (0.72)	2.82 (0.74)	2.85 (0.72)	2.77 (0.74)	2.79 (0.72)
Housing, fuel and furniture	σ_{μ}^{33}	8.20 (1.52)	12.82 (2.56)	7.00 (1.47)	8.26 (1.65)	7.74 (1.46)	8.10 (1.41)	7.27 (1.46)	7.70 (1.40)	7.19 (1.46)	7.63 (1.40)
Travel and recreation	σ_{μ}^{44}	15.75 (4.26)	36.76 (12.10)	15.82 (8.13)	7.53 (2.50)	10.32 (2.57)	10.49 (2.54)	9.24 (2.60)	9.40 (2.57)	8.98 (2.62)	9.15 (2.59)
Other goods and services	σ_{μ}^{55}	1.43 (0.34)	1.87 (0.44)	1.25 (0.37)	1.17 (0.33)	1.32 (0.33)	1.34 (0.33)	1.19 (0.33)	1.24 (0.33)	1.20 (0.33)	1.24 (0.33)
Food etc. vs. Clothing etc.	σ_{μ}^{12}	0.06 (0.73)	0.74 (0.62)	1.15 (1.54)	-0.89 (0.24)	-0.20 (0.51)	-0.59 (0.22)	-0.18 (0.51)	-0.59 (0.22)	-0.17 (0.51)	-0.59 (0.22)
Food etc. vs. Housing etc.	σ_{μ}^{13}	-0.12 (1.07)	1.09 (1.35)	-1.10 (1.50)	-2.75 (0.58)	-0.80 (0.76)	-1.96 (0.49)	-0.83 (0.76)	-1.99 (0.49)	-0.88 (0.76)	-2.01 (0.50)
Food etc. vs. Travel etc.	σ_{μ}^{14}	-7.85 (2.08)	-10.20 (3.22)	-9.71 (4.95)	-2.82 (0.77)	-4.94 (1.11)	-3.16 (0.75)	-4.70 (1.12)	-2.85 (0.75)	-4.61 (1.12)	-2.78 (0.75)
Food etc. vs. Others	σ_{μ}^{15}	0.01 (0.50)	0.85 (0.53)	0.09 (0.79)	-0.61 (0.23)	-0.29 (0.35)	-0.23 (0.16)	-0.29 (0.35)	-0.25 (0.17)	-0.30 (0.35)	-0.25 (0.17)
Clothing etc. vs. Housing etc.	σ_{μ}^{23}	-2.11 (0.72)	0.93 (0.94)	-2.32 (0.87)	-1.18 (0.46)	-2.19 (0.69)	-0.90 (0.37)	-2.19 (0.69)	-0.90 (0.37)	-2.18 (0.69)	-0.90 (0.37)
Clothing etc. vs. Travel etc.	σ_{μ}^{24}	-1.33 (1.31)	-6.15 (2.30)	-2.46 (2.50)	-0.80 (0.61)	-0.76 (0.93)	-1.61 (0.56)	-0.60 (0.93)	-1.40 (0.55)	-0.58 (0.94)	-1.34 (0.55)
Clothing etc. vs. Others	σ_{μ}^{25}	0.18 (0.33)	0.66 (0.38)	0.27 (0.43)	-0.01 (0.14)	0.14 (0.31)	0.04 (0.13)	0.15 (0.31)	0.03 (0.13)	0.16 (0.31)	0.03 (0.13)
Housing etc. vs. Travel etc.	σ_{μ}^{34}	-5.46 (2.05)	-15.94 (4.98)	-2.81 (2.38)	-3.84 (1.43)	-4.10 (1.52)	-4.91 (1.22)	-3.57 (1.53)	-4.46 (1.23)	-3.43 (1.54)	-4.37 (1.24)
Housing etc. vs. Others	σ_{μ}^{35}	-0.51 (0.49)	1.09 (0.74)	-0.77 (0.48)	-0.49 (0.34)	-0.65 (0.46)	-0.33 (0.27)	-0.68 (0.46)	-0.35 (0.28)	-0.70 (0.46)	-0.36 (0.28)
Travel etc. vs. Others	σ_{μ}^{45}	-1.11 (0.90)	-4.48 (1.91)	-0.84 (1.26)	-0.06 (0.47)	-0.52 (0.63)	-0.81 (0.42)	-0.37 (0.64)	-0.69 (0.43)	-0.36 (0.64)	-0.66 (0.43)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Variations and covariances of preference variables (μ) in M2 models^{a)}

Commodity/ income measure	Parameter	MODEL			
		Preference variables correlated with total expenditure		Preference variables uncorrelated with total expenditure	
		E3P3C2	E3P2C2	E3P3C1	E3P2C1
Food, beverages and tobacco	$\sigma_{\mu\mu}^{11}$	7.95 (1.00)	7.10 (0.88)	6.73 (0.85)	6.40 (0.82)
Clothing and footwear	$\sigma_{\mu\mu}^{22}$	3.53 (0.74)	3.61 (0.75)	3.53 (0.74)	3.52 (0.73)
Housing, fuel and furniture	$\sigma_{\mu\mu}^{33}$	8.58 (1.70)	8.66 (1.67)	7.32 (1.60)	7.41 (1.57)
Travel and recreation	$\sigma_{\mu\mu}^{44}$	20.67 (3.44)	19.75 (3.38)	12.52 (2.63)	12.29 (2.58)
Other goods and services	$\sigma_{\mu\mu}^{55}$	1.89 (2.41)	1.78 (0.35)	1.58 (1.78)	1.59 (0.34)
Food etc. vs. Clothing etc.	$\sigma_{\mu\mu}^{12}$	-0.05 (0.61)	-0.27 (0.23)	-0.15 (0.56)	-0.65 (0.21)
Food etc. vs. Housing etc.	$\sigma_{\mu\mu}^{13}$	0.88 (0.92)	-0.76 (0.46)	-0.38 (0.82)	-1.48 (0.43)
Food etc. vs. Travel etc.	$\sigma_{\mu\mu}^{14}$	-9.15 (1.49)	-0.95 (0.93)	-6.00 (1.16)	-3.96 (0.75)
Food etc. vs. Others	$\sigma_{\mu\mu}^{15}$	0.37 (0.43)	-0.12 (0.14)	-0.20 (0.38)	-0.31 (0.13)
Clothing etc. vs. Housing etc.	$\sigma_{\mu\mu}^{23}$	-2.08 (0.81)	0.10 (0.45)	-2.19 (0.79)	-0.64 (0.40)
Clothing etc. vs. Travel etc.	$\sigma_{\mu\mu}^{24}$	-1.40 (1.20)	-3.61 (0.84)	-1.14 (1.04)	-2.20 (0.68)
Clothing etc. vs. Others	$\sigma_{\mu\mu}^{25}$	0.00 (0.37)	0.18 (0.15)	-0.05 (0.36)	-0.03 (0.12)
Housing etc. vs. Travel etc.	$\sigma_{\mu\mu}^{34}$	-7.62 (2.03)	-8.17 (1.72)	-4.40 (1.68)	-5.09 (1.36)
Housing etc. vs. Others	$\sigma_{\mu\mu}^{35}$	0.24 (0.56)	0.18 (0.23)	-0.35 (0.53)	-0.21 (0.25)
Travel etc. vs. Others	$\sigma_{\mu\mu}^{45}$	-2.50 (2.15)	-2.02 (0.49)	-0.98 (1.62)	-1.04 (0.44)

^{a)} See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 8.3
Variances and covariances of preference variables (μ) in M3 models^{a)}

