

Howell, Bronwyn E.; Potgieter, Petrus H.

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Bundling information goods and access – simulating competition

Bronwyn E Howell* and Petrus H Potgieter†

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Abstract

We discuss the effect of pricing strategies by two firms on the total firm revenue, consumer and total welfare using simulation and numerical analysis. We consider pricing decisions for mixed bundling and where each firm offers two closely related products as well as a bundle. Bundling is a key feature for information goods (Bakos & Brynjolfsson, 1999; Shapiro & Varian, 1999) and we might assume that the market has two differentiated content products (each of which is a bundle of channels, for example, or a bundle of content titles to which access is sold). In many markets, this would be a basic entertainment product and then a sports product or a premium bundle with recent films etc. We can also consider this to be an access and a content product, to consider the issues around merger of content and access firms.

In the model for this paper, we introduce a principle of bounded rationality by limiting the ability of the firms to determine revenue-maximising pricing strategies. That means that the firms are able to reduce their effort to find a revenue optimum and will in general find a relatively good solution only but not necessarily an optimum one. Considering the effects of this approach might be useful for both regulators and firms. We also assume that the firms collude to maximise their joint revenue, which we regard as a realistic supposition in a duopoly market. The model can be extended to cover the case where one firm offers/bundles more than two products but this is a topic for future research.

1 Introduction

This paper expands on earlier work that examined the bundling and pricing decisions for a monopoly (Howell & Potgieter, 2017b). It considers a model for two firms, both of which have two highly similar (possibly, digital content) products. It is assumed the firms can bundle arbitrarily, the consumers' willingness-to-pay is known to the firms who then make pricing decisions so as to maximise their revenue. In the case of digital goods, this amounts to maximising joint profit. Our model could also be used to consider

- complimentary in the demand for the two content products; and
- different demand correlations between the content and the access products.

The model we have constructed combines the component pricing methodology of Chen & Riordan (2013) with the bundle pricing methodology of M. Armstrong (2013). Our simulations consist of rounds involving a scenario of random customer willingness-to-pay (WTP) values and then determining profit-maximising prices in each round for the two firms under the assumption that customers maximise their surplus. In each round, the firms are allowed to examine a fixed number (iterations) of randomly sampled prices and

*School of Management, Victoria University of Wellington, New Zealand, bronwyn.howell@vuw.ac.nz

†Department of Decision Sciences, University of South Africa, Pretoria, php@grensnet.com / potgiph@unisa.ac.za

determine what their revenue would have been, had prices been set like that. The firms then choose the prices that maximise their joint revenue.

The consumers are allowed to purchase any combination of products or bundles and are not constrained to purchasing only one of each item where we consider two similar products of different firms to constitute an *item*. This captures the complex interactions that occur on the demand side when consumers make their purchase decisions and we attempt to assess the likely effects of mixed bundling offers on revenue and welfare in different WTP scenarios. In this paper, we emphasise the diversity of outcomes based on 100 consumers who can be thought of as proxies for discrete groups of consumers making similar purchasing decisions.

Our earlier work based on a simple mixed bundling monopoly model revealed diverse welfare outcomes from the ability to bundle, depending on the underlying consumer demand where there is a monopoly provider of content and access. In that work, it was clearly shown that welfare outcomes depend very much on the structure of the underlying willingness-to-pay. We found total welfare was often significantly increased when bundling is possible and in many cases only a very small percentage of consumers were disadvantaged by forced bundling or tying. Notably, small changes in prices could sometimes lead to very large changes in consumer welfare and profitability, and large price changes have negligible effect, even though the samples were drawn from the same WTP distributions.

2 Background

Bundling – where consumers are offered a discount when buying several distinct products together – is both widely used and increasing in popularity as even-larger numbers of products are sold using digital interfaces (Bakos & Brynjolfsson, 1999; Shapiro & Varian, 1999). “Triple play” bundles of voice telephone, internet and video (pay television) content are long-established in telecommunications and media markets. Recent moves to expand these into quadruple, quintuple and even larger “plays” by adding mobile telephony, cellular data and access to WiFi hotspots are becoming commonplace, as are a range of sophisticated content bundling offers combining new forms of content (e.g. Netflix, Spotify and Binge-on) and applications (e.g. home security, electricity) with the purchase of fixed and/or mobile internet connections.

