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# Access and Cooperative Investment under Asymmetric Infrastructure Competition

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## Abstract

We introduce a model of asymmetric competition where two network operators with different investment costs may build an internet access infrastructure and where a virtual operator provides services through third-party access. We show that the virtual operator requests access from the low cost network operator, because it reduces the geographical area where both network operators build an infrastructure. We also show that potential entry of a virtual operator increases welfare if the access price and the cost asymmetry between the network operators is high enough. Finally, regardless of whether a virtual operator is allowed to enter the market or not, standard access regulation leads to broader infrastructure coverage than co-investment if and only if the profit of a network operator that builds a monopolistic infrastructure is higher than the total profit two network operators that both invest.

*Keywords:* Next generation networks; Access obligations; Asymmetric competition

*JEL codes:* L96; L51.

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# 1 Introduction

Investments in next-generation access networks (NGANs) are a priority for policymakers in the European Union. NGANs can provide superfast broadband access to the Internet, and are expected to bring large benefits to the economy and the society, in terms of increased productivity, venture creations, etc. Operators are therefore encouraged by public authorities to invest massively in these new infrastructures. However, to avoid the monopolization of the new networks by a few operators, different network-sharing provisions have been introduced or are envisioned to allow smaller players to remain active in the downstream markets, such as standard access or co-investment agreements.

In Europe and in many other countries, the deployment of superfast broadband infrastructures is also mainly achieved by telecommunications operators on the one hand and cable operators on the other. While telecom operators invest to expand their NGA coverage gradually, cable operators mainly keep their coverage fixed, but upgrade their old-technology cable networks to provide superfast broadband. In other words, telecom operators and cable operators face different deployment costs.

In this paper, we propose a framework to analyze the rollout of new infrastructures by incumbents with different deployment costs, when network-sharing obligations are in place.

More specifically, we study the coverage strategies of two incumbent operators that compete with an entrant, which does not possess any infrastructure, and can be only active via a network-sharing agreement. We consider two regimes of network sharing: the standard access regime and the co-investment regime. In the access regime, the entrant pays a per-unit access price to the access provider. In the co-investment regime, the entrant pays half of the investment cost to the incumbent that shares its network. We assume that the network-sharing obligation applies to only one of the two incumbents, and that this access provider is exogenously determined.

First, we consider a scenario where only the incumbent operator providing access (the “telco”) invests to deploy the new technology, whereas the rival incumbent (the “cable operator”) has a coverage which is exogenously determined. In this scenario, we show that the telco’s coverage is larger under the co-investment regime than under the access regime, unless the access price is high enough and products are sufficiently homogeneous.

Second, we analyze the scenario where both incumbents have to decide

how much of the country to cover with the new technology. In this scenario, we show that a lower access price reduces coverage in the access regime. Furthermore, a lower access price can yield a change in equilibrium, to an equilibrium where the access provider leads in investment to an equilibrium where it follows. We also show that total coverage tends to be larger under co-investment than under access if the access provider leads in investment, whereas total coverage is the same under the two regimes if the other incumbent leads.

This paper is related to the literature on access and investment in network industries.<sup>1</sup> In the context of next-generation access networks, the literature has investigated different questions: which access rules are the most appropriate to solve potential conflicts between static efficiency and investment incentives (Klumpp and Su, 2010; Nitsche and Wiethaus, 2011); how access to the legacy network affects investment incentives for the new networks (Bourreau, Cambini and Dogan, 2012; Inderst and Peitz, 2012); co-investment as a cost-sharing or risk-sharing agreement (Inderst and Peitz, 2014); the impact of co-investment on the possibility of collusion (Krämer and Vogelsang, 2016).

The present paper is more closely related to Bourreau, Cambini and Hoernig (2018). In this paper, the authors compare three different regimes of access – the standard access regime, co-investment, and co-investment with access – in a setting with one incumbent firm and one entrant. We extend their analysis by introducing a second incumbent firm and by analyzing the competition in coverage investments between the incumbents.

The rest of the paper is organized as follows. In Section 2, we set up the model. In Section 3 we analyze investment incentives in the access regime and in Section 4, we do the same for the co-investment regime. In Section 5, we solve the model in two scenarios: one scenario where only one incumbent invests in coverage; and one scenario in which both incumbents decide on coverage. Finally, Section 6 concludes.