Commodity/ income measure	Parameter	MODEL									
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure					
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1
Food, beverages and tobacco	σ_{μ}^{11}	12.29 (3.95)	9.33 (1.84)	10.74 (4.57)	7.10 (1.23)	6.58 (0.86)	6.19 (0.80)	6.58 (0.86)	6.17 (0.80)	6.56 (0.86)	6.16 (0.80)
Clothing and footwear	σ_{μ}^{22}	3.84 (1.08)	4.91 (1.10)	3.80 (0.99)	3.51 (0.73)	3.51 (0.74)	3.43 (0.72)	3.51 (0.74)	3.43 (0.72)	3.51 (0.74)	3.41 (0.72)
Housing, fuel and furniture	σ_{μ}^{33}	9.30 (3.60)	16.48 (4.83)	6.44 (1.68)	7.11 (1.64)	6.45 (1.61)	6.92 (1.51)	6.40 (1.61)	6.92 (1.51)	6.32 (1.61)	6.85 (1.51)
Travel and recreation	σ_{μ}^{44}	39.74 (17.78)	62.09 (25.14)	19.40 (11.68)	9.06 (2.75)	10.27 (2.68)	10.43 (2.63)	10.19 (2.70)	10.32 (2.65)	10.16 (2.71)	10.31 (2.66)
Other goods and services	σ_{μ}^{55}	2.03 (0.76)	2.69 (0.71)	1.45 (0.37)	1.44 (0.34)	1.44 (0.34)	1.51 (0.34)	1.43 (0.35)	1.50 (0.34)	1.45 (0.35)	1.52 (0.34)
Food etc. vs. Clothing etc.	σ_{μ}^{12}	1.29 (1.79)	1.73 (1.13)	1.02 (1.61)	-0.90 (0.21)	-0.08 (0.56)	-0.74 (0.22)	-0.08 (0.56)	-0.75 (0.22)	-0.06 (0.56)	-0.75 (0.22)
Food etc. vs. Housing etc.	σ_{μ}^{13}	3.34 (3.00)	4.15 (2.76)	-0.09 (1.95)	-2.27 (0.56)	-0.72 (0.82)	-1.73 (0.51)	-0.72 (0.83)	-1.75 (0.52)	-0.75 (0.83)	-1.73 (0.53)
Food etc. vs. Travel etc.	σ_{μ}^{14}	-18.43 (7.58)	-16.97 (6.44)	-11.68 (6.98)	-3.35 (0.81)	-5.45 (1.18)	-3.38 (0.77)	-5.44 (1.18)	-3.33 (0.77)	-5.41 (1.18)	-3.34 (0.77)
Food etc. vs. Others	σ_{μ}^{15}	1.51 (1.42)	1.76 (1.02)	0.01 (0.94)	-0.58 (0.19)	-0.34 (0.38)	-0.34 (0.17)	-0.34 (0.38)	-0.34 (0.17)	-0.34 (0.38)	-0.35 (0.17)
Clothing etc. vs. Housing etc.	σ_{μ}^{23}	-1.06 (1.40)	2.74 (1.86)	-1.87 (0.92)	-0.95 (0.47)	-2.04 (0.79)	-0.78 (0.38)	-2.03 (0.79)	-0.80 (0.39)	-2.00 (0.79)	-0.78 (0.39)
Clothing etc. vs. Travel etc.	σ_{μ}^{24}	-4.54 (4.15)	-10.54 (4.39)	-3.07 (2.68)	-1.53 (0.64)	-1.42 (1.04)	-1.82 (0.55)	-1.43 (1.05)	-1.80 (0.54)	-1.48 (1.05)	-1.78 (0.54)
Clothing etc. vs. Others	σ_{μ}^{25}	0.46 (0.68)	1.16 (0.69)	0.11 (0.43)	-0.12 (0.13)	0.02 (0.36)	-0.09 (0.12)	0.02 (0.36)	-0.09 (0.12)	0.03 (0.36)	-0.10 (0.12)
Housing etc. vs. Travel etc.	σ_{μ}^{34}	-12.17 (7.56)	-26.17 (10.70)	-3.78 (3.30)	-3.66 (1.49)	-2.99 (1.71)	-4.28 (1.32)	-2.93 (1.72)	-4.25 (1.32)	-2.85 (1.72)	-4.22 (1.33)
Housing etc. vs. Others	σ_{μ}^{35}	0.60 (1.20)	2.80 (1.64)	-0.70 (0.56)	-0.22 (0.34)	-0.71 (0.53)	-0.13 (0.30)	-0.73 (0.53)	-0.13 (0.31)	-0.72 (0.53)	-0.12 (0.30)
Travel etc. vs. Others	σ_{μ}^{45}	-4.60 (3.30)	-8.42 (3.94)	-0.88 (1.52)	-0.51 (0.50)	-0.41 (0.71)	-0.94 (0.44)	-0.39 (0.72)	-0.94 (0.44)	-0.42 (0.72)	-0.96 (0.45)

^{a)} See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 9
Variances of preference variables (σ)^{a)}

Commodity/ income measure	Parameter	MODEL											
		M1					M2		M3				
		E3P2C2	E2P2C2	E3P2C1	E2P2C1	E1P2C1	E3P2C2	E3P2C1	E3P2C2	E2P2C2	E3P2C1	E2P2C1	E1P2C1
Food, beverages and tobacco	σ_{α}^{11}	6.63 (1.13)	8.29 (1.48)	7.21 (1.12)	6.96 (1.13)	6.92 (1.13)	7.80 (1.07)	7.53 (1.07)	6.72 (1.19)	8.37 (1.31)	7.61 (1.14)	7.60 (1.14)	7.60 (1.14)
Clothing and footwear	σ_{α}^{22}	3.22 (0.95)	3.11 (0.93)	3.23 (0.92)	3.04 (0.92)	2.97 (0.92)	3.43 (0.94)	3.74 (0.94)	3.78 (0.94)	3.91 (0.92)	3.78 (0.91)	3.79 (0.91)	3.77 (0.90)
Housing, fuel and furniture	σ_{α}^{33}	11.53 (2.59)	12.82 (3.56)	11.13 (2.54)	10.79 (2.56)	10.78 (2.57)	8.94 (2.59)	9.06 (2.60)	9.88 (3.00)	9.33 (3.26)	8.65 (2.85)	8.68 (2.87)	8.51 (2.90)
Travel and recreation	σ_{α}^{44}	230.72 (179.36)	10.20 (6.34)	18.14 (6.08)	15.68 (5.90)	15.11 (5.89)	64.02 (17.97)	26.11 (8.78)	1202.22 (1845.84)	14.96 (7.86)	18.43 (6.36)	17.99 (6.23)	17.93 (6.21)
Other goods and services	σ_{α}^{55}	1.16 (0.45)	1.00 (0.43)	1.13 (0.41)	1.04 (0.41)	1.04 (0.42)	1.63 (0.41)	1.53 (0.40)	1.51 (0.47)	1.35 (0.43)	1.38 (0.42)	1.36 (0.43)	1.39 (0.43)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 10

Variances and covariance of preference variables (λ)^{a)}

Income measure	Parameter	M _i	MODEL									
			Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure					
			E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1
Income measure 1	$\sigma_{\lambda\lambda}^{11}$	M1	202.38 (19.88)	191.05 (16.85)	196.49 (22.52)	205.72 (23.43)	192.57 (16.68)	193.23 (16.71)	190.29 (16.78)	191.00 (16.81)	189.54 (16.77)	190.31 (16.80)
Income measure 2	$\sigma_{\lambda\lambda}^{22}$	M1	799.31 (79.39)	724.06 (60.03)	749.35 (91.96)	834.30 (105.27)	721.53 (58.80)	724.83 (58.95)	708.11 (59.19)	711.71 (59.35)	701.91 (59.18)	705.85 (59.35)
Income measure 1 vs. income measure 2	$\sigma_{\lambda\lambda}^{12}$	M1	304.33 (35.60)	275.38 (28.23)	287.19 (42.37)	315.09 (46.06)	276.53 (27.72)	278.01 (27.78)	271.08 (27.91)	272.69 (27.98)	268.92 (27.90)	270.67 (27.97)
Income measure 1	$\sigma_{\lambda\lambda}^{11}$	M2	231.56 (19.31)	232.04 (19.21)			219.63 (17.72)	219.63 (17.72)				
Income measure 2	$\sigma_{\lambda\lambda}^{22}$	M2	946.42 (71.99)	947.94 (72.19)			893.41 (66.70)	893.41 (66.70)				
Income measure 1 vs. income measure 2	$\sigma_{\lambda\lambda}^{12}$	M2	370.19 (33.45)	371.08 (33.58)			344.70 (30.67)	344.70 (30.67)				
Income measure 1	$\sigma_{\lambda\lambda}^{11}$	M3	183.67 (17.25)	182.41 (16.79)	189.40 (23.31)	204.72 (26.77)	187.36 (16.80)	188.10 (16.83)	184.42 (16.89)	185.12 (16.93)	183.85 (16.92)	184.62 (16.95)
Income measure 2	$\sigma_{\lambda\lambda}^{22}$	M3	678.03 (62.07)	672.45 (59.57)	705.91 (92.50)	821.59 (117.16)	690.41 (59.40)	693.85 (59.53)	675.32 (59.70)	678.54 (59.86)	668.97 (59.93)	672.47 (60.09)
Income measure 1 vs. income measure 2	$\sigma_{\lambda\lambda}^{12}$	M3	256.98 (7.34)	254.35 (28.08)	270.10 (43.52)	311.64 (52.60)	263.84 (28.01)	265.44 (28.08)	257.21 (28.17)	258.71 (28.24)	255.21 (28.24)	256.86 (28.32)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 11

Covariances between preference variables and latent total expenditure^{a)}

Commodity/ income measure	Parameter	M1 MODELS				M2 MODELS		M3 MODELS			
		E2P2C2	E3P2C1	E2P2C1	E1P2C1	E3P2C2	E3P2C1	E3P2C2	E2P2C2	E3P2C1	E2P2C1
Food, beverages and tobacco	$\sigma_{\lambda_1}^1$	20.78 (5.95)	17.56 (5.40)	28.73 (12.16)	19.77 (6.46)	18.22 (3.62)	12.94 (3.15)	35.47 (11.59)	24.63 (7.34)	30.19 (15.14)	16.73 (6.14)
Clothing and footwear	$\sigma_{\lambda_1}^2$	4.73 (4.88)	14.73 (3.70)	11.35 (7.51)	0.92 (3.75)	2.14 (3.33)	5.83 (3.00)	8.34 (9.28)	18.00 (4.94)	7.74 (7.92)	0.60 (4.02)
Housing, fuel and furniture	$\sigma_{\lambda_1}^3$	6.60 (7.00)	32.63 (7.40)	-0.36 (9.20)	-10.14 (6.68)	13.08 (4.99)	13.11 (4.69)	24.80 (14.03)	46.68 (10.96)	6.74 (10.61)	-2.01 (6.50)
Travel and recreation	$\sigma_{\lambda_1}^4$	-36.38 (10.20)	-75.93 (17.71)	-44.00 (19.07)	-11.97 (7.72)	-39.99 (6.85)	-37.39 (7.67)	-79.86 (23.65)	-105.35 (26.27)	-48.21 (24.29)	-17.31 (8.28)
Other goods and services	$\sigma_{\lambda_1}^5$	4.27 (3.31)	11.01 (3.18)	4.28 (4.41)	-1.42 (2.69)	6.55 (2.32)	5.51 (2.18)	11.25 (6.51)	16.04 (4.34)	3.54 (4.97)	1.99 (2.82)
Income measure 1	σ_{λ}^1	67.07 (19.84)	33.01 (13.52)	60.79 (25.86)	66.81 (26.10)	81.75 (14.00)	84.48 (14.33)	33.32 (13.94)	21.37 (10.23)	56.41 (28.04)	74.59 (28.47)
Income measure 2	σ_{λ}^2	163.97 (36.89)	68.99 (25.74)	138.07 (54.08)	181.74 (52.19)	193.73 (26.70)	198.78 (27.34)	69.32 (27.40)	38.80 (19.26)	121.69 (57.81)	191.02 (54.65)

