

Crampes, Claude; Haritchabalet, Carole; Jullien, Bruno

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# ADVERTISING, COMPETITION AND ENTRY IN MEDIA INDUSTRIES

CLAUDE CRAMPES  
CAROLE HARITCHABALET  
BRUNO JULLIEN

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# ADVERTISING, COMPETITION AND ENTRY IN MEDIA INDUSTRIES

## Abstract

This paper presents a model of media competition with free entry when media operators are financed both from advertisers and customers. The relation between advertising receipts and sales receipts, which are both complementary and antagonist, is different if media operators impose a price or a quantity to advertisers. When consumers dislike advertising, media operators are better off setting an advertising price than an advertising quantity. We establish a relationship between the equilibrium levels (advertising and entry) and the advertising technology. In particular, media operators' profit is not affected by the introduction of advertising when they impose advertising quantities and when advertising exhibits constant returns to scale in the audience size. Under constant or increasing returns to scale in the audience size, we find an excessive level of entry and an insufficient level of advertising.

JEL Code: L13, L82.

Keywords: media, advertising, free entry, two-sided markets.

*Claude Crampes*  
*University of Social Sciences*  
*Manufacture des Tabacs*  
*21, allée de Brienne*  
*31000 Toulouse*  
*France*  
*ccrampes@cict.fr*

*Carole Haritchabalet*  
*University of Toulouse I*  
*GREMAQ*  
*France*  
*carole.haritchabalet@univ-tlse1.fr*

*Bruno Jullien*  
*University of Social Sciences*  
*Manufacture des Tabacs*  
*21, allée de Brienne*  
*31000 Toulouse*  
*France*  
*bjullien@cict.fr*

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# 1 Introduction

For any firm that intends to market its product, buyers must be viewed as a scarce resource. To transform potential buyers into active buyers, the minimum required is to inform the former about the existence of the product and about its physical, spatial and economic characteristics. In some circumstances, this transmission of information can be performed with a high degree of precision through personalized advertising, particularly for intermediary products sold to firms. But in most cases, the would-be seller only has a fuzzy idea on who and where potential buyers are. Like a fisher in cloudy water who uses large nets, the seller will have to use large scale media to get some chances of catching buyers. This procedure has two drawbacks. First it has low return since it is random both at the emission stage (the advertiser does not know whether receivers are potential buyers) and at the reception level (all the potential buyers will not receive the message). The second drawback is that non potential buyers also receive the message and this will probably disturb them if they have paid the media for its content: the interruption of a movie by commercials is likely to divert couch potatoes from channels that abuse of interruption.

On the media side, advertising can be the principal way to collect the funds necessary to pay programming costs.<sup>1</sup> For example, the technology used by TV broadcasting does not allow to exclude viewers.<sup>2</sup> Consequently, the media operator cannot break even without the help of public financing or advertising.<sup>3</sup> For media operators, the dilemma is as follows: to devote a large room for advertising increases revenues from manufacturers but decreases revenues from readers or watchers who are only ready to pay for the entertainment or documentary content. Likewise, the manufacturers accept to pay for advertising on condition that the media they invest in do have enough buyers.

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<sup>1</sup>In 2003, according to a study by the french Senat, advertising accounted for around 40 % of the revenues for TV and the press, 55 % for radio. Total media revenue in France were 10.24 billion euros, with 4 b. for press and 3.75 b. for TV. Advertising on Internet is also increasing very fast, it is evaluated at \$9.6 billion for 2004 in the US (IAB, PriceWaterhouseCooper).

<sup>2</sup>Thanks to decoders, some channels can exclude viewers who do not contribute to the production budget. (See Canal+ in Europe). Note also that the digital terrestrial TV will allow to control the access to selected broadcast channels.

<sup>3</sup>There exist some exceptions of media operators that balance their budget with voluntary contributions.

We construct a free entry model to analyze the relation between advertising receipts and sales receipts in the sectors of newspapers, magazines, cable TV, coded broadcast TV, etc. where consumers have to pay to read articles or to view programs. In these activities, services and ads are both complementary (they increase the total revenue of the media owner) and antagonist (they can be hardly increased simultaneously). We consider the Salop's model of horizontal differentiation. In a first stage, the media operators must decide whether to enter the market or not; if they enter, they incur a fixed entry cost. In a second stage, media operators compete in subscription prices to attract consumers, and decide how much to advertise. As regards advertising, media operators can either impose an advertising level (quantity game) or an advertising price (price game). This distinction is important because the interaction between advertising revenues and sales revenues is different in the two cases. In the quantity game, media operators compete in advertising volumes and the advertising price is fixed by the market at the willingness to pay of the marginal advertiser. The demand for advertising is thus independent of the demand for media services. In the price game, the demand for advertising depends on both the advertising price and the audience of the media. An increase in any price has thus a complex impact due to the two-sided nature of the interaction between the customers and the supply of ads.<sup>4</sup> At equilibrium, the amount of advertising is the same in the two games but the equilibrium subscription price is different. In the price game, the subscription price is lower when consumers value advertising and higher when consumers dislike ads. In the latter case, the profit of the media operators is thus smaller when they use a pricing rule that ties the price of an ad to its true value.

One important feature of the model is that we allow for different advertising technologies in the sense that we define how an increase in the audience modifies the unit value of an ad. We consider three cases. In the benchmark case, advertising exhibits constant returns to scale in the audience size and the unit value of an ad is just a fixed value per customer. In this case, we show that, in the advertising quantity game, the entry is the same as without advertising and consumers are better off. The welfare analysis indicates that entry is always excessive (and is more excessive when media operators set an advertising price) and that the level of advertising is insufficient. We also

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<sup>4</sup>See Jullien (2005) for an introduction to two-sided markets. For more detailed analysis, see Caillaud-Jullien (2001, 2003), Rochet-Tirole (2003), Armstrong (2004).

consider increasing returns in the audience so that a larger audience implies a larger unit value of an ad. We obtain the same welfare results : entry is excessive and advertising is insufficient, and the quantity game does better than the price game. Third, we consider the opposite case where increasing the audience decreases the unit value on an ad. The effect on firms profit and consumer surplus is directly linked to this ad unit value. Compared to a world without advertising, media operators always benefit from advertising when there are decreasing returns in the audience. Welfare results are ambiguous. We may obtain excessive entry with either excessive or insufficient advertising or insufficient entry with insufficient advertising.

The literature on this topic includes Anderson and Coate (2003), Gabszewicz, Laussel and Sonnac (2001a,b), Armstrong (2004), Dukes and Gal-Or (2003), Ferrando, Gabszewicz, Laussel and Sonnac (2003), Peitz and Valletti (2004) or Reisinger (2004) who develop spatial models of broadcasting competition in which two media operators compete in both programming and advertising levels. Papers by Gabszewicz, Laussel and Sonnac (2001) and Dukes and Gal-Or (2003) analyze the impact of advertising on the level of differentiation. They find that advertising reduces differentiation between media operators. More recently, Peitz and Valletti (2004) compare both the advertising intensity and the level of differentiation when media operators offer free services and when the subscription price is positive. Our model does not allow for endogenous differentiation apart from the impact of entry, but we also conclude that free services are associated with larger advertising levels when customers dislike advertising (the reverse holds if the consumers like advertising). The analysis of Anderson and Coate (2003) focuses on welfare issues. They show that equilibrium advertising levels can be above or below socially optimal levels and that media operators can provide too many or too few programs.

Our equilibrium analysis is close to Armstrong (2004) who presents the comparison between an advertising quantity game and an advertising price game in the Hotelling model. His results are akin to those presented in this paper : setting an advertising price modifies the elasticity of the demand function. The equilibrium subscription price is lower (resp. higher) when media compete in advertising quantity rather than in advertising price when consumers dislike (resp. like) advertising.

These models do not allow to endogenize the number of active media. Choi (2003) uses the Salop's model of horizontal differentiation to analyze broadcast competition between media stations that are only financed through

advertising. In his model, customers freely access to stations. His result is similar to Anderson and Coate's : there is no clear-cut answer to the effect of entry and advertising. Our analysis extends Choi's work by introducing price competition to attract viewers/readers. The results obtained using a setting similar to Choi's (advertising quantity game and constant unit ad value) are rather different. In our model, and under the assumption of constant returns to scale, advertising does not affect firms' profit and the level of entry is thus without ambiguity excessive. The benefit of imposing prices to consumers depends on the returns to scale of advertising on the audience. When returns to scale are constant or increasing and when consumers dislike advertising, pay-media profits are higher than free-media profits when the subscription price is positive.

The paper is organized as follows. In section 2, we construct a simple monopoly model in order to exhibit the main trade-offs in the double financing of media and we extend it to the multistation case using the Salop's model of horizontal differentiation. In section 3, we analyze the long run equilibrium of an industry with advertising resources in the Salop's framework. We present the advertising quantity game and the advertising price game and provide a comparison of the advertising and quantity models with a free media model. Section 4 presents the welfare analysis. Section 5 concludes.

## 2 A simple monopoly model

To get a first insight in the problem of balancing sale revenues and advertising revenues in the media industry, consider the simple case of a media operator that is a monopolist for its services.

