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Nr. 111

A Note on the Effects of Degeneracy on
Cost Coefficients Ranges and an
Algorithm to Resolve Interpretation Problems

Gerhard Knolmayer



Kiel, im Juli 1982

A NOTE ON THE EFFECTS OF DEGENERACY ON COST COEFFICIENT RANGES
AND AN ALGORITHM TO RESOLVE INTERPRETATION PROBLEMS

Gerhard Knolmayer, University of Kiel

ABSTRACT

The problems of determining meaningful dual values and of performing meaningful sensitivity analysis in degenerate optima are reconsidered. For primal degenerate optima, earlier results on determining meaningful dual values are used to propose an efficient algorithm for determining meaningful cost coefficient ranges. It is further shown that the recently proposed determination of cost ranges at dual degenerate optima does not correspond to the usual definition of cost ranges.

Subject Areas: Linear Programming and Sensitivity Analysis.

INTRODUCTION

The main problem connected with primal degenerate optima is that the information about meaningful dual values and meaningful cost coefficient ranges is spread over several optimal tableaus. Evans and Baker /5/ accentuate the problem of sensitivity analysis in degenerate optima, concentrating on cost coefficient ranges for positive primal structural variables. They propose a procedure in which meaningful ranges are determined by taking the union of the individual cost coefficient ranges over several tableaus. In this procedure the original cost coefficient of the variable regarded is modified parametrically to create dual degenerate columns which allow iterations without losing dual feasibility. This must be done individually for the lower and upper bound on the cost coefficient and may be expensive in large-scale problems. Related procedures which individually change RHS-values to determine shadow prices are proposed in /1/ and /13,p.134/.

In this paper an efficient algorithm to determine meaningful cost coefficient ranges by using earlier results on finding meaningful shadow prices in primal degenerate optima is suggested. Furthermore, it is shown that at dual degenerate optima the application of the procedure recently proposed /5/ does not lead to results which correspond to the usual definition of cost coefficient ranges.

PRIMAL DEGENERACY AND DUAL VALUES

Earlier research has shown that the signs of the coefficients in degenerate rows are important for the interpretation of the dual values /8,p.255;9;10,pp.254-255/: If all coefficients of a primal slack variable in the degenerate rows are nonpositive (nonnegative), the accompanying dual value can be interpreted as a left-hand- (right-hand-) derivate of the objective function for modifying the value of the original RHS-element. If both signs exist in the degenerate rows, the dual value given cannot be interpreted as a derivate: The critical region for which the value holds (cf. e.g. /6,p.116/) diminishes to zero. The possibility of many optimal values existing for some structural variables of the corresponding dual problem has led to the idea that there may be "many-sided shadow prices" for resources /4/. However, "price" is an economic and not a mathematical notion: Although one may mathematically obtain many optimal values for a dual structural, at most two of them are meaningful from an economic point of view /2;3,p.130;7;9;10,p.255/.

The "reduced cost" of an nonbasic primal structural gives the amount by which the corresponding cost coefficient must be increased for this primal structural to become basic. In primal degenerate optima the reduced cost is meaningful if the corresponding column does not possess positive coefficients in degenerate rows /10,p.254/: The pivot element of a regular simplex iteration using this column would be in a nondegenerate row and the primal structural would become positive. Therefore, only one of the probably several optimal values of a dual surplus variable may be interpreted as reduced cost.

AN ALGORITHM FOR DETERMINING SEVERAL DUAL SOLUTIONS

In order to find another dual solution to a primal degenerate optimum, a dual simplex iteration can be performed with a degenerate row as pivot row /9;10,p.253;12,p.74-75/. Dual simplex iterations are usually carried out to reduce primal infeasibility and to maintain dual feasibility. In the application discussed, the primal solution remains unchanged and dual feasibility is maintained. The pivot element of a dual simplex iteration is negative and the sign of a coefficient which is inappropriate for the information wanted is changed. To obtain all meaningful dual values, usually only a (small) part of all optimal solutions must be generated. The algorithm sketched has been implemented as an ECL-program in connection with IBM's MPSX/370; the implementation is described in /11/. The ECL-program can be obtained from the author.

PRIMAL DEGENERACY AND COST COEFFICIENT RANGES

The optimality range of a cost coefficient, $\delta_j \leq c_j \leq \gamma_j$, represents the region over which the current primal solution remains optimal. If this range is computed from one primal degenerate tableau and the column which determines the tightest upper or lower bound on the cost coefficient has at least one positive coefficient in the degenerate rows, the bound obtained may not be meaningful: If the simplex criterion were violated in the column regarded, the entering variable would remain at zero after the regular simplex iteration. Therefore, only columns without positive coefficients in degenerate rows are relevant for the determination of cost ranges. If the algorithm described above is used to determine meaningful dual values, meaningful cost ranges can be computed with little additional effort for all positive structurals in one sequence of steps: Each time a column without positive coefficients in the degenerate rows is found, meaningful upper or lower bounds can be computed on all cost coefficients of basic variables with a nonzero coefficient in this column. If the bound obtained is tighter than the previous available bound, the former is replaced.

