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Innovation, Automation, and Inequality: Policy Challenges in the Race against the Machine

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Abstract. We analyze the effects of R&D-driven automation on economic growth, education, and inequality when high-skilled workers are complements to machines and low-skilled workers are substitutes for machines. The model predicts that innovation-driven growth leads to an increasing population share of college graduates, increasing income and wealth inequality, and a declining labor share. We use the model to analyze the effects of redistribution. We show that it is difficult to improve income of low-skilled individuals as long as both technology and education are endogenous. This is true irrespective of whether redistribution is financed by progressive wage taxation or by a robot tax. Only when higher education is stationary, redistribution unambiguously benefits the poor. We show that education subsidies affect the economy differently depending on their mode of funding and that they may actually reduce education. Finally, we extend the model by fair wage concerns and show how automation could induce involuntary low-skilled unemployment.

Keywords: Automation, Innovation-Driven Growth, Inequality, Wealth Concentration, Unemployment, Policy Responses.


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1. **Introduction**

Common wisdom in growth and labor economics suggests that technological progress is labor-augmenting. Accordingly, technological progress developed by market R&D and incorporated into new machines complements human work effort and makes workers more productive. In this paper, we look at the dark side of R&D-driven technological change and consider a situation in which new technologies complement only high-skilled workers but substitute for low-skilled workers.

In contrast to other recent studies on automation and growth (to be discussed below), we consider an overlapping generations structure of the population and focus on endogenous education decisions and the effects of policy responses that are often suggested as an answer to the challenges posed by automation. Since (at least at the current state of technology) high-skilled labor is more difficult to automate than low-skilled labor, people may avoid the downside of technological progress and enjoy its benefits by upgrading their skills. We show that an increase in the skill premium due to automation motivates an increasing share of people to obtain higher education in the form of a college degree. However, in a heterogeneous society, not everybody is able or willing to obtain higher education. Due to ability constraints, some individuals do not manage to acquire higher education and are left behind. In this way, R&D-based growth leads to increasing income and wealth inequality from one generation to the next and, in an extension of the model, to increasing involuntary unemployment of low-skilled individuals. We implement redistribution policies, financed by labor income taxes or robot taxes, and show why it is difficult to improve the disposable income of low-skilled individuals as long as both technology and education are endogenous.

The most popular discussion of skill-biased technological progress is perhaps provided by Goldin and Katz (2009), who argue that America has lost the “Race between Education and Technology” because high school completion rates have stagnated since the 1950s. However, as emphasized by Acemoglu and Autor (2012), this loss is relative because the underlying model assumes that low-skilled labor also benefits from innovation but to a lesser degree than high-skilled labor. Here, by contrast, we conceptualize individuals with tertiary education (a college degree) as high-skilled workers. These workers are complements to machines and their wages as well as their share in the workforce increased throughout the 20th century. The race against technology is lost by those individuals who do not obtain a tertiary education and whose wages do not benefit from automation.

Another important distinction from Goldin and Katz (2009) is that, in their study, education and technology are treated as being exogenous such that education could win the race against technology. In our framework, by contrast, both forces are endogenous. Skill-biased technical change promotes education and more education, in turn, leads to more R&D, more
innovation, and further advancements of technology. The outcome that education fails to keep pace with increasing skill-biased technological change derives from the fact that technology advances perpetually, whereas the labor share of high-skilled individuals converges to an upper limit.

The idea of labor-substituting technological progress has been popularized by Brynjolfsson and McAfee’s (2011) book on another race, the “Race against the Machine”. Brynjolfsson and McAfee claim that technological progress, understood as automation, makes people more innovative, productive, and richer but at the cost of increasing unemployment and (wealth) inequality. Early quantitative evidence for this view stems from Berman et al. (1998), who show that around 70 percent of the decline in production workers’ share of the wage bill is explained by R&D and computerization. Goos et al. (2009, 2014) and Autor and Dorn (2013), provide evidence for the phenomenon of wage polarization, i.e., that routine tasks in manufacturing are more and more performed by robots, which drives low-skilled workers out of manufacturing and into the service sector. Since service sector jobs are typically paid less well, this process leads to a hollowing out of the middle class.¹ More recently, Graetz and Michaels (2018) provide evidence that industrial robots lead to a reduction in the demand for low-skilled labor; Frey and Osborne (2017) argue that the average educational attainment of an occupation and the probability of this occupation to be automated are negatively correlated; Arntz et al. (2016, 2017) explain that low-skilled workers perform tasks that are typically much easier to automate than the tasks performed by high-skilled workers; and Acemoglu and Restrepo (2017) find that the increase in industrial robots in U.S. manufacturing had large negative effects on wages and employment across commuting zones with the strongest wage effects on workers with high school education or less.

To curb rising inequality of living standards, it is seemingly attractive to transfer income to the losers from automation, perhaps in the form of unconditional transfers financed by progressive labor income taxation or by taxing machine input, i.e., by imposing a so-called robot tax (Shiller, 2017; Gasteiger and Prettner, 2017). Here, we investigate the impact of these policies when both education and technology are endogenous. We show that both policies reduce education, R&D, and economic growth. Aside from its repercussions on innovation and growth, redistribution from the skilled to the unskilled may not improve disposable income of low-skilled individuals in the short and medium run. The reason for this perhaps non-obvious result is that both a robot tax and a progressive income tax reduce the potential income gain from higher education, which leads to a larger population share of

¹ For tractability and because we are less interested in modeling structural change, our framework does not feature a service sector that could absorb labor. Extending the model to include a service sector (with appropriate simplifications in other parts of the model) is a promising avenue for further research on automation.
low-skilled individuals. The increased low-skilled labor supply depresses the pre-tax wage of low-skilled workers. Redistribution to low-skilled workers improves their disposable income unambiguously only in the long run, when higher education is stationary.

By the same general equilibrium argument, low-skilled wages can benefit from a subsidy on the education of high-skilled workers. Due to the induced additional uptake of higher education, low-skilled labor supply declines and low-skilled wages increase. Only in the long run, when education is stationary, an education subsidy makes low-skilled workers unambiguously worse off. The impact of education subsidies on economic growth depends on the mode of their funding. If financed by robot taxation, an education subsidy may actually reduce education because it depresses R&D, growth, and the demand for high-skilled labor. If financed by an income tax, education subsidies increase education and growth. Although this particular policy constitutes a de facto redistribution from the unskilled poor to the skilled rich, it may not increase after-tax inequality because pre-tax inequality declines due to the induced higher supply of high-skilled labor.

Rising inequality may also trigger rising unemployment. To investigate this idea, we integrate Akerlof and Yellen’s (1990) fair wage theory into an extension of the model. Automation increases the productivity and income of high-skilled workers but leaves productivity of low-skilled workers unchanged. If low-skilled workers receive “neoclassical” wages according to their marginal product under full employment, they would perceive the increasing inequality as unfair and would not exert full effort at work. To elicit full effort, firms might let low-skilled workers participate in the productivity gains from automation and adjust their employment accordingly, causing involuntary unemployment among low-skilled workers. This mechanism, however, does not necessarily imply that unemployment increases with automation. The reason for this perhaps unexpected result is that automation also increases the skill premium and induces more higher education such that the supply of low-skilled labor declines. With both the supply and demand of low-skilled labor contracting, the effect on unemployment is ambiguous. Only when education is stationary, automation and fair wage concerns imply unambiguously rising unemployment.

Some recent articles have investigated automation in the context of long-run development. Héamous and Olsen (2016) and Acemoglu and Restrepo (2018) are perhaps most closely related to our contribution. Like us, both studies focus on R&D-based innovations and inequality in the process of economic growth. In both studies, the household side of the economy is simpler than in our case because there is no education decision and skills are taken as given by the infinitely living (representative) individual. The production side, however, is more complex in both studies. Specifically, final goods are assumed to be produced by a variety of intermediate goods (Héamous and Olsen) or a variety of tasks (Acemoglu and
Restrepo). Varieties are produced by labor and potentially by (low-skilled) labor replacing machines. R&D generates new varieties that start out as un-automated. Firms may then make investments in order to automate the production of their intermediate good. As a result, low-skilled wages benefit from R&D-based innovations and even automation could raise low-skilled wages because it encourages more R&D. It is perhaps fair to say that both theories focus on the production side of the economy and on the question under which conditions (low-skilled) workers could benefit from automation. This justifies a rather stylized household side of the economy. The race between education and technology, and the impact of redistribution policies are not the focus of their contributions.

