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## The Multi Sectoral Growth Model MSG 4 Formal Structure and Empirical Characteristics

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# Discussion Paper

Central Bureau of Statistics, P.B. 8131 Dep, 0033 Oslo 1, Norway

No. 8

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## *THE MULTI-SECTORAL GROWTH MODEL MSG-4 FORMAL STRUCTURE AND EMPIRICAL CHARACTERISTICS*

*BY*

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### ABSTRACT

In the first part of this paper a brief retrospective survey of the development of the MSG model is given, from it was first presented in Johansen (1960) until the present version MSG-4. Some principal choices to be made regarding the structure of an applied general equilibrium model are also discussed. The paper then presents main features of the formal structure of the MSG-4 model. Finally, some empirical characteristics of the present MSG version are demonstrated by presenting estimates of long-term total elasticities calculated by the MSG model.

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THE MULTI-SECTORAL GROWTH MODEL MSG-4  
FORMAL STRUCTURE AND EMPIRICAL CHARACTERISTICS

By

S. Longva, L. Lorentsen and Ø. Olsen<sup>\*)</sup>

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## 1 MODEL BACKGROUND AND HISTORY

The macro-economic model known as the MSG-model (abbreviated from Multi-Sectoral Growth) was first presented in Johansen (1960). This work represented the first successful implementation of an applied general equilibrium model without the assumption of fixed input-output coefficients, cfr. Jorgenson (1982). Johansen assumed fixed coefficients in modeling demand for intermediate goods, but applied Cobb-Douglas production functions in modeling the substitution between labour and capital services. Neutral technical change was assumed by adding time trends to the linear logarithmic production functions. Johansen replaced the normally applied assumption of fixed coefficients in household demand by a system of demand functions, based on Frisch (1959). Both producers and households behaviour were dependent on relative prices. The total supplies of capital and labour were assumed to be inelastic, i.e. exogenously given, and the equilibrium solution of Johansen's original 20 sector model was simultaneous in prices and quantities.

The Johansen study was an attempt to construct a model which covered important aspects of the process of economic growth, with particular emphasis on the explanation of differences in growth rates between various sectors of the economy. It was the explicit intention of the model's originator that the theoretical content should be kept simple enough for the model to be implemented by existing statistics and solved by means of computational equipment available around 1960. Linear logarithmic functions imply that the parameters describing substitution between labour and capital can be estimated from a single data point, as factor shares. The price elasticities of the Frisch demand system can also be determined from a single data point, given the expenditure elasticities and the elasticity of the marginal utility of total expenditure which must be estimated econometrically. As a concession to computational difficulties, the original study only included calculations of growth rates from a starting point, obtained by neatly partitioning and manipulating the matrix formulation of the model. During the 60-th and early 70-th, Johansen's MSG-model gave impetus to an extensive research effort at the Institute of Economics at the University of Oslo. For complete references, see Johansen (1974), which also presents a survey of general equilibrium

modeling through that date.

Some years after the original presentation, the Norwegian Ministry of Finance launched a project for revising the model and developing adequate computational and administrative routines for using the model in long term economic planning. Larsen and Schreiner (1985) contains a detailed account of the introduction of the MSG model in the Norwegian planning system. This new version of the model, called MSG-2F, became operational in 1968 and is described by Schreiner (1972) and Spurkland (1970). MSG-2F was extensively used for some years, mainly to calculate growth paths for the economic development 5-30 years ahead but also to solidify government reports and for ad hoc analyses.

In the early seventies another revision of the model became necessary mainly due to the introduction of a new system of national accounts, but also due to the growing magnitude of the Norwegian petroleum activities. This work was undertaken by the Central Bureau of Statistics in 1974-75 in close cooperation with the Ministry of Finance. The new version, MSG-3, is presented in Lorentsen and Skoglund (1976). Since this third generation of the MSG model, the Central Bureau of Statistics has been responsible for maintenance and further development of the model along with other models for government planning in Norway.

The fourth version, MSG-4, presented below, appeared in 1980. Through many years of administrative use the MSG models had proved to be useful not only for elaborating long term perspectives for the macro economic development, but also to some extent for sectoral planning. In addition to its traditional application MSG-4 was designed specifically to incorporate the interactions between economic growth and energy production and use, see Longva, Lorentsen and Olsen (1983). This fourth generation of the model also included alternative assumptions for the capital market, introduced new elements of neo-classical theory of production, some sector models were partly based on an engineering approach, econometric methods were applied to a greater extent than in previous versions in assessing model parameters and the computational work was greatly facilitated through the introduction of a powerful and flexible computer system.

Through these three major revisions, the size of the model has increased somewhat (the present version has 32 production sectors) and a number of changes have been made. However, the main theoretical content and structure of the original model have to a great extent been preserved from the original version. The usefulness of Johansen's approach is underlined both by the continuous use of the model for more than 15 years in Norwegian economic planning (see Larsen and Schreiner (1985)) and through

the international proliferation of the model. Gradually, the MSG type of model has become a concept in the literature on economic growth and planning, embracing a variety of multi-sectoral, neo-classical, long-term equilibrium models (Bergman (1985)).

## 2 CHOICE OF THEORETICAL CONTENT IN AN EQUILIBRIUM MODEL TO BE USED IN LONG-TERM MACROECONOMIC PLANNING

### Macroeconomic planning, theory and practice

The concept macroeconomic planning has been defined differently by different authors. Johansen (1977) arrived at the following eclectic definition:

Macroeconomic planning is an institutionalized activity by, or on behalf of a Central Authority for (a) the preparation of decisions and actions to be taken by the Central Authority, and (b) the coordination of decisions and actions by lower-order units of the economy, as between themselves and vis-a-vis the Central Authority, for the purpose of governing the development of the whole economy and its constituent parts so as to achieve certain (more or less detailed and more or less explicitly specified) goals for the economy and harmonize the development of the economy with broader non-economic goals.

At the most advanced level this concept of planning would imply the elaboration of strategies, i.e. sets of plans to meet different situations where the actions would be conditioned by the future outcome of uncontrolled variables. The longer the planning horizon, the more important becomes the strategy element. A theoretically satisfactory treatment of the planning problem would require the use of intertemporal optimization models, where the time profiles of the use of political instruments are determined from maximizing some time dependent welfare function. So far such models have only been used for illustrative purposes in academic settings rather than for decision making. The reasons are obvious; an intertemporal optimization model with an adequate representation of the economy and a maximand with several (conflicting) target variables is at

present unmanageable. And even if it were technically manageable, it would probably not be transparent enough to be accepted by decision makers. An iterative process between simple intertemporal optimization models and more traditional macroeconomic models is though possible and an attractive compromise. In practice, models used for planning purposes have been of the instrument-target type, operated by assessing time paths for instruments and other model exogenous variables. The preferable or optimal solution is then drawn from a mapping of several alternative model generated developments. In most models for long term macroeconomic planning, including MSG, the instruments are represented only indirectly and in aggregate terms. The main issue is to select feasible solutions, and the concern over future implementation problems is often paid little attention or left aside. However, the implicit economic policy constraints necessary to achieve the model generated developments should be derived to evaluate the results.

Even if we limit ourselves to comment upon the traditional instrument-target models, there are still some principal choices to be made about the theoretical content of a long-term model to be used in a general equilibrium planning process, influencing not only the model results but also their proper interpretations. Broadly classified, there are two model approaches :

The first approach is to try to model what is actually going to happen the next 10 to 20 years. In this case the model will realistically have to include some explicit or implicit elements of disequilibrium, allowing for low capacity utilization, delays of adjustments, probably some mismanagement etc. which altogether result in discrepancies between potential and actual growth. This does not mean that the model will have to trace business cycles, but that on average it will allow for some opportunities foregone.

The second approach is to try to model what might happen if everything is working smoothly, i.e. to model potential growth or steady state growth. The Cambridge Growth Project originally adapted this approach, having one model describing the movement of the economy from an initial situation towards a steady state path and another model describing the steady state path, see Stone (1964). This method has the advantage of being theoretically satisfactory, but the policy conclusions one can draw from it depend on the realism of the steady state path as a preferable and "achievable" goal.

In the Norwegian set-up of long term economic planning the MSG simulations have normally been given the interpretation of "neutral

projections". These projections have been interpreted as projections of how the economy actually will work, normally not as potential growth or steady state paths etc., although the intentions have not always been clearly stated. The idea has been to prolong the short and medium term development, given the assumption that external and internal conditions are not radically changed and that economic policies are reasonably successful.

In this approach the relation between the long-term path depicted by the model and the transition path on which current policy must be based is of course very important. As stressed by Bjerkholt and Tveitereid (1985) the underlying logic of the long-term equilibrium path is that medium-term policy should be transitory and directed towards reaching the long term path. In the short term and medium term planning of the Norwegian economy the multi-sectoral input - output based models, i.e. MODIS and MODAG, play a central role (see Bjerkholt and Longva (1980) and Cappelen and Longva (1984)). These models are oriented towards demand management and income policy, combining certain elements from the Scandinavian model of inflation, and Keynesian macrotheory. This is a contrast to the MSG-model where the factors of growth (growth in labour force, capital accumulation and technical progress), i.e. supply side factors, are the driving forces. The separate modeling approach for short and medium-term and for long-term planning partly reflect the fact that the explicit policy instruments in Norway are mostly related to demand management and income policy, while the instruments effecting the supply side are more indirect and have a longer time perspective. However, it also reflects that the coordination between medium-term and long-term policy and planning is hard to achieve. The "technical" solution applied is either to let the medium-term projection approach the long-term path, or to "force" the long-term path through the last year of the current medium-term projection.

It is important to notice that the projections produced in the Norwegian planning process have never been simple presentations of model calculations. Published projections, normally as addenda to the government long term programs, have been results of an iterative process, drawing on the information and experience of various agencies and experts. Once reliable base projections have been drawn, they have been extensively used in more detailed analyses - elaborating energy programs, deducing environmental consequences, regional analyses etc - consistent with the base projection, and as starting points for alternative projections.



### The modeling of labour and capital markets and of external trade

In the actual formulation of a general equilibrium model for the Norwegian economy there are some modelling issues that deserves special attention, namely the modelling of labour and capital market and the modelling of external trade.

In most economic growth models the total supply of labour is exogenous, i.e. inelastic. Hence, a change in the use of material input, energy or capital must change the equilibrium price of labour in real terms. This approach seems appropriate as an approximation to the long-run equilibrium in the Norwegian labour market, or in any economy where full employment is the first priority target, and has been chosen in the MSG-4 model.

The choice of an approximation for the long-run equilibrium in the capital market is less obvious. Two extreme alternatives offer themselves as convenient simplifications (Hogan (1979)):

- i) A fixed total input of capital, i.e. inelastic supply (MSG-4S).
- ii) Fixed real rate of return to capital, i.e. perfectly elastic supply (MSG-4E).

In case i) changes in other inputs - materials, labour and energy - will change the marginal productivity of capital. With a given total stock of capital the equilibrium rate of return to capital in real terms must also change. This may, over time, affect the willingness to save and invest, and the approximation of inelastic supply of capital may turn out to be implausible without some compensating capital policy or without some iterative mechanisms influencing the capital supply. The interplay of labour and capital at the macro level will be trivial, and the equilibrium factor prices need to be checked for realism.

In case ii) capital input is adjusted to changes in materials, labour and energy inputs so that the marginal productivity of capital is maintained. With this approximation of the long-run equilibrium of the capital market, a change for instance in the price of energy will change the total use of capital, materials and energy, the real price of labour and energy and gross output.

The assumption of a fixed real rate of return to capital is characteristic for steady state growth in a neo-classical growth, model while the assumption of an inelastic supply of capital is an appropriate short-run specification of such a model. Even though the MSG model is not used to trace out steady state paths in any strict sense this indicates that the

assumption of perfectly elastic supply of capital is most suitable in studying the long-run tendencies of the economy. When studying the transition path the assumption of inelastic supply of capital may be the most appropriate specification.

