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# Discussion Paper

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## Net National Product as a Welfare Indicator

Kjell Arne Brekke

### Abstract

Weitzman (1976) claimed that in the case of linear utility, NNP is proportional to welfare, defined as the discounted value of future utility. We first demonstrate that this theory only applies to economies with stationary economic policy and no technical progress. Furthermore Weitzman's result does not generalize to the case of nonlinear utility.

We also prove that only under the assumption of unchanged economic policy and with constant shadow value of investment is marginal changes in NNP a measure of marginal changes in welfare. Thus the theory gives no justification of attempts to maximize NNP growth. Finally we point out that levels or growth rates of NNP for different countries, is no indicator of relative welfare or the relative success of economic policy.

In the last part of the paper we discuss national wealth as a potential welfare measure. We point out that in a small open economy, welfare will be an increasing function of national wealth. Unfortunately, this will no longer be true if we relax the assumption that the economy is small and open.

# 1 Introduction

The main purpose of this paper is to consider the theoretical foundations for NNP as a welfare measure. This is an important basis for the discussion of extensions of NNP to include changes in environmental quality or the nations resource base.

Environmental resources are crucial for economic development. In spite of this fact, a standard measure of economic development, the national product, takes no account of changes in a country's resource base. This has led to the claim that NNP or GNP should be changed to account for the changes in environmental resource base. It is, however, not obvious that this is the best response to the problem. No single measure can give a complete account of the development of a complex body like a national economy. For simplicity and overview of the development, it is preferable with few measures. On the other hand, aggregating things that do not belong together will not add clarity.

The claim that NNP or GNP should be extended has got considerable attention from non-economists. Postel (1990) argues for including development of environmental variables in GNP, but she provides no economic theory to guide us to what kind of conclusions we can draw from this redefined GNP. Still the lack of account for environmental variables in GNP is taken as evidence that economist ignore environmental problems. This illustrates the symbolic importance of national accounting figures. The separation of national product and indicators of the development in environmental resource base, indicates that a good environment is a costly luxury good, rather than an important input for the economy. I consider this as a very important point, but a further discussion will unfortunately be beyond the scope of this paper.

To give an outline of the paper: In section 2 we will present a result due to Weitzman (1976). He proved that in an economy with infinite lived agents with perfect foresight and linear utility, NNP is proportional to welfare defined as the discounted value of the infinite stream of utility for the representative agent. We will demonstrate that this is only true with stationary technology, which excludes dynamic economic policy and technical progress. We will also point out that NNP as defined in Weitzman (1976) does not correspond to either real or nominal NNP, since it is calculated at current prices in utility units. We will also prove that the result cannot be

generalized to the case of non-linear utility.

In section 3 we consider the possibility of using marginal changes in NNP as a measure of marginal changes in welfare. We demonstrate that the change in NNP will not measure the changes in welfare due to a marginal change in policy. Furthermore, NNP growth is only a measure of changes in welfare under the assumption that economic policy is unchanged and that the rate of return is equal to the discount rate. Finally we show that we NNP from different countries is not a measure of relative welfare.

In section 4 we discuss the definition and properties of national wealth, and in section 5 we show that if all prices are exogenous, then welfare is a increasing function of wealth. In section 6 we will argue that the assumption of exogenous prices cannot be relaxed. Section 7 summarizes the results and concludes that NNP is not a welfare measure in any reasonable sense.

## 2 NNP and the level of welfare

We will start with the result from Weitzman (1976). The result states that in a Ramsey model with linear utility, NNP is proportional to the present value of the optimal consumption stream. I.e., Consider the problem of choosing consumption and investment to find

$$V_t(K_t) = \max \int_t^{\infty} C(s)e^{-r(s-t)}ds, \quad (1)$$

subject to

$$(C_t, \dot{K}_t) \in S(K_t), \quad (2)$$

where  $\dot{K} = \frac{dK}{dt}$ . The set  $S(K_t)$ , is the technology, which only depends upon  $K_t$ , and is independent of  $t$ . The problem can be interpreted as the optimal consumption problem for a representative agent with linear utility. Weitzman (1976) proved that there exist competitive prices  $p_t$  such that

$$rV(K_t) = C_t^* + p_t \dot{K}_t^* = Y_t^*, \quad (3)$$

where  $C^*$  and  $K^*$  is the optimal solution to (1) and (2).

