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SOEPpapers

on Multidisciplinary Panel Data Research

Do justice perceptions support the concept of equal sacrifice? Evidence from Germany

Maria Metzing

Do justice perceptions support the concept of equal sacrifice? Evidence from Germany*

MARIA METZING

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Abstract

The ability-to-pay approach assesses taxes paid as a sacrifice by the taxpayers. This raises the question of how to define and how to measure it: in absolute, relative, or marginal terms? U.S. respondents prefer a tax schedule that is either a pure (absolute) Equal Sacrifice or a mixture of Equal Sacrifice and Utilitarianism [Weinzierl, 2014]. To determine whether Germans prefer absolute, relative, or marginal Equal Sacrifice principle for their income taxation, I use a question item from the German Socio-Economic Panel (SOEP) to obtain information on the level of taxes individuals consider as fair. I estimate tax and transfer schedules with regard to three Equal Sacrifice definitions and analyze which one of the three best fits the data. The absolute and the relative Equal Sacrifice principle are the dominant candidates in terms of statistical fit.

JEL Classification: H21, D63

Keywords: Equal Sacrifice, Optimal Taxation, Fair Taxation

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1 Introduction

How we devise a fair tax schedule? According to Adam Smith [1776] the tax burden should depend on two principles of fairness: On the one hand, the tax burden should be calculated based on the benefits received - the so called *benefit principle*. People who benefit more from negative externalities, like pollution from their cars, should also pay more tax e.g. a fuel tax. On the other hand, the tax burden should also depend on the *ability-to-pay principle*: individuals with high ability should pay higher average tax rates than individuals with low ability.

Mill [1848] defined on the basis of the ability-to-pay approach the Equal Sacrifice principle. People with the same ability-to-pay should pay the same amount of taxes (horizontal equity) and the tax payment should rise with the ability to make an income (vertical equity). This raises the question of how the sacrifice should be measured. Three principles were therefore defined (see Musgrave and Musgrave [1973], Richter [1983], Young [1988]): (1) Absolute Equal Sacrifice (AES) is satisfied if everyone gives up the same amount of utility in remitting taxes. (2) Relative Equal Sacrifice (RES) is satisfied if everyone sacrifices the same percentage of utility in remitting taxes. (3) Marginal Equal Sacrifice (MES) is satisfied if the first derivative of the utility in paying taxes is the same for everyone.

Researchers, such as Young [1990] or Weinzierl [2014], use the Equal Sacrifice criteria to define the objective function of the social planner as an alternative to welfarism [Mirrlees, 1971].¹ For instance, Young [1990] finds that the U.S. tax schedule is in line with the absolute Equal Sacrifice principle.

But what do individuals consider fair when it comes to income taxation? When asked U.S. individuals directly, many prefer tax schemes that fit the Equal Sacrifice principles. Weinzierl [2014] let individuals choose between different taxation alternatives. Most respondents preferred a tax schedule that confirms either an absolute Equal Sacrifice or a mixture of absolute Equal Sacrifice. Existing studies used U.S. data, so that I am the first to employ German data.

In this paper, I examine a related research question: Do stated preferences on fair net and fair gross income confirm one of the three Equal Sacrifice

¹In the literature, optimal tax theory commonly assumes a utilitarian objective function [Mirrlees, 1971]. However, a number of alternative approaches are proposed in the literature: the Rawlsian Criterion, the Libertarian Principle, and Equal Opportunity (see Piketty and Saez [2013], Weinzierl [2014], Saez and Stantcheva [2016], Jessen et al. [2017]).

principles?² To identify if individuals' preferences are in line with one of the three Equal Sacrifice principles, I impose the CRRA (constant relative risk aversion) utility function to structure individuals' utility and check against the three sacrifice theorems. As the ability-to-pay differs for different household types, the CRRA utility function here depends on the *equivalized* gross and net income [Ebert and Moyes, 2000]. For the analysis, micro data on fair gross and net income is required. Therefore, I use question items from the German Socio-Economic Panel (SOEP) on fair perceived gross and net income in order to construct a social security and (income) taxation schedule on the basis of three Equal Sacrifice principles. One huge advantage is that respondents do not need any priming on optimal taxation theory. They only answer about what they think is a fair gross and a fair net income. A function of this difference is interpreted as the fair sacrifice and can be checked against the Equal Sacrifice principles.

Which of the Equal Sacrifice principle fits best is empirically assessed by the R^2 of the Equal Sacrifice tax schedule and its Mean Square Error (MSE), which indicates the deviation between the fitted and observed data points. I find that the principle of AES and RES yield the best fit by the fit statistics and, graphically, a remarkable fit is obtained. I also find that a fair tax schedule should be progressive.

The paper is structured as follows. Section 2 describes the theoretical framework, Section 3 gives an overview about the data and provides further statistics. In Section 4, I test AES, RES, and MES theories, while Section 5 concludes.

2 Theoretical Framework

On the basis of the ability-to-pay principle, Mill [1848] defined the rule of Equal Sacrifice, which imposes that all taxpayers have to bear the same sacrifice or the same reduction in welfare. The loss in welfare is related to a reduction in income and, hence, the welfare function depends on incomes in this context. If the level of welfare - as a function of income - is the same for all taxpayers, the Equal Sacrifice rule requires that individuals with the same ability-to-pay have to pay the same taxes.

To apply this, Equal Sacrifice requires two main assumptions: First, utility is cardinal, so that the absolute value and relative differences between the utilities are measurable. This assumption is indispensable for the interpretation of the sacrifice that is calculated in terms of utility. Second, the utility

²Since studying how well a utilitarian approach would fit requires a fundamentally different approach and is beyond the scope of this study.

function of equalized incomes is identical for all individuals. People with the same ability-to-pay have the same utility and therefore, should pay the same amount of taxes (horizontal equity). Moreover, the tax payment should rise with the ability-to-pay (vertical equity). The statement of vertical equity is subject to controversial discussions because it is not clear how high the tax burden for those with high incomes should be. Therefore, the definition of Equal Sacrifice is important as well as the function of the utility of income. First, I discuss three concrete definition of Equal Sacrifice (see Subsection 2.1) and second, I define the utility function of income (see Subsection 2.2).

2.1 Equal Sacrifice Theories

As described above, in the literature three Equal Sacrifice principle are discussed. Sidgwick [1883], defines the tax burden as the absolute level of sacrifice: every tax payer has to bear the same absolute sacrifice meaning that the loss in utility is equal for all individuals.

To reach the same absolute loss in utility for all individuals, the government revenue is divided as long as the utility loss due to taxation for all types is equal. The size of the tax burden depends on the assumption of the marginal utility of income. Having constant marginal utility leads to the same tax burden for all individuals, whereas decreasing marginal utility leads to a tax schedule increasing in income. Richter [1983] formally denotes AES as:

$$U(Y_i) - U(Y_i - T_i(Y_i)) = s_A \quad \forall i \in \{1, \dots, N\} \quad (1)$$

where the absolute difference between the utility before $U(Y_i)$ and after tax $U(Y_i - T_i)$ is equal to the sacrifice s_A . s_A is constant for all taxpayers. Y_i represents gross income, T_i the tax burden and $Y_i - T_i$ net income for individual i . Whether a tax schedule is regressive, proportional or progressive depends on the elasticity of the marginal utility of income with respect to the income. An elasticity above one indicates a regressive, equal to one a proportional, and below one a progressive tax schedule.

In contrast to AES, RES is defined as a sacrifice concept that is proportional to the taxpayers' gross income. The government revenue is divided as long as the relative utility loss is equal between all individuals. Richter [1983] formalizes RES as:

$$\frac{U(Y_i) - U(Y_i - T_i(Y_i))}{U(Y_i)} = s_R \quad \forall i \in \{1, \dots, N\} \quad (2)$$

and sacrifice s is the difference between the relative utility functions

of gross and net income proportional to the gross income. As for AES, taxation can be regressive, proportional, or progressive for RES. Constant marginal utility leads to a proportional tax schedule, whereas decreasing linear marginal utility leads to a progressive tax schedule. A generalization for a marginal utility function with a decreasing rate is difficult. The result depends on the level and slope of the marginal utility function, the initial income level, and the intended government revenue.