### 2.1 Model setting

There is an entry cost  $k$ , and with  $q$  media "customers" paying for the service, the operating cost is  $cq$ . Advertisers are producers of goods and services who are interested in selling their products to customers and post ads on the media support. We do not provide a detailed analysis of how the producers decide to advertise in order to inform/persuade the customers to buy their goods and services. We just assume that the relationship between customers and advertisers is synthesized by a demand  $a$  for advertising spots. This advertising demand is a function  $a = A(s, q)$  increasing with the audience

$q$  and decreasing with the advertising price  $s$ , leading to an inverse demand  $s(q, a)$  that represents the willingness to pay for one advertising message aimed at reaching the  $q$  customers of the media.

Let us denote

$$v(q, a) = \frac{s(q, a)}{q}$$

the willingness to pay for an ad per customer. Define also

$$\varphi(q, a) \equiv v(q, a)a,$$

the advertising revenue of the media operator per customer. The total revenue from advertising is then

$$r(q, a) = q\varphi(q, a) = s(q, a)a,$$

that is assumed to be concave in  $a$ . We also assume that for  $q \leq 1$ ,  $v_q$  has a constant sign and  $|v_q|$  and  $|v_{qq}|$  are uniformly bounded and small enough (see section 3.2). In particular  $r$  is increasing with  $q$ .

This advertising demand function can be generated as follows. There is a continuum of advertisers indexed by  $\gamma$  who want to advertise their products, where  $\gamma$  is uniformly distributed on  $[0, 1]$ . Each of them may post one ad. The ad of advertiser  $\gamma$  reaches the  $q$  customers of the media, which generates a net cash-flow  $qv(q, \gamma)$  to advertiser  $\gamma$ . The function  $v(q, \gamma)$  is an index of the average efficiency of advertising, decreasing with  $\gamma$ . It depends on the characteristics of the media and on the geographical and sociological dispersion of the households. At unit price  $s$ , advertiser  $\gamma$  will post an ad iff  $qv(q, \gamma) \geq s$ . Therefore the marginal advertiser is  $\hat{\gamma}$  such that  $qv(q, \hat{\gamma}) = s$ , and the total demand for ads is defined as  $a = \int_0^{\hat{\gamma}} d\gamma$  since density is equal to 1. We deduce that  $qv(q, a) = s$  so that the indirect demand function for advertising is defined by  $s(q, a) = qv(q, a)$ .

Depending on the nature of the media,  $v(q, a)$  can have different forms. In the benchmark case,  $v(q, a)$  is a constant in  $q$ . Since the willingness to pay for the product is the same over consumers, manufacturer  $\gamma$  is willing to pay up to  $qv(q, a) = qv(0, a)$  to post an ad.

If there are some increasing returns from the audience size, we have  $v_q(q, a) > 0$ . This is the case when potential buyers are scattered among the audience of the media and some network effects allow to transmit information by word of mouth.



By contrast, when  $v_q(q, a) < 0$ , there are decreasing returns in advertising because potential buyers are concentrated and well identified in the set of households. An increase in the size of the audience of the media will bring a lower average revenue to announcers.

Assuming that the demand for subscription by readers/viewers depends on the subscription price  $p$  and on the level of advertising  $a$ , denote by  $Q(p, a)$  this demand. The drawback of advertising is that the willingness to pay for the services supplied by the media operator may be decreasing with  $a$ , because most users are interested in the content of articles or programs, not in the incentives to buy other products. These people use pop-up blockers on web browsers and digital video recorders to skip the ads when watch recorded programs. In some specific cases however, it can be true that customers enjoy ads.<sup>5</sup> Here, we don't impose that the demand is decreasing and assume instead that the demand function for the media  $Q(p, a)$  is continuously differentiable, decreasing with the subscription price  $p$  and has an inverted U shape with respect to the number of ads  $a$ . A small amount of ads may thus raise demand but beyond some point the impact becomes negative.

## 2.2 Balancing the two sources of revenues

The media monopolist's problem is

$$\max_{p,s,a,q} pq + sa - cq \tag{1}$$

$$\text{s.t. } q = Q(p, a) \tag{2}$$

$$a = A(s, q) \tag{3}$$

The cost of maintaining the media is supposed to be fixed, whatever its content. Plugging (3) into (2), for given prices  $p$  and  $s$ , the demand and the supply of content services is defined by  $q = Q(p, A(s, q))$ , that we assume to have a unique solution for simplicity.

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<sup>5</sup>It is the case for magazines specialized in fashion or cars. It can also be true for some specific events: "the Super Bowl is a showcase for television commercials, and more than a quarter of viewers tune in just to watch the ads" (The Economist, April 2nd 2005, 'A survey of consumer power', p.3).

We solve the maximization problem by inverting relations (2) and (3), and expressing the prices as functions of  $q$  and  $a$  : we obtain  $p = p(q, a)$ , and  $s = s(q, a)$ . The profit is then

$$p(q, a)q + r(q, a) - cq = (p(q, a) + \varphi(q, a) - c)q$$

that must be maximized with respect to  $q$  and  $a$ . Assuming an interior solution, the first order conditions are<sup>6</sup>

$$p(q, a) + qp_q(q, a) = c - r_q(q, a) \quad (4)$$

$$\varphi_a(q, a) = -p_a(q, a) \quad (5)$$

Both equations depict the usual equality between marginal revenue and marginal cost. In equation (5), the cost of advertising is represented by the decrease in sales revenue per customer  $p_a$  due to a lower willingness to pay for content. In condition (4), the marginal revenue is equated to the opportunity cost of a customer: the unit cost per customer is decreased by the marginal revenue from advertising, reflecting the fact that each new customer of the media operator is a potential buyer for the producers that advertise, which allows to increase advertising revenue. Note that, as  $c$  is very low in some media markets (the Internet for instance), it can result from (4) that the media operator fixes a price  $p$  such that marginal revenue is negative. And if the media operator has a weak market power (as measured by  $qp_q(q, a)$  in (4)), it is even possible for the price to be negative, for example because the media operator attracts subscribers by means of gifts, and uses its subscribers list to increase the tariff of ads. If negative prices are not feasible, then the media will be distributed for free to the customers (see below).

In terms of prices we obtain

$$\frac{p - c - a \frac{A_q}{A_s}}{p} = \frac{1}{\epsilon_{Q/p}} \quad (6)$$

$$\frac{s - q \frac{Q_a}{Q_p}}{s} = \frac{1}{\epsilon_{A/s}} \quad (7)$$

where  $\epsilon_{A/s} \equiv -\frac{sA_s}{a} > 0$  and  $\epsilon_{Q/p} \equiv -p \frac{Q_p}{q} > 0$ .

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<sup>6</sup>The profit function is supposed to be strictly concave so that the first order conditions are sufficient to determine the global maximum. Moreover, we assume that  $\varphi_a(q, 0) + p_a(q, 0) > 0$  so that the optimal advertising expenditure is strictly positive.

These are standard formulas for multi-product pricing by a monopolist. One can see  $q \frac{Q_a}{Q_p}$  as the opportunity cost of ads, that is equal to the reduction in the media operator's revenues from sales when  $a$  increases and  $p$  is adjusted so as to maintain  $q$ . The interpretation is symmetric for ads: the term  $-a \frac{A_q}{A_s} > 0$  is the opportunity revenue from keeping  $a$  fixed by balancing changes in  $q$  and  $s$ . Note that by (6), at the equilibrium point the elasticity of the demand for the service is smaller than in the case where there is no advertising. Moreover, if  $c$  is small, it is possible that  $\epsilon_{Q/p} < 1$  at the equilibrium point, which never occurs in models without advertising resources.

### 2.3 The multistation monopoly

To prepare the analysis of competition in the next section, we now adopt the Salop's model of horizontal differentiation where the monopoly operates  $n$  websites, newspapers or TV stations. Media customers are uniformly distributed along a circle of length equal to 1. When the consumer located at point  $x$  subscribes to station  $i$  located at point  $x_i$ , his net utility is

$$u(x; x_i, a_i) - p_i = \bar{u} - t|x - x_i| - \lambda(a_i) - p_i \quad (8)$$

where  $\lambda(\cdot)$  is the disutility from ads (the "nuisance effect" as Anderson and Coate (2003) call it),  $t$  is the unit cost of transport,  $\bar{u}$  is the reservation utility,  $a_i$  is the level of advertising programmed by  $i$  and  $p_i$  the subscription rate to the service provided by site  $i$ . The function  $\lambda(\cdot)$  is assumed to be convex and  $\lambda'(a)$  tends to  $+\infty$  when  $a$  goes to infinity, moreover  $\lambda(0) = 0$ .<sup>7</sup>

A site is created at costs  $k$ , and it is operated at a constant marginal cost  $c$ . When a new site is created, the monopoly relocates the other sites evenly on the circle. For the moment we take the number of sites  $n$  as given.

Concerning advertising we assume that advertisers can post one ad per site. In this context the demand for advertising space at a given media site depends only on its price  $s_i$  and its mass of customers  $q_i$ .<sup>8</sup> Thus we focus on the impact of advertising on the allocation of customers, and ignore the issue of the allocation of advertisers. The demand of ads to station  $i$  is

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<sup>7</sup>The disutility  $\lambda(a)$  may however be negative for small values of  $a$  or for some group of media clients.

<sup>8</sup>Schmidtke (2005) considers a more complex relation between advertisers and medias. The demand of advertising space for a given media operator depends on its price and its mass of clients as well as on the price and the audience of the other medias.

then  $a_i = A(s_i, q_i)$ , leading to a revenue  $r(q_i, a_i) = q_i \varphi(q_i, a_i)$ . We assume throughout that  $\varphi_a(q, 0) > \lambda'(0)$  and  $\varphi_{aa} < 0$ , which ensures that there will be a positive finite level of advertising, uniquely defined.

