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## Depreciation Profiles and the User Cost of Capital

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# Discussion Paper

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## *DEPRECIATION PROFILES AND THE USER COST OF CAPITAL*

BY

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### ABSTRACT

The standard neo-classical formulae for the user cost of capital is based on the assumption that the retirement and decline in efficiency of the capital units with age follow an exponentially declining function (exponential decay). In the present paper, we generalize this specification to the case where the capital volume is defined in terms of a general survival function. The specification of the corporate tax system in this context is discussed. Three capital concepts are involved: the gross capital, the net capital, and the tax accounting capital. Conditions for neutrality of the tax system, which generalize previous results in the literature, are established. Numerical illustrations based on Norwegian data are reported.

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DEPRECIATION PROFILES AND THE USER COST OF CAPITAL<sup>\*)</sup>

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## 1. INTRODUCTION

The cost of using real capital as a factor of production is one of the principal determinants of the firm's investment decisions. It is also a useful tool in theoretical and empirical analysis of the system of corporate taxation. Seminal contributions to this literature were the articles by Jorgenson (1967) and Hall and Jorgenson (1967), in which the user cost of capital and its dependence on the corporate income tax system were integrated into a neo-classical model of producer's behaviour. A basic assumption in these articles, which has been more or less tacitly accepted by most researchers, is that the replacement investment (technical depreciation) is a constant, time invariant proportion of the capital stock. Constant rate of technical depreciation has also become a main ingredient in the growing literature on the optimality, or lack of optimality, of the corporate income tax.<sup>1)</sup>

Several authors have, however, contested this hypothesis, both theoretically and empirically.<sup>2)</sup> If we disregard the empirically uninteresting situation in which investment grows at a constant rate, it will be satisfied only in the special case where the survival rates are exponentially<sup>3)</sup> declining functions of the capital's age (exponential decay). Jorgenson, on the other hand, has attempted to justify this hypothesis as an approximate long-run description. (Jorgenson et al. (1963).) He invokes "a fundamental result in renewal theory that replacements for ..... an infinite [investment] stream approach a constant

1) See e.g. King (1975), Sandmo (1974), Boadway and Bruce (1979), and Atkinson and Stiglitz (1980, section 5.3).

2) Examples are Griliches (1963), Feldstein and Foot (1971), Eisner (1972), Feldstein and Rothschild (1974), and Hulten and Wykoff (1981).

3) When considering time as continuous. If time is discrete, the hypothesis implies geometrically declining survival rates.

proportion of capital stock for (almost) any distribution of replacements for a single investment and for any initial age distribution of capital stock". (Jorgenson (1963, p. 251).) Long-run constructs are, however, difficult to implement econometrically, and in any case, the relevance of Jorgenson's simple conclusion for *short-term* model building is questionable. The short-run variations in economic activity are usually accompanied by large fluctuations in gross investment, and it may be a drastic simplification to exclude *a priori* the possibility that these fluctuations affect the average annual depreciation rate.

The problem of defining and measuring the user cost of capital is closely related to the problem of defining and measuring the volume of the capital stock. They are, in a sense, *dual* problems. This implies that the underlying specification of the replacement process should be the same for the two variables if they are to be applied in the same analysis. It would, for instance, be inconsistent to combine user cost series constructed on the basis of a constant rate of technical depreciation with capital data computed by cumulating previous investment series and assuming linear depreciation or a 'one horse shay' (simultaneous exit) specification; the latter being a common procedure for constructing capital data in several countries.<sup>4)</sup>

Moreover, taking constant rate of technical depreciation as a maintained hypothesis will strongly restrict the class of tax systems which can be analyzed from the point of view of optimality (neutrality). Symptomatically, authors dealing with this issue have almost without exception considered only the *declining balance method* of calculating depreciation allowances for tax purposes.<sup>5)</sup> This is probably due to the formal

4) See, for instance, OECD (1982).

5) Examples are Sandmo (1974), Hartman (1978), Boadway (1980), and Bergström and Södersten (1982).

similarity between this depreciation scheme and the specification with a constant rate of technical depreciation, since it implies that a constant fraction of the firm's book value of the capital stock is written off in its accounts each year. But practically important depreciation schemes, like straight-line depreciation and the sum-of-the-years'-digits method, cannot be properly handled within this framework. So both theoretically and empirically the standard parametrization may be felt as something of a strait-jacket, and a generalization is well worth exploring.

In this paper, we attempt to generalize the specification with constant rate of technical depreciation to the situation where the capital volume is defined in terms of a general survival function on the basis of past investment. We start by defining the basic capital concepts (and some related terms) required for deriving the user cost of capital (section 2). It becomes essential to distinguish between the capacity 'dimension' and the wealth 'dimension' of the capital. The former represents the (potential) flow of capital services from a firm's equipment at a given point of time - it is the variable to be used as argument in a production function. The user cost of capital is the cost per unit of these capital services, or equivalently, the cost of using the capacity of the capital stock at a given point of time. The wealth dimension of the capital, on the other hand, is needed for defining corporate income, depreciation, depreciation allowances for tax purposes, and hence taxable income. In section 3, we derive, on the basis of these two capital concepts, a general expression for the user cost of capital in the presence of corporate taxation. In section 4, we consider more closely the effect of the tax system on the user cost via the rules for depreciation allowances, interest deductibility, and capital gains taxation. Our results appear as generalizations of previous conclusions in the literature confined to models with exponential depreciation. Finally, in section 5, we present some numerical results based on parametric survival profiles and Norwegian data.

## 2. PRELIMINARIES: GROSS CAPITAL, NET CAPITAL, REPLACEMENT (DETERIORATION), AND DEPRECIATION

Let  $J(t)$  denote the quantity invested by the firm at time  $t$ , where time is considered as continuous. To characterize the retirement of the capital units over time, we introduce the function  $B(s)$ , indicating the proportion of an investment made  $s$  years (periods) ago which still exists as productive capital. It represents both the loss in efficiency of existing capital units and physical disappearance of old capital goods. This function, which we call the '*technical survival function*', is non-increasing, with values between 0 and 1:

$$(1) \quad 0 \leq B(s) \leq 1, \quad B'(s) \leq 0 \quad (\text{if it exists}) \quad \text{for all } s \geq 0,$$

$$B(0) = 1, \quad B(\infty) = 0.$$

We assume that the units of measurements<sup>6)</sup> are chosen in such a way that *one capital unit 'produces' one unit of capital services per unit of time*. Then

$$(2) \quad K(t,s) = B(s)J(t-s)$$

represents both the volume of the capital which is  $s$  years of age at time  $t$  and the momentaneous flow of capital services produced at time  $t$  by capital of age  $s$ . The total capital volume at time  $t$ , in the following to be denoted as the '*gross capital stock*', is

$$(3) \quad K(t) = \int_0^{\infty} K(t,s) ds = \int_0^{\infty} B(s) J(t-s) ds.$$

6) And possibly also the definition of the functional form of the production function.