The two extreme ways of modelling the capital market have been embedded in two versions of the present MSG model. Exogenous total supply of capital has been a feature of previous MSG models, and this version is called MSG-4S. The version with elastic supply of capital is called MSG-4E. Except for this difference in the philosophy and modelling of the capital market, the two MSG versions are identical.

In academic models and textbooks, a (small) open economy is normally assumed to face a perfectly elastic supply of imports and a perfectly elastic demand for exports at given world market prices. If the economy consumes and is able to produce  $n$  different tradeables by means of  $m$  factors of production at constant returns to scale, (where  $n > m$ ) equilibrium conditions commonly imply that at most  $m$  goods will be produced and possibly exported. In such models only net exports of tradable goods are determined, see Samuelson (1953). This theory is, of course, not meant to be applied straightforwardly in an empirical model like MSG with only 2 primary production factors and 32 domestic production sectors of which around 20 produce tradeables. The theory reveals some equilibrium or optimum features of trade liberalization and specialisation, but there are many good reasons why it is empirically rejected. A production sector of the model contains many different activities, some of which will survive facing international competition, although many will not. In that case the production technology of the sector will change due to changed activity composition, but the remaining (and possibly expanding activities) may still be within the old sector classification. Assuming only a small number of different production factors is also a simplification; more realistically there are specific types of capital and skilled labour in each production sector. Although an equilibrium solution may imply a specialisation in the long run, immobility and different expectations will prevent it. For some sectors there may be nonproportionate returns to scale, in which case changes in scale will keep the rate of return to factors at a required level - as also technological improvements will do.

Similarly, if Samuelson's stylized small country assumption is replaced by the assumption of country-specific goods, i.e. the Armington assumption of price dependent exports and imports, adaption along demand and supply curves may allow for more than  $m$  survivors. However, the assumption that foreign goods are imperfect substitutes to domestically

produced tradeables may be difficult to accept in a long-term context, and he estimated elasticities will most probably be rather unreliable. Considerations of risk and uncertainty will also lead to hedging or diversification, even if calculations based on expectations suggest specialisation.

As pointed out by Johansen (1974) these complicated problems of trade are not artificial difficulties created by the formal representation of the economy in a model. They represent real problems, that are difficult to model adequately. Again, the MSG solution is a compromise. In MSG-4S and MSG-4E export volumes, non-competitive import prices and market shares of imports (estimated by commodity and receiving sector) are exogenous, while prices of competitive imports and exports are endogenous and cost determined. In MSG-4E one can optionally apply a balance of trade restriction, in which case export volumes and import shares are scaled proportionately from initially assessed developments to provide a given balance of trade at every point of time. This option is convenient in the actual use of the model, but the procedure is theoretically dubious unless the exogenous assessments of export volumes and import shares are based on support models or other supplementary information. The idea is that if the relative composition of production of tradeables can be determined by specific market analyses, the exchange rate policy and income policy must secure a competitiveness which scales the production of tradeables to a required level. This also means that the price levels of Norwegian tradeables, generated by MSG, are assumed to correspond to the international equilibrium levels.

### 3 ECONOMIC AND FORMAL STRUCTURE OF MSG-4

The fourth generation of the MSG model was constructed in order to study the overall long term prospects of the Norwegian economy and also more specifically the long term interactions between economic growth and energy supply and demand. The model is mainly used by the Ministry of Finance as a quantitative tool in macroeconomic planning, but other government bodies and research institutes also make use of it. The dimensions

of the model, 32 production sectors and 42 commodities, reflect a compromise between the ambitions to produce and apply detailed sector information and the need for a manageable model for the Ministry. In most industries the input aggregates labour, capital, energy and materials are substitutable according to neo-classical production functions. In addition, interfuel substitution is assumed within the energy group of each sector. In the terminology of the model these aggregates of commodities or primary inputs define activities, i.e. aggregates with fixed relative proportions. Thus, the model is based on an input-output description of the economy, where the substitution possibilities are defined between activities, comprising aggregate inputs. Labour and capital are assumed to be freely moveable and malleable, i.e. unconstrained in the allocation between sectors.

As discussed in section 2 there are two versions of the present MSG model, MSG-4S where total capital is exogenous and MSG-4E where the real rate of return to capital is exogenous. The development of the total production capacity of the economy is determined by the exogenously given growth of total labour force, sectoral assessments of technical change and total supply of capital (MSG-4S) or the exogenously given rates of return to capital (MSG-4E). In addition, the composition of production influences total productive capacity since sectors are not equally efficient.

The model is closed by letting the level of household consumption be endogenously determined in such a way that full capacity utilization is ensured. By omitting the macro consumption function, household consumption is determined by allocating to consumer activities what is left of production capacity over gross investments, government consumption and net exports. The model calculates the corresponding equilibrium prices for commodities, real wages and in the case of the MSG-4S version also the equilibrium real rate of return to capital. This does not necessarily mean that the model will show an economy running at "full employment" and full capital utilization and with general equilibrium prices; labour supply might be set below the available labour force, sectoral capacity utilization indices below one and mark-up rates may differ from one. The model will, however, trace out paths of balanced growth in the sense that there is a continuous balance between supply and demand of goods and factors of production, within the limits of available capacity. Some price indices, such as nominal wages, the prices of non-competitive imports, oil, gas, electricity, government fees and commodity taxes are exogenous to the model, and determine the nominal price level.

By manipulating exogenous demand assessments, sector specific rates of technical change, parameters for capacity utilization and mark-up rates, the model can be calibrated to coincide neatly with actual figures for one year or with period averages. From a disequilibrium starting point, the model can either be steered towards the long-term equilibrium (i.e. simulating the transition path) by normalizing parameters and exogenous growth rates, or it can be used to simulate a prolonged, partly malfunctioning actual development.

Such long-term equilibrium paths depicted by the model also show many important deviations from uniform and constant growth rates. Partly this is due to formal properties of the model such as exogenous supply, demand and price assessments, different demand elasticities for different goods and different rates of Hicks neutral technical change in production sectors. It may also be argued that 10 to 30 years is a too short period both to reveal and impose steady state growth properties when the focus is on "neutral projections".

The substitution parameters of the model are most properly interpreted as long term elasticities. In an equilibrium model with no lags, as in MSG, agents react immediately to adjust their allocations to changes in prices or other incentives. In the real world, it necessarily takes time for economic agents to adapt to changed incentives. The MSG model therefore "oversubstitutes" when predicting year by year fluctuations caused for instance by significant changes in input prices. The model more adequately predicts the average development over a period where changed incentives persist long enough to allow agents to adjust.

A simplified structure of the MSG-4E version of the model is depicted in figure 1. The MSG-4E version is easier to explain than MSG-4S, since the outside assessments of both wages and returns to capital and the assumption of constant returns to scale (or exogenous output-prices or production) make the model neatly recursive in a price model and a quantity model.

For a guidance through figure 1 assume that all industries produce at constant returns to scale, minimize costs, and set prices equal to unit costs. Start in the upper part of the diagram with given wage rates, returns to capital, trends of technical change and capacity utilization indices. The intersectoral price-cost relations, mark-up indices and the price dependent input demand functions then simultaneously determine the cost minimizing techniques in terms of input coefficients (labour, capital, materials and energy per unit of output), and the commodity prices that cover calculated costs. The capacity utilization and mark-up indices are used to adjust for deviations from normal or long run equilibrium

behaviour.

Given these variables the quantity side of the model may be solved as a traditional input-output model with fixed coefficients. The scale of production by industry is determined by demand assessments - which are partly exogenous, such as exports and government expenditures, and partly endogenous, such as private gross investments and household consumption - and by imports to intermediate and final use which are calculated from import shares, differentiated by commodity and purchasing sector. Private gross investments are determined in a closed loop with the scale of production by industry. The scale of production by industry determines the demand for capital services and thereby capital stock by industry and by kind of capital good. This again determines private gross investments by commodity. For given prices the commodity composition of household consumption depends only upon total household consumption expenditure, which is determined in such a way that the specified labour force is fully employed.

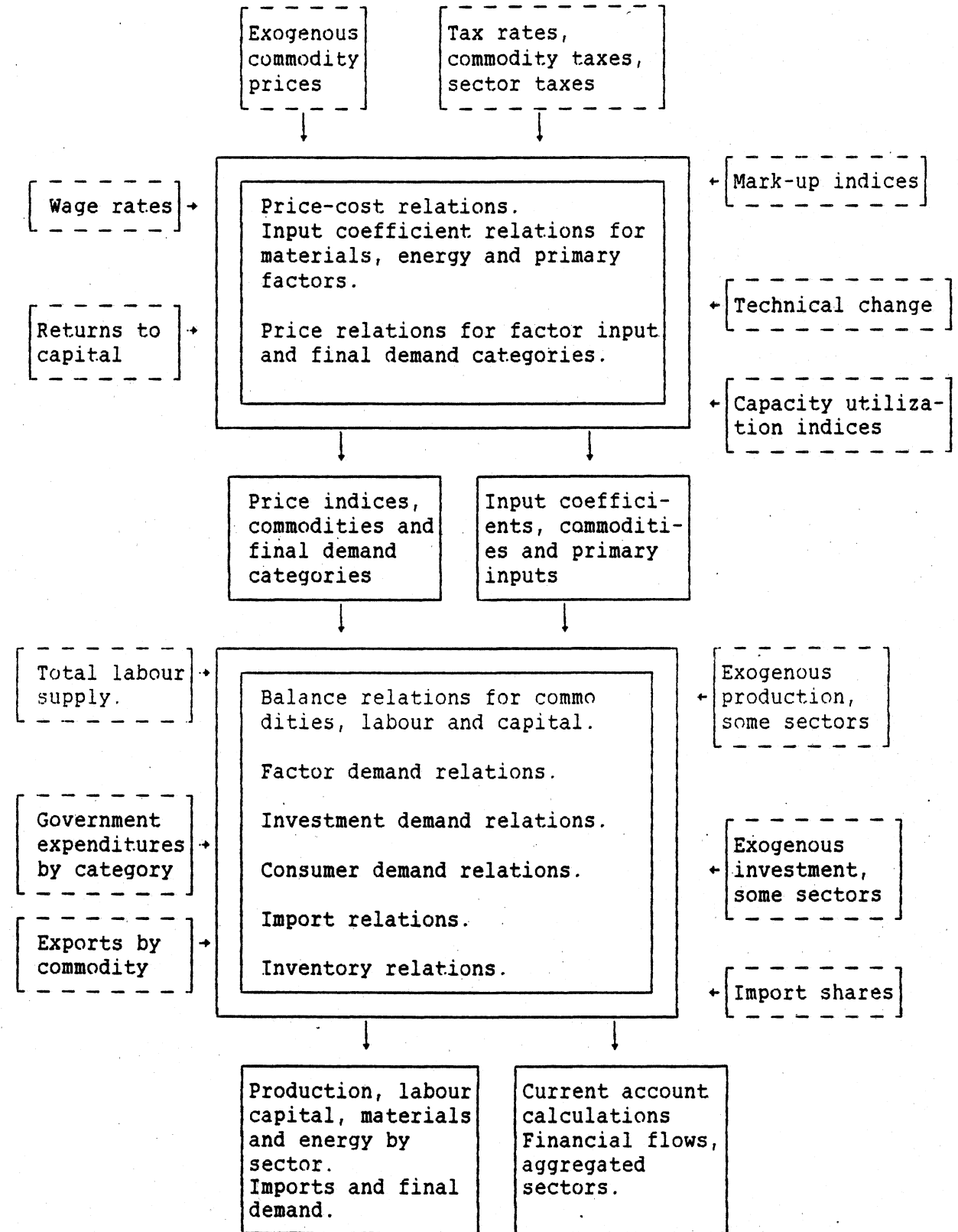
The total productive capacity for the economy as a whole is in MSG-4E determined by the exogenous total labour force, technical change, the capital stock consistent with the exogenously determined rate of return to productive capital, and the distribution of production between sectors.

