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Working Paper
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Chemnitz Economic Papers, No. 026

Provided in Cooperation with:
Chemnitz University of Technology, Faculty of Economics and Business Administration

Suggested Citation: Kuhn, Thomas; Pestow, Radomir; Zenker, Anja (2018) : On the Axiomatic Foundation of Ecological Footprint Indices, Chemnitz Economic Papers, No. 026, Chemnitz University of Technology, Faculty of Economics and Business Administration, Chemnitz

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http://hdl.handle.net/10419/190432

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On the Axiomatic Foundation of Ecological Footprint Indices

Thomas Kuhn
Radomir Pestow
Anja Zenker

Chemnitz Economic Papers, No. 026, November 2018
On the Axiomatic Foundation of Ecological Footprint Indices

Thomas Kuhn∗, Radomir Pestow†, Anja Zenker‡
Department of Economics, Chemnitz University of Technology

November 6, 2018

Abstract

The objective of this paper is to provide an axiomatic foundation to the concept of ecological footprint indices. For this purpose, we propose five axioms representing general properties which any ecological footprint measure should fulfill. It can be shown that there exists a unique index which is characterized by the given set of axioms. Its functional form is determined and an economic interpretation is given. We find that the proposed index may resolve or confirm some important issues discussed in the literature. First, it incorporates a trade component indicating the ecological footprint of economic activities embodied in the trade pattern of a country or region. Moreover, the productivity of land use in production as a means to mitigate the pressure on the ecological system is reflected. But most importantly, from a methodological point of view, there is no longer the need for designing ecological footprint indices ad-hoc, in particular for the sake of empirical application.

JEL classification: Q01, Q20, Q30, C43

Keywords: Ecological Footprint Indices, Axiomatic Foundation, Sustainable Welfare, Economic Inequality

∗e-mail: thomas.kuhn@wirtschaft.tu-chemnitz.de, phone: +49 371 531 34941
†e-mail: radomir.pestow@wirtschaft.tu-chemnitz.de, phone: +49 371 531 31742
‡e-mail: anja.zenker@wirtschaft.tu-chemnitz.de, phone: +49 371 531 39967
1 Introduction

Over the last three decades, sustainability has become a widely accepted and most important objective in policy. For being able to measure a sustainable utilization of the biosphere and its resources, different approaches have been developed to obtain qualitative and quantitative conclusions. One of these concepts is called the ecological footprint. It indicates the “pressure” on the natural resources exerted by the population of a region or country and its economic behavior.

For example, Wackernagel/Rees (1996), regarded as one of the pioneer works, and, therefore, one of the foundations of the ecological footprint concept, find that this index can offer an estimation of the land-use for consumption activities of humans as well as the assimilation of waste products by nature. The general application proposes that the more land-area is required by human activity or consumption, all other things equal, the higher is the pressure on the biosphere.

Hence, one of the major assumptions underlying the ecological footprint concept is that future global welfare highly depends on meeting the strong benchmarks of sustainability. As such it calls for maintaining the biosphere as a source for any kind of economic activity as well as a sink for the waste products generated alongside. Given that consumption is largely driven by the availability of renewable resources (Bicknell et al., 1998), the debate on sustainable lifestyle is strongly connected to the debate on climate change (Collins/Flynn, 2015). The ecological footprint concept thus generates a link between sustainability and consumption activities as it may estimate the pressure of the economic activity of a population on the available natural resources, in particular by means of land-area use.

