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## Early retirement and social security : a long term perspective

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# EARLY RETIREMENT AND SOCIAL SECURITY: A LONG TERM PERSPECTIVE

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# EARLY RETIREMENT AND SOCIAL SECURITY: A LONG TERM PERSPECTIVE

## Abstract

We provide a long-term perspective on the individual retirement behaviour and on the future of retirement. In a Markovian political economic theoretical framework, in which incentives to retire early are embedded, we derive a political equilibrium with positive social security contribution rates and early retirement. Aging has two opposite effects: it leads to lower taxes and fewer (early) retirees, while a poorer median voter will push for higher contributions. The model highlights the existence of crucial income effects: a decrease of the income of young people will induce them to postpone retirement and to vote for less social security.

JEL Code: H53, H55, D72.

Keywords: pensions, income effect, tax burden, politico-economic Markovian equilibrium.

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# 1 Introduction

Retirement decisions represent one of the hottest issues of the current social security debate. Several studies - see Blondal and Scarpetta (1998) and Gruber and Wise (1999 and 2003) among the most recent work - have suggested that individual retirement decisions are strongly affected by the design of the social security system. In particular, individuals tend to retire either as soon as they are given the opportunity, i.e., at early retirement age, or at normal retirement age. Moreover, most social security systems have been proven to provide strong incentives - in terms of large implicit taxes on continuing to work - to anticipate retirement. In taking their retirement decisions, most individuals prefer to enjoy generous early retirement benefits - and the leisure associated with an early exit from the labor market - rather than to continue working, since, in the latter case, their additional contributions to the system would not sufficiently increase their future pension benefits.

Several studies have made an additional step by arguing that the massive use of early retirement provisions has come at a cost: the deterioration of the financial sustainability of the system, already under stress because of population aging. In fact, several international organizations - such as the European Union at the 2001 Lisbon Meetings - have advocated an increase in the effective retirement age, or - analogously - the increase in the activity rate among individuals aged above 55 years, as a key policy measure to control the rise in social security expenditure. In a nutshell, the postponement of the retirement age has become common to all social security reform's proposals. Yet, whether these policy prescriptions will actually be adopted depends on the politics of early retirement (see Fenge and Pestieau, 2005, for a detailed discussion of early retirement issues, and Galasso and Profeta, 2002, for a more general survey of the political economy of social security).

In this paper we acknowledge the crucial role of income effects in the individual decision of postponing retirement<sup>1</sup>, and we emphasize also their relevance for the proposed policy measures. In particular, we suggest that reductions in lifetime income may facilitate the political sustainability of this largely advocated policy: postponing retirement.

In a simple two-period overlapping generations economy featuring a redistributive social security system, we introduce a Markovian politico-economic model to predict the equilibrium path of social security policies. We characterize political equilibrium sequences of social security tax rates and the associated use of the early retirement provisions, together with a stable social security equilibrium. To take a long term perspective on these social security issues, we examine the impact of aging and economic slowdowns on social security contribution rates and retirement age. Comparative statics suggest that aging has two opposite effects: it tends to decrease contributions and the use of early retirement provisions, since it makes the public pension system less profitable, but it makes the median voter poorer, and thus induces higher social security. In addition to aging, our analysis highlights the crucial role of economic slowdowns in determining the evolution of the retirement behavior and the political decisions over social security, through negative income effects. A decrease of the income of young people will in fact induce all young agents to postpone retirement and to prefer less social security.

There exists a vast literature on retirement decisions. Already two decades ago, Feldstein (1974) and Boskin and Hurd (1978) analyzing the determinants of the decline in the labor force participation of elderly workers pointed at two key parameters of social security systems: the income guarantee and the implicit tax on earnings. Endogenous retirement decisions have been analyzed by showing

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<sup>1</sup>Empirical support for the role of income effects in the decision of retirement can be found in Butler et al. (2005).

how pension systems introduce distortions in the labor supply choice (see among others Diamond and Mirrless, 1978, Hu, 1979, Crawford and Lilien, 1981, and Michel and Pestieau, 1999). A new literature has lately emerged on the political economy of early retirement (see Fenge and Pestieau, 2004, Lacomba and Lagos, 2000, Casamatta et al., 2002, Cremer and Pestieau 2000, Cremer et al. 2002, Conde-Ruiz and Galasso, 2003 and 2004), although generally neglecting the role of income effects. Markovian politico-economic models of social security have been recently studied by Azariadis and Galasso (2002), Hassler et al (2003), Gonzalez-Eiras and Niepelt (2004), Forni (2005). These models however focus on social security and neglect the role of retirement.

The paper is structured as follows. In the next section, we present a Markovian politico economic model. Section 3 analyzes the effect of aging on the steady state level of early retirement and social security, while Section 4 studies the income effects. Section 5 concludes.

## 2 A Politico-Economic Model

### 2.1 The Economic Environment

We introduce a simple two -period overlapping generations model. Every period, two generations are alive, we call them young and old. We consider a continuum of individuals heterogeneous in young and old wage income. The wage income of a type- $\delta$  old individual is  $w_t^y = \delta \bar{w}_t^y$  in youth, and  $w_t^o = \delta \bar{w}_t^o$  in old age, where  $\bar{w}_t^y$  and  $\bar{w}_t^o$  are respectively the average income of young and old workers. Individual types  $\delta$  are distributed according to some density function  $f(\delta)$  over an interval  $[\underline{\delta}, \bar{\delta}]$  with an average equal to 1 and cumulative density function  $F(\delta)$ .

Young individuals work: they receive a wage,  $w^y$ , pay a payroll tax,  $\tau$ , on labour income and save all their disposable income for old age consumption.

There exists a storage technology that transforms a unit of today's consumption into  $1 + r$  units of tomorrow's consumption. All private intertemporal transfers of resources into the future are assumed to take place through this technology. Old individuals decide what fraction,  $z$ , of the second period to spend working; in other words, they decide when to retire. An old individual who works a proportion  $z$  of the second period receives a net labor income equal to  $w^o(1 - \tau)$ , for the fraction  $z$  of the period, while he receives a pension  $p$  for the remaining fraction  $(1 - z)$ , during which he is retired. Population grows at a non-negative rate,  $n$ ; but we abstract from wage growth, so that  $\bar{w}_{t+1}^y = \bar{w}_t^y$ .