It is well known that the optimal solution for the representative agent is equal to the competitive solution in a decentralized economy. Hence the competitive prices  $p_t$  is equivalent the adjoint variables from the corresponding

optimal control problem, and hence  $Y_t^*$  is equal to the hamiltonian evaluated at the optimal policy.

$V(K_t)$  is the total discounted utility for the representative agent. We will refer to  $V$  as “welfare” though this definition of welfare is admittedly oversimplified. Defining  $V$  as welfare, we see that NNP is proportional to welfare, i.e., NNP is a welfare measure.

Weitzman also interpreted  $Y_t^*$  as NNP, and concluded that NNP proportional to welfare. There is one problem with this interpretation.  $p_t$  is current prices measured in utility units, which is different from nominal current prices. Thus  $Y_t$  cannot be interpreted as nominal NNP. Real NNP on the other hand is measured using a set of prices,  $p_0$ , from a base year. Thus  $Y_t^*$  cannot be interpreted as real NNP. This inaccuracy on the timing of prices turn out to be an important problem for the theory.

Let us disregard this problem for a moment. How can a measure that depends only upon the value of *current* consumption and investment reflect the infinite stream of *future* utility? The answer is that we have assumed that the representative agent have infinite horizon perfect foresight. His current choice of consumption and investment reflects the value of alternative future spending. Thus the assumption of perfect foresight is crucial. We have also assumed that the agents do not die, but this assumption can be relaxed. As shown in Barro (1974) it is possible to derive the same equilibrium if the next generations utility is an argument in the current generation’s utility function. The model also assumes full certainty, but this assumption can also be relaxed. For a further discussion on the properties of the market solution under different assumption, see Blanchard and Fisher (1989).

To summarize this discussion we can conclude that the theory of NNP as a welfare measure relies on very strong assumptions. On the other hand, is it usually impossible to derive an empirical measure without strong assumptions. The purpose of this paper is, however, to show that even if we accept these assumptions as a reasonable approximation of reality, the theory does not prove that NNP is a welfare measure.

## 2.1 Net national product and the design of economic policy

Welfare indicator is needed for evaluation of different economic policy proposals. To formalize this idea, we have to introduce governmental decision variables into the model. It is important for the proof of (3), that  $K_t$  and  $C_t$  are optimal, hence the only place in the model where a governmental decision variable can be introduced is the the production possibility set  $S(K_t)$ .

Let  $\theta_t$  be the governmental decision variables at time  $t$ . Suppose that the production possibility set is  $S(K_t, \theta_t)$ . Let  $\Theta$  denote a path of  $\theta_t$ . The optimal solution  $C^*$  and  $K^*$  will depend upon the economic policy. Let  $C_t^*(\Theta)$  and  $K_t^*(\Theta)$  denote the value of the optimal solution at time  $t$  given the economic policy  $\Theta$ . Suppose furthermore that the government wants to maximize total welfare. Then  $\Theta$  should be chosen to maximize  $V(K_t, \Theta)$ . By (3),

$$\max_{\Theta} V(K_t, \Theta) = \max_{\Theta} \left[ \int_t^{\infty} C_s^*(\Theta) e^{-r(s-t)} ds \right] = \frac{1}{r} \max_{\Theta} [Y_t^*(\Theta)]. \quad (4)$$

Thus maximizing NNP,  $Y_t^*(\Theta)$ , is equivalent to maximizing  $V(K_t, \Theta)$ . Note that only  $Y_t^*$  is relevant for evaluating the policy  $\Theta$ . The path of  $Y_s^*$  for  $s > t$  is totally irrelevant, which I think is contrary to most economist's intuition.