^{a)} See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 12.1

Marginal budget shares and income-consumption ratios in M1 models^{a)}

Commodity/ income measure	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expen- diture						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	b ₁	0.082 (0.025)	0.091 (0.022)	0.047 (0.052)	0.091 (0.028)	0.162 (0.012)	0.165 (0.012)	0.164 (0.012)	0.167 (0.012)	0.166 (0.012)	0.169 (0.012)	0.124 (0.016)	0.120 (0.016)	0.121 (0.016)
Clothing and footwear	b ₂	0.106 (0.021)	0.063 (0.014)	0.076 (0.033)	0.119 (0.018)	0.122 (0.011)	0.122 (0.010)	0.122 (0.011)	0.121 (0.011)	0.121 (0.011)	0.120 (0.011)	0.102 (0.015)	0.101 (0.015)	0.101 (0.015)
Housing, fuel and furniture	b ₃	0.237 (0.031)	0.130 (0.029)	0.279 (0.040)	0.314 (0.030)	0.268 (0.016)	0.264 (0.016)	0.276 (0.016)	0.272 (0.016)	0.279 (0.016)	0.275 (0.016)	0.301 (0.022)	0.297 (0.022)	0.297 (0.022)
Travel and recreation	b ₄	0.487 (0.044)	0.658 (0.071)	0.507 (0.082)	0.374 (0.035)	0.343 (0.021)	0.344 (0.021)	0.331 (0.022)	0.333 (0.022)	0.328 (0.022)	0.330 (0.022)	0.373 (0.027)	0.384 (0.027)	0.382 (0.027)
Other goods and services	b ₅	0.088 (0.015)	0.058 (0.013)	0.091 (0.019)	0.102 (0.013)	0.105 (0.007)	0.105 (0.008)	0.107 (0.007)	0.107 (0.007)	0.106 (0.008)	0.106 (0.007)	0.100 (0.013)	0.098 (0.011)	0.099 (0.011)
Income measure 1	e ₁	0.275 (0.080)	0.405 (0.066)	0.307 (0.118)	0.260 (0.114)	0.514 (0.053)	0.510 (0.053)	0.527 (0.055)	0.523 (0.054)	0.522 (0.055)	0.518 (0.055)	1.172 (0.077)	1.207 (0.079)	1.190 (0.079)
Income measure 2	e ₂	0.524 (0.136)	0.859 (0.122)	0.660 (0.239)	0.442 (0.217)	1.110 (0.100)	1.102 (0.100)	1.163 (0.103)	1.155 (0.103)	1.188 (0.105)	1.179 (0.105)	3.142 (0.169)	3.232 (0.173)	3.213 (0.173)

^{a)} See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 12.2

Marginal budget shares and income-consumption ratios in M2 models^{a)}

Commodity/ income measure	Parameter	MODEL				
		Preference variables correlated with total expenditure		Preference variables uncorrelated with total expen- diture		No individual differences in preferences
		E3P3C2	E3P2C2	E3P3C1	E3P2C1	E3P1C1
Food, beverages and tobacco	b ₁	0.084 (0.010)	0.098 (0.009)	0.121 (0.007)	0.125 (0.007)	0.134 (0.007)
Clothing and footwear	b ₂	0.113 (0.011)	0.101 (0.010)	0.119 (0.007)	0.117 (0.007)	0.117 (0.007)
Housing, fuel and furniture	b ₃	0.192 (0.016)	0.191 (0.014)	0.225 (0.010)	0.225 (0.010)	0.233 (0.010)
Travel and recreation	b ₄	0.548 (0.020)	0.544 (0.020)	0.456 (0.013)	0.454 (0.013)	0.434 (0.012)
Other goods and services	b ₅	0.063 (0.007)	0.066 (0.007)	0.079 (0.005)	0.079 (0.005)	0.082 (0.004)
Income measure 1	e ₁	0.107 (0.027)	0.105 (0.027)	0.170 (0.024)	0.170 (0.024)	0.353 (0.028)
Income measure 2	e ₂	0.185 (0.037)	0.182 (0.037)	0.297 (0.035)	0.297 (0.035)	0.785 (0.052)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 12.3

Marginal budget shares and income-consumption ratios in M3 models^{a)}

Commodity/ income measure	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expen- diture						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	b ₁	0.007 (0.049)	0.052 (0.030)	0.031 (0.068)	0.098 (0.027)	0.157 (0.013)	0.161 (0.012)	0.158 (0.013)	0.163 (0.013)	0.160 (0.013)	0.165 (0.013)	0.112 (0.015)	0.114 (0.015)	0.115 (0.015)
Clothing and footwear	b ₂	0.007 (0.040)	0.034 (0.019)	0.080 (0.035)	0.110 (0.019)	0.112 (0.012)	0.111 (0.011)	0.113 (0.012)	0.111 (0.011)	0.111 (0.012)	0.109 (0.011)	0.098 (0.015)	0.098 (0.015)	0.098 (0.015)
Housing, fuel and furniture	b ₃	0.177 (0.061)	0.082 (0.045)	0.261 (0.047)	0.292 (0.029)	0.283 (0.017)	0.279 (0.017)	0.288 (0.018)	0.283 (0.017)	0.291 (0.018)	0.287 (0.017)	0.296 (0.022)	0.302 (0.022)	0.303 (0.022)
Travel and recreation	b ₄	0.685 (0.102)	0.800 (0.110)	0.536 (0.110)	0.404 (0.038)	0.344 (0.023)	0.348 (0.022)	0.335 (0.023)	0.339 (0.023)	0.333 (0.023)	0.336 (0.023)	0.401 (0.028)	0.389 (0.028)	0.387 (0.028)
Other goods and services	b ₅	0.054 (0.028)	0.032 (0.013)	0.092 (0.022)	0.096 (0.008)	0.104 (0.008)	0.101 (0.008)	0.106 (0.008)	0.104 (0.008)	0.105 (0.008)	0.103 (0.008)	0.093 (0.010)	0.097 (0.010)	0.097 (0.010)
Income measure 1	e ₁	0.444 (0.078)	0.495 (0.067)	0.348 (0.137)	0.251 (0.130)	0.553 (0.058)	0.549 (0.058)	0.566 (0.059)	0.561 (0.059)	0.560 (0.060)	0.555 (0.060)	1.274 (0.107)	1.232 (0.100)	1.217 (0.101)
Income measure 2	e ₂	0.985 (0.153)	1.107 (0.127)	0.784 (0.278)	0.452 (0.241)	1.221 (0.111)	1.212 (0.111)	1.258 (0.113)	1.248 (0.113)	1.281 (0.116)	1.271 (0.116)	3.553 (0.260)	3.442 (0.241)	3.439 (0.244)

^{a)} See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 13.1
Engel elasticities in M1 models^{a)}

Commodity	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure				No individual differences in preferences				
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	E ₁	0.32 (0.10)	0.36 (0.08)	0.18 (0.20)	0.35 (0.11)	0.63 (0.05)	0.65 (0.05)	0.64 (0.05)	0.65 (0.05)	0.65 (0.05)	0.66 (0.05)	0.48 (0.06)	0.47 (0.06)	0.47 (0.06)
Clothing and footwear	E ₂	0.99 (0.20)	0.59 (0.14)	0.71 (0.31)	1.11 (0.17)	1.14 (0.11)	1.14 (0.10)	1.14 (0.11)	1.13 (0.11)	1.13 (0.11)	1.12 (0.11)	0.95 (0.14)	0.95 (0.14)	0.95 (0.14)
Housing, fuel and furniture	E ₃	0.95 (0.13)	0.53 (0.12)	1.13 (0.16)	1.26 (0.13)	1.08 (0.07)	1.07 (0.07)	1.11 (0.07)	1.10 (0.07)	1.12 (0.07)	1.11 (0.07)	1.21 (0.09)	1.20 (0.09)	1.20 (0.09)
Travel and recreation	E ₄	1.56 (0.14)	2.11 (0.23)	1.62 (0.27)	1.20 (0.11)	1.10 (0.07)	1.10 (0.07)	1.06 (0.07)	1.07 (0.07)	1.05 (0.08)	1.05 (0.07)	1.19 (0.09)	1.23 (0.09)	1.22 (0.09)
Other goods and services	E ₅	1.15 (0.20)	0.76 (0.17)	1.19 (0.25)	1.35 (0.18)	1.38 (0.10)	1.37 (0.11)	1.41 (0.10)	1.40 (0.10)	1.41 (0.11)	1.40 (0.10)	1.32 (0.18)	1.30 (0.15)	1.29 (0.15)