This is a technical concept, indicating the productive capacity of the capital stock at time  $t$ .

The volume of the capital worn out at time  $t$ , or the *replacement (deterioration)*, can now be written as

$$(4) \quad D(t) = J(t) - K(t) = \int_0^{\infty} b(s)J(t-s)ds,$$

where (if it exists)

$$(5) \quad b(s) = -B'(s) \quad (s \geq 0).$$

The function  $b(s)$  represents the share of an initial investment (expressed in efficiency units) which disappears  $s$  years after its instalment.<sup>7)</sup>

Let  $q(t)$  denote the investment price at time  $t$ . The current investment outlay is  $q(t)J(t)$ . The market value of an old capital object does not, in general, reflect its historic cost, but rather the service flow that it is likely to produce during its remaining life time. It is this property which is of interest to a potential purchaser (user) of capital goods.

The value of the capital vintage  $t-s$  at time  $t$  can be written as

$$(6) \quad V(t,s) = q(t,s)K(t,s),$$

where  $q(t,s)$  is the price of one capital (efficiency) unit of age  $s$  at time  $t$ , and  $K(t,s)$ , as defined in eq. (2), is the number of such units.

<sup>7)</sup> Here we interpret the functions  $B(s)$  and  $b(s)$  deterministically. They can also be interpreted within a stochastic framework:  $B(s)$  is then the probability that a new capital unit will survive in at least  $s$  years, and  $b(s)$  is the density function of its life time.



We make the specific assumption that the relative prices per unit of capital objects of different ages, at each point of time, *perfectly* reflect the differences in their prospective service flows. The total flow of capital services from one capital unit during its life time is

$$\Phi(0) = \int_0^{\infty} B(z) dz.$$

In general,

$$(7) \quad \Phi(s) = \frac{1}{B(s)} \int_s^{\infty} B(z) dz \quad (s \geq 0),$$

has the interpretation as the flow of *remaining* capital services to be produced per capital unit which has attained age  $s$ , since  $\int_s^{\infty} B(z) dz$  is the service flow produced after age  $s$  by one *initial* capital unit and  $B(s)$  is the share of this unit which attains age  $s$ .<sup>8)</sup> We can then express our assumption as follows:

$$(8) \quad \frac{q(t,s)}{\Phi(s)} = \frac{q(t)}{\Phi(0)} \quad \text{for all } s \geq 0.$$

Substituting (2), (7), and (8) in (6), we obtain

$$(9) \quad V(t,s) = q(t)G(s)J(t-s),$$

where

$$(10) \quad G(s) = \frac{B(s)\Phi(s)}{\Phi(0)} = \frac{\int_s^{\infty} B(z) dz}{\int_0^{\infty} B(z) dz} \quad (s \geq 0).$$

8) If the replacement process is interpreted stochastically (cf. footnote 7), it can be shown that  $\Phi(s)$  represents the expected remaining life time of a capital good which has attained age  $s$ .

The value of the capital vintage  $t-s$  at time  $t$  is thus the product of the replacement value of the original investment,  $q(t)J(t-s)$ , and the share of the total service flow which is produced by one capital unit after it is  $s$  years old,  $G(s)$ . Aggregation over capital vintages yields the following expression for the total capital value at time  $t$ :

$$(11) \quad V(t) = \int_0^{\infty} V(t,s)ds = \int_0^{\infty} q(t)G(s)J(t-s)ds.$$

This value can be separated into a price and a volume component in several ways. For our purpose, the following decomposition is convenient:

$$(12) \quad V(t) = q(t)K_N(t),$$

where

$$(13) \quad K_N(t) = \int_0^{\infty} G(s)J(t-s)ds.$$

We shall call  $K_N(t)$  the '*net capital stock*'. Like the gross capital  $K(t)$ , it is a volume concept constructed by aggregating the previous investment flow in volume terms, but the weighting system is basically different. The weight assigned to investment made  $s$  years ago in  $K_N(t)$ ,  $G(s)$ , is the share of the total service flow produced by one unit invested *after it is  $s$  years old*, whereas  $K(t)$  is based on the survival rates  $B(s)$ , or, what is equivalent, on the *instantaneous service flow at age  $s$* . From (10) and (1) it follows that  $G(s)$  has the same general properties as  $B(s)$ :

$$(14) \quad 0 \leq G(s) \leq 1, \quad G'(s) \leq 0 \quad (\text{if it exists}) \quad \text{for all } s \geq 0,$$

$$G(0) = 1, \quad G(\infty) = 0.$$

Differentiating (12) with respect to time, we get

$$(15) \quad \dot{V}(t) = q(t)\dot{K}_N(t) + \dot{q}(t)K_N(t).$$

We define *depreciation*, in volume terms,<sup>9)</sup> as the difference between the volume of the gross investment and the increase in the volume of the net capital stock:

$$(16) \quad D_N(t) = J(t) - \dot{K}_N(t) = \int_0^{\infty} g(s)J(t-s)ds,$$

where (if it exists)

$$(17) \quad g(s) = -G'(s) = \frac{B(s)}{\int_0^{\infty} B(z)dz} \quad (s \geq 0),$$

the last equality following from (10). The function  $g(s)$  has the same relation to depreciation as  $b(s)$  has to deterioration; the former is in a sense the economic counterpart to the latter, technical concept,

The (*net*) value of depreciation (the true economic depreciation) is the difference between the current investment outlay and the rate of increase of the capital value:

$$(18) \quad E(t) = q(t)J(t) - \dot{V}(t) = q(t)D_N(t) - \dot{q}(t)K_N(t) \\ = q(t) \int_0^{\infty} \left\{ g(s) - \frac{\dot{q}(t)}{q(t)} G(s) \right\} J(t-s) ds.$$

9) We here define depreciation as a volume concept. It can alternatively be defined "from the price side", i.e. in terms of the prices  $q(t,s)$ . (Cf. e.g. Hall (1968) and Jorgenson (1974).) The two interpretations can be shown to be equivalent.