Despite the promise of different digital distribution methods enabling unbundling of content from its previous delivery modes (e.g. single music tracks from a CD collection bundle; television programmes from broadcast channels for individual sale; news items “unbundled” from each other and the paper on which they were previously published and sold individually via web-based media), the tendency appears to be toward the repackaging of the components for sale in different and even larger bundles than before e.g. individual tracks or programmes or news stories available via content distribution networks (CDNs) using nonlinear pricing mechanisms such as flat-rate subscription pricing or free access, independent of the quantity used/accessed. This tendency has raised a number of competition and welfare concerns. First, are firms with some market power increasingly using bundling as a form of profit-maximising price discrimination (or nonlinear pricing) to appropriate larger profits than they would under stand-alone (component) pricing, at the expense of consumers (the ‘price discrimination’ argument)? Second, are firms using bundling and pricing choices strategically to foreclose competitors, thereby harming long-term welfare by enabling all or any of higher prices, lower quality and reduced product variety to prevail than would have been the case if bundling had not occurred (the ‘strategic foreclosure’ argument)?

A large body of theoretical literature has been developed to address these questions – much of it motivated by high-profile regulatory inquiries (e.g. ‘unbundling’ local loop telephony access from network operation; ‘zero-rating’ mobile data traffic used by specific applications purchased in a bundle) and antitrust cases (Microsoft’s bundling of its proprietary web browser and operating system on its browser rivals; the vertical merger between General Electric and Honeywell and its effects on the jet aircraft engine and avionics markets; the integration of video content production and distribution – cable and broadcast – networks, in multiple cases such as Time-Warner/Turner, Newscorp/DirectTV). Notable papers examining the profit and welfare consequences of bundling as price discrimination include: G. J. Stigler (1963); Adams & Yellen (1976); Schmalensee (1984); Schmalensee (1982); McAfee, McMillan, & Whinston (1989); Mark Armstrong (1996);

Bakos & Brynjolfsson (1999); Mark Armstrong & Vickers (2010); Chu, Leslie, & Sorensen (2011); Chen & Riordan (2013); M. Armstrong (2013); Mark Armstrong (2016). Papers addressing actual or potential strategic foreclosure include: Whinston (1990); Nalebuff (2004); Gans & King (2006); Choi (2008); Mialon (2014); Mantovani & Vandekerckhove (2016).

The theoretical models tend to be highly stylised, featuring only a very limited number of goods, no or limited competition and/or unrealistic consumer valuations chosen for their mathematical tractability. For example, most rely on assumptions of full market coverage, continuous distributions, linear and symmetric demands and the purchase of no more than one item of each category. These stylised assumptions are rarely reflected in the real-world examples that the models are intended to shine light upon. Nonetheless, they provide some justification for wondering whether, in some circumstances, consumers could be made better-off by regulations requiring firms to unbundle or antitrust orders precluding mergers (Lim & Yurukoglu, 2015).

The body of empirical literature is much less extensive than its theoretical counterpart. One limiting challenge is the lack of direct comparisons. Hence the models depend upon the welfare effects of unknown potential entrants who never actually enter (Lim & Yurukoglu, 2015) or assumptions about consumers' responses to products and services that do not as yet exist (Howell & Potgieter, 2017b). One (albeit speculative) approach is to create a realistic model of the relevant industry using parameters chosen by statistical methods to match observations observed or anticipated. The model can then be used to simulate outcomes either with or without bundling. Examples can be found in Crawford & Yurukoglu (2012), Chu et al. (2011), Chen & Riordan (2013) and Howell & Potgieter (2017a).

2.1 The original bundling literature

The classic early papers of G. Stigler (1973), Adams & Yellen (1976), Schmalensee (1982), Schmalensee (1984) and McAfee et al. (1989) consider a single firm with market power in at least one product which can be bundled with a second product. They establish the key findings that bundling increases firm profitability under a wide range of consumer demand conditions – particularly when demands for the products in the bundle are independent or negatively correlated. As bundle prices are less than the sum of component prices, bundling can also increase consumer welfare. Whereas pure bundling typically harms consumer welfare, as some consumers are required to purchase goods for which they have little (or even negative) value in order to obtain surplus from the higher-valued product, mixed bundling may increase consumer surplus, albeit at a cost to firm profits. Bakos & Brynjolfsson (1999) show that when consumer demands for components are independent and identically distributed, adding more products with negligible marginal cost to a bundle sold at a fixed price increases both profits and consumer welfare relative to component sales because the law of large numbers makes it easier to predict consumers' valuations for the bundle than for the components individually.

The seminal papers by Whinston (1990) and Choi (2008) focus on the effects of bundling on competition. In Whinston, firm 1 has a monopoly in market A, which it endeavours to lever into market B by tying sale of its product B1 in a bundle with A to foreclose duopoly rival firm 2's product B2. Limited numbers of products and firms restrict the number of relationships that must be considered. By considering only component sales (A, B1 and B2 sold separately) and the sale of a tied bundle to compete with the component B2, the number of own- and cross-effects to be considered is considerably reduced. Whinston demonstrates that when the merged firm 1 can precommit to offering only a pure bundle (AB1 – for example by using technical features to limit compatibility with the rival's offerings), the rival cannot remain. Forcing firm 1 to unbundle restores competition in the market for good B, and thereby enhances consumer welfare.