The life time budget constraint for an agent born at time  $t$  is equal to:

$$c_{t+1}^o = (1 - \tau_t) w_t^y (1 + r) + (1 - \tau_{t+1}) z_{t+1} w_{t+1}^o + p_{t+1} (1 - z_{t+1}) \quad (1)$$

where  $c_{t+1}^o$  is old age consumption at time  $t + 1$  and subscripts indicate the calendar time. Moreover,  $\tau_t$  and  $\tau_{t+1}$  are the payroll taxes respectively at periods  $t$  and  $t + 1$  and  $r$  is the exogenous interest rate.

Every individual's pension benefit depends on her contribution in youth and in old age. In particular, we assume the individual pension to be earning-related for the contributions paid in old age, but flat for the contributions paid in youth. The combination of these two instruments induces an element of within-cohort redistribution, from the rich to the poor. As in Tabellini (2000) and in Conde-Ruiz and Galasso (2005), this feature is crucial in our political game, since it may induce low ability young to support the social security system<sup>2</sup>. Moreover, the use of an old age earning related component allows to model the incentive effect created by current contributions and pension benefits in the retirement decision. The total pension benefits obtained by an individual in her old age at time  $t + 1$  can thus be divided into an earning related and a fixed component.

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<sup>2</sup>Evidence in favor of the existence of this within cohort redistribution can be found in Boskin et al. (1987) and Galasso (2002).

$$p_{t+1}(1 - z_{t+1}) = \tau_{t+1}w_{t+1}^o z_{t+1} + \bar{p}_{t+1} \quad (2)$$

Since we concentrate on budget balanced PAYG social security systems, this fixed component is

$$\bar{p}_{t+1} = \tau_{t+1}(1 + n_{t+1})\bar{w}_{t+1}^y \quad (3)$$

with

$$\bar{w}_{t+1}^y = \int_{\underline{\delta}}^{\bar{\delta}} \delta \bar{w}_{t+1}^y f(\delta) d\delta \quad (4)$$

In fact, in the aggregate, a balanced budget pay as you go (PAYG) social security system requires the sum of all pension transfers to equal the sum of all contributions. Thus, the aggregate social security budget constraint can be written as

$$\int_{\underline{\delta}}^{\bar{\delta}} p_{t+1}(1 - z_{t+1})f(\delta)d\delta = \tau_{t+1} \int_{\underline{\delta}}^{\bar{\delta}} \delta \bar{w}_{t+1}^o z_{t+1} f(\delta)d\delta + \tau_{t+1}(1 + n_{t+1}) \int_{\underline{\delta}}^{\bar{\delta}} \bar{w}_{t+1}^y f(\delta)d\delta \quad (5)$$

Agents maximize a logarithmic utility function, which depends on old age consumption and leisure:

$$U(c_{t+1}, z_{t+1}) = \ln c_{t+1}^o + \phi \ln(1 - z_{t+1}) \quad (6)$$

where  $\phi < 1$  measures the relative importance of leisure to the individuals.

Hence, an old agent at time  $t + 1$  maximizes eq. 6 with respect to  $z_{t+1}$  subject to the budget constraints at eq. 1.

The solution of the maximization problems yields the following optimal individual labor supply decision:

$$\hat{z}_{t+1} = \frac{1}{1 + \phi} - \frac{\phi}{1 + \phi} \frac{(1 - \tau_t)w_t^y(1 + r) + \bar{p}_{t+1}}{w_{t+1}^o} \quad (7)$$

or, equivalently,

$$\hat{z}_{t+1} = \frac{1}{1 + \phi} - \frac{\phi}{1 + \phi} \frac{(1 - \tau_t)(1 + r)\bar{w}_t^y}{\bar{w}_{t+1}^o} - \frac{\phi}{1 + \phi} \frac{\tau_{t+1}(1 + n_{t+1})\bar{w}_{t+1}^y}{\delta \bar{w}_{t+1}^o} \quad (8)$$



This individual retirement decision displays standard properties: an increase in the net labor income in youth induces all agents to retire early – due to a positive income effect – where an increase in the net labor income in old age, or a decrease in the pension benefits, would push them to postpone retirement – due to a positive substitution effect. Aging – by increasing the dependency ratio, thereby decreasing the pension benefits, for a given contribution rate – would lead to later retirement.

To ensure that no type- $\delta$  agent will end up either working the entire old age or retiring at the end of the youth – that is, to avoid corner solutions in the individual labor supply decision – some conditions have to be imposed. In particular, with no social security system in place, i.e., if  $\tau_t = 0 \forall t$ , no agent will even want to work the entire old age, and all agents will work for some period if  $1 + r < \bar{w}_t^o / \bar{w}_t^y \phi$ . We shall hence assume that this condition holds, also in a dynamically efficient economy ( $r > n$ ). For positive contribution rates, the condition that individual labor supply decisions lead to interior solutions, i.e.,  $\hat{z}_t \in [0, 1] \forall t$ , amounts to impose some restrictions on the dynamics of the contribution rates. In particular, we have that

$$\tau_{t+1} < \frac{\delta (\bar{w}_t^o - \phi \bar{w}_t^y (1 + r) (1 - \tau_t))}{(1 + n_{t+1}) \bar{w}_t^y}. \quad (9)$$

The mass of employed elderly in the economy<sup>3</sup> at time  $t + 1$  can easily be obtained by aggregating all individuals' retirement decisions:

$$Z_{t+1} = \int_{\underline{\delta}}^{\bar{\delta}} \hat{z}_{t+1} f(\delta) d\delta \quad (10)$$

which can also be written as

$$Z_{t+1} = \frac{1}{1 + \phi} - \frac{\phi}{1 + \phi} \frac{(1 - \tau_t)(1 + r) \bar{w}_t^y}{\bar{w}_{t+1}^o} - \frac{\phi}{1 + \phi} \frac{\tau_{t+1}(1 + n_{t+1}) \bar{w}_{t+1}^y \hat{\delta}}{\bar{w}_{t+1}^o} \quad (11)$$

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<sup>3</sup>Clearly,  $1 - Z$  defines the mass of (early) retirees.

with

$$\widehat{\delta} = \int_{\underline{\delta}}^{\overline{\delta}} \frac{1}{\delta} f(\delta) d\delta \quad (12)$$

Since individuals with different income display different retirement behaviors, the mass of retirees will depend on the distribution of income in the economy. In particular, due to the incentive effect embedded in the model, high income elderly workers will be induced to retire later than low income workers. Yet, this effect is not linear, and tends to magnify the importance of the agents who enjoy very low income in old age and hence have an incentive to retire very early. The parameter  $\widehat{\delta}$  captures this effect by weighting the mass of these low-income elderly with their retirement behavior. The larger – for instance – the share of low-income elderly, the larger this  $\widehat{\delta}$ ; and hence the larger the mass of (early) retirees  $(1 - Z)$ .