## 2 The model

Consider a country consisting of a continuum of areas  $z \in [0, \infty)$ , which have identical demand but different sunk costs to be covered with a new network infrastructure. There are two network operators (NOs), Firms 1 and 2, and

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<sup>1</sup>See Cambini and Jiang (2009) for a survey.

a virtual operator (VO), Firm 3. The NOs deploy their infrastructures over the country, whereas the VO needs to access the infrastructure of a NO to enter the market. For Firm  $i \in \{1, 2\}$ , the cost of covering an area  $z$  is  $c_i(z)$ , where  $c_i(z)$  is differentiable and strictly increasing in  $z$ . Covering the areas  $[0, z]$  then costs  $C_i(z) = \int_0^z c_i(x) dx$ .

We then study the following game. In the first stage, a regulatory authority sets the access price. As in most of the literature on access regulation, we consider a constant access price per unit of output. In the second stage, the VO decides which NO to request access from in the areas where both NOs will invest. In the third stage, the NOs simultaneously decide on the areas where they will deploy infrastructures. Finally, in the fourth stage, the firms compete in local areas and profits are realized.

### 3 Competition in local areas

We assume that firms can adjust prices in local areas, according to the market structure. The following market structures can be encountered: In the areas where both NOs have invested, the VO will request access from one of them and pay the access price  $a_3$ ; In the areas where only NO  $i$  has invested, it will provide access to NO  $j \neq i$  and the VO, at prices  $a_j$  and  $a_3$ , respectively; In the areas where no firm has invested, no internet access service is provided.

If, by convention, Firm 1 is the access provider, the local gross profits are

$$\begin{aligned}\pi_1(a_2, a_3) &= (p_1 - c_w - c_r)q_1 + (a_2 - c_w)q_2 + (a_3 - c_w)q_3, \\ \pi_2(a_2, a_3) &= (p_2 - a_2 - c_r)q_2,\end{aligned}$$

and

$$\pi_3(a_2, a_3) = (p_3 - a_3 - c_r)q_3,$$

where  $c_w$  and  $c_r$  are the wholesale and retail marginal costs, respectively. The firms may compete in prices  $p_i$  or in quantities  $q_i$ , and have symmetric (inverse) demands.

We consider that the equilibrium local profits satisfy the following property.

**Assumption 1.** *i. For all  $i \in \{2, 3\}$ , there exists an  $\tilde{a}_i$  such that  $\partial\pi_1/\partial a_i > 0$  iff  $a_i < c_w < \tilde{a}_i$ . ii. For all  $i, j \in \{2, 3\}$  and  $i \neq j$ ,  $0 < \partial\pi_i/\partial a_j < -\partial\pi_i/\partial a_i$ .*

Assumption 1 extends the standard hypothesis of the access regulation literature to the case where two NOs and one VO are in the market. Assumption 1-i. indicates that for all access price  $a_j$  charged to Firm  $j$ , there is an access price  $\tilde{a}_i > c_w$  charged to Firm  $i \neq j$  that maximizes the local profits of the access provider. Assumption 1-ii. indicates that when Firm  $i$  accesses at price  $a_i$  the infrastructure built by a NO, its profit decreases in  $a_i$  more than it increases in the access price  $a_j$  paid by the other firm that requests access.

Since the costs and (inverse) demands are symmetric, it is natural to assume that the regulator will request the access provider to charge the other firms with the same access price  $a_2 = a_3 \equiv a$ . Then, Assumption 1 implies that there is an access price  $\tilde{a}^2$  that maximizes the local profits of the access provider when both NOs invest, and an access price  $\tilde{a}^1$  that maximizes its profit when only one NO invests. We also consider that the access price set by the regulator is the same when only one NO and when both NOs invest.<sup>2</sup>

The profits in the different market configurations are denoted as follows. If only one NO invests, the profit of the access provider is  $\pi_P^1 \equiv \pi_1(a, a)$  and the profit of the other firms is  $\pi_S^1 \equiv \pi_2(a, a) = \pi_3(a, a)$ . If both NOs invest, the profit of the access provider is  $\pi_P^2 \equiv \pi_1(0, a)$ , the profit of the other NO is  $\pi_O^2 \equiv \pi_2(0, a)$ , and the profit of the VO is  $\pi_S^2 \equiv \pi_3(0, a)$ . The ranking of the local profits in the different market configurations is as follows.