Choi expands on Whinston's model by allowing mixed bundling – that is, firm 1 sells A1 and B1 in competition to firm 2's components A2 and B2 and bundle A2B2. However, to limit the number of own-and cross-price effects modelled, further assumptions are made. First, Choi assumes products A and B are complements purchased in strict proportion to each other. That is, if a consumer purchases an A product, one and only one B product must also be purchased. No B or A products are purchased alone (such as when an internet customer may purchase internet access, but not a cable TV subscription, or vice-versa). Neither will any consumer purchase two products in one category, even though the products offered by each firm may be differentiated (so no consumer will buy both firm 1 and firm 2's content distribution offerings, even though

each has some content not available on the other). Thus, only four possible products are sold: A1B1, A1B2, A2B1 and A2B2. Further assumptions also limit the mathematical complexity of the model – namely the demand functions for each component are linear, the demand system is symmetric, an equal increase in the prices of all composite goods reduces the demand of each composite good and all four composite (bundled) products are equally substitutable.

If all of these conditions hold, then sometimes (but not always) prices can be found at which it will be profitable for firm 1 to merge, decrease its bundle price and increase component prices relative to stand-alone pricing, thereby causing firm 2 to compete more aggressively (by lowering its prices and profits). Choi notes that the success of the strategy depends upon the extent to which each system can be viewed as a separate product. If there is little direct competition between the systems, then the merger may be welfare-enhancing, because consumers benefit from bundle discounts even as firm 1 revenues fall. As substitutability increases and competition between systems becomes more intense, beyond a certain point the effects on total welfare become unequivocally negative. If the fixed costs of firm 2 are sufficiently large that they cannot be offset by the lower profits induced by firm 1's pricing strategy, then firm 2 will not enter (is foreclosed) or if present will exit from the market.

In Choi's model with mixed bundling, strategic foreclosure is possible under some circumstances, but not inevitable as in Whinston's pure bundling model. However, the complementarity and proportionality assumptions limit its applicability to some very specific circumstances which only rarely apply in practice, and almost certainly do not prevail in the burgeoning bundles observed in telecommunications access and application markets. As historic "triple plays" in telecommunications markets have almost always been offered as mixed bundles with no limit on the minimum or maximum numbers of each product variants purchased, and current and future bundles attracting attention (e.g. zero rating) do not conform to the complementarity and proportionality assumptions, the ability to use bundling to achieve strategic foreclosure appears limited.

2.2 The complexity of mixed bundling

To address the computational complexity of mixed bundling as the number of products increases, Chu et al. (2011) propose bundle-size pricing (BSP) as a more computationally-efficient means for firms to set prices as the number of products in the bundle increases. They observe that despite the profit-maximising potential offered to firms by complex pricing structures, they are rarely implemented in practice. They suggest this is because it takes surprisingly few prices to obtain nearly as much profit as would be earned by pricing every possible bundle combination optimally. Using simulations, they show that a firm can compute optimal prices for bundles of any m products from amongst n components ($m \leq n$) that yield profits that almost always exceed component pricing (CP) and tend to quite closely approximate the profits from mixed bundling. This pricing strategy has been observed in the prices of cable television companies, online music stores, magazine newspaper and journal subscriptions, and subscriptions to seasons of sports games, music concerts and drama performances.

Whilst Chu et al. (2011) model is helpful in explaining why multi-product firms do not engage in highly-complex pricing strategies, it offers little assistance to policy-makers and competition authorities as it does not address the effects of the pricing policy on consumer and total surplus. Nor does it address the consequences of the competitive supply of at least one of the products in the bundle. In his more general theory of commodity bundling, M. Armstrong (2013) models bundle pricing as a variant of nonlinear pricing, where rather than calculating optimal prices for every possible bundle, the supplier offers a discount on the sum of the component prices if two or more products are purchased together.

Armstrong's pricing strategy is frequently observed when two firms offer discounts for joint purchase (e.g. petrol and groceries; airlines and car rental/accommodation; multiple tourist attractions offered in a 'city pass'). It is also a common feature of telecommunications bundles, where a further fixed discount is added for each product included in a bundle from a range, for example, fixed voice telephony, mobile voice telephony, fixed internet access, mobile internet access, WiFi hotspot access, application or content distribution subscriptions, retail electricity supply etc., where at least some of the products at least are supplied by partner firms (rather than by vertical integration), where the demands for some components are

certainly positively correlated (e.g. fixed internet, mobile internet and WiFi hotspot access; fixed internet access and retail electricity supply), one consumer may purchase more than one item in a given category (content distribution subscriptions) and many are supplied in at least duopoly or oligopoly markets.