We first assume that all customers subscribe to one media service and we determine how customers chose between station  $i$  and station  $i + 1$  located at  $x_{i+1} > x_i$ . Assuming that both sites have a positive demand, the consumer indifferent between subscribing at  $i$  or at  $i + 1$  is located at  $\tilde{x}_{i+1}$  defined by

$$\bar{u} - \lambda(a_i) - t(\tilde{x}_{i+1} - x_i) - p_i = \bar{u} - \lambda(a_{i+1}) - t(x_{i+1} - \tilde{x}_{i+1}) - p_{i+1}$$

From this condition we obtain

$$\tilde{x}_{i+1} = \frac{\lambda(a_{i+1}) - \lambda(a_i) + p_{i+1} - p_i}{2t} + \frac{x_{i+1} + x_i}{2}.$$

In the interval  $[x_i, x_{i+1}]$ , all the consumers with an address  $x \leq \tilde{x}_{i+1}$  are customers of site  $i$ . Their number is  $\tilde{x}_{i+1} - x_i$ . But station  $i$  also has a boundary with station  $i - 1$ . From this side of the market, it attracts  $x_i - \tilde{x}_{i-1}$  consumers, where

$$\tilde{x}_{i-1} = \frac{\lambda(a_i) - \lambda(a_{i-1}) + p_i - p_{i-1}}{2t} + \frac{x_i + x_{i-1}}{2}.$$

Consequently, the number of consumers of media site  $i$  is  $q_i = \tilde{x}_{i+1} - \tilde{x}_{i-1}$ .

We deduce that the demand to station  $i$  is given by

$$q_i = \frac{\lambda(a_{i+1}) + \lambda(a_{i-1}) - 2\lambda(a_i) + p_{i+1} + p_{i-1} - 2p_i}{2t} + \frac{x_{i+1} - x_{i-1}}{2}. \quad (9)$$

As we will consider only symmetric equilibria with  $n$  active media stations, let  $p_{i+1} = p_{i-1} = p_n$  denote the price fixed by the closest sites for the service they sell and let  $a_{i+1} = a_{i-1} = a_n$  their advertising quantity. The distance between the two closest sites of  $i$  is  $\frac{2}{n}$ . It results that

$$q_i = \frac{1}{n} + \frac{\lambda(a_n) - \lambda(a_i) + p_n - p_i}{t}.$$

which obviously reduces to  $q_i = \frac{1}{n}$  when the monopoly implements a uniform pricing and advertising policy through all its stations.

Consider now optimal prices  $p_n$  and advertising levels  $a_n$ . When the market is covered, the saturation of the participation constraint (8) of the most

remote customer (the one at distance  $\frac{1}{2n}$  from any station) gives the subscription price

$$p_n = \bar{u} - \frac{t}{2n} - \lambda(a_n). \quad (10)$$

By contrast, when the monopoly does not serve all the would-be customers, from (8) the marginal customer is the one at distance  $\frac{\bar{u}-p-\lambda(a)}{t}$  from any site. Therefore, the demand of service to a site is

$$q_n = 2 \cdot \frac{\bar{u} - p_n - \lambda(a_n)}{t} < \frac{1}{n} \quad \text{or} \quad p_n = \bar{u} - \frac{tq_n}{2} - \lambda(a_n). \quad (11)$$

We must determine the advertising price, the audience and the number of stations. Using (10) and (11), this reduces to

$$\max_{a,q,n} n\left(\bar{u} - \frac{tq}{2} - \lambda(a) - c + \varphi(q, a)\right)q - nk.$$

subject to

$$q \leq \frac{1}{n}.$$

Clearly the optimal level of advertising is

$$a^*(q) = \arg \max_a (\varphi(q, a) - \lambda(a)).$$

The first order conditions for  $a$  gives give

$$\varphi_a(q, a^*(q)) = \lambda'(a^*(q)) \quad (12)$$

Equation (12) states that the marginal advertising revenue per consumer must be equal to the marginal disutility of advertising. Increasing the amount of advertising  $a$  for a given clientele  $q$  generates a marginal revenue per customers  $\frac{r_a}{q} = \varphi_a$  and requires to reduce the subscription price by  $\lambda'$  so as to maintain the clientele. Notice that the level of advertising is independent of the transport cost. Notice also that it is not always the case that  $\lambda'(a) > 0$  at the optimal level of advertising: whenever the revenue per customer decreases with total advertising at the level that maximizes the utility of customers, the media chooses to restrict advertising beyond this level.

It is immediate that the market is covered ( $q_n = \frac{1}{n}$ ) if and only if  $n \geq \bar{n}$  where  $\bar{n}$  is defined by

$$\frac{1}{\bar{n}} = \arg \max_q \left( \bar{u} - \frac{tq}{2} - c + \max_a (\varphi(q, a) - \lambda(a)) \right) q.$$

Under our assumptions, it is always the case that  $\max_a (\varphi(q, a) - \lambda(a)) > 0$ , which implies that the profit per station is always strictly larger when advertising is used. Moreover the market is covered with advertising when it is covered without.

When the monopolist maximizes its profit by a full coverage of the market, the first order condition for  $n$  to be optimal is then

$$\frac{1}{n^2} \left( \frac{t}{2} - \varphi_q \left( \frac{1}{n}, a_n \right) \right) - k = 0 \quad (13)$$

Clearly, the effect of advertising on the multistation monopoly's choice of diversity depends on the sign of  $\varphi_q$ . With constant returns,  $n$  is the same with and without advertising. When there are decreasing returns to ads,  $\varphi_q$  is negative and the monopolist will choose a greater diversity with advertising than without. This is because reducing the size of the population of customers targeted by one ad (by raising the number of sites) raises the return from advertising and thus the advertising revenue. The reverse is true in the case where  $\varphi$  increases with the audience.

### 3 Oligopolistic competition

We now consider that there is competition in the media industry where customers are depicted by the Salop's model. From now on, the index  $i$  identifies an independent media operator (a media). Medias set prices  $p_i$  for the service they propose to customers, but we contrast two cases for the advertising market. In the first case, media fix the level  $a_i$  of advertising and let the price  $s_i$  adjust to maintain the quantity.<sup>9</sup> In the other case media set the price  $s_i$  for ads and let the quantity  $a_i$  adjust. The next two sections analyze these two possibilities.

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<sup>9</sup>In the case of constant returns ( $\varphi(q, a) = \varphi(a)$ ), setting the quantity  $a$  is equivalent to setting a price  $\frac{\varphi(a)}{a}$  per client so that  $r = q\varphi(a)$ . In the general case, a fixed quantity obtains with a non-linear tariff  $s(q) = \frac{r(q, a)}{a}$ .

### 3.1 Quantity Equilibrium

We first consider the case where media set the level of advertising. So we assume that they simultaneously set  $(p_i, a_i)$ , then the demand  $q_i$  is realized as described in section 2.3 and the price  $s_i$  adjusts to  $s(a_i, q_i)$ , leading to a revenue  $r(a_i, q_i)$ . A symmetric equilibrium obtains when the mass of customer is  $q = \frac{1}{n}$ , while the equilibrium price is  $p_n$  and the level of advertising is  $a_n$ . To derive the equilibrium conditions we focus on the first order conditions of the media operators' optimal strategies. We will derive second order and global existence conditions at the end of the section.

Consider first the optimal strategy of media  $i$  when there are  $n$  media operators and the  $(n - 1)$  other operators charge a price  $p_n$  and a quantity of advertising  $a_n$ .

Its objective is

$$\max_{a,p} \pi(a, p; a_n, p_n; n) = \left(\frac{1}{n} + \frac{\lambda(a_n) - \lambda(a) + p_n - p}{t}\right)(p - c) + r(q, a) \quad (14)$$

where  $r(q, a) = q\varphi(q, a)$ .

The first order conditions are evaluated at  $p_n = p$  and  $a_n = a$  to define the equilibrium level of price and the equilibrium level of advertising. This writes as

$$\begin{aligned} \frac{1}{n} - \frac{1}{t} \left( p_n - c + r_q\left(\frac{1}{n}, a\right) \right) &= 0, \\ r_a\left(\frac{1}{n}, a\right) - \frac{\lambda'(a)}{t} (p_n - c + r_q\left(\frac{1}{n}, a\right)) &= 0. \end{aligned}$$

We postpone the discussion of existence of a pure strategy equilibrium to the end of the section, where it is shown that the first order conditions are sufficient if  $\varphi_q \leq 0$  or  $\varphi_q$  not too large.

Before looking at the general case, consider the case where there are constant returns,  $\varphi(q, a) = \varphi(a)$  so that  $\varphi_q \equiv 0$  and the advertising revenue is  $r(q, a) = q\varphi(a)$ . Then the profit writes as

$$\left(\frac{1}{n} + \frac{\lambda(a_n) - \lambda(a) + p_n - p}{t}\right)(p - c + \varphi(a)).$$

Denoting by  $h = p + \lambda(a)$  the hedonic price for customers, the profit is

$$\left(\frac{1}{n} + \frac{h_n - h}{t}\right)(h - c + \varphi(a) - \lambda(a)).$$

It is then clear that the media will choose the advertising level:

$$a^* = \arg \max_a (\varphi(a) - \lambda(a)).$$

Concerning the subscription price, we can observe that the equilibrium hedonic price  $p_n + \lambda(a^*)$  is the same as in a Salop model with a cost  $c - \varphi(a^*) + \lambda(a^*)$ , leading to  $p_n = c + \frac{t}{n} - \varphi(a^*)$ .