**Lemma 1.** *For all  $a \geq c_w$ ,  $\pi_S^2 \leq \pi_S^1 \leq \pi_O^2 \leq \pi_P^2 \leq \pi_P^1$ .*

*Proof.* Straightforward from the definition of the profits and Assumption 1.  $\square$

Lemma 1 indicates that the highest local profit is that of a NO that builds an infrastructure when the other does not. It is followed, in increasing rank order, by the profit of the access provider and the profit of the other NO when both NOs invest. The latter is higher than the profit of the non-investing firms when only one NO invests, which in turn is higher than the profit of the VO when both NOs invest. This ranking is straightforwardly explained by the additional revenue earned by an access provider and the cost disadvantage incurred by a firm that requests access.

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<sup>2</sup>In general, when several NOs invest, competition to provide access to VOs may result in cost-oriented access prices. However, as shown by Bourreau et al. (2011), this is not the only equilibrium and NOs may set a high access price, unless the latter is regulated.

## 4 A benchmark: Symmetric investment costs

In this section, we consider the simple case where the NOs have symmetric investment cost, that is,  $c_1(z) = c_2(z) \equiv c(z)$  for all  $z$ . Then, the identity of the access provider does not matter and the second stage of the game can be omitted. We therefore determine the equilibrium of the investment subgame and study how it is affected by the access price. By convention, we consider that a firm that is indifferent between investing ( $I$ ) and not investing ( $N$ ) will build an infrastructure. We also assume that if the Nash equilibria are  $(I, N)$  and  $(N, I)$ , then the NOs coordinate and one of them builds an infrastructure. We denote as  $\Delta_k$  the difference between the profit of a NO when it invests at the same time as its rival and its profit when it does not invest. If the NO is the access provider when it invests, then  $k = P$  and  $\Delta_P \equiv \pi_P^2 - \pi_S^1$ . Otherwise,  $k = O$  and  $\Delta_O \equiv \pi_O^2 - \pi_S^1$ .

Then, we obtain the following result.

**Lemma 2.** *Both NOs invest if  $z \in [0, c^{-1}(\Delta_O)]$ . One NO invests if  $z \in (c^{-1}(\Delta_O), c^{-1}(\pi_P^1)]$ .*

*Proof.* If  $z \in [0, c^{-1}(\Delta_O)]$ , then investing is a dominant strategy for both NOs. If  $z \geq c^{-1}(\pi_P^1)$ , then investing is a dominated strategy for both NOs. Otherwise, there are two Nash equilibria:  $(I, N)$  and  $(N, I)$ .  $\square$

Lemma 2 indicates that both NOs build infrastructures in areas with low investment costs and refrain from investing in areas with high investment costs. In areas with intermediate investment costs, only one NO finds it profitable to build an infrastructure. Based on this result, the relation between the level of the access price and investment can be summarized as follows.

**Lemma 3.** *For all  $a < \min(\tilde{a}^1, \tilde{a}^2)$ ,  $\partial c^{-1}(\Delta_O)/\partial a > 0$  and  $\partial c^{-1}(\pi_P^1)/\partial a > 0$ .*

*Proof.* We find  $\partial c^{-1}(\Delta_O)/\partial a = (\partial \pi_P^2/\partial a - \partial \pi_S^1/\partial a)(c^{-1})'$  and  $\partial c^{-1}(\pi_P^1)/\partial a = (\partial \pi_P^1/\partial a)(c^{-1})'$ . From Assumption 1,  $\partial c^{-1}(\Delta_O)/\partial a > 0$  for all  $a < \tilde{a}^2$  and  $\partial c^{-1}(\pi_P^1)/\partial a > 0$  for all  $a < \tilde{a}^1$ .  $\square$

Lemma 3 indicates that a higher access price results in a broader infrastructure coverage and in an increase in the number of areas where both NOs invest. This standard result is explained by the fact that a higher access price increases the NOs' incentives to build infrastructures, regardless of the investment strategy chosen by their rival. Although this is not

modeled explicitly in this paper of the paper, the negative side of a higher access price is that it generally results in higher prices and lower quantities, because it increases the marginal cost of the firms that request access and softens competition for the firm that provides access (see Section 6 for an illustration).

## 5 Asymmetric investment costs

In the general case where the NOs have different investment costs, the VO has to decide who it will request access from in the areas where both NOs invest. By convention, consider that Firm 1 is the low cost NO ( $c_1(z) \leq c_2(z)$  for all  $z$ ). We then obtain the following result.

