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## A framework to study the role of structural transformation in productivity growth and regional convergence

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**ADB Working Paper Series**

**A FRAMEWORK TO STUDY THE ROLE  
OF STRUCTURAL TRANSFORMATION  
IN PRODUCTIVITY GROWTH  
AND REGIONAL CONVERGENCE**

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**Asian Development Bank Institute**

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**Abstract**

We show that  $\sigma$ -convergence in regional productivity growth can be approximated by  $\sigma$ -convergence in sectoral productivity growth and  $\sigma$ -convergence in structural transformation-led productivity growth. Applying this framework to Japanese prefecture-level data from 1874 to 2008, we find support for substantial convergence effects of structural transformation in the post-WWII years.

**Keywords:** structural transformation, labor productivity, regional convergence, Japan

**JEL Classification:** O40, O10

## Contents

1.	INTRODUCTION .....	1
2.	METHODOLOGICAL FRAMEWORK.....	1
3.	DATA AND EMPIRICAL EVIDENCE.....	7
3.1	Data .....	7
3.2	Some Stylized Facts: Structural Transformation, 1874–2008 .....	7
3.3	Convergence of Labor Productivity, 1874–2008 .....	9
3.4	Productivity Catch-up and Convergence through Structural Transformation .	9
4.	CONCLUSION .....	12
	REFERENCES .....	13
	APPENDIX 1.....	16
	APPENDIX 2: DISTRIBUTION OF THE ADJUSTMENT TERM.....	18

# 1. INTRODUCTION

Structural transformation has been regarded as a key mechanism for aggregate labor productivity growth<sup>1</sup> and convergence in regional labor productivity (Caselli and Coleman 2001; Duarte and Restuccia 2010; Hnatkovska and Lahiri 2012). In a multisector growth framework, a standard shift-share analysis decomposes aggregate labor productivity growth into the contribution of structural transformation (between-sector effect) and the contribution of sectoral productivity (within-sector effect). Even if structural transformation makes a positive contribution to aggregate labor productivity growth, it could also lead to regional divergence in labor productivity if the degree and contribution of structural transformation to economic growth varies across regions (McMillan, Rodrik, and Verduzco-Gallo 2014). In this paper, we offer a new decomposition framework to examine the role of structural transformation in regional convergence by addressing this concern.

We study productivity convergence using the notion of  $\sigma$ -convergence and measure  $\sigma$ -convergence regarding changes in the Gini coefficient for aggregate productivity (the sum of sectoral productivity and structural transformation) over time (O'Neill and Van Kerm 2008). As Yitzhaki (2003) points out, it is difficult to decompose the Gini index of the sum of two random variables unless certain assumptions are met. We derive the conditions under which  $\sigma$ -convergence (changes in the Gini coefficient) in aggregate productivity is closely approximated by a summation of changes in the Gini coefficient for productivity growth through sectoral productivity and changes in the Gini coefficient for productivity growth through structural transformation. We apply this framework to a novel historical data set on sectoral productivity and employment shares (across three sectors: primary, secondary, and tertiary) over ten benchmark years (1874–2008)<sup>2</sup> and across 47 Japanese prefectures. The empirical findings provide evidence that convergence in regional productivity is closely approximated by the sum of  $\sigma$ -convergence through sectoral productivity growth and  $\sigma$ -convergence through the growth led by structural transformation.

The rest of the paper is organized as follows. In Section 2, we describe the methodological framework. Section 3 provides the main findings on the relationship between structural transformation and regional convergence. Section 4 concludes.

# 2. METHODOLOGICAL FRAMEWORK

Consider a framework with three production sectors, primary ( $P$ ), secondary ( $S$ ), and tertiary ( $T$ ), as well as two regions,  $H$  (high productivity) and  $L$  (low productivity).<sup>3</sup> In the context of Japan,  $H$  can be thought of as Tokyo, while  $L$  represents the other prefectures. Production in  $P$ ,  $S$ , and  $T$  takes place in both regions. Labor is reallocated across sectors within each of the regions between two points in time,  $t$  and  $t + 1$ , and  $\theta_{ki}^t$  denotes the sectoral labor share of sector  $i$  in region  $k$  and period  $t$ . Following a variant of the canonical shift-share decomposition methodology (see Fabricant 1942 for the original decomposition, and de Vries, Timmer, and de Vries 2013 and

<sup>1</sup> Structural transformation through resource allocation can significantly impact growth and convergence as labor and other resources move from less productive to more productive sectors (Kuznets 1955).

<sup>2</sup> 1874, 1890, 1909, 1925, 1935, 1940, 1955, 1970, 1990, and 2008 (Fukao et al. 2015).

<sup>3</sup> To convey the main idea, we simplify the framework by considering only two regions. In our empirical analysis, we consider 47 regions (prefectures).

Foster-McGregor and Verspagen 2016 for the variant) we write changes in aggregate labor productivity between  $t$  and  $t + 1$  as follows:

$$\Delta V_k = \sum_{i=P,S,T} (\theta_{ki}^t) (\Delta V_{ki}) + \sum_{i=P,S,T} (\Delta \theta_{ki}) (V_{ki}^t) + \sum_{i=P,S,T} (\Delta \theta_{ki}) (\Delta V_{ki}), \quad (1)$$

where  $V_{ki}$  is the log of labor productivity in sector  $i$  (primary, secondary, or tertiary) and region  $k$ , and  $\theta_{ki}$  denotes the labor share in sector  $i$  in region  $k$ . On the right-hand side of equation (1), we have three terms. The first term shows the contribution of own-sector productivity growth due to capital accumulation, technological progress, or a reduction in the misallocation of resources among firms within a sector. The second term represents the static effect of the reallocation of labor through differences in the sectoral productivity level at the beginning of each period. Finally, the third term measures the covariance effect between the reallocation of labor across sectors and changes in sectoral productivity. The last two terms together measure the contribution of structural transformation to changes in aggregate labor productivity. Thus, productivity growth in region  $k$  (as well as aggregate productivity growth) can be decomposed as follows:

$$V_k^{t+1} - V_k^t = \Phi(WS)_k + \Phi(ST)_k, \quad (2)$$

where  $\Phi(WS)_k$  and  $\Phi(ST)_k$  represent labor productivity growth in region  $k$  due to within-sector productivity growth and structural transformation, respectively.