Note, however, that in (2),  $S$  is independent of  $t$ , thus  $S(\cdot, \theta_t)$  should be independent of  $t$  as well. This restriction excludes technical progress and dynamic economic policy. That this restriction in fact is essential is demonstrated by the following example, showing that when  $\theta$  is time dependent, (3) is wrong.

*Example 1.* Consider an economy with aggregated production function,

$$f(K_t, \theta_t) = AK_t^a + B\theta_t, \quad (5)$$

where  $0 < a, b < 1$ .  $\theta_t$  can be interpreted as the depletion of a finite publicly owned resource. The stock of the resource is denoted  $X_t$ , and  $\dot{X}_t = \theta_t$ . With a linear utility the optimal capital path will be independent of the extraction path  $\Theta$ , thus

$$Y_t^* = A(K_t^*)^a + p_t \dot{X}_t + B\theta_t. \quad (6)$$

Thus  $Y_t^*$  depends upon  $\Theta$  only through  $\theta_t$ , which may even be chosen equal to zero, while the rest of the path has obvious effects on the total welfare. This demonstrates that the assumption of a stationary technology is essential.  $\square$

The following proposition summarizes our findings.

**Proposition 1** *In an economy with: a) infinite lived agents with perfect foresight and where b) agents have linear utility, and the technology is stationary, NNP is a measure of the discounted value of all future utility. Assumption b) is essential for this conclusion.*

*To maximize welfare the government should choose a strategy that maximizes current net national product. The path of future development in NNP is irrelevant.*

The requirement of a stationary technology excludes considerations of dynamic policy and economies with technical progress. In the following we will only consider constant policies, denoted by  $\theta$  (instead of the path  $\Theta$ ).

The conclusion that only one point of the NNP path is relevant runs contrary to the standard intuition that economic growth is desirable. To illustrate the intuition behind this result, consider the following example:

*Example 1. (cont.)* We can bring the problem in example one over to a problem with constant policy, by assuming that the government decides the extraction rate, and that only constant rates of extraction are allowed, i.e., the extraction is

$$\theta_t = d \cdot X_t, \quad (7)$$

where  $X_t$  is the resource stock, and  $\underline{d} \leq d \leq \bar{d}$  is the extraction rate. Letting  $\underline{d}$  denote the policy, we must consider  $X_t$  as real capital, and the dis investment in resources must be added to  $Y_t^*$ . Still the rest of the economy will be unaffected of the extraction path. Let  $\tilde{Y}_t$  denote the NNP in the case  $X_t = 0$ , i.e. with no resources. Then net national product with resources ( $X_t > 0$ ) will be

$$Y_t(d) = \tilde{Y}_t + B \cdot X_t d - p_t \cdot X_t d \quad (8)$$

It is easily derived that  $p_t = \frac{Bd}{r+d} < B$ , and thus  $Y_t^*$  is increasing in  $d$ . On the other hand  $X_t$  is *decreasing* at the rate  $d$ , hence the growth rate of the economy is decreasing in the extraction rate,  $d$ . Thus if the government chooses the policy to optimize the growth rate of the economy, it will choose  $d = \underline{d}$  which minimizes the social welfare! To maximize welfare the government should choose  $d = \bar{d}$  which minimizes the growth rate.

The intuition behind this result is straightforward. With linear utility, the resource should be extracted as soon as possible. Any delay will be a loss, due to discounting. On the other hand, a rapid extraction of a finite resource, must lead to a rapid fall in NNP towards the level corresponding to no resource extraction.  $\square$

## 2.2 Nonlinear utility with many commodities

In the previous section we assumed linear utility, and only one commodity. A more general and reasonable assumption is to consider an economy with many commodities and assume a non-linear instantaneous utility function. I.e. we assume that the representative agent maximizes:

$$\int_t^{\infty} u(C_t) e^{-rt} dt \quad (9)$$

where  $C_t$  is a consumption vector.