Here we can interpret  $q(t)D_N(t)$  as the gross value of depreciation, and  $\dot{q}(t)K_N(t)$ , i.e. the part of the increase in the capital value which is due to changes in the current investment price, as the value of the appreciation of the capital. Their difference is the true economic depreciation. An equivalent way of stating this is that the weight  $g(s)$  assigned to capital vintage  $t-s$  when calculating the volume of depreciation, should be replaced by the 'inflation adjusted' weight  $g(s) - [\dot{q}(t)/q(t)]G(s)$  when calculating its value counterpart.

### 3. THE USER COST OF CAPITAL: A GENERAL FORMULA

Since one capital unit produces  $\Phi(0)$  units of capital services during its total life time, and since - in the absence of taxation - its (effective) purchase price is  $q(t)$ ,  $q(t)/\Phi(0)$  would be the price per unit of capital services at time  $t$  - or the user cost of capital - in the absence of *interest costs*. To account for such costs, we replace  $\Phi(0)$ , as defined in (7), by the corresponding service flow *discounted* at the real rate of interest  $\rho$ <sup>10)</sup>

$$(19) \quad \Phi_{\rho}(0) = \int_0^{\infty} e^{-\rho z} B(z) dz,$$

and define the user cost as

$$(20) \quad c(t) = \frac{q(t)}{\Phi_{\rho}(0)} = \frac{q(t)}{\int_0^{\infty} e^{-\rho z} B(z) dz}.$$

10) More precisely,  $\rho$  is the rate of interest forgone by a producer who owns the capital and uses its services instead of purchasing interest-bearing financial assets.

[Note that if we set  $\rho = r - \gamma$ , where  $r$  is the nominal interest rate and  $\gamma$  is the rate of increase of  $q$ , and if  $r$  and  $\gamma$  are constants, then (20) is equivalent to  $q(t) = \int_0^{\infty} e^{-rZ} c(t+z)B(z)dz$ .]

We now introduce corporate taxes into this framework. We proceed by first specifying an income tax function which comprises a wide class of tax systems as special cases, and expressing the user cost of capital in terms of the parameters of this general function. Then, in section 4, we consider some specific tax systems within this class.

Let  $X(t)$  denote the difference between the firm's output value and the total cost of all other inputs than capital at time  $t$ . The *tax function* is

$$(21) \quad T(t) = u[X(t) - q(t) \int_0^{\infty} \mu(s)J(t-s)ds],$$

where  $u$  is the income tax rate (assumed to be constant) and  $\mu(s)$  is a function representing the effect of the previous investment decisions on the current income tax base: an increase in the replacement value of an investment made  $s$  years ago by one unit reduces the current tax base by  $\mu(s)$  units. This is a general way of representing the depreciation allowances, the treatment of interest deductions and capital gains, and other factors determining the corporate taxable income. All tax systems we shall consider in section 4 can be written in this format.

The firm's *net cash-flow* at time  $t$  is

$$(22) \quad \begin{aligned} R(t) &= X(t) - q(t)J(t) - T(t) \\ &= (1-u)X(t) - q(t)[J(t) - u \int_0^{\infty} \mu(s)J(t-s)ds]. \end{aligned}$$

Let  $r$  be the (constant) rate of (nominal) interest at which the firm can discount receipts and outlays at different points of time, and assume that *the investment price grows at a constant rate  $\gamma$* . The present value of the net cash-flow can then, after some rearranging, be written as

$$(23) \quad W = \int_0^{\infty} e^{-rt} R(t) dt \\ = \int_0^{\infty} e^{-rt} (1-u) [X(t) - q(t) \frac{1-\lambda u}{1-u} J(t)] dt + W_0,$$

where

$$(24) \quad W_0 = u \int_0^{\infty} e^{-rt} q(t) \int_t^{\infty} \mu(s) J(t-s) ds dt$$

and

$$(25) \quad \lambda = \int_0^{\infty} e^{-(r-\gamma)z} \mu(z) dz.$$

Since  $W_0$  is affected only by investment decisions made before time  $t = 0$  it represents the part of  $W$  which is predetermined in relation to the firm's plans for the period  $[0, \infty)$ .

Eq. (23) shows an interesting correspondence between the income tax (21) and a tax on the firm's net cash-flow: A tax on the corporate income at the rate  $u$  is equivalent to defining a corrected investment price  $q^*(t) = q(t) (1-\lambda u)/(1-u)$  and taxing the resulting net cash-flow  $X(t) - q^*(t)J(t)$  at the rate  $u$  in each period  $t \in [0, \infty)$ .<sup>11)</sup> We shall refer to  $(1-\lambda u)/(1-u)$  as the fiscal factor in the following. This motivates us to modify the definition of the user cost of capital accordingly. We then get

11) This, of course, presumes the existence of a perfect financial market, by means of which the firm can transform payment streams between periods at the interest rate  $r$ .

$$(26) \quad c(t) = \frac{q^*(t)}{\phi_{r-\gamma}(0)} = \frac{q(t)}{\phi_{r-\gamma}(0)} \cdot \frac{1-\lambda u}{1-u},$$

or

$$\text{user cost of capital} = \frac{\text{investment price}}{\text{present value of capital service flow}} * \text{fiscal factor.}$$

This is our general formula for the user cost of capital in the presence of corporate taxes. It is more general than the expressions usually discussed in the literature since it applies to any specification of the survival function  $B(s)$  and any system of capital taxation which can be represented by the general weighting function  $\mu(s)$ .<sup>12)</sup>

#### 4. THE EFFECT OF THE TAX SYSTEM

Let us now consider, more specifically, how depreciation allowances, interest deductibility, and taxation of capital gains affect the form of the function  $\mu(s)$ , and hence the parameter  $\lambda$  and the user cost of capital  $c(t)$ . For this purpose, we introduce the concept *accounting capital*. This is the capital concept used by the firm (and the tax authorities) for accounting purposes in order to define depreciation allowances and, possibly, also for calculating interest deductions, and capital gains. We define the value of the accounting capital at time  $t$  as