**Lemma 4.** *i. If the access provider is the low cost NO: Both NOs invest if  $z \in [0, c_2^{-1}(\Delta_O)]$ ; One NO invests if  $z \in (c_2^{-1}(\Delta_O), c_1^{-1}(\pi_P^1)]$ ; No NO invests if  $z > c_1^{-1}(\pi_P^1)$ . ii. If the access provider is the high cost NO: Both NOs invest if  $z \in [0, \min(c_2^{-1}(\pi_P^1), c_1^{-1}(\pi_O^1))]$ ; One NO invests if  $z \in (\min(c_2^{-1}(\pi_P^1), c_1^{-1}(\pi_O^1)), c_1^{-1}(\pi_P^1)]$ ; No NO invests if  $z > c_1^{-1}(\pi_P^1)$ .*

*Proof.* i. If the low cost NO provides access, we have either  $c_2^{-1}(\Delta_O) < c_2^{-1}(\pi_P^1) \leq c_1^{-1}(\Delta_P) < c_1^{-1}(\pi_P^1)$  or  $c_2^{-1}(\Delta_O) < c_1^{-1}(\Delta_P) < c_2^{-1}(\pi_P^1) < c_1^{-1}(\pi_P^1)$ . In both cases, investing is a dominant strategy for both NOs if  $z \leq c_2^{-1}(\Delta_O)$  and a dominated strategy if  $z \geq c_1^{-1}(\pi_P^1)$ . Otherwise, there are two Nash equilibria:  $(I, N)$  and  $(N, I)$ . ii. If the high cost NO provides access, we have either  $c_2^{-1}(\Delta_P) < c_1^{-1}(\Delta_O) < c_2^{-1}(\pi_P^1) < c_1^{-1}(\pi_P^1)$  or  $c_1^{-1}(\Delta_O) < c_2^{-1}(\Delta_P) < c_2^{-1}(\pi_P^1) < c_1^{-1}(\pi_P^1)$ . In both case, investing is a dominant strategy for both NOs if  $z \leq \min(c_2^{-1}(\Delta_P), c_1^{-1}(\Delta_O))$  and a dominated strategy if  $z \geq c_1^{-1}(\pi_P^1)$ . Otherwise, there are two Nash equilibria:  $(I, N)$  and  $(N, I)$ .  $\square$

Lemma 4 shows that the outcome of the investment subgame depends on the identity of the access provider in areas where both NOs invest. In particular, it determines the number of such areas. The choice of the access provider matters for the VO, because it does not have the same local profits when one or two NOs build an infrastructure. We obtain the following result.

**Proposition 1.** *The virtual operator requests access from the low cost network operator in areas where both network operators invest.*

*Proof.* From Lemma 1, the VO is better off when only one NO invests. From Lemma 4, there are more areas where only one NO invests if the VO requests



access from the low cost NO. Indeed,  $c_2^{-1}(\Delta_O) < \min(c_2^{-1}(\Delta_P), c_1^{-1}(\Delta_O))$ .  $\square$

Proposition 1 is explained by the fact that the VO is better off when only one NO invests (Lemma 1). Indeed, in this case, only the investing NO has a cost advantage over the VO, whereas both NOs do when they both invest. Therefore, the VO requests access from the low cost NO in order to minimize the number of areas where both NOs invest. The welfare implication of this strategy is a priori ambiguous. Indeed, although investment by only one NO always improves local welfare, infrastructure duplication can either increase or reduce it (see Section 6 for an illustration).

## 6 Virtual operators and welfare: An illustration

In this section, we study whether entry of a VO increases welfare. On the one hand, entry of a VO can be expected to reduce prices and improve welfare in local areas. On the other hand, it may reduce the profits of the NOs and their incentives to build infrastructures. Moreover, as pointed out in Section 5, the VO requests access from the low cost NO, which reduces the number of areas where both NOs invest and ambiguously affect welfare.

We consider that firms compete in quantities for selling a homogeneous internet access service. We assume that the inverse demand and the investment costs are linear. More specifically, for all  $i \in \{1, 2, 3\}$ , the inverse demand is  $p_i = p = 1 - q$ , where  $q$  is the total quantity, and the investment costs are  $c_1(x) = x$  and  $c_2(x) = bx$ , where  $b \geq 1$ . The local profits are the same as defined in Section 3. If there is no VO, then  $q_3 = 0$ . We add the subscript  $D$  (duopoly) to the previous notations to indicate that there are only two NOs in the market and the subscript  $T$  (triopoly) when there are two NOs and a VO.