Next, to examine the mechanism through which structural transformation is linked with productivity growth, we consider the term  $\Phi(ST)_k$  from equation (1). By adding a time suffix to  $V(x)_k$ , and after some simple algebraic manipulations, the structural transformation effect is transformed into the sum of two factors:

$$\Phi(ST)_k = (\theta_{kT}^{t+1} - \theta_{kT}^t) (V_{kT}^{t+1} - V_{kP}^{t+1}) + (\theta_{kS}^{t+1} - \theta_{kS}^t) (V_{kS}^{t+1} - V_{kP}^{t+1}). \quad (3)$$

The first term on the right-hand side of equation (3) shows the change in the tertiary sector employment share multiplied by the productivity gap between the tertiary and the primary sector in region  $k$ . Meanwhile, the second term shows the same relationship between the secondary and the primary sector in region  $k$ . Using vector notation, the equation can be rewritten as  $V_k^{ST} = [\Delta \theta_k] \times [PG_k]$ , where  $\Delta \theta_k$  and  $PG_k$  represent the change in the nonprimary sector labor share and the productivity gap between the nonprimary and the primary sector in region  $k$ . If both of these vectors are either positive or negative, the contribution of structural transformation to productivity growth is positive.<sup>4</sup> However, reallocation of labor from the primary sector may lower the aggregate labor productivity level if labor productivity in the primary sector is higher than in the other two sectors. Moreover, if the sectoral productivities are equal, then labor reallocation does not lead to any change in aggregate productivity. The poor region ( $k'$ ) catches up with the rich region through structural transformation ( $k$ ) if  $[\Delta \theta_{k'}] \times [PG_{k'}] > [\Delta \theta_k] \times [PG_k]$ , which shows regional convergence.

<sup>4</sup> McMillan, Rodrik, and Verduzco-Gallo (2014) distinguish between growth-enhancing structural transformation (mostly in Asia) and growth-reducing structural transformation (as seen in many countries in Africa and Latin America).

As suggested by equation (2), in the context of a multisector model for each region or for the whole economy, structural transformation makes a partial contribution to aggregate productivity growth. The contribution of the within-sector effect to aggregate productivity growth is typically larger than that of the between-sector effect (Kaldor 1961; Syrquin 1988; Roncolato and Kucera 2014; Timmer and de Vries 2009).<sup>5</sup> Moreover, structural transformation may not lead to convergence if the degree and contribution of structural transformation to economic growth vary across regions (McMillan, Rodrik, and Verduzco-Gallo 2014). This implies that even if sectoral productivity growth and structural transformation both make a positive contribution to productivity growth, they could work in opposite directions in terms of regional convergence or divergence and hence (partially) offset each other.<sup>6</sup>

Next, let us construct a framework to decompose convergence in regional aggregate productivity into (1) the contribution of convergence in sectoral productivity growth and (2) the contribution of convergence in the growth effect of the reallocation of labor across sectors (structural transformation). To do so, we define  $V_{WS}^{t+1} = V^t + \Phi(WS)$ , where  $V^t$  represents productivity in period  $t$ ,  $\Phi(WS)$  represents the change in productivity due to the within-sector effect, and  $V_{WS}^{t+1}$  represents the hypothetical productivity level in period  $t + 1$  if productivity growth is driven only by the within-sector effect. To simplify our notation, we omit suffix  $k$  when this does not result in confusion. In a similar manner, we define  $V_{ST}^{t+1} = V^t + \Phi(ST)$  when productivity growth is driven only by the between-sector effect (structural transformation). Using the definitions of  $V_{WS}^{t+1}$  and  $V_{ST}^{t+1}$  and equation (2), we can write

$$V^{t+1} - V^t = V_{WS}^{t+1} - V^t + V_{ST}^{t+1} - V^t. \quad (4)$$

We use the Gini coefficient of regional labor productivity to measure regional disparities in labor productivity. In many studies, measures of income inequality are the coefficient of variation of GDP (Friedman 1992) or the standard deviation of log GDP (e.g., Sala-i-Martin 1996). The Gini coefficient is most similar to the variance and shares many properties with it (Yitzhaki, 2003). In addition, as Yitzhaki (2003) shows,

<sup>5</sup> These studies show that 75%–79% of aggregate labor productivity growth is explained by the within-sector effect.

<sup>6</sup> This table compares the link between productivity growth and regional convergence in a one-sector and a multisector model. The left-hand panel shows regional convergence in a one-sector model, while the right-hand panel shows the same in a multisector model (with two sources of productivity growth). The shaded cells show that the net impact on  $\sigma$ -convergence is jointly determined by  $\sigma$ -convergence in sectoral productivity growth and growth from structural transformation when the  $\sigma$ -convergence based on these two factors has the opposite sign.