Traditional economic theory focuses on the overall welfare that depends on the utilities of all individuals and not on justice of fairness (see e.g. Musgrave [1959, 2005]). With regard to the traditional economic theory, Edgeworth [1897] formalized the social welfare function where all individuals have the same concave increasing utility function and income is fixed. The government chooses the tax burdens T_i to maximize the utilitarian social welfare function W subject to the budget constraint $\sum_i T_i = R$. The government revenue is now divided as long as the marginal sacrifice, or marginal utility of income, for all individuals is equal. The assumption of the same utility function for all individuals leads to $U'_i(X_i) = U'(X_i)$ and results in the same income after tax X_i for all individuals in the optimum i.e. for all non-linear utility functions applies $Y_i - T_i = Y_j - T_j, \forall i, j$.³ To sum up, social welfare is maximized if net income have the same size for all individuals and total sacrifice is minimized.⁴ As a result, a decreasing function of marginal utility of income⁵ that requires the same sacrifice leads to a maximal progressive tax schedule - a marginal tax rate of 100 %. In this case, performance would never be rewarded and regardless of individual performance, income is equal for all. This is not a realistic case and assumes that MES is presumably not considered fair. Richter [1983] denotes MES by

$$U'(Y_i - T_i(Y_i)) = s_M \quad \forall i \in \{1, \dots, N\} \quad (3)$$

describing the marginal utility function of net incomes.

To sum up, MES produces the highest tax burden for the high income earner and the lowest tax burden for low income earner if utility is decreasing.

³Consequently, an average earner is taxed to the necessary extend to finance government revenue by $T_i(\bar{Y}) = \delta * \bar{Y}$ where δ describes the rate of government revenue. For all others, the tax is calculated by the deviation from the average income plus the tax burden of the average income type: $T_j(Y_i) = (Y_j - \bar{Y}) + \delta * \bar{Y}$.

⁴The leveling of income starts at the top until the needed government revenue is reached. For illustration, two tax-payers, 1 and 2, where 2 earns twice as much as 1. If 1 pays 100 Euro, 2 pays 200 Euro tax and the MES for 2 is much lower than for one. In that case we reduce the tax amount of 1 by 10 Euro and increase it with the same amount for 2. The reduction of equal sacrifice is much greater for 1 than the increase for person 2. With the same amount of taxes collected, we have a decreased sum of MES.

⁵The rate of the decreasing marginal utility is not important.

If AES or RES rule is more progressive than the RES for the low (high) income type, depends on the definition of the utility function, the initial income level, and the intended government revenue. However, AES and RES create more realistic tax schedules than MES.

2.2 Utility Function of Income

The above definitions rest on the concept of a utility function. In the literature, the function of constant relative risk aversion (CRRA) is the most common utility function [Young, 1988, Berliant and Gouveia, 1993, Weinzierl, 2014]. Constant relative risk aversion entails that one would spend the same share in risky assets with increasing available money.⁶ Researchers as Friend and Blume [1975] or Chiappori and Paiella [2011] show that CRRA is a good approximation as utility function for individuals. Thus, I define:

$$U(Y_i) = \frac{Y_i^{1-\varepsilon} - 1}{1-\varepsilon} \text{ so that } \varepsilon = -\frac{Y_i U''(Y_i)}{U'(Y_i)} \quad \forall i \in \{1, \dots, N\} \quad (4)$$

where ε stands for relative risk aversion, also known as Arrow-Pratt measure, and is constant [Pratt, 1964, Arrow, 1971]. For $\varepsilon=1$, CRRA is defined as:

$$U(Y_i) = \ln(Y_i). \quad (5)$$

Furthermore, the risk aversion parameter ε can be also interpreted as an inequality aversion parameter [Atkinson, 1970]. The preference for redistribution is increasing in ε .

3 Data

To examine if individuals in Germany prefer a tax schedule according to one of the Equal Sacrifice principles, I use a newly introduced question asked since 2015 (every two years) on the German Socio-Economic Panel (SOEP) [Goebel et al., 2018], where respondents are asked whether they consider their

⁶Other utility functions can belong to the classes: IRRA (increasing relative risk aversion), DRRRA (decreasing relative risk aversion), IARA (increasing absolute risk aversion), DARA (decreasing absolute risk aversion), and CARA (constant absolute risk aversion).

individual gross and net labor incomes to be fair.⁷ The detailed questions are:

67. Is the gross income that you earn at your current job just, from your point of view?	
No..... <input type="checkbox"/>	Yes <input type="checkbox"/> → <i>Question 69!</i>
↓	
68. How high would your gross income have to be in order to be just?	
Gross: <input type="text"/>	euros per month Don't know <input type="checkbox"/>
69. Is the net income that you earn at your current job just, from your point of your view?	
No..... <input type="checkbox"/>	Yes <input type="checkbox"/> → <i>Question 71!</i>
↓	
70. How high would your net income have to be in order to be just?	
Net: <input type="text"/>	euros per month Don't know <input type="checkbox"/>

Figure 1: Questionnaire

This question gives information on fair perceived gross and net income in order to construct a social security and (income) taxation schedule on the basis of three Equal Sacrifice principles. One advantage is that respondents do not need any priming on optimal taxation theory nor on the three Equal Sacrifice Principles. They only answer about what they think is a fair gross and a fair net income. A function of this difference is interpreted as their fair sacrifice and can be checked against the Equal Sacrifice principles.

With regard to the data, there is one major limitation. The wedge between fair gross and net income includes both social security contributions and income taxes. As a consequence, it is impossible to identify which share of the total tax burden would be apportioned to income taxes alone by the respondent. Therefore, I refrain from separating these two components.

First, I will give some informations on the sample and summary statistics (see Subsection 3.1) and second, I discuss how respondents have linked the answers on fair gross and net income to their tax burden (see Subsection 3.2).

⁷A question on fair income was asked from 2005 to 2013 (every second year) in the SOEP questionnaire and was inspired by a perceived justness of incomes formula developed by Jasso [1978]. Only respondents who think that their gross income is not fair were asked these questions. In 2015, this question was modified into four more specific questions that specifically ask if individuals are satisfied with their gross and their net incomes. Therefore, I only use the 2015 question.

3.1 Sample and summary statistics

A total of 27,183 individuals who responded the personal questionnaire in the 2015 wave. Since only working respondents were interviewed, only 16,361 individuals answered the question about fair gross income, 16,304 about fair net income and 16,274 both. While individuals who do not work in the survey year 2015 did not get the relevant questions, the calculations do not include the whole population, e.g. pensioners, the unemployed, or school children are not included; thus nothing can be said about their preferences. Conditioning on respondents giving an amount and having valid cross-section weights 15,245 individuals are still available. The main analysis builds upon these observations.

In Germany, the tax system allows income splitting, therefore the answers of the respondents might be motivated by higher tax burdens for the spouse with the lower income. This may be especially relevant for females who frequently are not the main breadwinner. Therefore, I construct tax units and identified 10,243 tax units. As argued before, the ability-to-pay may differ between household types. Therefore, I use equivalized incomes⁸ for all taxunits where the composition of individuals is clearly determinable (N=8,099). Furthermore, I create an indicator for five different household types: single households without children (N=2,988), single households with one child (N=372), married couples without children (N=1,326), married couples with one child (N=743), married couples with two children (N=1,138).

Table 1 presents descriptive statistics for the main sample (N=15,245). Around 59 % of the respondents think that their personal gross (Y) and net income (X) is fair, whereas 34 % of the respondents think that their gross and net incomes are unfair. Only 1 % of the respondents think that their net income is fair but their gross income is unfair, whereas 6 % of the respondents think that their gross income is fair but their net income is unfair. Compared to their 2015 SST burden, 41 % would like to have a different gross, net, or both (gross and net) income.

Table 2 presents the summary statistics of the relevant variables. Fair gross and net income is, on average, greater than current gross and net income. In addition, Table 2 presents the average tax rate (ATR) that is calculated by:

$$ATR_i = \frac{T_i(Y_i)}{Y_i}, \quad (6)$$

⁸To reflect the differences in household size the modified OECD scale is used. The first adult is counted by 1, the second and each subsequent person by 0.5 and children below 14 are counted by 0.3.

Table 1: Summary statistics

	Net labor income (X) is...		Total
	fair	unfair	
Gross labor income (Y) is fair	59 %	6 %	65 %
Gross labor income (Y) is unfair	1 %	34%	35 %
Total	60 %	40 %	

Source: SOEP v.32 (own calculations).