12) It can be shown formally that (26) is consistent with the conditions for maximization of  $W$  with respect to  $J(t)$ , subject to (3). The first order condition for this problem can be expressed as

$$q(t) \frac{1-\lambda u}{1-u} = \int_0^{\infty} e^{-rz} X'_K(t+z) B(z) dz,$$

where  $X'_K(t+z) = \partial X(t+z)/\partial K(t+z)$  is the value of the marginal product of capital services at time  $t+z$ . Since (26) implies

$$q(t) \frac{1-\lambda u}{1-u} = \int_0^{\infty} e^{-rz} c(t+z) B(z) dz,$$

the user cost of capital as defined above corresponds to the opportunity cost of holding capital goods in this constrained optimization problem.

$$(27) \quad V_A(t) = \int_0^{\infty} A(s)e^{\varepsilon s} q(t-s)J(t-s)ds,$$

where  $A(s)$  is the proportion of the original investment cost which is included in the accounting capital  $s$  years later, and  $e^{\varepsilon s}$  is an inflation adjustment factor:  $\varepsilon$  is the inflation rate which the firm is allowed to use for tax accounting purposes. If  $\varepsilon = \gamma$ , the accounting capital is based on replacement cost, if  $\varepsilon = 0$ , it is based on historic cost, etc. The function  $A(s)$ , which may be denoted as the *statutory survival function of the accounting capital*, is assumed to satisfy

$$(28) \quad 0 \leq A(s) \leq 1, \quad A'(s) \leq 0 \quad (\text{if it exists}) \quad \text{for all } s \geq 0,$$

$$A(0) = 1, \quad A(\infty) = 0,$$

i.e. it has the same general properties as  $B(s)$  and  $G(s)$ ; cf. (1) and (14).

The *depreciation allowances* at time  $t$  can be written as

$$(29) \quad D_A(t) = \int_0^{\infty} a(s)e^{\varepsilon s} q(t-s)J(t-s)ds,$$

where (if its exists)

$$(30) \quad a(s) = -A'(s).$$

The function  $a(s)$  represents the *statutory depreciation rates*, i.e. the weight assigned to capital invested  $s$  years ago when calculating the volume component of the depreciation of this capital vintage. Its price component is the original purchase price inflated by  $e^{\varepsilon s}$ .

From (27)-(30) we obtain

$$\dot{V}_A(t) = q(t)J(t) - D_A(t) + \varepsilon V_A(t),$$



which shows that  $D_A(t)$  has the character as the *gross* depreciation of the accounting capital. Its net value is obtained by subtracting the capital gains as recorded in the firm's accounts,  $\epsilon V_A(t)$ . Hence, in analogy with eq. (18), which gives the true economic depreciation of the firm's capital, we can define the *accounted net depreciation* as

$$(31) \quad E_A(t) = q(t)J(t) - \dot{V}_A(t) = D_A(t) - \epsilon V_A(t) \\ = \int_0^{\infty} \{a(s) - \epsilon A(s)\} e^{\epsilon s} q(t-s)J(t-s) ds.$$

Obviously, we have  $E_A(t) = E(t)$  (and  $V_A(t) = V(t)$ ) regardless of the time path of  $q$  and  $J$  if the following two conditions are satisfied: (i)  $A(s) = G(s)$  ( $\Rightarrow a(s) = g(s)$ ), i.e. the statutory survival function for the accounting capital coincides with the weighting function for the net capital, and (ii)  $\epsilon = \gamma = \dot{q}(t)/q(t)$ , i.e. the rate of inflation permitted for accounting purposes is equal to the rate of increase of the investment price.

Let  $m$  be the proportion of the (imputed) interests on the capital value which is deductible in the firm's income tax base, and  $n$  the proportion of the capital gains, defined as  $\dot{q}(t)K_N(t) = \gamma V(t)$ , which is included in taxable income. The tax function then becomes

$$(32) \quad T(t) = u[X(t) - D_A(t) - mrV(t) + n\gamma V(t)].$$

Inserting from (29) and (11), this implies that the function  $\mu(s)$  in (21) takes on the following specific form:

$$(33) \quad \mu(s) = a(s)e^{(\epsilon-\gamma)s} + \{mr - n\gamma\}G(s).$$

Define, for an arbitrary constant  $\rho$ , the functions

$$(34) \quad Y_{\rho} = \int_0^{\infty} e^{-\rho s} G(s) ds,$$

$$(35) \quad Z_{\rho} = \int_0^{\infty} e^{-\rho s} A(s) ds.$$

Inserting (33) in (25) and making use of (34)-(35), we find that the parameter  $\lambda$  in the fiscal factor can be written as

$$(36) \quad \lambda = 1 - (r-\epsilon)Z_{r-\epsilon} + \{mr-m\gamma\}Y_{r-\gamma}.$$

The resulting formula for the user cost of capital becomes

$$(37) \quad c(t) = \frac{q(t)}{\Phi_{r-\gamma}(0)} \cdot \frac{1}{1-u} [1-u\{1-(r-\epsilon)Z_{r-\epsilon} + \{mr-m\gamma\}Y_{r-\gamma}\}].$$

We have thus expressed the user cost of capital in terms of the investment price  $q(t)$  and its rate of increase, the interest rate  $r$ , the tax parameters  $u$ ,  $\epsilon$ ,  $m$ , and  $n$ , and the present values of the survival rates of the gross capital  $\Phi_{r-\gamma}(0)$ , the net capital,  $Y_{r-\gamma}$ , and the accounting capital,  $Z_{r-\epsilon}$ . Note that the first two present values are based on the market real interest rate  $r-\gamma$ , while the third is based in the "tax permitted" real interest rate  $r-\epsilon$ . From this formula we derive three conclusions:

1. The fiscal factor will be 1 when  $\lambda=1$ . This is thus, in general form, the condition for *neutrality* of the corporate tax system. It will be satisfied *uniformly* (i.e. for all values of  $u$ ,  $r$ ,  $\gamma$ , and  $G(s)$ ) in the following cases:

- (a) Depreciation allowances are based on replacement value ( $\epsilon=\gamma$ ), with the depreciation rates equal to the true rates of depreciation of the net capital ( $A(s) = G(s) \Rightarrow Z_{r-\gamma} = Y_{r-\gamma}$ ), full interest

deductibility is permitted ( $m=1$ ), and capital gains are fully included in taxable income ( $n=1$ ).