In the MSG-4S version of the model, with inelastic supply of capital, there is a crucial link between the price and quantity side of the model represented by the overall level of return to capital. Given the resource restriction on capital, the level of return to capital has to be endogenously determined. The equation systems of the two versions are equal, but MSG-4S is simultaneous in prices and quantities.

The MSG model also includes calculations and special features not indicated in the figure such as submodels for capital depreciation, indirect taxes, government consumption, energy supply and demand etc. Special options to "control" the model's results for the balance of trade by adjusting the exogenously given import shares and export volumes are introduced.

A number of support routines and models are linked to MSG. These models are either pre-calculations to provide exogenous estimates (labour force, population growth, oil investment and production profiles etc.) or post-calculations (demand for different types of skilled labour, industry pollution, financial variables etc.)

Figure 1. Structure of MSG-4E.



→ = Effects from/to

[ - - - ] = Exogenous variables

[ ] = Blocks of equations

[ ] = Endogenous variables

### 3.1 BASIC CONCEPTS AND BALANCE EQUATIONS FOR COMMODITIES AND PRICES 1)

The Norwegian national accounting system, which is in very close adherence to the revised SNA (see the United Nations (1968)), forms the conceptual framework of the MSG model. The main model includes an accounting system, i.e. balance equations and definitional relations, which to a great extent are identical with the real flows of the national accounts. The financial flows are not included in the main model except for some aggregated current account figures. However, a "post model" for financial flows has been constructed (see section 3.6).

The commodity flows of the MSG model may be described as flows between (functional) sectors. The inter-industry transactions of the economy form a central component of the model and the sector concept is first of all used for the classification of establishments and similar economic units into production sectors. The model has 32 production sectors, i.e. 27 industries and 5 general government production sectors. In addition to a classification of establishments, the sector concept is also applied to broad categories of goods and services classified by origin or use, i.e. sectors for imports, exports, household consumption, general government consumption, private investments, and general government investments.

The commodity classification is arrived at by adopting the "main producer" principle, i.e. letting all goods and services with the same industry as the main producer form one commodity. The classifications of industries and commodities are thus closely related. If strictly followed, this procedure will give the same number of domestically produced commodities as the number of industries 2). Commodities representing imports for which there is no domestic production (non-competitive imports) and marketed government services are included as separate commodities. Altogether there are 42 commodities in the model.

In addition to commodities, each production sector absorbs primary factors, i.e. labour and capital services. At present there is just one category of labour input, while the model distinguishes between three categories of capital goods ("buildings and constructions", "machinery" and "transportation equipment") 3).

The rather disaggregate representation of the commodity-by-sector flows makes it possible to focus both on the industrial and final demand



structure and on the industrial interdependences in a growth process. However, with respect to the specification of behavioural relations in the model it is hardly possible, nor essential for the quality of the model results, to introduce substitution possibilities between all inputs and outputs of each sector. To simplify, the detailed set of commodity and primary input flows of each sector is therefore partitioned into mutually exclusive and exhaustive subsets. Each subset defines an aggregate of input or output commodities or of primary inputs. Substitution possibilities in the production or utility functions are introduced only between these aggregates 4). Within each aggregate fixed proportions are assumed, i.e. the aggregator functions are simple Leontief functions. In the model these fixed coefficient commodity and primary input aggregates within each sector are called activities.

In each production sector, commodities and primary inputs are aggregated into five input activities, namely one for capital services (three types of capital goods), one for labour (one type only), one for materials (all non-energy commodities), one for electricity (electricity and distribution services and one for other energy inputs (petrol and fuel oil), for short called fuels. In the household consumption sector the individual input commodities are aggregated into 18 activities.

The different value concepts adopted in the model are essential in the modelling of the inter-industry transactions and in the modelling of substitution induced by changes in relative prices. The fixed coefficients within each activity are estimated from the national accounts for the base year of the model. This means that quantities of commodity flows are measured in unit prices of the base year, i.e. constant unit values. The principal concept for evaluating commodity flows in the model is (approximate) basic values 5). The basic value concept is preferred to producers' value or purchasers' value because the trade margins (including transport charges) and commodity tax rates may vary between receiving sectors of the same commodity and thus may cause a discrepancy between calculated total supply and total demand in constant unit values in producer or purchaser prices 6).

The activities are, however, evaluated in market values, computed as producers' value of commodity outputs and as purchasers' value of commodity inputs or primary inputs. The rationale behind this choice is that the substitution possibilities within each sector are specified between activities, not between commodities. Market prices of activities are then the relevant price concept in modelling the producers' and consumers' behaviour.

In matrix notation, the commodity balance equation in the MSG-4 model, including the assumption of fixed activity coefficients, is given by

$$(3.1) \quad \Lambda_I I + \Lambda_X X = \Lambda_M M + \Lambda_E E + \Lambda_F F + \Lambda_C C + \Lambda_J J + \Lambda_A A$$

In relation (3.1) the  $\Lambda$ 's are matrices of commodity-by-activity coefficients, where the elements are commodity flows relative to corresponding activity levels. The commodity flows are measured in basic value and the activity levels in producers' or purchasers' value. On the left hand side of (3.1) the  $\Lambda$ 's are combined with (column) vectors of activity levels for imports (I) and domestic production (X) to give total supply of goods. On the right hand side the commodity demand is separated into intermediate inputs of materials (M), electricity (E) and fuels (F) (input activities for commodities in production sectors, see section 3.2) and the final demand categories household consumption (C), gross investments (J) and exports (A).

On the price side of the model the separation of commodity flows into activities implies that the following set of activity price indices may be defined.

$$(3.2) \quad P_i = \Lambda_i' B \quad i=I, X, M, E, F, C, J, A$$

where the P's are (column) vectors of price indices for the commodity activities specified in (3.1), and B is a vector of commodity basic price indices, i.e. prices of commodity flows. (Superscript ' denotes transposed matrix).

(3.2) is the dual relations to (3.1), with the number of equations corresponding to the number of activities in the commodity balance equation. To simplify the specification of (3.2) we have in these equations omitted the commodity taxes, which are rather detailed specified in the equations of the model.

### 3.2 THE SUBMODEL FOR PRODUCTION 7)

While substitution possibilities in earlier versions of the MSG model were restricted to the primary inputs labour and capital, a more general specification of production structure is chosen in MSG-4. The model of producer behaviour in the present version includes substitution possibilities between the input activities labour (L), capital (K), electricity (E), fuels (F) and materials (M), while fixed coefficients are assumed within the activities.

The substitution responses are formally represented by Generalized Leontief (GL) cost functions, interpreted as second order approximations to the "real" production structure (Diewert (1971)). In most industries the production functions are linearly homogeneous in the aggregate inputs, and technical change is assumed to be Hicks neutral 8).

In addition to the separation of the industry inputs into activities a further separability condition is introduced restricting the substitution properties of the two energy inputs. Electricity and fuels are assumed to be weakly separable from the other aggregate inputs, implying that the energy goods are only substituted against other inputs via an aggregate for total energy input, in the following denoted by U.

Restricting this aggregate function to be linearly homogeneous, the overall cost function will be separable in the corresponding price indices (Berndt and Christensen (1973)), and the dual to the energy activity aggregate may be interpreted as a price index for energy (denoted by  $P_U$ ). A GL (unit) cost function is chosen as an approximation also for this relation.

For industry j the unit cost structure is represented by the following relations:

$$(3.3) \quad \frac{Q_j}{X_j} = h_j(t) \left[ \sum_k \sum_l \alpha_{kl} (P_{kj} P_{lj}) \right]^{\frac{1}{2}} \quad k, l = K, L, U, M$$

$$(3.4) \quad P_{Uj} = \left[ \sum_k \sum_l \beta_{kl} (P_{kj} P_{lj}) \right]^{\frac{1}{2}} \quad k, l = E, F$$

where the P's are prices of the input activities,  $Q_j$  denotes total costs and  $h_j(t)$  describes Hicks neutral technical change.

The estimation of the cost functions is based on time series of national accounting figures for the five aggregate inputs labour, capital,

materials, electricity and fuels, and price indices of the same inputs.

Applying "Shephards lemma" (Shephard (1953)) the factor demand system in terms of factor input coefficients may be derived as

$$(3.5) \quad z_{kj} \equiv \frac{d \frac{Q_j}{X_j}}{dP_{kj}} = h_j(t) \sum_l \alpha_{kl} \left( \frac{P_{lj}}{P_{kj}} \right)^{\frac{1}{2}} \quad k, l = K, L, U, M$$

$$(3.6) \quad z_{Ukj} \equiv \frac{dP_{Uj}}{dP_{kj}} = \sum_l \beta_{kl} \left( \frac{P_{lj}}{P_{kj}} \right)^{\frac{1}{2}} \quad k, l = E, F$$

$$(3.7) \quad z_{kj} = z_{Ukj} z_{Uj} \quad k = E, F$$

where the  $z_{kj}$ 's are input coefficients measuring aggregate input pr. unit of output, and the  $z_{Ukj}$ 's are energy coefficients measuring the input of electricity and fuels respectively pr. unit of total energy use.

The factor demand relations of industry j may then be written as:

$$(3.8) \quad \begin{matrix} K_j \\ L_j \\ M_j \\ E_j \\ F_j \end{matrix} \left. \vphantom{\begin{matrix} K_j \\ L_j \\ M_j \\ E_j \\ F_j \end{matrix}} \right\} \rightarrow = z_{kj} X_j \quad k = K, L, M, E, F$$

The producers are assumed to be profit maximizers, which implies that marginal costs equal the output price, i.e. for industry j:

$$(3.9) \quad P_{Xj} = \frac{dQ_j}{dX_j}$$

However, when the production function is linearly homogeneous, it may be said that profit maximization fails to determine a unique supply curve. In these industries it is assumed that output is priced in such a way that the price just covers average costs (equal to marginal costs). This means that (3.9) can be interpreted as a competitive market equilibrium condition rather than as a supply function. Cost minimization is then, together with this equilibrium condition, sufficient as a description of producer behaviour 9).

With the notation introduced above (3.9) - the price-cost relation for industry j - may be written as

$$(3.10) \quad P_{Xj} = Z_{Lj} P_{Lj} + Z_{Kj} P_{Kj} + Z_{Mj} P_{Mj} + Z_{Ej} P_{Ej} + Z_{Fj} P_{Fj}$$

where  $P_{Lj}$  is an index for wage costs pr. unit of labour input and  $P_{Kj}$  is the user cost of capital (10).

(3.10) gives, for each industry, the relation between the activity price indices defined in (3.2) and the production structure as measured by the input coefficients  $Z$ , and is the dual relation to (3.8). For given prices of the primary inputs ( $P_{Lj}$  and  $P_{Kj}$ ), the relation expresses the fact that the output prices are determined from the cost side.

While the wage rates ( $P_{Lj}$ ) are actually exogenous variables, the model contains expressions for user costs of capital that are non-trivial. Capital stock is assumed to follow an exponential survival curve (geometric depreciation). With the assumption of a constant composition of the capital equipment within each industry the user cost of capital in industry  $j$  is expressed as

$$(3.11) \quad P_{Kj} = \sum_{i=1}^m \kappa_{ij} (R_j + \delta_{ij}) P_{Ji}$$

where  $R_j$  is the rate of return to capital, the  $\kappa$ 's are fixed industry capital structure coefficients the  $\delta$ 's are the fixed rates of depreciation differentiated by kind of capital and industry.  $m$  is the number of capital categories (3 in most industries).

### 3.3 LABOUR AND CAPITAL MARKETS

As discussed in section 2 the total supply of labour is exogenous i.e. inelastic. The supply of labour, defined as man hours, is derived from estimates of population development and changes in working force participation rates by sex and age, and assumed changes in normal working hours. The development of nominal wage rate by industry is also exogenous in the model. This allows for wage differentials between industries even in long run equilibrium. When using the model the historical wage differentials, which have been rather stable in Norway, are normally assumed to prevail also in the future. This also means that the (common) change in wage rates may be interpreted as the "numeraire" of the model.