Note: Table includes all individuals that answered the questions and have a valid cross-sectional weight (N=15,245). Observations are weighted by the cross-sectional survey weights provided by the SOEP.

where Y_i is gross labor income of individual i . The variable T_i is the SST burden that is defined by the difference between Y_i and X_i , the net labor income of individual i . Therefore, ATR is the average tax rate and a relative measure. The ATR is significantly lower than this quotient in the German 2015 tax schedule, thus indicating that individuals prefer reduced taxation. Furthermore, the standard deviation of ATR^{fair} is much higher, implying a broad range of answers in regard to fair gross and net income. Table B.1 of the Appendix presents the same variables but income is transformed to equivalized gross and net labor income. As expected, equivalized gross and net labor income is much lower.

Figure 2 presents the ATRs of the fair perceived monthly gross income for five different household types. For all household types, the tax schedule is progressive. With a gross income of around 2000 Euro, the ATRs of all household types are in the same range around 0.3. For high income households with a gross income around 10,000 Euro, the fair perceived ATR is between 0.37 to 0.43: single households have the highest and married couples the lowest ATRs. For low income below 2,000 Euro the picture is the other way around: highest ATRs for married and lowest ATRs for singles. This indicate that different household with a different ability-to-pay prefer different tax schedule. Therefore, I will use equivalized income for my analysis.

3.2 Evidence for (un)fair perceived tax burden

The SOEP questions (see Figure 1) do not directly ask for the level of a fair tax burden and respondents who read the question on their fair gross income

Table 2: Additional summary statistics

Variable	Mean	Std. Dev.
Gross labor income (Y)	2683.77	2297.78
Net labor income (X)	1762.81	1340.16
ATR	0.29	0.14
Fair gross labor income (Y^{fair})	2993.15	2607.46
Fair net labor income (X^{fair})	2022.28	1592.37
ATR^{fair}	0.26	1.32

Source: SOEP v.32 (own calculations).

Note: Table includes all individuals that answered the questions and have a valid cross-sectional weight (N=15,245). Income is not equalized. Observations are weighted by the cross-sectional survey weights provided by the SOEP.

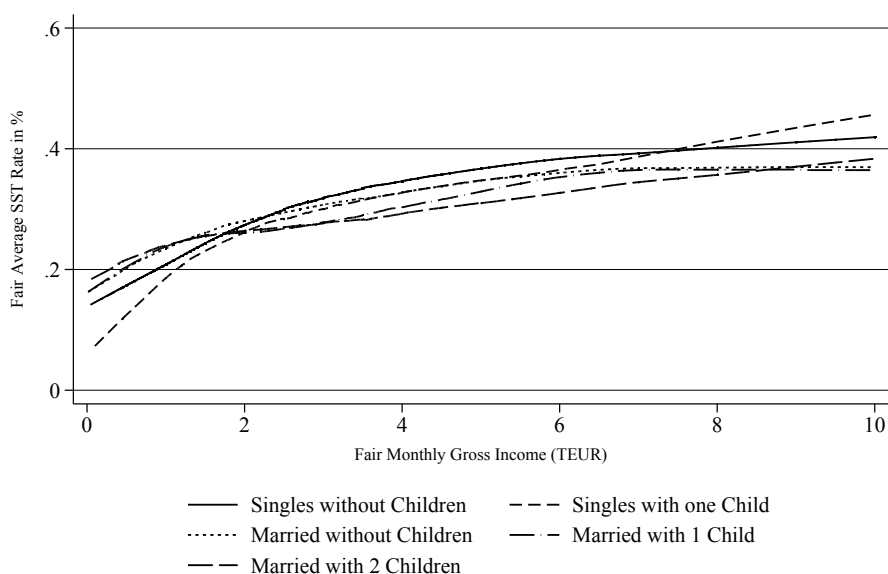
may think about just incomes. Therefore, these questions are often used for research on justice of incomes (see e.g. Jasso and Webster [1997], Liebig et al. [2010, 2012]). However, asking about fair gross and net incomes at the same moment implies a fair social security and tax (SST) schedule.

Respondents who answered that gross (net) income is fair but net (gross) is unfair are not satisfied with their SST burden in 2015 and answered these question to give a fair SST. With regard to Table 1, these are 7 % of the respondents.

For respondents who answered that gross and net income is unfair (around 34 % of the respondents), it is not clear if they think that their wage or the tax schedule is unfair. Therefore, I plot the fair perceived and the 2015 average tax rates (ATR) and marginal tax rates (MTR). The scatter plots of ATR and MTR⁹ (see Appendix Figure B.1 and B.2) show a wide spread, thus indicating that many respondents prefer a different SST schedule compared to the actual tax schedule in 2015 and not only on (un)fair income.

Furthermore, respondents who answered that their gross and net income

⁹While ATR is calculated as tax burden divided by gross income (see equation 6), MTR is the tax rate that is paid for the last earned Euro. The MTR is calculated by: $MTR_i = \frac{T_{p+1} - T_p}{Y_{p+1} - Y_p}$, where p defines the percentile in the distribution, T the tax burden and Y the labor gross income.



Source: SOEP v.32 (own calculations).

Note: For plotting, a lowess (locally weighted least square regression) regression is used. Income is not equivalized.

Figure 2: ATR of the fair perceived income for different household types

is fair (around 59 % of the respondents), may connect this question only to their incomes and not to the tax burden. To check whether this is the case, I use a different fairness question from the SOEP Innovation sample questionnaire (not part of my main sample). In 2015, the question on fair gross and net income was also asked in the innovation sample. In addition, respondents were also asked about their opinion on income redistribution.¹⁰ Respondents who think that their gross and net income are both fair, are satisfied with their current taxation. For these respondents, there should be no relation with the statement that rich or poor people should be taxed higher or lower. By using a χ^2 -Test and Cramer's V,¹¹ this hypothesis is not rejected, which means that I cannot find a significant relationship. However, I find a significant effect for people who think that their (gross and net) income

¹⁰The statements respondents were supposed to evaluate "Persons with high income should have an increase in the tax rate in the future" and "Persons with low income should have more transfers in the future".

¹¹Cramer's V is a χ^2 based test and gives an association between two nominally scaled variables (here: between two dummies).

is unfair and the preference for redistribution (see Table B.2 in Appendix) indicating a relation between unfair income and the statement that rich or poor people should be taxed higher or lower. These results underpin that respondents, if they perceive their income as fair or not, also understood the question in the sense of a sacrifice through taxation. For this purpose, I use all combinations of answers for my analysis.

4 Testing Equal Sacrifice Theories

I now test if one of the theories of Equal Sacrifice is in line with individuals' preferences for the data and if one of the principles could serve as objective function for a fair tax schedule.

4.1 Specifying the risk aversion parameter ε

As described in Section 2, I use the CRRA utility function (equation 4 and 5) to permametrize the three sacrifice definitions (equation 1,2 and 3). To use the CRRA utility function, ε , the measure of relative risk aversion, has to be calculated or estimated. Chetty [2006] argues that an ε under or equal 2 is reasonable. Furthermore, for risk aversion, a broad range of values has been estimated. Gourinchas and Parker [2002] estimate a relative risk aversion parameter between 0.51 to 1.39, whereas Kaplan [2012] estimates a value around 1.6 to 1.65 for the USA. For Germany, Dohmen et al. [2011] argue that relative risk aversion parameters between one and five are realistic and above 10 are unrealistic. With the lack of data on consumption for Germany and therefore no opportunity to estimate, it is also common to set the value for risk aversion (see e.g. Haan and Wrohlich [2010] set the relative risk aversion parameter to 1.5).

Therefore, I use three different ε for each of the Equal Sacrifice principles: I set ε to 1 and 2; the bounds derived by Chetty [2006] and estimate an ε that fits well for AES, RES, and MES separately.

For AES and as explained in Young [1990], I estimate ε with the help of the mean value theorem (see Appendix A). This is done by the following OLS regression:

$$\ln(T_i^{fair}) = \varepsilon * 0.5 * \ln(Y_i^{fair} * (Y_i^{fair} - T_i^{fair})) + e_i, \quad (7)$$

where T_i presents the tax burden, ε the coefficient describing the risk parameter, $0.5 * \ln(Y_i(Y_i - T_i))$ the independent variable including the gross income

Y_i , and e_i the error term. The independent variable $0.5 * \ln(Y_i(Y_i - T_i))$ defines the logarithm of the distance between the data points: $U(Y)$ and $U(Y-T)$. As a result, ε describes the slope of the utility function and can be used as the risk aversion parameter. I find an ε that is equal to 1.2 (see Appendix A). For RES and MES principle, and with regard to the mean value theorem that can be only used for absolute terms, this strategy does not apply.