In the general case where  $\varphi_q \neq 0$  we obtain the following equilibrium levels

$$p_n = c + \frac{t}{n} - r_q\left(\frac{1}{n}, a_n\right), \quad (15)$$

and

$$\varphi_a\left(\frac{1}{n}, a_n\right) = \lambda'(a_n). \quad (16)$$

In the general case, we have  $r_q > 0$  so the equilibrium price is below  $c + \frac{t}{n}$ . The subscription equilibrium price is lowered by an audience effect of advertising. Like in the multistation monopoly case, equation (16) states that the marginal advertising revenue per consumer must be equal to the marginal disutility of advertising. For a fixed number of firms, the level of advertising is thus the same in the quantity model and in the multistation monopoly : each media acts has a local monopoly in the advertising market.

When the number of media operators  $n$  becomes large,  $a_n = a^*\left(\frac{1}{n}\right)$  converges to  $a^*(0) > 0$ , and  $r_q\left(a_n, \frac{1}{n}\right)$  converges to  $\varphi(0, a^*(0)) > 0$ , so that the subscription price is below the marginal production cost  $c$ .

**Proposition 1** *Assume that media set the advertising quantity. For  $n$  large enough, the price is below marginal cost.*

A standard comparative static analysis adds the following result.

**Lemma 2**

$$\begin{cases} a_n > a^*(0) \text{ and } \frac{da_n}{dn} < 0 & \text{if } \varphi_{aq} > 0 \\ a_n < a^*(0) \text{ and } \frac{da_n}{dn} > 0 & \text{if } \varphi_{aq} < 0 \end{cases}$$

The proof is straightforward by totally differentiating the first order condition 16.



If we insert the first order conditions into (8), the equilibrium level of individual utility for the consumer located at  $x$  when there are  $n$  active media is then given by

$$u(n) = \bar{u} - c - t|x - x_i| - \frac{t}{n} + r_q\left(\frac{1}{n}, a_n\right) - \lambda(a_n)$$

where  $|x - x_i| \leq 1/2n$ .

Similarly, the equilibrium level of (variable) profit of the individual media operator is given by

$$\pi(n) = \frac{t}{n^2} - \frac{1}{n}r_q\left(\frac{1}{n}, a_n\right) + r\left(\frac{1}{n}, a_n\right).$$

As compared with a world without advertising, we see that the effect of advertising on profit and utility depends on the convexity of the advertising revenue function:

$$\begin{aligned} \pi(n) - \frac{t}{n^2} &= -\varphi_q\left(\frac{1}{n}, a_n\right)\frac{1}{n^2} \\ u(n) - (\bar{u} - c - t|x - x_i| - \frac{t}{n}) &= r_q\left(\frac{1}{n}, a_n\right) - \lambda(a_n). \end{aligned}$$

The effect of advertising on the media operator profit crucially depends on the advertising technology. Advertising increases the revenue of the media operator but it also has a negative impact on the subscription price to the media. The total effect is given by the sign of  $\varphi_q$ . When advertising exhibits constant return to scale in the audience, the total effect is null. Advertising has no impact on media operators' profits. If returns to scale are decreasing,  $\varphi_q < 0$ , each media operator earns more money than in the no advertising case.

For customers, the effect of advertising depends on the sign of  $(r_q - \lambda)$ . If the "price decrease effect"  $r_q$  is higher than the "nuisance effect"  $\lambda$ , the customers are better off when media can provide advertising. Notice that  $r_q - \lambda$  is the derivative with respect to  $q$  of  $\max_a (r(q, a) - \lambda(a)q)$ , which is the revenue net of the nuisance and thus the total surplus generated by advertising for the media and its customers. Thus the customers benefit from advertising whenever  $\max_a (r(q, a) - \lambda(a)q)$  is increasing with  $q$ .

The term  $r_q - \lambda$  is clearly positive if  $\lambda < 0$ . Now suppose that  $\lambda > 0$ . Then, using  $r = q\varphi$ , and  $\varphi = av(q, a)$  and since  $\frac{\lambda'}{n} = r_a$ , we have

$$r_q - \frac{\lambda}{\lambda'} \frac{r_a}{q} = \varphi \left(1 - \frac{\lambda}{a\lambda'}\right) + q\varphi_q - \frac{\lambda}{\lambda'} av_a.$$

Given that  $\lambda(a_n)$  is positive and  $\lambda(\cdot)$  is convex,  $1 > \frac{\lambda}{a\lambda'} > 0$ , and  $v_a < 0$ . Therefore the effect of advertising is positive when returns to scale in the audience are constant or increasing ( $\varphi_q \geq 0$ ). When returns to scale are decreasing, each oligopolist benefits from advertising only when  $\varphi_q$  is close to 0. When  $n$  is large, note that  $q\varphi_q$  converges to zero so that the effect is positive.

These results are summarized in the following proposition.

**Proposition 3** *The advertising technology determines the impact of advertising on media profits and consumers' surplus. As compared with the no-advertising case, for a fixed number of media operators:*

- i) under constant returns to scale in the audience ( $\varphi_q = 0$ ), media profits are not affected by advertising and the consumers' surplus is higher;*
- ii) under increasing returns to scale ( $\varphi_q > 0$ ), media profits are lower and the consumers' surplus is higher than without advertising;*
- iii) under decreasing returns to scale ( $\varphi_q < 0$ ), media profits are higher; moreover the consumers' surplus is higher when  $n$  is large enough, or when  $|\varphi_q|$  is small enough, or when  $\lambda(a_n) < 0$ .*

In the long run, the equilibrium number of media operators is given approximately by  $\pi(n) = k$ .

We assume that for all  $q$  and  $a^*(q) = \arg \max_a (\varphi(q, a) - \lambda(a))$ , the following condition holds:

$$t \geq 2\varphi_q + q\varphi_{qq} - q \frac{(\varphi_{aq})^2}{\varphi_{aa} - \lambda''}. \quad (17)$$

Condition (17), which guarantees the existence of the equilibrium, implies also that the profit is decreasing with respect to  $n$ . Thus there exists a unique equilibrium number of entrants.<sup>10</sup> Define  $n_C^*$  the level of entry in the quantity game. The number of active media in the long run equilibrium is given by

$$\frac{t}{n_C^{*2}} - k = \varphi_q\left(\frac{1}{n_C^*}, a_n\right) \frac{1}{n_C^{*2}}. \quad (18)$$

---

<sup>10</sup>Monotonicity of the equilibrium profit with respect to  $n$  is given by  $\frac{\partial \pi(n)}{\partial n} < 0$ , which imposes that  $2t > 2\varphi_q\left(\frac{1}{n}, a_n\right) + \frac{1}{n}\varphi_{qq}\left(\frac{1}{n}, a_n\right) - \frac{1}{n} \frac{\varphi_{qa}\left(\frac{1}{n}, a_n\right)^2}{\varphi_{aa}\left(\frac{1}{n}, a_n\right) - \lambda''(a_n)}$ .

Remind that without advertising, the level of entry is given by  $n = \sqrt{\frac{t}{k}}$ . Note that entry is thus similar to the entry in a world without advertising when there are constant return to scale in the audience. Moreover:

**Corollary 4** *The level of entry is higher with advertising than without if the price of advertising per customer decreases with the audience (decreasing returns to scale). It is lower under increasing returns to scale.*

### Global existence conditions

In the Salop model ( $a \equiv 0$ ), the profit function is concave and first-order conditions are sufficient for equilibrium. Notice that in the case  $\varphi_q \equiv 0$ , the level of advertising is  $a^*(0)$  for any price  $p$  set by the media, so that the model is equivalent to a Salop model with a marginal cost  $c - \varphi(a^*(0))$ . Thus first-order conditions are sufficient to prove existence.

This may not be the case in our general model due to nonlinearity in advertising resources which changes the gains from increasing the size of the clientele. The sufficient conditions for the existence of the equilibrium are given in the following lemma.

**Lemma 5** *Under condition 17,  $(p_n, a_n)$  is an equilibrium if and only if a media does not benefit when setting  $(p_i, a_i)$  such that  $q_i = \frac{3}{n}$ , which reduces to*

$$t \geq 3n \int_{\frac{1}{n}}^{\frac{3}{n}} \varphi_q(q, a^*(q)) dq - 2\varphi_q\left(\frac{1}{n}, a_n\right).$$

Observe that condition 17 is verified under constant return to scale in the audience. Else, it requires that the first and second derivatives of  $\varphi$  with respect to the audience be close enough to zero. For  $n$  large, the sufficient condition reduces to

$$t \geq 4\varphi_q(0, a^*(0)).$$

## 3.2 Price Equilibrium

We now suppose that the decision variable of the media is the price of advertising. Thus media  $i$  chooses a subscription price  $p_i$  and an advertising price  $s_i$ .