2.3 A multiproduct adaptation

In contrast to most other models, the Zhou (2017) model does not begin with a two-dimensional Hotelling framework of two firms and two products sold to consumers uniformly distributed on a unit square. Rather, it is a multiproduct adaptation of the random utility framework of Perloff & Salop (1985). Consumers' valuations for a firm's products are random draws from some distribution, independent across firms and consumers, reflecting the idea that firms sell differentiated products and consumers have idiosyncratic tastes. Its distributional assumptions echo those the Mark Armstrong (2016) competitive models, rather than the more complex stochastic demand interdependencies in the Chu et al. (2011) and Chen & Riordan (2013) single supplier models which use copula functions to represent the joint distribution of consumers' valuations for two products. That is, for $n \geq 2$ firms each supplying all of $m \geq 2$ products, the random utility match for firm j 's product i is independent and identically distributed across consumers (reflecting idiosyncratic consumer tastes) and for a given consumer is also independent and identically distributed across firms.

Furthermore, the m products are symmetric, and the market is fully covered – that is, each consumer buys one (and only one) of each of the m products. Also, the firms all offer variants of each of the m products, so that they compete globally with each other in every product category, differing only in the characteristics of the products offered and not by product ranges. Zhou (2017) relaxes the full market coverage assumption and finds the results largely unaltered, but the product symmetry and independent distribution assumptions along with the requirement that all n firms offer each of the m products remain.

The result is a model that generalises to m products and n firms, but with two products and two firms it converts to the two-dimensional Hotelling model. Beginning with an investigation of how dispersion of consumer valuations affects price competition in a single product environment, Zhou (2017) explores the ways bundling reduces consumer valuation heterogeneity, such that the per-product valuation density becomes more peaked and has thinner tails compared to the original single product valuation density. Whether a less-dispersed valuation leads to a lower market price depends on how dispersion is ranked and often also on the number of competitors. In the two-firm case, bundling intensifies price competition and lowers market prices, with the positive price effect often outweighing losses from reduced choice flexibility.

2.4 Summary of the theoretical background

In sum, the theoretical and empirical bundling literature identifies a wide number of dimensions in which real-world bundling scenarios can vary. These can be summarised as:

- the number of products offered in the bundle;
- the extent to which the products are perceived by consumers as identical items, differentiated commodities or completely separate products;
- the type of bundling strategies offered (e.g. component; pure; mixed; bundle-size);
- the degree of market coverage (do all consumers purchase all goods or only a subset of those available?);
- the nature of bundle price determination (optimal bundle pricing; discount on additive component prices);
- the extent to which consumers view the discrete products in the bundle as complements or substitutes;
- the extent to which search and shopping costs (brand preferences) influence consumers' choices;
- the extent to which consumers' demands for the items in the bundle are correlated, and if correlated, the stochastic nature of that correlation;

- the marginal benefit to consumers of buying more than one of each commodity or bundle;
- the number of firms (monopoly duopoly, other); and
- the form of competitive interaction (whether the products offered by different providers are substitutes or complementary).

Whilst some endeavours have been made to develop general theories of bundling (notably M. Armstrong, 2013; Mark Armstrong, 2016; Mark Armstrong & Vickers, 2010) or to extend monopoly and duopoly models into more general competitive engagements (e.g. M. Armstrong & Zhou, 2015; Zhou, 2017), these too rely upon a number of simplifying assumptions that are rarely observed in real-life cases (e.g. independent and identically-distributed consumer preferences; symmetric demands; single purchases of individual commodities). Exploration of any one of these effects comes at the expense of necessary simplifications of the others. Hence, for example Chen & Riordan (2013) and Chu et al. (2011) explore effects of the stochastic interdependence of consumer valuations of the different components that can contribute to a bundle, but are limited to considering only monopoly provision. Meanwhile, Choi (2008), Mark Armstrong (2016) and Zhou (2017) consider different forms of competitive interaction, but are constrained by assumptions of full market coverage or single product purchase, i.i.d consumer preferences and symmetric demands in order to render their analyses tractable.

3 A simulation model

If insight is to be gained into the outcomes in real-life bundling examples, then it may not always be possible to rely upon the findings of highly stylised mathematical models. An alternative approach is to use simulations using models capturing the specific characteristics of the case under review. The results may not be generalisable to other situations, but nonetheless can provide some guidance for the specific decision at hand - e.g. a merger application or competition inquiry.