For RES, I calculate an ε with the help of the best numerical fit. I minimize the sum of all squared differences of the fair T_i^{fair} and new calculated T_i^{ES} for ε between 1 and 2 in 0.01 steps. Thus, the ε is equal to 1.013 for RES and slightly lower than the estimated ε for AES.

By using the same strategy for MES as for RES identifies an ε of 1 which is the bound. Therefore, I choose the middle of the bounds, an ε of 1.5, to have also three scenarios for MES.¹²

4.2 Results of Equal Sacrifice Theories

With the help of the estimated ε , I check whether one of the Equal Sacrifice principles is consistent with the fair perceived tax for the entire distribution. Therefore, I use the fair net and fair gross income within the three sacrifice theories to calculate the sacrifice s , take the mean of s , and calculate the tax schedules for all three sacrifice theorems. The SST schedule can be calculated by rearranging the specific Equal Sacrifice definitions (see equation 1, 2, and 3). Table 3 presents the formulas to identify the SST schedule T. Since ε of 1 requires a different utility function, I have six different tax formulas.

Subsequently, I check which of the three sacrifice theorems and which risk parameter ε (as explained in section 4.1) best with the data. Table 4 presents the results of each sacrifice definition: the mean μ of the sacrifice s , the standard derivation σ of s , and ATR for different income levels. The mean μ of the sacrifice s is calculated by plugging in the individual fair gross Y_i and fair net X_i incomes into the equation 1, 2, and 3. As explained before, for the definition of the utility functions I use CRRA (see equation 4 and 5). Out of all sacrifices s , I calculate the mean μ and σ . As assumed in the theoretical Section 2, the lowest average sacrifice s can be found for MES principle. However, with increasing ε , the average sacrifice s decreases.

The lower part of Table 4 presents tax schedule with ATR for all nine scenarios and different gross income levels. For calculating the ATR, I plug

¹²Risk aversion is a deep parameter and should not depend on the definition of the Equal Sacrifice Principle. However, the best fitting tax schedules produces different risk aversion parameters that are in the same range.

Table 3: Social Security and Tax (SST) Formulas

	Equal Sacrifice Theories		
	AES	RES	MES
$\varepsilon \neq 1$	$T_i = Y_i - (Y_i^{1-\varepsilon} - (1 - \varepsilon) * s)^{\frac{1}{1-\varepsilon}}$	$T_i = Y_i - ((1 - s) * (Y_i^{1-\varepsilon} - 1) + 1)^{\frac{1}{1-\varepsilon}}$	$T_i = Y_i - s^{\frac{1}{\varepsilon}}$
$\varepsilon = 1$	$T_i = Y_i - e^{-s} * Y_i$	$T_i = Y_i^{1-s} * (Y_i^s - 1)$	$T_i = Y_i - \frac{1}{s}$

Note: The SST schedule can be calculated by rearranging the specific Equal Sacrifice definitions by plugging in the utility function (see equation 4 and 5) to identify the tax burden T . Y indicates gross income, s the sacrifice, and ε the risk aversion parameter.

Table 4: Equal Sacrifice and ATRs

	Equal Sacrifice Theories								
	AES			RES			MES		
ε	1	1.200	2	1	1.013	2	1	1.500	2
$\mu(s)$	0.36	0.09	$0.45 * 10^{-3}$	$0.49 * 10^{-1}$	$0.47 * 10^{-1}$	$0.44 * 10^{-3}$	$0.23 * 10^{-2}$	$0.24 * 10^{-3}$	$0.51 * 10^{-4}$
$\sigma(s)$	0.245	0.071	0.003	0.048	0.046	0.003	0.007	0.002	0.001
Y	Average tax and social security rate= $(T(y)+S(y))/y$								
500	0.30	0.26	0.18	0.26	0.26	0.18	0.15	0.49	0.72
1000	0.30	0.29	0.31	0.29	0.29	0.31	0.57	0.74	0.86
2000	0.30	0.32	0.47	0.31	0.31	0.47	0.79	0.87	0.93
4000	0.30	0.36	0.64	0.33	0.34	0.64	0.89	0.94	0.96
10000	0.30	0.41	0.82	0.36	0.37	0.82	0.96	0.97	0.99

Note: Y indicates equalized gross income, s the sacrifice, ε the risk aversion parameter, σ the is the coefficient of variation, μ the mean, and ATR the average tax rate.

in the individual fair gross Y_i and fair net X_i incomes into equation 6. The ATR for AES and RES are very similar. Furthermore, with an increasing parameter of risk aversion (ε), I find a more progressive SST schedule, indicating a higher level of redistribution. In the case of MES, high tax rates of 99 percent for the very rich and, therefore, the highest degree of progressivity in the chosen scenarios.¹³ These findings underpin the assumptions from Section 2 that the MES principle leads to an extremely high progressive tax schedule. Only for AES, with a risk aversion parameter ε equal to 1, I find a proportional tax schedule. In the other eight scenarios of Table 4, the tax schedule calculated by the three defined Equal Sacrifice principles are progressive. Table 3 explains this: if the relative risk aversion parameter ε is one, the tax function rearranges to $T = Y - e^{-s} * Y$ where e^{-s} is a constant and ends up in a linear tax schedule.

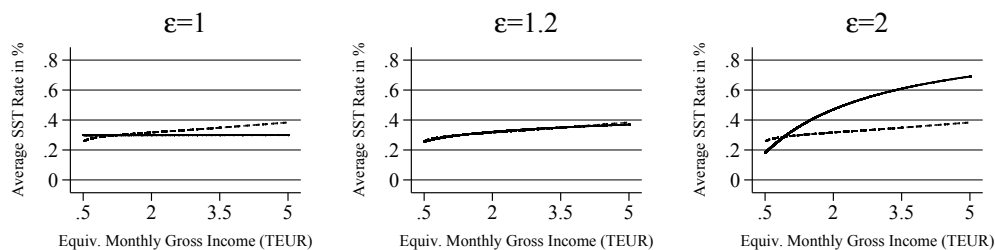
4.3 Graphical and numerical fit

If stated preferences on fair net and gross income confirm one of the three defined Equal Sacrifice principles will be discussed in this step. Therefore, I compare the new calculated tax (Equal Sacrifice SST) schedule with the original data (Fair SST). Figure 3 presents the tax rates of both schedules (see Appendix Figure B.3 for a broader range of equivalized monthly gross income).¹⁴ The MES differs most from the fair perceived SST; the AES and RES theories with the estimated and calculated ε (scenarios in the middle) have the best fit.

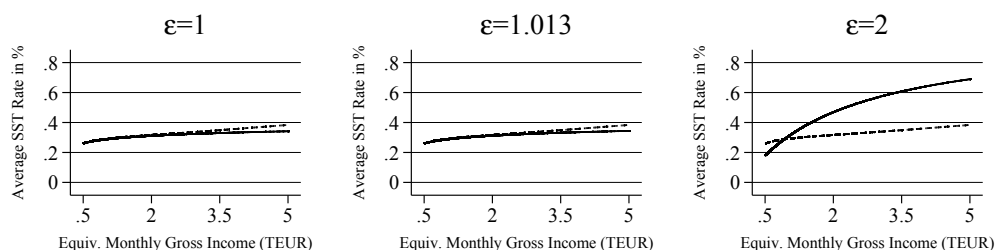
¹³In the case of MES, I also find transfers to the working poor.

¹⁴For plotting the surveyed difference between gross and net income, a lowess regression is used. Lowess regression is a locally weighted least square regression and helps to smooth graphs.

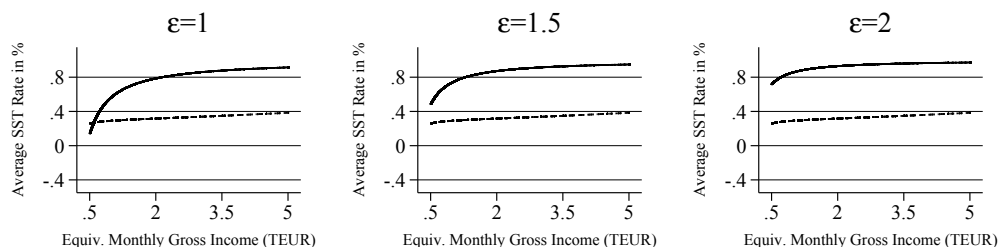
Absolute Equal Sacrifice (AES)



Relative Equal Sacrifice (RES)



Marginal Equal Sacrifice (MES)



— Equal Sacrifice SST Schedule - - - Fair SST Schedule

Source: SOEP v.32 (own calculations).

