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A simulation study of simplification strategies in the development of optimization models

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Nr. 96

A SIMULATION STUDY OF SIMPLIFICATION
STRATEGIES IN THE DEVELOPMENT OF
OPTIMIZATION MODELS

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A Simulation Study of Simplification Strategies in the Development of Optimization Models

Summary

A major decision in the development of optimization models concerns the degree of accuracy to be used in the model. In practice, this simplification decision is widely met in an ad hoc fashion. Recently, several ways of computing (worst-case) bounds for simplifying linear programming (LP) models have been developed. On the other hand, the need for empirical studies of simplification effects has been expressed.

This paper discusses LP-models for product-mix planning in a job-shop in which manufacturing options exist. In this decision situation some simplification (compared to the data used in nonsimultaneous planning) is often necessary to obtain a computationally manageable LP model. Several simplification strategies are defined and applied in the simulation study.

Properties of the solutions obtained after applying the different simplification strategies are compared to those of the "real" optimum. Significant differences of the objective function values allow ranking of the simplification strategies and giving recommendations for practical model development.

A Simulation Study of Simplification Strategies in the Development of Optimization Models

1. Introduction

Several decisions in the development of optimization models determine benefits and costs of such models /13,p.8;15/. Multiple objectives influence the decision for the construction of a certain model /14/. In recent years several recommendations were developed to guide the practical modelling effort. These recommendations may be based on deductive or inductive research. A typical example of deductive research is complexity theory (surveyed e.g. in /23/) which compares e.g. the efficiency of different algorithms. Deductive results have been obtained primarily for rather simple problems such as searching, sorting etc. Hence, most recommendations are derived empirically and have to be qualified as "rules of thumb".

Recommendations based on practical experience may differ and are usually not derived scientifically. Therefore, simulation studies of the typical actions available in the development of optimization models have gained some importance: In countless papers the computational properties of optimization algorithms giving the same solution have been experimentally compared; a survey on such

experiments in mathematical programming is given in /8/. Different possibilities of implementing a given algorithm have been collated /27/ and several formulations giving the same optimum have been discussed in mathematical programming /5; 13; 15; 18; 22; 31; 32; 33,p.183; 34/. In all these cases a cost-comparison is adequate because benefits are implicitly assumed to coincide. The cost-comparison is usually approximated by comparing CPU-times.

Sometimes benefits of different heuristic algorithms are implicitly compared to their costs by giving solution values and CPU-times (e.g./35; 36/). However, very few comparisons in mathematical programming quantitatively discuss the benefits of different problem formulations. In these papers /1; 24/ only one data constellation is analysed, only a few optimization runs are executed and no statistical analysis is performed. Hence, further results on the consequences of using differently formulated LP-models may be of interest.

Recently, much deductive research has been done on the effects of aggregation in optimization models. This research provides several methods of computing a-posteriori and a-priori bounds on the differences between the maximum values of the objective functions (MVOF) of micro and aggregate models /3; 4; 10; 17; 19; 20; 30; 37; 38; 39; 40/. In contrast to these remarkable results obtained from sophisticated analyses, the practical importance of such bounds may be smaller if

- atypical model structures are assumed to compute the bounds
- the computed bounds are not tight
- the computation of the bounds is inconvenient in practical applications.

The need for empirical studies in aggregation theory is emphasized by Geoffrion /4/. The present paper gives empirical results for the development of linear programming (LP) models for product-mix planning. The aim of such models is to determine how much of various products should be manufactured within a certain planning period. Especially in job-shop technologies, a "micro" LP model using all the information available in the elementary form in which it is stored in a data base would need many thousands of balance equations and is therefore unsolvable even with today's powerful optimization software /13,p.132; 26,p.51/: Some kind of aggregation is absolutely necessary even in this most frequently cited optimization application.

In applications usually a non-simplified description and therefore a "true" micro model is missing /25/. In the application discussed, it is obvious to use a LP-model containing the data used for nonsimultaneous planning as the planner's micro model.