Further research on ecological footprints and its controversial debate in the literature created a broad variety of indices proposed for quantifying a specific human’s footprint measure (e.g. Galli et al., 2012, Čuček et. al., 2012, Wang et al., 2008, Venetulis et al., 2008, and Fang et. al., 2014 provide a comprehensive overview and well-founded comparison of different sustainability indicators). However, to our knowledge, an axiomatic foundation to specify the characteristics considered as being appropriate for ecological footprint measures in general has not been proposed yet. Therefore, to this day, ecological footprint indices primarily have been developed through the adaption of the groundbreak-
ing concepts provided by Wackernagel and Rees (i.a. Rees, 1992, Wackernagel/Rees, 1996, Wackernagel et al., 1999, Bicknell et al., 1998, Ferguson, 1999, Venetulis et al., 2008, Wang et al., 2008, Galli et al., 2011, 2012, 2016, Borucke et al., 2013, Collins/Flynn, 2015, and Lin et al., 2018) in a more or less ad-hoc fashion. The reasoning on their mathematical appearance is often alike with certain existing similarities related to the variables employed. It is undisputable that the intended ecological interpretation is one important reason for the defined structure of such indices and, furthermore, that satisfying the specific requirements of their practical application is self-evident. Yet, none of the ecological indices used in the literature have been further reviewed for their axiomatic foundations while the majority of economic indices such like price indices or indices of economic inequality is thoroughly analyzed and is subject to general mathematical principles. This fact is all the more surprising considering that ecological footprint data is commonly reflected in the official statistics of various international organizations and have been included in those institutions’ methodologies on environment statistics as an indicator for sustainable land use (see, e.g., OECD, 2008, UNEP, 2010, UN Statistics Division, 2013, 2018) As a consequence, the appropriateness of ecological footprint indices is still to be discussed and established as far as their axiomatic characterization is concerned.

This paper therefore intends to provide an axiomatic approach to a general ecological footprint measure. By analyzing the features which footprint indices proposed in the literature might have in common, their appropriate characteristics (such as variables and their interdependencies) are identified. Then, we define a set of fundamental axioms representing the theoretical concept we have in mind of what an ecological footprint should measure. The advantage of this approach lies in the proposition of a few stylized facts for which we have a clear idea on how the index should respond to. But most importantly, once the formula of the index has been determined it can be applied to any real-world situation, irrespective of the particular values the independent variables may take. Following this procedure we can show that a unique index exists which meets the axiom system proposed. Subsequently, its features are discussed in detail on the background of well-known indices in the literature.

The outline of the paper is as follows: In section 2 we try to give a short review of the existing footprint-literature with its various proposed measures. From there, in section 3, we are able to emphasize some particular properties of economic footprint measures, represented by an axiom system which any index should meet. Finally, an ecological footprint
2 Ecological Footprint Indices in the Literature

The origin of ecological footprint indices is an academic work of Rees (1992). Initially, the term 'appropriated carrying capacity' first defined how a given environment can support a population of maximum size which also included the biological sphere. The term 'ecological footprint' was added later. At that time, disputes arose how area is used as a variable to cover the demands of an urban region. Before, urbanization focused solely on the area used for living and providing goods and services for the citizen inside the urbanized structure (Collins/Flynn, 2015). Rees (1992) argues that it actually needs far more area to satisfy the demand since goods are traded from and to other regions. Thus, the area needed for production 'elsewhere' has to be included when researching urbanization. This statement was a clear distinction compared to common research before.

Later on, the purpose of the ecological footprint changed such that it measured the burden imposed on the biosphere by human activity Wackernagel et al. (1999). They argue that the impact of an activity on nature correlates with the area needed to perform it. That means the used natural capital needs to be expressed in (bio-)physical units in order to enable a quantitative measurement. The authors emphasize that a monetary analysis is inappropriate because it implies substitutability, enables future-related discounting and examines marginal rather than absolute values. Moreover, they state that their footprint measure is determined by basically four variables, namely the ecological productivity, the population size, living standards and technology. The ecological footprints of people cannot overlap which indicates a competition for ecological space. (Wackernagel et al., 1999)