Finally, by substituting the individual decision at eq.7 and the social security budget constraint, we can easily derive the indirect utility respectively of a type- $\delta$  young and old individual at time  $t$ , which we denote by  $v_t^y(\tau_t, \tau_{t+1}; \delta)$  and  $v_t^o(\tau_{t-1}, \tau_t; \delta)$ .

## 2.2 The Political Equilibrium

The purpose of this paper is to propose a theoretical framework in which to analyze the link between early retirement provision and the size of the social security system. As already showed at eq. 7, early retirement behavior may be induced by specific features of the social security system, such as the size of contribution rates and pension benefits. Here, we study the determination of this social security contribution rate within the political arena. Elections take place every year in which the current social security contribution rate is determined. All young and old agents participate at the elections. Yet, their preferences over the contribution rate may differ – typically according to their income ( $\delta$  type) and age. We follow a well established tradition in political

economics by concentrating on the median voter decision. Moreover, due to the intergenerational nature of the system, we allow for some interdependence between current and future political decisions. In particular, we analyze Markov perfect equilibrium outcomes<sup>4</sup> of a repeated voting game over the social security contribution rate. Since we want to examine the possible link between the use of early retirement provisions and the size of the social security system, we base our notion of Markov equilibrium on the idea that current voters – in taking their policy decisions – expect future policy-makers to base their political decisions on social security on the mass of early retirees – or employed elderly – in the economy. These expectations will clearly be validated in equilibrium.

More specifically, at every period  $t$ , the median voter in each generation of voters – hence typically a young individual – decides her most favorite social security system (i.e., the tax rate  $\tau_t$ ). In taking her decision, she expects her current decision to have an impact of future policies. In particular, her expectations about the future social security tax rate – and hence about her pension benefits – depend on the current level of employed elderly, according to a function  $\tau_{t+1} = q^e(Z_t)$ . Hence, future contribution rates depend on the current level of labor force participation by the elderly, which is in turn affected by the current voter's decision over the social security contribution rate. Therefore, the median voter's optimal decision can be obtained maximizing her lifecycle utility with respect  $\tau_t$  and given expectations on the next period policy function  $\tau_{t+1} = q^e(Z_t) = Q(Z_t(\tau_t))$ :

$$\max_{\tau_t} v_t^y(\tau_t, \tau_{t+1}; \delta) = \max_{\tau_t} v_t^y(\tau_t, Q(Z_t(\tau_t)); \delta) \quad (13)$$

We can now define the Markov political equilibrium as follows

**Definition 1** *A Markov political equilibrium is a pair of functions  $(Q, Z)$ , where*

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<sup>4</sup>For examples of Markov equilibria, see Krusell et al.(1996), Grossman and Helpman (1998), Bassetto (1999), Azariadis and Galasso (2002), Hassler et al. (2003), Gonzalez-Eiras and Niepelt (2004), Forni (2005).

$Q : [0, 1] \rightarrow [0, 1]$  is a policy rule,  $\tau_t = Q(Z_{t-1})$ , and  $Z : [0, 1] \rightarrow [0, 1]$  is an aggregation of private decision rules,  $Z_t = \int_{\underline{\delta}}^{\bar{\delta}} \hat{z}_t f(\delta) d\delta$ , such that the following functional equations hold:

$$\begin{aligned} i) \quad & Q(Z_{t-1}) = \arg \max_{\tau_t} v_t^y(\tau_t, \tau_{t+1}; \delta) \text{ subject to } \tau_{t+1} = Q(Z(\tau_t)). \\ ii) \quad & Z(\tau_t, \tau_{t+1}) = \frac{1}{1+\phi} - \frac{\phi}{1+\phi} \frac{(1-\tau_t)(1+r)\bar{w}_t^y}{\bar{w}_{t+1}^y} - \frac{\phi}{1+\phi} \frac{\tau_{t+1}(1+n_{t+1})\bar{w}_{t+1}^y \hat{\delta}}{\bar{w}_{t+1}^y} \end{aligned}$$

The first equilibrium condition requires that  $\tau_t$  maximizes the objective function of the median voter – a type- $\delta^m$  young individual – taking into account that the future social security system tax rate,  $\tau_{t+1}$  depends on the current social security tax rate,  $\tau_t$ , via the mass of elderly employed and thus the private labor supply decision of the elderly. Furthermore, it requires  $Q(z_{t-1})$  to be a fixed point in the functional equation in part i) of the definition. In other words, if agents believe future benefits at any time  $t+j$  to be set according to  $\tau_{t+j} = Q(z_{t+j-1})$ , then the same function  $Q(z_{t-1})$  has to define the optimal voting decision today. The second equilibrium condition requires that all old individuals choose their labor supply optimally.

In order to compute the Markov political equilibrium, we have to consider the optimal social security tax rate chosen by the median voter at time  $t$  who maximizes the indirect utility function with respect to  $\tau_t$  and subject to  $\tau_{t+1} = Q(Z_t(\tau_t))$ .

The corresponding first order condition is:

$$-w_t^y(1+r) + \frac{\partial \tau_{t+1}}{\partial \tau_t}(1+n_{t+1})\bar{w}_{t+1}^y = 0 \quad (14)$$

where the first element represents the current cost to the median voter in terms of higher contributions, while the second term may represent the future benefits corresponding to a higher pension, if a higher current contribution leads to a higher contribution rate also tomorrow:  $\partial \tau_{t+1} / \partial \tau_t > 0$ . The redistributive design of the social security system yields the usual result that – in perfect

financial markets – the most preferred contribution rate of a young individual is decreasing in her income; whereas the elderly most preferred social security contribution rate does not depend on their type and is always larger than any young's. These features command the usual distribution of social security preferences among the voters, which is displayed in the next proposition.

The solution of the maximization problem of the median voter yields the optimal fiscal policies, as summarized in the following proposition.