The gross welfare in local areas is given by  $w_i^j \equiv \int_0^{q_i^j} (1 - x)dx$ , where  $q_i^j$  is the total quantity in market configuration  $i \in \{D, T\}$ , when  $j \in \{1, 2\}$  NOs have invested.

**Lemma 5.** *Entry of a VO improves welfare in local areas where at least one NO invests.*

*Proof.* For all market configurations, welfare is an increasing function of the total quantity. If there are only two NOs in the market, the total equilib-

rium quantity is  $q_D(a) = (2 - a)/3$  when one NO invests and  $q_D(0) = 2/3$  when both NO invest. We also find that the equilibrium quantities are non negative iff  $a \leq 1/2$ . If there are two NO and a VO in the market, the total equilibrium quantity is  $q_T^1 = (3 - 2a)/4$  when one NO invests and  $q_T^2 = (3 - a)/4$  when both NOs invest. We also find that the equilibrium quantities are non negative iff  $a \leq 1/3$ . Since  $q_D(0) \leq q_T^2$  for all  $a \leq 1/3$  and  $q_D(a) < q_T^1$ , entry of a VO improves welfare in areas where at least one NO invests.  $\square$

The fact than entry improves welfare under homogeneous Cournot competition among retailers is a well-know result. Lemma 5 indicates that, as long as the inverse demand is linear, this remains valid when a retailer (the NO that provides access) is also a supplier.

**Lemma 6.** *Entry of a VO reduces the number of areas where at least one NO builds an infrastructure. It also reduces the number of areas where both NOs invest, unless the access price belongs to a narrow, specific range ( $a \in [10/37, 5/8]$ ).*

*Proof.* The number of areas where at least one NO builds an infrastructure is determined by the profit of a monopolistic access provider. The difference between these profits when there are only two NOs in the market and when there are two NOs and a VO is given, for all  $a \leq 1/2$ , by  $\pi_{PD}^1 - \pi_{PT}^1 = 7(1 - 2a)^2/144 \geq 0$ . If there are only two NOs in the market, the number of areas where both NOs invest is determined by the difference between the profit a NO would earn when both NOs invests and the profits it would earn without investing when its rival does. For all  $a \leq 1/2$ , it is given by  $\Delta_{OD} \equiv \pi_{OD}^2 - \pi_{SD}^1 = -(4(-1 + a)a)/(9b)$ . Likewise, if there are two NOs and a VO, the number of areas where both NOs invest is determined by  $\Delta_{OT}(a) = \pi_{OT}^2 - \pi_{ST}^1 = -(3(-2 + a)a)/(16b)$  for all  $a \leq 1/3$  and  $\Delta_{OT}(1/3)$  otherwise. From the difference between  $\Delta_{OD}$  and  $\Delta_{OT}$ , we find that entry of a VO strictly reduces the number of areas where both NOs invest, unless  $a \in [10/37, 5/8]$ .  $\square$

The combination of the effects described in Lemmas 5 and 6 leads to an indetermination. On the one hand, entry of a VO improves welfare in the areas where at least one NO has invested. On the other hand, it reduces the infrastructure coverage. In most cases, it also reduces the number of areas where both NOs invest, which has ambiguous welfare implications.

Welfare with two NOs only and two NOs and a VO is given by

$$W_D \equiv \int_0^{c_2^{-1}(\Delta_{OD})} (w_D^2 - c_2(x))dx + \int_{c_2^{-1}(\Delta_{OD})}^{c_1^{-1}(\pi_{PD}^1)} (w_D^1 - c_1(x))dx$$

and

$$W_T \equiv \int_0^{c_2^{-1}(\Delta_{OT})} (w_T^2 - c_2(x))dx + \int_{c_2^{-1}(\Delta_{OT})}^{c_2^{-1}(\pi_{PT}^1)} (w_T^1 - c_1(x))dx,$$

respectively. The following result ensue from the comparison between (1) and (2).