The rate of return to capital in industry  $j$  is given by the equation

$$(3.12) \quad R_j = \rho_j \bar{R}$$

where  $\rho_j$  is the relative rate of return of industry  $j$  and  $\bar{R}$  is the rate of returns to capital in the economy as a whole. The  $\rho$ 's are exogenous variables, while  $\bar{R}$ , as mentioned in section 2, is exogenous in the model version with perfectly elastic supply of capital (the price-quantity recursive version), but is endogenously determined in the version with inelastic supply of capital (the price-quantity simultaneous version). The assumption of return differentials between industries is explained by traditional differences in profit requirements, investment risks, average size of the firms, degree of monopolization etc. within the various industries (Johansen (1960)). Relative rates of returns for the different industries are actually estimated from data of operating surplus (the residually determined capital income). Following Strøm (1967) it is suggested that there is a convergent development in the observed relative rates of returns, and base year values for the  $\rho$ 's are calculated as the steady state solutions of these magnitudes 11).

#### 3.4 THE SUBMODEL FOR HOUSEHOLD CONSUMPTION 12)

As in the original version of the MSG model developed by Leif Johansen there is no aggregate consumption function in MSG-4. Total consumption is determined residually as what is left of total capacity output over gross investments, government consumptions and net exports. A system of household demand functions is however a central part of the model, determining the commodity composition of household demand from relative prices and the level of total consumption. More precisely the demand system determines the allocation of demand by consumption activities, while commodity demand follows from the assumption of fixed coefficients within each of these aggregates.

The chosen system of demand functions has been directly specified rather than derived from an explicit specification of either the direct or the indirect utility function. It is important for the use within the context of the MSG model that the system has reasonable long run properties. For reasons of transparency it is advantageous that the para-

meters of the demand functions have fairly straightforward interpretations.

The demand for consumption activity (category)  $i$  is written as:

$$(3.13) \quad C_i = \eta_{Ci} (\theta V)^{\epsilon_i} \prod_j P_{Cj}^{\gamma_{ij}}$$

where  $V$  is total expenditure,  $\theta$  is an auxiliary variable,  $P_{Cj}$  is the price of consumption activity  $j$  and  $\eta_{Ci}$ ,  $\epsilon_i$  and  $\gamma_{ij}$  are parameters. The system can be interpreted as a first-order logarithmic approximation of any complete system of demand functions. The auxiliary variable  $\theta$  is introduced to ensure that the budget constraint

$$(3.14) \quad \sum_i P_{Ci} C_i = V$$

is fulfilled for every combination of prices and demand. The specification of  $\theta$  into the demand system (3.13) is commonly denoted "horizontal adjustments of Engel curves" (13). If the demand system is adjusted to fit the data in a base year (i.e.  $\theta$  is normalized to one) the  $\epsilon$ 's and  $\gamma$ 's have straightforward interpretations as Engel and Cournot elasticities respectively.

In the estimation of the demand system (3.13-3.14), which is based on national accounting data, rather strong restrictions are placed on the underlying utility function. The "complete scheme" approach of Frisch (1959) assumed want independence (additive utility function), i.e. strong separability between every single consumption good. In MSG-4 the energy orientation of the model structure has led us to introduce want dependence within two groups of consumption activities where energy use is strongly related to the consumption of other goods (Housing and Transportation services), while the assumption of strong separability between these two groups and the other consumption activities is retained.

### 3.5 OTHER MAIN PARTS OF THE MODEL

#### Private Investments

Optimal capital stock pr. unit of output in each industry is determined by the cost minimizing procedure underlying the (unit) input demand functions (3.5) and (3.6). The MSG-4 model thus includes relations

describing neo-classical investments behaviour. In deriving investments and commodity demand from changes in capital stocks, the model distinguishes between a number of capital-/investment categories. For each category an investment activity,  $J_i$ , is defined, "demanding" deliveries of commodities in constant proportions. The activity level of investment activity (category)  $i$  is determined by the relations

$$(3.15) \quad J_i = \sum_{j=1}^n \kappa_{ij} \left[ (K_j - K_j^{(-1)}) + \delta_{ij} K_j \right]$$

where  $n$  is the number of industries. As mentioned in section 3.2 the fixed  $\kappa$ 's indicate the assumed constant composition of the capital equipment within each industry.

### External trade

Export activities - one for each commodity of which there is domestic production - are exogenously determined. Import activity levels are derived from simple import demand relations, including import shares differentiated by the various intermediate and final demand activities (categories). The import relations are thus written as

$$(3.16) \quad \Lambda_I I = (S_M \circ \Lambda_M) M + (S_E \circ \Lambda_E) E + (S_F \circ \Lambda_F) F + (S_C \circ \Lambda_C) C + (S_J \circ \Lambda_J) J + (S_A \circ \Lambda_A) A$$

where the  $S$ 's are import shares matrices. (The symbol  $\circ$  denotes matrix multiplication element by element). In the present versions of the model changes in the import shares by commodity are exogenous variables.

In MSG-4E and MSG-4S, the balance of trade is endogenously determined, following from export volumes and import shares assessed by the model user and mainly endogenous prices. In a special version of MSG-4E, MSG-4ET, the user's preliminary assessments of export volumes and the import volumes implied by the import shares are endogenously scaled to achieve a given target path for the balance of trade. Some commodities are excepted from the scaling procedure in MSG-4ET; for example the export assessments for oil, gas and shipping services are retained at the user determined values. Thus, in MSG-4ET, the traditional export/import industries have to restructure to attain the required external balance. The balance of trade restriction in MSG-4ET first of all provides a convenient procedure in the finetuning of a model run, it does not provide a procedure for determining the composition of tradeables.



### General government consumption and investments

The description of general government activities in the MSG-4 model is very simple. In the present version there are five government production sectors. In these sectors gross (investments and thereby capital stock), employment and material and energy inputs are determined exogenously. Government consumption is calculated as gross total wages, material expenditures and depreciation less marketed government services, i.e. in accordance with the national accounting practice.

### 3.6 SPECIAL FEATURES OF THE MSG-4 MODEL

The above description of the formal structure outlines the main features and economic content of the MSG-4 model. However, in this outline a number of details in the actual equation system, exceptions from the general treatment of sectors and commodities and other specific properties are excluded. The omission of commodity taxes in equation (3.2) and industry taxes in equation (3.10) is already mentioned. To complete the presentation of the model the most important of these special features are discussed below.

#### Net additions to stocks

Relations describing net additions to stocks by commodity are also included in the model structure. Changes in stocks are related to changes in supply by a vector of fixed coefficients. Net additions to stocks are then of course also included in the commodity balance equation (3.1).

#### The specification of electricity flows

As emphasized above the principal concept for evaluating commodity flows in the model is basic values. However, in the MSG-4 model special attention is given to the specification of value flows for electricity. The single basic value flows for electricity in the national accounts are divided into two model commodities, electricity and distribution services with two corresponding production sectors. The two commodities are constructed by deducting user differentiated distribution costs and calculated rates of price differentiation from the basic value flows in the

accounts. The resulting constant value flow defines the volume concept for electricity in the model, referred to as "constant standard value". The price differentiation terms are specified explicitly in the model as artificial "taxes" or "subsidies" with differentiated rates. On the demand side of the model the two commodities, electricity and distribution services, are assumed to be used in fixed but purchaser differentiated proportions. In the model language they thus constitute one commodity activity in each sector 14).

The specification of production structure in the two "electricity supply sectors" furthermore differs from the general formulation outlined above. The cost structure is specified in order to benefit from calculations of future long term marginal costs in electricity supply. This kind of data are provided by the Norwegian Water Resources and Electricity Board 15).

#### Oil activities and ocean transport

Crude oil and gas production and ocean transport are large and important sectors in the Norwegian economy, with the activity levels having particularly important impacts on the trade balance. These industries are completely "exogenous sectors" in the present MSG-model, as investments (and thereby capital stock), employment, production and material input requirements must be given by the model user. For the oil sector the exogenous treatment may be motivated by the dominating role of central government in these activities and the limited number and diversity of oil and gas fields in actual production or to be developed in the next 20 years. The activity level in ocean transport is clearly dependent on international trade, and exports of these services are, as mentioned above, given exogenously in MSG-4.

#### Industries with decreasing returns to scale

As noted in the description of the general production model, the production technologies in most sectors are assumed to be homogeneous of degree one in the specified inputs. Exceptions from this specification - in addition to the two electricity sectors - are agriculture, fishing and mining. Within the general formulation of GL cost functions, in these three sectors decreasing returns to scale are assumed, based on the argument that these are extractive activities. The production levels are exogenously given, motivated by the strong government influence on the development of these industries 16).

### Corrections for disequilibrium

MSG-4 is formulated as an equilibrium model. Perfect mobility and given utilization rates of capital and labour are assumed, and the prices may be interpreted as equilibrium prices. The estimated parameters of the model are supposed to be long run parameters. The economy itself is, however, normally not in equilibrium and there is therefore no reason to expect that a simulation on the model automatically would imply that endogenous variables coincide with actual figures, neither in the past nor in the future.

However, since the MSG model is a model for practical use some of the main "sources" of the discrepancies between long run equilibrium and actual performance are identified and parameterized in the model. The most important of these adjustment parameters are

- capacity utilization rates (short run demand fluctuations)
- mark-up rates on prices (price setting or monopoly behaviour in the short run)
- "temperature-corrections" for energy use (climatic conditions)
- differences between long and short run demand adjustments (partial adjustment or price-lags in the demand relations in the short run)

From the estimation of the submodels for producer behaviour and household consumption these adjustment parameters normally can be derived for past years including the base year of the model. Given the base year estimates of these adjustment parameters, the model may then - from a disequilibrium starting point - be steered towards an equilibrium path by normalizing these parameters 17).

### Calculations of financial flows

MSG-4 contains relations between real flows of the economy. These flows are traced between functional sectors. Perfect mobility and a given utilization of labour and capital are assumed, and the model then calculates the long run development of volume figures such as production by industry, household consumption and investment and the corresponding equilibrium prices. Equations describing financial flows between institutional sectors and relations between income and demand (e.g. a "Keynesian" consumption function) are not explicitly specified in the main model. A common interpretation of this is that an equilibrium path traced by the MSG model tacitly assumes that incomes and financial flows between different sectors are distributed in such a way that the calculated development may be realized.

However, from a user point of view it is clear that calculations of incomes and financial flows are very useful in order to evaluate the realism in an economic development (in real terms) simulated by MSG-4. In addition to calculation of national account figures in constant and current prices a "post model" for financial flows (called MINK) have therefore been constructed and linked to the MSG model (Bergan (1984)). The MINK model contains relations between 6 institutional sectors of the Norwegian economy. The equations in the model may be interpreted as simplified income and capital accounts for these sectors. Starting out from calculations of incomes and expenditures (including transfers) total savings for each sector may be estimated. Financial investments are defined as the difference between total savings and real investments. Accumulating the figures of financial investments, the development of the stock of financial assets in the various sectors may be calculated.

Income and expenditure figures used as input to the financial calculations are partly taken from a simulation on the MSG (main) model (e.g. wages, operating surplus and indirect taxes) and partly given exogenously by the user (e.g. transfers and direct taxes). In distributing various income flows from MSG-4 on the six (institutional) sectors in the MINK model, fixed coefficients are applied. Incomes/expenditures in terms of interest flows are however dependent on the stocks of financial assets.

As mentioned above the MINK model may be used to examine the consistency between the development of real values from the MSG model and the corresponding financial flows. In addition to a comparison between figures for household consumption (from MSG) and disposable income and savings by households (from MINK), the balance of current accounts, government incomes and expenditures and the relation between savings and investments in private enterprises can be evaluated.

#### 4 EMPIRICAL CHARACTERISTICS OF MSG-4 ILLUSTRATED BY LONG-TERM TOTAL ELASTICITIES

The MSG-4 model is meant to be a practical tool in the long-term planning process. Both the formal structure and the empirical content are

decisive for the actual usefulness of the model. The close conceptual and empirical links to the national accounts are, as already mentioned, a main feature of the model. The parameters of the production and consumption submodels are econometrically estimated from national accounts time series. The fixed coefficients of the activities, i.e. the input-output coefficients, are estimated from the national accounts for the base year of the model. The model is regularly updated; normally the base year only lags one or two years behind the present year.