^{a)} See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 13.2
Engel elasticities in M2 models^{a)}

Commodity	Parameter	MODEL				
		Preference variables correlated with total expenditure		Preference variables uncorrelated with total expenditure		No individual differences in preferences
		E3P3C2	E3P2C2	E3P3C1	E3P2C1	E3P1C1
Food, beverages and tobacco	E ₁	0.33 (0.04)	0.38 (0.04)	0.47 (0.03)	0.49 (0.03)	0.53 (0.03)
Clothing and footwear	E ₂	1.05 (0.10)	0.94 (0.10)	1.11 (0.07)	1.09 (0.07)	1.09 (0.07)
Housing, fuel and furniture	E ₃	0.77 (0.06)	0.77 (0.06)	0.91 (0.05)	0.91 (0.05)	0.94 (0.04)
Travel and recreation	E ₄	1.75 (0.07)	1.74 (0.07)	1.46 (0.05)	1.45 (0.05)	1.39 (0.05)
Other goods and services	E ₅	0.82 (0.10)	0.86 (0.10)	1.04 (0.07)	1.04 (0.07)	1.08 (0.06)

^{a)} See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 13.3
Engel elasticities in M3 models^{a)}

Commodity	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	E ₁	0.03 (0.19)	0.20 (0.12)	0.12 (0.27)	0.38 (0.11)	0.61 (0.05)	0.63 (0.05)	0.62 (0.05)	0.64 (0.05)	0.63 (0.05)	0.64 (0.05)	0.44 (0.06)	0.45 (0.06)	0.45 (0.06)
Clothing and footwear	E ₂	0.72 (0.38)	0.32 (0.18)	0.75 (0.33)	1.03 (0.18)	1.05 (0.11)	1.03 (0.11)	1.05 (0.12)	1.04 (0.11)	1.04 (0.12)	1.02 (0.11)	0.91 (0.14)	0.92 (0.14)	0.92 (0.14)
Housing, fuel and furniture	E ₃	0.71 (0.25)	0.33 (0.18)	1.05 (0.19)	1.18 (0.12)	1.14 (0.07)	1.12 (0.07)	1.16 (0.08)	1.14 (0.07)	1.17 (0.08)	1.15 (0.07)	1.19 (0.09)	1.22 (0.09)	1.22 (0.09)
Travel and recreation	E ₄	2.19 (0.33)	2.56 (0.36)	1.72 (0.35)	1.29 (0.13)	1.10 (0.08)	1.11 (0.08)	1.07 (0.08)	1.09 (0.08)	1.06 (0.08)	1.08 (0.08)	1.28 (0.09)	1.24 (0.09)	1.24 (0.09)
Other goods and services	E ₅	0.71 (0.37)	0.42 (0.17)	1.20 (0.29)	1.27 (0.11)	1.36 (0.11)	1.33 (0.11)	1.39 (0.11)	1.35 (0.11)	1.38 (0.11)	1.35 (0.11)	1.22 (0.14)	1.26 (0.14)	1.26 (0.14)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 14.1
Demographic effects in M1 models^{a)}

Commodity/ income measure	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expen- diture						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
	No. of children													
Food, beverages and tobacco	c ₁₁	1.328 (0.191)	1.279 (0.178)	1.513 (0.315)	1.282 (0.199)	0.906 (0.148)	0.888 (0.145)	0.895 (0.149)	0.876 (0.146)	0.891 (0.149)	0.873 (0.146)	1.108 (0.149)	1.129 (0.149)	1.126 (0.149)
Clothing and footwear	c ₂₁	0.194 (0.163)	0.423 (0.145)	0.355 (0.213)	0.126 (0.151)	0.109 (0.132)	0.114 (0.131)	0.113 (0.132)	0.116 (0.131)	0.121 (0.132)	0.123 (0.131)	0.216 (0.140)	0.221 (0.141)	0.222 (0.140)
Housing, fuel and furniture	c ₃₁	-0.167 (0.237)	0.393 (0.246)	-0.394 (0.274)	-0.574 (0.242)	-0.330 (0.192)	-0.312 (0.193)	-0.373 (0.193)	-0.353 (0.194)	-0.382 (0.193)	-0.362 (0.194)	-0.503 (0.204)	-0.482 (0.205)	-0.480 (0.204)
Travel and recreation	c ₄₁	-1.253 (0.324)	-2.152 (0.471)	-1.356 (0.494)	-0.655 (0.284)	-0.492 (0.253)	-0.501 (0.252)	-0.429 (0.256)	-0.438 (0.255)	-0.430 (0.256)	-0.438 (0.256)	-0.650 (0.257)	-0.709 (0.256)	-0.713 (0.255)
Other goods and services	c ₅₁	-0.102 (0.111)	0.057 (0.107)	-0.118 (0.130)	-0.179 (0.104)	-0.193 (0.090)	-0.189 (0.089)	-0.206 (0.090)	-0.201 (0.090)	-0.200 (0.090)	-0.196 (0.090)	-0.171 (0.118)	-0.159 (0.101)	-0.155 (0.101)
Income measure 1	f ₁₁	-0.126 (0.739)	-0.812 (0.693)	-0.302 (0.867)	-0.055 (0.859)	-1.384 (0.673)	-1.366 (0.673)	-1.461 (0.679)	-1.442 (0.679)	-1.401 (0.679)	-1.381 (0.679)	-4.877 (0.727)	-5.052 (0.739)	-4.883 (0.735)
Income measure 2	f ₂₁	2.963 (1.366)	1.196 (1.300)	2.333 (1.704)	3.387 (1.649)	-0.122 (1.270)	-0.080 (1.270)	-0.416 (1.283)	-0.372 (1.284)	-0.471 (1.286)	-0.426 (1.287)	-10.885 (1.583)	-11.345 (1.615)	-11.022 (1.611)
	No. of adults													
Food, beverages and tobacco	c ₁₂	1.551 (0.360)	1.436 (0.325)	1.981 (0.674)	1.444 (0.387)	0.569 (0.236)	0.526 (0.231)	0.545 (0.238)	0.502 (0.233)	0.523 (0.239)	0.480 (0.234)	1.037 (0.256)	1.084 (0.259)	1.073 (0.259)
Clothing and footwear	c ₂₂	0.245 (0.308)	0.777 (0.246)	0.618 (0.437)	0.089 (0.270)	0.048 (0.210)	0.059 (0.207)	0.057 (0.212)	0.064 (0.209)	0.072 (0.212)	0.076 (0.210)	0.297 (0.242)	0.308 (0.244)	0.308 (0.244)
Housing, fuel and furniture	c ₃₂	-1.145 (0.446)	0.155 (0.444)	-1.666 (0.546)	-2.087 (0.449)	-1.526 (0.307)	-1.485 (0.307)	-1.621 (0.310)	-1.576 (0.310)	-1.658 (0.311)	-1.611 (0.311)	-1.924 (0.352)	-1.876 (0.354)	-1.886 (0.355)
Travel and recreation	c ₄₂	-0.649 (0.620)	-2.735 (0.954)	-0.899 (1.057)	0.733 (0.520)	1.122 (0.409)	1.101 (0.407)	1.258 (0.414)	1.237 (0.413)	1.301 (0.417)	1.282 (0.416)	0.747 (0.443)	0.613 (0.442)	0.636 (0.444)
Other goods and services	c ₅₂	-0.002 (0.209)	0.367 (0.195)	-0.034 (0.262)	-0.179 (0.190)	-0.213 (0.143)	-0.201 (0.142)	-0.239 (0.145)	-0.227 (0.144)	-0.238 (0.145)	-0.227 (0.145)	-0.157 (0.206)	-0.129 (0.174)	-0.131 (0.175)
Income measure 1	f ₁₂	12.402 (1.288)	10.812 (1.159)	12.010 (1.663)	12.575 (1.630)	9.474 (1.066)	9.514 (1.066)	9.314 (1.081)	9,358 (1.080)	9.372 (1.087)	9.420 (1.087)	1.459 (1.257)	1.044 (1.282)	1.192 (1.282)
Income measure 2	f ₂₂	18.291 (2.315)	14.192 (2.162)	16.633 (3.323)	19.289 (3.117)	11.111 (2.011)	11.206 (2.011)	10.471 (2.042)	10.570 (2.042)	10.157 (2.060)	10.263 (2.061)	-13.630 (2.737)	-14.718 (2.797)	-14.627 (2.806)

^{a)} See table 1 and 2 for model descriptions. Standard deviations in parentheses.

