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# Forecast Heuristics, Consumer Expectations, and New-Keynesian Macroeconomics: A Horse Race

by Tae-Seok Jang and Stephen Sacht



# Forecast Heuristics, Consumer Expectations, and New-Keynesian Macroeconomics: A Horse Race

Tae-Seok Jang\* and Stephen Sacht†

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## Abstract

This study extends the hybrid version of the baseline New-Keynesian model with heterogeneous agents who may adopt various forecast heuristics. With a focus on consumer expectations, we identify the most appropriate pairs of forecast heuristics that can lead to an equivalent fit to the data compared with the model specification under rational expectations. The competing specifications are estimated using the simulated method of moments. Our empirical results suggest that expectations under bounded rationality in the United States are grounded on consumers' emotional state, while for the Euro Area they are technical in nature. This observation questions the need for a hybrid model specification under rational expectations.

**Keywords:** Consumer Expectations; Forecast Heuristics; New-Keynesian Model; Simulated Method of Moments.

**JEL classification:** C53, D83, E12, E21, E32.

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# 1 Introduction

The past decade has seen the adoption of simple forecasting heuristics to explain the importance of the expectations formation process in relation to the business cycle. In hindsight, rational expectation models failed to predict the crisis in the US housing market in late 2007. Although many macroeconomic models find stable equilibrium paths under the rational expectation hypothesis, however, the predictions are particularly inconsistent with crisis periods. This finding suggests that individuals do not necessarily forecast future economic dynamics given full information on the underlying structure of the economy and properties of the exogenous shocks.

For the majority of economic decisions, individuals discard certain information and allow emotions to drive their behaviors. This may be expected partly because agents are aware that information is not fully available. Instead, they process past information and/or fundamental values through backward-looking rule-of-thumb behavior (i.e., via forecast heuristics). This view (while already previously addressed by various authors) was boosted by Akerlof and Shiller (2009), who analyzed the mechanism behind the early stage of the global financial crisis. Following this line of argument gives rise to the so-called bounded rationality models that account for the behavior of heterogeneous agents.

The phenomena of emotional decision making as well as the discarding of information can be expected of agents who do not forecast future developments in a professional way. As an example, for the latter, investors may rely more on a broad range of information to apply technical forecasting techniques. Regarding private consumption, however, empirical evidence suggests that this contributes the most to the level of GDP, while it is often influenced by the lagged or lead variables of confidence indicators: there exists a strong correlation between consumer confidence and private household expenditure. Therefore, private households may account for their confidence when predicting their periods ahead consumption possibilities.

In the macroeconomic literature, modeling expectations formation, especially the estimation of bounded rationality models, has gained broad attention in recent years. With a focus on consumer confidence, whether this is crucial to explain business cycle dynamics is the key question of this study. We argue that consumers' expectations do not necessarily reflect a rational forecasting strategy. Instead, agents' decision-making process relies either on their willingness to forecast future consumption based on historic patterns and/or on the fundamental values for consumption or even their emotional state alone. While the former forecast strategy is rather technical in nature, the latter is described by waves of optimism and pessimism in the spirit of Keynes' (1936) concept of animal spirits.

In this study, we consider various forecast heuristics for consumer expectations in the baseline New-Keynesian model (NKM henceforth). Households choose

from a set of emotional- and technical-oriented forecast heuristics that display the relationship between consumer confidence and private household expenditure. In each period, the distribution of the proportions of emotional and technical forecasters changes because of the endogenous switching process. As heuristics can be seen as a proxy for agents' expectations formation processes, we seek to find if they could lead to an equal or even better description of a standard macroeconomic model to empirical data compared with the same model under rational expectations. In the case of the equivalency or dominance of bounded-rational forecast heuristics in terms of fitting the data, we seek to identify if they are technical- or emotional-oriented. We then investigate the effect of confidence on the economic decision-making process and hence macroeconomic dynamics. Finally, based on our results, we claim that the need for purely rational expectations macroeconomic models is disputable. The novelty of our contribution therefore lies in the comparison of oppositional types of forecast heuristics and their effect on consumer behavior from a macroeconomic perspective. To the best of our knowledge, such a theoretical and empirical investigation on the competing forecast strategies of consumers under bounded rationality is a novel contribution to this strand of the literature.

We consider a bounded-rational variant of the NKM with rule-of-thumb possessing agents. We then estimate the model parameters via the simulated method of moments (SMM) approach where the parameter estimates can be used for the calibration of bounded-rational dynamic stochastic general equilibrium (DSGE) models. Our investigation reveals that from an empirical point of view, the selected specifications of the bounded rationality model lead, in fact, to an equal or even better fit to the empirical data compared with the model under rational expectations. The results provide explicit information on expectations formation where the heuristics being applied can be emotional- or technical-oriented for two economic regions of interest. Based on this, we can then examine what kinds of bounded rationality models stand out as alternatives to their rational expectations counterparts.

The remainder of this paper is structured as follows. In the following section, we provide an overview of the related literature and how we contribute to it. Section 3 examines the role of consumer confidence as a proxy for households' expectations in the United States and the Euro Area based on descriptive statistics. In Section 4, we describe the structure of the NKM and present several forecast heuristics applied in a heterogeneous agent-based version of the model given the discrete choice mechanism. The parameter estimates of the various model specifications are evaluated according to their fitness criteria in Section 5 using the SMM approach. The latter, as the empirical estimation method of choice introduced by Franke et al. (2015), is presented in the Appendix. Finally, Section 6 concludes.

## 2 Contribution to the Related Literature

Various types of *forecast heuristics* have been widely used to model the expectations formation process in DSGE models. This work was pioneered by Brock and Hommes (1997) and Gaunersdorfer et al. (2008) among others in financial economics. Agents sort themselves into categories related to both their emotional states and specific professional (i.e., technical) forecasting rules based on heuristics. In all subsequent periods, agents are allowed to switch between groups based on the discrete choice approach. As a prominent example, De Grauwe (2011) was one of the first to incorporate this kind of mechanism into the NKM framework in which economic agents are, in fact, either boundedly rational or exhibit perfect rational expectations. This scheme also applies to the expectation of future house prices and consumption of non-durable goods, as presented by Bofinger et al. (2013).

In this study, we examine the validity of competing rule-of-thumb behavior in expectations formation processes. To show this, we adopt a *horse race* exercise to evaluate the performance of different heuristics using a well-defined objective function. As the point of departure, we consider the estimation of the De Grauwe (2011) model by Jang and Sacht (2016). In particular, we seek to find specific pairs of heuristics that lead to the smallest deviation of the model generated second moments from their empirical counterparts. A similar contribution, but with a different scope on rational expectations only, was investigated by Anderson (2008) with respect to the solution methods of DSGE models.

Our contribution is also closely related to Cornea-Madeira et al. (2017), who examined the validity of simple rules-of-thumb. In their framework, differences in group behavior (i.e., fundamentalists and chartists) play a key role in explaining US inflation dynamics. Their results revealed that endogenous switching between both groups depends on the realization of agents' performance when predicting future outcomes. In this study, we take a different perspective by considering the importance of *consumer confidence* in the business cycle instead of inflation expectations. Therefore, we consider different types of heuristics regarding consumption expectations and only one bounded-rational specification for the New-Keynesian Phillips curve (NKPC).

Previous studies have shown ambiguous and inconclusive results about the relationship between consumer confidence and economic activity. In an early work on animal spirits, Blanchard (1993) examined empirically the extent to which consumer confidence reflects a negative consumption shock during the 1990/91 recession in the United States. He found evidence for agents with limited information processing ability. The author suggested that perfect foresight does not fully explain the drop in consumers' expectations, which allows some scope to interpret confidence as a driver of agents' decision-making process. This view, however, was questioned by Barsky and Sims (2012). They examined the impact of news shocks and only a noise-ridden signal of that kind of shock, where the latter can be regarded as a type of animal spirits. According to their empir-

ical results, animal spirits only contribute weakly to the observed relationship between confidence and economic activity.

Our approach, which focuses on forecast heuristics, differs from those found in the literature on learning. With a focus on DSGE modeling, the concepts of agents' learning ability with a constant gain and adaptive learning (see Evans and Honkapohja (2001) for an overview) are strongly considered. As an example of the former, Orphanides and Williams (2007) performed stochastic simulations with a focus on monetary policy intervention under the misperception of natural rates. The authors suggested an augmented monetary policy rule under a constant gain, which accounts for a more aggressive response to inflation volatility and inertia in the interest rate itself. As a result, inflation persistence will be dampened and, therefore, the objective of price stability is fulfilled. Evans and Ramey (2006) considered a Cagan-Friedman adaptive expectation formula with a decreasing gain. They showed that in the Nash equilibrium, agents select a common value of the gain that minimizes the mean squared error of the inflation forecast. When facing switching in the inflation regime being observed infrequently, applying their proposed adaptive expectation rule turns out to be optimal in the spirit of the Lucas critique. A drawback of the learning approach is that agents' perceived law of motion does not distinguish between emotional- and technical-oriented forecasting methods. In addition, switching between groups of heterogeneous agents is, in general, not considered explicitly.

*Persistence* in the dynamics of consumption (and hence, output and inflation) is observed empirically. Various studies have aimed to modify this baseline NKM to account for this stylized fact. The result is the so-called hybrid version of the model under rational expectations with leads and lags in the dynamic IS equation and the NKPC (cf. Smets and Wouters (2005) and Christiano et al. (2005) among others). The corresponding backward-looking components predominantly stem from habit formation in consumption and price indexation.

Cornea-Madeira et al. (2017) reviewed the mixed evidence on the role of forward-looking and backward-looking components in DSGE models. The contributions by Kozicki and Tinsley (2002, 2005) and Roberts (2005) as well as Boivin and Giannoni (2006) highlighted the need for various specifications of shocks to introduce inertia in the standard NKM. Ball (2000) proposed an expectations formation process in which agents apply univariate forecasts. The latter reflect the optimality of processing all information available on inflation dynamics, while those for other relevant variables such as output and interest rates are ignored. The author showed that his bounded-rational rule-of-thumb leads to a better match of observed US inflation persistence than a purely rational expectations one. Milani (2005) showed that sources of persistence (e.g., habit formation in consumption and price indexation) are not essential under bounded rationality. This holds if agents gain knowledge of the unknown model parameters by updating their beliefs using the constant gain approach. In the absence of the latter in our study, we show that emotional- and technical-oriented fore-

cast strategies, as substitutes for purely rational expectations, are sufficient to ensure persistence — with no need for additional backward-looking components. While Milani (2005) discussed the results of his empirical study as he applied the Bayesian estimation technique, we consider the *simulated method of moments* approach for our investigation. Therefore, we tie into this field of "non-rational expectations econometrics" (Ireland (2003)).