Note: The bounds for the ϵ are set to 1 (left graphs) and 2 (right graphs). The ϵ for scenarios in the middle differ: For AES ϵ is estimated by the Young [1990] method (see Appendix A) and is $\epsilon=1.2$. MSE is reduced to 0.220 for $\epsilon=1.2$ (MSE is 0.249 for $\epsilon=1$ and 0.422 for $\epsilon=2$). While the method for AES does not apply, I calculate an ϵ which has the best numerical fit for RES and ϵ is 1.013. MSE is 0.222 (MSE is 0.222 for $\epsilon=1$ and 0.422 for $\epsilon=2$). For MES the best numerical fit lies out of the bounds, I set ϵ to 1.5. MSE of 0.951 is lowest for $\epsilon=1$ (MSE for $\epsilon=1.5$ is 1.031 and for $\epsilon=2$ is 1.166). For plotting, a loess (locally weighted least square regression) regression is used.

Figure 3: Equal Sacrifice Tax Schedule vs. Fair Tax Schedule

To test the numerical fit, I calculate the correlations between the Equal Sacrifice principles and the Fair SST Schedule by an ordinary least square regression without a constant and in logs:

$$\ln(T_i^{ES}) = \beta \ln(T_i^{fair}) + e_i, \quad (8)$$

where T^{ES} presents the calculated tax burden of the three Equal Sacrifice principles, T^{fair} the tax burden that is considered as fair by the respondents, concrete, fair gross labor income minus fair net labor income, and e_i the error term. As Figure 3 shows an exponential course for the both tax schedules, I use the log form in the least square regression.

Table 5 presents the β , R^2 and Mean Square Error (MSE) for all three theories and risk aversion parameters. A β around one, a high R^2 and a low MSE indicate high consistence, a high level of explained variance and a small difference between the fitted line and the data points.

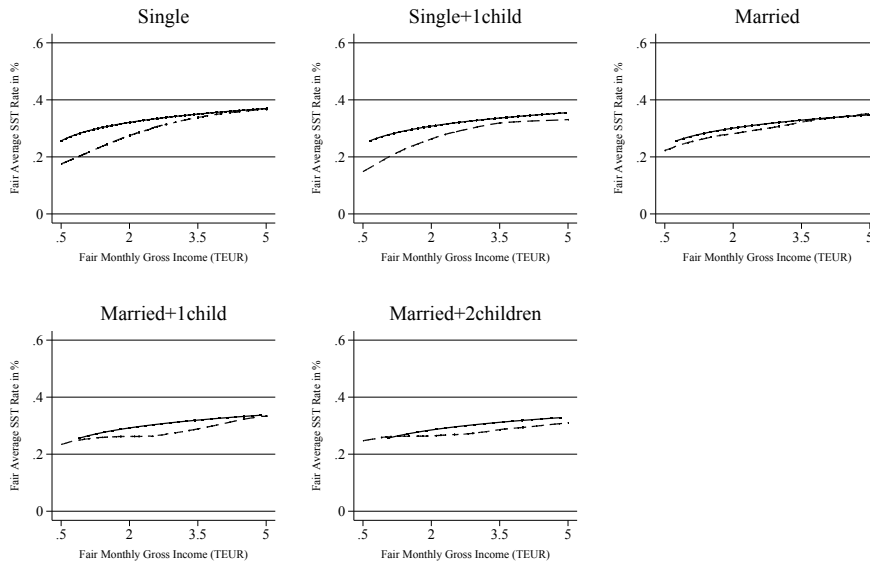
Table 5: OLS regression for the three Equal Sacrifice SST and Fair SST Schedule (in logs and without a constant)

	$\epsilon=1$			$\epsilon = \{1.2; 1.013; 1.5\}$			$\epsilon=2$		
	β	R^2	MSE	β	R^2	MSE	β	R^2	MSE
AES	1.048	0.865	0.249	1.045	0.881	0.220	0.950	0.772	0.422
RES	1.034	0.880	0.222	1.035	0.880	0.222	0.946	0.772	0.422
MES	1.008	0.447	0.951	1.552	0.419	1.031	2.368	0.355	1.166

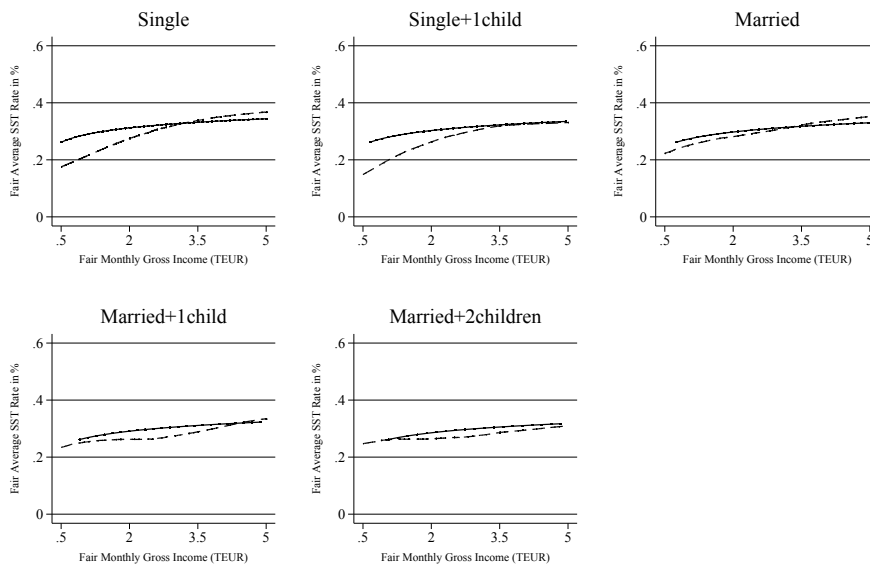
Source: SOEP v.32 (own calculations).

The highest R^2 can be observed for the AES and RES, especially for the estimated ϵ . The lowest R^2 can be found for the MES principle. In addition, the lowest MSE and, therefore, a small distance between the fitted line and the data points is found for AES and RES for the estimated ϵ . The MSE for RES is minimal smaller than for AES. These results confirm the graphical results. With regard to the risk aversion parameter ϵ , the parameter estimated by the method of Young [1990] for AES and the calculated ϵ for RES produces the lowest MSE indicating that these schedules are most similar compared to the fair tax data.

Absolute Equal Sacrifice with $\epsilon = 1.2$



Relative Equal Sacrifice with $\epsilon = 1.013$



— Equal Sacrifice SST Schedule - - - Fair SST Schedule

Source: SOEP v.32 (own calculations).

Note: For plotting the fair SST schedule, a lowess (locally weighted least square regression) regression is used. Income is not equalized.

Figure 4: Equal Sacrifice Tax Schedule vs. Fair Tax Schedule for different household types

Figure 3 and Table 5 show that AES and RES with the estimated ϵ (scenario in the middle) fits best with the fair perceived SST schedule. As argued before, the ability-to-pay for different household types may differ and therefore, income is equalized in the estimations before. Figure 4 shows whether the calculated Equal Sacrifice SST schedules also agree with the preferences of different household types and presents the best fitting (see Figure 3 and Table 5 the szenario in the middle for AES and RES) Equal Sacrifice Schedule vs. the fair answered SST schedule for different household types.¹⁵ For household types with a married couple, the stated preferences on fair net and gross income confirm the AES and RES principle. In this case, the OECD equivalence scale seems to be in good agreement with the preferences. For singles, the fit is not quite as good, especially in the lower income ranges, respondents prefer a lower tax. This may indicate that the ratio of the currently selected equivalence scale does not necessarily coincide with the desired preferences with regard to taxation. As shown in van de Ven et al. [2017], empirical calculated tax implicit equivalence scales varies with gross income that may explain the relatively worse fit for the single household types. Nevertheless, for most parts of the income distribution, the fit seems to be good whereas the graphical fit for AES seems to be slightly better than for RES.