MODEL

Commodity/ income measure	Parameter	Preference variables correlated with total expenditure		Preference variables uncorrelated with total expen- diture		No individual differences in preferences
		E3P3C2	E3P2C2	E3P3C1	E3P2C1	E3P1C1
	No. of children					
Food, beverages and tobacco	c ₁₁	1.316 (0.148)	1.244 (0.142)	1.122 (0.138)	1.105 (0.136)	1.054 (0.116)
Clothing and footwear	c ₂₁	0.162 (0.131)	0.222 (0.130)	0.131 (0.124)	0.140 (0.124)	0.141 (0.112)
Housing, fuel and furniture	c ₃₁	0.068 (0.196)	0.073 (0.194)	-0.103 (0.182)	-0.104 (0.182)	-0.145 (0.165)
Travel and recreation	c ₄₁	-1.578 (0.266)	-1.557 (0.263)	-1.096 (0.234)	-1.088 (0.233)	-0.980 (0.210)
Other goods and services	c ₅₁	0.032 (0.091)	0.018 (0.089)	-0.054 (0.084)	-0.053 (0.084)	-0.070 (0.076)
Income measure 1	f ₁₁	0.757 (0.656)	0.769 (0.656)	0.431 (0.638)	0.431 (0.638)	-0.521 (0.481)
Income measure 2	f ₂₁	4.749 (1.266)	4.765 (1.267)	4.166 (1.232)	4.166 (1.232)	1.629 (0.885)
	No. of adults					
Food, beverages and tobacco	c ₁₂	1.519 (0.227)	1.351 (0.217)	1.064 (0.203)	1.024 (0.201)	0.915 (0.175)
Clothing and footwear	c ₂₂	0.168 (0.209)	0.310 (0.203)	0.096 (0.184)	0.117 (0.184)	0.119 (0.168)
Housing, fuel and furniture	c ₃₂	-0.600 (0.312)	-0.589 (0.304)	-1.002 (0.271)	-1.004 (0.271)	-1.101 (0.247)
Travel and recreation	c ₄₂	-1.396 (0.416)	-1.346 (0.413)	-0.264 (0.347)	-0.244 (0.346)	0.009 (0.316)
Other goods and services	c ₅₂	0.309 (0.144)	0.274 (0.139)	0.106 (0.125)	0.107 (0.125)	0.058 (0.114)
Income measure 1	f ₁₂	14.443 (0.946)	14.471 (0.948)	13.675 (0.913)	13.675 (0.913)	11.440 (0.722)
Income measure 2	f ₂₂	22.420 (1.789)	22.457 (1.790)	21.050 (1.737)	21.050 (1.737)	15.090 (1.330)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

Table 14.3
Demographic effects in M3 models^{a)}

Commodity/ income measure	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expen- diture						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
	No. of children													
Food, beverages and tobacco	c ₁₁	1.724 (0.307)	1.486 (0.217)	1.598 (0.393)	1.245 (0.196)	0.935 (0.149)	0.912 (0.147)	0.926 (0.150)	0.901 (0.147)	0.920 (0.150)	0.896 (0.147)	1.172 (0.138)	1.161 (0.138)	1.156 (0.137)
Clothing and footwear	c ₂₁	0.353 (0.244)	0.578 (0.166)	0.334 (0.223)	0.176 (0.154)	0.165 (0.133)	0.173 (0.132)	0.162 (0.134)	0.170 (0.133)	0.172 (0.134)	0.179 (0.132)	0.243 (0.132)	0.240 (0.132)	0.238 (0.132)
Housing, fuel and furniture	c ₃₁	0.147 (0.369)	0.647 (0.317)	-0.300 (0.303)	-0.461 (0.236)	-0.415 (0.196)	-0.391 (0.196)	-0.440 (0.198)	-0.415 (0.198)	-0.449 (0.198)	-0.424 (0.198)	-0.480 (0.199)	-0.515 (0.200)	-0.511 (0.199)
Travel and recreation	c ₄₁	-2.298 (0.615)	-2.902 (0.684)	-1.511 (0.631)	-0.812 (0.296)	-0.502 (0.255)	-0.522 (0.254)	-0.452 (0.258)	-0.473 (0.256)	-0.454 (0.258)	-0.473 (0.257)	-0.807 (0.249)	-0.742 (0.250)	-0.740 (0.249)
Other goods and services	c ₅₁	0.074 (0.171)	0.191 (0.130)	-0.121 (0.142)	-0.148 (0.106)	-0.183 (0.091)	-0.172 (0.091)	-0.196 (0.092)	-0.183 (0.091)	-0.189 (0.092)	-0.178 (0.091)	-0.128 (0.090)	-0.144 (0.090)	-0.143 (0.090)
Income measure 1	f ₁₁	-1.024 (0.724)	-1.287 (0.694)	-0.525 (0.943)	-0.008 (0.922)	-1.594 (0.685)	-1.572 (0.685)	-1.670 (0.692)	-1.647 (0.691)	-1.601 (0.693)	-1.576 (0.693)	-5.430 (0.929)	-5.220 (0.897)	-5.024 (0.895)
Income measure 2	f ₂₁	0.528 (1.376)	-0.113 (1.309)	1.568 (1.856)	3.330 (1.739)	-0.716 (1.297)	-0.667 (1.297)	-0.926 (1.310)	-0.875 (1.310)	-0.958 (1.316)	-0.906 (1.315)	-13.104 (2.253)	-12.546 (2.160)	-12.198 (2.173)
	No. of adults													
Food, beverages and tobacco	c ₁₂	2.465 (0.645)	1.913 (0.421)	2.172 (0.862)	1.357 (0.379)	0.635 (0.241)	0.580 (0.236)	0.615 (0.244)	0.558 (0.238)	0.591 (0.245)	0.535 (0.239)	1.174 (0.242)	1.149 (0.242)	1.144 (0.243)
Clothing and footwear	c ₂₂	0.609 (0.520)	1.132 (0.296)	0.564 (0.464)	0.199 (0.280)	0.174 (0.216)	0.192 (0.213)	0.168 (0.218)	0.185 (0.214)	0.191 (0.219)	0.207 (0.215)	0.351 (0.232)	0.343 (0.232)	0.346 (0.234)
Housing, fuel and furniture	c ₃₂	-0.412 (0.787)	0.747 (0.622)	-1.443 (0.623)	-1.819 (0.435)	-1.717 (0.320)	-1.663 (0.318)	-1.772 (0.323)	-1.714 (0.320)	-1.813 (0.325)	-1.756 (0.322)	-1.865 (0.348)	-1.942 (0.350)	-1.961 (0.353)
Travel and recreation	c ₄₂	-3.069 (1.313)	-4.470 (1.438)	-1.250 (1.387)	0.372 (0.555)	1.099 (0.417)	1.054 (0.413)	1.209 (0.423)	1.162 (0.419)	1.249 (0.426)	1.200 (0.422)	0.404 (0.436)	0.550 (0.438)	0.573 (0.441)
Other goods and services	c ₅₂	0.407 (0.365)	0.678 (0.251)	-0.043 (0.292)	-0.109 (0.193)	-0.191 (0.148)	-0.163 (0.147)	-0.220 (0.150)	-0.191 (0.148)	-0.218 (0.151)	-0.186 (0.149)	-0.064 (0.158)	-0.100 (0.158)	-0.102 (0.159)
Income measure 1	f ₁₂	10.327 (1.259)	9.713 (1.161)	11.503 (1.872)	12.689 (1.803)	8.993 (1.105)	9.043 (1.104)	8.834 (1.120)	8.888 (1.119)	8.902 (1.129)	8.960 (1.128)	0.278 (1.650)	0.780 (1.575)	0.860 (1.589)
Income measure 2	f ₂₂	12.658 (2.420)	11.163 (2.198)	15.110 (3.742)	19.167 (3.373)	9.748 (2.101)	9.859 (2.099)	9.300 (2.125)	9.415 (2.123)	9.015 (2.151)	9.136 (2.148)	-18.482 (4.000)	-17.146 (3.787)	-17.389 (3.854)

a) See table 1 and 2 for model descriptions. Standard deviations in parentheses.