The new feature is that unlike the previous case, the demand to media  $i$  does not depend only on the strategies of its two adjacent competitors,

but on all the prices of all the media operators. Starting from a symmetric equilibrium, suppose that media  $i$  changes its pricing strategy. From (9), we see that it modifies the demand for its service as well as the demand addressed to the adjacent media. Additionally, since the demand for advertising at media  $i$  is

$$a_i = A(s_i, q_i), \quad (19)$$

a change in  $s_i$  will change the demand for advertising to media  $i$  as well as to the adjacent media operators. As a consequence, the demand of service to the next media operators  $i + 2$  and  $i - 2$  will incur an additional change, and it will be the same for the demand for advertising they face. Following this reasoning we see that the demands for both services and advertising to all media are affected. We refer to this as a propagation effect, along the circle.

Therefore, when one price changes, in order to determine the final effect on sales and advertising we have to solve for the complete allocation. The following lemma establishes the relations between all these variations, in the case of a marginal change in prices.

**Lemma 6** *Starting from a symmetric situation where all prices are  $p$  for services and  $s$  for ads, and quantities are  $q = \frac{1}{n}$  and  $a$ , suppose that media  $i$  changes its prices by  $dp_i$  and  $ds_i$ . Then the changes in demands for service and for advertising at media  $i$  are linked by the relations*

$$dq_i = -\frac{dp_i + \lambda'(a)da_i}{t\theta(n, a)} \quad (20)$$

$$da_i = A_q dq_i + A_s ds_i \quad (21)$$

where  $\theta(n, a)$  is given by

$$\theta(n, a) = \frac{1 + \alpha(n, a)^n}{(1 - \alpha(n, a))(1 - \alpha(n, a)^{n-1})}$$

and  $\alpha(n, a)$  is a root of

$$0 = 1 - 2 \left( 1 - \frac{t}{\lambda'(a)} \frac{s_a(\frac{1}{n}, a)}{s_q(\frac{1}{n}, a)} \right) x + x^2.$$

We have

$$\begin{cases} \theta > 1 & \text{when } \lambda'(a) > 0 \\ \theta < 1 & \text{when } 0 > \lambda'(a) > \frac{t}{2} \frac{s_a(\frac{1}{n}, a)}{s_q(\frac{1}{n}, a)}. \end{cases}$$

The difference with the case where media set advertising quantities is the term  $\theta$  in (20). When consumers dislike advertising ( $\lambda'(a) > 0$ ), the subscription demand is less sensitive to the subscription price (for a given level of  $a_i$ ). To understand this effect, suppose that media  $i$  increases its subscription price by 1 unit. Absent advertising, this would reduce the demand for its service by  $\frac{1}{t}$ . The immediate effect is to raise the demand of customers to adjacent operators. As a consequence the levels of advertising at adjacent operators also increase. But this reduces the attractiveness of adjacent operators so that the final reduction is smaller than  $\frac{1}{t}$ . There is a feedback effect through the adjustment of advertising demands. When consumers slightly like advertising ( $\frac{s_a(\frac{1}{n}, a)}{2s_q(\frac{1}{n}, a)} < \lambda'(a) < 0$ ), the feedback effect reinforces the demand effect so that the final reduction in the demand is larger than  $\frac{1}{t}$ . If the value of  $\lambda'(a)$  is too small so that consumers hugely value advertising, the feedback effect becomes too important to guarantee the existence of the equilibrium.

The marginal effects of prices are obtained by combining the feedback effect with the propagation effect discussed above.

Now consider the optimal pricing strategy of media  $i$  when there are  $n$  media operators and the  $(n - 1)$  competitors charge prices  $s_n$  and  $p_n$ . We differentiate the profit function with respect to  $q$  and  $a$ , rather than to  $p$  and  $s$ . The first order condition with respect to  $q$  is  $p + q \frac{\partial p}{\partial q} + r_q - c = 0$  and the first order condition with respect to  $a$  is  $q \frac{\partial p}{\partial a} + r_a = 0$ . But, since from 20 the final change in quantities is given by

$$-t\theta dq - \lambda' da = dp,$$

we have  $\frac{\partial p}{\partial q} = -t\theta$  and  $\frac{\partial p}{\partial a} = -\lambda'$ .

Now, evaluate the solution at  $a = a_n$  and  $q = \frac{1}{n}$  to obtain the symmetric equilibrium. It results that the subscription price  $p_n$  and the individual advertising volume  $a_n$  are given by

$$p_n = c + \frac{t}{n}\theta(n, a_n) - r_q\left(\frac{1}{n}, a_n\right) \quad (22)$$

$$\varphi_a\left(\frac{1}{n}, a_n\right) = \lambda'(a_n) \quad (23)$$

where the term  $\theta(n, a_n)$  captures the fact that the price charged by competitors for advertising is fixed.

There are now two effects on the subscription price. First  $r_q > 0$ , and, like in the quantity game, the subscription equilibrium price is lowered by the audience effect of advertising. But a second indirect effect depending on the value of the disutility from ads arises. When consumers dislike ads, because the volume of advertising in media competitors is endogenous, it is harder to capture market shares. The changes in volumes of advertising mitigate the price effects. As discussed above, the elasticity of the final residual demand is lower with advertising than without. This effect, that appears through  $\theta > 1$  in (22) tends to raise the equilibrium price and media are better off in the "price game" as compared with the "quantity game". When consumers slightly like advertising, we obtain a different result because the indirect effect reinforces the price effect ( $\theta < 1$  in (22)). In this case, the elasticity of the final residual demand is higher with advertising than without. As a consequence, media are worse off in the "price game" as compared with the "quantity game". Note also that in the particular case where consumers are indifferent about advertising, the quantity and the price model do not differ.

These results are summarized in the following proposition.

**Proposition 7** *When media set the price of advertising, the equilibrium level of advertising is the same as when they set the volume of advertising, but the subscription price is larger when consumers dislike advertising and lower when they slightly like advertising.*

*For  $n$  large enough the subscription price is below the marginal cost:*  
 $p_n < c$ .

This difference between a price model and a quantity model is also emphasized in Armstrong (2004). With a two firm model, he obtains a similar relation between profits and consumers' taste for advertising : media profits increase (resp. decrease) when setting an advertising price compared to setting an advertising quantity when consumers dislike (resp. like) advertising.

As before, when compared with a world without advertising, we see that the effect on profit and utility depends on the convexity of the advertising revenue function.

$$\begin{aligned}\pi(n) - \frac{t}{n^2} &= \frac{t}{n^2}(\theta - 1) + r - qr_q \\ &= \frac{t}{n^2}(\theta - 1) - \frac{\varphi_q(\frac{1}{n}, a_n)}{n^2}.\end{aligned}$$

Consider first that consumers dislike advertising. With constant or decreasing returns in the audience size ( $\varphi_q \leq 0$ ), the price game gives the same result than the quantity game : each media operator earns more money than in the no advertising case. When  $\varphi_q > 0$ , the negative effect on profits is alleviated by an increase in the subscription price. We show in the appendix that when  $n$  increases,  $\theta$  evolves as  $\sqrt{n}$ . This implies that when the number of media operators is large, media operators make more profits than in the no-advertising case, which was not the case in the quantity game.

For the consumer, the effect of advertising depends on the sign of  $(r_q - \lambda) - \frac{t}{n^2}(\theta - 1)$ . When  $n$  is large,  $r_q - \lambda$  is strictly positive, while we have seen that  $\frac{\theta-1}{n}$  goes to zero. The following proposition summarizes these results.

**Proposition 8** *Assume that consumers dislike advertising and that the media set the price of advertising. For  $n$  large enough, the media operators' profits and consumers' surplus are higher than without advertising.*

Consider now the case where consumers like advertising. In this case, the feedback effect reinforces competition between media so that the profit can decrease compared with the no advertising game. Medias lose profits when there are constant or increasing returns to scale in the audience. Under decreasing returns, media operators can earn more money than in the no advertising case if the transportation cost is small enough.

Let us now evaluate the level of entry in the price game and compare it to the level of entry in the quantity game. Define  $n_B^*$  the level of entry in the price game. Entry is given by

$$\frac{t\theta(n_B^*, a_n)}{n_B^{*2}} - k = \varphi_q\left(\frac{1}{n_B^*}, a_n\right)\frac{1}{n_B^{*2}}. \quad (24)$$

Like in the quantity model, there exists a unique equilibrium number of entrants. When consumers dislike advertising, since profits are higher when media impose an advertising price, we obtain a larger number of entrants in the price model than in the quantity model.

### 3.3 Free versus pay media

In the above analysis, we have concluded that the subscription price can be negative. A negative price can correspond to gifts or can result from bundling

some other good with the media. For example, publishers can bundle CDs or DVDs with magazines. In other cases it is difficult to support negative prices, for instance when the concerned population is small and the rest of the population can acquire the good at no cost and decide not to consume it (with thus no benefit to advertisers). In this case the equilibrium subscription price will be zero.

In this part we allow consumers to have free access to media services so that media are only financed through advertising, and we study how the pricing of media services impacts market performance. In particular we compare the level of advertising and the level of media profits under pay and free media services. For this part we fix the number of media operators.

When consumers can freely access to media services, results can be derived by adjusting our first order condition to the constraint  $p_n = 0$ . The equilibrium level of advertising  $a_F$  is given by<sup>11</sup>

$$r_a(a_F, \frac{1}{n}) + \frac{dq}{da} \left( r_q(a_F, \frac{1}{n}) - c \right) = 0, \quad (25)$$

where  $\frac{dq}{da} = -\frac{\lambda'}{t}$  when media set advertising quantities, and  $\frac{dq}{da} = -\frac{\lambda'}{\theta t}$  when media set an advertising price.