The estimation of the submodels for production and consumption are presented in Longva and Olsen (1983a, 1983c) and Bjerkholt and Rinde (1983), respectively. Instead of repeating the discussion of these empirical findings we shall present the empirical characteristics of the complete model through estimates of long-term total elasticities. However, all modelbuilders have to cut some corners in order to keep the model at a manageable level. As clearly indicated in the discussion in section 3, the MSG model is not a complete model of the working of the economy. Several important groups of variables which obviously are endogenously determined in the economy are treated as exogenous in the model. This means that the model will, at least for some types of sensitivity analysis, yield unrealistic, counterintuitive or even adverse results. A presentation of the empirical characteristics of MSG-4, as given below, will only illustrate the functioning of the model as such and not necessarily the working of the economy. The actual usefulness of MSG-4 can therefore only be reviewed when the model is regarded in its proper setting, i.e. as a tool in a planning or a policy analysing process. An example of how to use the model as a tool in describing and understanding how the economy actually works and to make projections is given by Bjerkholt and Tveitereid (1985).

#### 4.1 SOME ELASTICITY CONCEPTS 18)

Elasticities can only be given a precise interpretation with reference to a specified model. In general, elasticities refer to measures of the responsiveness of the endogenous variables to changes in the exogenous variables. The most common and well known examples are the elasticities defined from a single demand equation (partial elasticities). In this case the own-price elasticity is the percentage change in quantity demanded resulting from a one percent change in the price of the good in question, assuming that all other (specified) determinants of demand remain con-

stant. Elasticities with respect to other prices (cross-price-elasticities) and income or output (income or output elasticities) are similarly defined. In simultaneous models one can also define elasticities from a single equation as the percentage change in one endogenous variable resulting from a one percent change in one of the exogenous variables, assuming all other exogenous - and all other endogenous variables - remain constant. However, in a simultaneous model a change in one of the exogenous variables will in general effect all endogenous variables. An initial shock in equation  $j$  will have repercussions through the model system back to the endogenous variables of equation  $j$ . The total elasticity may then be defined as the percentage change in an endogenous variable resulting from a one percent change in one of the exogenous variables, assuming that all other exogenous variables remain constant - but allowing all endogenous variables to attain their new equilibrium values. The magnitudes of partial and total elasticities defined in the same model may be strikingly different. Consider for instance the impacts of an increase in the electricity price on electricity demand in a MSG production sector. The partial elasticity, derived from a factor demand function, may be low if the elasticities of substitution for electricity against other input factors are low. The total elasticity, derived from solving the whole model, may on the other hand be quite high - since low substitution elasticities on the input side mean that the increased energy cost is to a great extent passed over to the output price, reducing demand and hence scaling down both the output level and energy use of the sector.

In static, linear models the total elasticities follow straight-forwardly as the derivatives of the reduced form model. In dynamic, non-linear models the total elasticities are clearly both dependent on time and on the reference scenario. The classic example is the effects on consumption from increasing investments in a growth model of the MSG type. The short term effect is a reduction in consumption, but as investments accumulate the productive capacity of the economy increases and allows for both higher investments and consumption than in the reference scenario. The short term elasticity is thus negative, the medium term elasticity passes through zero and the long term elasticity is positive. Furthermore, if the return to capital is a decreasing function of the stock of capital, the magnitude of the elasticity is at each point of time dependent on the initial stock of capital (or in general dependent on all of the variables in the reference scenario).

Elasticities as defined above are single measures of how a specific

model is working. Keeping in mind that the elasticities may be defined in several ways, changing signs and magnitudes over time and being dependent on the reference scenario, a set of model elasticities may be of great value to the model user. In addition to the educational or pedagogical value of the elasticities, they give useful information when preparing new rounds of model calculations. The availability of model elasticities may even save rounds of full numerical recalculations, since elasticities may be used to approximate new solutions by adjusting a reference scenario. This procedure is particularly useful when the model is (close to) static and linear, in which case the reduced form coefficients give a full set of (approximately correct) elasticities or table of effects, (Cappelen, Holm and Sand (1980)).

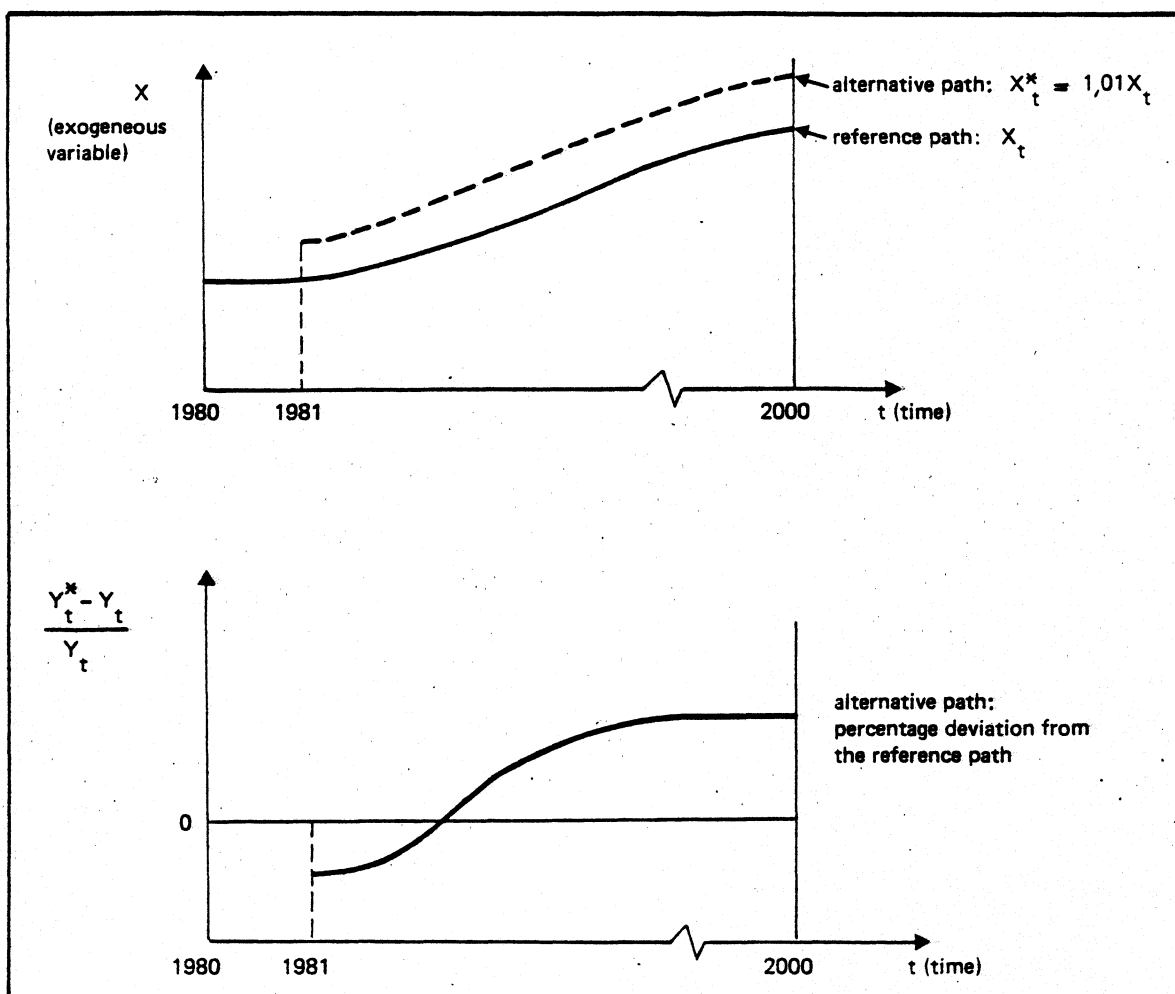
#### 4.2 ELASTICITIES WITH RESPECT TO CHANGES IN THE GROWTH POTENTIAL 19)

Most of the MSG-elasticities reported below are long-term total elasticities. They are calculated by increasing the value of one exogenous variable with one percent at each point of time compared to a reference scenario, and then calculating the deviation in the resulting endogenous variables at a terminal point, where the model is assumed to be in long term equilibrium. The reference scenario covers the period 1980 to 2000. The procedure is illustrated in figure 2.

Where relevant the elasticities are calculated for all three versions of the MSG-4 model, i.e. the simultaneous version, MSG-4S, the price-quantity recursive version, MSG-4E and the trade balance restricted versions of MSG-4E, MSG-4ET 20). This comparison of elasticities reveals important differences in the theoretical contents of the three model versions, although their equation systems are - with minor exceptions for MSG-4ET - exactly the same.

The elasticities for some key economic aggregates with respect to changes in main economic growth factors (employment, technical change and capital stock) are displayed in table 4.1 for the three models. Table 4.2 gives some specific elasticities for labour and capital input in private industries, and illustrates in more detail the substitution effects of the model. The comments below are grouped by kind of exogenous change.

Figure 2. Procedure for estimating long-term total elasticities.



### Increased total employment

An increase in total employment increases the productive capacity of the economy. Thus, gross domestic product (GDP) increases in all three model versions. There is, however, a marked difference between the impact on the activity level in MSG-4S - i.e. when assuming a complete inelastic supply of real capital - and the corresponding effects calculated when using the MSG-4E or the MSG-4ET version - where the supply of capital is assumed to be infinitely elastic. In the latter cases a one per cent increase in total employment leads to increases in GDP and total capital stock of approximately the same magnitude. The reason is rather obvious: In most industries of MSG the input coefficients - and thus the relations between labour and capital - are functions only of prices (as the production functions are linearly homogeneous). In MSG-4E as well as in MSG-4ET, where prices are independent of quantity variables, this means that the input coefficients are not influenced by changes in total employment; the capital stock will change proportionally to the change in employment. The fact that both the elasticity of the total capital stock and the GDP-elasticity with respect to employment slightly exceeds



one in the simulation on the MSG-4E model is a result of a change in the composition of industries in favour of relatively capital intensive consumer good industries.

In the simultaneous version of the model (MSG-4S) total capital stock is exogenous and thus unaffected by the increase in total employment. This is sufficient to explain why the impact on total output of the economy is markedly less in this case than when the change in employment is accompanied by a corresponding change in the capital stock. The input structure in industries changes, and the production techniques become more labour intensive and less capital intensive. Changes in the input structure require, as mentioned above, changes in (relative) input prices. As wage rates are exogenously given other prices must increase in order to make labour relatively cheaper. It is seen from table 4.1 that the most important effect on prices is a significant increase in the rate of return to capital. This makes labour input relatively cheaper, thus motivating the producers to apply more labour intensive techniques, and at the same time counteracting the increased demand for capital caused by the higher activity level in the economy. The implicit elasticity for total labour input with respect to the real product wage (nominal wage deflated by the GDP deflator) is slightly above -2. With respect to real wage (nominal wage deflated by consumption prices) this elasticity is considerably lower in absolute terms (close to -1,2). The interpretation is that along this growth path the real wage will have to decrease with 2 per cent (alternatively 1,2 per cent) to absorb a 1 per cent increase in labour supply.

Turning to the effects on final demand categories it is seen from table 4.1 that the strongest impact of increasing total employment is an increase in private consumption. This applies both to the simultaneous version and the price quantity recursive version, particularly when no restriction on the balance of trade is imposed. The highest elasticity for private consumption is reasonably enough derived from the MSG-4E version since this model involves the strongest long run impact on the activity

Table 4.1. Effects of changing the growth potential of the economy: impacts on main economic aggregates and corresponding prices derived from simulations on different model versions. Elasticities.

MSG-4S: The price-quantity simultaneous version.

MSG-4E: The price-quantity recursive version.

MSG-4ET: The price-quantity recursive version with restricted trade balance.