Overall, Table 4 and Figure 3 show that none of the three Equal Sacrifice principles fit perfectly with the data on fair perceived gross and net income of the employed, but, the principle of AES and RES have the best fit. As shown in Figure 3, there is almost no graphically difference between these Equal Sacrifice principles and Fair SST schedule. With regard to the risk aversion parameter ϵ , the parameter estimated by the method of Young [1990] for AES and the ϵ for RES produces an Equal Sacrifice tax that is most similar compared to the fair tax data. These results underpin that two of the defined Equal Sacrifice principles are in line with the fair perceived income taxation in Germany. Nevertheless, this result is estimated from the fair perceived income of the employed and does not include the whole population, e.g. pensioners, the unemployed, or school children are not included.

4.4 Government Revenue

Besides a preference for an Equal Sacrifice principle it is also interesting to examine how much government revenue is generated compared to the 2015 tax schedule. To identify if one of the Equal Sacrifice tax schedules satisfy the 2015 budget constraint, Table 6 presents the rate of the government

¹⁵See Appendix Figure B.4 for a broader range of monthly gross income.

consumption level Δ compared to the level of the 2015 SST schedule that is calculated by:

$$\Delta = \frac{\sum_{i=0}^N (Y_i - X_i)}{\sum_{i=0}^N (Y_i^{ES} - X_i^{ES})} - 1$$

where X presents the net labor income, Y the gross labor income and the subscript ES presents the incomes within the Equal Sacrifice SST schedules. In the scenario of MES, the government consumption level is much higher with the Equal Sacrifice SST schedule compared to the 2015 SST schedule, where government consumption reduces in the first two scenarios of AES and RES tax schedules.¹⁶ Therefore, the budget of the government expenditures would be reduced for the two best fitting scenarios (AES with $\epsilon=1.2$ and RES with $\epsilon=1.013$) indicating that the overall German working population would like to pay less tax.

Table 6: Equal Sacrifice and Government Consumption

	Equal Sacrifice Theories								
	AES			RES			MES		
ϵ	1.000	1.2	2.000	1.000	1.013	2.000	1.000	1.500	2.000
δ G in pp	-0,19	-0,08	0,29	-0,13	-0,13	0,29	0,51	0,63	0,65

Δ presents the governments consumption level compared to the 2015 social security and tax schedule in percentage points.

5 Conclusions

The basic idea of this paper was to use two novel questions from the SOEP questionnaire on fair gross and net income and transform them into an indicator for a fair social security and income tax rate, which is then used to develop a fair social security and (income) tax scheme. While the ability-to-pay differs for different household types equalized household income is used. The scheme is then compared for its fit with absolute, relative, and marginal Equal Sacrifice principles. Unique to the approach of this study is that respondents did not have to choose between given taxation scheme alternatives. Respondents were asked directly to determine their own fair gross and net incomes.

¹⁶With an ϵ of 1.751 or 1.749, the budget constraint of the government is binding in the case of AES or RES. However, in this case, the fit is worse.

The general finding is that none of the three Equal Sacrifice principles fits perfectly with the survey data. However, the principle of AES and RES yield the best fit by numbers and, in the graphical plot, a remarkable fit is obtained. I also find that a fair tax schedule should be progressive. For optimal tax theory and also for the social planner, this result implies that two of the Equal Sacrifice principles qualify as an alternative to the welfarism. The related question of how well the optimal taxation schedule in accordance with Mirrlees [1971] fits would require a different approach however and exceeds the scope of this study. Also left for further research is the question whether the Equal Sacrifice principles even hold if respondents (including non-working) are asked explicitly about a fair taxation scheme.

References

- Kenneth Joseph Arrow. *Essays in the theory of risk-bearing*. Markham Pub. Co Chicago, 1971.
- Anthony B Atkinson. On the measurement of inequality. *Journal of Economic Theory*, 2(3):244 – 263, 1970.
- Marcus Berliant and Miguel Gouveia. Equal sacrifice and incentive compatible income taxation. *Journal of Public Economics*, 51(2):219–240, 1993.
- Raj Chetty. A new method of estimating risk aversion. *American Economic Review*, 96(5):1821–1834, 2006.
- Pierre Chiappori and Monica Paiella. Relative Risk Aversion Is Constant: Evidence From Panel Data. *Journal of the European Economic Association*, 9(6):1021–1052, 2011.
- Thomas Dohmen, Armin Falk, David Huffman, Uwe Sunde, Jürgen Schupp, and Gert G. Wagner. Individual risk attitudes: Measurement, determinants, and behavioral consequences. *Journal of the European Economic Association*, 9(3):522–550, 2011.
- Udo Ebert and Patrick Moyes. Consistent income tax structures when households are heterogeneous. *Journal of Economic Theory*, 2000.
- Francis Y. Edgeworth. The pure theory of taxation. *The Economic Journal*, 7:46–70, 1897.
- Irwin Friend and Marshall E Blume. The demand for risky assets. *American Economic Review*, 65(5):900–922, 1975.
- Jan Goebel, Markus Grabka, Stefan Liebig, Martin Kroh, David Richter, Carsten Schröder, and Jürgen Schupp. The German Socio-Economic Panel (SOEP). *Jahrbücher für Nationalökonomie und Statistik*, 2018.
- Pierre-Olivier Gourinchas and Jonathan Parker. Consumption Over the Life Cycle. *Econometrica*, 70(1):47–89, 2002.
- Peter Haan and Katharina Wrohlich. Optimal Taxation: The Design of Child-Related Cash and In-Kind Benefits. *German Economic Review*, 11: 278–301, 08 2010.
- Guilermína Jasso and Murray Jr. Webster. Double standards in just earnings for male and female workers. *Social Psychology Quarterly*, 60:66–78, 1997.

- Guillermina Jasso. On the Justice of Earnings: A New Specification of the Justice Evaluation Function. *American Journal of Sociology*, 83:1398–1419, 1978.
- Robin Jessen, Maria Metzger, and Davud Rostam-Afschar. Optimal Taxation Under Different Concepts of Justness. *Discussion Papers Free University Berlin*, 26, 2017.
- Greg Kaplan. Inequality and the life cycle. *Quantitative Economics*, 3(3): 471–525, 2012.
- Stefan Liebig, Peter Valet, and Jürgen Schupp. Perceived income justice depends on the economy. *Weekly Report - Deutsches Institut für Wirtschaftsforschung (DIW)*, 6:187–192, 2010.
- Stefan Liebig, Carsten Sauer, and Jürgen Schupp. The justice of earnings in dual-earner households. *Research in Social Stratification and Mobility*, 30(2):219 – 232, 2012. ISSN 0276-5624.
- James Stuart Mill. *Principle of Political Economy*. Longmans Green & Co., London, 1848.
- James A. Mirrlees. An Exploration in the Theory of Optimum Income Taxation. *Review of Economic Studies*, 38(2):175–208, 1971.
- R. A. Musgrave and P. B. Musgrave. *Public finance in theory and practice*. McGraw-Hill, New York, 1973.
- Richard A. Musgrave. *The Theory of Public Finance*. McGraw-Hill, 1959.
- Richard A. Musgrave. Fairness in taxation. In Robert D. Ebel Joseph J. Cordes and Jane G. Gravelle, editors, *The Encyclopedia of Taxation and Tax Policy*. Urban Institute Press, 2005.
- Thomas Piketty and Emmanuel Saez. Optimal Labor Income Taxation. In M. Feldstein A. Auerbach, R. Chetty and E. Saez, editors, *Handbook of Public Economics*, chapter 9. Elsevier-North Holland, 2013.
- John W. Pratt. Risk Aversion in the Small and in the Large. *Econometrica*, 32(1/2):122–136, 1964.
- Wolfram F. Richter. From Ability to Pay to Concepts of Equal Sacrifice. *Journal of Public Economics*, 20:211–229, 1983.

- Emmanuel Saez and Stefanie Stantcheva. Generalized social marginal welfare weights for optimal tax theory. *American Economic Review*, 106(1):24–45, 2016.
- Henry Sidgwick. *Principle of Political Economy*. Macmillan, London, 1883.
- Adam Smith. *An Inquiry into the Nature and Causes of the Wealth of Nations*. 1776.
- Justin van de Ven, Nicolas Herault, and Francisco Azpitarte. Identifying tax implicit equivalence scales. *Journal of Economic Inequality*, 2017.
- Matthew Weinzierl. The promise of positive optimal taxation: normative diversity and a role for equal sacrifice. *Journal of Public Economics*, 118(C):128–142, 2014.
- Peyton Young. Distributive justice in taxation. *Journal of Economic Theory*, 44:321–335, 1988.
- Peyton Young. Progressive taxation and sacrifice. *American Economic Review*, 80:253–266, 1990.