- (b) Immediate deduction of capital purchases in taxable income is permitted ( $Z_{r-\epsilon} = 1$ ), no interest deductibility is permitted ( $m=0$ ), and capital gains are not subject to taxation ( $n=0$ ).

Conclusion (a) generalizes the conclusions of Sandmo (1974, sections 4, 6 and 7), King (1975, p. 275), and Boadway (1980, pp. 254-255) to the situation with general survival functions of gross capital, net capital, and accounting capital. These authors consider the case with an exponentially declining survival profile ( $B(s) = e^{-\delta s}$ ) and the declining balance method of depreciation ( $A(s) = e^{-\alpha s}$ ) only. Conclusion (b) confirms the neutrality of the cash-flow tax.

2. These conclusions rest essentially on the assumption that the tax-permitted interest deductions, and the calculation of capital gains are based on the capital value  $V(t)$  as defined in (12). If, however, these components are calculated on the basis of the accounting capital  $V_A(t)$ , as defined in (27), the tax function (32) changes to

$$T(t) = u[X(t) - D_A(t) - mrV_A(t) + neV_A(t)]$$

and we get

$$\lambda = 1 - \{(1-m)r - (1-n)\epsilon\} Z_{r-\epsilon}.$$

In this case, the condition for neutrality ( $\lambda=1$ ) is simply  $m=n=1$ , regardless of the values of  $\epsilon$  and  $Z_{r-\epsilon}$ , and hence of  $A(s)$ . The interpretation of this is that with full interest deductibility and full inclusion of capital gains, the tax system - including the form of the accounting capital function - will not interfere with the firm's optimizing conditions, *provided that the same accounting capital concept is used for calculating depreciation allowances, interest deductions, and capital gains.* This generalizes the

conclusion in Boadway (1980, pp. 255-256), which is confined to a model with exponential depreciation allowances.

3. The distinction between the gross and the net capital concepts is indispensable for the derivation of the general user cost formula (37). In one particular case, however, their values are equal, namely the familiar *exponential* case  $B(s) = e^{-\delta s}$ . Then  $\Phi(s) = 1/\delta$  and  $G(s) = B(s)$  for all  $s$ ; cf. (7) and (10). In this degenerate case, we have

$$\phi_{r-\gamma} = Y_{r-\gamma} = 1/(r+\delta-\gamma).$$

If, for instance, depreciation allowances are calculated according to the declining balance scheme,  $A(s) = e^{-\alpha s}$  ( $\alpha > 0$ ), based on historic cost ( $\epsilon=0$ ), i.e.

$$Z_{r-\epsilon} = 1/(r+\alpha),$$

and if  $m=1$  and  $n=0$ , eq. (37) gives the same expression for the user cost of capital as the one discussed in Boadway (1980, p. 257):

$$c(t) = q(t)(r+\delta-\gamma) \left[ 1 + \frac{ur(\delta-\gamma-\alpha)}{(1-u)(r+\delta-\gamma)(r+\alpha)} \right].$$

## 5. NUMERICAL ILLUSTRATIONS

The above results hold for any specifications of the survival function for gross capital,  $B(s)$ , and of the statutory survival function for accounting capital,  $A(s)$ , which satisfy (1) and (28). In this section, we consider a selection of parametrizations of these functions to illustrate (i) the relationship between the curvature of  $B(s)$  and the user cost of capital, and (ii) the sensitivity of the user cost with respect to the tax parameters  $m$ ,  $n$ , and  $\epsilon$ .

For this purpose, we specify two classes of survival functions for gross capital,  $B(s)$ . Both have two parameters, the first, denoted by  $N$ , representing the (maximal) life time of the capital, the second indicating the 'curvature' of the survival profile.

### Class I

The first class has the form

$$(38) \quad B(s) = B^I(s; N, \tau) = \begin{cases} \left(1 - \frac{s}{N}\right)^\tau & \text{for } 0 \leq s \leq N \\ 0 & \text{for } s > N, \end{cases}$$

where  $N$  and  $\tau$  are positive<sup>13)</sup> constants,  $\tau$  integer. Inserting this in (5), (10), and (17), we obtain

$$(39) \quad b(s) = \frac{\tau}{N} \left(1 - \frac{s}{N}\right)^{\tau-1} = \frac{\tau}{N} B^I(s; N, \tau-1),$$

$$(40) \quad G(s) = \left(1 - \frac{s}{N}\right)^{\tau+1} = B^I(s; N, \tau+1),$$

$$(41) \quad g(s) = \frac{\tau+1}{N} \left(1 - \frac{s}{N}\right)^\tau = \frac{\tau+1}{N} B^I(s; N, \tau), \quad \text{for } 0 \leq s \leq N,$$

respectively.

<sup>13)</sup>  $B(s)$ ,  $G(s)$ , and  $g(s)$  are defined also for  $\tau=0$ , but  $b(s)$  is undefined.

Class II

The survival function in the second class is

$$(42) \quad B(s) = B^{II}(s; N, \sigma) = \begin{cases} 1 - \left(\frac{s}{N}\right)^\sigma & \text{for } 0 \leq s \leq N \\ 0 & \text{for } s > N, \end{cases}$$

where  $N$  and  $\sigma$  are positive constants,  $\sigma$  integer. This parametrization implies

$$(43) \quad b(s) = \frac{\sigma}{N} \left(\frac{s}{N}\right)^{\sigma-1} = \frac{\sigma}{N} [1 - B^{II}(s; N, \sigma-1)],$$

$$(44) \quad G(s) = 1 - \frac{s}{N} - \frac{s}{\sigma N} \left[1 - \left(\frac{s}{N}\right)^\sigma\right] = \left(1 + \frac{1}{\sigma}\right) B^{II}(s; N, 1) - \frac{1}{\sigma} B^{II}(s; N, \sigma+1),$$

$$(45) \quad g(s) = \frac{\sigma+1}{\sigma N} \left[1 - \left(\frac{s}{N}\right)^\sigma\right] = \frac{\sigma+1}{\sigma N} B^{II}(s; N, \sigma), \quad \text{for } 0 \leq s \leq N.$$