Growth factors:	Total employment			Technical change			Total capital stock	Rate of return	
	MSG-4S	MSG-4E	MSG-4ET	MSG-4S	MSG-4E	MSG-4ET		MSG-4S	MSG-4E
<u>Volumes</u>									
GDP .....	0,6	1,1	1,0	1,5	2,2	2,1	0,4	-0,07	-0,06
Export surplus *) ..	-1,0	-1,8	0,0	-1,6	-2,9	-0,6	-0,6	0,12	0,02
Domestic use .....	1,0	1,9	0,9	2,1	3,4	2,3	0,6	-0,12	-0,07
Private consumption	2,1	3,1	1,4	4,2	6,0	3,7	0,8	-0,17	-0,07
Investments .....	-0,1	1,0	0,8	-0,1	1,5	1,3	0,9	-0,11	-0,11
Government cons. ...	-0,1	-0,1	-0,1	-0,1	-0,2	-0,1	-0,1	0,01	0,01
Total employment	1,0	1,0	1,0	-	-	-	-	-	-
Total capital stock	-	1,3	1,0	-	2,1	1,7	1,0	-0,19	-0,17
Total energy demand	1,4	2,4	1,5	2,3	3,9	2,8	0,7	-0,16	-0,09
<u>Prices</u>									
GDP .....	0,5	-0,1	-0,1	-0,9	-1,7	-1,7	-0,4	0,07	0,07
Domestic use .....	0,6	0,0	0,0	-0,9	-1,9	-1,9	-0,5	0,08	0,08
Private consumption	0,8	0,0	0,0	-0,9	-2,1	-2,1	-0,6	0,11	0,11
Investments .....	0,5	0,0	-0,0	-1,5	-2,2	-2,3	-0,4	0,08	0,06
Government cons. ...	0,2	0,0	0,0	-0,6	-0,9	-0,9	-0,1	0,03	0,03
real product wage ..	0,5	-	-	0,9	1,7	1,7	0,4	-0,07	-0,07
Rate of return to capital .....	7,2	-	-	11,6	-	-	-5,6	1,00	1,00
Real product energy price .....	-0,5	-	-	0,9	1,7	1,7	0,4	-0,07	-0,07

\*) Change in export surplus in constant prices relative to total export in the reference scenario.

Table 4.2. Effects of changing the growth potential of the economy: impacts on employment and capital stock in various industries. Elasticities

Growth factors:	Relative shares *)	Total employment		Technical change		Total capital stock	Rate of retur
		MSG-4S	MSG-4E	MSG-4S	MSG-4E	MSG-4S	MSG-4E
<u>Employment</u>							
Primary industries .....	5,9	2,9	0,1	1,3	-3,0	-2,1	0,4
Energy intensive industries ..	2,0	0,7	0,1	-1,6	-2,6	-0,5	0,1
Other industries and mining .	15,3	1,3	0,9	-0,8	-1,4	-0,3	0,0
Construction .....	6,6	0,7	1,5	-1,7	-0,7	0,6	-0,1
Electricity supply .....	2,0	1,0	1,9	1,1	2,7	0,7	-0,1
Domestic transportation .....	11,3	0,8	1,3	0,2	1,0	0,3	-0,1
Private service industries ..	32,6	1,4	1,7	0,4	1,0	0,3	-0,1
Public service industries ...	22,2	0,0	0,0	0,0	0,0	0,0	0,0
Ocean transport .....	1,5	0,0	0,0	0,0	0,0	0,0	0,0
Oil- and gas production .....	0,6	0,0	0,0	0,0	0,0	0,0	0,0
All industries .....	100,0	1,0	1,0	0,0	0,0	0,0	0,0
<u>Capital stock</u>							
Primary industries .....	5,8	-1,4	0,1	-2,1	0,3	1,2	-0,2
Energy intensive industries ..	1,9	-0,7	0,1	-1,7	-0,4	0,6	-0,1
Other industries and mining .	7,7	-1,2	0,9	-1,7	1,7	1,7	-0,3
Construction .....	1,1	-1,3	1,5	-1,0	3,4	2,2	-0,3
Electricity supply .....	9,2	1,1	2,2	0,4	2,2	0,8	-0,2
Domestic transportation .....	5,7	-0,1	1,3	-0,5	1,7	1,1	-0,2
Private service industries ..	31,2	0,3	2,9	0,9	5,0	2,0	-0,4
Public service industries ...	20,4	0,0	0,0	0,0	0,0	0,0	0,0
Ocean transport .....	5,6	0,0	0,0	0,0	0,0	0,0	0,0
Oil- and gas production .....	11,4	0,0	0,0	0,0	0,0	0,0	0,0
All industries .....	100,0	0,0	1,3	0,0	2,1	1,0	-0,2

\*) Relative shares in the year 2000 in the reference scenario

level. In the two model versions with no restriction on the trade balance (MSG-4S and MSG-4E) the increased domestic use of goods and services - and particularly in private consumption - leads to increased imports (determined by exogenously given import shares). With export volumes being kept unchanged this implies a deterioration of the balance of trade. When the trade balance (in current prices) is assumed to be kept unchanged from the reference scenario (i.e. the simulation with the MSG-4ET version), imports are reduced and exports are increased. The main impact is a reduction of total domestic use revealed through the elasticity for private consumption, which decreases markedly (from 3.14 in MSG-4E to 1.37 in MSG-4ET). Real resources will have to be allocated from production of consumption goods to production of tradeables to meet the balance of trade constraint imposed on MSG-4ET.

Commodity prices are influenced by changes in employment only in the simulation with the MSG-4S version, where there are two-way links between the price- and quantity sides of the model. (The minor changes in the price deflators of economic aggregates observed in table 4.1 also in the simulations with the price-quantity recursive versions are caused by changes in the composition of these aggregates.) The most important price effect in the simultaneous version is the significant increase in the rate of return to capital. The impacts on the price deflators for GDP and total domestic use measured by elasticities are both close to 0,5. The impact on the price index for private consumption is stronger than the relative change in the price deflator for investments and government consumption, reflecting partly the relatively higher capital intensity in important consumer good industries than in industries producing investments goods.

#### Increased Hicks-neutral technical change

A one per cent increase in total productivity in all private industries (except Ocean transport and Oil- and gas production) reduces proportionally the input requirements per unit of output for every given combination of factor prices. Thus, more outputs can be produced with the same amounts of inputs, which means that the production potential is increased. A general increase in productivity has a direct effect on price levels as unit costs of production are reduced. An important implication for relative prices is that real wages increase, since the prices of produced goods decrease. This effect, in combination with the assumption of a given supply of labour that must be absorbed in production activities,

imply that the total demand for real capital is increased.

It is seen from table 4.1 that the expansive effects on main economic aggregates as GDP, total domestic use and private consumption in the various simulations with increased productivity are close to twice the percentage impacts on the same variables in corresponding simulations with increased labour supply. For GDP the elasticity exceeds 2 in both simulations with the price-quantity block recursive model where the capital stock is determined from the demand side, while the elasticity is reduced to 1,5 when the growth path is restricted by exogenously given capital. The stronger impact by increased productivity than by increased labour supply is due to the fact that technological progress is assumed to be Hicks-neutral. This means that all inputs - including raw materials, labour and capital - immediately become more "productive". Following the growth process step by step there is a "first-order direct effect" on output in each sector of one per cent and on GDP by more than one per cent even with the input levels unchanged. However, since the productivity increase comprises also sectors producing intermediate inputs and capital goods, this situation cannot represent a new equilibrium solution; this second order effect of increased efficiency in the supply of productive resources gives over time additional expansion of the economy.

The most significant long run impact of higher productivity is the effect on the volume of private consumption, which even when domestic expenditure is restricted by maintaining the balance of payments, is increased by 3,75 percent. Also in these simulations the long run changes in investments are somewhat less than the corresponding changes in capital stocks, reflecting that the composition of the capital stock is changed in the direction of capital categories with lower depreciation rates (buildings and constructions).

The observed relative differences between the elasticities (for volume figures) calculated by the various model versions when productivity is increased are approximately the same as in the case of changed employment, and the differences may also be explained by the same mechanisms: In MSG-4S total capital supply is given. In MSG-4E the increased activity levels and the change in relative prices induced by employment or productivity growth imply that the demand for capital is also increased, and consequently the impacts on production and expenditure are stronger than in MSG-4S. Applying the MSG-4ET model with a restriction on the development of the trade balance increases the export surplus and decreases the impact on private consumption compared to MSG-4E. The

elasticities for GDP and investments/capital stock are also higher in MSG-4E than in MSG-4ET, reflecting the relatively high capital intensity in some industries producing private consumption goods.

The increased productivity in private industries leads to reductions in unit costs of production and consequently to lower prices of produced goods and services. The impacts on prices are obviously strongest when the returns to capital are frozen and the increased productive capacity is followed by an increased demand for capital. In the simultaneous version, MSG-4S, where the capital stock is assumed to remain unchanged, the capital demand is restricted by a strong increase in the rate of returns and thus in the user cost of capital. This counteracts the downward pressure on price levels caused by the increased productivity.

#### Increased total capital stock

The effects of increased total capital supply is only simulated on the MSG-4S version. An increase in the total capital stock implies a rise in the productive capacity of the economy. It is seen, however, that for the economy as a whole the marginal elasticity of capital is less than the marginal elasticity of labour (the total GDP elasticities are 0,4 and 0,6 respectively). The impacts on the domestic expenditures naturally also differ compared to the simulations with changes in labour supply and productivity. Since the capital stock is increased (by one percent) for every year in the simulation period, a corresponding effect is reflected in total private investments. As a result, relatively less resources are left for the production of consumption goods in this case than in the previous two simulations on the MSG-4S model.

The increased supply of capital implies that the "price" of capital services is reduced; from table 4.1 it is seen that the rate of return to capital decreases with 5,6 per cent. This implies a substitution towards more capital intensive techniques. In combination with the increased activity levels in production sectors these substitution effects ensure that the new capital stock is fully utilized in the economy.

The decrease in the rate of return to capital furthermore causes a downward pressure on commodity prices in the model. From table 4.1 it is revealed that on average the price indices for domestic use decrease with nearly 0,5 per cent; the price impacts being strongest for (capital intensive) consumption goods also in this simulation.

### Increased rate of return to capital

An increase in the overall rate of return to capital in the MSG-4E version has the effect that real capital becomes relatively more expensive, the total capital stock demanded by industries is decreased and the productive capacity of the economy reduced. The elasticity for the total capital stock with respect to the rate of return to capital is estimated to  $-0,2$ , and consequently the impacts (in absolute values) on other main figures as GDP, private consumption and investments are markedly weaker in this simulation than when the capital stock is increased exogenously by one per cent in the MSG-4S version. It may be noted that the proportions between the elasticities are very much the same.

The reduced activity level involves an increase in the export surplus (imports are reduced) in the simulation with the MSG-4E model. When restricting the effect on the trade balance (in current prices) to be zero, as is the case in MSG-4ET, export volumes are reduced and more real resources are left for the production of consumption goods.

The magnitudes of the elasticities derived by changing the rate of return by one per cent should not be compared with the elasticities derived from a corresponding change in e.g. total employment. From the elasticities presented in table 4.1 one may deduce - assuming that the elasticities are approximately constant over some range of variation in the rate of return - that in order to obtain the same impacts on the economy by changing the rate of return (in the MSG-4E version) as in the case of increasing total capital stock exogenously by one per cent (in the MSG-4S version), the rate of return must be lowered by 5 per cent. As the rate of return may be interpreted as an interest rate, which itself is commonly expressed in per cent, this may correspond to a reduction from e.g. 7,0 to 6,6 per cent.

### 4.3 ENERGY DEMAND ELASTICITIES

Several empirical studies during the last years has addressed the question of the relation between economic growth, energy prices and the demand for energy. MSG-4 is meant to be a tool also for analysing this type of questions. In this section we shall illustrate how the interactions between the energy demand and the rest of the economy are depicted by the MSG model. Energy demand elasticities are only studied for the price-quantity recursive versions of the model, i.e. the versions with perfectly elastic capital supply (MSG-4E and MSG-4ET). As argued by Hogan (1979) a general equilibrium model with inelastic supply of labour and

elastic supply of capital seems to be most appropriate when studying long-term energy-economy interactions.