The empirical evidence for the type of bounded rationality model presented in this paper is so far ambiguous. Jang and Sacht (2016) found that the bounded-rational version of the NKM for the Euro Area exhibits a similarly good fit to the data as the one with rational expectations being assumed. Liu and Minford (2014) showed evidence for the opposite. Therefore, the investigation into the role of agents' bounded-rational behavior is not trivial. Notably, several statistical techniques have been developed over the past two decades to take the DSGE models to data. Moment-based estimation via SMM is as efficient as maximum likelihood (ML) as long as the moment conditions encompass the true data-generating process (Carrasco and Florens (2002) as well as Fernández-Villaverde and Rubio-Ramírez (2007)). This suggests that ML has desirable theoretical properties for statistical inference; however, the complexity of DSGE models poses challenges for the estimation of behavioral parameters using the ML method (Canova and Sala (2009) and Canova (2011)). If the data-generating process is subject to non-linear specifications in modeling, the Monte Carlo Markov Chain (MCMC) method is often used to handle irregular and multimodal posteriors in the parameter space. However, while the MCMC method can be advantageous for the evaluation of forecast heuristics in bounded rationality models (Herbst and Schorfheide (2015)), prior information on some behavioral parameters is not available. These difficulties have led to the development of simulation-based inference in macroeconomic models.

To apply SMM, we utilize the robust optimization ("interior point") method to find a global minimum for the parameter estimation. This seems to be a practical way to estimate the behavioral parameters since the complexity of the models and computation of conditional moments can be easily obtained by simulations by powerful computers. Moreover, when researchers have a clear picture of how the economy works with specific moment conditions, partial information methods may prove to be more useful than the full information ML method. Indeed, moment matching might be useful in terms of its robustness to both the distribution of shocks and the risk of misspecification. Hence, this study focuses on matching the covariance structure of inflation, interest, and consumption to evaluate different behavioral specifications. In other words, the goal of this study is to evaluate the fit of the models along the dimensions of the moment conditions of interest to find the parameter estimates associated with the best fit of the models with alternative expectations formation mechanisms.



### 3 On the Link between Consumer Expectations and Household Expenditure

The main concern of business cycle analysis is to find the reason why the economy is constantly going through periods of booms and busts. In general, the question arises of how business cycles come about. In theory, the answer is simply given by the decomposition of GDP into its main components of private consumption, aggregate investment, and net exports based on the system of national accounts. Empirically, private consumption contributes the most to the level of GDP. More precisely, according to recent data on total household expenditure provided by the OECD (2016a), private consumption amounts to around 68 and 56 percent relative to US and Euro Area GDP, respectively, excluding housing and government transfers. By including the latter, these numbers range around 75 and 69 percent for the United States and Euro Area, respectively. Correlation coefficients of 0.828 (for the United States) and 0.712 (for the Euro Area) in both cases reveal the strong co-movement between household expenditure and GDP in both economic regions. The timespans cover the period from 1971 to 2014 for the United States, while the Euro Area data are limited to 1996 to 2014 because of partial data availability.

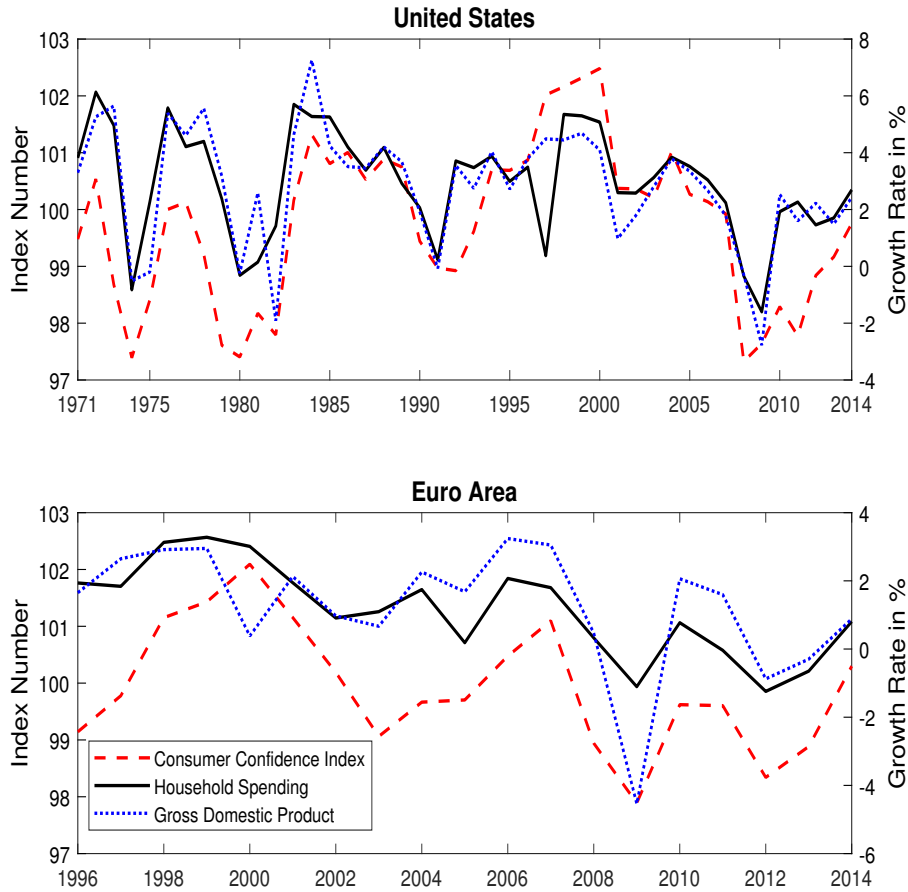
Empirical evidence also suggests a significant relationship among consumer confidence, household expenditure, and GDP. Figure 1 compares the annual growth rates of household expenditure (OECD (2016a)) and GDP (OECD (2016c)) with the index numbers of the consumer confidence index (CCI henceforth; OECD (2016b)) for the US economy and Euro Area. Since the time series of the CCI consists of monthly data, we calculate and report the annual index numbers on average.<sup>1</sup>

Two observations are worth mentioning. First, we find a strong correlation between the time series. While we have already mentioned the strong correlations between household expenditure and GDP, the same is indeed true for the relationship between the CCI and household expenditure as well as GDP. The correlation coefficients for the US economy (upper panel of Figure 1) are 0.662 (CCI to household expenditure) and 0.659 (CCI to GDP). For the Euro Area (lower panel of Figure 1), the coefficients are 0.841 (CCI to household expenditure) and 0.645 (CCI to GDP). This is in line with De Grauwe (2012), who reported a correlation coefficient of 0.600 for the index numbers of the Michigan Consumer Confidence Indicator and US output gap growth rate from 1970 to 2009. For the US economy, the co-movements in all the indicators becomes apparent, especially in times of economic slowdown given by very low or even

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<sup>1</sup>The CCI is provided on a monthly basis by "The Conference Board", a US non-profit business membership and research group. A survey of 5,000 households in the United States and Euro Area, respectively, consists of five questions, each related to current and future business and employment conditions as well as the prediction of future household income. Answers can be positive, neutral, and negative. The index values are calculated based on the relation of each question's positive responses to the sum of its positive and negative responses. For more information, see <https://www.conference-board.org>.

**Figure 1:** Development of the CCI, total household expenditure, and GDP for the United States and Euro Area



Source: OECD, authors' own calculations.

Note: The solid, dashed, and dotted lines depict the growth rates of total household expenditure, the CCI, and GDP, respectively. Realizations are given as an annual magnitude. The CCI numbers are calculated on average based on the underlying monthly series. The latter is amplitude-adjusted with a long-term average of 100. Household expenditure is measured as the annual growth rate and GDP growth as the percentage change from the previous quarter and from the same quarter of the previous year, respectively.

negative growth rates. For example, this holds for both oil crises in 1973 and 1979, at the beginning of the Great Moderation period around 1980, during the Gulf War in 1990/91, the bursting of the dot-com bubble in 2001, and, most clearly, in the case of the bursting of the US housing bubble in 2007. In this regard, we observe a (discontinuous) upswing in the CCI during the Great Recession period (i.e., from 2008 onward). This can be explained by the central bank's zero lower bound interest rate, which might have stimulated consumption. We can make similar comments on the observations for the Euro Area.

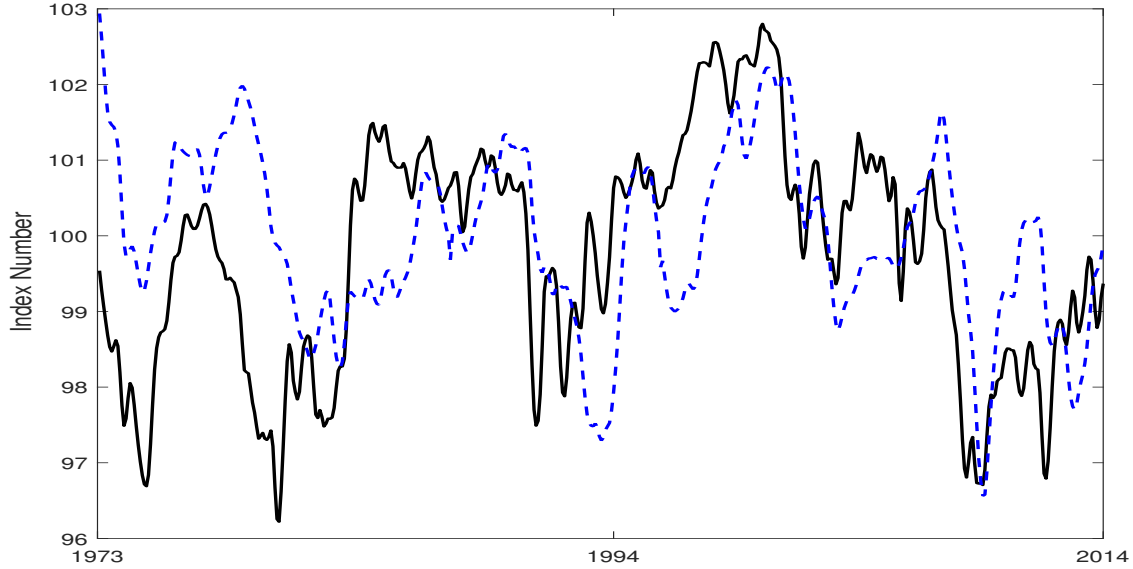
Second, the CCI and household expenditure move closely together. For example, in the Euro Area, both time series behave synchronously with one-to-one overlapping peaks and troughs. This holds primarily in more recent periods such as 2009 and at the peak of the sovereign debt crises from 2010 to 2012. This gives rise to the question of whether the CCI can be identified as a leading indicator used by governments and firms to determine economic policy and business decisions, respectively. Cross-correlation patterns for both the United States and the Euro Area, however, reveal that the highest correlation coefficients between the CCI and household expenditure (given by the values above) appear at the zero lag. This finding suggests that ups and downs in household expenditure do not follow those of the CCI but rather overlap at the same point in time.<sup>2</sup>

Overall, these strong and contemporaneous co-movements in the time series suggest that consumer confidence plays a crucial role in the determination of household expenditure and the pass-through to GDP fluctuations. This view is not entirely new. For example, using a bounded-rational macroeconomic model, Milani (2014) found that so-called sentiment shocks (in terms of shifts from optimistic to pessimistic expectations and vice versa) account for roughly 40 percent of GDP fluctuations in the US economy. Golinelli and Parigi (2004) focused on the forecast performance of consumer sentiment. They showed that a restricted VAR model with consumer confidence outperforms an unrestricted model without one. Similarly, Sacht (2015, p. 13) pointed out that the recovery of the Spanish economy since 2014 has been mainly grounded on the reversal in consumer confidence from a pessimistic to an optimistic view on future economic developments. Van Aarle and Moons (2017, p. 242) showed that economic activity is caused by confidence in retail sales in the Euro Area. Evidence of Granger causality between household expenditure and consumer confidence was also reported by Dees and Soares Brinca (2013) for both the United States and the Euro Area. The authors claimed that the predictive power of consumer confidence on household expenditure is increasing in periods such as the Great Recession for which large fluctuations in the corresponding index numbers are observed. This ties in with the analysis of Fuhrer (1993). He showed that the predictive power of a forecast activity is statistically significant but modest.