It has been shown that the number of rows in a micro job-shop product-mix LP can often be reduced to a manageable size by coupling activities in such a way that balance equations are eliminated /13,p.135/. Balance equations are rows with RHS-elements equal zero. They coordinate the levels of processes which use an item with the levels of processes which provide this item. Activity coupling results in a "perfect" aggregation because the same value of the objective function and the same optimal solution can be derived from the transformed model. Such a perfect aggregation has been recommended by Müller-Merbach and Wiggert (e.g. in /22; 31/) especially for situations in which no manufacturing options exist. Such a technology can be described by the well-known gozinto-graphs /28, p.429/. Under this technological condition, perfect aggregation reduces the number of rows and the number of structurals by an identical number. Using the available commercial software, the resulting models can be optimized without computational problems although matrix generation may be quite expensive.

However, in industrial decision making, manufacturing options or purchase-make options are often available and are one of the main reasons for using LP-models. The existence of manufacturing options led to the development of special storage techniques in data bases for bills of materials and operations lists /6; 7/ as well as to the development of generalized gozinto graphs with several node types /29/. If many options are available in several

production stages, perfect aggregation needs such an enormous number of structural variables that matrix generation and optimization may be unsuccessful /13,p.106; 15/. In this case one could decide against perfect aggregation and accept an aggregation loss which results from the use of a simplified model. In the practical development of optimization models, simplification is usually applied in an ad hoc fashion. This paper determines exact effects of using differently simplified product-mix models by simulation.

2. The Design of the Simulation Study

2.1. The Decision Situation

Object of the study are static product-mix decisions in which 20 final products, 60 intermediate products, 20 materials and 15 capacities are involved. The numbers given are small compared to practical job-shop situations in which several hundred final products and many ten-thousands of intermediates and materials may exist. The model size was chosen to allow the optimization of the micro model even if several options are available for many production processes. The size of the micro model was restricted by the comparatively small computer system and by the non-commercial optimization program COSIMA /2/ available; both together imposed rather tight constraints on the number of rows and structurals manageable. Furthermore, the results presented are based on more than 1000 optimization runs and the costs of the experiments were to be kept reasonable.

Some details of the structure (e.g. purchase and sale of intermediate products, bounds for purchasing and selling activities) and the numerical coefficients of the micro model are randomly generated. An earlier (but only slightly modified) version of the problem generator is described in /13,p.227/. The structure as well as the numerical coefficients of the micro model are attempted to be realistically determined. Each production process requires three different materials or inter-

mediates to manufacture exactly one item. The number of capacities needed for one production process is controlled by the parameter CAPCOFF whose values in different decision situations are given in Fig. 1. "Reasonable" coefficients in the objective functions are generated by using rough estimates of the dual values of resources and items. The "average" micro model of the experiments performed is shown in Table 1. All upper bounds were treated as rows because the program used does not include an upper bounding technique.

<u>Rows:</u>	1	objective function
	15	capacity constraints
	20	balance equations for final products
	60	balance equations for intermediate products
	20	balance equations for materials
	12	upper bounds on the sale of final products
	2	upper bounds on the purchase of intermediates
	4	upper bounds on the sale of intermediates
	<u>134</u>	rows

<u>Structurals:</u>	20	sales variables for final products
	6	sales variables for intermediate products
	6	purchase variables for intermediate products
	20	purchase variables for materials
	20	production variables for 20 final products
	40	production variables for 40 intermediate products without manufacturing options
	20.v	production variables for 20 intermediate products with manufacturing options
	<u>132+20.(v-1)</u>	structural variables