**Proposition 2** *The set of feasible fiscal policies  $\{\tau_t^*\}_{t=s}^\infty \in [0, 1]$  which can be supported by a Markovian politico-economic equilibrium satisfies:*

$$\tau_{t+1} = Q(Z_t) = A - \frac{(1 + \phi) \delta_t^m (1 + r) \overline{w}_t^o}{\phi(1 + n_t)(1 + n_{t+1}) \widehat{\delta \overline{w}}_t^y} Z_t$$

where  $\delta_t^m$  the identity of the median voter at time  $t$  solves the following equation

$$1 + (1 + n_t) F(\delta_t^m) = 1 + n_t/2$$

and  $A$ , the free parameter pinned down by the first median voter's expectation of future policies, is restricted to the support  $A \in [\frac{\overline{w}_t^o \delta_t^m}{\phi(1+n_t)(1+n_{t+1}) \widehat{\delta \overline{w}}_t^y}, 1 - \frac{(1+r)\delta_t^m}{(1+n_{t+1})} \left(1 + \frac{1}{(1+n_t)\widehat{\delta}} \left(1 + r - \frac{\overline{w}_t^o}{\phi \widehat{\overline{w}}_t^y}\right)\right)]$

**Proof.** See Appendix.

The result in the above proposition points to the existence of a positive link, in the political arena, between the current use of the early retirement provisions – that is, the mass of (early) retirees  $(1 - Z)$  – and the future social security contribution rate. This link complements the economic channel running from the social security contribution rate to the current labor supply decision of the elderly, as described at eq. 7. In particular, a current increase in the social security contribution rate – by reducing the opportunity cost of retirement – leads to more current retirees, which in turn creates expectations of higher future social security contributions – and hence more early retirees in the future.

By exploiting this double link between contribution rate and mass of retirees – given the expression of  $Z_t$  at eq.11 – the dynamics for the equilibrium policy function can be described as follows:

$$\begin{aligned}\tau_{t+1} = & \frac{\delta_t^m(1+r)}{(1+n_t)(1+n_{t+1})}\tau_t + \frac{\delta_t^m(1+r)^2}{(1+n_t)(1+n_{t+1})\widehat{\delta}}(1-\tau_{t-1}) + \\ & + A - \frac{\overline{w}_t^o \delta_t^m(1+r)}{\phi(1+n_t)(1+n_{t+1})\widehat{\delta}\overline{w}_t^y}\end{aligned}\quad (15)$$

Interestingly, the dynamics of the contribution rate involves more than just one period, as the contribution rate at time  $t+1$  depends – positively – on the tax rate at time  $t$ ; but negatively on the tax rate at time  $t-1$ . This is due to the impact that the contribution rates at time  $t$  and  $t-1$  have on the retirement decision at time  $t$ , which represent respectively a positive substitution effect and a negative income effect.

It is now convenient to consider a constant demographic dynamics, with  $n_t = n_{t+i} = n \forall i$ , and to define  $\alpha = (1+r)/(1+n)$  as the performance of the PAYG social security system relatively to the saving (storage) technology; since we assume the economy to be dynamically efficient, then  $\alpha > 1$ .

The next proposition examines the dynamic properties of the sequence of contribution rates.

**Proposition 3** *If  $\alpha\delta^m < 1$  and  $\alpha < \widehat{\delta}$ , the Markovian politico-economic equilibrium path converges to a stable steady state corresponding to*

$$\overline{\tau} = \frac{\overline{w}^y \phi(1+n)\widehat{\delta}A - \overline{w}^o \delta^m \alpha + \alpha^2 \delta^m \phi(1+n)\overline{w}^y}{\overline{w}^y \phi(1+n)\widehat{\delta} \left(1 - \alpha\delta^m + \alpha^2 \frac{\delta^m}{\widehat{\delta}}\right)}.$$

*At this steady state, the mass of employed elderly is*

$$\overline{Z} = \frac{1}{1+\phi} - \frac{\phi}{1+\phi} \frac{(1+r)\overline{w}^y}{\overline{w}^o} - \frac{\phi(1+n)}{1+\phi} \frac{\overline{w}^y}{\overline{w}^o} \left[\widehat{\delta} - \alpha\right] \overline{\tau}$$

**Proof.** See Appendix.

The above proposition suggests that – even in this dynamically efficient economy – a stable steady state with a positive level of the social security contribution rate may emerge as an equilibrium of the Markovian political game, if two conditions are satisfied. The first condition,  $\alpha\delta^m < 1$ , is relatively standard in the social security literature and requires the young type- $\delta^m$  median voter to obtain a better deal from social security than from alternative assets, due to the redistributive nature of the social security system. In order for this condition to be satisfied, together with a highly redistributive social security system, the economy has to feature a high level of income inequality, as measured by the density function  $f(\delta)$ . Yet, unlike most systems analyzed in this literature, here we allow the agents to choose their retirement age. The second condition,  $\alpha < \hat{\delta}$ , amounts to assume that a large number of individuals will retire early. In particular, as shown by the equation in the proposition above, if this condition is satisfied, the impact of the (steady state) contribution rate on the (steady state) mass of employed elderly is negative, as higher taxes lead to more early retirees; thereby validating – even at steady state – the result of the current contribution rate at eq. 11. In the remaining sections, we will hence assume that these two conditions are always satisfied.

### 3 Aging, Social Security and Early Retirement

The equilibrium policy function obtained in the previous section allows us to analyze the effects of aging on the social security tax rate and on the use of early retirement. In our model, aging has economic and political effects on the steady state social security tax rate. First, aging reduces the profitability of the PAYG pension system with respect to alternative savings; second, for a given contribution rate, an increase in the share of elderly in the population reduces the pension benefits, thereby inducing the elderly to postpone retirement. Finally,

aging tends to change the identity of the median voter, who becomes poorer, and may decide to modify her political decision on social security.

The first economic effect and the political impact of aging identified in our model arise also in other standard political economy models of social security (for a survey, see Galasso and Profeta, 2002); and typically push in opposite directions. In fact, while a decrease in the profitability of the social security system may convince the median voter to downsize the system – in order to increase her private provision of retirement income through alternative private assets- aging will also reduce the income of the median voter, who will hence be keener on increasing the contribution rate.