One-sector model		Multisector model			
<div> <div><math>\sigma</math>-conv</div> <div>YesNo</div> </div>				Sectoral productivity growth (within-sector)	
				$\sigma$ -conv	
				Yes	No
		Structural transformation (between-sector)	$\sigma$ -conv	Yes	Yes
				No	No



the Gini mean difference<sup>7</sup> can be more informative about the properties of distributions that are nearly normal, such as stochastic dominance between two distributions and stratification (when the overall distribution is decomposed into subpopulations). The Gini coefficient of regional labor productivity is written as

$$G(V) = 1 - 2 \int_{\alpha}^{\beta} [1 - F(V)] \frac{V}{\mu} f(V) dx, \quad (5)$$

where  $\mu$  is the mean value of labor productivity ( $V$ ),  $\alpha$  and  $\beta$  are the lower and upper bounds of  $V$ ,  $F$  is the cumulative distribution of  $V$ , and  $f$  is the density function of  $V$ . The Gini coefficient represents the weighted average of mean-normalized productivity ( $\frac{V}{\mu}$ ), where the weights,  $1 - F(V)$ , are determined by the relative rank of each region's labor productivity. By adding a time suffix to  $G(V)$ , changes in inequality between  $t$  and  $t + 1$  can be written as

$$\Delta G(V) = G^{t+1}(V^{t+1}) - G^t(V^t). \quad (6)$$

From equation (4), we can write  $V^{t+1} = V_{WS}^{t+1} + V_{ST}^{t+1} - V^t$ . Based on the properties of the Gini coefficient of the sum of two or more random variables (Yitzhaki 2003),  $G^{t+1}(V^{t+1})$  can be approximated as

$$G^{t+1}(V^{t+1}) = G^{t+1}(V_{WS}^{t+1}) + G^{t+1}(V_{ST}^{t+1}) - G^t(V^t) + \varphi^t, \quad (7)$$

where  $\varphi^t$  denotes the adjustment term of this approximation. The detailed derivation of equation (7) is provided in Appendix 1. If we subtract  $G^t(V^t)$  from both sides of equation (7), we obtain

$$(V^{t+1}) - G^t(V^t) = \{G^{t+1}(V_{WS}^{t+1}) - G^t(V^t)\} + \{(G^{t+1}(V_{ST}^{t+1}) - G^t(V^t))\} + \varphi^t. \quad (7')$$

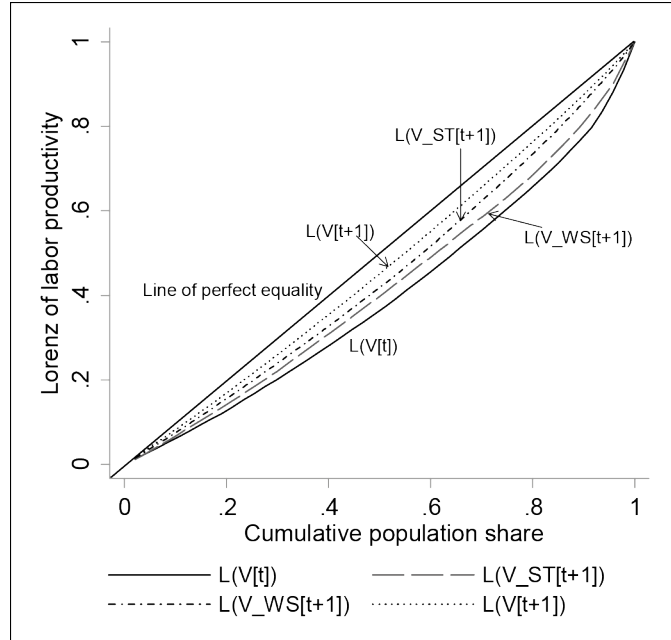
Equation (7') implies that given a smaller value of  $\varphi^t$ ,  $\sigma$ -convergence in labor productivity (a drop in the left-hand side of equation (7')) can be approximated by the net sum of  $\sigma$ -convergence due to the within-sector effect (a drop in the difference in the first two terms on the right-hand side of equation (7')) and  $\sigma$ -convergence due to structural transformation (a drop in the difference in the last two terms on the right-hand side of equation (7')). Figure 1 provides a graphical representation of this argument using some hypothetical Lorenz curves and assuming that the value of  $\varphi^t$  is equal to zero. Using the Lorenz curves of labor productivity,  $\sigma$ -convergence in labor productivity is represented by the area between  $L(V[t+1])$  and  $L(V[t])$ .  $\sigma$ -convergence due to the within-sector effect is represented by the area between  $L(V_{WS}[t+1])$  and  $L(V[t])$ , and  $\sigma$ -convergence due to structural transformation is represented by the area between  $L(V_{ST}[t+1])$  and  $L(V[t])$ .

We next provide a theoretical explanation of the size of the approximation error,  $\varphi$ . In Appendix 1, we show that the magnitude of the approximation error  $\varphi$  becomes large if the Gini correlation coefficients are far from 1. In addition, the size of  $\varphi$  becomes small if the expected values of the four key variables,  $E(V^{t+1})$ ,  $E(V_{WS}^{t+1})$ ,  $E(V_{ST}^{t+1})$ , and  $E(V^t)$ , are similar in magnitude. If these terms differ greatly, then the magnitude of  $\varphi$  becomes large. In order to check how the stochastic dynamic process of these factors affects the

<sup>7</sup> The Gini mean difference and the Gini coefficient are defined as  $G_{MD} = 4Cov(x, F(x))$  and  $G(x) = \frac{Cov(x, F(x))}{E(x)}$ , respectively (where  $x$  is a random variable and  $F$  is the cumulative distribution of  $x$ ). Thus, the relationship between these two terms becomes  $G_{MD} = 4 G(x)E(x)$ .

distribution of  $\varphi$  across different periods, we perform a  $t$ -test of the null hypothesis that  $\varphi = 0$ . Empirically, the value of  $\varphi$  for each period can be calculated for any time period as long as  $G^{t+1}(V^{t+1}) - G^t(V^t)$ ,  $[G^{t+1}(V_{WS}^{t+1}) - G^t(V^t)]$ , and  $[G^{t+1}(V_{ST}^{t+1}) - G^t(V^t)]$  are measured separately. We use these values to test the above hypothesis about  $\varphi$  using the benchmark years from 1874 to 1955 and then annual figures for the rest of the period from 1955 to 2008.