To generalize Weitzman's result to this case, let  $\tilde{C}_t = u(C_t)$ , and define the production possibility set:

$$\tilde{S}(K_t, \theta) = \{(\tilde{C}_t, \dot{K}_t) | \tilde{C}_t = u(C_t); (C_t, \dot{K}_t) \in S(K_t, \theta)\} \quad (10)$$

Now the competitive solution is formally equivalent to the solution above with  $C$  and  $S$  replaced by  $\tilde{C}$  and  $\tilde{S}$ . Thus by Weitzman's result we know that

$$\tilde{C}_t^*(\theta) + p_t(\theta) \dot{K}_t^*(t, \theta) = r \int_t^{\infty} \tilde{C}_s^*(\theta) e^{-r(s-t)} ds. \quad (11)$$

Note that both prices and quantities will depend upon the policy  $\theta$ . This right hand side of this equation can be rewritten to:

$$rV(K_t, \theta) = u(C_t^*) + p_t d\dot{K}_t^* = H_t^* \quad (12)$$

Where  $H_t^*$  is the Hamiltonian of the optimization problem. This equation gives us immediately the following proposition.

**Proposition 2** *To measure the level of welfare, we should count the current utility of consumption, not the consumption level. When the utility function is unknown, the level of NNP gives no information about welfare.*



### 3 Growth and marginal changes in welfare

It is unreasonable to assume that the utility function is known, thus the previous proposition is a crucial objection against using level of NNP as a welfare measure. On the other hand, the utility function can locally be approximated as a linear function. Thus it is natural to consider the possibility that marginal changes in NNP is a measure of marginal changes in welfare.

By (12), and the equilibrium condition  $p_{jt} = \frac{\partial u}{\partial C_{jt}}$ , the effect of a marginal change in  $\theta$  is:

$$r \frac{\partial V}{\partial \theta} = \sum_{j=1}^n \left[ p_{jt} \left( \frac{\partial C_{jt}^*}{\partial \theta} + \frac{\partial \dot{K}_{jt}^*}{\partial \theta} \right) + \frac{\partial p_{jt}}{\partial \theta} \dot{K}_{jt} \right] \quad (13)$$

This gives a linear approximation of  $rV(K_t, \theta)$ , around a given policy  $\bar{\theta}$ :

$$rV(K_t, \theta) \approx \bar{V}_t + \Delta NNP_t(\bar{p}_t) + \Delta p^T \dot{K}_t \quad (14)$$

where  $\bar{V} = rV(K_t, \bar{\theta})$  and  $\bar{p}_t$  is equilibrium prices at  $\bar{\theta}$ .  $\Delta NNP$  is changes in net national product measured with  $\bar{p}_t$  prices. Note that if  $\frac{\partial p}{\partial \theta} = 0$ , the last term in (14) vanishes, and changes in NNP is a measure of changes in welfare. But is the assumption that  $\frac{\partial p}{\partial \theta} = 0$  reasonable? Unfortunately not.

Consider a case where an increase in  $\theta$  will be equivalent to a technical progress, but that only at a capital level higher than  $K_t$ , i.e.  $S(K, \theta)$  is independent of  $\theta$  for  $K = K_t$ . We will denote  $(\Delta C, \Delta K)$  as marginally feasible if  $(C^*, \dot{K}^*) + \alpha(\Delta C, \Delta K) \in S(K_t, \bar{\theta})$ , for  $\alpha$  sufficiently small. Since  $C_t^*$  and  $\dot{K}_t^*$  maximizes the hamiltonian within  $S(K_t, \bar{\theta})$  for  $\bar{p}_t$  given, we know

$$\frac{\partial u}{\partial C} \Delta C + \bar{p}_t \Delta \dot{K} \leq 0. \quad (15)$$

Since a change in  $\theta$  does not change  $S(K_t)$ , the first order effect on  $(C^*, \dot{K}^*)$  must be marginally feasible, and hence,  $\Delta NNP \leq 0$ .

Still  $V(K_t, \theta)$  must obviously be increasing in  $\theta$ , since increasing  $\theta$  is equivalent to a technical progress. Since  $H_t^*$ , by (12) is equal to  $rV$  we conclude that  $\Delta p^T \dot{K}_t^* > 0$ .