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<sup>2</sup>Potential properties for consumer confidence cannot be entirely ruled out. Cross-correlation values of 0.631 (for the United States) and 0.446 (for the Euro Area) at the first lag in the CCI imply that the latter indeed exhibits some explanatory power on contemporaneous household expenditure. This is in line with the findings of Dees and Soares Brinca (2013) among others.

**Figure 2:** Development of the CCI in monthly magnitudes for the United States and the Euro Area (1973:M1–2014:M1)



*Source:* OECD

*Note:* The *solid* and *dashed* lines depict the index values of the CCI in monthly magnitudes for the United States and Euro Area, respectively. For a clear arrangement, on the x-axis, we indicate only 1973, 1994, and 2014, which are recorded as the corresponding values in January.

However, it is not trivial to understand the psychological concepts behind the establishment of confidence as a driving force of real economic fluctuations. This holds especially with respect to a theoretical foundation. In this study, we try to bridge the gap between the empirical observations and model-based expectations. First, we examine the degree of autocorrelation of the CCI time series, as consumer confidence exhibits a high degree of persistence, which contributes to an increase in confidence itself according to a backward-looking expectations formation scheme. Intuitively, the latter should account for the high degree of inertia in the time series of household expenditure for the strong correlation between the two indicators being observed. In this regard, our emphasis is on the lag behavior of the CCI.

Figure 2 depicts the development of the CCI for the United States and Euro Area. The data are now given as a monthly magnitude (OECD (2016b)). Since the data for the Euro Area are only available from January 1973, we focus on the period starting in this month until January 2014.<sup>3</sup> As the amplitude of

<sup>3</sup>Different timespans are used in Figure 1 because the information on household expenditure and GDP growth provided by the OECD is not available before 1971 for the US economy and 1996 for the Euro Area.

the time series is adjusted with a long-term average of 100, index values above (below) that value indicate an increase (decline) in consumer confidence in future economic developments. Indeed, the index values fluctuate around the long-term trend. Most importantly, the degree of autocorrelation at the first lag is 0.988 and 0.983 for the United States and Euro Area, respectively. This finding strengthens our view that consumer confidence is highly persistent over time.<sup>4</sup> We claim that the persistence in the CCI contributes to the (moderate) degree of autocorrelation in the growth rates of household expenditure given by 0.412 (for the United States) and 0.639 (for the Euro Area). This statement relies on our observations (and those found in the literature) of a strong and contemporaneous relationship between consumer confidence and household expenditure. In the following section, we account for this empirical evidence by considering a set of forecasting rules (i.e., heuristics) that account for the aspects of backward-looking expectations formation.

## 4 Expectations Formation in the Baseline NKM

### 4.1 New-Keynesian Workhorse Model

The baseline NKM in its hybrid variant reads as follows:

$$c_t = \frac{1}{1+\chi} \tilde{E}_t^j c_{t+1} + \frac{\chi}{1+\chi} c_{t-1} - \tau(r_t - \tilde{E}_t^j \pi_{t+1}) + \varepsilon_{c,t} \quad (1)$$

$$\pi_t = \frac{\nu}{1+\alpha\nu} \tilde{E}_t^j \pi_{t+1} + \frac{\alpha}{1+\alpha\nu} \pi_{t-1} + \kappa c_t + \varepsilon_{\pi,t} \quad (2)$$

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)(\phi_\pi \pi_t + \phi_c c_t) + \varepsilon_{r,t} \quad (3)$$

$$c_t = y_t \quad (4)$$

where the superscript  $j = \{\text{RE}, \text{BR}\}$  refers to the rational expectations (RE) and the bounded rationality (BR) model specifications, respectively. The corresponding expectations operator is  $\tilde{E}_t^j$ , which has to be specified for both models in quarterly magnitudes. All the variables are given in gap notation, i.e.,  $s_t = \hat{s}_t - \bar{s}$  holds, where we consider the deviation in the contemporaneous realization of the variables from their steady-state values denoted by  $\hat{s} = \{\hat{c}, \hat{\pi}, \hat{r}\}$  and  $\bar{s} = \{\bar{c}, \bar{\pi}, \bar{r}\}$ , respectively. In the following, we omit the expression "gap" to provide a clear arrangement if not necessary otherwise.

In equation (1), private consumption expenditure stems from the intertemporal optimization of consumption and saving, which leads to consumption smoothing (based on the realizations of the real interest rate gap denoted by  $r_t - \tilde{E}_t^j \pi_{t+1}$ ). The parameter  $\tau \geq 0$  denotes the inverse intertemporal elasticity of substitution in consumption behavior. Equation (2) represents the NKPC, where aggregate consumption ( $c_t$ ) acts as the driving force of inflation ( $\pi_t$ ) dynamics due to monopolistic competition and the Calvo-type sticky price setting scheme. The

<sup>4</sup>Figure 1 shows that the degree of autocorrelation in the CCI (annual magnitude) amounts to 0.753 (for the United States) and 0.553 (for the Euro Area). The relatively low degree of autocorrelation is explained by the loss of information from the aggregation of monthly data.

slope of the NKPC is given by the parameter  $\kappa \geq 0$ .  $\nu$  measures the discount factor ( $0 < \nu < 1$ ). The microfoundations allow for a hybrid structure of the economy in the demand and supply framework, namely the parameters for habit formation  $0 \leq \chi \leq 1$  and price indexation  $0 \leq \alpha \leq 1$ , respectively. We consider the well-known three-equation representation of the baseline New-Keynesian workhorse model for a closed economy in its log-linearized form. This model specification addresses the so-called the "persistence anomaly" discussed by Chari et al. (2002), who stated that only shock-driven models without intrinsic persistence cannot account for inertia in the data.

According to the ad-hoc Taylor rule with interest rate smoothing given by equation (3), the nominal interest rate gap ( $r_t$ ) is a predetermined variable with the corresponding persistence parameter  $0 \leq \phi_r \leq 1$ . The monetary authority reacts directly to contemporaneous movements in consumption ( $\phi_c \geq 0$ ) and inflation ( $\phi_\pi \geq 0$ ). We assume that the exogenous driving forces of the model variables follow specific shocks  $\varepsilon_{s,t}$  to demand, supply, and the monetary policy instrument, which are independent and identically distributed around mean zero and variance  $\sigma_s^2$  with the indices  $s = \{c, \pi, r\}$ . As already mentioned, as a main characteristic of linearized DSGE models, the dynamics are described by the deviations from the steady states, where consumption expenditure equals output in the equilibrium. Hence, equation (4) implies that equation (1) expresses only the standard dynamic IS curve. That becomes even more apparent as equation (4) stands for the national income identity in the absence of private investment, the trade balance, and government expenditure as assumed for our prototype model here.

The concept of animal spirits is linked to firms' investment decisions, as primarily stated by Keynes (1936). However, based on our argument above, we do not investigate investors' expectations formation in our macroeconomic model for two reasons. First, we present an intuition that the index numbers for consumer confidence serve as a fundamental formulation of consumer forecast heuristics. The focus is therefore more on households' behavior. Second, investment decisions, if substantial, have an impact on long-run dynamics. Since we focus on the development over the business cycle in the short run under the consideration of stationary steady states, investors do not belong to the group of agents in the economy. Nonetheless, it would be fruitful to shed light on this topic in future endeavors. It follows from this that as we express the dynamic IS equation (1) in terms of the output gap  $y_t$ , we rely on the notation for the consumption gap given by  $c_t$  instead. The reason is that we want to avoid any confusion for the reader, because we focus on consumption and, in particular, consumers' expectations. The appearance of  $c_t$  in the NKPC (2) and Taylor rule (3) is justified from a theoretical point of view under the consideration of the equilibrium condition (4); see Galí (2015, Chapter 3) for more details.

## 4.2 Specifications of the Forecast Heuristics

Under RE, the forward-looking terms include expectations with respect to consumption and inflation at time  $t + 1$  in equations (1) and (2):

$$\tilde{E}_t^{RE} z_{t+1} = E_t z_{t+1} + E_t \tilde{\varepsilon}_{z,t} \quad (5)$$

with  $z = \{c, \pi\}$ .  $E_t$  denotes the expectation operator conditional on the information at time  $t$ . In a stochastic environment, the model includes the random error term  $\tilde{\varepsilon}_{z,t}$ . The latter is independent of the future realizations in  $z$  and has an expected value of zero, i.e.,  $E_t \tilde{\varepsilon}_{z,t} = 0$  holds. According to the RE hypothesis, it is therefore assumed that agents' expectations are not systematically biased and that those agents process all the available information in an optimal way.

Under the BR specification, we distinguish between expectations formation with respect to consumption and inflation. In particular, we apply the heuristics adopted by Gaunersdorfer et al. (2008) and De Grauwe (2011). Regarding consumption expectations, agents can sort themselves into four groups of forecasters expressed through the following heuristics:

$$E_t^F c_{t+1} = \bar{c} + \psi_c(c_{t-1} - \bar{c}), \quad (6)$$

$$E_t^C c_{t+1} = c_{t-1} + \xi_c(c_{t-1} - c_{t-2}), \quad (7)$$

$$E_t^O c_{t+1} = \frac{1}{2}(\beta + \delta \lambda_{c,t}), \quad (8)$$

$$E_t^P c_{t+1} = -\frac{1}{2}(\beta + \delta \lambda_{c,t}), \quad (9)$$

where  $\bar{c} = 0$  holds in the steady state. Equations (6) to (9) reflect consumers' forecast heuristics. The use of the latter is, in general, motivated by the premise that the structure of the economy is observable, whereas the interactions of the relevant variables are barely understandable (Munier et al. (1999)). Individuals recognize the building blocks of the economy (equations (1) to (4)), but expectations can be systematically biased under bounded rationality according to heuristics (6) to (9).<sup>5</sup>

In the absence of the RE hypothesis, two groups of agents apply forecasting rules (6) and (7). These consist essentially of backward-looking elements. We assume that *fundamentalists* (F) and *chartists* (C) display professional forecast behavior (i.e., the absence of emotional states with limited information). Fundamentalists have regressive expectations over the steady-state value  $\bar{c}$  with the speed of convergence given by  $0 \leq \psi_c \leq 1$ . A quick (slow) movement is observed in the case where  $\psi_c$  is close to 0 (1). Chartists form their expectations based on historic patterns in the time series. Given the past realization and relation

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<sup>5</sup>One can hardly argue against this view as it would imply otherwise that, in particular, households do not understand their own consumption reaction function stemming from their corresponding utility maximization approach. If this were true, it displays random agents' decision processes under chaos (which might be considered in future research).

between the first and second lags, this type of agent either extrapolates the last change in  $c_t$  ( $\xi_c > 0$ ) or expects a reversal instead ( $\xi_c < 0$ ). In other words, these heuristics are *technical* in nature.