**Proposition 2.** *i. If the investment cost asymmetry between NOs is high, entry of a VO increases welfare if and only if the access price is high enough.*  
*ii. Otherwise, entry of a VO increases welfare for intermediate levels of access prices and reduces welfare for low and high access prices.*

*Proof.* See the appendix.  $\square$

Proposition 2 is illustrated graphically in Figure 1. It can be explained as follows. The difference between welfare in the two market configurations is given by  $\Delta \equiv W_T(a) - W_D(a)$ . If  $a \in [10/37, 5/8]$ ,

$$\begin{aligned} \Delta &= \Delta_D^2 (w_T^2 - w_D^2) + (\Delta_T^2 - \Delta_D^2) (w_T^2 - w_D^1) + \\ &(\Delta_T^1 - \Delta_T^2) (w_T^1 - w_D^1) + (\Delta_D^1 - \Delta_T^1) (-w_D^1). \end{aligned}$$

Otherwise,

$$\begin{aligned} \Delta &= \Delta_T^2 (w_T^2 - w_D^2) + (\Delta_D^2 - \Delta_T^2) (w_T^1 - w_D^2) + \\ &(\Delta_T^1 - \Delta_D^2) (w_T^1 - w_D^1) + (\Delta_D^1 - \Delta_T^1) (-w_D^1). \end{aligned}$$

If the investment cost asymmetry is high, then the proportion of areas where both NOs invest is small. Indeed, if building infrastructures is much more expensive for the high cost than the low cost NO, there is only a small proportion of areas where investing is a dominant strategy for both NOs (the first and second terms of  $\Delta$  are close to zero). Then, the comparison between welfare with and without a VO is mainly driven by the following effects. On the one hand, when only one NO invests, local welfare is higher with than without a VO (the third term of  $\Delta$  is positive). On the other

hand, coverage is narrower with and without a VO (the forth term of  $\Delta$  is negative). The latter effect dominates for low access prices. Otherwise, the former effect dominates, which explains Proposition 2-i.

If the cost asymmetry is lower, then there is a significant proportion of areas where both NOs invest, both with and without a VO. This affects the comparison between the two market configurations, as shown by the first and second terms of  $\Delta$ . However, unless the access price is very high, similar effects as for high cost asymmetry prevail, and the benefits of entry of a VO dominate its costs if and only if the access price is high enough. Nevertheless, if the access price is higher than  $1/3$ , then the benefits of allowing a VO to enter the market are limited, because the NOs foreclose the market. This explains why welfare is higher without a VO if the access price is very high and the investment cost asymmetry is not too high (Proposition 2-ii).

Finally, Proposition 2 implies that if the regulator sets welfare maximizing access prices, then allowing a VO to enter the market improves welfare if and only if the cost asymmetry is high enough, that is, if the proportion of areas where both NOs invest is small (see Figure 1).

## 7 Co-investment

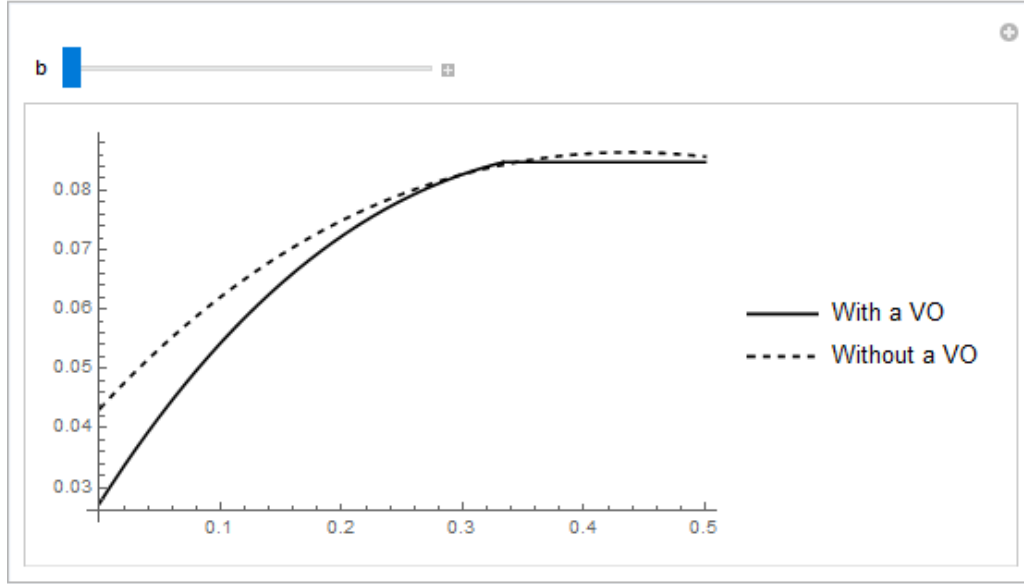
If there are only two NOs in the market, an alternative to access regulation is co-investment. Then, the two NOs share the investment cost and the infrastructure. If  $x$  areas are covered and if the proportion of the investment cost paid by NO 1 is  $\alpha$  and that paid by NO 2 is  $1 - \alpha$ , then the total profits of NO 1 and 2 are  $x\pi_{OD}^2 - \alpha C_1(x)$  and  $x\pi_{OD}^2 - (1 - \alpha)C_1(x)$ , respectively. Under standard access regulation, if there are  $y$  areas where both NOs invest and if, by convention, the investing firm is NO 1 when the Nash equilibria of the investment game are  $(I, N)$  and  $(N, I)$ , the profits of NOs 1 and 2 can be written as  $x\pi_{PD}^1(a) + y(\pi_{OD}^2 - \pi_{PD}^1(a)) - C_1(x)$  and  $x\pi_{SD}^1(a) + y(\pi_{OD}^2 - \pi_{SD}^1(a)) - C_2(y)$ , respectively. The following result ensues.