The additional economic impact of aging featured in our model goes in the same direction as the previous economic effect, thereby reinforcing it. In fact, in our Markovian political system, the increase in the retirement age due to the negative incentive effect of a lower pension benefit will lead to a reduction in the mass of (early) retirees and thus to a reduction in the contribution rate. As suggested by the next two propositions, our model delivers similar predictions to the ones traditionally obtained by the political economy literature on social security, despite the introduction of an additional – political – link between the retirement decisions and social security. While these results validate the empirical plausability of the model, the next section will explore additional interesting implications of our theoretical framework to address the long run feature of social security and early retirement.

The next proposition summarizes the impact of these economic effects, by addressing the effect of aging on the steady state social security contribution rate, for a given median voter type.

**Proposition 4** *For a given median voter type,  $\delta^m$ , if  $\hat{\delta} \in (0, \bar{w}^o / (\bar{w}^y \phi (1 + n)))$ , aging (corresponding to a reduction in the population growth rate) decreases the*



steady state social security contribution rate,  $\partial\bar{\tau}/\partial n > 0$ , and increases the mass of employed elderly at steady state,  $\partial\bar{Z}/\partial n < 0$ .

**Proof.** See Appendix.

Hence, aging has the expected impact on the social security contribution rate at steady state, provided that  $\hat{\delta}$  is below a threshold, and thus the mass of (early) retirees is not too large. As the population growth rate drops, the implicit return from a PAYG social security system decreases as well. Median voters will modify the policy function by making it more responsive – in absolute terms – to the mass of employed elderly. Eventually, at the new steady state, the contribution rate decreases leading to fewer early retirees.

Yet, aging has also a political effect, which we now turn to. An increase in the share of elderly in the voting population modifies the identity of the median voter, who becomes poorer. The next proposition shows that – in accordance with the existing literature – a poorer median voter will prefer more social security if  $\hat{\delta}$  is small – i.e., below the same threshold as in the previous proposition – and hence the mass of (early) retirees is not too large.

**Proposition 5** *If  $\hat{\delta} \in (0, \bar{w}^o / (\bar{w}^y \phi(1+n)))$ , the equilibrium steady state social security contribution rate depends negatively on the income type of the median voter:  $\partial\bar{\tau}/\partial\delta^m < 0$ .*

**Proof.** See Appendix.

These two propositions suggest that the political effect of aging on the identity of the median voter is opposed to an economic effect of aging. Which effect will dominate remains an empirical question to be settled (for instance, Galasso and Profeta, 2004, simulate the political effect to prevail).

## 4 Income effects, Social Security and Early Retirement

In this section we highlight the role of income effects on retirement decisions and thus on the social security equilibrium tax rate. Many studies on retirement have uniquely concentrated on the role of the incentives (substitution effects) provided by the impact of the labor tax on continuing to work. Possible income effects – leading poorer individuals to work longer, i.e., to retire later – have typically been abstracted from. Yet, several authors (see for instance Costa, 1998) have suggested that the long lasting decreasing trend in the retirement age may – at least partially – be due to the major improvements in economic conditions that increased the demand for leisure, and hence for early retirement. In a recent empirical paper, Butler et al. (2005) find evidence for the existence of income effects in the retirement decisions in Switzerland.

Our model includes the role of income effects on early retirement and social security through the impact of  $\bar{w}_t^y$  on  $Z$  and  $\tau$ .

**Proposition 6** *A decrease of the wage income in youth leads to a reduction of the steady state social security contribution rate  $\partial\bar{\tau}/\partial\bar{w}^y > 0$  and to an increase in the steady state mass of employed elderly  $\partial\bar{Z}/\partial\bar{w}^y < 0$ .*

**Proof.** See Appendix.

A reduction in the wage income at youth induces individuals to postpone retirement, through a negative income effect – since their lifetime income decreases – and a substitution effect – since the pension benefits, which are based on the workers' wage, decrease as well. Although the overall profitability of the social security system needs not to change due to a drop in wage income in youth, and the identity of the median voter is not affected, the steady state social security contribution rate decreases, since most individuals tend to postpone retirement, thereby reducing the share of (early) retirees. Interestingly, the magnitude of

the adjustment in the retirement age driven by a change of the young income is decreasing in income, thus implying that low income workers will react more actively to a negative income effect. In our political equilibrium, this increase in the overall fraction of employed elderly will command a lower contribution rate. The reduction of early retirees at steady state depends instead both on the direct negative income and substitution effects driven by the reduction of  $\bar{w}_t^y$  and on an indirect substitution effect due to the decrease of the social security tax rate, which leads to fewer (early) retirees.

This proposition provides an interesting insight on the future of the early retirement provisions, which complements the results obtained in the previous section. When the effects of changes in income or wealth on the retirement behavior are taken into account, a reduction in the young wage income induces individuals to postpone retirement. We argue that – to the extent that this reduction in the young wage income may proxy for a drop in the life-time labor income – this may prove a crucial result to understand the future evolution of the early retirement provision. Societies characterized by economic stagnation or raise in lifetime inequality that increase the share of low-income individuals may thus be associated with a less pervasive use of these early retirement provisions.

## 5 Conclusions

Since recent studies by Blondal and Scarpetta (1998) and Gruber and Wise (1999 and 2003) provided evidence that individual retirement decisions are strongly affected by the design of the social security system, measures to postpone the effective retirement age have become a milestone in all social security reform's proposals.

We concentrate on the long term determinants of the retirement decisions and the evolution of social security system and early retirement provisions. In

our politico-economic Markovian environment, every period a young low-income median voter determines the social security contribution by considering the evolution of the early retirement behavior. We emphasize the role of substitution and income effects in these retirement decisions. The incentive effects have been analyzed by a large empirical literature, which shows how (at the margin) non-actuarially fair pension systems may induce rational agents to retire early, by reducing the opportunity cost of leisure. Income effects have instead generally been neglected in models of retirement and social security, despite the empirical evidence suggesting that variation in lifetime income may modify retirement decisions.

In line with the implications found in the political economy literature (see Galasso and Profeta, 2002), we found that aging – through its negative impact on the profitability of PAYG social security systems – is expected to lead to lower social security contributions and to less use of early retirement. However, as aging also modifies the identity of the median voter, who becomes poorer, an opposite effect arises, leading to higher social security. The overall effect of aging will hence depend on which effect dominates. Our model also suggests that a decrease in the wage income in youth leads to lower social security tax rate and fewer early retirees. To the extent that this change in young wage income may proxy for a change in the net life-time income, we believe that this may represent a suggestive result for the evolution of the early retirement provisions, since it represents an additional channel that reduces the use of early retirement provisions and hence postpones retirement.