While Hémous and Olsen and Acemoglu and Restrepo assume that R&D creates new intermediate inputs or tasks in production that start out un-automated, we conceptualize R&D as the process that creates patents for automation capital. This view is supported by Mann and Püttermann (2017) who analyzed granted US patents from 1976 to 2014 and identified automation innovations. They find that the share of automation patents in total patents increased from 25 percent to 67 percent during the observation period. Acknowledging that R&D probably generates both un-automated new tasks and machines that substitute for low-skilled labor, our theory complements the existing literature.²

An earlier related study is provided by Krusell et al. (2000), who discuss capital-skill complementarity and inequality in a growth model without R&D and TFP growth. Growth is driven by the accumulation of equipment capital, which substitutes for low-skilled labor and augments high-skilled labor. The study focuses on mechanisms on the production side of the economy and considers neither household decisions nor redistribution policies.

Sachs and Kotlikoff (2012) provide a model that discusses, like us, the interaction between automation and education in an overlapping generations context. However, their framework as well as their conclusions are different from ours. In their framework, all individuals are assumed to start their working life as being low-skilled and may later in life invest in education and physical capital. When exogenous technical advances increase the productivity

² Other, for various reasons less related studies on automation and macroeconomic performance are provided by Zeira (2006), Steigum (2011), Peretto and Seater (2013), Benzel et al. (2015), Restrepo (2015), Sachs et al. (2015), Abelianzky and Prettnner (2017), Gasteiger and Prettnner (2017), Prettnner (2017), Cords and Prettnner (2019), Guimarães and Mazeda Gil (2019), Lankisch et al. (2019), and Venturini (2019). Most of these studies do not explain technological progress endogenously. Exceptions are Zeira (2006) and Peretto and Seater (2013), which, however, do not address inequality issues and redistribution. The interaction between technology, wages, and education relates our paper to the unified growth literature, where one of the core mechanisms is the rise of education triggered by technological progress (Galor and Weil, 2000; Galor and Moav, 2002; Galor, 2005; 2011). In contrast to this literature, we focus on tertiary education, R&D-based growth, and automation through new technologies. In an earlier study (Strulik et al., 2013), we constructed an overlapping generations version of the Romer (1990)–Jones (1995) R&D-based growth model with an endogenous education and fertility decision to discuss long-run adjustment processes. However, we did not consider automation and the evolution of inequality.
of machines that substitute for low-skilled labor, young individuals respond by investing less in education. Instead of a race between education and technology, the study thus investigates a case where the two “runners” move in opposite directions.

Our paper also contributes to the long-standing debate on the interaction between inequality and economic growth. While the earlier theoretical literature focused mainly on the causality running from inequality to growth, where empirical studies found a negative association (Persson and Tabellini, 1994; Alesina and Rodrik, 1994; Aghion et al., 1999), the literature related to skill-biased technical change (cited above) argues in favor of the causality running from growth to inequality and suggests a positive association. Recently, Piketty (2014) has popularized the view that economic growth reduces inequality in the context of the neoclassical growth model. Here, we argue that R&D-based growth theory in conjunction with automation provides a “non-Pikettarian” result: ceteris paribus, faster growth is predicted to lead to more inequality in labor income and wealth. This finding, however, does not imply that there is no threat from automation when the growth rate of factor productivity declines. As long as R&D-based growth is positive, automation causes inequality to rise.

The paper proceeds as follows. In Section 2, we set up the basic model of R&D-driven automation. In Section 3, we take the education system as given and provide a series of analytical results on economic growth and various aspects of inequality along the balanced growth path. The full model with growth–education interaction is investigated in Section 4. We calibrate the model to the U.S. economy in which economic growth is roughly constant during the 20th century and mildly declining in the long run. While our main results hold also true for the knife-edge case of gradual convergence towards a positive long-run growth rate (see the Appendix), the scenario of mildly declining growth is better suited to capture the actual long-run trends. We use this version to investigate the various forms of redistribution policies mentioned above. In Section 5, we extend the model by fair wage concerns and involuntary unemployment of low-skilled workers. Section 6 concludes the paper.

2. The Model

2.1. Basic Assumptions. Consider an overlapping generations economy in which individuals live for two periods. They enter the economy as young adults and are equipped with secondary education (high school or less). Young adults are endowed with one unit of time and decide whether or not to spend a certain amount $\eta$ of their available time studying in order to acquire greater skills in the form of a college degree. The remaining time of young adulthood is supplied on the labor market. Young adults save for the second period of their life when they are retired. After the second period, individuals die with certainty. Time $t$
evolves discretely with each time step capturing one generation. The working population is of size $L_t$ and constant. In an earlier version of this paper (Prettner and Strulik, 2017), we showed that the main results also hold when there is a growing population. Employing the argument of finite space on earth, it can be argued that the only meaningful long-run steady state is associated with a stationary population (Strulik, 2005).

Individuals are heterogeneous with respect to their ability to graduate from college, which is expressed as the disutility (or effort) entailed by studying. As a result, the society is divided into high-skilled and low-skilled workers: high-skilled labor, denoted by $L_{H,t}$, is, as conventionally assumed, complemented by machines, whereas low-skilled labor, denoted by $L_{L,t}$, is substituted by machines. For simplicity, we ignore the potential automation of some high-skilled jobs by artificial intelligence. Including this feature would provide more realism to our stylized model but would not change the main mechanisms and results.

There are three production factors, the two types of labor described above and physical capital in the form of machines and robots. Low-skilled workers are employed in the final goods sector for tasks that can also be performed by machines. High-skilled workers are either employed as workers in the final goods sector responsible for tasks that cannot be easily automated (engineers) or as workers in the R&D sector for developing new technologies (scientists). In the basic model we ignore the possibility of involuntary unemployment such that all labor markets clear. The government taxes wage income and/or the use of machines (i.e., it imposes a “robot tax”) at a constant rate and uses the revenues for redistribution.

2.2. Individuals. Individuals experience utility from consumption in working age and old age. In period $t$, the remaining lifetime utility of working-age individuals of type $j = L, H$ is given by

$$u_t = \log(c_{j,t}) + \beta \log(\bar{R}s_{j,t}) - 1_{[j=H]} v(a),$$

where $c_{j,t}$ is consumption of young adults, $\beta$ is the discount factor, $1_{[j=H]}$ is the indicator function attaining a value of one for those who obtain a college degree, $v(a)$ is the disutility from obtaining a college degree of an individual with idiosyncratic ability level $a \geq 0$, and $\bar{R}$ is the gross interest rate on savings $s_{j,t}$ such that $c_{j,t+1} = \bar{R}s_{j,t}$ refers to consumption in the second period of life. For simplicity, we assume that the economy is comparatively small and open to international capital flows to an extent that the interest rate is determined at the world market.

Higher education requires a constant investment of time such that the time spent on education is $\eta_L = 0$ without tertiary education and $\eta_H > 0 = \eta$ with tertiary education, where $\eta_H < 1$. Realistically, not all members of a society are willing or capable to obtain

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3 See Strulik and Weisdorf (2008) for a unified growth model that endogenously generates convergence towards a stationary population.
a college degree. We model this aspect by assuming that the disutility (or effort) to be incurred by higher education is declining in ability $a$. Formally, $v(a) \geq 0$ and $\partial v / \partial a < 0$ for all individuals who invest in a college degree, while $v(a) = 0$ for individuals who choose not to invest in a college degree. Moreover, there exists a pole for ability, $\lim_{a \to a^+} v(a) = \infty$, which captures the notion of a lower bound of ability below which graduation from college is associated with infinite disutility (or requires infinite effort). Consequently, and realistically, there are always some individuals in society who remain without a college degree.

Instead of considering effort, we could have implemented heterogeneity in education by time constraints. Individuals of lower ability would then need more time to obtain higher education and the fact that time (lifetime in working age) is finite would prevent higher education of some individuals of low ability. These features would produce qualitatively similar results for the predicted evolution of education and technology. They would, however, imply an implausibly large variation of graduation time across individuals and a secular increase of average graduation time in the course of economic development.