We develop model that addresses three features that are not easily accommodated into the theoretical models discussed above:

- consumers with asymmetric willingness-to-pay for products in the same category produced by different providers (feature 1),
- purchasing none, one or more than one product in each category depending on their maximum surplus (feature 2),
- in a market where providers (or regulators and policy-makers) have knowledge of the mean demands for each product but not the details of how consumers' demands for the different products may be correlated (feature 3).

The questions of the potential to use strategic pricing or bundling are secondary to these primary considerations that arise from the nature of the products and consumer tastes and preferences.

Simulations using a common (or, several) scenarios of consumer valuations for the products can then be run, varying one model parameter at a time, to gain insights into the likely effects of different provider or policy/regulator decisions (e.g. forced unbundling; different bundling choices) on key decision variables (e.g. joint provider profit; individual provider profit; total welfare; consumer welfare). By utilising information about the actual distributions of consumer valuations in the samples evaluated, it will also be possible to provide additional analysis of the distributions of consumer surplus across consumers of different types, thereby providing a richer analysis of the distributions of welfare between providers and consumers than is possible in the stylised theoretical cases.

The model is simple, but is constructed in a manner that will enable it to be customised and/or calibrated to capture a range of effects not easily incorporated into the theoretical models. In this paper, we focus only on the analysis of the basic outcomes in terms of revenue, consumer surplus and total welfare.

3.1 Description of the model

The model looks at pricing outcomes for digital goods in a market with two colluding firms that seek to maximise their joint revenue through setting the prices for their individual projects and bundles. Quasi-optima are determined through simulation by randomly selecting from the available prices for a fixed number of iterations and then picking the best among the options evaluated.

For a large number of iterations, this will likely cover all the possibilities and be equivalent to an exhaustive search through the discrete space of possible prices. We assume that the individual products have integer prices (but not the bundles, the prices of which are determined by a fixed formula). In our view, this is a particularly realistic aspect of the model and it has the additional advantage of side-stepping the Bertrand paradox (Bertrand, 1883).

We have n customers (in this paper $n = 100$), 2 firms and 2 items, both items being sold in differentiated form (called products) by each firm. Both firms also offer a bundle consisting of their own two products, offered at a price that is $0.90^2 = 81\%$ of the sum of the two individual products' prices. The finite number of customers implies a "chunkiness" in the valuations, which we believe might be close to reality and/or to firms' assumptions.

Customers may purchase any number of products or product bundles and their willingness-to-pay (WTP) for each product is assumed to be an integer and is determined as the sum of the following.

1. A random underlying integer WTP of w_i for the basic item i (regardless of the firm offering the product) of at least 0 and at most 9.
2. A premium, also random and integer, of p_{ji} for the product offered by firm j for each j that is at least 0 and at most 3.

Consumers' WTP are assumed to be integer-valued, randomly selected and uniformly distributed for the present paper. Uniform distribution is not an essential feature of the model and can easily be modified.

The WTP for a bundle is the sum of the WTP of its components. No complementarity between the products in the bundle is assumed. Consumers have a WTP for a collection of bundles that is the sum of the underlying WTP for each unique item in the collection as well as the firm premiums. If the products offered by firm 1 are labelled 11 (the firm 1 version of item 1) and 12 (the firm 1 version of item 2) and the corresponding products of firm 2 are 21 (its version of item 1) and 22 (its version of item 2), respectively, the following might be a typical customer WTP for the available purchase decisions.

Purchase decision	WTP	Remark
{11}	11	Firm 1, product 1
{12}	4	Firm 1, product 2
{11, 12}	15	Firm 1, bundle
{21}	11	Firm 2, product 1
{22}	6	Firm 2, product 2
{21, 22}	17	Firm 2, bundle
{11, 12}, {21, 22}	20	Firm-specific premium seen in bundles
{11, 12}, {21}	18	Firm-specific premium
{12}, {11}	15	Direct sum for two items, same firm
{12}, {21}	15	Direct sum for two items, different firms
{12}, {21}, {11}	18	Firm specific premium
{21, 22}, {11}	20	Firm-specific premium
{21, 22}, {12}	17	No firm-specific premium for 12
{21, 22}, {12}, {11}	20	Firm specific premium
{21}, {11}	14	Firm-specific premium in item 1
{22}, {11, 12}	17	Firm-specific premium
{22}, {11, 12}, {21}	20	Firm specific premium
{22}, {11}	17	Direct sum for two items, different firms
{22}, {12}	6	Two products, one item

Purchase decision	WTP	Remark
{22}, {12}, {11}	17	Firm-specific premium
{22}, {12}, {21}	17	No evident firm-specific premium for item 2
{22}, {12}, {21}, {11}	20	As expected, the same as above
{22}, {21}	17	Direct sum for two items, same firm
{22}, {21}, {11}	20	Firm-specific premium

As expected, the table above shows identical WTP for the decisions {11, 12}, {21} (firm 1's bundle and product 1 from firm 2) and {12}, {21}, {11} (firm 1's two individual products as well as product 1 from firm 2). The data is more than enough to determine the 8 underlying random integer values described above but this is left as an exercise for the reader.