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## 6 Appendix

### 6.1 Proof of proposition 2

The first order condition of the median voter is:

$$-w_t^y(1+r) + \frac{\partial \tau_{t+1}}{\partial \tau_t}(1+n_{t+1})\overline{w}_{t+1}^y = 0 \quad (16)$$

or, equivalently

$$-\delta_t^m \overline{w}_t^y(1+r) + \frac{\partial \tau_{t+1}}{\partial \tau_t}(1+n_{t+1})\overline{w}_t^y = 0 \quad (17)$$

where

$$\frac{\partial \tau_{t+1}}{\partial \tau_t} = Q' \frac{\partial Z_t}{\partial \tau_t} \quad (18)$$

with

$$Q' = \frac{\partial Q}{\partial Z_t} \quad (19)$$

and

$$\frac{\partial Z_t}{\partial \tau_t} = -\frac{\phi}{1+\phi} \frac{(1+n_t)\overline{w}_t^y}{\overline{w}_t^o} \int_{\underline{\delta}}^{\overline{\delta}} \frac{1}{\delta} f(\delta) d\delta = -\frac{\phi}{1+\phi} \frac{(1+n_t)\overline{w}_t^y}{\overline{w}_t^o} \widehat{\delta} \quad (20)$$

Substituting eq. 20 and eq. 19 into eq. 18 and using it into the first order condition at eq. 17 we obtain

$$Q' = -\frac{(1+\phi) \delta_t^m (1+r) \overline{w}_t^o}{\phi(1+n_t)(1+n_{t+1}) \widehat{\delta} \overline{w}_t^y} \quad (21)$$

Integrating the above equation with respect to  $Z_t$  we obtain

$$\tau_{t+1} = Q(Z_t) = A - \frac{(1+\phi) \delta_t^m (1+r) \overline{w}_t^o}{\phi(1+n_t)(1+n_{t+1}) \widehat{\delta} \overline{w}_t^y} Z_t \quad (22)$$

where  $A$  is a constant of integration.

Since  $\tau_{t+1} = Q(Z_t)$  represents a tax rate, it has to be that  $Q \in [0, 1]$ . Using eq. 11 for  $Z_t$ , it is easy to see that a sufficient condition for  $\tau_{t+1}$  to be positive is that  $\tau_{t+1}$  is positive for  $\tau_{t-1} = 1$  and  $\tau_t = 0$  which implies to have

$$A > \underline{A} = \frac{\overline{w}_t^o \delta_t^m}{\phi(1+n_t)(1+n_{t+1}) \widehat{\delta} \overline{w}_t^y} > 0. \quad (23)$$



Moreover, it is easy to see that a sufficient condition for  $\tau_{t+1} < 1$  is to have  $\tau_{t+1} < 1$  for  $\tau_t = 1$  and  $\tau_{t-1} = 0$ , which implies to have

$$A < \bar{A} = 1 - \frac{(1+r)\delta_t^m}{(1+n_{t+1})} \left( 1 + \frac{1}{(1+n_t)\widehat{\delta}} \left( 1 + r - \frac{\overline{w}_t^o}{\phi\overline{w}_t^y} \right) \right) \quad (24)$$

It is easy to check that, given the assumption  $(1+r) < \frac{\overline{w}_t^o}{\phi\overline{w}_t^y}$ , it is  $\underline{A} < \bar{A}$  if  $\frac{1+r}{1+n_{t+1}}\delta^m < 1$ , which is a condition required by the next proposition 3 for the stability of the system.

Finally, to determine the identity of the median voter, notice that – by equation 17 – the most preferred social security contribution rate among the young is weakly decreasing in their income; and that the old always command a higher tax rate than the any young. For non-negative population growth rates, the median voter is among the young and has a type  $\delta_t^m$ , which divides the distribution of preference in halves:  $1 + (1+n_t)F(\delta_t^m) = 1 + n_t/2$ .

## 6.2 Proof of proposition 3

Eq..15 can be rewritten as

$$\tau_{t+1} - \alpha\delta_t^m\tau_t + \alpha^2\frac{\delta_t^m}{\widehat{\delta}}\tau_{t-1} = A - \frac{\overline{w}_t^o\delta_t^m\alpha}{\phi(1+n)\widehat{\delta}\overline{w}_t^y} + \alpha^2\frac{\delta_t^m}{\widehat{\delta}} \quad (25)$$

It is easy to see that three cases arise in the solution of this second order differential equation, depending on the sign of the determinant of the associated characteristic equation

$$b^2 + a_1b + a_2 = 0 \quad (26)$$

with

$$a_1 = -\alpha\delta^m, \quad a_2 = \alpha^2\frac{\delta^m}{\widehat{\delta}}. \quad (27)$$

In fact, we obtain

$$b_1, b_2 = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2} = \frac{\alpha\delta^m \pm \sqrt{\alpha^2(\delta^m)^2 - 4\alpha^2\frac{\delta^m}{\widehat{\delta}}}}{2} = \frac{\alpha\delta_t^m}{2} \pm \alpha\sqrt{\delta^m \left( \frac{\delta^m}{4} - \frac{1}{\widehat{\delta}} \right)} \quad (28)$$

Depending on the sign of  $\Delta^2 = \delta^m \left( \frac{\delta^m}{4} - \frac{1}{\delta} \right)$ , there are 3 possible cases: (i) real and distinct roots  $b_1$  and  $b_2$ ; (ii) real and equal roots  $b_1 = b_2$ ; and (iii) complex roots  $b_1$  and  $b_2$ . If  $\alpha\delta < 1$  and  $\alpha < \hat{\delta}$ , then, regardless of which of the three cases apply, the condition for a stable convergence towards the steady state are always satisfied.