Let $w_j$ denote the wage per unit of labor supply of type $j = L, H$, $\tau_w$ the tax rate on wage income, and $T_j$ the lump-sum transfers to type $j$. The budget constraint that each individual faces is then given by

$$(1 - \tau_w)(1 - \eta_j)w_{j,t} + T_j = c_{j,t} + s_{j,t}.$$  

(2)

Maximizing utility (1) subject to the budget constraint (2) leads to optimal consumption and optimal savings as

$$c_{j,t} = \frac{(1 - \tau_w)(1 - \eta_j)w_{j,t} + T_j}{1 + \beta}, \quad s_{j,t} = \frac{\beta [(1 - \tau_w)(1 - \eta_j)w_{j,t} + T_j]}{1 + \beta}.$$  

(3)

where $\beta/(1 + \beta)$ is the savings rate of both types of workers.

2.3. Education Decision. By inserting (3) into (1), we obtain the indirect utility function conditioned on education. Individuals compare utility with and without a college degree and obtain higher education if

$$v(a) \leq (1 + \beta) \log \left[ \frac{(1 - \tau_w)(1 - \eta)w_{H,t} + T_H}{(1 - \tau_w)w_{L,t} + T_L} \right] \equiv \tilde{w}_t,$$  

(4)

in which $\tilde{w}_t$ denotes the net skill premium in terms of utility. Solving (4) with equality, we obtain the ability level at the threshold above which individuals achieve tertiary education, denoted by $a_t^*$:

$$v^{-1}(\tilde{w}_t) = a_t^*.$$  

(5)

The threshold is moving over time and declines with increasing skill premium. These considerations are visualized in the upper part of Figure 1. Suppose that the lowest ability level
is given by zero. Let the cumulative distribution function of ability be denoted by \( F(a) \). An example, is shown in the lower part of Figure 1. A skill premium of \( \tilde{w}_t \) is, via the threshold level \( a^*_t \), associated with a population share \( F(a^*_t) \) of low-skilled individuals. A rising skill premium gradually motivates individuals of lower ability to obtain tertiary education such that \( F(a^*_t) \) declines.

**Figure 1. The Education Threshold**

![Figure 1](image)

Individuals with ability \( a \) below the threshold \( a^*_t = v^{-1}(\tilde{w}) \) remain without tertiary education. A share \( F(a^*_t) \) of individuals remains without tertiary education.

2.4. **Final Goods Production.** The production side of the economy builds upon Romer (1990) and Jones (1995). Aggregate output is produced with physical capital in the form of machines and with both types of labor according to the production function

\[
Y_t = L_{H,Y,t}^{1-\alpha} \left( L_{L,t}^\alpha + \sum_{i=1}^{A_t} x_{i,t}^\alpha \right),
\]

where \( L_{H,Y,t} \) is the part of high-skilled labor that is employed in the final goods sector, \( x_{i,t} \) are machines of the specific type \( i \), \( \alpha \in (0, 1) \) denotes the elasticity of output with respect to human labor that can easily be automated, and \( A_t \) is the stock of specific blueprints.
available for the associated machines of type $i$, i.e., it represents the technological frontier of the country under consideration. We conceptualize technological progress (a growing technological frontier $A$, which is TFP growth) as an increase in the variety of machines in the production process.\footnote{Alternatively, we could have used a quality-ladder model (following Aghion and Howitt, 1992), which, in reduced-form, would be equivalent to the variety approach.}

Let $p_{i,t}$ denote the net price of a unit of a machine of type $i$ and $\tau_R$ the ad-valorem tax on machine input in production (the robot tax). The factor rewards are then given by

\begin{align}
  w_{H,Y,t} &= (1 - \alpha)L_{H,Y,t}^{-\alpha} \left( L_{L,t}^\alpha + \sum_{i=1}^{A_t} x_{i,t}^\alpha \right) \Leftrightarrow w_{H,t} = (1 - \alpha) \frac{Y_t}{L_{H,Y,t}}, \\
  w_{L,t} &= \alpha (L_{H,Y,t}/L_{L,t})^{1-\alpha}, \\
  (1 + \tau_R)p_{i,t} &= \alpha L_{H,Y,t}^{1-\alpha} x_{i,t}^{\alpha-1},
\end{align}

The key difference with respect to the conventional growth literature is that technological progress (rising $A$) has a different impact on the two types of labor. As commonly assumed, it increases the productivity of complementing labor $L_H$ and is in this sense quasi labor-augmenting. However, it leaves the productivity of substitutable labor $L_L$ unaffected such that the relative importance of this type of labor declines with technological progress.

The key difference to related studies on automation (Acemoglu and Restrepo, 2018; Hemous and Olson, 2016) is that we provide an aggregate view according to which machines do not replace low-skilled labor in aggregate production. In our notation, the alternative view would be formalized by a production function $Y_t = L_{H,Y,t}^{1-\alpha} \left( L_{L,t} + \sum_{i=1}^{A_t} x_{i,t} \right)^\alpha$ and, depending on factor prices, production would employ either machines or low-skilled labor. Obviously, this approach requires a disaggregated view on the production process, where labor-replacing machines are only available in some sectors of the economy.

Here, by contrast, low-skilled labor is always in demand. Intuitively, when new varieties of machines enter the production process ($A$ rises) and perform tasks formerly performed by low-skilled labor, low-skilled workers find employment elsewhere in the economy. We thus maintain the notion of quasi labor-augmenting technological progress from conventional growth theory. The only difference is that technological progress does not increase the productivity of low-skilled labor. This can be seen in the low-skill wage (8), which is independent from technology and determined just by the relative skill supply. High-skilled labor, by contrast, benefits from technological progress and wages increase at the rate of aggregate output growth when labor supply stays constant [see (7)]. Finally, note that the demand for machines depends negatively on the robot tax $\tau_R$ [see (9)].
2.5. **R&D Sector.** The R&D sector produces blueprints for new machines by employing scientists, which are recruited from high-skilled labor. To focus on our research questions, we abstract from governmentally-funded basic research and from R&D tax credits for applied research (see, for example, Prettner and Werner, 2016; Minniti and Venturini, 2017; for the importance of R&D policies in determining long-run economic growth). The production function of the R&D sector is given by

\[ A_t - A_{t-1} = \bar{\delta}_t L_{H,A,t}, \]  
(10)

where \( L_{H,A,t} \) denotes scientists employed in the R&D sector and \( \bar{\delta} \) is the productivity of these scientists. The productivity level of scientists itself depends on intertemporal knowledge spillovers (the standing-on-giants-shoulders externality) and on congestion effects (the stepping-on-toes externality) as described by Jones (1995). We follow the standard approach and write

\[ \bar{\delta}_t = \frac{\delta A_{t-1}}{L_{1-\lambda}^{H,A,t}}, \]  
(11)

where \( \phi \in (0, 1] \) measures the strength of intertemporal knowledge spillovers and \( 1 - \lambda \) with \( \lambda \in [0, 1] \) measures the strength of the congestion externality.

Profits in the R&D sector are given by the revenue that R&D firms generate by selling their patents net of the costs for the scientists that they employ,

\[ p_{A,t} \bar{\delta}_t L_{H,A,t} - w_{H,A,t} L_{H,A,t}, \]  
(12)

where \( p_{A,t} \) is the price of blueprints and \( w_{H,A,t} \) denotes the wage rate of scientists. Due to the competitive labor market, the wage rate of scientists attains the same level as the wage rate of high-skilled workers in the final goods sector. R&D firms maximize profits by choosing optimal R&D employment, which provides the optimality condition \( w_{H,A,t} = \bar{\delta}_t p_{A,t} \).

Our overlapping generations structure allows us to introduce a finite patent length of one generation, which is reasonably close to the actual patent duration of approximately 20 years (United States Patent and Trademark Office, 2017).