Table 1: Structure of the "Average" Micro Model

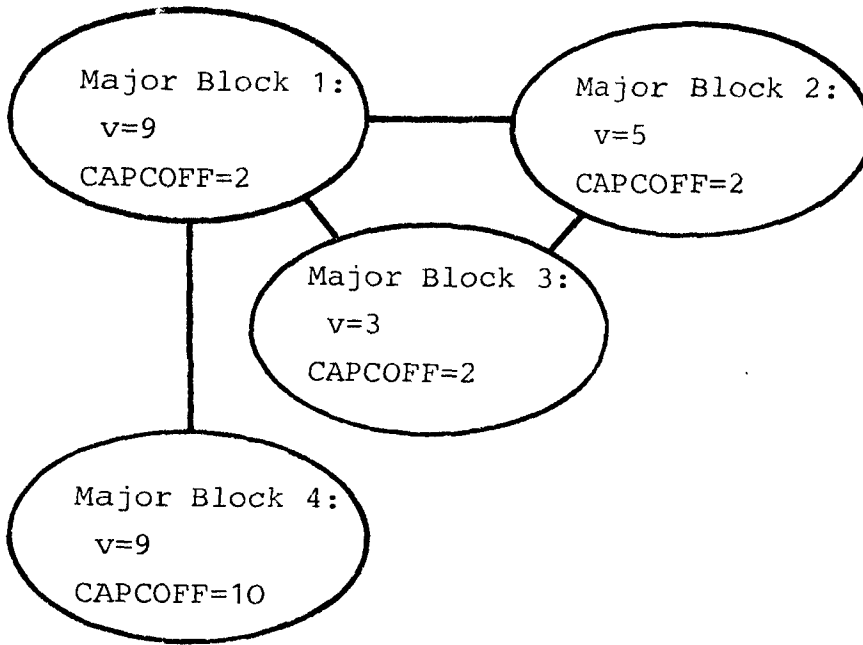


Fig.1: Problem Classes Examined

The 4 major blocks illustrated in Fig. 1 have been examined. The arcs connect nodes which differ in only one parameter value. Different major blocks were chosen to consider the influence of parameter variations on several criteria. For each major block t , $n=40$ micro models were generated, optimized, differently simplified and again optimized.

Effects of some reformulations for the structure of the "average model" of major blocks 1 and 4 are shown in Table 2.

Model	Description of the Model	Number of	
		Rows	Columns
1	Micro Model	134	292
2	Perfectly aggregated model by eliminating all balance equations from Model 1	34	many millions
3	Perfectly aggregated model by eliminating balance equations from Model 1 for those items for which no options exist	58	216
4	Simplified model of Model 1 with one manufacturing variable for each item	134	132
5	Perfectly aggregated model of the simplified Model 4	34	32

Table 2: Alternative Model Structures for the "Average Model"

Of the first three formulations described in Table 2, only Formulation 3 may be computationally manageable in a practical application with many options available. However, even in Formulation 3, too many rows may exist if several thousand items can be differently provided. Thus, it depends on technological conditions and the EDP-system available whether simplification is absolutely necessary in developing a product-mix LP for a job-shop.

To have better understanding of the model and especially its dual solution, a unique sign convention is recommended for all balance equations /13,p.94/. Using an item is specified by positive coefficients and providing an item by negative coefficients in the models generated. Under this sign convention all dual values of the balance equations will be nonnegative because there are no joint products. The model does not contain any greater-equal constraints: Therefore all dual variables will be nonnegative in the optimum and each model has feasible solutions.

All v options generated for manufacturing one product require an equal number of the three items and also use the same capacities. However, the coefficients in these capacity constraints and in the objective function differ considerably between the manufacturing options. The coefficients vary independently in such a way that each coefficient of the first process generated is the mean value of a slightly curtailed normal distribution. Thus, the options generated will probably offer more substitution possibilities than those available in practice and simplification will be more incisive than in many real-world applications.

Generating the coefficients of the options independently may result in inefficient processes. A process r is called inefficient if

$$(1) \quad \exists s: \begin{pmatrix} -c_s \\ a_{is} \end{pmatrix} \leq \begin{pmatrix} -c_r \\ a_{ir} \end{pmatrix}.$$

The optimal dual values $u_i^* \geq 0$ will result in

$$0 \leq \sum_i a_{is} u_i^* + c_s \leq \sum_i a_{ir} u_i^* - c_r.$$

The use of process r can never give a better result than the use of process s : Process r can be dropped from the LP model as inefficient.

2.2. The Simplification Strategies

In this study it is assumed that due to computational requirements only one manufacturing variable for each product is incorporated into the simplified product-mix model. This simplification allows for a perfect aggregation in a second step in which the balance equations can be eliminated without obtaining additional structurals. The main goal of the simplification is to reduce the number of rows; this is achieved by reducing the number of columns first. This procedure preserves feasibility which is usually lost in direct row aggregation /37,p.63; 39/.

To allow for the size reduction obtained with Model 5 of Table 2, the options available in manufacturing one item must in some way be aggregated. This is a typical case where the situation modelled implies the group of variables to be aggregated as will often happen in structured micro models of real-world applications. Some of the simplification strategies represent "typical" aggregation in assigning positive weights to several variables. Others only select a variable from each group and may therefore be regarded as a "degenerate" aggregation using only one positive weight in each group. For this study the notion "simplification" includes the notions "aggregation" and "selection".