Note that the essential assumption made in the argument above was that  $S(K, \theta)$  is independent of  $\theta$  for *one* point ( $K = K_t$ ) in the space of possible capital vectors. Thus changes in NNP does not measure the effect of changes

in production possibilities outside this point. On the other hand, changes in technology outside this point, is obviously important for changes in welfare.

Since changes in prices usually are second order effects that can be ignored in a marginal analysis, it may seem contra intuitive at first glance that it is the price changes that captures the welfare changes. The price changes would have been a second order effect if we were considering a marginal exogenous changes in investment and consumption, but in the analysis above we were considering a marginal change in *policy*,  $\theta$ , and this will have a first order effect on both prices and quantities.

An alternative linear approximation of the welfare function will be useful in the following. This can be derived by linearizing the utility function. Note that the un observable term in (12) is  $u(C_t^*)$ . Thus alternatively we may use a linear approximation of  $u$ . By a Taylor expansion of  $u$  we get:

$$u(C_s) \approx u(C_t^*) + \sum_{i=1}^n \frac{\partial u}{\partial C_i}(C_t^*) \cdot (C_{is} - C_{it}). \quad (16)$$

Using the equilibrium condition  $p_{it} = \frac{\partial u}{\partial C_i}(C_t^*)$  we get:

$$rV(K_s) \approx u(C_t^*) + NNP_{s,t} + (p_s - p_t)I_{ts} \quad (17)$$

where  $NNP_{s,t}$  is the net national product at time  $s$  measured by prices at time  $t$  and  $I_{ts}$  is average level of investment from time  $t$  to  $s$ .

We noted in section 2 that there was a problem in indentifying  $Y_t^*$  as either nominal or real NNP. An interesting observation at this point is that  $NNP_{s,t}$  corresponds to the real NNP, measured at time  $s$  with base prices from time  $t$ . I will thus use the approximation in (17) in the following.

### 3.1 Evaluating NNP growth

In the case of linear utility we demonstrated that it may be optimal to minimize economic growth. But in that example the resource stock was declining over time, and we may expect welfare to decline as well. It may still be true that changes in NNP over time is a measure of changes in welfare.

Equation (17) implies

$$r(V(K_s) - V(K_t)) = NNP_{t,s} - NNP_{t,t} + (p_s - p_t)I_{ts}. \quad (18)$$

It follows immediately that changes in NNP measures changes in welfare only if shadow prices on capital are unchanged. We know that with changes in policy from  $s$  to  $t$ , we must expect prices to change, but what are the conditions for constant shadow prices when the policy is unchanged?

**Proposition 3** *If the shadow price of investments is constant and economic policy is unchanged, growth in welfare is proportional to NNP growth.*

*NNP will not measure the welfare effect of changes in  $S(K, \theta)$  outside the current point in the capital space, i.e. for  $K \neq K_t$ . Thus the theory gives no justification of attempts to maximize economic growth.*

How restrictive is the assumption that the shadow price on investment is constant? From optimal control theory we know that  $\dot{p}_t = rp_t - \frac{\partial H_t}{\partial K_t}$ , where  $H_t$  is the hamiltonian. In the one-good-case with linear utility and where  $f(K_t) = C_t + \dot{K}_t$ , we have that  $NNP_t = f(K_t)$ , and thus  $\frac{\partial H_t}{\partial K_t} = f'(K_t)p_t$ , or

$$\dot{p}_t = p_t(r - f'(K_t)). \quad (19)$$

Thus the shadow prices of capital will not be constant until we have reached the golden rule level of capital. But at that point there will be no changes in NNP. On the other hand, if  $f'(K_t) \approx r$  or  $I_t \approx 0$ , we may ignore the last term in (18), and NNP-growth will be approximately proportional to welfare changes.

### 3.2 Comparing NNP figures from different countries

Suppose that we are comparing the NNP development in two countries, which start at the same NNP level in period  $t$ . Suppose furthermore that NNP is growing faster in one country than in the other. Can we conclude that the government in the country with highest growth is using the best economic policy?