2.6. **Intermediate Goods Sector.** The intermediate goods sector uses physical capital as variable input factor to produce machines. The production function is linear with a unitary capital input coefficient such that \( x_{i,t} = k_{i,t} \), where \( k_{i,t} \) is the amount of physical capital employed by each intermediate goods producer. There are two types of firms in the intermediate goods sector. Producers of the latest vintage of machines use a blueprint (patent) from the R&D sector as fixed input. There are two types of firms in the intermediate goods sector. Producers of the latest vintage of machines use a blueprint (patent) from the R&D sector as fixed input. These firms have a certain degree of market power and free entry into the intermediate goods sector implies that operating profits in period \( t \), \( \pi_{i,t} \), are equal to the entry costs consisting of the price that has to be paid up-front.
for the blueprint such that
\[ \pi_{i,t} = p_{A,t}. \] (13)

Producers of older vintages of machines are no longer protected by patent law and free entry ensures that a zero profit condition holds. Let variables associated with the latest vintage of machines be indexed by \( i \) and variables associated with earlier vintages by \( j \). Operating profits for producers of the latest vintage are given by
\[ \pi_{i,t} = p_{i,t}(x_{i,t})x_{i,t} - \bar{R}x_{i,t}. \] (14)

Profit maximization implies
\[ p'_{i,t}(x_{i,t})x_{i,t} + 1 = \frac{\bar{R}}{p_{i,t}} \Rightarrow p_{i,t} = \frac{\bar{R}}{\alpha}. \] (15)

Producers of the latest vintage of machines charge a markup over marginal cost and the production of machines of type \( i \) adjusts (due to capital inflow/outflow) to the point at which \( \bar{R} = \alpha L_{H,Y,t}^{1-\alpha} x_{i,t}^{\alpha-1}/(1 + \tau_R) \). Producers of older vintages charge prices at marginal cost \( p_{j,t} = \bar{R} \) for all \( j \) such that the production of machines of type \( j \) adjusts to the point at which \( \bar{R} = \alpha L_{H,Y,t}^{1-\alpha} x_{j,t}^{\alpha-1}/(1 + \tau_R) \). Combining both demand functions provides the input ratio
\[ x_{j,t} = \alpha \frac{1}{\alpha - 1} x_{i,t}, \] (16)

implying that the demand for older vintages is higher because their price is lower. Aggregating over all vintages and using (16) we obtain
\[ \sum_{j=1}^{A_t-1} x_{j,t}^{\alpha} + \sum_{i=A_{t-1}+1}^{A_t} x_{i,t}^{\alpha} = \bar{A}_t x_{i,t}^{\alpha}, \quad \bar{A}_t \equiv \left( \alpha^{\frac{\alpha}{\alpha - 1}} - 1 \right) A_{t-1} + A_t. \] (17)

Using the new notation, we can rewrite final goods production as
\[ Y_t = L_{H,t}^{1-\alpha} [L_{L,t}^{\alpha} + \bar{A}_t x(i)^{\alpha}]. \]

2.7. Equilibrium. In the basic model, we abstract from unemployment. Since the population share of low-skilled individuals amounts to \( F(a_t^*) \), see Figure 1, low-skilled employment is given by \( L_{L,t} = F(a_t^*) L_t \). Since high-skilled individuals spend a fraction \( \eta \) of their time on education, aggregate supply of high-skilled labor amounts to \( L_{H,t} = [1 - F(a_t^*)](1 - \eta) L_t \).

Labor market clearing requires that employment of high-skilled labor in the final goods sector and in R&D add up to the total supply of high-skilled labor such that
\[ L_{H,t} = L_{H,Y,t} + L_{H,A,t}. \] (18)

The market-clearing wage rate is given by
\[ w_{H,A,t} = w_{H,Y,t} \quad \Leftrightarrow \quad p_{A,t} L_{H,A,t}^{1-\alpha} = (1 - \alpha) \frac{L_{L,t}^{\alpha} + \bar{A}_t x_{i,t}^{\alpha}}{L_{H,Y,t}^{\alpha}}. \] (19)
From Equation (9), we get demand for intermediates as

\[ x_{i,t} = \left[ \frac{\alpha}{p_{i,t}(1 + \tau_R)} \right]^{\frac{1}{1-\alpha}} L_{H,Y,t}. \]  

(20)

Plugging (13) and (20) into (19) provides

\[ \pi_{i,t} = (1 - \alpha) \bar{R} x_{i,t} / \alpha. \]  

Inserting this expression, the price \( p_{i,t} = \bar{R} / \alpha \), and equations (18) and (20) into (21), we obtain an implicit function for the employment level of scientists \( L_{H,A,t} \). If an interior solution with R&D exists, the employment level of scientists solves the equation

\[ \frac{\bar{R}}{\alpha} \left[ \frac{\alpha^2}{R(1 + \tau_R)} \right]^{\frac{1}{1-\alpha}} (L_{H,t} - L_{H,A,t})^{\frac{1}{1-\lambda}} \delta A_{t-1} = \left( \frac{L_{L,t}}{L_{H,t} - L_{H,A,t}} \right)^\alpha + \tilde{A}_t \left[ \frac{\alpha^2}{R(1 + \tau_R)} \right]^{\frac{1}{1-\alpha}}. \]  

(22)

Finally, the government runs a balanced budget. Low-skilled individuals receive a fraction \( \kappa \) of total transfers such that

\[ T_{L,t} = \kappa T_t / L_{L,t} \]  

and

\[ T_{H,t} = (1 - \kappa) T_t / L_{H,t} \]  

with

\[ T_t = \tau_W w_{H,t} L_{H,t} + \tau_W w_{L,t} L_{L,t} + \tau_R p_{i,t} x_{it} \left[ \left( \frac{\alpha}{1-\alpha} - 1 \right) A_{t-1} + A_t \right]. \]  

(23)

3. Analytical Results

The full model is recursive: individuals need to form expectations on their future wages to decide upon their education. Future wages, however, depend on the education decision. We solve this problem numerically in Section 4. Here, we focus on the long-run results and assume that the outcome of the education decision, \( L_{H,t} \), is given as a positive pre-determined state variable at any time \( t \) (implying \( 0 < L_{H,t} < L_t \)). Suppose, for now, that \( \tau_R = \tau_W = 0 \). Then, solving for the equilibrium boils down to solving one equation, namely (22), for one unknown variable, employment in R&D, \( L_{H,A,t} \).

3.1. Equilibrium R&D Employment. Inspection of (22) provides the following result.

**Proposition 1.** At any time \( t \), the equilibrium employment level in the R&D sector exists and it is positive and unique.

For the proof notice that, due to the assumed positivity constraints on parameters and state variables \( \bar{R} > 0, \delta > 0, \phi \in (0, 1), \alpha \in (0, 1), \lambda \in (0, 1), A_{t-1} > 0, L_{L,t} > 0, \) and \( L_{H,t} > 0 \), the left-hand side (LHS) of Equation (22) is strictly decreasing in \( L_{H,A,t} \), while the...
right-hand side (RHS) is strictly increasing in $L_{H,A,t}$. Furthermore, we have that

$$\lim_{L_{H,A,t} \to 0} LHS = \infty, \quad \lim_{L_{H,A,t} \to 0} RHS = \text{const.} > 0,$$

$$\lim_{L_{H,A,t} \to L_{H,t}} LHS = 0, \quad \lim_{L_{H,A,t} \to L_{H,t}} RHS = \infty.$$  

As a consequence, the equilibrium level of scientists in the R&D sector is positive and unique. Once $L_{H,A,t}$ has been found, we can solve for all other variables.

3.2. Balanced Growth Path. Along the balanced growth path, $a$ has reached its steady state level $\bar{a}$ and the population shares of workers stay constant. The long-run economic growth rate rises if there are more scientists employed in R&D and if these scientists have a higher productivity level ($\delta$), and it decreases with the extent of the duplication externality. Let $L^*_{H,A}$ denote the solution of (22) at $a^*_t = \bar{a}$. The balanced growth rate is obtained from (10) and (11) as $g = \delta(L^*_{H,A})^\lambda$ for $\phi = 1$ and as $g = 0$ for $\phi < 1$. As is well known, when the population and the education level per person stay constant, positive balanced growth exists only for a knife-edge case. Off the balanced growth path, however, the case of zero asymptotic growth is compatible with meaningful adjustment dynamics, as shown below.

3.3. Wage inequality. Inspection of (7) and (8) shows that high-skilled workers enjoy wage growth when the economy is growing (growing $Y_t$). By contrast, wages of low-skilled workers are constant on the balanced growth path because factor shares are constant. This leads directly to the next result.