The strategies defined in Table 3 were applied in the experiments. A few comments on the strategies might be appropriate. Strategy S_1 serves as reference for the comparisons: Are there strategies which can significantly improve random selection? Strategy S_2 is based on the idea that in the absence of further information equal weights for all options may be plausible. It is often used in literature (e.g./17,p.126; 20; 37,p.57; 40/). However, further information is available and used in the following strategies. S_3 and S_4 exclude evidently unreasonable modelling by first applying the efficiency criterion (1). Strategy S_5 only regards direct costs while neglecting opportunity costs. Strategy S_6 implicitly assumes high and common opportunity costs for all capacities: Direct costs can then be neglected.

Strategies S_7 to S_9 take the numerical values of the objective function coefficient as well as those of the capacity coefficients into account. The way this is realized differs: Strategy S_7 combines both influencing factors by multiplication. Strategy S_8 uses an estimated dual value which is supposed to hold for all capacity constraints; a kind of rudimentary simplex criterion is computed under this assumption. Strategy S_9 uses individual estimates of the dual values y_i to build the simplex-like selection criterion. In contrast to strategies S_1 to S_7 the strategies S_8 and S_9 employ subjective elements: The results for S_8 and S_9 depend on the estimates chosen.

S ₁	<p>"Random selection":</p> <p>Randomly select one of the options available.</p>
S ₂	<p>"Simple aggregation":</p> <p>Select an "average" process which is computed using common weights for all options available.</p>
S ₃	<p>"Random selection of efficient processes":</p> <p>Randomly select one of the efficient options available.</p>
S ₄	<p>"Simple aggregation for efficient processes":</p> <p>Select an "average" process which is computed by using common weights for all efficient options available.</p>
S ₅	<p>"Direct-cost-oriented selection":</p> <p>Select the process of which the objective coefficient is best.</p>
S ₆	<p>"Capacity-oriented selection":</p> <p>Select the process for which the sum of the capacity coefficients is minimal.</p>
S ₇	<p>"Multiplicative direct-cost-capacity-oriented selection":</p> <p>Select the process which minimizes the product</p> $ c_j \cdot \sum_{i \in \text{CAP}} a_{ij} \quad .$
S ₈	<p>"Simplex-oriented selection with equal opportunity costs":</p> <p>Select the process which minimizes</p> $ c_j + y \cdot \sum_{i \in \text{CAP}} a_{ij}$ <p>where y is an opportunity cost equal for all capacities.</p>
S ₉	<p>"Simplex-oriented selection with individual opportunity costs":</p> <p>Select the process which minimizes</p> $ c_j + \sum_{i \in \text{CAP}} a_{ij} \cdot y_i \quad .$

Table 3: Definition of Simplification Strategies

In the construction of the micro models uniformly distributed rough estimates $0 \leq y_i \leq 80$ were used for 80 % and $y_i = 0$ were used for 20 % of the capacities. These rough estimates y_i were employed in strategy S_9 . For a fair comparison the mean value $y = 32$ was used in S_8 as a common estimate for all dual values of the capacity constraints. Strategy S_9 will give better results than S_8 if good estimates y_i exist. However, the analysis of the optimal solutions of the micro models showed only a modest correlation between the y_i and the actual dual values ($\rho = .22$). In reality better estimates of the dual values may be available from an earlier optimization run.

In the experiments described no recursive optimization procedures are employed: The consequences of iterative "column generation" by using dual values of the earlier optimization run are not discussed here. One reason for this is that in practice the "perfect" aggregation necessary after the simplification may be quite expensive compared to the small improvement that will be possible after using an appropriate simplification strategy.

2.3. The Areas Examined

The goal of the study is to find an answer to the question "How does the application of simplification strategies modify properties of the LP solutions ?". Properties discussed are the MVOF, capacity utilization and the number of final products manufactured.

It is interesting that by deductive reasoning only the ranking $S_4 \succeq S_2$ can be derived: Equal weights only for efficient processes will never give a worse result than equal weights for all processes. Many other rankings like $S_3 \succeq S_1$ are plausible but do by no means hold for all individual cases. Therefore it is of interest whether significant differences among strategies can be experimentally derived.