MODEL

Commodity/ income measure	Parameter	Preference variables correlated with total expenditure				Preference variables uncorrelated with total expen- diture						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	a ₁₁	2.790 (0.524)	2.712 (0.509)	3.083 (0.676)	2.717 (0.525)	2.125 (0.475)	2.096 (0.469)	2.108 (0.477)	2.079 (0.471)	2.091 (0.477)	2.061 (0.471)	2.440 (0.462)	2.471 (0.463)	2.466 (0.463)
	a ₁₂	2.554 (0.558)	2.448 (0.535)	2.952 (0.786)	2.445 (0.566)	1.650 (0.484)	1.611 (0.478)	1.627 (0.486)	1.587 (0.480)	1.604 (0.487)	1.565 (0.480)	2.079 (0.477)	2.122 (0.479)	2.113 (0.479)
Clothing and footwear	a ₂₁	-0.542 (0.454)	-0.178 (0.452)	-0.287 (0.509)	-0.648 (0.445)	-0.675 (0.428)	-0.667 (0.428)	-0.668 (0.429)	-0.664 (0.429)	-0.659 (0.430)	-0.657 (0.429)	-0.506 (0.436)	-0.499 (0.437)	-0.498 (0.436)
	a ₂₂	-0.811 (0.483)	-0.317 (0.465)	-0.465 (0.568)	-0.954 (0.465)	-0.991 (0.436)	-0.981 (0.435)	-0.983 (0.437)	-0.977 (0.436)	-0.970 (0.437)	-0.967 (0.437)	-0.763 (0.450)	-0.753 (0.452)	-0.752 (0.451)
Housing, fuel and furniture	a ₃₁	3.164 (0.658)	4.052 (0.716)	2.810 (0.695)	2.522 (0.685)	2.908 (0.620)	2.936 (0.624)	2.842 (0.622)	2.873 (0.627)	2.815 (0.623)	2.848 (0.628)	2.630 (0.634)	2.661 (0.635)	2.660 (0.634)
	a ₃₂	3.144 (0.699)	4.348 (0.749)	2.662 (0.760)	2.272 (0.723)	2.795 (0.632)	2.833 (0.636)	2.706 (0.634)	2.748 (0.638)	2.671 (0.635)	2.714 (0.639)	2.419 (0.655)	2.463 (0.656)	2.460 (0.656)
Travel and recreation	a ₄₁	-4.981 (0.879)	-6.404 (1.158)	-5.152 (1.068)	-4.039 (0.824)	-3.785 (0.818)	-3.799 (0.816)	-3.690 (0.825)	-3.704 (0.824)	-3.656 (0.828)	-3.668 (0.827)	-4.023 (0.798)	-4.112 (0.793)	-4.108 (0.793)
	a ₄₂	-4.517 (0.940)	-6.449 (1.279)	-4.749 (1.240)	-3.238 (0.866)	-2.891 (0.834)	-2.910 (0.833)	-2.762 (0.843)	-2.781 (0.841)	-2.717 (0.845)	-2.735 (0.844)	-3.218 (0.825)	-3.340 (0.820)	-3.331 (0.820)
Other goods and services	a ₅₁	-0.431 (0.307)	-0.182 (0.312)	-0.454 (0.326)	-0.552 (0.303)	-0.573 (0.292)	-0.566 (0.292)	-0.592 (0.293)	-0.584 (0.294)	-0.591 (0.294)	-0.584 (0.294)	-0.541 (0.312)	-0.521 (0.313)	-0.520 (0.312)
	a ₅₂	-0.370 (0.232)	-0.030 (0.327)	-0.400 (0.358)	-0.525 (0.318)	-0.563 (0.297)	-0.553 (0.297)	-0.588 (0.299)	-0.577 (0.299)	-0.588 (0.299)	-0.577 (0.300)	-0.517 (0.323)	-0.492 (0.323)	-0.490 (0.323)
Income measure 1	d ₁₁	-0.317 (2.175)	-1.404 (2.118)	-0.581 (2.283)	-0.197 (2.297)	-2.291 (2.138)	-2.263 (2.139)	-2.404 (2.150)	-2.373 (2.151)	-2.375 (2.151)	-2.342 (2.152)	-7.828 (2.264)	-8.129 (2.295)	-7.943 (2.283)
	d ₁₂	2.764 (2.259)	1.288 (2.178)	2.404 (2.454)	2.926 (2.457)	0.078 (2.176)	0.115 (2.177)	-0.075 (2.190)	-0.034 (2.191)	-0.030 (2.192)	-0.014 (2.194)	-7.421 (2.341)	-7.824 (2.375)	-7.590 (2.362)
Income measure 2	d ₂₁	-8.718 (4.107)	-11.521 (3.974)	-9.842 (4.365)	-8.035 (4.414)	-13.566 (4.025)	-13.501 (4.028)	-14.011 (4.057)	-13.943 (4.060)	-14.247 (4.065)	-14.175 (4.068)	-30.619 (4.937)	-31.409 (5.030)	-31.120 (5.011)
	d ₂₂	-6.889 (4.237)	-10.692 (4.082)	-8.417 (4.732)	-5.693 (4.715)	-13.483 (4.097)	-13.395 (4.099)	-14.086 (4.133)	-13.994 (4.136)	-14.396 (4.144)	-14.297 (4.147)	-36.588 (5.105)	-37.647 (5.204)	-37.301 (5.182)

a) Standard deviations in parentheses.

Table 15.2

Constant terms in Engel functions and income-consumption relations in M2 models^{a)}

Commodity/ income measure	Parameter	MODEL					
		Preference variables correlated with total expenditure		Preference variables uncorrelated with total expen- diture			No individual differences in preferences
		E3P3C2	E3P2C2	E3P3C1	E3P2C1	E3P1C1	
Food, beverages and tobacco	a ₁₁	2.769 (0.486)	2.654 (0.471)	2.459 (0.463)	2.432 (0.458)	2.351 (0.402)	
	a ₁₂	2.526 (0.492)	2.370 (0.476)	2.105 (0.466)	2.068 (0.461)	1.958 (0.406)	
Clothing and footwear	a ₂₁	-0.594 (0.425)	-0.497 (0.427)	-0.643 (0.420)	-0.628 (0.420)	-0.627 (0.386)	
	a ₂₂	-0.881 (0.433)	-0.750 (0.433)	-0.948 (0.423)	-0.928 (0.423)	-0.927 (0.389)	
Housing, fuel and furniture	a ₃₁	3.536 (0.637)	3.543 (0.638)	3.262 (0.617)	3.261 (0.618)	3.195 (0.569)	
	a ₃₂	3.649 (0.648)	3.659 (0.647)	3.277 (0.621)	3.275 (0.623)	3.185 (0.574)	
Travel and recreation	a ₄₁	-5.491 (0.870)	-5.457 (0.860)	-4.721 (0.792)	-4.707 (0.789)	-4.535 (0.727)	
	a ₄₂	-5.210 (0.883)	-5.163 (0.873)	-4.164 (0.798)	-4.145 (0.795)	-3.911 (0.733)	
Other goods and services	a ₅₁	-0.220 (0.295)	-0.243 (0.293)	-0.357 (0.286)	-0.358 (0.286)	-0.384 (0.263)	
	a ₅₂	-0.084 (0.301)	-0.116 (0.297)	-0.270 (0.288)	-0.270 (0.288)	-0.305 (0.266)	
Income measure 1	d ₁₁	1.077 (2.185)	1.096 (2.187)	0.555 (2.136)	0.555 (2.136)	-0.967 (1.662)	
	d ₁₂	4.656 (2.195)	4.682 (2.197)	3.946 (2.143)	3.946 (2.143)	1.881 (1.676)	
Income measure 2	d ₂₁	-5.899 (4.240)	-5.874 (4.243)	-6.832 (4.130)	-6.832 (4.130)	-10.889 (3.059)	
	d ₂₂	-3.062 (4.250)	-3.028 (4.253)	-4.328 (4.139)	-4.328 (4.139)	-9.837 (3.085)	

^{a)} Standard deviations in parentheses.

Commodity/ income measure	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expen- diture						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	a ₁₁	3.413 (0.699)	3.038 (0.566)	3.213 (0.779)	2.659 (0.517)	2.171 (0.473)	2.133 (0.468)	2.157 (0.474)	2.118 (0.468)	2.137 (0.475)	2.099 (0.469)	2.539 (0.421)	2.521 (0.421)	2.514 (0.420)
	a ₁₂	3.400 (0.794)	2.891 (0.611)	3.129 (0.938)	1.376 (0.556)	1.712 (0.483)	1.661 (0.477)	1.693 (0.484)	1.641 (0.478)	1.668 (0.486)	1.615 (0.479)	2.212 (0.438)	2.188 (0.437)	2.179 (0.437)
Clothing and footwear	a ₂₁	-0.293 (0.542)	0.064 (0.487)	-0.324 (0.517)	-0.572 (0.444)	-0.590 (0.427)	-0.578 (0.426)	-0.594 (0.427)	-0.583 (0.426)	-0.578 (0.428)	-0.567 (0.426)	-0.465 (0.404)	-0.471 (0.404)	-0.473 (0.403)
	a ₂₂	-0.473 (0.624)	0.011 (0.509)	-0.515 (0.583)	-0.853 (0.466)	-0.877 (0.436)	-0.860 (0.434)	-0.882 (0.437)	-0.866 (0.434)	-0.860 (0.437)	-0.845 (0.435)	-0.707 (0.420)	-0.715 (0.419)	-0.717 (0.419)
Housing, fuel and furniture	a ₃₁	3.664 (0.812)	4.454 (0.823)	2.963 (0.723)	2.707 (0.671)	2.781 (0.630)	2.818 (0.633)	2.744 (0.632)	2.783 (0.635)	2.710 (0.633)	2.748 (0.636)	2.660 (0.607)	2.608 (0.610)	2.609 (0.610)
	a ₃₂	3.822 (0.936)	4.895 (0.892)	2.870 (0.807)	2.523 (0.707)	2.622 (0.643)	2.672 (0.646)	2.572 (0.646)	2.624 (0.648)	2.527 (0.647)	2.580 (0.650)	2.463 (0.630)	2.392 (0.634)	2.391 (0.634)
Travel and recreation	a ₄₁	-6.631 (1.342)	-7.588 (1.546)	-5.393 (1.244)	-4.290 (0.828)	-3.805 (0.819)	-3.835 (0.815)	-3.730 (0.824)	-3.761 (0.820)	-3.694 (0.827)	-3.724 (0.822)	-4.259 (0.760)	-4.157 (0.763)	-4.151 (0.762)
	a ₄₂	-6.757 (1.552)	-8.056 (1.761)	-5.057 (1.502)	-3.578 (0.878)	-2.916 (0.837)	-2.957 (0.833)	-2.814 (0.843)	-2.857 (0.838)	-2.769 (0.846)	-2.811 (0.841)	-3.538 (0.789)	-3.401 (0.792)	-3.390 (0.792)
Other goods and services	a ₅₁	-0.153 (0.377)	0.002 (0.348)	-0.459 (0.336)	-0.504 (0.303)	-0.557 (0.292)	-0.538 (0.292)	-0.577 (0.293)	-0.557 (0.293)	-0.575 (0.293)	-0.556 (0.293)	-0.475 (0.275)	-0.501 (0.276)	-0.499 (0.275)
	a ₅₂	0.008 (0.435)	0.259 (0.373)	-0.409 (0.376)	-0.468 (0.318)	-0.541 (0.298)	-0.516 (0.298)	-0.569 (0.299)	-0.542 (0.299)	-0.566 (0.300)	-0.539 (0.299)	-0.430 (0.286)	-0.464 (0.286)	-0.463 (0.286)
Income measure 1	d ₁₁	-1.727 (2.138)	-2.151 (2.121)	-0.920 (2.348)	-0.113 (2.354)	-2.606 (2.153)	-2.572 (2.153)	-2.713 (2.169)	-2.677 (2.169)	-2.695 (2.173)	-2.655 (2.173)	-8.796 (2.819)	-8.428 (2.750)	-8.169 (2.736)
	d ₁₂	0.848 (2.219)	0.274 (2.182)	1.942 (2.571)	3.038 (2.555)	-0.354 (2.198)	-0.308 (2.198)	-0.500 (2.216)	-0.451 (2.215)	-0.465 (2.221)	-0.411 (2.221)	-8.692 (2.939)	-8.200 (2.858)	-7.798 (2.842)
Income measure 2	d ₂₁	-12.551 (4.011)	-13.579 (3.983)	-10.866 (4.485)	-8.108 (4.482)	-14.467 (4.059)	-14.391 (4.059)	-14.770 (4.097)	-14.691 (4.096)	-15.025 (4.114)	-14.942 (4.114)	-34.387 (6.850)	-33.404 (6.643)	-33.000 (6.650)
	d ₂₂	-12.095 (4.178)	-13.488 (4.100)	-9.812 (4.964)	-6.065 (4.846)	-14.715 (4.147)	-14.612 (4.147)	-15.128 (4.187)	-15.021 (4.186)	-15.452 (4.209)	-15.339 (4.208)	-41.581 (7.141)	-40.268 (6.903)	-39.854 (6.907)