**Proposition 2.** Technological progress is skill-biased. In an economy populated by high-skilled workers who are complementary to machines and low-skilled workers who are substitutes to machines, higher growth implies higher wage inequality.

The intuition for this result is straightforward. Technological progress raises the productivity of high-skilled workers by the introduction of new machines. At the same time, however, new machines do not raise the productivity of low-skilled workers. Another way to illustrate the effect of technological progress on low-skilled workers is to consider the labor share in aggregate income and to decompose it between high-skilled workers and low-skilled workers.

**Proposition 3.** Along a path of positive balanced growth, the total labor share is declining towards $(1 - \alpha)$. The low-skilled labor share is declining to zero.

For the proof, we compute the labor share as

$$(1 - \alpha) + \frac{w_{LL,t}L_{LL,t}}{Y_t}, \quad (24)$$
where \((1 - \alpha)\) is the high-skilled labor share. We then note that, along a path of positive balanced growth, \(L_{\ell,t}\) and \(w_{\ell,t}\) are constant because population shares are constant, whereas \(Y_t\) is growing at a positive rate. The decline in the labor share in the course of automation is consistent with the empirical evidence for the U.S. pointing out that the labor share was constant until the 1970s, but declined by almost 6 percentage points since then (Karabarbounis and Neiman, 2014).

3.4. Wealth Inequality. The declining relative income of low-skilled labor has, furthermore, a clear inequality-enhancing effect on the distribution of wealth.

**Proposition 4.** In a growing economy without redistribution, the share of wealth held by high-skilled workers increases and converges to one asymptotically. Ceteris paribus, faster economic growth leads to a faster increase of wealth inequality.

For the proof, we insert wages (7) and (8) into savings (3) for \(T_H = T_L = 0\) and obtain relative wealth held by high-skilled workers \(\tilde{s}\):

\[
\tilde{s} = \frac{(1 - \eta)(1 - \alpha)\frac{Y_t}{L_{H,Y,t}}}{(1 - \eta)(1 - \alpha)\frac{Y_t}{L_{H,Y,t}} + \alpha \left(\frac{L_{H,Y,t}}{L_{\ell,t}}\right)^{1-\alpha} \frac{L_{\ell,t}}{L_{H,Y,t}}}. \tag{25}
\]

Along the balanced growth path, the population shares stay constant, while \(Y_t/L_{H,Y,t}\) grows perpetually. This implies that the second term in the denominator becomes gradually less important from a quantitative point of view such that \(\tilde{s}\) converges to 1. Clearly, wealth inequality increases faster when \(Y_t\) grows at a higher rate. Notice that, off the steady state, rising higher education (declining \(L_L\) and increasing \(L_H\)) reinforces wealth inequality during the transition towards the steady state.

In our stylized framework, rising wealth inequality is a product of growing wages of high-skilled workers, stagnating wages of low-skilled workers, and constant saving rates. In a less stylized framework, utility functions could take into account subsistence needs or status concerns in consumption. These mechanisms would, however, further amplify wealth inequality because they imply lower saving rates for the poor. The result of Proposition 4 deviates from the findings of Piketty (2014), who argues that, ceteris paribus, faster economic growth reduces inequality. The reason for this difference is the simplifying assumption of Piketty (2014) that all wages (those of low-skilled and of high-skilled individuals) grow at the same rate, while we allow for differential growth that drives a wedge between the wages of high-skilled workers and the wages of low-skilled workers to the extent that also their savings diverge from one another.
4. The Race between Education and Technology

4.1. Preliminary Considerations. We next consider adjustment dynamics off the steady state and the interaction between education and technology. Qualitatively, it is straightforward to see the impact of technology on education.

Proposition 5. With technological progress, the share of high-skilled labor in the population increases and converges to $\bar{a}$.

The proof is obvious from Proposition 2 and inspection of Equation (5) and Figure 1. As technology advances and more machines are used in production, the wage of high-skilled workers increases relative to low-skilled workers. An increasing skill premium motivates more individuals to obtain higher education. Increasing supply of high-skilled labor leads, ceteris paribus, to relatively higher wages for low-skilled workers [see Equations (7) and (8)], and, thus, suppresses the skill premium. This interplay captures the “race between education and technology” only that, in its original setup, Goldin and Katz (2009) treated education as exogenous, whereas here, education is endogenous and triggered by technological advancements. Given that technological advances are the dominating force, the skill premium is gradually increasing, and a larger share of the society obtains higher education. Ultimately, society converges towards a situation in which a population share of $F(\bar{a})$ remains without higher education because their ability is too low to obtain a college degree.

4.2. Calibration for the United States. To fully assess the interactions in the race between education and technology, we calibrate the model and solve it numerically. In so doing we try to fit U.S. trends for the second half of the 20th century and beyond for TFP growth (Fernald, 2015), the population share with a college degree (U.S. Census, 2019b), the Gini coefficient of before-tax monetary income (U.S. Census, 2018a), and the R&D expenditure to GDP ratio (Ha and Howitt, 2007). We start the computation of adjustment dynamics in the year 1900 and convert the predicted growth rates per generation into annual rates.

We assume that a generation takes 25 years and set $\bar{R} = 1.56$, which implies an annual real interest rate of close to 4 percent. We assume that the time preference rate equals the interest rate and set $\beta = 1/\bar{R}$. Assuming that high-skilled individuals enter college at age 19, complete their education after 5 years and leave the workforce at age 63, we obtain the estimate of $\eta = 0.11 = 5/(63 - 19)$. We assume a parsimonious parameterization of the subutility function $v(a); v(a) = [\theta/(a - \bar{a})]^{\psi}$.

We approximate the ability distribution with the IQ-distribution (truncated at 0). This choice is mostly motivated by data availability and should be broadly interpreted since,
clearly, ability and educational success depend on a host of factors, which are not all correlated with IQ. IQ test scores are approximately normally distributed and, by construction, have a mean of 100 and a standard deviation of 15 (Encyclopedia Britannica, 2019). The implied partial distribution function is 

\[ f(a) = \frac{1}{15\sqrt{2}} e^{-\frac{(a-100)^2}{2 \cdot 15^2}} \]

and the cumulated distribution function is given by

\[ F(a) = 0.5 \left[ 1 + \text{erf} \left( \frac{a - 100}{15 \sqrt{2}} \right) \right], \]

in which \( \text{erf} \) denotes the error function. We set \( \bar{a} \) to 0.5, implying that – in the limit – half of the population obtains a college degree. While college graduation is highly correlated with IQ, the correlation is not perfect and there are likely some individuals with an IQ score below 100 who graduate from college. On the other hand, many individuals with an IQ above 100 do not obtain a college degree due to credit constraints, family background, illness, and a host of other reasons including unfavorable non-cognitive skills or just lack of interest. These factors are not represented in our stylized model, which assumes a strict division of educational attainment of society by ability. Hence, our macroeconomic calibration target is not a precise picture of the stratification of society but “only” to get the population share of college graduates about right.

Another possibility to motivate the ability curve and its threshold is through the supply side of education. A crucial determinant of college admission are SAT scores, which are also about normally distributed and strongly correlated with IQ such that formulas have been suggested to convert SAT scores to IQ (e.g. Frey and Dettermann, 2014). The admission threshold is college-specific and higher SAT scores are needed for admission at more selective universities. Top colleges aim for SAT scores of 1400 or higher. Less selective public institutions and small liberal arts colleges accept applicants in the 950-1050 SAT score range but it is very unlikely to enter college with a score of 700 or below (McCammom, 2016). Using Frey and Dettermann’s (2014) conversion formula, a SAT score of 700 is converted into an IQ of 105.

We set population size (arbitrarily) to 1,000 and then calibrate the initial technology level \( A(0) \), the technology parameters \( \alpha, \delta, \phi, \) and \( \lambda \) and the education parameters \( \psi \) and \( \theta \) to fit the following seven calibration targets: a roughly constant TFP growth rate of about 2 percent (Fernald, 2015); an R&D share that increases from 0.8% in 1953 to 1.6 percent at the end of 20th century (Ha and Howitt, 2007); a share of college graduates among the male population that rises from 12.8% in 1967 to 33.7% in 2017 (US Census, 2018a); a Gini coefficient of income that rises from 40% in 1967 to 48% in 2017 (US Census, 2018b). This leads to the estimates \( \alpha = 0.63, \phi = 0.70, \psi = 0.0625, \theta = 8 \cdot 10^8, \lambda = 0.2, \delta = 1.5, \) and
$A(0) = 14$. In the benchmark case, there is no redistribution such that $\tau_L = \tau_R = 0$. The parameters used in the calibration are provided below Figure 2.