If manufacturing options are "lost" by simplification of the model, one could assume that generally there will be more idle capacity in the optimal solution of the simplified model than in the micro model. Furthermore, one could assume that in simplified models the number of final products manufactured rises because final products, which are unattractive from the micro models view, allow a better use of the available resources. No deductive results can be obtained for these effects.

2.4. Some Statistical Aspects of the Study

In the terminology of statistical literature, the application of a certain simplification strategy is regarded as "treatment". This terminology goes back to the literature on experimental design (surveyed e.g. in Kleijnen /11,p.287/) which emerged from agricultural research.

The results of the experiments will be described by comparing the MVOF z_{it} of the micro model to the MVOFs z_{itj} resulting from the application of strategy S_j . Often the difference $z_{it} - z_{itj}$ is used as evaluation criterion for aggregate models /4; 17,p.119; 37,p.57; 39; 40/. The percentual aggregation result

$$p_{itj} = \frac{z_{itj}}{z_{it}} \cdot 100 \%$$

may be more meaningful if the z_{it} differ remarkably with i . We denote the average aggregation result as

$$\bar{z}_{tj} = \frac{\sum_{i=1}^n z_{itj}}{n} \cdot 100 \%$$

the average percentual aggregation result as

$$\bar{p}_{tj} = \frac{\sum_{i=1}^n p_{itj}}{n} \cdot 100 \% ,$$

and the rank derived from ordering z_{itj} ($\forall j$) as r_{itj} .

As already mentioned, a two-way layout is used in the experiments. Such a design is characterized by blocks with identical resp. similar initial conditions. In the experiment described, blocking was used in two ways: First, the above mentioned major blocks of decision situations were determined by the parameters used to generate the micro model. The minor blocks result from a certain starting value of the random number generator which (together with the parameters describing a major block) determines a unique micro model. Each case i constitutes a minor block. Several simplified models are deduced from one micro model by applying the strategies described above.

The observations for each case i depend on common random numbers which are used to compare systems "under the same circumstances" /11,p.200/. The advantage of using common random numbers is shown in the following example: Suppose that the application of two strategies S_u and S_w results in

$z_{1t} = 100$	$z_{2t} = 200$
$z_{1tu} = 95$	$z_{2tu} = 180$
$z_{1tw} = 97$	$z_{2tw} = 186.$

Using individual observations, one might obtain $p_{1tu} = 95\%$ and $p_{2tw} = 93\%$. This would suggest that S_w is worse than S_u . However, the ranks $r_{itu} = 2$ and $r_{itw} = 1$ ($i=1,2$) show that for both cases S_w performs better than S_u . By the use of common random numbers the sample size is reduced but statistical analysis of the results may become more complicated.

In the experiments described a typical analysis of variance would test the null hypothesis

$$H_0: \bar{z}_{t1} = \bar{z}_{t2} = \dots = \bar{z}_{t9}.$$

However, it is often evident that at least one significant difference exists between treatments: The experimenter is more interested in which of these treatments differ significantly or whether the different treatments can be classified into subsets of similar properties. Therefore, multiple comparison procedures have been developed for comparing several treatments simultaneously. Kleijnen /12/ mentions that these techniques have thus far been extremely rarely applied in the analysis of simulation experiments. This study employs a sign test for multiple comparison of the simplification strategies. This nonparametric test avoids the assumptions of normality and homogeneous variances and controls the error rate per experiment (and not, as is often done, for the individual comparisons). The analogous signed-rank test is not employed because this test is not really distribution-free /21,p.140/. We follow the recommendation that when a "sign test... is powerful enough to do the job requested, why fool around with the rank test" /21,p.176/.