In addition, with respect to forecasting rules (8) and (9), we follow directly the specifications proposed by Jang and Sacht (2016) to quantify the divergence in beliefs. Here, we assume that agents may adopt either an *optimistic* or a *pessimistic* (indicated by the superscripts  $O$  and  $P$ , respectively) attitude toward movements in future consumption. Hence, both types of agents are uncertain about the associated future dynamics and therefore predict a subjective mean value of  $c_{t+1}$  measured by  $\beta \geq 0$ . However, this kind of subjective forecast is generally biased and therefore depends on the volatility in consumption, which is given by the unconditional standard deviation  $\lambda_{c,t} \geq 0$ . The corresponding parameter  $\delta \geq 0$  measures the degree of divergence in the movement of economic activity. We consider symmetry with respect to  $\beta$  and  $\delta$ : optimists expect that the consumption will differ positively from the steady-state value  $\bar{c}$  given by the value of  $\beta/2$  percent, while pessimists expect a negative deviation of the same magnitude. We refer to these heuristics as *emotional*.

Given the expectations formation process, we identify different scenarios (i.e., the specifications of the model). Hence, the horse race consists of six scenarios, where the corresponding heuristics (5) to (9) are considered. In the first scenario, the model is estimated based on the RE hypothesis according to equation (5) only. The second scenario focuses on the so-called pure technical block (PTB), that is, equations (6) and (7) hold. The third scenario consists of the so-called *pure emotional block* (PEB), where equations (8) and (9) only are applied. As a mixture of both specifications, we introduce the emotional-fundamental block (EFB) and emotional-chartist block (ECB) as the fourth and fifth scenarios, respectively. The former uses heuristics (6), (8) and (9), while the latter consists of (7) to (9). Finally, the sixth scenario is labeled the *cognitive aggregate block* (CAB), where we allow for the existence of all four groups of heterogeneous agents.

These scenarios are useful for comparing the fit of the model specifications with respect to the nature of expectations formation in the spirit of our horse race exercise. In particular, the emotional blocks account for animal spirits (i.e., the waves in optimistic and pessimistic beliefs), while the technical blocks include forecasting rules where the emotional states are absent. By considering these pairs of heuristics, we account for the observations from the previous section. Therefore, we investigate the following three questions. First, does a BR model specification based on heuristics alone account for the inertia in the time series or is a structural representation of an RE model with leads and lags strictly required? Second, related to this, which of the BR scenarios considered could lead to the best description of the data? Finally, which combinations of heuristics could account the most for the high degree of autocorrelation in consumer confidence and hence, expectations?



Under BR, individuals adopt one of the forecasting strategies. Switching from one group to the other is based on discrete choice theory described as follows. The expression for the market forecast regarding consumption across the four groups is given by

$$\tilde{E}_t^{BR} c_{t+1} = \sum_{i=1}^4 (\alpha_{c,t}^{k\{i\}} \cdot E_t^{k\{i\}} c_{t+1}) \quad (10)$$

with  $k = \{O, P, F, C\}$ . The probability  $\alpha_{c,t}^k$  indicates the stochastic behavior of agents who adopt a particular forecasting rule (i.e., of heuristics (6) to (9)). More precisely,  $\alpha_{c,t}^k$  can be interpreted as the probability of being an optimist, pessimist, fundamentalist, or chartist with respect to the development of consumption in period  $t$ . The selection of forecasting rules (6) to (9) depends on the forecast performances of each group given by the mean squared forecasting error  $U_t^k$ . This utility measure for forecast performances can be updated in every period as (cf. Brock and Hommes (1997))

$$U_{c,t}^k = \rho U_{c,t-1}^k - (1 - \rho)(E_{t-2}^k c_{t-1} - c_{t-1})^2 \quad (11)$$

where  $\rho$  measures symmetrically the memory of the four types of agents ( $0 \leq \rho \leq 1$ ). Here,  $\rho = 0$  suggests that agents have no memory of past observations, while  $\rho = 1$  means that they have an infinite memory instead. Under discrete choice, agents can revise their expectations given their forecast performances. The different types of performance measures can then be utilized for  $\alpha_{c,t}^k$  as follows:

$$\alpha_{c,t}^{\tilde{k}} = \frac{\exp(\gamma U_t^{\tilde{k}})}{\sum_{i=1}^4 \exp(\gamma U_t^{k\{i\}})}; \quad (12)$$

$$\alpha_{c,t}^C = \frac{\exp(\gamma U_t^C)}{\sum_{i=1}^4 \exp(\gamma U_t^{k\{i\}})} \cdot \exp \left[ -\frac{(c_{t-1} - \bar{c})^2}{\varpi} \right] \quad (13)$$

where the last part of equation (13) stands for the penalty term (with  $\varpi > 0$ ) that ensures that the deviations in past consumption from its steady-state value remain bounded. The parameter  $\gamma \geq 0$  denotes the intensity of choice. Equations (10) to (13) have to be adjusted conditional on the expectations formation scenario being considered. Of course, the probability of being a fundamentalist is then given by

$$\alpha_{c,t}^F = 1 - \sum_{i=1}^3 \alpha_{c,t}^{\tilde{k}\{i\}} \quad (14)$$

with  $\tilde{k} = \{O, P, C\}$ . Again, according to the different scenarios considered, the specification in equation (14) will differ accordingly.

We distinguish the probabilities of the subgroups  $\tilde{k} = \{O, P\}$  in (12) from the one regarding the chartists  $C$  in (13). In comparison with the groups of optimists, pessimists, and fundamentalists, the forecast heuristic of the chartists

given by equation (7) shows that they react to the historic pattern of consumption by up to two lags but not the steady state. It follows that as the market performance of chartists turns out to be the highest among all groups, this forecasting strategy becomes the dominant one. The expected deviation in past realizations in consumption over two lags then contributes heavily to the volatility of economic dynamics if chartists have an overall higher weight. As a result, an ongoing deviation in consumption from its steady-state value will be observed, which leads to divergent adjustment paths and hence instability. Since we are interested in a convergent solution in favor of future policy analysis, we account for the penalty term in equation (13) only. According to Hommes (2010), the latter can be seen as the transversality condition in the model with heterogeneous agents: large deviations in consumption from its steady-state value result in  $\exp\left[-\frac{(c_{t-1}-\bar{c})^2}{\varpi}\right]$  becoming smaller. This leads to an increase in the attractiveness of the other groups' forecasting heuristics and ensures that "speculative bubbles" cannot last forever (cf. Gaunersdorfer et al. (2008)).

As we will compare the empirical fits of the RE and BR specifications with the data, for the latter, we consider a purely bounded-rational approach (i.e., we also incorporate non-rational expectations formation with respect to *inflation*). The central bank seeks to stabilize inflation via the interest channel of monetary policy. In particular, the monetary authority anchors expectations by announcing a target for inflation given by  $\bar{\pi}$ . Inflation fundamentalists consider this pre-commitment strategy to be fully credible. The corresponding forecasting rule then becomes

$$E_t^F \pi_{t+1} = \bar{\pi} \quad (15)$$

with a target rate for the inflation gap of  $\bar{\pi} = 0$  for simplicity. This implies that the central bank seeks to minimize the deviation in its (realized) target rate of inflation from the corresponding steady-state value. The deviation should then be zero in the optimum. Inflation chartists expect that the future value of the inflation gap is given by

$$E_t^C \pi_{t+1} = \pi_{t-1}. \quad (16)$$

Hence, we adopt the same heuristics with respect to fundamentalists and chartists as before (see equations (6) and (7) above) but with  $\psi_\pi = 0$  and  $\xi_\pi = 0$  instead. We use these constraints on the heuristics to examine the impact of consumer confidence in isolation, while following the description of the so-called inflation targeters and extrapolators imposed by De Grauwe (2011). Equations (10) to (14) have to be adjusted for the inflation expectations formation process. The memory parameter given by  $\rho$  remains the same for consumption as well as for inflation. Further, while we vary the heuristics with respect to consumption expectations, those for inflation are always the same and given by the specifications in equations (15) and (16) under BR.

**Table 1:** Design of the horse race: RE and BR scenarios

Scenario	Heuristics w.r.t. $c_t$	$\alpha_{c,0(-)}^0$	$\alpha_{c,0(-)}^P$	$\alpha_{c,0(-)}^F$	$\alpha_{c,0(-)}^C$	Set of Information w.r.t. $\tilde{E}_t^j c_{t+1}$
RE	(5)	—	—	—	—	$S_1(c_{t+1})$
PTB	(6),(7)	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$S_2(c_{t-1}, c_{t-2}, \bar{c})$
PEB	(8),(9)	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$S_3(\lambda_{c,t})$
EFB	(6),(8),(9)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$S_4(\lambda_{c,t}, c_{t-1}, \bar{c})$
ECB	(7),(8),(9)	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$S_5(\lambda_{c,t}, c_{t-1}, c_{t-2})$
CAB	(6),(7),(8),(9)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$S_6(\lambda_{c,t}, c_{t-1}, c_{t-2}, \bar{c})$

Table 1 shows the design of the horse race. The structure of the baseline NKM is given by equations (1) to (4). For the six scenarios (one for RE and five for BR), we use the heuristics regarding consumption according to the equations mentioned in the second column and in the text. Expectations in the RE case are determined by equation (5). For the BR specifications, equations (6) to (16) are used to simulate consumption and inflation. Some heuristics for  $c_t$  are ruled out according to the scenario being considered. This is also mimicked by the zero entries in the third to sixth columns that display the proportions of the different groups of agents  $\alpha_{c,\tilde{t}}^k$  at the beginning of the estimation. Period  $\tilde{t} = 0(-)$  indicates the point in time before the economy is hit by the shocks. The entries, which are not set to zero, indicate a uniform distribution of proportions a priori. The last column contains agents' information sets (S) at time  $t$  used for their forecasts. For completeness, the latter depend on the proportions  $\alpha_{c,t}^k$  which are computed at the beginning of each period. Again, the heuristics for inflation dynamics remain unchanged and are given by equations (15) and (16) together with  $U_{\pi,t}^F$ ,  $U_{\pi,t}^C$ ,  $\alpha_{\pi,t}^F$  and  $\alpha_{\pi,t}^C$ .