**Figure 2. Adjustment Dynamics of the Model and Observed Data for the United States**

Solid lines in Figure 2 refer to the predicted adjustment dynamics. Dashed lines refer to the underlying data. The first panel shows that the calibration supports an almost constant and eventually mildly falling trend of TFP growth during the 20th and 21st century. In the second panel, we see how the rising skill premium induces an increasing share of the population to acquire higher education. The model approximates the actual increase of college graduates well. It predicts that the population share of high-skilled workers reaches its steady-state level $(L_H^A/L)^* = (1 - \eta)\bar{a} = 0.45$ in the middle of the 21st century.

The middle panel in Figure 2 shows that the model predicts a somewhat too fast increase of R&D intensity during the 20th century. On the other hand, one could argue that TFP growth at the end of the 20th century is overestimated by the model. These shortcomings
are partly due to the simple structure of the model, which provides no degrees of freedom to adjust TFP growth independently from investment in R&D. Another reason is that TFP growth is an imperfect measure of technological progress.

The fourth panel shows that the model matches the increasing trend in inequality well. The Gini coefficient is easily obtained as \( s_w - L_{H,t}/L_t \), where \( s_w \) is the income share of high-skilled individuals, \( s_w \equiv w_{H,t}L_{H,t}/(w_{H,t}L_{H,t} + w_{L,t}L_{L,t}) \). As the economy grows, the income share of high-skilled individuals converges to 100\%, whereas the share of high-skilled workers converges toward 45\%. Hence the Gini coefficient gradually rises towards its steady-state level of 55\%. The computational experiment clearly refutes the view that declining or constant productivity growth is incompatible with increasing automation and increasing inequality. As explained above, for these trends to be simultaneously observed, we only need positive TFP growth, i.e., further innovation in automation. In the Appendix, we show that these transitional dynamics are fully robust to changes in the parameters even to the extent of the knife edge case \( \phi = 1 \).

4.3. Redistributive Taxation. Since only high-skilled individuals benefit from technological progress, it seems appropriate to support the losers in the race between education and technology by means of redistribution and taxation. Aside from redistribution through progressive income taxation, the implementation of a robot tax has recently been proposed. However, when technology and education are endogenous, redistributive taxation can entail less benign outcomes in a dynamic context than suggested by thought experiments that treat education and technology as given. In a fully dynamic context, redistribution has the potential to be counterproductive such that it does not help to improve disposable income of low-skilled workers. Before we discuss redistribution in the calibrated economy, we develop some intuition for this result by means of an analytical discussion.

**Proposition 6.** As long as \( a_t^* > \bar{a} \), an increasing wage tax or an increasing transfer to low-skilled labor reduces education.

The proof is obtained by differentiating the wage premium in monetary terms, \( \hat{w} \equiv [(1 - \tau_w)(1 - \eta)w_{H,t} + T_H] / [(1 - \tau_w)w_L + T_L] \), for the case where \( T_L \geq T_H > 0 \). Intuitively, higher taxes on wage income or higher transfers \( T_L \) reduce the incentive to increase wages through higher education. In a dynamic context, this means that along the transition path, i.e., as long as the low-skilled labor share has not yet settled at \( F(\bar{a}) \), redistribution slows down the take up of higher education, and thereby innovation and economic growth.

\(^5\) If the ability threshold \( \bar{a} \) were (much) smaller than 100, the high-skilled elite would eventually comprise a majority of society and an increasing wage premium would lead to declining inequality as measured by the Gini coefficient. Other measures of inequality, like, for example, the skill premium or the absolute Gini coefficient (Chakravarty, 1988) would continue to rise.
The response of employment in R&D (and thus economic growth) to an increase in the robot tax can be discussed with the help of the equilibrium condition (22). From visual inspection, the response is seemingly ambiguous. On the one hand, an increase of $\tau_R$ leads to less machine production, lower profits in the machine producing sector, a lower equilibrium price for the blueprints of new machines, and, thus, lower wages and employment in R&D. Formally, this implies that the left-hand side of (22) declines. On the other hand, the reduction of machine input leads to lower wages for high-skilled workers in goods production and therefore to a shift of high-skilled labor towards R&D. Formally, this implies that the right-hand side of (22) declines. By implicitly differentiating (22), it can be shown that the negative effect through profits in the machine sector dominates.

**Proposition 7.** An increasing robot tax $\tau_R$ leads to less employment in R&D and slower growth.

The proof of Proposition 7 is stated in the Appendix. While the static responses of education and innovation to redistributive taxation can be assessed analytically, the impact on low-skilled income and income inequality can only be assessed numerically in the dynamic equilibrium. For that purpose, we take the model as calibrated in Figure 2 and assume a redistributive policy reform in the year 2025. We focus on the case in which only low-skilled individuals receive income transfers, i.e., $\kappa = 1$ such that $T_L = T$ and $T_H = 0$. The results are shown in Figure 3. To facilitate comparisons, solid (blue) lines replicate the benchmark case without redistribution from Figure 2.

Adjustment dynamics for a robot tax of $\tau_R = 0.1$ are shown by green (dashed) lines in Figure 3. Confirming Proposition 7, the robot tax depresses R&D and TFP growth (top panel). As a result, wages for high-skilled labor increase at a lower rate, which causes a slower high-skilled labor expansion (second panel). Redistribution reduces the high-skilled labor supply also in the long-run. Since, in the limit, transfers are growing at the rate of technological progress, some individuals with abilities above but close to $\bar{a}$ are permanently discouraged from obtaining a college degree. In the limit, $L_H/L$ converges to 0.436 (instead of 0.45).

Because the population share of rich college graduates expands slower with rising technology, pre-tax inequality increases (fourth panel) such that, in the short run, redistribution has hardly any effect on after-tax inequality (bottom panel). As time passes, however, the effect of redistribution dominates the general equilibrium effect on gross wages. In the limit, transfers $T_L$ grow at the rate of high-skilled income, whereas low-skilled wages stagnate such that, in the long run, low-skilled income is dominated by transfers, $\lim_{t \to \infty} w_L/T_L = 0$. In...
the long-run, the after-tax Gini coefficient settles at a lower level, because, through redistribution, low-skilled individuals participate in the gains from automation and economic growth.

Alternatively, transfers could be financed by a labor income tax. Red (dash-dotted) lines in Figure 3 show the outcome for $\tau_w = 0.1$ (and $\tau_R = 0$). In contrast to the robot tax, the wage tax has a direct effect on education and an indirect effect on technological progress and growth. Lower net wages reduce the gains from education such that higher education increases at a lower rate with economic development. As a result of reduced high-skilled labor supply, there is less employment in R&D, and thus less innovation and lower productivity growth. The labor income tax reduces the progress of technology and education similarly to the robot tax and it therefore has similar consequences for the adjustment dynamics of low-skilled income and inequality (panels 2 to 5 in Figure 3).
Next, we consider an education subsidy implemented in the year 2000. This conforms to the case of $\kappa = 0$ such that all transfers are channeled to high-skilled individuals. The results are shown in Figure 4, where again blue (solid) lines replicate the benchmark run from Figure 2, green (dashed) lines reflect the case of $\tau_R = 0.1$ and red (dash-dotted) lines reflect the case of $\tau_W = 0.1$.

**Figure 4. Adjustment Dynamics: Education Subsidy ($\kappa = 0$)**

![Figure 4](image)

Parameters as for Figure 2: blue (solid) lines: $\tau_R = \tau_W = 0$ (replication of Figure 2); policy reform in 2000: green (dashed) lines: $\tau_R = 0.1$; red (dashed-dotted) lines: $\tau_W = 0.1$.