**Figure 1: Lorenz Curves Illustrating the Decomposition of Labor Productivity Growth**



Up to this point, we have mainly focused on  $\sigma$ -convergence. However, as many studies on convergence have shown (e.g., Barro and Sala-i-Martin 1992), analysis based on  $\beta$ -convergence is also useful and provides important insights into the dynamic process of convergence. As a next step, we incorporate the mechanism of  $\beta$ -convergence into our decomposition framework of structural transformation and productivity convergence. Following the lead of Jenkins and Van Kerm (2006) and O'Neill and Van Kerm (2008), we extend the relationship between  $\sigma$ -convergence and  $\beta$ -convergence in the context of a multisector model. We rewrite equation (6) as

$$G^{t+1}(V^{t+1}) - G^t(V^t) = [G^{t+1}(V^{t+1}) - C_t^{t+1}(V^{t+1}, V^t)] - [G^t(V) - C_t^{t+1}(V^{t+1}, V^t)], \quad (8)$$

where  $C_t^{t+1}(V^{t+1}, V^t) = 1 - 2 \int_{\alpha}^{\beta} \int_{\alpha}^{\beta} [1 - F^t(V^t)] \frac{V^{t+1}}{\mu^{t+1}} h(V^{t+1}, V^t) dV^{t+1} dV^t$  is the concentration index (Schechtman and Yitzhaki 2003; Lambert 2001) indicating the distribution of regional productivity levels in period  $t + 1$ , with the regions being arranged according to the productivity ranking in period  $t$ , where  $h$  is the bivariate density function of productivity in periods  $t$  and  $t + 1$ . In general, the concentration index reveals the relationship between two random variables. Unlike the Gini coefficient, which measures the cumulative shares of a variable plotted against the cumulative frequencies of that variable, the concentration coefficient shows the degree of association between two variables, and its value lies in the range  $[-1, 1]$ . Equation (8) shows that changes in the Gini index between two periods can be decomposed into

two factors. The last two terms on the right-hand side of equation (8) show the change in the Gini index caused by productivity catch-up between  $t$  and  $t + 1$  keeping the ranking of the regions as in period  $t$ . We express this part by  $Progress(V^{t+1}, V^t)$ . If the productivity growth of a poorer region is higher than that of a richer region, then the value of  $Progress(V^{t+1}, V^t)$  becomes negative. The first two terms show the change in the Gini index caused by the reranking of regions in terms of the aggregate productivity level. We express this part by  $Rank(V^{t+1}, V^t)$ . If there is no change in the ranking of regions between  $t$  and  $t+1$ , then the value of  $Rank(V^{t+1}, V^t)$  becomes zero. If there is a change in the ranking, then it has a positive value. Therefore,  $Rank(V^{t+1}, V^t) \geq 0$ , implying that the reranking of regions dampens the pace of  $\sigma$ -convergence.

Thus, a change in the inequality of labor productivity ( $\sigma$ -convergence) between two points in time can be decomposed into the effect of productivity catch-up ( $\beta$ -convergence) and the effect of reranking:

$$G^{t+1}(V^{t+1}) - G^t(V^t) = Rank(V^{t+1}, V^t) - Progress(V^{t+1}, V^t). \quad (8')$$

O'Neill and Van Kerm (2008) have shown that  $[G^{t+1}(V^{t+1}) - G^t(V^t)]$  can be interpreted as an indicator of the magnitude of  $\sigma$ -convergence and the term  $Progress(V^{t+1}, V^t)$  can be interpreted as an indicator of the magnitude of  $\beta$ -convergence.<sup>8</sup> Using this decomposition framework, we can find the contribution of  $\beta$ -convergence to  $\sigma$ -convergence net of the reranking of regions.

In a similar manner, we define the concentration index for  $V_{WS}^{t+1}$  as

$$C_t^{t+1}(V_{WS}^{t+1}, V^t) = 1 - 2 \int_{\alpha}^{\beta} \int_{\alpha}^{\beta} [1 - F^t(V^t)] \frac{V_{WS}^{t+1}}{\mu_{WS}^{t+1}} h(V_{WS}^{t+1}, V^t) dV_{WS}^{t+1} dV^t, \quad (9)$$

where  $\mu_{WS}^{t+1}$  is the mean of labor productivity ( $V_{WS}^{t+1}$ ),  $\alpha$  and  $\beta$  are the lower and upper bounds of  $V_{WS}^{t+1}$  and  $V^t$ ,  $F$  is the cumulative distribution of  $V$ , and  $f$  is the density function of  $V$ . The concentration index is a weighted average of mean-normalized productivity ( $\frac{V_{WS}^{t+1}}{\mu_{WS}^{t+1}}$ ), where the weights,  $1 - F^t(V^t)$ , are determined by the relative rank of each region's labor productivity in period  $t$ . Moreover,  $h$  is the bivariate density function of productivity in periods  $t$  and  $t + 1$ . We use  $C_t^{t+1}(V_{WS}^{t+1}, V^t)$  to replicate the decomposition shown in equation (8) for  $G^{t+1}(V_{WS}^{t+1}) - G^t(V^t)$ :

$$G^{t+1}(V_{WS}^{t+1}) - G^t(V^t) = Rank(V_{WS}^{t+1}, V^t) - Progress(V_{WS}^{t+1}, V^t). \quad (10)$$