Appendix

A Calculation of the risk aversion parameter by Young [1990]

As discussed in Section 2, I use the CRRA as utility function (see equation 4), plug it into equation 1 and now I follow Young [1990]:

$$\frac{Y^{1-\varepsilon} - 1}{1 - \varepsilon} - \frac{(Y - T)^{1-\varepsilon} - 1}{1 - \varepsilon} = s \quad (9)$$

The SST burden T and the gross income Y is available but ε has to be estimated.

We know from the definition of ε of the CRRA function (see equation 4):

$$\varepsilon = -\frac{zU''(z)}{U'(z)} = -\frac{dU'(z)}{U'(z)} : \frac{dz}{z} = -\frac{d(-\ln U'(z))}{d(\ln z)} = \frac{-\% \Delta U'(w)}{\% \Delta w} \quad (10)$$

where the rate of change is described by $-\ln U'(z)$ with respect to $\ln(z)$. Thus, I need to calculate w which defines distance between $U(Y)$ and $U(Y-T)$ for estimating ε .

Therefore, the mean value theorem is used and helps to rearrange the equation:

$$\frac{U(Y) - U(Y - T)}{Y - Y + T} = U'(w) \Leftrightarrow \frac{Y^{1-\varepsilon} - 1}{1-\varepsilon} - \frac{(Y-T)^{1-\varepsilon} - 1}{1-\varepsilon} = w^{-\varepsilon} \quad (11)$$

to:

$$\frac{w}{Y} = \left(\frac{(\varepsilon - 1) * \frac{T}{Y}}{(1 - \frac{T}{Y})^{1-\varepsilon} - 1} \right)^{\frac{1}{\varepsilon}} \quad (12)$$

The w and ε are unknown, but the relationship between T and Y can be defined. As starting point and done in Young [1990], I set $\frac{T}{Y}=0.2$ and plug this into equation 12. Now, I can identify an approximation for ε that is used to simplify equation 12:

ε	w/Y	
3	0.893	
2.5	0.894	$\Rightarrow \varepsilon = 2 \Rightarrow w = \sqrt{Y(Y - T)}$.
2	0.894	
1.5	0.895	
1.1	0.896	

Thus the distance w is defined as:

$$w = \sqrt{Y(Y - T)}. \quad (13)$$

By using equation 9 and 11, we find following relationship:

$$\frac{U(Y) - U(Y - T)}{T} = U'(w) = \frac{s}{T}$$

Without loss of generality Young [1988] is taking s equal to 1 and the logarithm and yields in:

$$-\ln U'(w) = -\ln U'(\sqrt{Y(Y - T)}) = -\ln T \quad (14)$$

Remember equation 10 and I set them equal to:

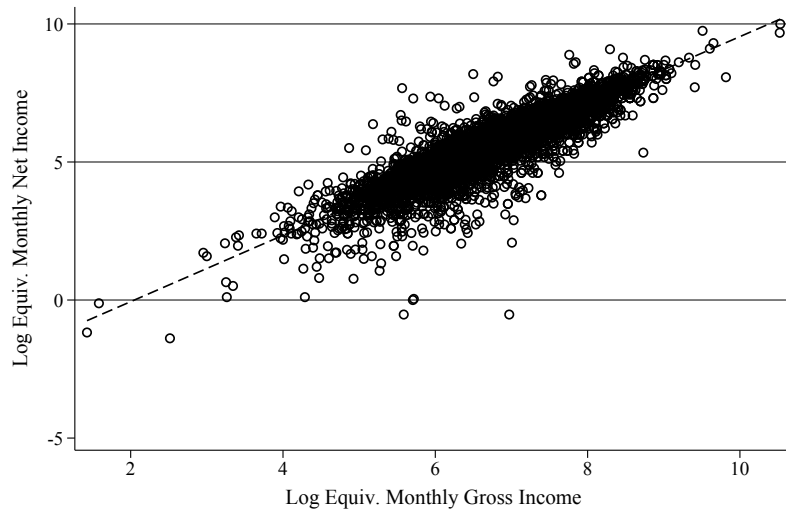
$$\varepsilon = -\frac{zU''(z)}{U'(z)} = -\frac{d(-\ln U'(z))}{d(\ln z)} = \frac{y}{x} \quad (15)$$

where this equation can be rearranged to the relationship: $y = \varepsilon * x$. The dependent variable y is described by $\ln(T)$ (see equation 14) and x by $\ln(w)$ that is equal to $\ln(0.5 * \ln(Y(Y - T)))$ (see equation 13). To identify ε , the following OLS regression is estimated:

$$\ln(T_i) = \varepsilon * 0.5 * \ln(Y_i(Y_i - T_i)) + e_i, \quad (16)$$

where $\ln(T_i)$ is the dependent variable, ε the coefficient describing the risk parameter, $\ln(Y_i(Y_i - T_i))$ the independent variable, and e_i the error term.

Figure A.1 presents the slope estimate for the utility function. The R^2 is equal to 0.73 meaning that 73 % of the variance can be explained by the linear model. The estimated risk aversion parameter $\hat{\varepsilon}$ is equal to 1.2.



Source: SOEP v.32 (own calculations).

Note: The figure presents the slope estimate for the utility function and identify the risk aversion parameter ε where $\hat{\varepsilon}=1.2$ and $R^2=0.73$.

Figure A.1: Regression for estimating the risk aversion parameter ε by the method of Young [1990]

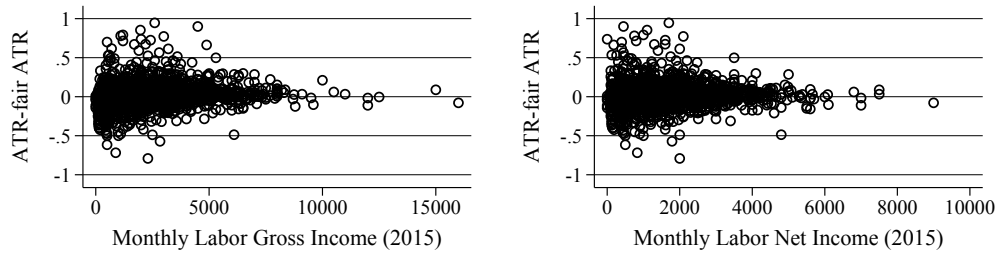
B Tables and Figures

Table B.1: Additional summary statistics

Variable	Mean	Std. Dev.
Equiv. Gross Labor Income (Y)	1541.28	1568.30
Equiv. Net Labor Income (X)	991.80	914.02
ATR	0.30	0.15
Fair Equiv. Gross Labor Income (Y^{fair})	1725.33	1143.30
Fair Equiv. Net Labor Income (X^{fair})	1143.30	1245.71
ATR^{fair}	0.26	1.35

Source: SOEP v.32 (own calculations).

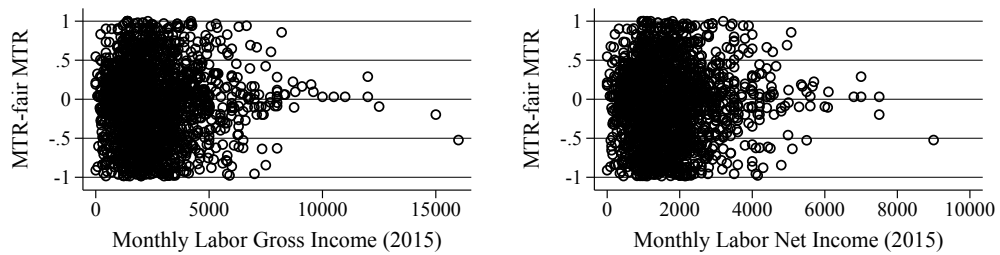
Note: (Fair) gross labor and (fair) net labor income is equivalized by the modified OECD scale. Observations are weighted by the cross-sectional survey weights provided by the SOEP.



Source: SOEP v32 (own calculations).

Note: Figures include all individuals on individual level that answered the questions and have a valid cross-sectional weight (N=15,245).