3. Influences of Simplification on Solution Properties

3.1. The Maximum Value of the Objective Function

3.1.1. Description

In Table 4 the \bar{p}_{tj} are given for different major blocks t and strategies S_j . In major blocks 1 and 2 all 9 simplification strategies were used. However, some of the ineffective strategies were not applied in blocks 3 and 4. Let $J(t)$ be the set of all strategies S_j used in major block t . Table 4 shows that the best and the worst strategy are identical for all major blocks:

$$\bar{p}_{t8} \geq \bar{p}_{tj} \quad j \in J(t), \quad \forall t$$

$$\bar{p}_{t2} \leq \bar{p}_{tj} \quad j \in J(t), \quad t=1,2.$$

Thus, the simple aggregation strategy S_2 often considered in literature performs very poorly.

t	1	2	3	4
S_1	82.30	87.92	-	-
S_2	82.24	87.42	-	-
S_3	87.76	90.81	-	-
S_4	88.25	91.64	-	-
S_5	91.39	94.95	-	-
S_6	91.60	93.55	-	-
S_7	95.07	96.79	98.72	97.34
S_8	96.93	97.68	98.81	98.18
S_9	95.62	97.17	98.41	97.95

Table 4: Average Percentual Objective Function Value Obtained for Simplification Strategy S_j in Major Block t

3.1.2. Test

A distribution-free multiple comparison procedure based on signs and described by Miller /21,p.138/ is used to test the null hypothesis whether the treatment effects

$$(H_0-1) \quad \tau_1 = \tau_2 = \dots = \tau_9$$

are equal for all cases of the major blocks 1 and 2 at an approximate experimentwise error rate α . For each of the $\binom{9}{2} = 36$ pairs of strategies S_u and S_w , a statistic S_{uw} is calculated and tested against a critical constant c which depends on the error rate and the number of treatments compared. If at least one statistic $S_{uw} \geq c$ exists the null hypothesis (H_0-1) is rejected and the performance of the strategies S_u and S_w can be considered as significantly different at level α . Table 5 marks the differences regarded as significant at the usual levels

$\alpha = .001$, $\alpha = .01$ resp. $\alpha = .05$ as $<<<$, $<<$ resp. $<$. Those pairs of strategies which are not significantly different at $\alpha = .05$ are signed as \sim .

The individual rankings in the major blocks 1 and 2 correlate highly. However, the smaller samples don't show all differences as clearly as the joint analysis does.

	S ₂	S ₃	S ₄	S ₅	S ₆	S ₇	S ₈	S ₉
S ₁	~	<<<	<<<	<<<	<<<	<<<	<<<	<<<
S ₂		<<<	<<<	<<<	<<<	<<<	<<<	<<<
S ₃			~	<<<	<<<	<<<	<<<	<<<
S ₄				<<<	<<<	<<<	<<<	<<<
S ₅					~	<<<	<<<	<<<
S ₆						<<<	<<<	<<<
S ₇							<<	~
S ₈								~

Table 5: Significance of the Differences between the MVOFs for all Pairs of Simplification Strategies in Major Blocks 1 and 2

The results are summarized verbally as follows:

(H₀-1) is rejected.

There are far more significant differences than undecided rankings. Therefore the latter are mentioned:

No significant ranking is possible between S₁ and S₂.

No significant ranking is possible between S₃ and S₄.

This shows that the simple aggregation strategies often considered do not lead to significantly better results than those obtained by random selection!

No significant ranking is possible between S₅ and S₆ which both use only part of the information available when selecting the variables for the simplified model.

The strategies S_7 , S_8 and S_9 , which use all the information available, worked significantly better ($\alpha = .001$) than all other strategies. Therefore, S_1 to S_6 were dropped from the comparisons in major blocks 3 and 4. A joint comparison of the rankings obtained in the three major blocks with sparse capacity constraints shows that $S_8 \succ S_7$ holds at $\alpha = .01$ while the ranking $S_8 \succ S_9$ holds only at $\alpha = .05$; S_7 and S_8 do not significantly differ at $\alpha = .05$. In major block 4 both strategies S_8 and S_9 perform significantly better than S_7 at $\alpha = .01$ but no significant difference between S_8 and S_9 exists.

These results are illustrated in Fig. 2 in which the simplification strategies are grouped into subsets of similar performance. The exact position of S_9 depends on the major block regarded: In major blocks 1 to 3, the performance of S_9 more closely resembles the performance of S_7 than that of S_8 . In major block 4 individual dual values of the capacities are more important than in other major blocks because of the large number of nonzeros and S_9 is therefore closer to S_8 .