These two parametrizations contain several specifications discussed in the literature as special cases. The case in which all capital objects retain their full productivity during  $N$  periods and are then completely scrapped (simultaneous exit, 'one horse shay'), corresponds to class I with  $\tau=0$ , or class II with  $\sigma \rightarrow \infty$ . The case with a linearly decreasing survival function  $B(s)$  is obtained by letting  $\tau=1$  in class I or  $\sigma=1$  in class II. In this case, the survival function for net capital is simply  $G(s) = (1-s/N)^2$ , with  $g(s) = (2/N)(1-s/N)$ , which follows from eqs. (40) and (41) (or (44) and (45)). We recognize the latter as the depreciation rates implied by the sum-of-the-years'-digits method. Furthermore, class I with  $\tau \rightarrow \infty$  (and  $N$  finite) implies momentaneous scrapping of the capital once it has been installed (for practical purposes, this is equivalent to a situation with

a service life of one year). If, however,  $\tau$  and  $N$  both go infinity while their ratio is a finite constant  $\delta$ , then class I degenerates to the standard exponential case,  $B(s) = e^{-\delta s}$ .

All members of class I in which  $\tau \geq 2$  have (strictly) *convex* survival functions for both gross capital and net capital (i.e.  $b'(s) < 0$ ,  $g'(s) < 0$ ). In class II,  $\sigma \geq 2$  implies a (strictly) *concave* survival function for gross capital (i.e.  $b'(s) > 0$ ), but a (strictly) convex survival function for net capital (i.e.  $g'(s) < 0$ ). There is thus no conflict between the assumption that the technical *deterioration* of the capital is *increasing* with age and the assumption that the *depreciation* (decline in capital value) is *decreasing*.

Function values of  $B(s)$  and  $G(s)$  for  $N=20$ ,<sup>14)</sup> with different values of  $\sigma$  and  $\tau$ , are given in table 1. Corresponding values of the relative user cost of capital,  $c/q$ , with the fiscal factor set to unity (i.e.  $\lambda=1$ ), are recorded in table 2. The user cost depends strongly on the curvature of the survival profile, as characterized by  $\sigma$  and  $\tau$ , the sensitivity is larger the lower is the technical life time.

Table 3 illustrates the sensitivity of the user cost with respect to the interest deductibility parameter  $m$ , the share of the capital gains subject to taxation  $n$ , and the inflation adjustment parameter  $\varepsilon$ . For simplicity, we consider only the case where the survival function of the gross capital is of the 'simultaneous exit' type ( $\tau=0$ , or  $\sigma \rightarrow \infty$ )—implying a linearly declining survival function for net capital — and where the depreciation allowances are linear over  $T$  years, where  $T$  may be different from  $N$ . The fourth column of the bottom part of the table ( $T=N=20$ ,  $m=n=1$ , and  $\varepsilon=\gamma$ ) corresponds to the neutral tax system referred to in section 4. We observe that other constellations of the tax parameters  $T$ ,  $m$ ,  $n$ , and  $\varepsilon$  may give large departures from neutrality, in particular when the inflation rate is high.

14) Since (38), (40), (42), and (44) are homogeneous of degree zero in  $s$  and  $N$ , it is straightforward to compute similar function values for other values of  $N$  from this table.

An interesting question for econometric work with investment equations is to which extent the standard parametrization with constant rate of (technical) depreciation is an acceptable approximation for practical purposes. From our formulae, we can throw some light on this issue. In a stationary situation with constant (gross) investment, the rate of deterioration (i.e.  $D/K$ ) will be equal to  $\delta_0 = 1/\int_0^{\infty} B(z)dz$ . Eqs. (38) and (42) give, in particular,

$$\delta_0 = \frac{\tau + 1}{N} \quad (\text{class I}),$$

$$\delta_0 = \frac{\sigma + 1}{\sigma N} \quad (\text{class II}).$$

Let us use this as an approximation to the actual deterioration rate in situations with fluctuating investment. Assume that the actual survival profile is of the simultaneous exit type ( $\tau=0, \sigma \rightarrow \infty$ ), which implies  $\delta_0 = 1/N$ . This suggests approximating  $\Phi_{r-\gamma}(0)$  and  $Y_{r-\gamma}$  by  $1/(r+1/N-\gamma)$  in eq. (37), since  $\Phi_{r-\gamma}(0) = Y_{r-\gamma} = 1/(r+\delta-\gamma)$  holds exactly when the deterioration rate is constant and equal to  $\delta$ , i.e. in the case with exponential survival profile. (Confer conclusion 3 in section 4 above.)

In table 4, we compare - for different values of the tax parameters  $m$ ,  $n$ , and  $\varepsilon$  - these two ways of calculating the relative user cost ( $c/q$ ). The data employed for  $u$ ,  $r$ , and  $\gamma = \dot{q}/q$  are Norwegian annual data for the income tax rate (joint-stock companies), the interest rate (loans from commercial banks to companies) and the rate of increase of the price of investment in machinery and equipment for the years 1965-1980. We find that the series calculated from the exact formula are significantly different



from those based on the approximate formula and the latter have a tendency to exaggerate the fluctuations. In most cases, however, they move in the same direction from one year to the next.

The results in table 4 indicate that a first order approximation based on a constant rate of technical depreciation may lead to inadequate estimates of the variations in the capital cost for policy analysis and prediction. This conclusion will probably hold *a fortiori* if  $\delta_0$  is replaced by a time function equal to the observed ratio between the replacement and the gross capital stock - a common practice in empirical investment analysis. The formulae for  $\Phi_{r-\gamma}(0)$  and  $Y_{r-\gamma}$ , as well as their inverses, are, in general, highly non-linear expressions. In periods with fluctuating interest and inflation rates - as several countries have experienced during the last 15 years. the standard treatment of the replacement component in the user cost of capital may not be as innocent a simplification for empirical research as it may first seem.

TABLE 1. Survival profiles for gross and net capital.