<sup>8</sup> In the growth literature,  $\beta$ -convergence represents the catching up by poorer regions and  $\sigma$ -convergence shows changes in the dispersion of income across regions. Thus,  $\beta$ -convergence is a necessary but not a sufficient condition for  $\sigma$ -convergence to occur. Using our framework, this can be shown as follows:

No  $\beta$ -convergence and no  $\sigma$ -convergence  $\begin{cases} \text{if } \Delta G(x) = 0 \text{ \& } Progress(x) = 0 \\ \text{if } \Delta G(x) > 0 \text{ \& } Progress(x) < 0 \end{cases}$   
 $\beta$ -convergence but no  $\sigma$ -convergence  $\text{if } \Delta G(x) < 0 \text{ \& } Progress(x) > 0 \text{ \& } |Rank(x)| > |Progress(x)|$   
 $\beta$ -convergence and  $\sigma$ -convergence  $\begin{cases} \text{if } \Delta G(x) < 0 \text{ \& } Progress(x) > 0 \text{ \& } Rank(x) = 0 \\ \text{if } \Delta G(x) < 0 \text{ \& } Progress(x) > 0 \text{ \& } |Rank(x)| < |Progress(x)| \end{cases}$

Intuitively, equation (10) shows the relationship between  $\sigma$ -convergence and  $\beta$ -convergence when  $\Phi(ST) = 0$ . In a similar manner, when  $\Phi(WS) = 0$ , the relationship between  $\sigma$ -convergence and  $\beta$ -convergence can be written as

$$G^{t+1}(V_{ST}^{t+1}) - G^t(V^t) = Rank(V_{ST}^{t+1}, V^t) - Progress(V_{ST}^{t+1}, V^t). \quad (11)$$

With the help of equations (10) and (11), we can separately analyze the contribution of sectoral productivity growth and structural transformation to  $\beta$ -convergence and  $\sigma$ -convergence.

### 3. DATA AND EMPIRICAL EVIDENCE

#### 3.1 Data

The data set on sectoral productivity and employment shares comprises nine benchmark years (1874, 1890, 1909, 1925, 1940, 1955, 1970, 1990, and 2008) spanning almost 135 years. We use three broad sectors of production, primary, secondary, and tertiary, which cover the whole economy. The primary sector consists of agriculture, forestry, and fishery, while the secondary sector consists of mining, manufacturing, and construction. The tertiary sector covers all other sectors. The data on real aggregate labor productivity (calculated as the gross prefectural domestic product over the number of workers) for the period 1874–1940 (in yen) are measured in 1934–36 prices and for the period 1955–2008 (in 1,000 yen) in 2000 prices. For this reason, we do not compare the figures on productivity between 1940 and 1955. By-employment is considered while calculating sectoral employment shares in the postwar period. See Fukao et al. (2015) for a detailed discussion of the data estimation methodology.<sup>9</sup>

#### 3.2 Some Stylized Facts: Structural Transformation, 1874–2008

The process of structural transformation in Japan started during the Meiji era (1868–1912). A number of early initiatives helped the reallocation of labor across sectors. There were the abolition of barrier stations and the caste system (in which society was divided into four classes: samurai, farmers, merchants, and craftsmen) in 1868 and the granting of official permission in 1872 to farmers to engage in commercial activities. Restrictions on the selection of occupation and residence from the Tokugawa period were also removed. In the period from 1874 to 1890, the share of manufacturing activities increased substantially in all prefectures. As we will show later, the national average labor productivity in the secondary sector remained at almost the same level as that in the primary sector. Therefore, it seems that the expansion of the manufacturing sector during this period was mainly driven by the expansion of traditional manufacturing activities such as food processing, wood products, labor-intensive textile production, etc. An important exception was Osaka, where capital-intensive industries such as the heavy chemical industry and the machinery industry started. During the Edo period, Osaka had been the hub of nationwide wholesale and banking networks. In addition, Osaka borders on Kyoto and Hyogo. Kyoto had been Japan's capital until the Meiji Restoration and the center of traditional manufacturing activities. Hyogo had the most important seaport for Japan's imports, Kobe, and import

<sup>9</sup> Detailed descriptions of the data and estimation techniques are available in Fukao et al. (2015). Note that data for Okinawa for the period 1955–1970 are not available.

substitution activities developed around this area. In the case of East Japan, manufacturing activities expanded particularly in the silk-reeling prefectures of eastern Japan (Gunma, Nagano, and Yamanashi).<sup>10</sup> Around this time, new industrialized areas also arose with specializations in heavy industry, machinery, shipbuilding, etc., in Fukuoka, Nagasaki, and Akita, which had international seaports (Fukao et al. 2015).

In addition, traditional manufacturing activities expanded throughout Japan through the abolition of protectionist measures introduced by feudal clans during the Edo period, the expansion of nationwide trade activities, and international trade without tariff autonomy. For example, the traditional production of candles, paper, and salt in Yamaguchi, which was governed by an influential feudal clan during the Edo period, declined substantially through domestic and international competition (Nishikawa 1985). Later on, the turn of the 20th century saw further expansion of high-productivity manufacturing sectors, which were located mainly in the urbanized areas (Tanimoto 1998; Nakabayashi 2003; Nakamura 2010). Heavy manufacturing-based industrialization evolved with the extensive use of electricity, chemicals, metals, and machinery (Fukao et al. 2015). The labor force in the primary sector declined from 15.4 million in 1874 to 13.1 million in 1909. At the same time, the dependency ratio (the ratio of nonworking to working people) rose from 60% in 1874 to 92% in 1909 as a result of significant population growth from 40 million in 1874 to 49 million in 1909.