According to these considerations, the ecological footprint of a country or region, $EF$, is calculated as the sum of the footprint components of all $N$ sectors in a country or region multiplied by a so called equivalence factor $EQF_i$:

$$EF = \sum_{i=1}^{N} FC_i \cdot EQF_i$$  \hspace{1cm} (1)

where $FC_i$ is the footprint component of sector $i$ ($i = 1, \ldots, N$) determined by
\[ FC_i = \frac{Y_i \gamma_i^n}{\gamma_i^n \gamma_i^w} \]  
(2)

\[ \gamma_i^w = \frac{Y_i^w}{A_i^w} \]  
(3)

\[ \gamma_i^n = \frac{Y_i^n}{A_i^n} \]  
(4)

with \( Y_i \) denoting the product output of sector \( i \), \( \gamma \) being the average product yield (Wackernagel et al., 1999), and \( n, w \) stand for a country and the world, respectively. The ecological footprint is typically measured in global area units (e.g. Fang et. al., 2014, Čuček et. al., 2012, and Galli et al., 2012). A country’s footprint can thus be interpreted as the total land-area appropriated to output production at world average productivity (Wackernagel et al., 1999). If the numerator is being replaced by \( C_i + XS_i \) with \( XS_i \) denoting a country’s net exports in sector \( i \), we can easily derive the ecological footprint of consumption. This index offers insights on how much area is used for the specific consumption of one region, it is therefore regarded as an indicator of consumption-based accounting.

While the ecological footprint mainly focuses on the pressure of human demand on the biosphere’s regenerative capacity, other footprint measures have been developed to specifically examine the impact of human activity on other ecosystem compartments such as the carbon footprint (atmosphere), the water footprint (hydrosphere), and fossil energy footprint (lithosphere) etc. Therefore, the notion of the ‘footprint family’ has been recently introduced in the literature. Although there are indeed overlaps between the footprint approaches which give rise to concerns of double counting, most authors recognize a certain complementarity within the footprint family (e.g. Fang et. al., 2014, Čuček et. al., 2012, and Galli et al., 2012) and suggest a joint application of the ecological, carbon and water footprint with the ecological footprint being narrowed down to a ‘land footprint’ to avoid double counting.

A different approach constitutes the Sustainable Process Index (SPI) which was developed by Krotscheck/Narodoslawsky (1996). It evaluates the operability of economic activity based on a life-cycle assessment, provided that sustainability is ensured. The SPI is a part of ecological engineering and it is argued that sustainability of the economy is achieved if a sustainable flow of solar energy is provided. However, area is required for the conversion of solar energy into services and is thus a limiting factor. The SPI is calculated as the ratio of the specific service area, \( A_{tot}/S_{tot} \), and the per-inhabitant area in the region.
which is relevant for the production process, \(a_{in}\):

\[
SPI = \frac{A_{tot}/S_{tot}}{a_{in}} \tag{5}
\]

with \(A_{tot}\) denoting the total area assigned to embed a process sustainably in the ecosphere and \(S_{tot}\) denotes the number of unit-services such as the units of output which are supplied by the process within one reference period. Measuring the area needed to provide the necessary material and energy proportional to the available area for the economic activity of one human being is the procedure of the SPI (Krotscheck/Narodoslawsky, 1996). Given that the area is included in the numerator as well as in the denominator, the SPI is expressed in capita per service unit. Once again, the main variables are area and energy related to population size similar to the approach of Wackernagel et al. (1999).

Another approach is the component-based model of ecological footprinting developed by Simmons/Lewis/Barrett (2000) as an alternative ecological footprint. The purpose of study was to calculate the impact of different human lifestyles and organizations on nature on the domestic level. Hence, it is a bottom-up approach usable for small entities such as households, organizations and regions within nations. The calculation of the component-based ecological footprint, CBEF, itself is similar to the footprint of Wackernagel et al. (1999), but area use and economic process get subdivided into smaller (sector-specific) parts as long as data is available (Simmons/Lewis/Barrett, 2000):

\[
CBEF_i = \frac{\sum X A_X}{Y_i} \tag{6}
\]