Changes in the activity level of the economy induced by one of the main "growth factors" of table 4.1 imply changes in the energy demand of the economy. The implicit energy/GDP-elasticities (impacts on energy demand normalized by changes in GDP) estimated from the various simulations of the price-quantity recursive versions of MSG-4 are presented in table 4.3. These results serve to illustrate that the relation between energy demand and economic growth is highly dependent on how this growth arises.

Table 4.3. Energy/GDP elasticities caused by (one per cent) changes in the various growth factors

Growth factors:	Total employment		Technical change		Rate of return to capital	
	MSG-4E	MSG-4ET	MSG-4E	MSG-4ET	MSG-4E	MSG-4ET
Total Energy....	2,1	1,5	1,7	1,4	2,4	1,5
Electricity...	2,0	1,6	1,3	1,1	2,4	1,7
Fuels.....	2,2	1,5	1,9	1,5	2,3	1,3

A general feature of the estimated energy/GDP-elasticities is that they exceed one. This means that when the total activity level of the economy increases, total energy demand increases more than proportionately. The explanation is obviously that in all simulations private consumption increases more than GDP (cf. the discussion above), and thus strongly affects the energy use in the household sector. The highest energy/GDP-elasticity (in absolute value) is obtained when the rate of return to capital is changed in the simulation with the MSG-4E version (2,4). Since energy and capital are complements in most industries, less energy intensive techniques are applied in the production sectors. The lowest energy/GDP-elasticities are obtained, as is rather obvious, in the cases when the increased activity level (measured by GDP) is caused by a general increase in productivity. As mentioned above technical change in the MSG model is assumed to be "neutral" and therefore all input



coefficients - also the inputs of energy pr. unit of outputs - are reduced when productivity increases.

Another interesting feature of the results in table 4.3 is the differences in the impacts on electricity and fuels demand in simulations with the MSG-4E version and simulations with the MSG-4ET version. When restricting the trade balance by running the latter model, the demand for both electricity and fuels is reduced compared to the MSG-4E version as a result of the dampened effect on private consumption in these simulations (for electricity this effect is counteracted by increased electricity demand in the production sectors due to the relative increase in exports and the production of energy intensive goods).

When analysing energy-economy interactions also the effects of price changes on energy demand are of general interest. Finally in this section we shall therefore study how the MSG-model visualizes the effects of changes in energy prices on the energy market themselves and on the rest of the economy. This may be viewed as an example of how the model can be used as an applied general equilibrium model in policy oriented analysis in addition to studying the more traditional growth oriented questions addressed above 21).

In the MSG-model the price of crude oil (and natural gas) as well as the electricity price are specified as exogenous variables. More specifically, the basic prices (see section 3.1) of these commodities are exogenous. In Energy Modeling Forum (1980) it is stressed that demand choices are made at the retail level and that it is therefore desirable to measure price elasticities as close to consumption as possible. The energy price elasticities presented below are therefore measured relatively to changes in purchasers' prices of energy.

In analysing price sensitivity the substitution responses are of course of central interest. However, in the case of Norway, induced income effects may be very important when studying changes in the prices of crude oil and natural gas since the production of these goods amount to about one fifth of the Norwegian GDP.

In table 4.4 we present estimates of energy price elasticities, both for energy demand and main economic aggregates. When no restriction on the balance of payment is imposed, i.e. when possible effects of changes in terms of trade are not accounted for (MSG-4E), we see from table 4.4 that the overall energy-capital complementarity causes a reduction in the total production level and total capital stock. GDP is thus slightly reduced compared to the reference scenario. Since electricity has little weight both in exports and imports the main effect of a partial increase in the

electricity price is a fall in investments, implying that more real resources are available for private consumption. When the crude oil price is raised and the increased revenues from exports are not "used" in the economy, a negative real income effect causes a reduction in domestic demand and production, while the balance of payments is considerably improved.

Table 4.4. Effects of changes in energy prices on volumes of main economic aggregates and energy demand. Elasticities

Increases in :	Electricity prices		Prices of crude oil and natural gas	
	MSG-4	MSG-4ET	MSG-4E	MSG-4ET
Model version:				
GDP	-0.02	-0.02	-0.01	0.01
Domestic use	-0.02	-0.02	-0.02	0.13
Private consumption	0.06	0.05	-0.04	0.22
Investments	-0.13	-0.13	-0.03	0.07
Total capital stock	-0.10	-0.10	0.00	0.03
Total energy demand	-0.16	-0.17	-0.21	-0.05
Electricity demand	-0.55	-0.56	0.13	0.24
Industries	-0.65	-0.64	0.02	-0.21
Households	-0.53	-0.54	0.27	0.80
Fuels demand	0.13	0.12	-0.46	-0.27
Industries	0.04	0.03	-0.42	-0.49
Households	0.24	0.23	-0.68	-0.11

The estimates presented in table 4.4 show that effects of a change in the price of electricity are approximately the same with and without restrictions on the balance of payments. This simply reflects the fact that changes in the electricity price does not significantly influence the terms of trade (22). However, when the price of crude oil is increased, the change in terms of trade allows for an increase in domestic demand, and particularly private consumption is raised. The reallocation of resources is also seen to have a positive effect on GDP.

For energy demand we see that the direct price elasticity for electricity is close to -0.5 in both model versions. The cross price elasticities are also practically identical.

The fuel price elasticities are, however, markedly different in the two model versions. In households the very strong terms of trade effects

reduce considerably the absolute values of the elasticities of fuels demand and increase markedly the elasticities of electricity demand. The terms of trade effects furthermore induce changes in the production structure as resources are reallocated from the energy intensive export oriented manufacturing industries to industries producing consumption goods and services. As a consequence energy demand - and particularly the demand for electricity in the enterprise sector - is reduced.

#### 4.4 TRANSITION PATHS AND LONG RUN PROPERTIES

The numerical estimates presented above are calculated after a simulation period of 20 years. It may therefore be of some interest to study how the elasticities change over time, i.e. the transition path, and whether the "final" elasticities actually represent long run properties of the model. In the following some examples of time profiles for elasticities are presented. The chosen examples are all calculated on the MSG-4ET model, i.e. the model with fixed real rate of returns to capital and a balance of trade restriction included. This model version may be regarded as a long-term neoclassical general equilibrium growth model.

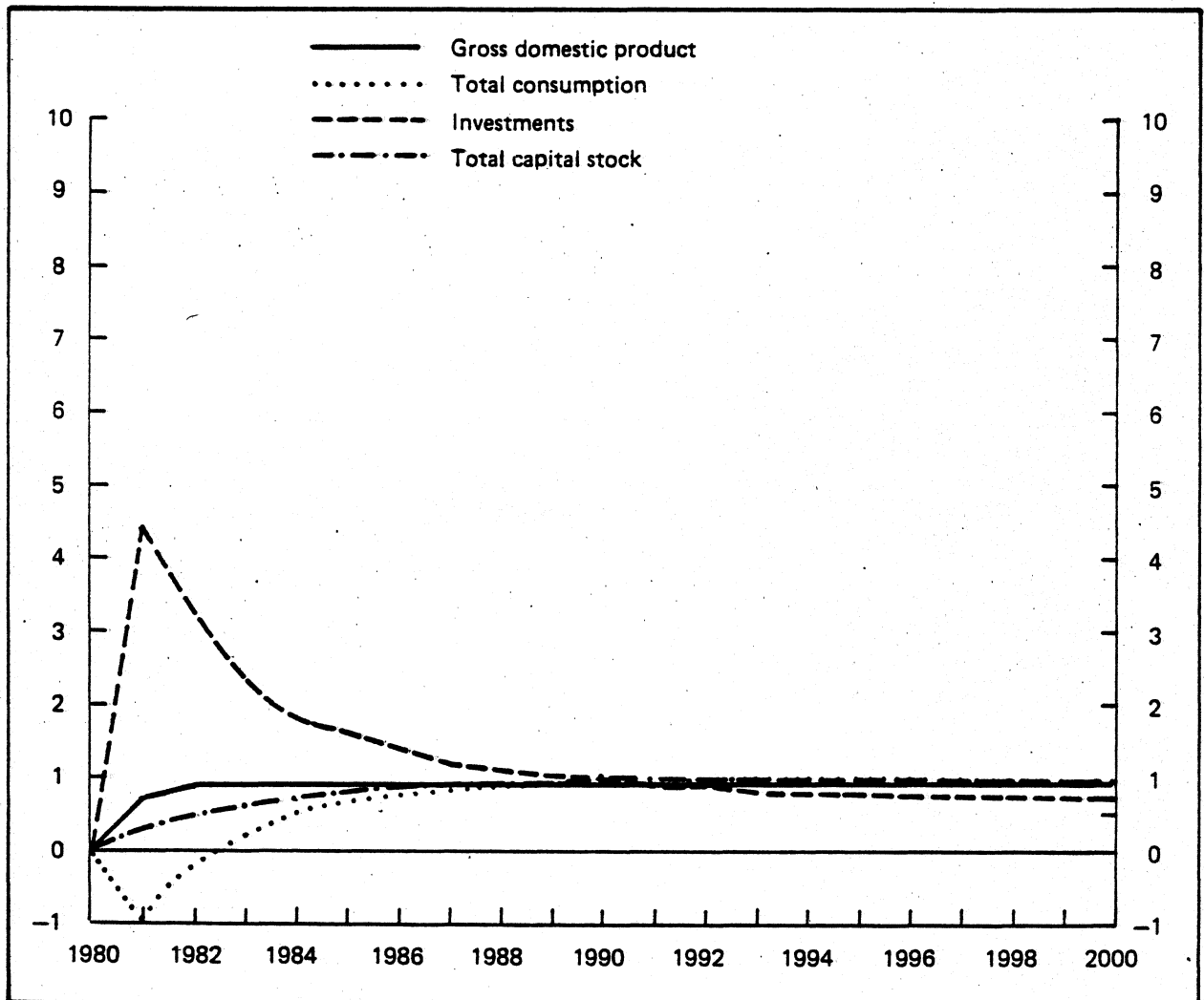
In figure 3 time profiles for the elasticities of GDP, consumption, investments and capital stock with respect to employment are drawn. The figure illustrates how the relative impacts on private consumption and investments change markedly from the year when employment is increased (1981) to the year when the elasticities presented in the previous tables are measured (2000). It may be noted that the elasticities do not reach their "long run equilibrium values" until after at least 10-15 years of simulation.

The strong, immediate effect on investments occurs as a result of the fact that capital stocks in each production sector must increase proportionally to the labour inputs. In order to reach the new equilibrium levels for the capital stock the increase in labour supply will have to be engaged for some years mainly in increased production of investment goods. In the first couple of years there are therefore not much real resources available for increased consumption, c.f. that the elasticity is close to zero in 1981. As capital stocks are gradually built up, real resources (labour input) are removed from the investment good sector to the production of consumption goods. As opposed to the impacts on the expenditure variables the elasticity for GDP reaches its long run level almost immediately. The variation over time in this elasticity is only a

result of differences in efficiency and primary input returns between the various industries.

After 20 years the relative increases in both GDP, capital stock, investments and the consumption potential are (somewhat below) 1 percent, i.e. close to the increase in labour supply. For the main aggregates the elasticities are approximately constant and uniform in the long run. When changing the growth potential by increasing labour supply the "long run" macroeconomic development depicted by the model thus resembles that of steady state growth.