DUAL DEGENERACY AND COST COEFFICIENT RANGES

At a dual degenerate and primal nondegenerate optimum for a problem with objective function coefficients $c_1=1$ and $c_2=1$, Evans and Baker /5/ present two different primal solutions and compute cost coefficient ranges $\infty \geq c_1 \geq 0$ and $\infty \geq c_2 \geq 0$ by taking the union of the individual ranges. These results do not correspond to the usual definition of cost ranges because two different primal solutions are considered and sensitivity analysis usually provides information about cost changes which do not modify the optimal primal solution. Dual degeneracy alone does not prevent the usual sensitivity analysis: The only speciality is that a degenerate column provides an upper (lower) bound equal to the original cost coefficient for each basic variable with a negative (positive) coefficient in the degenerate column.

PRIMAL AND DUAL DEGENERACY AND COST COEFFICIENT RANGES

In /10,pp.256-260/ it is shown that dual values obtained by the procedure discussed are not influenced by dual degeneracy. The meaningful determination of cost coefficient ranges for a given primal degenerate solution is possible even if several alternative primal optima exist. Thus, dual degeneracy does not cause additional problems in finding meaningful dual values and cost coefficient ranges.

EXAMPLE

Consider the linear program P:

$$\begin{aligned}
 \text{Max } Z &= 100 x_1 + 150 x_2 + 160 x_3 \\
 \text{s.t. } & \frac{1}{2} x_1 + 1 x_2 + 1 x_3 \leq 125 \\
 & 1 x_1 + \frac{1}{2} x_2 + 1 x_3 \leq 100 \\
 & 1 x_1 + 1 x_2 + 1 x_3 \leq 150 \\
 & 1 x_2 + 1 x_3 \leq 100 \\
 & x_1, x_2, x_3 \geq 0
 \end{aligned}$$

Four primal degenerate optimal tableaus T1 to T4 for problem P are given in Table 1. The cost coefficient ranges obtained indiscriminately from the individual tableaus are shown in Table 2, those determined by taking only the appropriate columns of the different tableaus into account, are given in Table 3. A comparison shows that all meaningful cost bounds result from columns with nonpositive coefficients in degenerate rows. There is no finite upper bound on c_2 although the traditional computation carried out by standard software would claim so in the tableaus T1, T3 and T4.

CONCLUSION

A lot of confusion in sensitivity analysis at primal degenerate optima can only be avoided by considering several optimal tableaus. The appropriate supplementation of standard software is highly recommended to encourage the practical use of dual values and sensitivity analysis in decision making. This seems to be far more valuable than the mere change in notation recommended in /1/.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	0	0	$20/3$	$400/3^*$	$100/3$	0	0	20 000
T1	1	0	$2/3$	$-2/3$	$4/3$	0	0	50
	0	1	$2/3$	$4/3$	$-2/3$	0	0	100
	0	0	$-1/3$	$-2/3$	$-2/3$	1	0	0
	0	0	$1/3$	$-4/3$	$2/3$	0	1	0
	0	0	40^*	0	100^*	0	100^*	20 000
T2	1	0	$1/2$	0	1	0	$-1/2$	50
	0	1	1	0	0	0	1	100
	0	0	$-1/2$	0	-1	1	$-1/2$	0
	0	0	$-1/4$	1	$-1/2$	0	$-3/4$	0
	0	0	0	0	20	80^*	60	20 000
T3	1	0	0	0	0	1	-1	50
	0	1	0	0	-2	2	0	100
	0	0	1	0	2	-2	1	0
	0	0	0	1	0	$-1/2$	$-1/2$	0
	0	0	0	120	20	20	0	20 000
T4	1	0	0	-2	0	2	0	50
	0	1	0	0	-2	2	0	100
	0	0	1	2	2	-3	0	0
	0	0	0	-2	0	1	1	0

Table 1: Four primal degenerate optima. Columns suited to compute a cost coefficient bound are marked by *.

	δ_1	γ_1	δ_2	γ_2	δ_3	γ_3
T1	90	300	140	200	$-\infty$	500/3
T2	20	300	110	∞	$-\infty$	200
T3	20	160	110	160	150	200
T4	90	160	140	160	150	500/3
Union	20	300	110	∞	$-\infty$	200

Table 2: Cost coefficient bounds derived indiscriminately from four primal degenerate optima and their unions.

	δ_1	γ_1	δ_2	γ_2	δ_3	γ_3
T1	-	300	50	-	-	-
T2	20	300	110	-	-	200
T3	20	-	110	-	-	-
T4	N o t c o n s i d e r e d					
Meaningful	20	300	110	∞	$-\infty$	200

Table 3: Cost coefficient bounds determined by taking only appropriate columns into account.

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