a) Standard deviations in parentheses.

Table 16. Estimation results for model E3P2CIMI with different degrees of winsorization. Standard deviations in parentheses

Parameter	Degree of winsorization			Parameter	Degree of winsorization		
	2 %	1 %	0 %		2 %	1 %	0 %
b_1	0.165 (0.012)	0.154 (0.012)	0.132 (0.012)	σ_{aa}^{11}	7.21 (1.12)	8.29 (1.29)	9.40 (1.59)
b_2	0.122 (0.010)	0.114 (0.010)	0.094 (0.010)	σ_{aa}^{22}	3.23 (0.92)	3.25 (0.99)	3.75 (1.12)
b_3	0.264 (0.016)	0.255 (0.016)	0.267 (0.017)	σ_{aa}^{33}	11.13 (2.54)	13.27 (3.11)	3.96 (4.47)
b_4	0.344 (0.021)	0.363 (0.021)	0.367 (0.026)	σ_{aa}^{44}	18.14 (6.08)	22.87 (7.53)	45.92 (14.36)
b_5	0.105 (0.007)	0.114 (0.008)	0.140 (0.014)	σ_{aa}^{55}	1.13 (0.41)	1.35 (0.65)	-3.20 (2.79)
c_{11}	0.888 (0.145)	1.012 (0.155)	1.307 (0.174)	$\sigma_{\lambda\lambda}^{11}$	193.23 (16.71)	190.411 (16.52)	203.42 (17.92)
c_{21}	0.114 (0.131)	0.137 (0.135)	0.231 (0.145)	$\sigma_{\lambda\lambda}^{12}$	278.01 (27.78)	280.77 (28.26)	310.85 (31.39)
c_{31}	-0.312 (0.193)	-0.269 (0.211)	-0.280 (0.243)	$\sigma_{\lambda\lambda}^{22}$	724.83 (58.95)	754.53 (61.66)	823.80 (68.65)
c_{41}	-0.501 (0.252)	-0.637 (0.271)	-0.837 (0.354)	$\sigma_{\lambda\lambda}$	380.56 (33.72)	434.97 (39.10)	545.15 (53.10)
c_{51}	-0.189 (0.089)	-0.243 (0.109)	-0.421 (0.204)	σ_{aa}	15.40 (4.63)	19.14 (5.85)	19.25 (9.14)
c_{12}	0.526 (0.231)	0.649 (0.247)	0.805 (0.286)	q_2	1.104 (0.030)	1.103 (0.032)	1.115 (0.036)
c_{22}	0.059 (0.207)	0.119 (0.215)	0.304 (0.237)	$\sigma_{\lambda\lambda}^1$	8.80 (1.40)	9.06 (1.50)	9.14 (1.72)
c_{32}	-1.485 (0.307)	-1.477 (0.338)	-1.783 (0.403)	$\sigma_{\lambda\lambda}^2$	10.00 (1.09)	10.63 (1.17)	11.97 (1.35)
c_{42}	1.101 (0.407)	0.989 (0.438)	1.267 (0.594)	σ_w^{11}	9.82 (0.72)	11.58 (0.85)	15.97 (1.15)
c_{52}	-0.201 (0.142)	-0.280 (0.173)	-0.593 (0.337)	σ_w^{22}	13.14 (0.93)	14.26 (1.01)	16.12 (1.14)
e_1	0.51 (0.05)	0.47 (0.05)	0.40 (0.05)	σ_w^{33}	26.98 (1.97)	34.50 (2.50)	60.25 (4.32)
e_2	1.10 (0.10)	1.03 (0.10)	0.86 (0.09)	σ_w^{44}	88.32 (6.13)	104.11 (7.27)	184.43 (12.72)
				σ_w^{55}	5.31 (0.38)	9.08 (0.65)	57.50 (3.98)
f_{11}	-1.37 (0.67)	-1.20 (0.66)	-0.93 (0.69)	σ_{ee}^{11}	57.43 (4.44)	56.01 (4.37)	57.31 (4.48)
f_{21}	-0.08 (1.27)	0.07 (1.29)	0.82 (1.35)	σ_{ee}^{12}	53.56 (5.58)	54.71 (5.74)	57.28 (6.07)
f_{12}	9.51 (1.07)	9.62 (1.06)	10.22 (1.13)	σ_{ee}^{22}	92.82 (9.11)	96.44 (9.68)	103.26 (10.50)
f_{22}	11.21 (2.01)	11.93 (2.05)	13.36 (2.22)				
χ^2	183.47	186.84	459.98	AIC	267.47	270.84	543.98

Table 17.1
Covariance matrix of the observed variables after 2 per cent winsorization^{a)}

	Food, beverages and tobacco		Clothing and footwear		Housing, fuel and furniture		Travel and recreation		Other goods and services	
	y ₁₁	y ₁₂	y ₂₁	y ₂₂	y ₃₁	y ₃₂	y ₄₁	y ₄₂	y ₅₁	y ₅₂
y ₁₁	33.866399									
y ₁₂	23.931881	34.277795								
y ₂₁	9.771840	9.894488	22.692229							
y ₂₂	10.362220	12.385392	9.556499	23.523477						
y ₃₁	16.573568	17.280461	7.069731	11.628316	54.839144					
y ₃₂	17.656239	19.497959	6.757210	13.738555	30.243543	61.781544				
y ₄₁	21.432868	21.064339	14.633581	14.944365	24.033823	22.290424	131.351104			
y ₄₂	26.029033	32.328722	15.823002	26.467252	34.579921	31.770744	55.026045	179.365061		
y ₅₁	6.995257	6.929788	5.202998	5.008590	7.746768	8.136831	14.012352	14.429180	9.590906	
y ₅₂	7.414759	9.006666	4.324339	6.445988	9.069422	10.505322	11.609091	15.243447	5.218842	11.862711
w ₁₁	51.588551	48.147343	32.549316	31.866398	59.113226	60.929099	127.655839	116.312647	29.988223	28.188578
w ₁₂	57.612568	58.135318	35.658711	40.761958	63.376137	64.759473	133.288826	149.462801	32.373895	30.914929
w ₂₁	107.289966	97.009444	64.431758	67.096449	140.906382	133.627689	238.857808	234.022386	57.959994	50.511329
w ₂₂	113.944568	108.692940	69.767003	78.773507	141.262465	136.863114	235.569617	278.668461	62.880572	55.233889
z ₁	2.912353	3.029978	1.318979	1.289102	1.469587	2.181751	1.699158	3.172564	0.540539	0.760104
z ₂	2.197420	2.294813	1.334488	1.339847	1.310812	1.752556	4.525261	4.460283	0.836724	0.995516

^{a)} Measurement unit: 1000 Norwegian 1974 kroner.