Technical life time:  $N = 20$ .

age s	simult. exit		class II, $\sigma = 10$		class II, $\sigma = 5$		class II, $\sigma = 2$	
	gross	net	gross	net	gross	net	gross	net
00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
01	1.00000	0.95000	1.00000	0.94500	1.00000	0.94000	0.99750	0.92506
02	1.00000	0.90000	1.00000	0.89000	0.99999	0.88000	0.99000	0.85050
03	1.00000	0.85000	1.00000	0.83500	0.99992	0.82000	0.97750	0.77669
04	1.00000	0.80000	1.00000	0.78000	0.99968	0.76001	0.96000	0.70400
05	1.00000	0.75000	1.00000	0.72500	0.99902	0.70005	0.93750	0.63281
06	1.00000	0.70000	0.99999	0.67000	0.99757	0.64015	0.91000	0.56350
07	1.00000	0.65000	0.99997	0.61500	0.99475	0.58037	0.87750	0.49644
08	1.00000	0.60000	0.99990	0.56000	0.98976	0.52082	0.84000	0.43200
09	1.00000	0.55000	0.99966	0.50502	0.98155	0.46166	0.79750	0.37056
10	1.00000	0.50000	0.99902	0.45005	0.96875	0.40313	0.75000	0.31250
11	1.00000	0.45000	0.99747	0.39514	0.94967	0.34554	0.69750	0.25819
12	1.00000	0.40000	0.99395	0.34036	0.92224	0.28933	0.64000	0.20000
13	1.00000	0.35000	0.98654	0.28588	0.88397	0.23508	0.57750	0.16231
14	1.00000	0.30000	0.97175	0.23198	0.83193	0.18353	0.51000	0.12150
15	1.00000	0.25000	0.94369	0.17922	0.76270	0.13560	0.43750	0.08594
16	1.00000	0.20000	0.89263	0.12859	0.67232	0.09243	0.36000	0.05600
17	1.00000	0.15000	0.80313	0.08173	0.55629	0.05543	0.27750	0.03206
18	1.00000	0.10000	0.65132	0.04138	0.40951	0.02629	0.19000	0.01450
19	1.00000	0.05000	0.40126	0.01188	0.22622	0.00702	0.09750	0.00369
20	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 1. (Cont.)

age s	linear deterioration		class I, $\tau = 2$		class I, $\tau = 5$		class I, $\tau = 10$	
	gross	net	gross	net	gross	net	gross	net
00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
01	0.95000	0.90250	0.90250	0.85737	0.77378	0.73509	0.59874	0.56800
02	0.90000	0.81000	0.81000	0.72900	0.59049	0.53144	0.34868	0.31331
03	0.85000	0.72250	0.72250	0.61412	0.44371	0.37715	0.19687	0.16734
04	0.80000	0.64000	0.64000	0.51200	0.32768	0.26214	0.10737	0.09590
05	0.75000	0.56250	0.56250	0.42187	0.23730	0.17798	0.05631	0.04224
06	0.70000	0.49000	0.49000	0.34300	0.16807	0.11765	0.02825	0.01977
07	0.65000	0.42250	0.42250	0.27462	0.11603	0.07542	0.01346	0.00875
08	0.60000	0.36000	0.36000	0.21600	0.07776	0.04666	0.00605	0.00363
09	0.55000	0.30250	0.30250	0.16637	0.05033	0.02768	0.00253	0.00139
10	0.50000	0.25000	0.25000	0.12500	0.03125	0.01562	0.00098	0.00049
11	0.45000	0.20250	0.20250	0.09112	0.01845	0.00830	0.00034	0.00015
12	0.40000	0.16000	0.16000	0.06400	0.01024	0.00410	0.00010	0.00004
13	0.35000	0.12250	0.12250	0.04287	0.00525	0.00184	0.00003	0.00001
14	0.30000	0.09000	0.09000	0.02700	0.00243	0.00073	0.00001	0.00000
15	0.25000	0.06250	0.06250	0.01562	0.00098	0.00024	0.00000	0.00000
16	0.20000	0.04000	0.04000	0.00800	0.00032	0.00006	0.00000	0.00000
17	0.15000	0.02250	0.02250	0.00338	0.00008	0.00001	0.00000	0.00000
18	0.10000	0.01000	0.01000	0.00100	0.00001	0.00000	0.00000	0.00000
19	0.05000	0.00250	0.00250	0.00013	0.00000	0.00000	0.00000	0.00000
20	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 2. User cost of capital for different survival profiles. Per cent of investment price.

Fiscal factor = 1 (i.e.,  $c/q = 1/\phi_\rho(0)$ ).

Technical life time (max) N	Real rate of interest $\rho^a$ Per cent	Simul- taneous exit: Class II, $\sigma=\infty$ Class I, $\tau=0$	Class II: $B(s) = 1 - (\frac{s}{N})^\sigma$			Linear Deterio- ration: Class II, $\sigma=1$ Class I, $\tau=1$	Class I: $B(s) = (1 - \frac{s}{N})^\tau$		
			$\sigma=10$	$\sigma=5$	$\sigma=2$		$\tau=2$	$\tau=5$	$\tau=10$
6	0	16.67	18.33	20.00	25.00	33.33	50.00	100.00	183.33
	5	19.29	20.97	22.68	27.90	36.75	53.81	104.31	187.92
	10	22.16	23.83	25.56	30.98	40.32	57.71	108.66	192.52
20	0	5.00	5.50	6.00	7.50	10.00	15.00	30.00	55.00
	5	7.91	8.39	8.92	10.60	13.59	18.92	34.36	59.61
	10	11.57	11.97	12.46	14.22	17.62	23.13	38.83	64.27
50	0	2.00	2.20	2.40	3.00	4.00	6.00	12.00	22.00
	5	5.45	5.59	5.77	6.48	7.90	10.13	16.44	26.64
	10	10.07	10.12	10.22	10.83	12.48	14.72	21.10	31.38
90	0	1.11	1.22	1.33	1.67	2.22	3.33	6.67	12.22
	5	5.06	5.09	5.16	5.51	6.41	7.65	11.20	16.91
	10	10.00	10.01	10.02	10.25	11.25	12.40	15.99	21.71
$\infty$	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
	10	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00

a) Continuous time.