where \(A_X\) denotes the area needed for the production of the intermediate good \(X\) which is used for the production of the final product \(Y\) and \(Y_i\) indicates the service quantity of the final product \(Y\) in (service) sector \(i\). Consequently, the component-based subindex is typically expressed in area per functional service unit such as hectares per passenger kilometer. The main variables are land use for the production of the intermediate, a final good, and a region for which the analysis is performed. These subindices can then be aggregated over different consuming entities (e.g. a population, organization or region) with given amounts of consumption \(c_i\) of service or product \(i\).
Given the different approaches to measure a relative concept which have been customized to their respective research focus as well as to their field of application, it is essential to create an axiomatic foundation to reveal the properties an appropriate consumption-based footprint index should exhibit, in particular with respect to the functional form mapping the input variables. Thereby, it could not only be assessed whether the footprint indices proposed in the literature satisfy the properties considered but, above all, how an appropriate index should look like mathematically. Our analysis thus should address such an axiomatic system in the following section.

3 The Axiom Set

As argued above, there is a need for an axiomatic foundation of the ecological footprint concept. Hence, it is to be determined which properties such an index must meet for proper empirical applications to measuring the sustainable land use within an economy. In this section, we therefore develop a system of axioms for ecological footprint indices (EFI) comprising properties generally accepted in the relevant literature. First, we have to give a general definition.

Definition 1. Let $D = \mathbb{R}^4_+$ be a set of the ecological states of a country or a region, where $x = (A, C, P, Y) \in D$ is a vector representing ecological variables composed of the land-use in production $A$, the domestic consumption $C$, the population size $P$, and the domestic production $Y$. Then, an ecological footprint index is a mapping:

$$f : D \mapsto \mathbb{R}_+, \quad x \mapsto f(x)$$

with the meaning that the footprint of a country with state $x_1 \in D$ is higher than the footprint of a country with state $x_2 \in D$

$$f(x_1) > f(x_2)$$
An ecological footprint index $f$ should satisfy the following set of axioms.

**Axiom 1 (Norm).** *The function $f$ takes the value 1 if each argument takes the value 1:*

$$f(1,1,1,1) = 1$$  \hfill (10)

*Remark 1.* This axiom is setting the unit of the scale. It is reasonable to propose that the index should take the value 1 if all variables take the value 1.

**Axiom 2 (Monotonicity).** *The function $f$ is strictly increasing in $C$ and strictly decreasing in $Y$:*  

$$f(A,C,P,Y) < f(A,\overline{C},P,Y) \quad \text{for} \quad C < \overline{C}$$ \hfill (11)

$$f(A,C,P,Y) > f(A,C,P,\overline{Y}) \quad \text{for} \quad Y < \overline{Y}$$ \hfill (12)

*Remark 2.* It seems to be natural to assume that the ecological footprint is higher for higher levels of consumption, all other things equal, and that it is lower for lower levels of production. While the first property is evident and perfectly in line with what an EFI should measure from the view of sustainable resource use, the second property needs some further explanation: Since the ecological footprint measures the land area required to satisfy the consumption needs of one human being in a country, a rise in production implies a higher productivity of land use in production. However, such an increase does not necessarily drive up domestic consumption as the domestically produced excess supply by definition gets exported to satisfy other countries’ consumption needs. In this regard, the ecological footprint, at any rate, is supposed to be a consumption-oriented, not a production-oriented index. The same kind of reasoning holds for the case of excess demand.