The comparison between  $a_F$  and  $a_n$  is immediate when we observe that equation (16) can be rewritten as

$$r_a(a_n, \frac{1}{n}) + \frac{dq}{da} (r_q(a_n, \frac{1}{n}) - c) = -\frac{dq}{da} p_n. \quad (26)$$

Given that the sign of  $-\frac{dq}{da}$  is the sign of  $\lambda'$ , we have the following immediate result.

**Proposition 9** *Assume that  $a_F$  is uniquely defined. The advertising level under free media ( $a_F$ ) is larger (resp. lower) than the advertising level under pay media ( $a_n$ ) when  $\lambda'(a_n)p_n > 0$  (resp.  $< 0$ ).*

Thus, when customers dislike advertising, pay-media will lead to less advertising when equilibrium prices are positive.

The next question is whether media operators benefit from imposing positive subscription prices to consumers. We show that the benefit of imposing prices to consumers depends on the advertising returns to scale in the audience.

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<sup>11</sup>A detailed analysis for the case of quantity setting and constant returns to scale in the audience can be found in Choi (2003).



**Proposition 10** *Assume that media set the volume of advertising, and that  $\lambda'(\cdot) > 0$  and  $a^*(q)$  is non-decreasing ( $\varphi_{aq} \geq 0$ ). Free-media profits are smaller than pay-media profits if and only if  $p_n > 0$ .*

The case where the equilibrium subscription cannot be negative raises interesting issues. Suppose first that prices cannot fall below zero. Then either the equilibrium subscription price is positive or the service is free. In this circumstances, media can only be hurt by a collective move toward free services.

## 4 Welfare analysis

We now establish the optimal number of media operators  $n^o$  and the optimal advertising quantity  $a^o$ .

To derive the welfare, we assume that there is no informational externality between consumers related to advertising such as words of mouth. We also assume that the goods offered by the producers are not competing. Welfare can be computed as the sum of the consumers surplus, the advertisers' surplus and the media profit. Social welfare can then be written as follows:

$$W(n, a) = [\bar{u} - 2n \int_0^{\frac{1}{2n}} t x dx - \lambda(a)] + \int_0^a v\left(\frac{1}{n}, \gamma\right) d\gamma - nk - c. \quad (27)$$

The expression in the square bracket represents consumers' benefits from consuming the media services given a number  $n$  of media operators and a quantity of advertising  $a$ . The second term  $\int_0^a v\left(\frac{1}{n}, \gamma\right) d\gamma$  represents manufacturers' benefits (the benefit from advertising).

The first order conditions with respect to  $n$  and  $a$  that define  $n^o$  and  $a^o$  are :

$$\frac{t}{4(n^o)^2} - k = \frac{1}{(n^o)^2} \int_0^{a^o} v_q\left(\frac{1}{n^o}, \gamma\right) d\gamma \quad (28)$$

and

$$v\left(\frac{1}{n^o}, a^o\right) = \lambda'(a^o) \quad (29)$$

The optimal level of advertising is such that the willingness to pay for the marginal advertiser (the marginal social benefit of advertising) equals the marginal disutility of advertising (the marginal social cost).

A first relation between the socially optimal levels and the equilibrium levels is stated in the following lemma. Denote  $n_C^*(a)$ ,  $n_B^*(a)$  and  $n^o(a)$  the respective solutions of equations (18), (24) and (28) for a given value of  $a$ . Denote  $a^o(n)$  the welfare maximizing level of advertising for a given  $n$ , solution of equation (29).

**Lemma 11** *We have that*

- i)  $a^o(n) > a_n$  for all  $n$ .
- ii)  $n^o(a) < n_C^*(a)$  for all  $a$  when  $v_{qa} \leq 0$ .
- iii)  $n^o(a) < n_C^*(a) < n_B^*(a)$  for all  $a$  when  $\lambda'(a) > 0$  and  $v_{qa} \leq 0$ .

The first part results from the exercise of market power by media on the market for advertising. The second part is standard with the Salop model. Because of a business stealing effect, for a given level of advertising, there would be too many entrants. The third part comes from the analysis conducted in section 3 : there is more entry in the price game than in the quantity game only when consumers dislike advertising.

The next proposition characterizes the welfare analysis for the case of constant and increasing returns to scale in the audience.

**Proposition 12** *Assume that consumers dislike advertising. When  $a^*(q)$  is non-decreasing ( $\varphi_{qa} \geq 0$ ) and  $v_{qa} \leq 0$ , entry is excessive and advertising is insufficient. We have  $n_B^* > n_C^* > n^o$  and  $a_B^* \leq a_C^* < a^o$ .*

This result of underprovision of advertising, as pointed out by Armstrong (2004), comes from the fact that each media acts has a local monopoly in the advertising market. When consumers are not charged for services, this result of excessive entry and insufficient advertising is not evident. Choi (2003) shows in a Salop model of spatial competition where firms are only financed by advertising revenues (with constant unit ad values) that entry and advertising can be either excessive or insufficient. This result of excessive entry and insufficient advertising is important when comparing the quantity and the price model.

**Corollary 13** *Assume that consumers dislike advertising. When  $\varphi_{qa} \geq 0$  and  $v_{qa} \leq 0$ , the quantity model is socially preferable to the price model.*

A simple case is when the average efficiency of advertising  $v$  is separable  $v(q, a) = g(q)G(a)$  with  $G(a)$  decreasing. If  $\lambda(a)$  is increasing, we can restrict the analysis to levels of advertising such that  $\varphi_a \geq 0$ . Then  $\varphi_{qa} = \frac{g'}{g}\varphi_a$  and  $v_{qa} = g'G'$ . This means that the conditions are verified when there a constant or increasing return. More generally, given that  $\varphi_{qa} = v_q + av_{qa}$ , the conditions require that  $v_q \geq 0$ .

This non-ambiguous result of excessive entry and insufficient advertising is more difficult to obtain when there are decreasing returns to scale in the audience. We may obtain excessive entry, which results in either excessive or insufficient advertising. If we have an insufficient entry, then advertising is without ambiguity insufficient. Under decreasing returns to scale, the choice between the two models is more difficult. In particular, if entry is excessive, we may obtain too much entry in the price game in comparison of the advertising game but we may obtain an advertising level closer to the socially optimal one in the advertising price game.

## 5 Conclusion

We have presented a model of media competition with free entry where media are financed both from advertisers and viewers. The relation between advertising receipts and sales receipts is analyzed through two particular features : on one hand the choice of the advertising price versus the advertising space by the media operator, and on the other hand non linearity in the advertising technology

Medias' profits are higher when media operators set the advertising price rather than the advertising quantity. As compared with a no-advertising benchmark, when consumers dislike advertising and media set the advertising space, media profits are higher under decreasing returns to scale, and they are lower under increasing returns to scale. When they fix the advertising price, their profits can be higher than without advertising if returns to scale are increasing.

Is it socially better to let media to set the advertising space or the advertising price? When consumers dislike advertising, profits and prices are higher when media choose the advertising price and, with a fixed market structure, the level of advertising is the same in the two models. This results in more entry in the advertising price game as compared with the advertis-

ing quantity game. Entry is excessive under constant or increasing returns to scale. We also have found an insufficient amount of advertising larger in the price model than in the quantity model. We thus obtain that, under constant or increasing returns to scale in the audience, the quantity model is socially superior to the price model. Under decreasing returns to scale, the social advantage of one model over the other is more difficult to assess. In particular, we may obtain that the equilibrium level of entry is closer to the socially optimal one in the quantity game but that the equilibrium level of advertising is closer to the social optimum in the advertising price game.

**Global existence condition**  
**Proof of lemma 5**

Consider the symmetrical equilibrium with  $n$  active media operators. Denote  $p_n$  and  $a_n$  the equilibrium price and advertising levels.

Define

$$\hat{\pi}_1(q, t) = (t(\frac{1}{n} - q) - c + p_n + \lambda(a_n))q + \max_a(r(q, a) - \lambda(a)q).$$

Let us now consider the deviation of firm  $i$ . Consider first a deviation leading to  $q < \frac{2}{n}$ . Then firm  $i$  faces a demand

$$q = \frac{\lambda(a_n) - \lambda(a) + p_n - p}{t} + \frac{1}{n}.$$

The maximal profit it can obtain while selling  $q < \frac{2}{n}$  is then given by  $\hat{\pi}_1(q, t)$ . If the function  $\hat{\pi}_1(q, t)$  is concave, deviations with  $q < \frac{2}{n}$  are not profitable. Notice that

$$\hat{\pi}_1(q, t) = \hat{\pi}_1(q, \frac{t}{2}) + \frac{t}{2} \left( \frac{1}{n} - q \right) q$$

is concave if  $\hat{\pi}_1(q, \frac{t}{2})$  is concave.