That is, in our model the consumer has a willingness to pay for the undifferentiated items, with the premium reflecting the additional value placed on the differentiated product offered by each firm. When a second product of the same type is purchased, the basic valuation of the decision increases only by the firm differentiation premium. For example, there could be a basic WTP of w_1 for a vanilla content distribution network, but the additional value of the product from firm 1 is p_{11} . The consumer's total WTP is then $w_1 + p_{11}$. But when a second CDN is purchased, only the differentiation value p_{21} will be acquired.

Increasing the range of the firm-specific premium, which was chosen to be quite modest for this paper, increases the extent of differentiation between the products. As it stands, the differentiation is not large, hence the propensity to buy more than one item of the same type is very low for the values used here. Thus, our model captures product differentiation only. It does not capture any additional benefit from buying two products from one firm relative to buying them from separate firms (one-stop shopping, or a shopping cost penalty imposed for buying from two firms). In that case, we are looking at a variation of Zhou (2017), albeit allowing mixed bundling.

Firms may price their products as they wish but bundles are priced identically, using a bundle discount factor of 0.9 to the power of the number of items in the bundle, so if products priced at 4 and 6 are bundled, the bundle will be priced at 8.10 and this principle holds for both firms. We assume that prices are discrete, specifically that the prices of the products are integers. An example of a pricing decision by the firms might result in the following costs to consumer for the purchase options.

Purchase decision	Cost to consumer
{11}	9.00
{12}	7.00
{11, 12}	12.96
{21}	7.00
{22}	9.00
{21, 22}	12.96
{11, 12}, {21, 22}	25.92
{11, 12}, {21}	19.96
{12}, {11}	16.00
{12}, {21}	14.00
{12}, {21}, {11}	23.00
{21, 22}, {11}	21.96
{21, 22}, {12}	19.96
{21, 22}, {12}, {11}	28.96
{21}, {11}	16.00
{22}, {11, 12}	21.96
{22}, {11, 12}, {21}	28.96
{22}, {11}	18.00
{22}, {12}	16.00
{22}, {12}, {11}	25.00

Purchase decision	Cost to consumer
{22}, {12}, {21}	23.00
{22}, {12}, {21}, {11}	32.00
{22}, {21}	16.00
{22}, {21}, {11}	25.00

The firms are assumed to be able to collude and attempt to maximise their joint revenue by setting prices of their underlying products. We assume that the firms are able to predict the revenue outcomes for a specific set of prices accurately but that they are able to do so only for a fixed number of randomly chosen pricing scenarios. A low number of simulations correspond to a limited ability or willingness of the firms to expend effort to find the overall optimum. Consumers make a purchase decision based on maximising their individual surplus and may purchase a specific item in the form of differentiated products from different firms – on its own or in bundles.

The following commentary on our terminology might be useful in what follows.

Term	Comment
item	one kind of <i>product</i>
product	a firm-specific, differentiated <i>item</i>
scenario	one specific randomly generated WTP set
iteration	one sample of random prices and calculation of consumer choices and firm revenue in a single scenario
round	a fixed number of iterations of the simulation, for a single scenario, with a maximal revenue determination

4 Simulation results

Our simulations were run in Python and the results post-processed using a Python script. Outcomes for every round were stored and the final results were analysed in LibreOffice Calc and in Microsoft Excel. All the calculations for the paper were able to run in under 24 hours of total computer time, on a good but standard laptop computer.

We considered a market with 100 consumers and initially generated 100 independent WTP scenarios for these consumers with the parameters described above. Then we looked at firms using 1000, 10 000 and 100 000 iterations (of randomly chosen price assignments) respectively. For our ranges of values, 100 000 is more than the total number of possible price assignments but there is a relatively small chance that the absolute optimum will not be found by examining 100 000 cases and in our results show that it is not terribly infrequently the case that 10 000 iterations on the same underlying WTP scenario produce a better outcome (for the firms) than 100 000 iterations. This is because 100 000 randomly sampled price assignments can miss a revenue maximum that is picked up in an independent round of 10 000 random assignments.