Case i) A sufficient condition to guarantee stability is that  $b_1 < 1$  and  $b_2 < 1$ .  $b_2 = \frac{\alpha\delta^m + \sqrt{\alpha^2(\delta^m)^2 - 4\alpha^2\frac{\delta^m}{\delta}}}{2} < 1$  if  $\alpha\delta^m + \sqrt{\alpha^2(\delta^m)^2 - 4\alpha^2\frac{\delta^m}{\delta}} < 2$ . If  $\alpha\delta^m < 1$ , then both terms are less than 1. Moreover,  $b_1 = \frac{\alpha\delta^m - \sqrt{\alpha^2(\delta^m)^2 - 4\alpha^2\frac{\delta^m}{\delta}}}{2} < 1$  because  $0 < b_1 < b_2 < 1$ .

Case ii) The stability property depends on  $\frac{-a_1}{2} = \frac{\alpha\delta^m}{2}$ . For  $\alpha\delta^m < 2$  the system converges to the steady state.

Case iii) The stability property depends on  $R = \sqrt{a_2} = \alpha\sqrt{\frac{\delta_t^m}{\delta}} = \sqrt{\alpha\delta_t^m\frac{\alpha}{\delta}}$ . If  $R < 1$  the system converges to the steady state, through fluctuations. Clearly, this is satisfied for  $\alpha\delta^m < 1$  and  $\alpha < \hat{\delta}$ .

The steady state value of the tax rate corresponds to the particular solution of eq.15  $\tau_{t+1} = \tau_t = \tau_{t-1} = \bar{\tau}$

This becomes

$$\bar{\tau} - \alpha\delta^m\bar{\tau} + \alpha^2\frac{\delta^m}{\delta}\bar{\tau} = A - \frac{\bar{w}^o\delta^m}{\phi(1+n)\hat{\delta}\bar{w}^y}\alpha + \alpha^2\frac{\delta^m}{\delta} \quad (29)$$

$$\bar{\tau} = \frac{A - \frac{\bar{w}^o\delta^m}{\phi(1+n)\hat{\delta}\bar{w}^y}\alpha + \alpha^2\frac{\delta^m}{\delta}}{1 - \alpha\delta^m + \alpha^2\frac{\delta^m}{\delta}} \quad (30)$$

recalling that  $\alpha = \frac{(1+r)}{(1+n)}$ , we have

$$\bar{\tau} = \frac{\bar{w}^y\phi(1+n)\hat{\delta}A - \bar{w}^o\delta^m\alpha + \alpha^2\delta^m\phi(1+n)\bar{w}_t^y}{\bar{w}^y\phi(1+n)\hat{\delta}\left(1 - \alpha\delta^m + \alpha^2\frac{\delta^m}{\delta}\right)} \quad (31)$$

Notice that the denominator of  $\bar{\tau}$  is always positive since  $1 > \alpha\delta^m(1 - \frac{\alpha}{\delta})$ , while the numerator is positive because  $A > \underline{A}$ , as defined at Eq.23

Imposing  $\tau_{t+1} = \tau_t = \tau_{t-1} = \bar{\tau}$  Eq.11 leads, after some simple manipulation, to the expression of the steady state level of employed elderly:

$$\bar{Z} = \frac{1}{1+\phi} - \frac{\phi}{1+\phi} \frac{(1+r)\bar{w}^y}{\bar{w}^o} - \frac{\phi(1+n)}{1+\phi} \frac{\bar{w}^y}{\bar{w}^o} [\hat{\delta} - \alpha] \bar{\tau} \quad (32)$$

It is easy to show after some algebra that  $\bar{Z} \in [0, 1]$ .

### 6.3 Proof of proposition 4

In order to prove propositions 4 and 5, it is convenient to introduce the following lemma.

**Lemma 7** if  $\hat{\delta} \in (0, \frac{\bar{w}^o}{\bar{w}^y \phi(1+n)})$ ,  $K = \frac{\bar{w}^o - \alpha \phi(1+n)\bar{w}^y}{\bar{w}^y \phi(1+n)(\hat{\delta} - \alpha)} > \bar{A}$ .

Proof: Substituting  $n_t = n_{t+1} = n$  and the definition of  $\alpha$  into the expression for  $\bar{A}$  at 24, we need to show that

$$K = \frac{\bar{w}^o - \alpha \phi(1+n)\bar{w}^y}{\bar{w}^y \phi(1+n)(\hat{\delta} - \alpha)} > \bar{A} = 1 - \alpha \delta^m - \frac{\alpha \delta^m}{\hat{\delta}} \left( \alpha - \frac{\bar{w}^o}{\phi(1+n)\bar{w}^y} \right)$$

After some algebra this condition can be rewritten as

$$(1 - \alpha \delta^m) \left( \frac{\frac{\bar{w}^o}{\bar{w}^y \phi(1+n)} - \alpha}{\hat{\delta} - \alpha} - 1 \right) + \alpha^2 \delta^m \left( \frac{\frac{\bar{w}^o}{\bar{w}^y \phi(1+n)} - \alpha}{\hat{\delta}(\hat{\delta} - \alpha)} \right) > 0 \quad (33)$$

Since we assumed that  $\frac{\bar{w}^o}{\bar{w}^y \phi(1+n)} - \alpha > 0$  and  $\hat{\delta} - \alpha > 0$ , a sufficient condition to guarantee the above inequality is that  $\hat{\delta} < \frac{\bar{w}^o}{\bar{w}^y \phi(1+n)}$ . **Q.e.d.**

We can not turn to studying the sign of  $\partial \bar{\tau} / \partial n$ .

Call  $\beta = \alpha(1+n)$ , the steady state level of the tax rate becomes

$$\bar{\tau} = \frac{A(1+n)^2 - \frac{\bar{w}^o \delta^m \beta}{\bar{w}^y \phi \hat{\delta}} + \beta^2 \frac{\delta^m}{\hat{\delta}}}{(1+n)^2 - \beta(1+n)\delta^m + \beta^2 \frac{\delta^m}{\hat{\delta}}}.$$