In the medium run, an education subsidy reduces pre-tax inequality but through different channels, depending on the mode of finance. If the education subsidy is financed by a labor income tax, it reduces inequality because it encourages education. The supply of high skill labor increases and the supply of low skill labor declines such that the wage premium and inequality decline, compared to laissez faire. In the long run, however, education and high-skilled labor supply converge towards their steady state levels and the education subsidy is
pure redistribution from the poor to the rich. Consequently, after-tax inequality is higher in the long run (red dash-dotted lines in Figure 4).

The education subsidy financed by the robot tax fails to boost education (green dashed lines in Figure 4). As explained above, the robot tax reduces the demand for machines and innovations and slows down TFP growth (top panel). The effect from reduced high-skilled income through lower growth dominates the positive direct effect of the transfer on education such that the supply of high-skilled labor increases at a slightly lower rate than in the benchmark run (second panel). As a result of the robot tax and the lower supply of high-skilled labor, the incentive for R&D declines and the R&D intensity grows at a lower rate than in the benchmark case.

5. Automation, Fair Wages, and Involuntary Unemployment

In principle, there are several gateways for rising unemployment due to automation in such a framework. While integrating a labor market with search and matching frictions and automation (see Cords and Prettner, 2019; Guimarães and Mazeda Gil, 2019) would complicate the model substantially, we introduced unemployment via the social welfare system and a reservation wage in an earlier version of this paper (Prettner and Strulik, 2017). The notion that automation causes voluntary unemployment through frictions on the labor supply side is, however, not entirely compelling. In the following, we integrate involuntary unemployment based on Akerlof and Yellen’s (1990) fair wage theory into our model. Akerlof and Yellen motivate the fair wage theory from different perspectives, including equity theory, relative deprivation theory, and social exchange theory, which all seem particularly relevant in the present context, where only one group of workers benefits from technological progress.

The basic idea is that workers compare their payment with that of coworkers and exert full effort at work only when they perceive their remuneration as fair. Specifically, effort at work $e_j$ is given by $e = \min(w_j/w_j^*, 1)$, in which $w_j^*$ is the wage that is perceived as fair by workers of group $j$. In our context, fair wage considerations are made only in the final goods sector because this is the sector in which high-skilled workers and low-skilled workers meet. In equilibrium, high-skilled workers receive the same wage in R&D as in goods production. To include effort considerations, we rewrite the goods production function such that $e_j$ is taken into account:

$$Y_t = \left( e_{H,t} \cdot L_{H,Y,t} \right)^{1-\alpha} \left[ (e_{L,t} \cdot L_{L,t})^\alpha + \sum_{i=1}^{A_t} x_{i,t}^\alpha \right].$$
Following Akerlof and Yellen, we assume that the fair wage of group \(j\) is a weighted average of the wage received by the reference group and the market-clearing wage

\[
\begin{align*}
    w^*_i, t &= \mu w_i, t + (1 - \mu) w^c_i, t, \\
    w^*_j, t &= \mu w_j, t + (1 - \mu) w^c_j, t,
\end{align*}
\]  

(26)

where \(w^c_j, t\) is the market clearing wage of group \(j\) and \(\mu\) measures the strength of wage comparisons. The market-clearing wage is defined as the wage that would clear the market for group-\(j\) labor in the neoclassical case (i.e., when workers unconditionally exert full effort).

Let \(L_j, t\) denote labor supply of group \(j\). We then have

\[
\begin{align*}
    w^c_{H, t} &= (1 - \alpha)(L_{H, t})^{-\alpha} (\bar{L}_{L, t} + \bar{A}_t x^\alpha_{L, t}), \\
    w^c_{L, t} &= \alpha L_{H, t}^{1 - \alpha} (\bar{L}_{L, t})^{\alpha - 1},
\end{align*}
\]  

(27)

where we made use of the shorthand notation \(\bar{A}_t\) according to (17). In principle, multiple equilibria are possible on the labor market. Here, we follow Akerlof and Yellen (1990) and consider the so-called integrated equilibrium as the most relevant one. At this equilibrium, all workers exert full effort. High-skilled workers receive a wage that exceeds the fair wage and full employment results such that \(L_{H, t} = \bar{L}_{H, t}\). Low-skilled workers receive a fair wage that exceeds the market-clearing wage and low-skilled unemployment results. The indirect demand function for low-skilled labor is given by

\[
w_{L, t} = \alpha (L_{H, t})^{\alpha - 1} \equiv L_D(w_{L, t}).
\]  

(28)

Inserting (27) into (26) and using \(w_{H, t} = w^c_{H, t}\) provides the “fair wage constraint”, i.e., the wage at which low-skilled workers exert full effort:

\[
w_{L, t} = \mu (1 - \alpha)(L_{H, t})^{-\alpha} (\bar{L}_{L, t} + \bar{A}_t x^\alpha_{L, t}) + (1 - \mu)\alpha (L_{H, t})^{1 - \alpha} (\bar{L}_{L, t})^{\alpha - 1} \equiv F_w(w_{L, t}).
\]  

(29)

Labor market equilibrium prevails when (28) and (29) hold simultaneously. It is depicted in Figure 5. Labor demand according to (28) is represented by the falling curve \(L_D\). The upward-sloping curve \(F_w\) represents the fair wage constraint. Low-skilled workers perceive it as fair to earn more as their employment rises. The full-employment constraint is represented by the vertical \(\bar{L}_L\) line. The initial labor market equilibrium is obtained at \(L_{L, 0}\) and initial unemployment is obtained as \(\bar{L}_L - L_{L, 0}\).

The implications of the race between education and technology for unemployment can be qualitatively deduced from Figure 5. Technological progress (rising \(\bar{A}_t\)) shifts the fair wage constraint upwards to \(F_{w, 1}\). Low-skilled workers demand a higher wage because they notice that wages for high-skilled workers rise with increasing automation. Due to the rising reference level, they perceive a higher wage for themselves as fair. Intuitively, low-skilled workers want to take part in the advances from technology, although their labor productivity does not improve. Without a pay rise, however, labor productivity would deteriorate due to declining effort. The new equilibrium employment level is at \(L_{L, 1}\).
Technological progress shifts the fair wage constraint $F_W$ upwards and the low-skilled workforce $L_L$ inwards.

Technological advances, however, also trigger higher education such that the workforce of low-skilled individuals declines. The workforce constraint shifts to the left to $\bar{L}_{L,1}$. Taking both moves together, it is ambiguous as to whether low-skilled unemployment, given by $\bar{L}_L - L_L$, increases or decreases due to high-skilled labor-augmenting technological progress. However, once education has converged to its upper bound, only the effect of technology on fair wages remains effective and low-skilled unemployment increases inevitably as automation technology advances.

**Proposition 8.** Once higher education converged towards its upper bound, more innovation and faster economic growth lead to more involuntary unemployment.

The proof is obvious from Figure 5. When education is stationary, the vertical $L_L$-line does not move. More innovation leads to a faster increasing TFP level $\bar{A}$ and a faster upward movement of the fair wage constraint. As a result, low-skilled wages increase at a higher rate and employment declines faster. The result is intuitive. More innovation and higher growth of high-skilled wages leads to a faster increase of the reference level for fair wage considerations and low-skilled workers need a faster increase of wages to induce full effort. Since low-skilled productivity does not rise, firms respond with reduced demand for low-skilled labor.

Finally, we consider the evolution of unemployment along the transition. For that purpose, we employ the model as calibrated for Figure 2 and set $\mu$ such that fair wage considerations generate an unemployment rate of 5 percent at the end of the 20th century. This leads to the estimate of $\mu = 0.004$. The importance of the reference wage is thus estimated to be
Figure 6. Adjustment Dynamics: Unemployment

Parameters as for Figure 2 and $\mu = 0.004$: blue (solid) lines: $\tau_R = \tau_W = 0$; green (dashed) lines: $\tau_R = 0.1$; red (dashed-doted) lines: $\tau_W = 0.1$.

relatively small. Since fair wage concerns reduce economic growth, we increase the initial value $A(0)$ to 25 in order to match the calibration targets for education and inequality in the 20th century. Figure 6 shows the implied adjustment dynamics.