The Appendix explains the SMM approach used to estimate the *structural* ( $\chi, \alpha, \tau, \kappa, \phi_r, \phi_\pi, \phi_c$ ) and *bounded rationality* ( $\beta, \delta, \psi_c, \xi_c$ ) parameters as well as the corresponding *standard deviations* ( $\sigma_c, \sigma_\pi, \sigma_r$ ) of the shocks.

## 5 Empirical Application

In this section, we estimate a hybrid version of the RE model and model under the BR scenarios without the induced backward-looking components (i.e., lags in the dynamic IS curve and NKPC). Hence, the intrinsic persistence parameters  $\chi$  and  $\alpha$  are estimated in the hybrid RE scenario only. For all the BR scenarios, we set  $\chi = \alpha = 0$  instead. This comparison exercise allows us to examine the similarities in and differences between backward-looking expectations formation schemes (via forecast heuristics) as well as the sources of intrinsic persistence (according to consumption habits and price indexation) under RE.

More generally, we circumvent the two criticisms related to the number of parameters to be considered and the purely forward-looking version of the model. First, given the structure of the specifications, 10 (for RE) and 10-12 (for BR) "degrees of freedom" are estimated in the parameter estimates. We use a similar number of parameters to avoid any tautological argument about the performance of these different models in matching the covariance structure given the empirical data.

Second, the backward-looking components play a key role in ensuring that the model dynamics match the persistence in the data being observed. While the BR scenarios already account for the past realization of the variables to be considered in the forecast heuristics, a purely forward-looking RE model exhibits no persistence in the absence of autocorrelated shock processes. Hence, we decide to consider non-autocorrelated shocks only to restrict the number of parameter estimates. As mentioned earlier, the RE model then fails to replicate the empirical data according to the "persistence anomaly". It follows that any statement about a potentially better fit of one or more of the BR model specifications to the data compared with the RE one would again be tautological. We are rather interested in the question of whether there is a need for hybridity in the structure of the model. In other words, do behavioral specifications account for autocorrelated time series alone? Can the backward-looking elements in the hybrid RE model be neglected if the forward-looking terms are described by BR forecast heuristics?

## 5.1 Data

The US dataset is taken from the webpage of the Federal Reserve Bank of St. Louis (<https://fred.stlouisfed.org>). The sample spans 1975:Q1 to 2009:Q4.<sup>6</sup> Inflation is measured using the seasonally adjusted consumer price index with 2009 as the base year. Output is obtained from seasonally adjusted real GDP based on billions of chained 2009 dollars. The effective federal funds rate is used to measure the short-term nominal interest rate in the United States.

We retrieve the Euro Area dataset from the 10th update of the Area-Wide Model quarterly database (<http://www.eabcn.org/page/area-wide-model>; see Fagan et al. (2001)). To be consistent with the timespan for the US economy, the sample covers 1975:Q1 to 2009:Q4 only. The consumption deflator is used to measure inflation in the Euro Area. The short-term nominal interest rate and real GDP are used to measure the gaps in the nominal interest rate and output in the Euro Area. The time series in the Area-Wide Model database have the following abbreviations: consumption deflation = PCD, short-term nominal interest rate = STN, and real GDP = YER. A standard smoothing parameter of  $\lambda = 1600$  is used to estimate the trend of the observed data from the Hodrick-Prescott filter for output, inflation, and the nominal interest rate.

<sup>6</sup>For the robustness of the empirical application with respect to different timespans for the US economy, we consider the period until 2016:Q4. See Section 5.2.3 for details.

According to the equilibrium condition  $c_t = y_t$ , we consider the output gap to be a proxy for the private consumption gap (because of the limited data availability of the latter) in our empirical analysis. This is also in line with our observations on the co-movements discussed in Section 3. In the estimation procedure, we take the model to the second-moment conditions derived from the gaps based on the dataset. The details of the estimation approach can be found in the Appendix.

To interpret the point estimates, we set the intensity of choice parameter  $\gamma$  to 1. This implies that agents sort themselves into one of the groups randomly a priori. Switching behavior remains persistent over time, where switching from one forecast heuristic to the other is smooth and slow in every period. Jang and Sacht (2016) reported broad confidence intervals for the bounded rationality parameters for an almost purely stochastic switching process with  $\gamma = 0.1$ . This observation is reasonable as, if the intensity of the choice parameter is set close to its lower bound, expectations do not influence the contemporaneous realizations of consumption or inflation. Like Jang and Sacht (2016), we also try  $\gamma$  values of 10 and 100, which imply strong switching processes. Neither calibration provides a good approximation of the data-generating process and therefore both are ruled out here. These results are available upon request. In general, given the highly non-linear structure of the BR model specification, pinning down the switching parameter becomes difficult (cf. Kukacka et al. (2018)).

Across all the empirical applications, the parameters of the discount factor  $\nu$  and memory parameter  $\rho$  are calibrated to 0.99 and 0, respectively. In the former case, we simply follow the literature, where the same value for  $\nu$  is the estimation result in an overwhelming majority of studies. In the latter case, we refer to our results, which show that this parameter is not significant across all the scenarios estimated here. These results are also available upon request.

Furthermore, we follow Gaunersdorfer and Hommes (2007) and assume that  $\varpi$  is set to 1800, which implies that the penalty term part of equation (13) is close to zero when large deviations in past consumption from its steady-state value occur. This suggests that large deviations in  $(c_{t-1} - \bar{c})$  ultimately result in  $\alpha_{c,t}^C = 0$  to rule out any kind of divergence.

## 5.2 Results

### 5.2.1 US Economy

Table 2 presents the parameter estimates from the US economy for all six scenarios. The values of the quadratic objective function  $J$  can be found in the penultimate row.  $J$  measures the degree of matching of the simulated time series to the empirical ones according to equation (A4) in the Appendix. This suggests that the lower the value of  $J$ , the better the fit of the model to the data will be. In addition, we report the  $p$ -value which reflects the critical value for the respective distributions.

**Table 2:** Estimation results based on  $US$  data (RE and BR scenarios)

	Hybrid RE	CAB	PEB	PTB	EFB	ECB
$\chi$	1.000 -	-	-	-	-	-
$\tau$	0.032 0.015 - 0.048	0.128 0.000 - 0.285	0.369 0.000 - 1.370	0.184 0.000 - 0.395	0.371 0.222 - 0.520	0.321 0.000 - 0.675
$\sigma_c$	0.554 0.394 - 0.714	0.287 0.000 - 0.626	0.556 0.000 - 1.351	0.408 0.0 - 0.895	0.543 0.267 - 0.818	0.378 0.000 - 0.833
$\alpha$	0.914 0.803 - 1.000	-	-	-	-	-
$\kappa$	0.030 0.019 - 0.040	0.205 0.149 - 0.261	0.216 0.130 - 0.302	0.185 0.140 - 0.229	0.213 0.175 - 0.252	0.243 0.000 - 0.527
$\sigma_\pi$	0.293 0.153 - 0.434	0.253 0.000 - 0.507	0.261 0.060 - 0.461	0.206 0.000 - 0.520	0.240 0.018 - 0.461	0.212 0.000 - 0.432
$\phi_\pi$	1.573 1.000 - 2.228	2.051 1.369 - 2.733	1.918 1.000 - 3.046	2.274 1.295 - 3.254	1.914 1.080 - 2.747	1.946 1.318 - 2.574
$\phi_c$	0.785 0.253 - 1.317	0.363 0.000 - 0.903	0.417 0.000 - 1.157	0.589 0.000 - 1.630	0.709 0.011 - 1.407	0.299 0.000 - 0.867
$\phi_r$	0.831 0.766 - 0.895	0.719 0.478 - 0.959	0.570 0.078 - 1.000	0.781 0.637 - 0.924	0.808 0.660 - 0.956	0.644 0.319 - 0.970
$\sigma_r$	0.464 0.133 - 0.796	0.358 0.000 - 0.862	0.314 0.000 - 0.850	0.221 0.000 - 0.576	0.151 0.000 - 0.417	0.304 0.000 - 0.965
$\beta$	-	3.220 1.474 - 4.967	2.253 0.878 - 3.628	-	3.282 1.598 - 4.967	2.527 0.809 - 4.244
$\delta$	-	0.635 0.000 - 1.755	0.577 0.000 - 1.445	-	0.531 0.000 - 1.550	0.775 0.000 - 2.024
$\psi_c$	-	0.737 0.534 - 0.940	-	0.897 0.564 - 1.229	0.951 0.657 - 1.0	-
$\xi_c$	-	1.373 0.367 - 2.419	-	0.758 -0.362 - 1.878	-	0.435 -0.798 - 1.668
$J$	47.33	42.47	212.40	39.28	43.29	61.28
$p$	0.973	0.989	0.000	0.998	0.989	0.674

*Note:* We use 78 moments (two years) based on the SMM approach. The 95 percent confidence intervals are given with a smaller size. The value of the objective function and  $p$ -value are denoted by  $J$  and  $p$ , respectively. For the hybrid RE, the degrees of freedom for the  $\chi^2$  distribution amount to 68. The 5 percent critical value for 68 degrees of freedom is 88.25. For the BR scenarios, we obtain 66, 67, 68, and 70 degrees of freedom. The corresponding 5 percent critical values are 85.96, 87.11, 88.25, and 90.53, respectively. No memory is assumed in the BR scenarios ( $\rho = 0$ ). The discount factor  $\nu$  is calibrated to 0.99. We set  $\varpi$  to 1800.

In the first step, we compare the fit of the model under RE with those of the others. For the RE model (with respect to both consumption and inflation),  $J = 47.33$  with  $p = 0.973$  holds. The highest values of  $J$  are observed for the ECB and PEB scenarios with 61.28 and 212.40, respectively. This is also confirmed by looking at the corresponding  $p$ -values for both cases, which are 0.674 and 0, respectively. The equality in the matching performance of all the remaining BR scenarios in relation to that under RE is remarkable. Therefore, the  $J$ -values for the PTB, CAB, and EFB are given by 42.47, 39.28, and 43.29, respectively. The corresponding  $p$ -values also turn out to be high, around unity.

The performance of these BR scenarios exhibit an equal fit to the data as that under RE. This finding confirms the results discussed in Jang and Sacht (2016). We infer from this that the forward-looking components do not play a crucial role in describing consumption expenditure and macroeconomic dynamics in general. In addition, the forecast heuristics considered in these BR scenarios provide novel insights into backward-looking behavior in the absence of rational expectations compared with simple habit formation and price indexation being assumed to hold.

In the second step, the question then arises: which kind of BR scenario should the interested researcher consider for her or his explorations? We identify the EFB with  $J = 43.29$  as the most promising scenario to be assumed for the specification of the model to hold. Here, consumers sort themselves into the groups of optimists, pessimists, and fundamentalists. While the overall market forecast is heavily grounded on their emotional state, one-third of the population (a priori) prefer a rule-of-thumb with respect to the fundamental value of  $c$  (i.e., consumption in the steady state instead). As mentioned before, the PTB and CAB scenarios provide a similar quality fit to the data. However, as shown in Table 2, most of the estimated parameters in both alternative scenarios are non-significant (five in the PTB and six in the CAB but only two in the EFB). In general, a model specification in which most of the parameters (especially  $\tau$  and  $\kappa$ ) are estimated to be non-significant does not provide reliable or practical suggestions for policymaking. Hence, we consider the EFB scenario to be our preferred choice across the BR specifications.