At  $q = \frac{2}{n}$ , there is a discontinuity as if a customer at the distance  $\frac{1}{n}$  buys from the media, so do all customers at a distance below  $\frac{3}{2n}$ . Therefore  $q$  jumps at  $\frac{3}{n}$ . Assume now that a consumer located at a distance  $x \in [\frac{j-1}{n}, \frac{j}{n}]$  from firm  $i$  is indifferent between purchasing from firm  $i$  and purchasing from the firm located at the distance  $\frac{j}{n}$  from firm  $i$ . We have that

$$\lambda(a_n) + p_n + t(\frac{j}{n} - x) = \lambda(a) + p + tx.$$

Then firm  $i$  faces a demand

$$q = \frac{\lambda(a_n) - \lambda(a) + p_n - p}{t} + \frac{j}{n}.$$

Thus the media can sell  $q \in [\frac{2j-1}{n}, \frac{2j}{n}]$  when announcing a price  $p = t(\frac{j}{n} - q) + \lambda(a_n) - \lambda(a) + p_n$ . The profit of the deviating firm is thus given by

$$\hat{\pi}_j(q) = (t(\frac{j}{n} - q) - c + p_n + \lambda(a_n))q + \max_a(r(q, a) - \lambda(a)q).$$

This can be rewritten as

$$\begin{aligned}\hat{\pi}_j(q) &= \hat{\pi}_1(q, \frac{t}{2}) + \frac{t}{2}(\frac{2j-1}{n} - q)q \\ &\leq \hat{\pi}_1(q, \frac{t}{2}),\end{aligned}$$

with equality if  $q = \frac{2j-1}{n}$ .

Observe next that  $\hat{\pi}_1(\frac{1}{n}, t) = \hat{\pi}_1(\frac{1}{n}, \frac{t}{2})$ , so that comparing the equilibrium profit with the deviation profit when  $q > \frac{3}{n}$  amounts to compare  $\hat{\pi}_1(\frac{1}{n}, \frac{t}{2})$  and  $\hat{\pi}_1(q, \frac{t}{2})$ .

Notice that  $\hat{\pi}_1(q, \frac{t}{2})$  is increasing with  $q$  at  $q = \frac{1}{n}$ . When  $\hat{\pi}_1(q, \frac{t}{2})$  is concave  $\hat{\pi}_1(q, \frac{t}{2}) \leq \hat{\pi}_1(\frac{1}{n}, \frac{t}{2})$  for all  $q \geq \frac{3}{n}$  if this is true for  $\frac{3}{n}$ .

The sufficient conditions to guarantee that firm  $i$  prefers not to deviate are thus

- i) The function  $\hat{\pi}_1(q, \frac{t}{2})$  is concave.
- ii)  $\hat{\pi}_1(\frac{1}{n}, \frac{t}{2}) \geq \hat{\pi}_1(\frac{3}{n}, \frac{t}{2})$ .

Let us now establish the condition for the concavity of  $\hat{\pi}_1(q, \frac{t}{2})$ . We evaluate  $\frac{\partial^2 \hat{\pi}_1(q, \frac{t}{2})}{\partial q^2}$  at  $a = a^*(q)$  where  $a^*(q)$  is defined by  $\varphi_a(q, a^*(q)) = \lambda'(a^*(q))$ . Using  $\frac{da^*(q)}{dq} = -\frac{\varphi_{aq}}{\varphi_{aa} - \lambda''}$ , we obtain

$$\frac{\partial^2 \hat{\pi}_1(q, \frac{t}{2})}{\partial q^2} = -t + 2\varphi_q + q\varphi_{qq} + q\varphi_{qa} \left( -\frac{\varphi_{aq}}{\varphi_{aa} - \lambda''} \right),$$

which gives the condition (17). Moreover the second condition writes

$$t + 2\varphi_q(\frac{1}{n}, a_n) \geq 3n \left( \max_a(\varphi(\frac{3}{n}, a) - \lambda(a)) - \max_a(\varphi(\frac{1}{n}, a) - \lambda(a)) \right)$$

which give the condition in the proposition.

### Proof of lemma 6

Suppose that media 1 changes its prices  $p_1$  and  $s_1$ . By symmetry, the consequences of these changes will be the same for media  $i$  and  $n + 2 - i$ .

Using the demand functions for services (9) and the demand function for advertising (19) we obtain

$$\begin{aligned} dq_1 &= \frac{\lambda'(a)da_2 - dp_1 - \lambda'(a)da_1}{t} \\ da_1 &= A_s ds_1 + A_q dq_1 \end{aligned} \quad (30)$$

$$\begin{aligned} dq_2 &= \frac{dp_1 + \lambda'(a)da_1 + \lambda'(a)da_3 - 2\lambda'(a)da_2}{2t} \\ da_2 &= A_q dq_2 \end{aligned} \quad (31)$$

$$\begin{aligned} dq_j &= \frac{\lambda'(a)da_{j-1} + \lambda'(a)da_{j+1} - 2\lambda'(a)da_j}{2t} \\ da_j &= A_q dq_j \text{ for } j \geq 3. \end{aligned} \quad (32)$$

Assume first that there are  $n = 2m$  media, then media  $m + 1$  is facing  $m$  and  $m + 2 = n + 2 - m$ . In that case, we can write  $da_{m+2} = da_{n-m} = da_m$  and by (32)

$$\begin{aligned} dq_{m+1} &= \frac{2\lambda'(a)da_m - 2\lambda'(a)da_{m+1}}{2t} \\ da_{m+1} &= A_q dq_{m+1} \end{aligned}$$

Assume now that there are  $n = 2m + 1$  media, then media  $m + 1$  is facing  $m$  and  $m + 2 = n + 2 - (m + 1)$ . In that case, we have  $da_{m+2} = da_{n-m} = da_{m+1}$  so that by (32)

$$\begin{aligned} dq_{m+1} &= \frac{\lambda'(a)da_m - \lambda'(a)da_{m+1}}{2t} \\ da_{m+1} &= A_q dq_{m+1} \end{aligned}$$

We can rewrite condition (31) as

$$-\frac{dp_1}{\lambda'(a)} = da_1 - 2 \left( \frac{t}{\lambda'(a)A_q} + 1 \right) da_2 + da_3 \quad (33)$$

and condition (32) as

$$0 = da_{i-1} - 2 \left( \frac{t}{\lambda'(a)A_q} + 1 \right) da_i + da_{i+1}, \quad i = 3, \dots, m. \quad (34)$$

Consider media  $m + 1$ :

- if  $n = 2m$ , we know that  $da_{m+2} = da_m$ ; therefore

$$\left(\frac{t}{\lambda'(a)A_q} + 1\right) da_{m+1} = da_m \quad (35)$$

- if  $n = 2m + 1$ , we know that  $da_{m+2} = da_{m+1}$ ; therefore

$$2\left(\frac{t}{\lambda'(a)A_q} + 1\right) da_{m+1} = da_m + da_{m+1} \quad (36)$$

Denote by  $\alpha$  (resp.  $\beta$ ) the smaller (resp. larger) root of  $0 = 1 - 2\left(\frac{t}{\lambda'(a)A_q} + 1\right)x + x^2$ . Observe that when  $\lambda'(a) > 0$ , the equation admits two (positive) roots, and when  $\lambda'(a) < 0$ , the equation admits two (negative) roots only when  $\lambda'(a) > \frac{t}{2A_q}$ . Let

$$da_i = x_i \alpha^{i-1} + y_i \beta^{i-1} \quad (37)$$

From (33)

$$-\frac{dp_1}{\lambda'(a)} - da_1 = (-2\left(\frac{t}{\lambda'(a)A_q} + 1\right)x_2\alpha + x_3\alpha^2) + (-2\left(\frac{t}{\lambda'(a)A_q} + 1\right)y_2\beta + y_3\beta^2).$$

From (34), for  $i = 3, \dots, m$

$$0 = (x_{i-1} - 2\left(\frac{t}{\lambda'(a)A_q} + 1\right)x_i\alpha + x_{i+1}\alpha^2)\alpha^{i-2} + (y_{i-1} - 2\left(\frac{t}{\lambda'(a)A_q} + 1\right)y_i\beta + y_{i+1}\beta^2)\beta^{i-2}$$

For  $m + 1$ , from (35), (36) and (37)

$$\begin{aligned} 0 &= \left(x_m - \left(\frac{t}{\lambda'(a)A_q} + 1\right)x_{m+1}\alpha\right)\alpha^{m-1} \\ &\quad + \left(y_m - \left(\frac{t}{\lambda'(a)A_q} + 1\right)y_{m+1}\beta\right)\beta^{m-1} \text{ if } n = 2m \end{aligned}$$

$$\begin{aligned} 0 &= \left(x_m - \left(\frac{2t}{\lambda'(a)A_q} + 1\right)x_{m+1}\alpha\right)\alpha^{m-1} \\ &\quad + \left(y_m - \left(\frac{2t}{\lambda'(a)A_q} + 1\right)y_{m+1}\beta\right)\beta^{m-1} \text{ if } n = 2m + 1 \end{aligned}$$



Set  $x_i = x$  and  $y_i = y$ . We obtain

$$-\frac{dp_1}{\lambda'(a)} - da_1 = -x - y \quad (38)$$

$$\begin{aligned} 0 &= x \left( 1 - \left( \frac{t}{\lambda'(a)A_q} + 1 \right) \alpha \right) \alpha^{m-1} + y \left( 1 - \left( \frac{t}{\lambda'(a)A_q} + 1 \right) \beta \right) \beta^{m-1} \text{ if } n = 2m \\ 0 &= x \left( 1 - \left( \frac{2t}{\lambda'(a)A_q} + 1 \right) \alpha \right) \alpha^{m-1} + y \left( 1 - \left( \frac{2t}{\lambda'(a)A_q} + 1 \right) \beta \right) \beta^{m-1} \text{ if } n = 2m + 1 \end{aligned}$$

or

$$x \left( \frac{1 - \alpha^2}{2} \right) \alpha^{m-1} = -y \left( \frac{1 - \beta^2}{2} \right) \beta^{m-1} \text{ if } n = 2m \quad (39)$$

$$x (\alpha - \alpha^2) \alpha^{m-1} = -y (\beta - \beta^2) \beta^{m-1} \text{ if } n = 2m + 1 \quad (40)$$