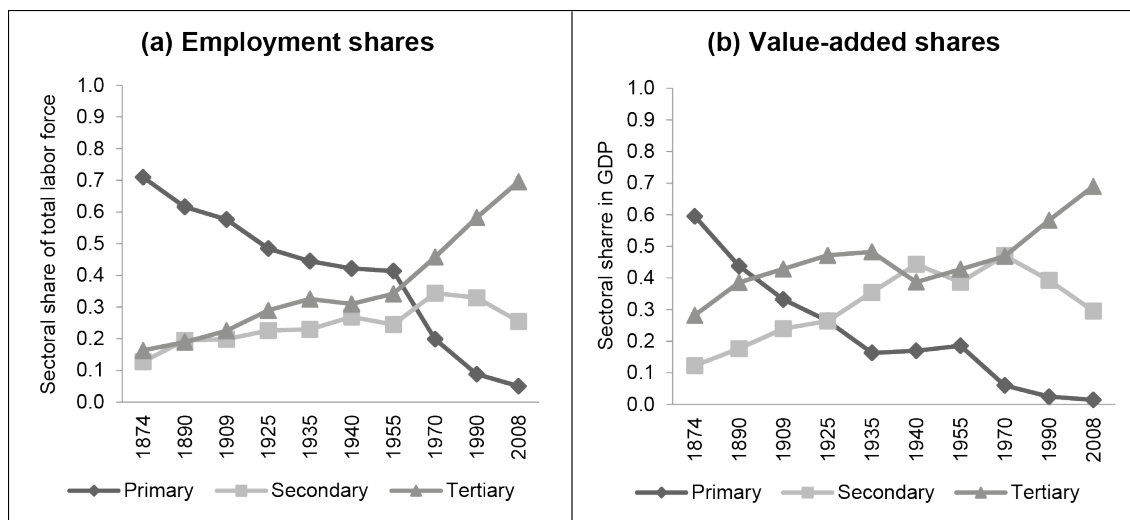
As depicted in Figure 2(a), employment shares in Japan based on labor input data show a steady fall for the primary sector, a steady increase for the tertiary sector, and a hump shape for the secondary sector. Over the 135 years from 1874, the employment share of the primary sector fell from 72% to 5%, whereas that of the tertiary sector rose from 16% to 69%. During the same period, the secondary sector's employment share grew from 14%, peaked at 34% in the 1970s, and then eventually dropped to 26% in 2008. The value-added trends in sectoral shares in GDP (Figure 2(b)) are consistent with the literature on growth and structural transformation in early industrialized countries.<sup>11</sup>

Before we conclude this section, we briefly mention a few factors that have slowed down the labor reallocation process in Japan. One of the factors partly responsible for the slowdown, according to Nakamura (1983), is the opening of new foreign markets for Japanese silk and tea. Saito (1998) showed that the level of income across peasant households wielded a decisive influence on migration as peasants were able to earn from both agriculture and cottage industries that had sprung up in the course of proto-industrialization<sup>12</sup> during the Tokugawa period, which provided less incentive for agricultural workers to reallocate to nonagricultural activities. Other factors that perhaps also contributed to the slow process of structural transformation include institutional barriers related to agriculture (Hayashi and Prescott 2008), the reallocation of capital to war industries and labor to the munitions industry (Okazaki 2016), and cost linkages between inputs and suppliers of inputs between prefectures (Davis and Weinstein 2002).

<sup>10</sup> After the abolition of strict regulations on international trade in 1954, Japan enjoyed comparative advantage in silk products and suffered from a disadvantage in cotton products. Consequently, prefectures that specialized in cotton products—such as Aichi and Osaka—suffered.

<sup>11</sup> See the recent survey by Herrendorf, Rogerson, and Valentinyi (2014).

<sup>12</sup> Proto-industrialization refers to pre-modern industrialization without energy- and capital-intensive modern factories. See Saito (1983) and Smith (1988) for details on proto-industrialization in Japan.

**Figure 2: Structural Transformation in Japan**

Note: By-employment is considered while calculating man-hour input shares. See Fukao et al. (2015) for a detailed discussion on the data estimation methodology. Sectoral shares in GDP are calculated using real GDP in constant 1934–36 prices for 1874–1940 and constant 2000 prices for 1955–2008.

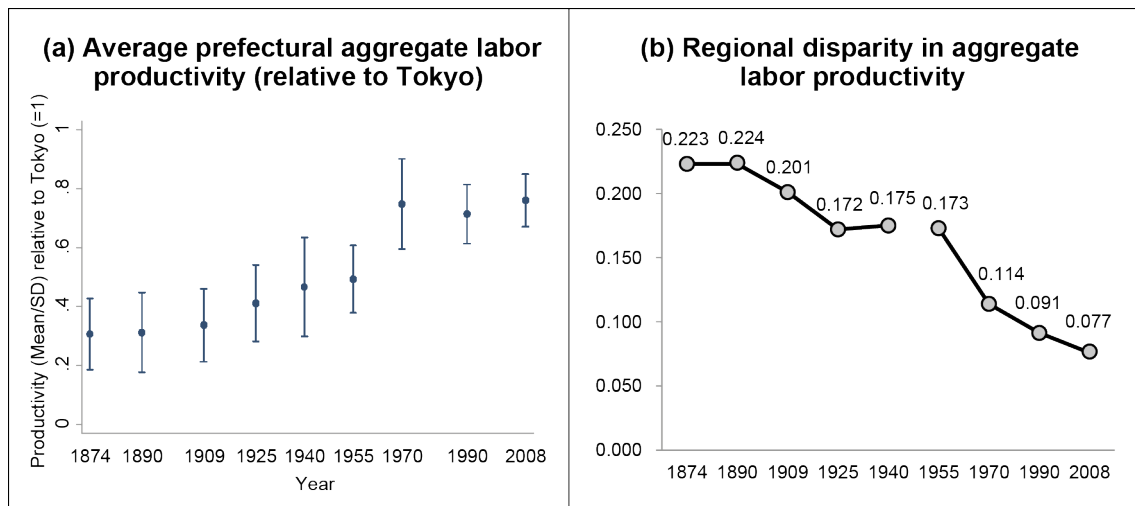
### 3.3 Convergence of Labor Productivity, 1874–2008