For each WTP scenario, we ran 10 rounds (each) of 1000 and 10 000 iterations and one round only of 100 000 (which was quite slow). In the analysis below, we consider mainly the average outcome of the 10 rounds for the 1000 and 10 000 iterations case.

We see the number of iterations as a proxy for the amount of effort the firms put into maximising revenue. The pricing strategy chosen by the firms in a specific simulation round therefore depends (in part) on chance and the number of iterations in the round depends on how lazy the firm is. As we shall see, it is not unreasonable for the firms to be a little slothful.

4.1 Revenue increases with effort but only on average

In addition to the iterations rounds, we have considered another pricing strategy for both firms where they simply set the prices for the products equal to the average WTP of the consumers. This is labelled “0 iterations” in our output files and in the tables below. In all of the 100 cases examined in this paper, the outcome was worse for the firms than in the iteration rounds where they pick the most advantageous of the randomly sampled pricing assignments – even for just 1000 iterations.

The 10 rounds for each WTP instance for the option of 1000 and 10 000 iterations were run independently. The average revenue for the firms (over the 10 independent rounds) for 1000 iterations is better for every WTP set than for the case of “0 iterations” and the average revenue for 10 000 iterations was always higher than for 1000. In 99 of the 100 WTP sets, the single instance of 100 000 iterations achieved a better result than the average of 10 000 iterations.

In as many as 24% of the WTP scenarios, there was at least one round of 1000 iterations which achieved the maximal revenue observed for the rounds (of any number of iterations). This rises to 94% for 10 000 iterations, showing that it is relatively likely that a medium-effort revenue maximisation will do as well as a high-effort maximisation.

Additional effort	Revenue improvement
“0” to 1000 iterations	17.08%
1000 to 10 000	5.23%
10 000 to 100 000	2.05%

Considering the gain from average WTP pricing (“0 iterations”) to a low-effort maximisation with 1000 rounds, there appears to be relatively modest incentives for firms to engage in high-effort maximisation of revenue.

4.2 Consumer surplus decreases with effort but not uniformly

As is to be expected, the consumer surplus is generally the highest for the low-pricing “0 iterations” strategy. However, in 7 of the 100 instances the consumer surplus is maximal where firms have made the most effort to extract revenue, i.e. with 100 000 iterations.

Effort (iterations)	Consumer surplus maximal
“0”	91
1000	1
10 000	1
100 000	7

This suggests that it is not uniformly the case that firm’s effort to extract revenue necessarily harms consumers.

4.3 Total welfare more frequently maximised with effort

Computing the number of times that total welfare (revenue plus consumer surplus) is achieved with a certain number of iterations, we see that it does increase with effort, in the instances that we have examined.

Effort (iterations)	Maximal total welfare
“0”	12
1000	13

Effort (iterations)	Maximal total welfare
10 000	23
100 000	52

The average total welfare improvement, for increased effort, is however modest.

Additional effort	Welfare improvement
“0” to 1000 iterations	0.65%
1000 to 10 000	1.97%
10 000 to 100 000	1.36%

Unsurprisingly, in terms of percentage improvement, we see that the firms themselves have a higher incentive than the society as a whole to increase the effort, the result of which is (on average) greater welfare and occasionally but not infrequently, a maximal consumer surplus.

4.4 The structure of the underlying WTP matters tremendously

We now consider the total firm revenue, in relation to the effort invested by the firms, averaging over the 100 WTP scenarios (and the 10 rounds for the 1000 and 10 000 iteration efforts). As one would expect, the average total revenue increases with the amount of effort.

Effort (iterations)	Average total revenue
“0”	701.90
1000	821.32
10 000	864.34
100 000	882.16

Nevertheless, the maxima and minima over the same 100 WTP scenarios reveal considerably more complexity. Since the 100 000 iterations effort almost always finds the actual optimum, we see that there is an WTP scenario in which the “0” iteration average pricing does better (795.52) than an almost exhaustive search did in another WTP scenario (751.32).

Effort (iterations)	Max total revenue	Min total revenue
“0”	795.52	585.52
1000	995.50	685.46
10 000	1023.68	708.96
100 000	1023.68	751.32

In every one of our WTP scenarios, 100 000 iterations is superior to “0” for the firms but the improvement ranges from 13% to 46%. However, the underlying WTP scenario can effect the outcome for effort and also with regard to the total welfare. In the following two distinct WTP scenarios, sharply different outcomes emerge from situations that feature identical pricing strategies. Product prices are given here and elsewhere as: product 1; product 2 of firm 1; product 1; product 2 of firm 2.