We now examine the corresponding parameter estimates for the EFB scenario. The estimated values for  $\sigma_r$  and  $\delta$  are not significant. Hence, we cannot rule out the possibility that the variance in the nominal interest rate shock as well as the degree of divergence are equal to zero. In the former case, the Federal Reserve Bank's monetary policy strategy does not rely on exogenous disturbances. In the latter case, consumers' subjective forecasts may become unbiased with respect to the unconditional standard deviation in consumption expenditure. The pass-through of changes in the real interest rate on consumption dynamics (measured by  $\tau$ ) and consumption on inflation dynamics (measured by  $\kappa$ ) turn out to be 0.371 and 0.213, respectively. These parameter estimates are higher than those reported in the majority of studies that investigate the estimation of RE models. However, the estimated value for  $\kappa$  of 0.213 suggests that

prices remain unchanged for about eight months on average, which is in line with the empirical microevidence provided by Nakamura and Steinsson (2008).<sup>7</sup>

The standard deviations  $\sigma_c$  and  $\sigma_\pi$  are 0.543 and 0.240, respectively, which show moderate exogenous shocks to consumption and inflation. With respect to the monetary policy parameters, the Federal Reserve Bank follows an ambitious strategy of inflation stabilization ( $\phi_\pi = 1.914$ ), while it also reacts strongly to consumption movements ( $\phi_c = 0.709$ ). The Taylor rule exhibits a large degree of interest rate smoothing ( $\phi_r = 0.808$ ).

The peculiarity of bounded rationality can be measured by the parameter  $\beta$ , which is estimated to be 3.282. This result suggests that optimistic consumers in the United States believe in a deviation in the future value of consumption from its steady state of around  $1.6 (= \beta/2)$  percent on average and upward. Owing to the symmetry of the heuristics applied, pessimists assume a negative deviation of around minus 1.6 percent on average and downward over the underlying time interval.

The estimation result for the speed of convergence reveals almost a corner solution with  $\psi_c = 0.951$ . According to the corresponding heuristic, equation (6) collapses to  $E_t^F c_{t+1} \approx c_{t-1}$ . The economic interpretation of the results is that US consumers who adopt this rule-of-thumb behavior do not believe in a convergence of the future value in consumption toward its steady state. Instead, they judge the past realization of  $c_t$  with one lag as the most reasonable forecast of this variable. The label "fundamentalists" seems to be misleading in this case, as the expectations formation scheme of this group remains independent of the fundamental value itself but becomes purely backward-looking instead.

As already discussed, our result suggests that the model under the PTB scenario leads to an equal (truly) good fit to the data as in the EFB case. Therefore, for the PTB scenario, it is shown that  $\psi_c$  and  $\xi_c$  are also estimated to be close to one (cf. heuristics (6) and (7) for details). This reinforces our view that a purely backward-looking (or let's say "naive") expectations formation scheme will dominate: as the fitting of the EFB scenario (besides the CAB and PTB) matches that of the RE scenario, rational expectations can be empirically rejected in favor of BR forecast heuristics. The estimates of the hybrid RE model with respect to the intrinsic parameters confirm this statement. These parameters are estimated to be close and even identical to the (theoretical) upper bounds since  $\chi = 1$  and  $\alpha = 0.914$  hold. This finding suggests that the forward-looking components do not play a major role in describing the data. This observation is at odds with the RE hypothesis (i.e., the cornerstone of standard macroeconomic modeling).

<sup>7</sup>To see this, the probability of not adjusting the price must be computed given the explicit expression for  $\kappa$ . The latter is given by  $\kappa = (1-\zeta)(1-\zeta\nu)/\zeta$ , where  $\zeta$  denotes the corresponding Calvo parameter. It follows that  $\zeta = 0.635$  holds. The frequency of price adjustments on a monthly magnitude can then be obtained by applying the formula  $m = 3/(1-\zeta)$  since  $\zeta$  follows a Poisson distribution. We then find a value of  $m = 8.223$ , or roughly eight months.



### 5.2.2 Euro Area

Table 3 presents the results for the Euro Area. As in the United States case, the RE scenario exhibits an almost equal fit to the data as the BR scenarios with  $J = 56.30$  and a  $p$ -value close to unity. This holds except for the PEB scenario, which stands out with a value of  $J$  of 261.71 and a  $p$ -value of zero.<sup>8</sup> For the other BR scenarios, we observe slightly better performances implied by the  $J$ -values of 37.96, 38.15, 42.11, and 46.85 for the PTB, CAB, EFB, and ECB scenarios, respectively. Not surprisingly, in all these cases, the  $p$ -values are also significantly close to unity.

The BR scenario, which we claim is suitable for policymaking, is given by the PTB one. As for the US economy, the associated  $J$ -value is close to that for the CAB scenario. The latter includes all four heuristics for consumers' expectations formation, while the dynamics obtained in the PTB scenario are driven by those for fundamentalists and chartists only. In the PTB scenario, none of the parameters is non-significant. On the contrary, we have three non-significant estimates in the CAB scenario: the one for  $\tau$  as well as the BR ones (i.e.,  $\beta$  and  $\delta$ ). The same is true for the EFB and ECB scenarios in which we find three parameter estimates that are not significantly different from zero. This limits the policymaking use of all the BR scenarios — except for the PTB one. In line with the results for the US economy, scenarios that consist of technical heuristics outperform the PEB one with emotional ones being considered only. Once again, it becomes apparent that the first- and second-order lags in the associated forecast heuristics are non-negligible components in the business cycle.

Now, we turn to the interpretation of the parameter estimates for the PTB scenario. In particular, the parameter estimates of  $\tau = 0.144$  and  $\kappa = 0.152$  suggest lower degrees of pass-through on consumption and inflation than in US case. These values are therefore close to those under RE obtained in various studies. In particular, the degree of observed price stickiness has an average frequency of price adjustments of roughly 9.5 months ( $m = 9.416$ ; see footnote 7). This result is supported by those of Fabiani et al. (2005). The standard deviations of the shock to consumption and inflation are moderate with  $\sigma_c = 0.413$  and  $\sigma_\pi = 0.360$ , respectively. The nominal interest rate shock is estimated to be close to these values with  $\sigma_r = 0.444$ , while the Taylor rule exhibits a much lower degree of persistence with  $\phi_r = 0.426$  than in the US case. With respect to the latter, the European Central Bank reacts less aggressively to changes in consumption ( $\phi_c = 0.325$ ) but strongly (as the US Federal Reserve Bank) to inflation fluctuations ( $\phi_\pi = 1.593$ ).

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<sup>8</sup>It is interesting to compare this result with that presented in Jang and Sacht (2016). That study investigated the empirical performance of RE and PEB only, finding consistently lower (higher) values of  $J$  ( $p$ ) for the PEB scenario. The difference between both observations is heavily grounded on the fact that the authors considered a hybrid specification of the PEB model. In the absence of forecast heuristics with the scope of backward-looking terms in the IS and NKPC, as considered in our study, empirical performance turns out to be worse. This finding strengthens our statements throughout the text.

**Table 3:** Estimation results based on *Euro Area* data (RE and BR scenarios)

	Hybrid RE	CAB	PEB	PTB	EFB	ECB
$\chi$	1.000 -	-	-	-	-	-
$\tau$	0.079 0.022 - 0.136	0.154 0.0 - 0.312	0.321 0.0 - 0.824	0.144 0.005 - 0.284	0.569 0.356 - 0.782	0.319 0.135 - 0.503
$\sigma_c$	0.561 0.430 - 0.693	0.370 0.118 - 0.622	0.471 0.0 - 1.155	0.413 0.206 - 0.619	0.839 0.716 - 0.962	0.474 0.131 - 0.817
$\alpha$	0.765 0.630 - 0.900	-	-	-	-	-
$\kappa$	0.035 0.021 - 0.049	0.159 0.121 - 0.197	0.225 0.082 - 0.368	0.152 0.125 - 0.178	0.166 0.133 - 0.200	0.171 0.111 - 0.230
$\sigma_\pi$	0.275 0.159 - 0.390	0.354 0.172 - 0.536	0.326 0.160 - 0.492	0.360 0.213 - 0.507	0.401 0.214 - 0.587	0.382 0.207 - 0.558
$\phi_\pi$	1.288 1.000 - 1.918	1.612 1.075 - 2.150	1.571 1.0 - 2.151	1.593 1.056 - 2.129	1.262 1.0 - 1.609	1.360 1.0 - 1.816
$\phi_c$	0.497 0.124 - 0.870	0.309 0.026 - 0.593	0.336 0.030 - 0.642	0.325 0.039 - 0.611	0.571 0.345 - 0.797	0.476 0.196 - 0.756
$\phi_r$	0.604 0.479 - 0.729	0.420 0.222 - 0.618	0.381 0.124 - 0.638	0.426 0.229 - 0.623	0.602 0.498 - 0.706	0.505 0.331 - 0.679
$\sigma_r$	0.421 0.072 - 0.769	0.429 0.035 - 0.824	0.288 0.0 - 0.662	0.444 0.078 - 0.809	0.258 0.0 - 0.518	0.331 0.0 - 0.680
$\beta$	-	1.861 0.0 - 4.427	2.162 1.099 - 3.224	-	3.093 0.0 - 6.504	1.509 0.0 - 3.594
$\delta$	-	1.709 0.0 - 4.397	0.591 0.0 - 1.352	-	2.000 0.0 - 5.393	1.225 0.0 - 3.790
$\psi_c$	-	0.570 0.139 - 1.0	-	0.762 0.526 - 0.998	0.945 0.630 - 1.0	-
$\xi_c$	-	1.500 0.683 - 2.316	-	1.010 0.574 - 1.447	-	0.798 -0.132 - 1.728
$J$	56.30	38.15	261.71	37.96	42.11	46.85
$p$	0.844	0.998	0.000	0.999	0.993	0.971

*Note:* See Table 2.

Overall, the estimation results for the BR parameters reveal a high degree of backward-looking expectations formation. Fundamentalists believe in a moderate convergence rate of consumption's future value toward its steady state given  $\psi_c = 0.762$  in contrast to the US case of a purely backward-looking scheme. However, chartists' heuristic for the expectations formation process turns out to be a corner solution with  $\xi_c = 1.010$ : this type of agent purely extrapolates past realizations of consumption; in other words, the corresponding expression  $E^C c_{t+1} \approx 2c_{t-1} - c_{t-2}$  (cf. heuristic (7)) holds.