Both regional convergence in productivity and the decline in the employment share in agriculture in Japan<sup>13</sup> started in the late 19th century (Fukao et al. 2015) when the process of industrialization gained momentum (see Figure 3(a)). The average labor productivity (over 46 prefectures) benchmarked to the level of Tokyo increased from 32% in 1874 to almost 77% in 1970. During the period of the postwar growth miracle from 1955 to 1970, Japan's aggregate productivity rose remarkably, but the regional disparity in productivity also narrowed to an unprecedented level in this phase. Since the 1970s, the average prefectural labor productivity level (excluding Tokyo) has remained in the vicinity of 75% of that of Tokyo. The Gini coefficient for labor productivity also continued to drop in the second half of the 20th century, and did so at a faster rate than in the pre-WWII period (Figure 3(b)).

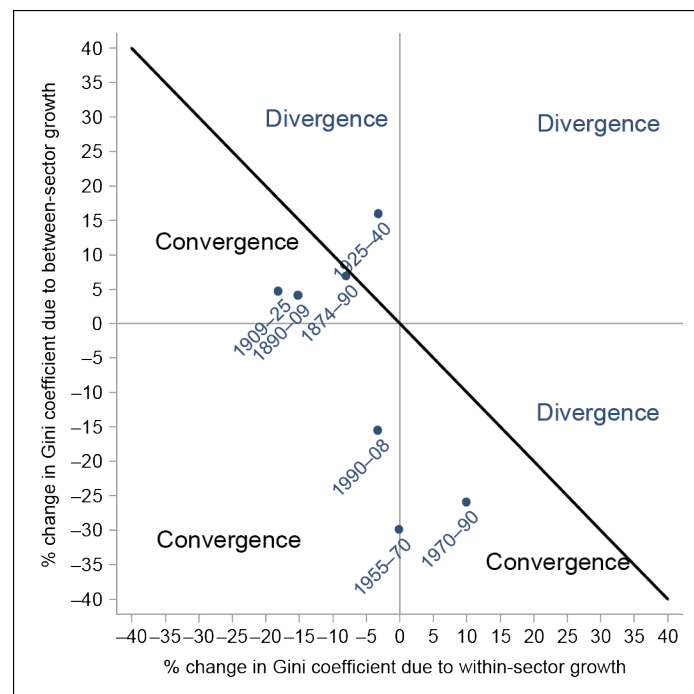
### 3.4 Productivity Catch-up and Convergence through Structural Transformation

In this section, we examine the role of structural transformation in productivity convergence. Figure 4 provides a graphic summary of the main results and indicates that there were two distinct patterns of regional convergence. Specifically, during the prewar period, it was primarily the within-sector effect that led to regional convergence, while during the postwar period it was the between-sector effect (i.e., structural transformation). In other words, convergence was the result of two countervailing forces: within-sector productivity growth and productivity growth driven by structural transformation. Appendix Figure 1 shows that apart from a few periods, the distribution of the adjustment term is close to zero. We conduct a *t*-test, which accepts the null hypothesis that  $\varphi = 0$  at the 10% significance level.

<sup>13</sup> For developments in the United States, see Easterlin (1960), Barro and Sala-i-Martin (1992), Kim (1998), and Mitchener and McLean (2003).

**Figure 3: Convergence of Aggregate Labor Productivity, 1874–2008**

Notes: In both figures, real GDP figures are in constant 1934–36 prices for 1874–1940 and constant 2000 prices for 1955–2008. In panel (a), the points indicate the average and the vertical range represents the spread (2 standard deviations) around the mean.

**Figure 4: Contribution of Structural Transformation and the Within-sector Effect to Regional Convergence ( $\sigma$ ) in Labor Productivity**

Note: This figure only shows the sign of the  $\sigma$ -convergence of aggregate productivity (resulting from the magnitudes and signs of  $\sigma$ -convergence of the within-sector and the between-sector effects). It does not show the actual measure of  $\sigma$ -convergence of aggregate productivity. The vertical and horizontal axes represent the percentage change in the Gini coefficient (of the initial year of each period) in regional labor productivity due to the between-sector and within-sector effects, respectively.

Table 1 reports the detailed empirical results of the decomposition of the change in the Gini coefficient. The top panel shows the results for the decomposition for  $\sigma$ -convergence in labor productivity, while the second and third panels show the results for the decomposition of  $\sigma$ -convergence in the between-sector and within-sector effects. Labor productivity converged across regions in all periods except in 1874–1890<sup>14</sup> and 1925–1940. The second column in each of the panels shows the change in productivity in terms of the percentage change in the Gini coefficient from the starting year of each period to the end year. Panel A suggests that  $\beta$ -convergence in the postwar era was much larger than in the prewar era. Our estimates show that the Gini coefficient, on average, dropped by almost 35% in the postwar periods compared to only 10% in the pre-war periods. The highest rate of productivity catch-up was observed in the high-speed growth era from 1955 to 1970. The estimates for Rank (the reranking of prefectures) were also higher for the postwar era, but the difference is less pronounced than in the case of  $\beta$ -convergence.