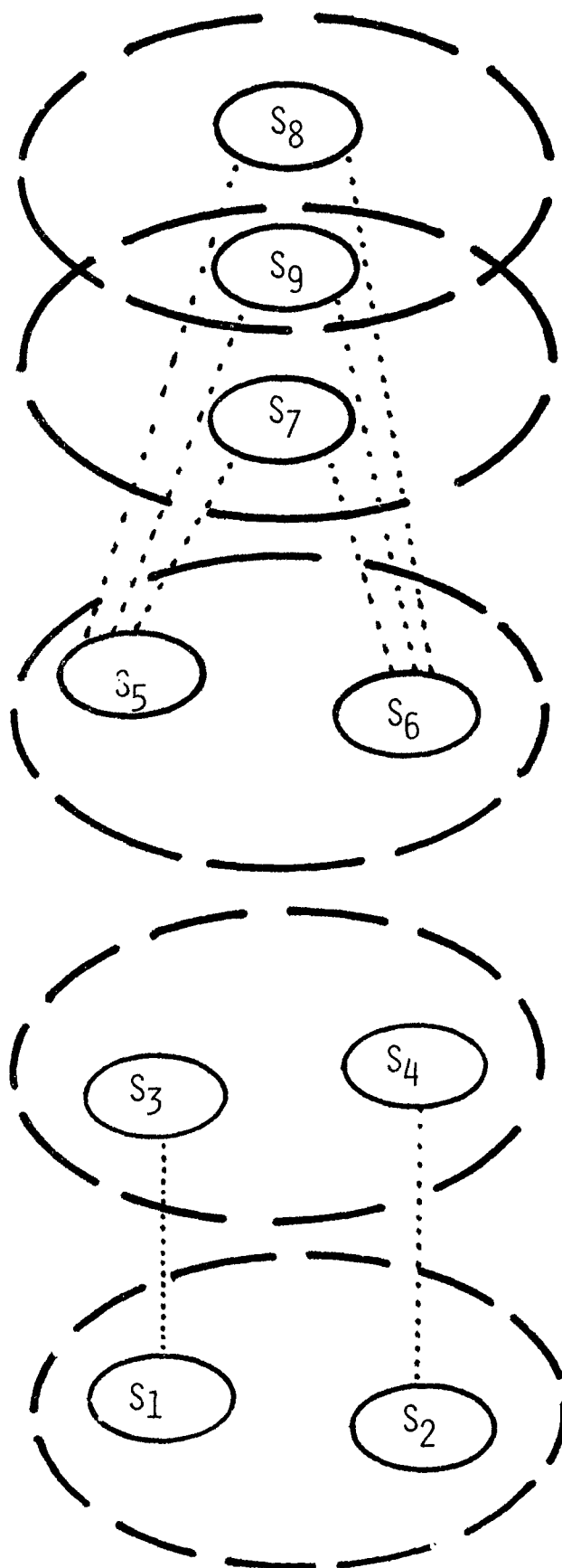


Fig.2: Ranking the Simplification Strategies into Subsets with Similar resp. Significantly Different Performances

3.2. Capacity Utilization

3.2.1. Description

For ease of presentation the micro model is called S_{10} in the following. Let f_{itj} denote the amount of idle capacities in the optimal solution resulting from the application of strategy S_j ($j=1, \dots, 10$) and F_{it} the sum of capacities available. The average capacity utilization obtained by strategy S_j for certain major blocks $T \subseteq \{1, 2, 3, 4\}$ is given by

$$\bar{f}_{Tj} = \frac{\sum_{i=1}^n \sum_{t \in T} f_{itj}}{\sum_{i=1}^n \sum_{t \in T} F_{it}} \cdot 100 \% .$$

The values \bar{f}_{Tj} obtained for the single major blocks t are given in Table 6. The micro models allow a better capacity utilization than the simplified models. In major block 4 the dense capacity constraints allow a better capacity utilization than the sparse constraints in the other major blocks.

t	1	2	3	4
S ₁	91.55	90.83	-	-
S ₂	92.09	91.07	-	-
S ₃	90.62	90.72	-	-
S ₄	91.93	91.22	-	-
S ₅	91.39	91.06	-	-
S ₆	90.15	91.02	-	-
S ₇	91.97	91.19	91.69	96.40
S ₈	91.12	91.08	91.66	96.15
S ₉	91.27	91.29	91.66	96.43
S ₁₀	95.26	93.25	93.14	98.35