From (38) and (39) we can compute  $x$  and  $y$  corresponding to the case  $n = 2m$ . Then injecting those values into (37) for  $i = 2$ , we obtain:

$$da_2 = - \left( \frac{dp_1}{\lambda'(a)} + da_1 \right) \left( \frac{\alpha (\beta^{m-1} - \beta^{m+1}) - \beta (\alpha^{m-1} - \alpha^{m+1})}{(\alpha^{m-1} - \alpha^{m+1}) - (\beta^{m-1} - \beta^{m+1})} \right) \quad (41)$$

if  $n = 2m$

$$da_2 = - \left( \frac{dp_1}{\lambda'(a)} + da_1 \right) \left( \frac{\alpha (\beta^m - \beta^{m+1}) - \beta (\alpha^m - \alpha^{m+1})}{(\alpha^m - \alpha^{m+1}) - (\beta^m - \beta^{m+1})} \right) \quad (42)$$

if  $n = 2m + 1$

Notice that  $\alpha\beta = 1$ . So we can also write (41) and (42) as

$$\begin{aligned} \lambda'(a)da_2 &= (\lambda'(a)da_1 + dp_1) \frac{\alpha^{\frac{n}{2}-1} + \beta^{\frac{n}{2}-1}}{\alpha^{\frac{n}{2}} + \beta^{\frac{n}{2}}} \\ &= (\lambda'(a)da_1 + dp_1) \frac{\alpha^{n-1} + \alpha}{\alpha^n + 1} \text{ for any } n, \text{ even or odd.} \end{aligned}$$

Using (30), we write the variation of quantity as

$$\begin{aligned}
tdq_1 &= \lambda'(a)da_2 - dp_1 - \lambda'(a)da_1 \\
&= -(1 - \frac{\alpha^{n-1} + \alpha}{\alpha^n + 1})(\lambda'(a)da_1 + dp_1) \\
&= -(\frac{\alpha^n + 1 - \alpha^{n-1} - \alpha}{\alpha^n + 1})(\lambda'(a)da_1 + dp_1) \\
&= -\frac{(1 - \alpha)(1 - \alpha^{n-1})}{\alpha^n + 1}(\lambda'(a)da_1 + dp_1)
\end{aligned}$$

The result can be extended to  $i \neq 1$ .

### Proof of proposition 7

Assume  $\lambda'(a) > 0$ . Consider what happens when  $n$  is large. Recall that  $a_n$  is defined by (23) as

$$\varphi_a(\frac{1}{n}, a_n) = \lambda'(a_n) \quad (43)$$

As  $n$  gets large,  $a_n$  converges to  $a^*(0)$ , while the price

$$s_n = \frac{v(\frac{1}{n}, a_n)}{n} \quad (44)$$

is equivalent to  $\frac{1}{n}v(0, a^*(0))$ .

We know that

$$A_q = -\frac{s_q}{s_a} = -\frac{nv(\frac{1}{n}, a_n) + v_q(\frac{1}{n}, a_n)}{v_a(\frac{1}{n}, a_n)}.$$

Using (44), when  $n$  increases,  $\lambda'(a)A_q$  evolves as  $-n\frac{v(0, a^*(0))}{v_a(0, a^*(0))}\lambda'(a^*(0))$  and thus tends to infinity. This implies that  $\alpha(n, a_n)$  converges to 1 with  $1 - \alpha \sim \sqrt{\frac{2t}{\lambda'(a)A_q}} \sim \frac{\beta}{\sqrt{n}}$  where  $\beta = \sqrt{-\frac{2tv_a(0, a^*(0))}{v(0, a^*(0))\lambda'(a^*(0))}}$  and  $\theta(n, a_n)$  goes to infinity,

$$\theta \sim \frac{\sqrt{n} \left(1 + \left(1 - \frac{\beta}{\sqrt{n}}\right)^n\right)}{\beta \left(1 - \left(1 - \frac{\beta}{\sqrt{n}}\right)^{n-1}\right)} \sim \frac{\sqrt{n}}{\beta}$$

The term  $\frac{t\theta}{n}$  thus converges to zero, but at the same rate as  $\frac{1}{\sqrt{n}}$  instead of  $\frac{1}{n}$ . The term  $r_q$  converges to  $\varphi(0, a^*(0)) > 0$ . Thus the price is below marginal cost for  $n$  large.

**Proof of proposition 8**

As shown in the preceding proof, the term  $\frac{t\theta}{n}$  converges to zero at the same rate as  $\frac{1}{\sqrt{n}}$ . The term  $\frac{\varphi_q(\frac{1}{n}, a_n)}{n^2}$  converges to 0 at the same rate as  $\frac{1}{n^2}$ . As a result,  $\frac{t}{n^2}(\theta - 1) - \frac{\varphi_q(\frac{1}{n}, a_n)}{n^2}$  is positive when  $n$  is large.

**Proof of proposition 10**

Both for the free and the pay media we have:

$$p - c = \frac{t}{\lambda'(a)} r_a(a, \frac{1}{n}) - r_q(a, \frac{1}{n})$$

and profits

$$\begin{aligned} \pi &= r + \frac{1}{n} (p - c) \\ &= r + \frac{1}{n} \left( \frac{t}{\lambda'} r_a - r_q \right) \\ &= \frac{1}{n} \varphi + \frac{1}{n} \left( \frac{t}{\lambda'} \frac{1}{n} \varphi_a - \varphi - \frac{1}{n} \varphi_q \right) \\ &= \frac{1}{n^2} \left( \frac{t}{\lambda'} \varphi_a - \varphi_q \right) \end{aligned}$$

We then have

$$\frac{\partial}{\partial a} \left( \frac{t}{\lambda'} \varphi_a - \varphi_q \right) = \left( \frac{t}{\lambda'} \varphi_{aa} - \frac{t\lambda''}{\lambda'^2} \varphi_a - \varphi_{aq} \right) < 0$$

**Proof of lemma 11**

For a given  $n$ , the advertising equilibrium level in the quantity and in the price game satisfies equation

$$v\left(\frac{1}{n}, a_n\right) + a_n v_a\left(\frac{1}{n}, a_n\right) = \lambda',$$

and the socially optimal level is given by

$$v\left(\frac{1}{n}, a^o(n)\right) = \lambda'.$$

Since  $v$  is decreasing with  $a$ , we obtain that  $a_n < a^o(n)$ .

Recall that the level of entry in the quantity model  $n_C^*(a)$  is given by

$$\frac{t}{n^2} - k = av_q\left(\frac{1}{n}, a\right) \frac{1}{n^2}.$$

The condition defining  $n^o(a)$  is

$$\frac{t}{4n^2} - k = \frac{1}{n^2} \int_0^a v_q\left(\frac{1}{n}, \gamma\right) d\gamma \quad (45)$$

Observe that when  $v_{qa} \leq 0$ ,  $av_q(\frac{1}{n}, a) \leq \int_0^a v_q(\frac{1}{n}, \gamma) d\gamma$  so that  $\frac{t}{n^2} - av_q(\frac{1}{n}, a) \frac{1}{n^2} > \frac{t}{4n^2} - \frac{1}{n^2} \int_0^a v_q(\frac{1}{n}, \gamma) d\gamma$ . And thus  $n_C^*(a) > n^o(a)$ .

### Proof of proposition 12

Consider first the benchmark case  $v_q(\cdot) \equiv 0$  : the equilibrium and socially optimal number of media are independent  $n_C^* = \sqrt{\frac{t}{k}}$ ,  $n_B^* = \sqrt{\frac{t\theta}{k}}$  and  $n^o = \sqrt{\frac{t}{4k}}$  and the equilibrium level and the socially optimal level of advertising are independent of  $n$  :  $a_C^* = a_B^* = a^*(0) < a^o$ .

Assume  $v_{qa} \leq 0$  and  $\varphi_{qa} \geq 0$ . We use lemma 11 together with lemma 2 to obtain the result. The sign of  $\frac{dn_C^*}{da}$  is given by the sign of  $-\frac{\frac{\partial \pi(n)}{\partial n}}{\frac{\partial \pi(n)}{\partial a}}$  which is negative, smaller than  $\frac{\partial a_n}{\partial n} < 0$ . This implies that for  $n \geq n_C^*$ ,  $n_C^*(a_n) \leq n$ . Then  $a^o(n) > a_n$  implies that if  $n \geq n_C^*$ , then  $n > n_C^*(a_n) > n_C^*(a^o(n)) > n^o(a^o(n))$ . Thus we must have  $n^o < n_C^*$  and as a consequence  $a^o = a^o(n^o) > a_{n^o} > a_C^*$ . Moreover since  $\frac{\partial a_n}{\partial n} < 0$  and  $n_C^* < n_B^*$ , we have  $a_C^* > a_B^*$ .

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