Given our previous findings, a negative answer should come as no surprise. To demonstrate that the answer is negative, consider the case in Example 1. Suppose that both countries have the same production function. The only difference is that country 1 has reserves  $X_{1t}$  that is higher than the resource reserves for country 2,  $X_{2t}$ . Suppose furthermore that  $\underline{d} \cdot X_{1t} = \bar{d} \cdot X_{2t}$ , and that country 1 actually uses the policy  $\underline{d}$  while 2 uses  $\bar{d}$ . Then initial NNP is equal, and as utility is linear, the two countries are initially equally

well off though  $X_{1t} > X_{2t}$ . As 1 uses an inferior policy, her growth rate will be highest.

**Proposition 4** *In comparing NNP between countries, differences in NNP level does not indicate differences in welfare level, and differences in NNP growth do not indicate the relative success of the economic policy.*

## 4 National wealth

To discuss possible definition of national wealth, we first have to know what to mean by “wealth”. The word is commonly used in financial economics. In these applications the “wealth” has a very specific structure. If we can define a national wealth with the same structure, we could use wealth management results from financial theory, to evaluate the development in national wealth. Let us thus first consider the structure of a financial wealth.

Let  $p_t$  be an  $n$ -dimensional vector of prices at time  $t$  on  $n$  different assets, and let  $\delta_t$  be the vector of dividends payed to the asset holders. Let  $\theta_t \in R^n$  be the number of assets an agent owns at time  $t$ . The wealth  $W_t$  is the total value of all assets:

$$W_t = p_t^T \theta_t \quad (20)$$

Prices and dividends are usually considered as exogenously given to the agents, especially the value of the different assets is independent of the agent’s utility function.

The wealth is of no interest if we cannot spend it for consumption. An agent holding  $\theta_{t-1}$  assets when entering period  $t$ , and choosing the portfolio  $\theta_t$  in period  $t$ , will get consumption  $C_t$ , given as:

$$C_t = p_t^T (\theta_{t-1} - \theta_t) + \delta_t^T \theta_{t-1} \quad (21)$$

Finally, explicit solution of wealth management problems usually requires no restriction on  $\theta_t$ , except that wealth must be positive.

We have identified three central properties of a financial wealth.

1. It is the total value of all assets.
2. It can be spent on consumption, and its size is independent of the utility of the wealth owner.

3. The owner of the wealth can determine how much on the wealth to place on each available asset.

## 5 National wealth in an open economy

An example of an economic model where we can define a wealth with the properties outlined in the previous section is the following.

$$\max u(c_0, c_1, c_2, \dots) \quad (22)$$

under the constraints

$$\begin{aligned} A_t &= p_t^T (F(K_t, X_t) - I_t) - C_t \\ B_{t+1} &= (B_t + A_t)(1 + r) \\ K_{t+1} &= d(K_t, X_t, I_t) \\ (X_t, I_t) &\in M(K_t) \\ C_t &= p_t^T c_t \\ \lim_{t \rightarrow \infty} B_t(1 + r)^{-t} &\geq 0 \end{aligned} \quad (23)$$

$p_t^T$  is the transposed of the vector of world market prices.  $c_t$  is a vector of consumption and  $C_t$  its value on the world market.  $A_t$  is net export.  $F(\cdot, \cdot)$  is the production function where  $K_t$  is capital, possibly including natural resources, and  $X_t$  is other variables, possibly extraction of resources, distribution of total investment etc..  $B_t$  is foreign bonds and  $I_t$  is total investment.  $d(\cdot, \cdot, \cdot)$  is a function describing the dynamics of the capital accumulation process. Finally is  $M(K_t)$  the set of possible values for  $X_t$  and  $I_t$ . We assume that  $u(\cdot)$  is strictly increasing.