Blue (solid) lines in Figure 6 reflect the case without income redistribution. The bottom panel shows that unemployment actually declines mildly during the 20th century, indicating that the effect of reduced low-skilled labor supply (the inward movement of the $T_L$-curve in Figure 5) dominates the effect of reduced labor demand due to technological progress and rising reference levels of the fair wage. Eventually, however, before labor supply becomes stationary, unemployment begins to rise as the levels of TFP and wages of high-skilled individuals increase further. Due to the fair-wage constraint, low-skilled wages benefit, at least partially, from technological progress and economic growth. The higher low-skilled
wages motivate some individuals close to the ability threshold $\bar{a}$ to abstain from tertiary education. Reduced skilled-labor supply leads to less R&D and lower growth.

The impact of fair wage constraints on inequality is, in principle, ambiguous. In the third panel of Figure 6 we see that fair-wage considerations increase inequality for the calibrated model by about 4 percentage points. The inequality dampening effect from growth participation of unskilled workers is dominated by the inequality boosting effects of (i) the presence of a group that receives no labor income, (ii) a smaller population share of high-skilled workers who benefit fully from economic growth.

We next reconsider the effects of redistributive taxation and assume that individuals compare gross wages and not their disposable income after redistribution. We assume that all transfers go to low-skilled labor and again consider a 10 percent robot tax (dashed, green lines) and a 10 percent labor income tax (dash-dotted, red lines). The response of TFP growth and education to redistribution is familiar and analyzed in detail in the discussion of Figure 3. Here, it is interesting to see that redistribution increases unemployment. This response is not driven by labor supply considerations of low-skilled individuals because transfers are granted irrespective of the employment status and labor supply is inelastic by assumption. Instead, low-skilled unemployment rises because of the depressing effect of redistribution on education. As a result of reduced high-skilled labor supply, high-skilled wages are higher at any level of technology, and thus low-skilled individuals consider a higher wage as fair. In order to elicit full effort, firms pay higher wages and reduce labor demand.

6. Conclusion

In this paper, we propose a model of endogenous technological progress and economic growth according to which R&D-based innovations in machine technology lead to more automation, a higher skill premium, and more inequality in terms of income and wealth. The model predicts that more sophisticated technology induces more education but only to a certain degree because, eventually, some individuals will be left behind who do not manage to obtain a college degree due to ability constraints. The feature that low-skilled labor does not benefit from automation creates rising inequality because the wages of high-skilled individuals increase at the rate of technological progress. Considering the other big race mentioned in the introduction, the model suggests that it could be difficult and eventually impossible to “run with the machine” (as suggested by Brynjolfsson and McAfee, 2011).

Similar to the related R&D-based growth literature, we focus on the manufacturing sector, which, in principle, leaves the loophole that non-routine, low-skilled labor finds employment in an expanding service sector. However, one could argue that the service sector is represented in reduced-form in our model. According to the aggregate production function,
low-skilled labor is not benefiting from automation, but it is also not made redundant by the arrival of new machines. It could be conceptualized as moving to different tasks. Moreover, in general equilibrium, low-skilled labor benefits indirectly from technological progress. Skill-biased technological progress induces more higher education in the population, and thus a relative decline of low-skilled labor supply and an increase of low-skilled wages. Of course, this positive indirect effect vanishes when technology no longer triggers increasing higher education. Likewise, the model could be refined by assuming that more recent vintages of machines are able to substitute, to an increasing degree, high-skilled labor.

In our overlapping generations setting, increasing wage inequality explains a secular decline of the aggregate saving rate and a secular increase of wealth inequality. These effects are stronger at higher rates of technological progress. Our theory therefore contrasts with the conventional view that high economic growth is conducive to lower inequality (Piketty 2014), which is mainly due to the fact that there is only one type of labor in the framework of Piketty (2014). We have also shown that it is difficult to improve income of low-skilled individuals as long as both technology and education are endogenous. This is true irrespective of whether redistribution is financed by progressive wage taxation (which reduces higher education and growth through lower high-skilled labor supply) or by a robot tax (which reduces demand for machines and growth through less R&D). Designing redistribution policies that circumvent the repercussions through adjustments of education and technology appears to be a serious challenge for the future. Only when higher education is stationary, does redistribution unambiguously benefit the poor. In this situation, redistribution schemes similar to the ones sketched out in our contribution might be needed to ensure that those who lost the race against the machine would nevertheless share in some of the gains from automation.

In an extension of the model, we consider the impact of automation on involuntary unemployment of low-skilled workers through fair wage constraints. This approach appears to be natural when automation benefits only one group at the workplace. If workers refuse to exert full effort when they are not allowed to share in the gains from technological progress, unemployment results. Interestingly, as long as higher education is non-stationary, technological progress does not necessarily lead to more technological unemployment. The reason is that it also triggers more higher education, and thus reduces the low-skilled workforce. For brevity, we focused on the integrated equilibrium of the fair wage model. In future work, it could be interesting to investigate segregated equilibria as well, i.e., conditions where automation under consideration of fair wage and effort concerns motivates firms to abandon the employment of low-skilled labor. It would also be interesting to investigate a case in
which workers compare disposable income rather than wages, which could motivate the introduction of a universal basic income or other redistribution schemes from the winners to the losers of the race between education and technology.
Appendix

Proof of Proposition 7. To analyze the impact of education on R&D and economic growth we re-write (22) as the implicit function

\[ F \equiv \frac{\bar{R}}{\alpha} \left[ \frac{\alpha^2}{\bar{R}(1 + \tau_R)} \right]^{1/\alpha} \left( L_{H,t} - L_{H,A,t} \right) \frac{\delta A_{t-1}^\phi}{L_{H,A,t}^{1-\lambda}} - \left( \frac{L_{L,t}}{L_{H,t} - L_{H,A,t}} \right) - \tilde{A}_t \left[ \frac{\alpha^2}{\bar{R}(1 + \tau_R)} \right]^{1/\alpha} = 0. \]

Differentiation of \( F \) provides and \( \frac{\partial F}{\partial L_{H,A,t}} < 0 \) and

\[
\frac{\partial F}{\partial (1 + \tau_R)} = -\frac{1}{1 - \alpha} \frac{1}{1 + \tau_R} \bar{R} \left[ 1 - \alpha \frac{\alpha^2}{\bar{R}(1 + \tau_R)} \right]^{1/\alpha} \left( L_{H,t} - L_{H,A,t} \right) \frac{\delta A_{t-1}^\phi}{L_{H,A,t}^{1-\lambda}} + \frac{1}{1 - \alpha} \frac{1}{1 + \tau_R} \tilde{A}_t \left[ \frac{\alpha^2}{\bar{R}(1 + \tau_R)} \right]^{1/\alpha} < 0
\]

and thus, by the implicit function theorem, \( \frac{dL_{H,A,t}}{d\tau_R} < 0 \). Then, from (10) and (11), \( \frac{dg_{A,t}}{d\tau_R} < 0 \).

Robustness and Positive Steady-State Growth. This section shows that the general results of our model are robust to re-specifications of the parameters, in particular, to having positive growth in the long run. The results are shown in Figure 7, where the parameter values used for this simulation are reported below the figure. Blue (solid) lines reflect adjustment dynamics of the benchmark case. As the economy grows and skill-biased technological progress unfolds (first panel), more individuals are motivated to acquire a college education (second panel). The increase in college graduates renders high-skilled labor less scarce and more high-skilled labor is allocated to R&D (third panel). This, in turn, further amplifies technological progress such that the economy takes off with initially increasing growth rates. After a while, however, the stepping-on-toes effect becomes noticeable and the gain in growth rates levels off as the economy adjusts towards the steady state.

The bottom panel of Figure 7 shows inequality measured by the Gini coefficient of wage income. To show our “non-Pikettarian” result, we next increase \( \delta \) to 0.5 (from 0.43) and keep all other parameters and initial values from the benchmark run. The results are shown by dashed (red) lines in Figure 7. Due to the higher productivity in R&D, the alternative economy grows at a faster rate, initially and everywhere along the adjustment path (panel 1). The higher rate of skill-biased technological progress induces faster growth of income.
for the high-skilled population and triggers more education (panel 2). A better educated workforce provides more labor supply for R&D (panel 3), which further spurs innovation and economic growth. Since low-skilled labor is not benefiting from these trends, inequality increases faster in the high-growth economy (panel 4). Individuals who suffer from ability constraints in learning and fail to achieve college graduation are left behind earlier than in the benchmark run.