Table 6: Average Capacity Utilization Obtained for Simplification Strategy S_j in Major Block t

3.2.2. Test

The procedure described in section 3.1.2. is used to test

$$(H_0-2) \quad \bar{f}_{T1} = \bar{f}_{T2} = \dots = \bar{f}_{T,10} \quad T = \{1, 2\}$$

at an (approximate) experimentwise error rate α . The results can be summarized as follows:

(H₀-2) is rejected only because significant differences between $\bar{f}_{T,10}$ and \bar{f}_{Tj} (j= 1,...,9) exist. Although all kinds of simplified models give a significantly smaller capacity utilization than micro models, no pair of simplified models significantly differs in capacity utilization at $\alpha = .05$.

3.3. Number of Final Products Manufactured

3.3.1. Description

Let h_{ijt} denote the number of final products manufactured in the optimal solution resulting from the application of strategy S_j ($j=1, \dots, 10$). The number of final products manufactured in the solution obtained after application of strategy S_j compared to the maximum number of final products possible is given by

$$\bar{h}_{Tj} = \frac{\sum_{i=1}^n \sum_{t \in T} h_{itj}}{n \cdot |T|}$$

where $|T|$ is the number of major blocks regarded. The values \bar{h}_{Tj} obtained for the single major blocks are given in Table 7.

Contrary to the above assumptions, there seems to be a slight tendency to manufacture fewer final products in simplified models. The significance of the observed differences is tested in the next subsection. It is interesting to note that the dense capacity constraints of major block 4 result in many more final products manufactured than the sparse constraints of the other major blocks.

t	1	2	3	4
S ₁	10.38	10.23	-	-
S ₂	10.58	10.03	-	-
S ₃	10.68	10.20	-	-
S ₄	10.55	10.13	-	-
S ₅	10.53	10.05	-	-
S ₆	10.98	10.38	-	-
S ₇	10.63	10.35	10.85	15.53
S ₈	10.63	10.40	10.63	15.80
S ₉	10.33	10.48	10.58	15.53
S ₁₀	11.00	10.60	10.85	15.75

Table 7: Average Number of Final Products Manufactured after Application of Simplification Strategy S_j in Major Block t

3.3.2. Test

The procedure described in section 3.1.2. is used to test

$$(H_0-3) \quad \bar{h}_{T1} = \bar{h}_{T2} = \dots = \bar{h}_{T,10} \quad T = \{1,2\}$$

at an (approximate) experimentwise error rate α . The results can be summarized as follows:

(H₀-3) is not rejected at $\alpha = .05$. There is neither a significant difference between the number of final products in any pair of simplified models nor between any type of simplified models and micro models.

4. Conclusion

This paper describes a simulation study in which several strategies are used to simplify linear programming models. It is shown that "clever" selections of variables give significantly better results than simple aggregations of variables. A selection is clever if all "reliable" information available is used. The use of "aggregate information" (as the average estimate y in S_8) may be more reliable than detailed information of low quality (as the estimates y_i in S_9). The use of the information available should generally be coordinated with the subsequent use of the result: The simplex-based selection strategies S_8 and S_9 perform better than S_7 which uses the information available in a multiplicative and therefore non-simplex-oriented way.

If a second optimization run is performed to reduce the danger of poor results, the optimal dual values y_i of the first run are expected to be of high quality and S_9 is expected to operate best in a second simplification process. The model developer must decide whether he is willing to meet the cost of further matrix generation and optimization runs when the exact optimum may not be far more valuable than a near-optimal solution (cf. /9/).

Further analysis shows that by applying simplified models the solutions may be systematically biased. In the experiments this is the case with capacity utilization which is more sensitive to the use of simplified models than objective function values. Therefore, switching between similar processes (which is typical for the micro model's solution) can be avoided at a relatively low objective function loss. This is of special interest if the LP-assumption of being able to combine different processes at no additional cost ("Additivity postulate" /cf. 16,p.36/) is not fully justified by the technological conditions. However, no significant differences in the number of final products manufactured have been found between simplified and micro models. Thus, the empirical study shows significant differences between two assumptions about properties of the solutions which may have a priori been regarded as equally plausible.

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