**Lemma 7.** *Without a VO, coverage is higher under access regulation than co-investment if and only if  $\pi_{PD}^1(a) > 2\pi_{OD}^2$ .*

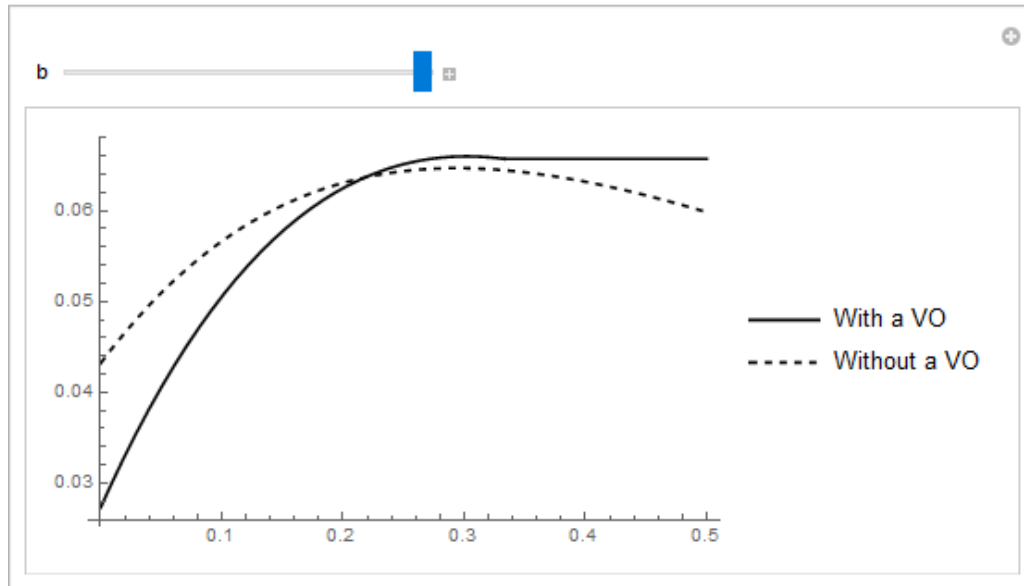
*Proof.* Under co-invest, the equilibrium coverage corresponds to the value of  $x$  that maximizes the total profit of the NOs, that is,  $x = c_1^{-1}(2\pi_{OD}^2)$ . Under access regulation, it is given by the value of  $x$  that maximizes the profit of NO 1, that is,  $x = c_1^{-1}(\pi_{PD}^1(a))$ .  $\square$

Figure 1: Entry of a virtual operator and welfare

(a) Symmetric investment costs ( $b = 1$ )



(b) Strong investment costs asymmetry ( $b = 10$ )



Lemma 7 indicates that coverage is higher under access regulation than co-investment if and only if the local profit of a NO when it builds a monopolistic infrastructure and provides access to the other NO is higher than the total local profit of the NOs when both of them invest. In the case of homogeneous Cournot competition studied in Section 6, this condition is satisfied if and only if  $a > (5 + \sqrt{5})/10$ .

When there are both NOs and VOs in the market, the NOs co-invest and provide access to the VOs. With two NOs and one NO, and if, by convention, the access provider is NO 1, the total profits of NO 1 and 2 are  $x\pi_{PT}^2 - \alpha C_1(x)$  and  $x\pi_{OT}^2 - (1 - \alpha)C_1(x)$ , respectively. If there is no co-investment but only access regulation, the total profits of NO 1 and 2 are  $x\pi_{PT}^1(a) + y(\pi_{PT}^2 - \pi_{PD}^1(a)) - C_1(x)$  and  $x\pi_{ST}^1(a) + y(\pi_{OT}^2 - \pi_{ST}^1(a)) - C_2(y)$ , respectively. , respectively. We obtain the following result.