Figure B.1: Scatter plot of ATR-fair ATR over income



Source: SOEP v32 (own calculations).

Note: Figure includes all individuals on individual level that answered the questions and have a valid cross-sectional weight (N=15,245).

Figure B.2: Scatter plot of MTR-fair MTR over income

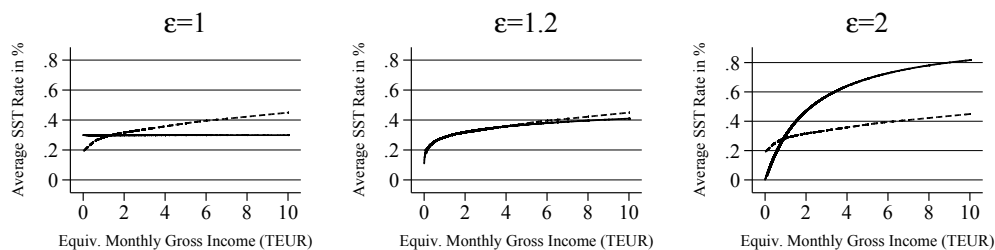
Table B.2: Cramer's V

gross and net is ...	less		higher	
	tax for rich	transfer for poor	tax for rich	transfer for poor
fair	-0.01	-0.02	0.03	0.01
unfair	0.07***	0.07***	-0.10***	-0.05***

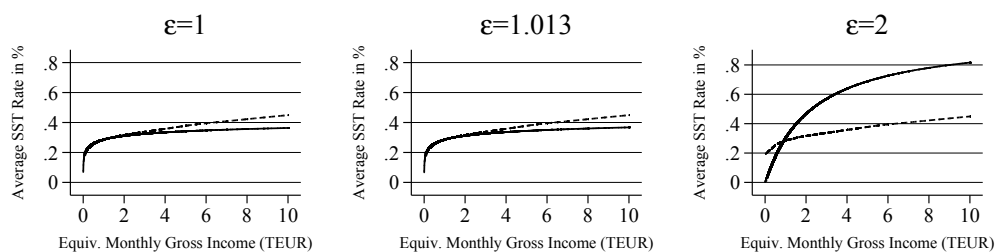
Source: SOEP-IS v.2015.1 (own calculations).

Note: This table contains the Cramer's V and checks the relationship between dummy variables. The dummy tax for rich/transfer to poor comes from the 5 point scale questions *Persons with high income should be taxed more in the future* and *Persons with low income should prospectively receive larger income*. *Higher* includes individuals that fully or rather disagree with the question and *less* includes individuals that completely or rather agree with the question.

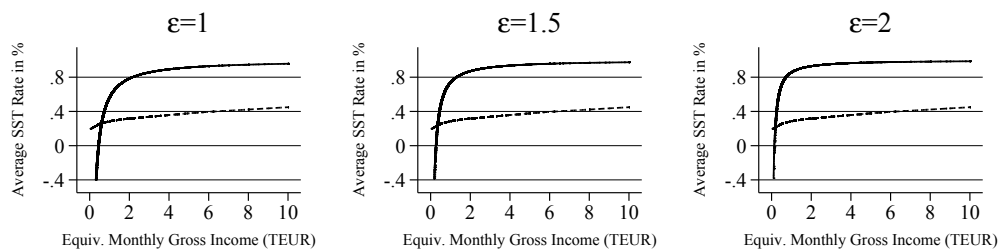
Absolute Equal Sacrifice (AES)



Relative Equal Sacrifice (RES)



Marginal Equal Sacrifice (MES)



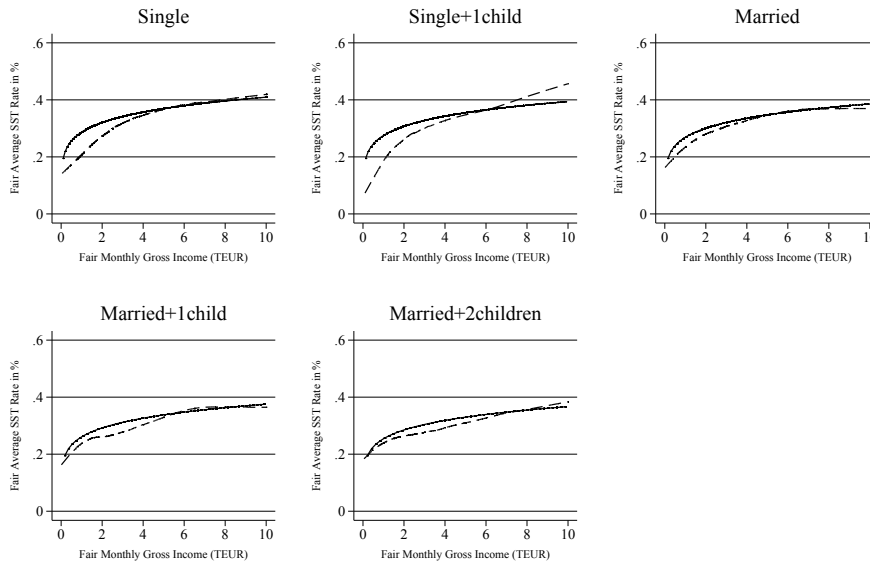
— Equal Sacrifice SST Schedule - - - Fair SST Schedule

Source: SOEP v.32 (own calculations).

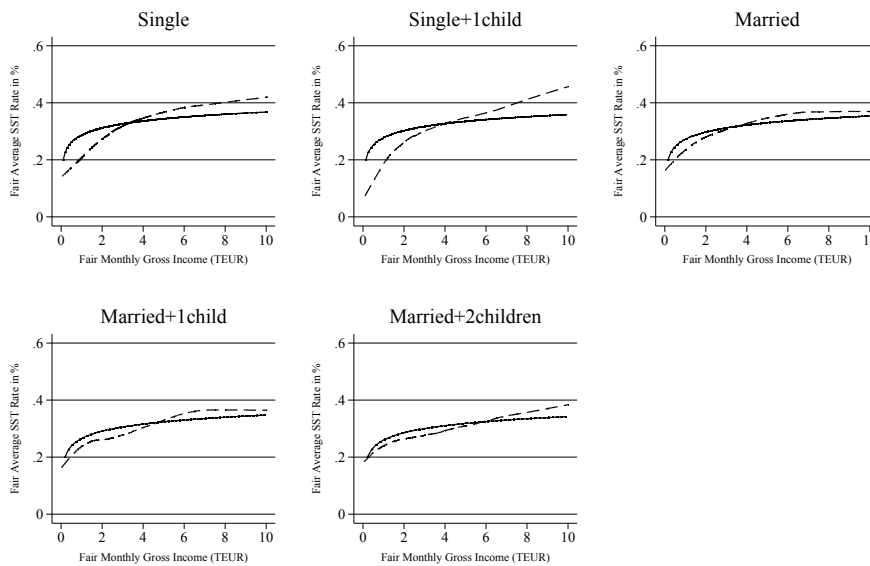
Note: The bounds for the ε are set to 1 (left graphs) and 2 (right graphs). The ε for scenarios in the middle differ: For AES ε is estimated by the Young [1990] method (see Appendix A) and is $\varepsilon=1.2$. MSE is reduced to 0.220 for $\varepsilon=1.2$ (MSE is 0.249 for $\varepsilon=1$ and 0.422 for $\varepsilon=2$). While the method for AES does not apply, I calculate an ε which has the best numerical fit for RES and ε is 1.013. MSE is 0.222 (MSE is 0.222 for $\varepsilon=1$ and 0.422 for $\varepsilon=2$). For MES the best numerical fit lies out of the bounds, I set ε to 1.5. MSE of 0.951 is lowest for $\varepsilon=1$ (MSE for $\varepsilon=1.5$ is 1.031 and for $\varepsilon=2$ is 1.166). For plotting, a loess (locally weighted least square regression) regression is used.

Figure B.3: Equal Sacrifice Tax Schedule vs. Fair Tax Schedule

Absolute Equal Sacrifice with $\epsilon = 1.2$



Relative Equal Sacrifice with $\epsilon = 1.013$



— Equal Sacrifice SST Schedule - - - Fair SST Schedule

Source: SOEP v.32 (own calculations).

Note: For plotting the fair SST schedule, a lowess (locally weighted least square regression) regression is used. Income is not equalized.

Figure B.4: Equal Sacrifice Tax Schedule vs. Fair Tax Schedule for different household types