Iterations	Total revenue	Consumer surplus	Total welfare	Product prices
100000	948.72	387.36	1336.08	6.00; 6.00; 6.00; 6.00
100000	868.08	334.12	1202.20	6.00; 6.00; 6.00; 6.00

Although our results show a negative correlation between consumer surplus and total revenue, the example above shows that the underlying WTP can also affect this relationship with both the higher revenue and the higher consumer surplus arising from the first scenario. Future work will look in more detail at the connection to the underlying WTP structure.

It is also not the case that high prices imply high revenue. Here the consumers' WTP structure also plays a considerable role. The average unit product price chosen by the firms in these simulations, is just over 7.00 which seems reasonable since the expected WTP is 6.00. If we consider total firm revenue as a multiple of the average unit product price, it can be observed that this varies very widely – without taking the “0” iterations case into account.

Measure	Revenue over average unit price
Average	118.30
Maximum	165.98
Minimum	81.05

Considering, for example, just the rounds with average unit product price (AUPP) of 7.00 we observe the following range of values for total firm revenue.

Measure	Total revenue with AUPP of 7.00
Average	836.88
Maximum	956.40
Minimum	692.64

This wide range is produced by two things: differences in the underlying WTP scenarios and the (bad) luck of lazy firms.

4.5 A lot depends on chance when firms are (even a bit) lazy

Moderately lazy firms using 10 000 iterations instead of 100 000 generally underachieve the latter case. In our model, the extent to which this is the case depends on how lucky the firm was in the specific round. We have compared the best to the worst outcome in the 10 rounds of 10 000 iterations for the different WTP scenarios and call the difference a “premium for luck” on the total revenue (being of the order of 800 to 900).

Measure	Premium for luck in 10 rounds
Average	40.32
Maximum	90.31
Minimum	12.04

We have no proof that this is not mere random fluctuation. Nevertheless, this shows that the actual revenue position for firms can depend appreciably on luck. The market share of the largest firm (C_1) is, on average not much more than 50% but here also considerable variation, attributable to chance, can be observed.

Effort (iterations)	Average C_1	Maximal C_1	Minimal C_1
“0”	54.57%	62.59%	50.07%
1000	55.63%	94.57%	50.00%
10 000	53.81%	71.81%	50.00%
100 000	53.78%	63.88%	50.00%

It is not strange that a market that is evenly split between the two firms can appear but the results demonstrate that strong market dominance can arise from pure chance when the firms are lazy. Even in the case of moderately lazy firms at 10 000 iterations, heavy market dominance can occur and for the case of (nearly) optimal revenue, a dominant firm is not unusual.

The revenue dominance of a single firm does not necessarily follow from as simple a cause as high prices. In the 100 000 iteration case with a C_1 of 63.88%, the product prices are 8.00; 8.00; 9.00; 7.00. That is, the difference between the two firms lies not in the pricing but possibly in their ability to use pricing to exploit the underlying WTP structure.

4.6 Consumer outcomes are also highly dependent on chance

We now consider, for a single WTP scenario and 1000 iterations, how consumer surplus and total firm revenue fluctuate. Outcomes have been arranged by increasing firm revenue, for the sake of convenience only.

Total firm revenue Tot	al consumer surplus
812.18	200.82
833.01	213.99
857.72	157.60
858.11	199.74
865.55	296.05
885.98	395.73
885.98	395.73
903.28	253.52
924.96	229.84
939.92	243.80

It is evidently not the case that consumer surplus is inversely related to firm revenue. In fact, the lowest consumer surplus observer (the two cases below 200) coincide with relatively low total firm revenue. The highest consumer surplus of 395.73 appear, indeed, where firm revenue is relatively high.

5 Conclusion

We have used a relatively simple simulation model to illustrate the complex outcomes that occur when two firms are presumed to not necessarily be able to find a revenue optimal and using realistic (integer) basic prices. We also show how the optima (if the firms are able to find them) deliver different outcomes that are highly dependant on randomly chosen WTP scenarios. We see that consumer surplus and revenue vary widely within each WTP scenario, depending only on chance, and that it is quite possible that firms and consumers are lucky the same time. It is also observed that market outcomes (in terms of concentration) for the same underlying WTP is heavily influenced by luck when firms are unable to evaluate all possible price points.

Our model should inform both firms and regulators as to the complexity of underlying market forces. Future work will include examining these outcomes with regard to the characteristics of the underlying willingness to pay instances. We do not exaggerate the verisimilitude of our model but we propose that the assumption that firms always find optima is far from realistic. Further, we suggest that a model with integer prices and a finite number of consumers (as a proxy for groups of consumers), in contrast to a continuum as in Nalebuff (2004) and many other papers, offers many insights.

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