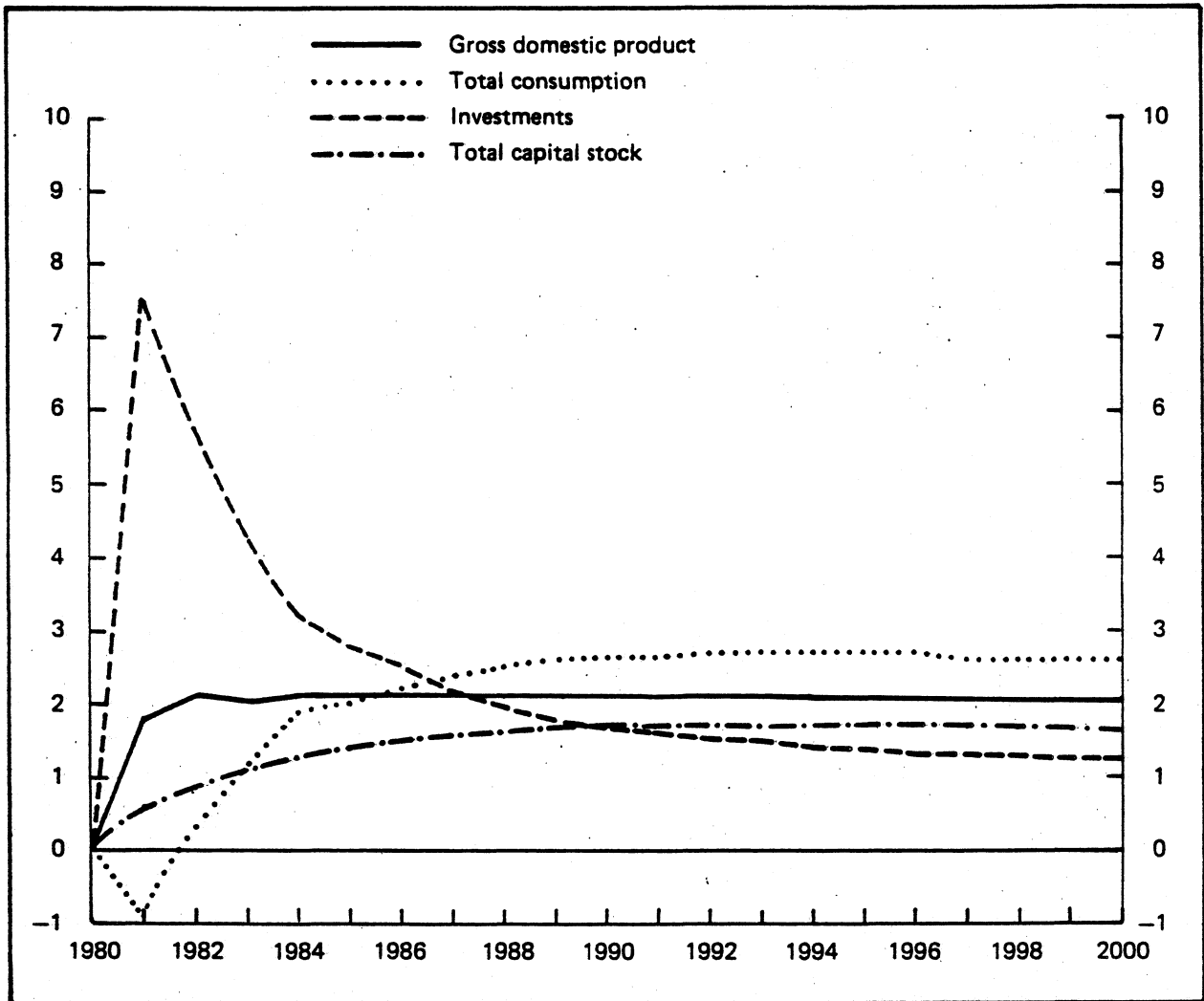
Figure 3. Total elasticities MSG-4ET. Transition paths and long run values. Increased total employment.



The development over time of the elasticities for main economic variables in simulating effects of technical change is rather similar to the development of impacts in the case of increased employment. In figure 4 we present time profiles for technical change elasticities. As for changes

in employment the figure illustrates that it is essential for the measurement of long run model properties to simulate the model some years ahead and that the elasticities seem to approach constant values. The deviations from uniform rates is mainly due to the assumption of Hicks-neutral technical change.

Figure 4. Total elasticities MSG-4ET. Transition paths and long run values. Increased technical change.



## N O T E S :

- 1) A more comprehensive discussion of these concepts is given in Bjerkholt and Longva (1980) and Longva, Lorentsen and Olsen (1983).
- 2) This does not mean that there is a one-to-one correspondence between commodities and industry outputs. At the chosen level of aggregation there will still be significant non-zero off-diagonal elements in the commodity-by-industry output matrix, i.e. multiple output in industries.
- 3) In addition, capital in shipping and three kinds of capital in crude oil production form separate categories.
- 4) Formally this means that these functions are assumed to be weakly separable in the defined subsets, see Berndt and Christensen (1973).
- 5) The Norwegian national accounting system includes a set of value notions, as recommended in A System of National Accounts, United Nations (1968).
- 6) Note that, apart from trade margins and commodity taxes, there may be genuine price differentiation in the base year. This bias in the base year weights may be a source of error in the model computations. Price differentiation is however explicitly corrected for in the case of electricity, see section 3.6.
- 7) For a more detailed and complete presentation, including estimation methods and numerical results, see Longva and Olsen (1983a, 1983c).
- 8) In the estimation of the cost functions the less restrictive assumption of a homothetic production structure was imposed.
- 9) Alternatively, producer equilibrium under constant returns to scale may be said to define a "horizontal" supply function.
- 10) To simplify, the terms for industry taxes, rates of capacity utilization and mark-up indices are omitted.
- 11) Since estimated rates of returns in general will deviate from the observed rates of return in the base year, total incomes will not be equal

to total costs, i.e. relation (3.10) will not automatically be fulfilled in the base year. In the model structure this is solved by introducing mark-up indices and capacity utilization rates in these equations (see also section 3.6). The difference between observed and estimated (expected) rates of return are therefore assumed to have been caused partly by less than full capacity utilization and partly by deviation between actual prices and (long run) marginal costs.

12) For a more detailed presentation of this submodel, including a description of estimation procedure and estimation results, see Bjerkholt and Rinde (1983).

13) A further understanding of this correction method may be gained from the general expression of Engel elasticities,  $E_i$  derived from (3.13-3.14), i.e.  $E_i = \epsilon_i / \sum_j a_j \epsilon_j$ , where the a's are budget shares.

14) For a further discussion of the specification of value flows for electricity, see Longva and Olsen (1983b).

15) The data series of future marginal costs have also been utilized to estimate the cost structure in electricity supply, see Rinde and Strøm (1983). Two different marginal cost functions have been estimated; one based on projects ranked according to the succession of the official plans, while the other is based on projects ranked according to increasing costs. To facilitate the use of the model to study various alternatives, marginal input coefficients for real capital are exogenous in the model.

16) The decreasing returns to scale would also otherwise have introduced a linkage between the "price" side and the "quantity side" of the model.

17) As mentioned in section 2 the equation system of the model also contains a set of parameters that may (and most commonly are) residually determined in such a way that the model "passes through the base year". These parameters correct for stochastic disturbances and constant price changes (rebasings of the variables) in the econometrically estimated relations.

18) A more comprehensive discussion of the elasticity concepts presented below is given in Longva, Olsen and Rinde (1983)

- 19) A more detailed presentation of elasticity calculations from MSG-4 is given in Offerdal (1985).
- 20) Obviously a "trade balance version" of the MSG-4S model might have been specified. However, at present no such operational model version exists.
- 21) More detailed and comprehensive discussions of these issues are given in Longva, Olsen and Rinde (1983) and Longva, Olsen and Strøm (1985).
- 22) Electricity intensive products are important export commodities. However, the induced increases in the prices of these products are very moderate.



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## A P P E N D I X

## A SIMPLIFIED EQUATION SYSTEM OF MSG-4

In this appendix a simplified version of the equation system of the MSG-4 model is presented. The equation system is structured in accordance with figure 1 of section 3.

The relations are formulated in matrix notation, referring to the equations of the various submodels presented in the main text. All vectors are column vectors. To simplify we

- (i) assume the same number of commodities and industries
- (ii) ignore government consumption and investment
- (iii) assume the same number of import - and export activities as commodities
- (iv) ignore all the special features of the model discussed in section 3.6.

THE PRICE PART

Price-cost relations by industries (equation (3.19)):

$$(1) \quad P_X = \hat{Z}_L P_L + \hat{Z}_K P_K + \hat{Z}_M P_M + \hat{Z}_E P_E + \hat{Z}_F P_F \quad n_X \text{ equations}$$

Factor input coefficient relations by industry written in general form (equations (3.5), (3.6) and (3.7)):

$$(2) \quad Z_i = Z_i(P_K, P_L, P_M, P_E, P_F; t) \quad i=K, L, M, E, F \quad 5 n_X \text{ equations}$$

Factor input prices for capital services by industry (user cost of capital, assuming only one capital (and investment) category, equation (3.11)):

$$(3) \quad P_K = (R+\delta)P_J \quad n_X \text{ equations}$$

Relations for relative rates of return to capital by industry (equation (3.12))

$$(4) \quad R = \rho \bar{R} \quad n_X \text{ equations}$$

Factor input prices for labour by industry (wage rates)

$$P_L = P_L^*, \text{ i.e. exogenously given wage rates}$$

Prices of imports, production, commodity input and final demand categories (commodity activity prices, equation (3.2)):

$$(5) \quad \begin{aligned} P_I &= \Lambda_I' B \\ P_X &= \Lambda_X' B \\ P_M &= \Lambda_M' B \\ P_E &= \Lambda_E' B \\ P_F &= \Lambda_F' B \\ P_C &= \Lambda_C' B \\ P_J &= \Lambda_J' B \\ P_A &= \Lambda_A' B \end{aligned} \quad 6n_X + n_C + n_J \text{ equations}$$

### THE QUANTITY PART

Factor demand relations by industry, (equation (3.8)):

$$(6) \quad \begin{aligned} K &= \hat{Z}_K X \\ L &= \hat{Z}_L X \\ M &= \hat{Z}_M X \\ E &= \hat{Z}_E X \\ F &= \hat{Z}_F X \end{aligned} \quad 5 n_X \text{ equations}$$

Commodity balance relations, i.e. the dual of the commodity activity price relation (equation (3.11)):

$$(7) \quad \lambda_I I + \lambda_X X = \lambda_M M + \lambda_E E + \lambda_F F + \lambda_C C + \lambda_J J + \lambda_A A \quad n_X \text{ equations}$$

Import relations by commodity (equation (3.16)):

$$(8) \quad \lambda_I I = (S_M \circ \lambda_M) M + (S_E \circ \lambda_E) E + (S_F \circ \lambda_F) F + (S_C \circ \lambda_C) C + (S_J \circ \lambda_J) J + (S_A \circ \lambda_A) A$$

$n_X$  equations

Household consumption demand by consumption activity, written in general form (equations (3.13) and (3.14)):

$$(9) \quad \begin{aligned} C &= C(P_C, V) \\ P_C' C &= V \end{aligned} \quad n_C \text{ (independent) equations}$$

Private investment demand, assuming only one capital (and investment) category (equation (3.15)):

$$(10) \quad J = K - K(-1) + \delta K \quad n_J \text{ equations}$$

Exports by commodity

$$A = A^*, \text{ i.e. exogenously given exports}$$

Primary factor balance relations

$$(11) \quad e'L = \bar{L}^*, \text{ i.e. exogenously given total supply of labour}$$

$$(12) \quad e'K = \bar{K}$$

(i) In MSG-4S  $\bar{K}$  is exogenously given while  $\bar{R}$  is endogenously determined (inelastic supply of capital)

(ii) In MSG-4E  $\bar{R}$  is exogenously given while  $\bar{K}$  is endogenously determined (perfectly elastic supply of capital)

The MSG model above has  $2n_x + 2n_c + 2n_j + 2$  independent equations between the same number of endogenous variables. In this simplified model only  $P_L^*$  (wage rates),  $L^*$  (labour),  $\bar{K}$  (capital stock) or  $\bar{R}$  (rate of return to capital) and  $A$  (exports) are exogenously given.



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