It is easy to see that the sign of  $(\partial \bar{\tau} / \partial n)$  is equal to the sign of the following expression:

$$2A(1+n)((1+n)^2 - \beta(1+n)\delta^m + \beta^2 \frac{\delta^m}{\hat{\delta}}) - \left( A(1+n)^2 - \frac{\bar{w}^o \delta^m \beta}{\bar{w}^y \phi \hat{\delta}} + \beta^2 \frac{\delta^m}{\hat{\delta}} \right) (2(1+n) - \beta \delta^m) \quad (34)$$

which can be written as

$$\frac{A(1+n)^2}{\hat{\delta}} (2\alpha - \hat{\delta}) + (\bar{w}^o - \alpha(1+n)\bar{w}^y \phi) (2 - \alpha \delta^m) \frac{1+n}{\bar{w}^y \phi \hat{\delta}} > 0 \quad (35)$$

where we define  $B_1 = 2\alpha - \hat{\delta}$ ,  $B_2 = \bar{w}^o - \alpha(1+n)\bar{w}^y \phi$ , and  $B_3 = 2 - \alpha \delta^m$ . Notice that  $B_3 > 0$  since  $\alpha \delta < 1$  and  $B_2 > 0$  since  $\alpha < \frac{\bar{w}^o}{(1+n)\bar{w}_t^y \phi}$ . Therefore we have two possible cases:

- for  $\alpha > \hat{\delta}/2$  (and  $B_1 > 0$ ),  $\frac{\partial \bar{\tau}}{\partial n} > 0$  if  $A > -\frac{B_2 B_3}{B_1(1+n)\bar{w}^y \phi} (< 0)$ , which is always true since  $A > \underline{A} > 0$ .
- for  $\alpha < \hat{\delta}/2$  (and  $B_1 < 0$ )  $\frac{\partial \bar{\tau}}{\partial n} > 0$  if  $A < -\frac{B_2 B_3}{B_1(1+n)\bar{w}^y \phi} = M (> 0)$ . Clearly,  $A < M$  if  $M > K = \frac{\bar{w}_t^o - \alpha \phi(1+n)\bar{w}_t^y}{\bar{w}_t^y \phi(1+n)(\hat{\delta} - \alpha)}$ , where  $K$  was defined in the lemma above, and  $K > \bar{A}$ , which is satisfied – according to the lemma above – if  $\hat{\delta} \in (0, \bar{w}_t^o / (\bar{w}_t^y \phi(1+n)))$ . After simple algebra we have that  $M > K$  if

$$\frac{(2 - \alpha \delta^m)}{(\hat{\delta} - 2\alpha)} > \frac{1}{(\hat{\delta} - \alpha)} \quad (36)$$

or

$$\alpha^2 \delta^m + \hat{\delta}(1 - \alpha \delta^m) > 0 \quad (37)$$

which is always satisfied, since we assumed  $1 > \alpha \delta^m$ . Thus  $\partial \bar{\tau} / \partial n > 0$ .

Finally, the impact of aging on the steady state level of early retirement depends on the direct effect of  $n$  and on the change in  $\bar{\tau}$  induced by  $n$ :

$$\frac{d\bar{Z}}{dn} = -\frac{\phi}{1+\phi} \frac{\bar{w}^y}{\bar{w}^o} [\hat{\delta} - \alpha] \bar{\tau} - \frac{\phi}{1+\phi} \frac{\bar{w}^y}{\bar{w}^o} [\hat{\delta} - \alpha] \frac{\partial \bar{\tau}}{\partial n} \quad (38)$$

which is always negative since we assumed that  $\hat{\delta} - \alpha > 0$  and we have just proved that  $\partial \bar{\tau} / \partial n > 0$ .

## 6.4 Proof of proposition 5

Using the definition at Proposition 3, we can define the steady state social security contribution rate as follows:

$$\bar{\tau} = \frac{\bar{w}_t^y \phi(1+n) \hat{\delta} A(1/\delta^m) - \bar{w}_t^o \alpha + \alpha^2 \phi(1+n) \bar{w}_t^y}{\bar{w}_t^y \phi(1+n) \hat{\delta} \left( (1/\delta^m) - \alpha + \alpha^2 \frac{1}{\delta} \right)} = g(1/\delta^m) \quad (39)$$

Thus,

$$\frac{\partial \bar{\tau}}{\partial \delta^m} = \frac{\partial g(1/\delta^m)}{\partial (1/\delta^m)} \left( -\frac{1}{(\delta^m)^2} \right) \quad (40)$$

and the sign of  $\partial \bar{\tau} / \partial \delta^m$  is the opposite of sign of  $\partial g(1/\delta^m) / \partial (1/\delta^m)$ . Simple algebra delivers the following expression:

$$\frac{\partial g(1/\delta^m)}{\partial (1/\delta^m)} = \frac{\phi(1+n) \left( A\alpha^2 - \hat{\delta} A\alpha - \alpha^2 \right) + \frac{\bar{w}^o}{\bar{w}^y} \alpha}{\phi(1+n) \hat{\delta} \left( (1/\delta^m) - \alpha + \alpha^2 \frac{1}{\delta} \right)^2} \quad (41)$$

Thus,  $\frac{\partial \bar{\tau}}{\partial \delta} > 0$  if

$$\phi(1+n) \left( A \left( \hat{\delta} - \alpha \right) + \alpha \right) - \frac{\bar{w}^o}{\bar{w}^y} < 0 \quad (42)$$

with the above inequality being satisfied for

$$A < K = \frac{\bar{w}^o - \alpha \phi(1+n) \bar{w}^y}{\bar{w}^y \phi(1+n) \left( \hat{\delta} - \alpha \right)}. \quad (43)$$

Using the lemma above, we know that  $K > \bar{A}$ , and thus  $A < K$  always, if  $\hat{\delta} \in (0, \bar{w}_t^o / (\bar{w}_t^y \phi(1+n)))$ .

## 6.5 Proof of proposition 6

From the equation for  $\bar{\tau}$  at proposition 3, it is immediate to see that  $\partial \bar{\tau} / \partial \bar{w}^y > 0$ .

Whereas from the equation for  $\bar{Z}$  at proposition 3, we have that

$$\frac{\partial \bar{Z}}{\partial \bar{w}^y} = -\frac{\phi}{1+\phi} \frac{(1+r)}{\bar{w}^o} - \frac{\phi(1+n)}{1+\phi} \frac{1}{\bar{w}^o} \left[ \hat{\delta} - \alpha \right] \tau_{EE} - \frac{\phi(1+n)}{1+\phi} \frac{\bar{w}^y}{\bar{w}^o} \left[ \hat{\delta} - \alpha \right] \frac{\partial \bar{\tau}}{\partial \bar{w}^y} < 0 \quad (44)$$

since all terms are negative, because  $\hat{\delta} - \alpha > 0$  by assumption and  $\partial \bar{\tau} / \partial \bar{w}^y > 0$ .

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