Table 17.1 (continued)
Covariance matrix of the observed variables after 2 per cent winsorization

	Income measure 1		Income measure 2		Number of children	Number of adults
	w ₁₁	w ₁₂	w ₂₁	w ₂₂	z ₁	z ₂
w ₁₁	487.977561					
w ₁₂	467.259940	573.080053				
w ₂₁	766.034038	774.003609	1626.371358			
w ₂₂	752.773693	890.587960	1624.143500	1851.203952		
z ₁	2.281886	4.351582	9.495276	12.433148	1.579200	
z ₂	12.660054	13.595467	20.156360	21.558213	0.078527	0.826605

Table 17.2
Covariance matrix of the observed variables after 1 per cent winsorization^{a)}

	Food, beverages and tobacco		Clothing and footwear		Housing, fuel and furniture		Travel and recreation		Other goods and services	
	y_{11}	y_{12}	y_{21}	y_{22}	y_{31}	y_{32}	y_{41}	y_{42}	y_{51}	y_{52}
y_{11}	37.083714									
y_{12}	26.070993	38.754505								
y_{21}	10.457512	10.468384	23.981277							
y_{22}	10.880761	13.085178	9.875489	25.198604						
y_{31}	17.230204	19.355597	7.092011	12.414434	64.474046					
y_{32}	19.202983	21.449040	7.370664	14.823993	33.952493	74.658158				
y_{41}	23.962853	23.178140	16.420520	16.232114	26.460718	24.777224	153.072583			
y_{42}	29.701490	35.863010	17.057238	29.853559	39.624948	36.425201	66.735199	218.808089		
y_{51}	8.488374	8.301978	6.865526	6.563520	9.384731	10.578911	19.047614	20.140193	16.187128	
y_{52}	8.109101	9.879358	4.376455	7.105149	10.915826	11.930919	13.281646	19.802805	6.910877	16.227970
w_{11}	53.274029	48.807302	33.771887	32.272193	58.358227	65.722202	137.423828	130.018809	35.390336	29.895059
w_{12}	59.094530	59.178121	36.043018	41.130028	63.585732	67.422451	142.553619	163.957810	36.551716	32.663576
w_{21}	112.342879	99.530366	66.795335	68.837307	145.896759	143.731423	261.019357	258.295381	71.807275	52.129141
w_{22}	120.159534	111.560866	72.162533	81.172361	145.802677	145.896493	258.273181	307.271843	75.194603	57.261566
z_1	3.009598	3.215560	1.331987	1.283013	1.488006	2.268794	1.780066	3.190930	0.579093	0.765442
z_2	2.312962	2.359651	1.389607	1.377734	1.292559	1.929162	4.767048	4.872700	1.031393	1.004983

^{a)} Measurement unit: 1000 Norwegian 1974 kroner.

Table 17.2 (continued)
Covariance matrix of the observed variables after 1 per cent winsorization

	Income measure 1		Income measure 2		Number of children	Number of adults
	w_{11}	w_{12}	w_{21}	w_{22}	z_1	z_2
w_{11}	484.768909					
w_{12}	460.715671	560.604116				
w_{21}	775.496403	774.333449	1673.355172			
w_{22}	763.656299	895.884195	1672.963231	1913.212811		
z_1	2.273339	4.363816	9.378550	12.293874	1.579200	
z_2	12.681641	13.475754	20.535430	22.114384	0.078527	0.826605

Table 17.3
Covariance matrix of the observed variables without winsorization^{a)}

	Food, beverages and tobacco		Clothing and footwear		Housing, fuel and furniture		Travel and recreation		Other goods and services	
	y_{11}	y_{12}	y_{21}	y_{22}	y_{31}	y_{32}	y_{41}	y_{42}	y_{51}	y_{52}
y_{11}	42.477565									
y_{12}	28.582066	46.822764								
y_{21}	10.939213	10.993651	25.638616							
y_{22}	11.520809	13.769723	10.107788	27.471930						
y_{31}	18.243772	24.705786	7.797902	13.624753	94.018550					
y_{32}	23.505460	26.006577	9.399248	17.448324	39.131261	107.967340				
y_{41}	29.999529	29.803956	20.298965	17.511155	34.579605	37.717573	271.160692			
y_{42}	33.756106	38.007857	22.370669	37.347154	43.193144	42.537537	89.217839	318.255988		
y_{51}	9.734404	9.669147	9.024012	7.554708	9.451253	13.843342	28.371503	23.131183	29.039444	
y_{52}	8.714969	13.658228	5.762853	10.002066	52.568080	20.950290	14.868362	24.875550	9.676115	103.527574
w_{11}	55.791909	49.844387	34.738560	33.497269	58.067830	73.531188	171.216443	154.808704	39.731596	24.489810
w_{12}	63.284179	60.843330	37.613784	42.542622	66.403590	78.478098	184.661670	194.650084	39.993964	32.475337
w_{21}	119.613784	101.830287	68.221329	71.646604	151.426933	158.871886	326.759172	283.906099	84.092929	53.775923
w_{22}	127.464632	111.996251	74.488824	84.601056	149.661987	163.270118	322.929001	339.648259	84.668418	53.590410
z_1	3.057959	3.737485	1.304484	1.269694	1.621674	2.316486	1.610502	2.864910	0.570612	0.669228
z_2	2.444938	2.437939	1.450132	1.460207	1.345447	2.413960	5.606104	5.638486	1.255497	1.252492

^{a)} Measurement unit: 1000 Norwegian 1974 kroner.

Table 17.3 (continued)
Covariance matrix of the observed variables without winsorization

	Income measure 1		Income measure 2		Number of children	Number of adults
	w_{11}	w_{12}	w_{21}	w_{22}	z_1	z_2
w_{11}	492.489522					
w_{12}	479.146509	591.015725				
w_{21}	801.751429	820.038973	1746.283299			
w_{22}	788.745461	941.759555	1741.041065	1980.047191		
z_1	2.203275	4.187152	9.210435	12.114696	1.579200	
z_2	12.878874	13.961632	21.236450	22.753770	0.078527	0.826605

FOOTNOTES

¹ An earlier version of this paper was presented at the Econometric Society European Meeting, Bologna, August 29 - September 2, 1988, and at seminars at the University of Oslo and the Central Bureau of Statistics of Norway. We are grateful for useful comments at the seminars and for particularly helpful comments from Robert Porter and two referees. Liv Daasvatn has given valuable programming assistance in the preparation of the data. This research has been financed in part by NORAS (project no. 2141802).

² Autonomy is here used in the sense of Haavelmo (1944, section 8).

³ Allowing for correlation between (μ, λ) and z would have made the coefficient matrices C and F unidentified. This can be seen by, for instance, combining (4) and (6) with the relationships $E(\mu|z) = A(z - E(z))$ and $E(\lambda|z) = B(z - E(z))$, where A and B are matrices of constants of dimension $I \times M$ and $K \times M$, respectively, the columns of A adding to zero. Then $C + A$ and $F + B$ would have been identifiable, but not C , A , F , and B .

⁴ Assuming v_t and ε_t to be stationary, this implies that $y_t - a_t$ and $w_t - d_t$ are both cointegrated with ξ_t .

⁵ The standard LES model implies constancy of the marginal budget shares at current prices, while we have used expenditure at constant prices in our empirical analysis. Since the relative prices changed only to a small degree in our data set we consider this as an acceptable approximation.

⁶ Observe that (13) implies homoscedasticity across time periods and no autocorrelation of measurement errors, which may be questionable, in particular for durable goods. A few experiments indicated that the conclusions drawn in this paper on the appropriate specification along the four dimensions are robust to such dynamic extensions of the model. Further experiments indicated that a systematic and satisfactory treatment of dynamic specifications in our context is demanding, and perhaps impossible, without a more flexible computer program (permitting inequality constraints on parameter values) and without data with a longer time span than $T=2$ years. Thus we have left such dynamic extensions of the analysis for future research.

⁷ These conclusions were supported by the observations that the computer program LISREL 7 (cf section 3) had no problems of obtaining convergence and of calculating the information matrix for each of the models we had found to be identified, while this was not the case for the non-identified models.

⁸ To calculate standard errors for parameters not obtained directly from

the LISREL output, we have applied the approximate asymptotic variance formula based on Taylor series expansions (cf e.g. Kmenta (1986,p.486)).

⁹ By e.g. a 'M3 model' we understand here and in the following a model containing M3 as one of its characteristics.

¹⁰ The symbol $\sigma_{\varepsilon\varepsilon}^{ij}$ denotes the ij -th element in $\Sigma_{\varepsilon\varepsilon}$, and correspondingly for other matrices.

¹¹ The parameters ρ^i , like all other population parameters, are estimated by the maximum likelihood method. This contrasts with the LISREL program which automatically produces a "squared multiple correlation", which can be looked upon as a "bastard" estimator of our parameter ρ^i , combining the the ML-estimator of σ_{vv}^{ii} and the sample counterpart of σ_{yy}^{ii} .

¹² To test and discuss this hypothesis thoroughly would require a separate study, and we have not focused on this aspect in the present paper. But tables 16-17 do not contradict the hypothesis and it does not seem unreasonable to conclude that the substantial empirical results in the present paper do not essentially depend on our specific winsorization procedure. Among the empirical results in table 16-17 one may note the following. (1) The most important changes occur when going from a 0 to a 1 per cent winsorization, the choice between 1 and 2 per cent does not matter so much. This indicates that it is important to choose some type of robust estimator, but its exact specification is not so important. (2) The variances of the observed variables decrease very much when going from the nonwinsorized to the winsorized case, e.g. the variance for Travel and recreation in period 1 is reduced from 271 to 131, which to a large extent is captured by the reduced variances of the measurement errors, e.g. the estimate for Travel and recreation decreased from 184 to 88. (3) The estimates of the marginal budget shares, on the other hand, are rather insensitive to the degree of winsorization. (4) The estimates of the variances of the preference variables are affected much more, but not in a systematic way, and one of the estimated variances became negative in the nonwinsorized case, indicating that estimates of the preference variation are not reliable when including the most extreme observations.

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