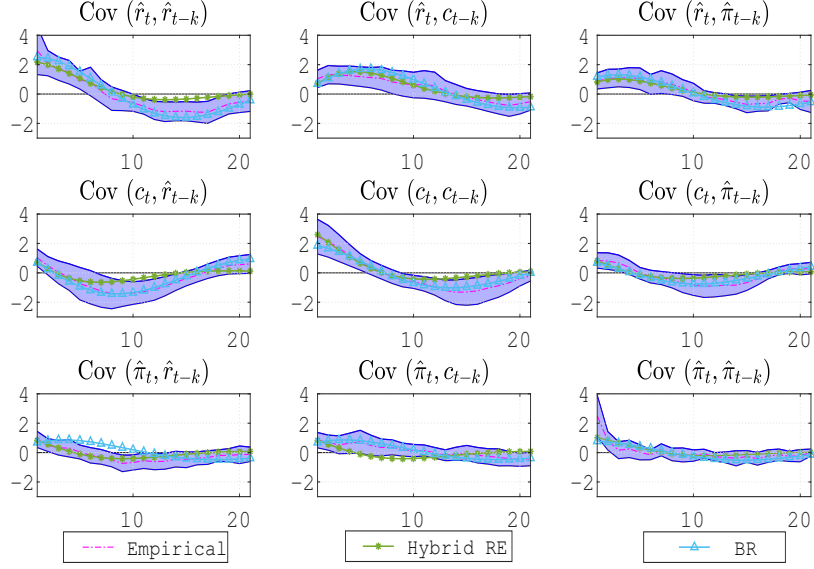
This becomes most apparent as the BR specifications exhibit a significantly better fit to the data than the purely forward-looking NKM. The model under this RE scenario (with  $\chi = \alpha = 0$ ) exhibits the highest  $J$ -values with 213.53 (for the US economy) and 230.49 (for the Euro Area), where all  $p$ -values are equal to zero. The results are available upon request. Hence, when considering the forward-looking elements only, which stem from the RE hypothesis, those are insufficient to establish a good fit to the data. In general, the identified near-optimality of such "naive" expectations is not surprising because the time series for consumption and output display a near-unit root process. This well-known fact has been intensively discussed in the literature (see Lanne (2001), Henry and Shields (2004), and Narayan and Narayan (2010) among others).

To sum up, we support rule-of-thumb behavior for the United States and Euro Area data. Approximately purely backward-looking (instead of rational) expectations formation can be identified as an appropriate choice for modeling consumer confidence and hence forecast expectations. The difference between these economic regions is shown by the influence of emotional states on decision making in the United States compared with the Euro Area, where expectations formation is more technical in nature.

The remarkable goodness-of-fit of the BR scenarios (i.e., EFB and PTB) indicates that optimism and pessimism as well as the trend following fundamental-oriented forecasting behavior may play a major role in expectations formation. Figure 3 shows the auto- and cross-covariances for US data compared to their empirical counterparts. A good fit is mimicked by the observation that the simulated auto- and cross-covariances are placed within the confidence bands of the empirical ones. The hybrid RE model specification is, in the majority of cases, not successful at matching the moments, while the BR one (under EFB) can approximate the empirical moments to a high degree. For example, the RE model fails to generate persistence in the covariance profiles for consumption and inflation. On the contrary, the BR model specification generates persistent behavior for consumption and inflation based on backward-looking expectations.

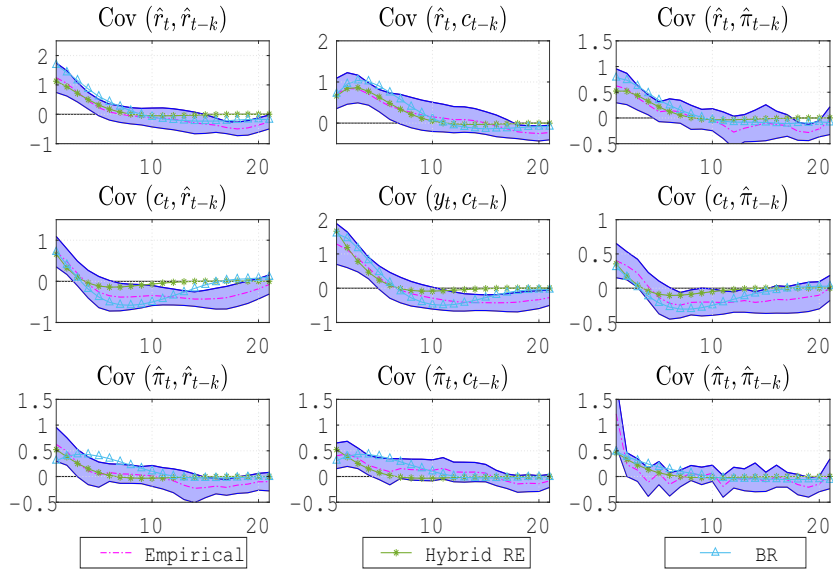
Next, we consider the PTB scenario for the Euro Area data. Figure 4 plots the corresponding simulated auto- and cross-variances, showing that the performances of the RE and BR models are both qualitatively similar to the results based on US data.

**Figure 3:** Model covariance profiles for the United States



*Note:* The estimation results obtained under the EFB are used to simulate the auto- and cross-covariance (designated COV) in the BR scenario.

**Figure 4:** Model covariance profiles for the Euro Area



*Note:* The estimation results obtained under the PTB are used to simulate the auto- and cross-covariance (designated COV) in the BR scenario.

### 5.2.3 Robustness Exercise under Different Monetary Policy Regimes

This section examines the robustness of our empirical application using SMM when the data sample is segmented. We consider several monetary policy regimes for the US economy only: the Great Inflation (GI, 1960:Q1-1979:Q4 with 80 observations), the Great Moderation (GM, 1980:Q1-2009:Q4 with 120 observations), and the Great Moderation and Great Recession (GM+GR, 1980:Q1-2016:Q4 with 148 observations). This can enhance the reliability and sensitivity of the parameter estimates of the models according to these structural changes in the economy. This approach is motivated by the study of Kukacka et al. (2018) who also consider this kind of data segmentation when evaluating a BR model via the SML estimation procedure.

First, we report the summary statistics for the three subperiods with focus on the first-order autocorrelations and variances in consumption, inflation, and the nominal interest rate. The corresponding numbers are given in the third, fifth, and seventh columns of Table 4. The results show that consumption persistence remains similar across the regimes with a value around 0.85 on average. However, among the subperiods, consumption volatility reduced from 3.09 (GI period) to 1.69 (GM+GR period). The same patterns but for both persistence and volatility are observed for inflation and the nominal interest rate. For example, the US economy experienced a high degree of inflation volatility (1.87) in the GI period, while we find a significantly lower value (0.73) for the GM+GR period. These results show high macroeconomic uncertainty in the 1970s and relatively low uncertainty from the 1980s until the 2010s. The US economy has faced large fluctuations in consumption and the nominal interest rate over the past three decades (i.e., in the GM+GR period), while inflation has remained relatively moderate.

To investigate which of the two model specifications is most suitable to capture the observations in persistence and volatility, we first estimate them using SMM. In particular, we once again pin down the parameter estimates of the hybrid RE and BR model specifications. For the latter, we consider the EFB scenario based on our discussions in this section so far for the US economy. Although we do not present the parameter estimates here in great detail (available upon request), few of them (about one to three) are not significant among the six estimations. The entries in last two rows of Table 4 reveal that the BR model specification exhibits a better fit to the covariance structure of the economy than the RE one. This finding holds for all the subperiods according to the low  $J$ -value for the EFB scenario.

While the overall superiority of the BR model specification in fitting the data is apparent, there is no clear matching of persistence and volatility. Based on our empirical results, we calculate the simulated values for autocorrelation and variance in Table 4 and compare them with their empirical counterparts. The results suggest that consumption persistence is in favor of the BR model specification, as the corresponding values are close to the empirical ones. The

**Table 4:** Empirical vs. simulated first-order autocorrelations and volatility

		GI: US (80)			GM: US (120)			GM+GR: US (148)		
		<i>Empirical</i>	RE	EFB	<i>Empirical</i>	RE	EFB	<i>Empirical</i>	RE	EFB
$c_t$	AR (1)	<i>0.83</i>	0.87	<b>0.84</b>	<i>0.86</i>	0.70	<b>0.85</b>	<i>0.87</i>	0.72	<b>0.86</b>
	Volatility	<i>3.09</i>	<b>3.01</b>	3.24	<i>2.00</i>	<b>1.71</b>	1.33	<i>1.69</i>	<b>1.47</b>	1.06
$\pi_t$	AR (1)	<i>0.50</i>	0.71	<b>0.64</b>	<i>0.49</i>	<b>0.86</b>	0.90	<i>0.44</i>	<b>0.80</b>	0.89
	Volatility	<i>1.87</i>	<b>2.10</b>	1.07	<i>0.80</i>	<b>0.65</b>	0.37	<i>0.73</i>	<b>0.50</b>	0.32
$r_t$	AR (1)	<i>0.82</i>	0.71	<b>0.89</b>	<i>0.67</i>	<b>0.94</b>	<b>0.95</b>	<i>0.67</i>	<b>0.94</b>	<b>0.95</b>
	Volatility	<i>2.88</i>	2.25	<b>2.35</b>	<i>2.91</i>	<b>2.14</b>	1.54	<i>2.38</i>	<b>1.87</b>	1.12
SMM	$J$	—	67.38	<b>36.72</b>	—	<i>48.84</i>	<b>40.05</b>	—	49.89	<b>44.07</b>
	$p$	—	0.498	0.999	—	0.962	0.997	—	0.951	0.989

*Note:* The expressions AR(1) and Volatility denote first-order autocorrelation and variance, respectively. In the simulation exercise, AR(1) and Volatility are calculated using 1,000 simulated series. The underlying parameter estimates for the RE and BR model specifications (EFB scenario) are available upon request. The closest matches of the simulated observations to the empirical observations are given in bold type, as are the lowest values of the objective function  $J$  under SMM.

opposite holds for consumption volatility, where the RE model specification outperforms the BR one in terms of matching. The same pattern holds for inflation persistence and volatility in the GI period. However, for the other two periods, the RE model is the most suitable at capturing both inflation persistence and volatility. A mirror image of this observation is obtained for the nominal interest rate. Here, the BR model specification exhibits the best matching to the values of interest in the GI period. In the GM and GM+GR periods, the autocorrelation values do not differ significantly for either model specification. In terms of volatility, the RE model specification is the most appropriate framework for describing this stylized fact.

Based on this robustness exercise, we can address two important issues. Given the different policy regimes for the US economy, our previous statement on the equivalency of the BR model in terms of fitting the data holds. This kind of model even significantly outperforms the RE model in the GI period, which can be seen from a comparison of the corresponding J-values. Furthermore, the mixed results for matching the empirical values for persistence and volatility reveal that the BR model structure has to be adjusted accordingly. This holds especially with respect to (almost) all kinds of volatility as well as inflation persistence for the GM and GM+GR periods.