**Axiom 3 (Proportionality and Inverse Proportionality).** *A scale increase in consumption by factor $\lambda$ leads to a proportional increase in the value of $f$ by the same factor, and a scale*
increase in production by factor $\lambda$ leads to an inverse-proportional decrease in the value of $f$ by the same factor:

$$f(A, \lambda C, P, Y) = \lambda f(A, C, P, Y) \quad \text{for} \quad \lambda > 0 \quad (13)$$

$$f(A, C, P, \lambda Y) = \frac{1}{\lambda} f(A, C, P, Y) \quad \text{for} \quad \lambda > 0 \quad (14)$$

Remark 3. Axiom 3 states that the ecological footprint index is a linear function of the domestic consumption as well as an inverse-proportional function of output. In our opinion, this axiom is reasonable. If a society consumes on a larger scale relative to another society, then its ecological footprint value should indicate this larger scale one to one. The same arguing holds for the scale of production: if a society produces on a larger scale, all other things being equal, then production is more land-efficient and, moreover, there is obviously excess supply to world markets. Hence, the value of the ecological footprint index should reflect the larger scale of production one to one. Scale changes in production and consumption are thus directly translated into the respective changes of the EFI values.

Axiom 4 (Dimensionality). An equally proportional change in consumption and population size does not change the value of the index. Likewise, the value of the index is not affected by an equally proportional change in land-area and production-output.

$$f(A, \lambda C, \lambda P, Y) = f(A, C, P, Y) \quad \text{for} \quad \lambda > 0 \quad (15)$$

$$f(\lambda A, C, P, \lambda Y) = f(A, C, P, Y) \quad \text{for} \quad \lambda > 0 \quad (16)$$

Remark 4. Axiom 4 addresses the dimensionality of an EFI index, indicating that the ecological footprint is unaffected if, on the one hand, product consumption as well as the population size, or, on the other hand, output production and the land area, respectively, are magnified by the same factor. In the first case, the per-capita consumption remains unchanged, and so does the ecological footprint. In the latter case, the productivity of
land area remains unchanged since a proportional change in land input brings about an equally proportional change in output.

Axiom 5 (Commensurability). The index value remains the same if a proportional increase in the per-capita consumption is offset by an equally proportional increase in the productivity of the land area used.

\[ f\left(\frac{1}{\lambda}A, \lambda C, P, Y\right) = f(A, C, P, Y) \quad \text{for} \quad \lambda > 0 \tag{17} \]

\[ f(A, C, \frac{1}{\lambda}P, \lambda Y) = f(A, C, P, Y) \quad \text{for} \quad \lambda > 0 \tag{18} \]

Remark 5. Axiom 5 finally presents properties associated with the commensurability of the EFI by stating that a proportional change in per-capita consumption can be offset by an increase in productivity, in the same proportion, brought about either by a proportional increase in output or by a proportional decrease in land input, .

In the following section, we will derive an index which satisfies the five axioms simultaneously.

4 Existence and Uniqueness of the Ecological Footprint Index

We will now state the following proposition.

Theorem (Existence and Uniqueness). Axioms 1 through 5 characterize the following unique index:

\[ \text{EFI}(A, C, P, Y) := \frac{C}{Y} \cdot \frac{A}{P} = \frac{C/p}{Y/A} \tag{19} \]

Proof. See Appendix.
The EFI may have different meanings: first of all, it is given by the ratio of the domestic per-capita consumption, \( C/P \), and the productivity of the land use, \( Y/A \). Here, an increase in the productivity of land in production mitigates the pressure put on the ecological system by human economic activity. Secondly, the EFI can be interpreted as the per-capita land area, \( A/P \), used to satisfy consumption needs adjusted by a factor which reflects the excess supply, \( C/Y \). Hence, if like in Wackernagel et al. (1999) the gap between production and consumption is interpreted as a country’s excess supply and excess demand, respectively, then the EFI turns out to be the per-capita land area appropriated adjusted by a trading coefficient:

\[
 EFI := \left(1 - \frac{XS}{Y}\right) \cdot \frac{A}{P}
\]

(20)
given by a country’s consumption share on total production where \( XS \) is denoting net exports. Therefore, if the country is a net exporter, i.e. \( XS > 0 \), it’s ecological footprint is decreasing (increasing) if the export share \( XS/Y \) is going up (down). A similar reasoning holds if a country is a net importer, i.e. \( XS < 0 \).