Let  $I^*$ ,  $X^*$  and  $C^*$  be the optimal policy. Let  $K^*$  be the corresponding path of capital, and let  $\pi_t^*$  be income:

$$\pi_t^* = p_t^T (F(K_t^*, X_t^*) - I_t^*) \quad (24)$$

We can prove that  $\pi_t^*$  is the path that maximizes net present value of

future income, i.e. if  $\hat{\pi}$  is a feasible income path, then<sup>1</sup>

$$\sum_{t=0}^{\infty} \hat{\pi}_t (1+r)^{-t} \leq \sum_{t=0}^{\infty} \pi_t^* (1+r)^{-t}. \quad (25)$$

The interpretation of this result is that in a centrally planned economy, the government should choose production to maximize present value of future income, the planner do not have to consider optimal consumption simultaneously.<sup>2</sup>

We define national wealth at time  $t$  as:

$$W_t = \sum_{s=t}^{\infty} \pi_s^* (1+r)^{t-s} + B_t \quad (26)$$

It is easily seen from the equations above that:

$$W_{t+1} = (W_t - C_t)(1+r). \quad (27)$$

There is no other restriction to the choice of  $C_t$  than  $\sum_{s=t}^{\infty} C_s (1+r)^{t-s} \leq W_t$ , thus the nation is free to spend this wealth. Furthermore it is independent of the utility function.

We define the aggregate utility function  $U$  as:

$$U(C_0, C_1, \dots) = \begin{array}{l} \max \quad u(c_0, c_1, \dots) \\ \text{s.t.} \quad p_t^T c_t \leq C_t \text{ for } t \geq 0 \end{array} \quad (28)$$

We can then prove that the optimal consumption path is the one that maximizes  $U(C_0, C_1, \dots)$ , subject to:

$$\sum_{t=0}^{\infty} C_t (1+r)^{-t} \leq W_0 \quad (29)$$

---

<sup>1</sup>This result is not deep. With dated commodities, the budget equation is  $p^T c \leq p^T y$ , where  $p$  are prices,  $c$  consumption and  $y$  production, all of which are vectors in dated and possibly state contingent commodities. With no interconnection between  $c$  and  $y$  it is obvious that we should maximize  $p^T y$ . The formulation in the text has the advantage of being more specific on the definition of national wealth.

<sup>2</sup>In a decentralized economy we can prove a similar result by assuming that the government has to sets of instruments. One set solely affecting the production side of the economy, while the other set solely affects the demand side. Then the optimal policy can be found by first choosing production instruments to maximize the wealth. The government do not have to consider demand side effects in this optimization. Then in the second step the optimal demand side policy is chosen.

We can now define the welfare function  $V$  as a function of wealth, by

$$V(W_0) = \max U(C_0, C_1, \dots) \quad (30)$$

s.t. (29). If  $u$  is strictly increasing,  $V$  is obviously strictly increasing too. Note also that if two countries have the same utility function<sup>3</sup>,  $u$ , there aggregated utility function  $V$ , will be equal too. This gives the following conclusion.

**Proposition 5** *In a small open economy, national welfare is an increasing function of national wealth. Moreover, for two countries with equal utility function, the country with highest per capital wealth has highest per capital welfare.*

## 6 National wealth in a large economy

In the previous section we considered an economy where all prices were exogenously given. This is not the case for most national economies. Furthermore, an important reason for the renewed interest in NNP as a welfare measure is the possibilities of extending the definition of NNP to include changes in environmental resources. But services from the environment, like unpolluted air, are not tradeable, and hence prices will be determined domestically. Thus, it is important to extend of Proposition 5 to economies where domestic supply and demand effects prices. Unfortunately this is not possible.

*Example 2* To illustrate the problem, consider an one period, two good economy, where the price,  $p_1$ , on good 1 is given exogenously, while good two is produced and consumed domestically. Suppose the utility function of the representative consumer is  $u = c_1^\alpha c_2^{1-\alpha}$ , where  $c_i$  is consumption of good  $i$ . Since there is no trade in good two,  $y_2 = c_2$ , and since the model is static, the budget restriction implies  $y_1 = c_1$ . The equilibrium condition

$$\frac{p_2}{p_1} = \frac{u'_2}{u'_1} = \frac{1 - \alpha y_1}{\alpha y_2},$$

guaranties that this is incentive compatible for a price taking consumer. Using this equation, we can derive the following equation for the value of total production:

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<sup>3</sup>For countries with different utility function we cannot compare welfare anyhow.

$$y_1 \cdot p_1 + y_2 \cdot p_2 = \frac{1}{\alpha} y_1 \cdot p_1. \quad (31)$$

With only one period, the wealth is equal to the value of total production. But, by this formula, the wealth is *independent of the size of the production of good 2*. The reason for this is very simple. When prices are determined domestically, the equilibrium price on good two is higher, the less the production is. In this specific case, the two effects cancels out, leaving the value of the production of good two independent of the size of the production.  $\square$

This example demonstrates that economic welfare is not an increasing function of wealth unless the economy is so small that all prices can be taken as exogenously given. On the other hand, comparing wealth at the same point in time, and the same set of prices but for different policies, is equivalent to a cost benefit analysis. Under the assumption that makes benefit minus cost in a CB analysis a welfare measure, changes in wealth is a measure of changes in welfare. To extend this to changes in wealth from one year to an other, other assumptions most likely have to be added. Furthermore, calculation in Aslaksen et.al. (1990) shows that changes in wealth include a huge stochastic term, independent of economic policy. A useful welfare measure would have to correct for this. A throughout analysis of these problems are beyond the scope of this paper.

## 7 Summary and Conclusions

Let us first summarize our findings before we turn to the conclusion about NNP or wealth as welfare measures. Proposition 1 states that the assumption of stationary technology is essential. This already excludes economies with technical changes and consideration of dynamic economic policy. Some of these problems may be overcome by introducing new capital component, e.g. we may introduce technical change through human capital. These extra capital components may be hard to measure. More important is the exclusion of dynamic policy. It is hard to see how to resolve that problem.

Proposition 1 also states that under the assumption of stationary policy and linear utility, the level of *current* NNP is proportional to welfare. We noted that the result was unclear as to whether this referred to real or nominal NNP.



Proposition 2 states that the results derived under the assumption of linear utility, cannot be generalized to non-linear utility. Thus generally the level of NNP is not proportional to welfare. Even if we accept the discounted value of future consumption as a reasonable concept of economic welfare, this observation poses a problem.

Proposition 3 consider the possibility of using *changes in* NNP as a measure of changes in welfare. The proposition states that changes in *real* NNP is proportional to changes in welfare only under the assumption that economic policy is unchanged and that the shadow price of investments is unchanged over time. Furthermore, most welfare effects of changes in economic policy is not measured by changes in NNP. These conclusions are true even for linear utility.

Finally, Proposition 4 states that comparing level or growth of NNP for different countries, does not allow us to draw conclusion about relative welfare or success of economic policy.

Given these results it is hard to see that NNP can be a welfare indicator in any reasonable sense. Note however, that we do not claim that NNP is *un* correlated to economic welfare. NNP is likely to be correlated to  $u(C_t)$ , and thus correlated to welfare as defined in this paper.

The last proposition states that in a small open economy, welfare is an increasing function of national wealth. A small open economy is defined as an economy where all prices are exogenously given. Unfortunately, this assumption seems to be essential.

The ultimate critique of using NNP as a welfare measure, is to point at a better alternative. Does there exist any better alternative? A national economy is very complex. The changes in the economy from one year to another range from unemployment, investment, productivity and technical progress to changes in the environmental resource base. Moreover, closely related to development in economic welfare are areas like culture, science or crime. It is not at all obvious that it will make much sense to try to make *one* measure that should account for total economic development. An obvious alternative to NNP as a welfare measure is to have separate indicators for consumption, development in environment and resources.

I will conclude by underlining that to let welfare only depend upon  $C_t$  is an unreasonable narrow concept of welfare. Important aspects of welfare are disregarded, like distribution, unemployment, quality of environment, and political freedom, just to mention some. Instead of searching for a single

indicator for this narrow concept of welfare, it would be more interesting to discuss extensions of this concept (see Dasgupta (1991)), and to derive a set of indicators for this extended concept of welfare.

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