TABLE 3. User cost of capital for different tax systems<sup>a)</sup>. Per cent of investment price.  
 Survival profile: Simultaneous exit, N=20 years.  
 Depreciation allowances: Linear, T=10 or 20 years.  
 Income tax rate:  $u=0.508$ <sup>b)</sup>.  
 Nominal interest rate:  $r=10$  per cent<sup>c)</sup>

A. No taxation of capital gains :  $n=0$ <sup>a)</sup>

Rate of inflation, $\gamma$ , per cent <sup>c)</sup>	T=20				T=10			
	m=0		m=1		m=0		m=1	
	$\epsilon=0$	$\epsilon=\gamma$	$\epsilon=0$	$\epsilon=\gamma$	$\epsilon=0$	$\epsilon=\gamma$	$\epsilon=0$	$\epsilon=\gamma$
0	18.71	18.71	11.75	11.75	16.42	16.42	9.46	9.46
3	15.04	14.02	8.49	7.47	13.20	12.34	6.65	5.79
6	11.72	9.79	5.63	3.71	10.29	8.79	4.20	2.71
9	8.83	6.10	3.23	0.51	7.75	5.84	2.15	0.25

B. Full taxation of capital gains:  $n=1$ <sup>a)</sup>

Rate of inflation, $\gamma$ , per cent <sup>c)</sup>	T=20				T=10			
	m=0		m=1		m=0		m=1	
	$\epsilon=0$	$\epsilon=\gamma$	$\epsilon=0$	$\epsilon=\gamma$	$\epsilon=0$	$\epsilon=\gamma$	$\epsilon=0$	$\epsilon=\gamma$
0	18.71	18.71	11.75	11.75 <sup>d)</sup>	16.42	16.42	9.46	9.46
3	17.00	15.99	10.45	9.44 <sup>d)</sup>	15.16	14.30	8.61	7.76
6	15.37	13.45	9.29	7.36 <sup>d)</sup>	13.94	12.45	7.85	6.36
9	13.86	11.14	8.27	5.54 <sup>d)</sup>	12.78	10.88	7.19	5.29

a) The figures in this table are based on a discrete time version of eq. (37).  
 The alternatives considered are:

- $m = 0$ : No interest deductibility. (Or debt ratio = 0.)
- $m = 1$ : Full interest deductibility. (Or debt ratio = 1.)
- $\epsilon = 0$ : Depreciation allowances based on historic cost.
- $\epsilon = \gamma$ : Depreciation allowances based on replacement cost.

b) Norwegian tax rate for joint-stock companies.

c) Pro anno rate.

d) Neutral tax system.

Table 4. User cost of capital for machinery and equipment calculated from exact<sup>a)</sup> and approximate<sup>b)</sup> formulae.  
Per cent of investment price.

Survival profile: Simultaneous exit, N = 20 years.

Depreciation allowances: Linear, T = 10 years.

A. Full interest deductibility: m = 1

Year	No taxation of capital gains: n=0				Full taxation of capital gains: n=1			
	Depreciation allowances based on: orig. cost: $\epsilon=0$		repl. cost: $\epsilon=\gamma$		Depreciation allowances based on: orig. cost: $\epsilon=0$		repl. cost: $\epsilon=\gamma$	
	exact	approx.	exact	approx.	exact	approx.	exact	approx.
1965	4.80	3.59	3.97	2.62	6.59	6.75	5.76	5.77
1966	5.53	4.90	4.96	4.21	6.73	6.95	6.16	6.26
1967	6.39	6.36	6.15	6.06	6.87	7.16	6.64	6.87
1968	7.27	7.72	7.42	7.91	6.97	7.24	7.12	7.43
1969	3.86	1.63	2.55	0.12	6.85	6.98	5.55	5.48
1970	0.74	-6.33	-1.04	-6.55	5.90	5.20	4.12	4.98
1971	3.85	1.72	2.62	0.34	6.71	6.91	5.48	5.53
1972	5.00	3.91	4.09	2.82	7.02	7.40	6.11	6.31
1973	6.70	6.81	6.33	6.35	7.47	8.06	7.10	7.60
1974	0.86	-5.70	-1.00	-6.56	6.16	5.62	4.29	4.76
1975	2.70	-0.77	1.06	-2.49	6.81	6.90	5.17	5.17
1976	2.23	-1.69	0.48	-3.47	6.81	6.85	5.06	5.06
1977	2.90	-0.33	1.22	-2.20	7.12	7.35	5.44	5.48
1978	1.04	-3.84	-1.04	-5.93	7.11	7.13	5.03	5.03
1979	8.32	8.92	7.79	8.25	9.51	10.72	8.98	10.05
1980	1.92	-2.38	-0.13	-4.67	7.49	7.75	5.44	5.47

Table 4. (cont.)

B. No interest deductibility:  $m = 0$ 

Year	No taxation of capital gains: $n=0$				Full taxation of capital gains: $n=1$			
	Depreciation allowances based on: orig. cost: $\epsilon=0$		repl. cost: $\epsilon=\gamma$		Depreciation allowances based on: orig. cost: $\epsilon=0$		repl. cost: $\epsilon=\gamma$	
	exact	approx.	exact	approx.	exact	approx.	exact	approx.
1965	8.30	9.75	7.47	8.78	10.09	12.91	9.26	11.94
1966	9.23	11.23	8.66	10.53	10.43	13.27	9.86	12.58
1967	10.27	12.79	10.04	12.50	10.76	13.60	10.52	13.30
1968	11.53	14.56	11.67	14.75	11.23	14.09	11.38	14.27
1969	8.24	9.46	6.93	7.96	11.23	14.82	9.92	13.31
1970	3.79	0.48	2.00	0.25	8.94	12.01	7.16	11.78
1971	7.62	8.56	6.39	7.18	10.49	13.75	9.26	12.37
1972	9.05	10.89	8.14	9.80	11.07	14.38	10.16	13.29
1973	11.08	13.93	10.72	13.47	11.85	15.19	11.49	14.73
1974	4.50	2.09	2.64	1.22	9.80	13.41	7.93	12.54
1975	7.15	7.53	5.51	5.81	11.26	15.20	9.62	13.48
1976	6.94	7.08	5.19	5.29	11.52	15.61	9.77	13.82
1977	7.97	8.90	6.30	7.03	12.19	16.57	10.52	14.70
1978	7.17	7.24	5.09	5.14	13.24	18.20	11.16	16.10
1979	15.83	20.23	15.30	19.56	17.02	22.02	16.50	21.35
1980	8.33	9.29	6.28	7.00	13.90	19.42	11.85	17.14

a) The discrete time version of (37).

b) Discrete time version of (37) with  $\phi_{r-\gamma}(0)$  and  $Y_{r-\gamma}$  replaced by  $1/(r+1/N-\gamma)$ .

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