Compared to the footprint indices existing, in particular the compound based footprint used by the global footprint network, it should be emphasized that the index characterized in the theorem stated above applies to just one type of land area and to one particular geographic area, may it be a region, country, or the world as whole. The aggregation over different land-area types as well as different geographic areas still remains an unresolved issue. In case of the ad-hoc design of indices this problem is dealt with in the literature for instance by constructing a common scale for different land area types in terms of world average hectares, and a common scale for the different land productivities among geographic areas in terms of global hectares. However, whether a common scale is necessary at all in the aggregation procedure, using an axiomatic approach, is the scope for future research.

5 Concluding Remarks

The purpose of this paper was to establish an axiomatic foundation to the concept of ecological footprint indices. We first identified the characteristics of generally accepted indices and discussed the relations between the various input variables. We then proposed
five axioms which we considered appropriate for constructing a mathematical formula for footprint indices in general. It has been shown that there exists a unique index which is meeting all the axioms simultaneously, and its functional form has been derived.

We find that the proposed index is resolving some important issues discussed in the literature. First, it incorporates the impact of a country’s trade pattern on the sustainability of consumption and human economic activities. In a sense, the footprint embodied in the imports and exports of goods and services is accounted for. Moreover, the importance of land productivity as a means to mitigate the pressure on the ecological system is reflected by the measure. But most important, from a methodological point of view, ecological footprint measures no longer must be introduced ad-hoc, but are given a theoretical foundation. It has been shown that there exists a unique footprint index being characterized by the given set of axioms considered being appropriate. This is of particular interest for empirical applications.

However, in our belief, a future field of research will open up for examining the potential existence of even more footprint indices and their properties to be based on alternative axiom systems. In this line, the index proposed in this paper may serve as just one example of measures following reasonable mathematical postulates. Moreover, the aggregation procedure is still open to discussion, since the restrictive construction of a common scale like global hectares might be dispensable.

**Appendix**

*Proof of the Existence and Uniqueness Theorem.* First, \( EFI(A,C,P,Y) := \frac{C}{Y} \cdot \frac{A}{P} \) satisfies axioms 1 to 5. This can be seen by a straightforward calculation after substituting \( EFI \) for \( f \) in the axioms, which proves the existence of an index that satisfies the given axioms.

It remains to show the uniqueness of the index. For this we will derive the \( EFI \) from axioms 1, 3 and 4.

By axiom 3 we have:
\[ f(A,C,P,Y) = C \cdot f(A,1,P,Y) \]
\[ = \frac{C}{Y} \cdot f(A,1,P,1) \]  
\[ (21) \]

Using the first equation from axiom 4 with \( \lambda = P \) and then axiom 3 we get:

\[ \ldots = \frac{C}{Y} \cdot f(A,\frac{1}{P},1,1) \]
\[ = \frac{C}{YP} \cdot f(A,1,1,1) \]  
\[ (22) \]

Applying the second equation from axiom 4 with \( \lambda = \frac{1}{A} \) and axiom 3 yields:

\[ \ldots = \frac{C}{YP} \cdot f(1,1,1,\frac{1}{A}) \]
\[ = \frac{CA}{YP} \cdot f(1,1,1,1) \]  
\[ (23) \]

Finally, we obtain by using axiom 1:

\[ f(A,C,P,Y) = \frac{C}{Y} \cdot \frac{A}{P} \]
\[ (24) \]

As is known from the first part of the proof, this index also satisfies the remaining axioms 2 and 5.

Thus, axioms 1 through 5 define the unique index \( EFI(A,C,P,Y) \).

As the proof shows, axioms 1, 3 and 4 are already sufficient to uniquely define \( EFI \), while the remaining axioms (2 and 5) are implications of these. Trivially, the remaining axioms are therefore redundant but, again, are consistent with axioms 1, 3 and 4.
References