**Lemma 8.** *With a VO, coverage is higher under access regulation than co-investment if and only if  $\pi_{PT}^1(a) > \pi_{PT}^2 + \pi_{OT}^2$ .*

*Proof.* Under co-investment, the equilibrium coverage is  $x = c_1^{-1}(\pi_{PT}^2 + \pi_{OT}^2)$ . If there is no co-investment but only access regulation, it is  $x = c_1^{-1}(\pi_{PT}^1(a))$ .  $\square$

According to Lemma 8, the principle that coverage is higher under access regulation than co-investment if and only if the local profit of a NO when it builds a monopolistic infrastructure is higher than the total local profit of the NOs when both of them invest remains valid if there is a VO in the market. In the case of homogeneous Cournot competition studied in Section 6, the condition expressed in Lemma 8 is satisfied if and only if  $a > 1 + 2^{-1/2}$ .

## 8 Conclusion

In this paper, we have introduced a model of asymmetric competition where two network operators with different investment costs may build an internet access infrastructure and where a virtual operator provides services through third-party access. We have shown that the virtual operator requests access from the low cost network operator, because it reduces the geographical area where both network operators build an infrastructure. In a simple model of homogeneous quantity competition, we have also shown that potential entry of a virtual operator increases welfare if the access price and the cost asymmetry between the network operators is high enough. Finally, regardless of

whether a virtual operator is allowed to enter the market or not, standard access regulation leads to broader infrastructure coverage than co-investment if and only if the profit of a network operator that builds a monopolistic infrastructure is higher than the total profit two network operators that both invest.

## Appendix: Proof of Proposition 2

*Proof.* Welfare with two NOs only and two NOs and a VO is given by

$$W_D \equiv \int_0^{c_2^{-1}(\Delta_{OD})} (w_D^2 - c_2(x))dx + \int_{c_2^{-1}(\Delta_{OD})}^{c_1^{-1}(\pi_{PD}^1)} (w_D^1 - c_1(x))dx \quad (1)$$

and

$$W_T \equiv \int_0^{c_2^{-1}(\Delta_{OT})} (w_T^2 - c_2(x))dx + \int_{c_2^{-1}(\Delta_{OT})}^{c_2^{-1}(\pi_{PT}^1)} (w_T^1 - c_1(x))dx, \quad (2)$$

respectively. From (1) welfare when there are only two NOs is

$$W_D(a) = \frac{3(a-2)(a(21a-26)-2)ab-9(a-2)^2a^2}{512b^2} + \frac{(a(a(3(108-35a)a-388)+152)+14)b^2}{512b^2}$$

if  $a \leq 1/2$  and  $W_D(1/2)$  otherwise. From (2) welfare when there are two NOs and a VO is

$$W_T(a) = \frac{4(a-1)(3a(3a-4)-2)ab-16(a-1)^2a^2}{162b^2} + \frac{(a(a(-36a^2+87a-82)+28)+7)b^2}{162b^2}$$

if  $a \leq 1/3$  and  $W_T(1/3)$  otherwise.

For all  $a \leq 1/3$ , there exists a unique  $\tilde{a}$  such that  $W_T(a) > W_D(a)$  iff  $a > \tilde{a}$ . Indeed,  $W_D(0) - W_T(0) = 329/20736$ ,  $W_D(1/3) - W_T(1/3) = (1841 + 9(115 - 477b)b)/(3359232b^2) < 0$ , and both  $W_D(a)$  and  $W_T(a)$  are strictly concave. For all  $a \in [1/3, 1/2]$ , we find there exists a unique  $\tilde{b} = (l + m)/n$ , where

$$l \equiv 1024(a-1)a(3a(3a-4)-2) - 1125,$$

$$m \equiv [2004525 - 1024a(a(a(2a(512a(a(3a(21a - 86) + 547) - 732)$$

$$+287075) - 209595) + 13090) + 10800)]^{1/2}$$

and

$$n \equiv 2(821 + 256a(-28 + a(82 - 87a + 36a^2))),$$

such that  $W_T(1/3) > W_D(a)$  iff  $b > \tilde{b}$ . This proves Proposition 2.  $\square$

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