## 6 Conclusion

In this study, we empirically examine the baseline NKM with heterogeneous agents who may adopt various heuristics to forecast future movements in consumption. In this framework, consumption expectations result from a discrete choice in the specific heuristics. Based on their favorable expectation scheme, agents sort themselves into the four groups of optimists, pessimists, fundamentalists, and chartists, who adopt backward-looking expectations formation using rule-of-thumb behavior. The competing non-linear specifications of the BR model are estimated using the SMM approach. Then, we seek to pin down the most appropriate set of forecast heuristics that can better fit the empirical data. We argue that this study contributes to the estimation of the BR models used in macroeconomic research.

The "wilderness" (Sims (1980)) of the bounded rationality problem makes the formulation of this kind of model a non-trivial one. This statement relies on the fact that agents' ability and willingness to possess information pose challenges for modeling underlying neurological processes. According to experimental studies, for example, Kryvtsov and Petersen (2013) and Pfajfar and Zakelj (2014) found strong evidence for a backward-looking expectations formation scheme. These authors reported that 10 to 15 percent of subjects adjusted their expectations based on previous experiments, while 25 to 35 percent considered decision rules with trend extrapolation. Regarding expectations formation and forecasting, it is not only important to investigate how corresponding heuristics are conducted but also whether those are empirically relevant. While some kind

of wilderness manifests itself in a high degree of freedom to develop BR models, this study focuses on modeling and validating consumers' expectations along the emotional and technical dimensions.

As the most interesting result, we favor the BR model specification, which is as good as the RE one with leads and lags. This finding is also confirmed by a robustness exercise that considers different monetary policy regimes in the United States. Furthermore, the corresponding heuristics exhibit a high degree of backward-looking behavior. The collapse of these expressions into corner solutions with the associated parameters is estimated to be close to unity. Our results suggest that the BR models for forecast heuristics developed in this study can be regarded as an alternative explanation of the expectations formation process in terms of intrinsic persistence. This observation questions the need for a hybrid specification of DSGE models under a purely RE formation scheme. While the standard NKM under RE allows for sound mathematical tractability (especially for economic forecasting), our contribution offers new insights into human decision making and behavior. It is therefore worthwhile studying the impact of shocks in a bounded-rational environment and emphasizing the different adjustments over the business cycle when agents process information in a rational manner only.

We interpret our results such that expectations formation in the United States is grounded on agents' emotional state (with respect to optimism and pessimism), while for the Euro Area it is most likely to be technical in nature (with respect to fundamentalists and chartists). By sorting agents into one of these groups, however, we cannot conclude that they act entirely emotionally or professionally (in terms of the forecasting techniques adopted by this group). In practice, consumers' expectations are influenced by many factors that might not be fully captured by the heuristics considered in this study. More precise rules of consumption behavior should thus be assessed using survey data. This calls for an empirical validation of the feedback effects between business cycles and economic behavior in future studies.

Our observation reveals that for policymaking, a BR model serves as a reliable alternative approach. The question then arises of whether the results obtained through our horse race exercise are sufficient to examine different heuristics for bounded rationality. Similarly, such an experimental setting should account for agents' confidence in investment decisions as well. Therefore, additional moment conditions besides the auto- and cross-covariance profiles (e.g., the raggedness of the time series; see Franke (2018) for more details) can be considered when estimating DSGE models using the SMM approach.

Future attempts should also focus on the relationship between forecast heuristics and the stability of the model. In general, stability analysis in any kind of model in which highly non-linear heuristics are considered is crucial for conducting reliable policy operations — especially in macroeconomic models that focus on optimal monetary and fiscal policy. Future attempts might also shed



light on the role of bounded-rational expectations formation when the central bank operates a zero lower bound interest rate. This step must accompany a reformulation of the NKPC and Taylor rule (e.g., as presented in Galí (2015) and Cochrane (2017)). We leave all these important topics to future research.

## Appendix: The SMM Approach

In this study, we seek to match the model-generated covariances across all the scenarios of consumption, inflation, and the nominal interest rate (all in gap notation) with their empirical counterparts. Statistical inference on market behavior is based on these model parameter values. The parameter estimates are considered to be the result of minimizing the distance between the model generated and empirical second moments in SMM. The moments include the variances of the model's variables (i.e., their absolute volatility), while pro- and countercyclical movements in the different aggregates are captured by (not only the first-order but) an entire profile of the auto- and cross-covariances. Hence, we can use all the (unconditional) co-movement statistics characterized by the estimation. See Franke et al. (2015) for a detailed introduction to this method and Jang and Sacht (2016) for an application to a DSGE model under bounded rationality.

More generally, the moment conditions account for the distributional properties of empirical data  $X_t$  with  $t = 1, \dots, T$ ; where  $T$  denotes the sample size. The sample covariance matrix at lag  $k$  is defined by:

$$m_T(k) = \frac{1}{T} \sum_{t=1}^{T-k} (X_t - \bar{X})(X_{t+k} - \bar{X})' \quad (\text{A1})$$

where  $\bar{X} = (1/T) \sum_{t=1}^T X_t$  is the vector of the sample mean. The average of discrepancy in the sample between the model-generated and empirical moments is denoted as

$$g(\theta; X_t) \equiv \frac{1}{T} \sum_{t=1}^T (m_T^* - m_T) \quad (\text{A2})$$

where  $\theta$  is an  $l \times 1$  vector of unknown structural parameters.  $m_T^*$  and  $m_T$  are the *empirical* and *model generated* moment functions, respectively (cf. equation (A1)).

The main goal of this study is to compare the performance of several behavioral macroeconomic specifications (as described in the previous section) based on observations' auto- and cross-covariances at a (fixed) lag  $k$  with  $k = 0, \dots, n$ . After selecting an appropriate number of  $j$  variables for the lag length, we compute the corresponding  $p$ -dimensional vector of the (empirical and simulated) moment conditions:

$$p = p(k, j) = (j \cdot k - 1) \cdot j. \quad (\text{A3})$$

We avoid double counting at the zero lags in the cross-relationships by considering the term  $(j \cdot k - 1)$ . The Delta method is used to construct a confidence interval for the auto- and cross-covariance moments. The lag length is then given by  $n = k^{max} = 8$  since repeating patterns in the time series do not exhibit additional information, while the model has three variables ( $j = 3$ ). Therefore, we consider  $p = 78$  moments to be an appropriate choice according to equation (A3) and the underlying model structure (see also Jang and Sacht (2016)).

Matching these moment conditions affects both the parameters and the empirical aspects of interest, and we can estimate the model parameters by minimizing the following quadratic objective function:

$$J(\theta) = \min_{\theta} g(\theta; X_t)' \widehat{W} g(\theta; X_t) \quad (\text{A4})$$

where more importance is attached to particular moment conditions according to the weighting matrix  $\widehat{W}$ . The kernel estimator has the following general form with the covariance matrix of the appropriately standardized moment conditions given by

$$\widehat{\Gamma}_T(h) = \frac{1}{T} \sum_{t=h+1}^T (m_T - \bar{m})(m_T - \bar{m})' \quad (\text{A5})$$

where  $\bar{m}$  once again denotes the sample mean. The popular choice of  $h \sim T^{1/3}$  is used to find an appropriate lag length, that is,  $h = 5$  for estimating the covariance matrix in the Euro Area (i.e., the Hansen-White covariance estimator):

$$\widehat{\Omega} = \widehat{\Gamma}_T(0) + \sum_{h=1}^5 \left( \widehat{\Gamma}_T(h) + \widehat{\Gamma}_T'(h) \right). \quad (\text{A6})$$

The weighting matrix  $\widehat{W}$  is computed from the inverse of the estimated covariance matrix  $\widehat{\Omega}$ . However, the estimated covariance exhibits singularity at the point where high correlations between the moment conditions occur. This leads to an increase in the correlation between the moment conditions and weighting matrix. The singularity problem of the covariance matrix is a major issue for small sample data (Altonji and Segal (1996)). To circumvent the econometric issues, we use the diagonal matrix entries as the weighting scheme, while the off-diagonal components of the matrix  $\widehat{W} = \widehat{\Omega}^{-1}$  are ignored.

Now, we examine the properties of the sample distribution for the parameter estimation. In particular, under certain regularity conditions, we arrive at the following asymptotic distribution of the model parameters:

$$\sqrt{T}(\widehat{\theta}_T - \theta_0) \sim N(0, \Lambda) \quad (\text{A7})$$

where  $\Lambda = [(DW D')^{-1}] D' W \Omega W D [(DW D')^{-1}]'$  holds.  $D$  is the gradient vector

of the moment functions evaluated around the point estimates. This can be written as

$$\hat{D} = \frac{\partial m(\theta; X_T)}{\partial \theta} \Big|_{\theta=\hat{\theta}_T}. \quad (\text{A8})$$

Under RE, we obtain the simple analytic moment conditions of the model as described above. However, the analytic expressions of the BR model are not available, because the non-linear structure of the expectations formation process constrains the model dynamics. To circumvent this problem, we simulate the BR model and estimate the behavioral parameters. In particular, SMM is suited to a situation in which the model is easily simulated by replacing theoretical moments. Then the model-generated moments in equation (A4) are replaced by their simulated counterparts:

$$m_T = \frac{1}{S} \sum_{s=1}^S \tilde{m}_T. \quad (\text{A9})$$

The above equation provides the approximation of the theoretical moments ( $m_t$ ) with the simulated data of  $\tilde{m}_t$ . The simulation size is denoted by  $S$ . Under certain regularity conditions, the SMM estimator is asymptotically normal (Duffie and Singleton (1993), Lee and Ingram (1991)):

$$\sqrt{T}(\hat{\theta}_{SMM} - \theta_0) \sim N(0, \Lambda_{SMM}), \quad (\text{A10})$$

where  $\Lambda_{SMM} = [(B'WB)^{-1}]B'W(1 + 1/S)\Omega WB[(B'WB)^{-1}]'$  holds, i.e., the covariance matrix of the SMM estimates. A gradient vector of the moment function is defined as  $B \equiv E\left[\frac{\partial m_t}{\partial \theta} \Big|_{\theta=\hat{\theta}}\right]$ .

However, the model estimation contains simulation errors, preventing us from accurately approximating the non-linear expectation formation processes in the BR model. Alternatively, we compute the standard errors by using the following steps:

1. The BR model is estimated using a simulation size of  $S = 10$ .
2. The estimation is iterated over 100 times, while different random seeds are used to obtain the point estimates of the model parameters for each iteration.
3. We take 100 estimates to compute the mean and standard error of the parameter estimates.

Indeed, the above iterative method is regarded as equivalent to a single estimation of the model based on a simulation size of 1,000. Note that the iteration approach can take the benefit of low simulation errors.

Finally, we use the  $J$ -test to evaluate the compatibility of the moment conditions:

$$\bar{J} \equiv T \cdot J(\hat{\theta}) \xrightarrow{d} \chi_{p-l}^2, \quad (\text{A11})$$

where  $l$  denotes the number of parameters to be estimated. In general, the  $J$ -statistic is asymptotically  $\chi^2$ -distributed with  $(p - l)$  degrees of freedom. In this study, the lag length for the covariance is set to two years. Hence, the number of moment conditions exceeds the model parameters and we consider this particular case as overidentification.

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