Next, panel B shows the decomposition results for the structural transformation effect. Here, let us focus on the column labeled “(-) Progress,” which represents productivity catch-up or  $\beta$ -convergence. The figures indicate that while there was  $\beta$ -divergence (positive figures) in the prewar period, the postwar period is characterized by  $\beta$ -convergence (negative figures). The estimates for Rank (the reranking of prefectures) show slightly higher values in the postwar period than in the prewar period. The results on regional convergence ( $\sigma$ -convergence) closely follow the productivity catch-up trend ( $\beta$ -convergence). Between 1955 and 1970, structural transformation-led growth alone contributed almost 30% to the drop in the Gini coefficient for aggregate productivity.

Finally, panel C presents the decomposition results for the within-sector effect. The figures indicate that Japan experienced a productivity catch-up of lagging regions through within-sector productivity growth in all periods. However, the pattern is the opposite of that observed for the between-sector effect, namely, the high rate of productivity catch-up was observed only in the postwar period. The within-sector effect made a particularly prominent contribution to regional convergence ( $\sigma$ -convergence) during the prewar era. This was driven by many factors. These include the introduction of motors at small factories in rural Japan (Minami 1976) as well as the transfer of management skills through mergers and acquisitions (Braguinsky et al. 2015). Overall, the sum total of  $\sigma$ -convergence in the within-sector effect (sectoral productivity growth) and  $\sigma$ -convergence in the reallocation effect (structural transformation-led productivity growth) provides a good approximation of the regional convergence in labor productivity.

Our results suggest that the contribution of structural transformation to regional convergence varies over time, as already highlighted by McMillan, Rodrik, and Verduzco-Gallo (2014). In addition, depending on the period, the contributions of the between-sector effect on growth and within-sector growth to regional convergence potentially offset each other.

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<sup>14</sup> This is the only period for which the change in the Gini index and the sum total of the decomposed factors have the opposite sign. This is because the magnitude of the approximation error was relatively large. However, the magnitude of convergence in labor productivity was negligible (only 0.5% of the Gini coefficient of labor productivity in 1874).



**Table 1: Evidence on Productivity Catch-up and Convergence**

	Change in Gini Index	Rank	(-) Progress	$\beta$ -convergence	$\sigma$ -convergence
A. Decomposition results for $\sigma$ -convergence in labor productivity					
1874–1890	0.5	9.3	-8.8	Yes	No
1890–1909	-11.6	3.7	-15.4	Yes	Yes
1909–1925	-14.4	3.2	-17.6	Yes	Yes
1925–1940	1.3	5.4	-4.1	Yes	No
1955–1970	-36.8	11.6	-48.4	Yes	Yes
1970–1990	-19.5	12.5	-32.0	Yes	Yes
1990–2008	-14.1	19.0	-33.2	Yes	Yes
B. Decomposition results for $\sigma$ -convergence in the structural transformation effect					
1874–1890	6.9	1.2	5.7	No	No
1890–1909	4.1	0.5	3.6	No	No
1909–1925	4.7	0.3	4.4	No	No
1925–1940	16.0	3.5	12.6	No	No
1955–1970	-29.9	8.3	-38.2	Yes	Yes
1970–1990	-25.9	2.9	-28.7	Yes	Yes
1990–2008	-15.5	0.6	-16.0	Yes	Yes
C. Decomposition results for $\sigma$ -convergence in the within-sector effect					
1874–1890	-8.0	9.2	-17.2	Yes	Yes
1890–1909	-15.2	3.6	-18.8	Yes	Yes
1909–1925	-18.1	3.8	-21.9	Yes	Yes
1925–1940	-3.2	15.3	-18.5	Yes	Yes
1955–1970	-0.1	8.0	-8.1	Yes	Yes
1970–1990	10.0	11.8	-1.9	Yes	No
1990–2008	-3.3	13.8	-17.2	Yes	Yes

Note: All figures are given as a percentage of the Gini index in the initial year of each period.

## 4. CONCLUSION

The primary purpose of this study was to estimate the potential role that the process of structural transformation played in regional productivity convergence in Japan. Using a novel data set for 47 Japanese prefectures spanning a period of nearly 135 years (from 1874 to 2008), and based on a simple theoretical framework, we find that the process of structural transformation played a crucial role in aggregate productivity growth, productivity catch-up, and regional convergence, especially in the second half of the 20th century. However, after the early 1970s, the pace of convergence slowed down as convergence in the growth effect of structural transformation was frequently offset by the divergence effect of within-sector productivity growth.

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## APPENDIX 1

Following Yitzhaki (2003), we define two additional terms: the Gini mean difference,  $G_{MD} = 4Cov(x, F(x))$ , where  $x$  is a random variable that represents labor productivity ( $x$ ), and  $F$  is the cumulative distribution of  $x$ , and the Gini correlation coefficient between two random variables,  $Y_{xy} = \frac{Cov(x, F(y))}{Cov(y, F(y))}$ , where  $x$  and  $y$  are two random variables.

**Lemma 1.** A necessary and sufficient condition for two Gini correlation coefficients to be equal, i.e.,  $Y_{xy} = Y_{yx}$ , is  $C_x^y = C_y^x$ , where  $C_x^y$  represents the area enclosed by the concentration curve of  $x$  with respect to  $y$ , and similarly  $C_y^x$  represents the area enclosed by the concentration curve of  $y$  with respect to  $x$  (Yitzhaki, 2003).

Since by construction  $V^{t+1} = V^t + \Phi(WS) + \Phi(ST)$ , using the definitions of  $V_{WS}^{t+1}$  and  $V_{ST}^{t+1}$ , we can write the linear relationship  $V^{t+1} = V_{WS}^{t+1} + V_{ST}^{t+1} - V^t$ .

Assuming that Lemma 1 holds, we can express the Gini mean difference of  $V^{t+1}$  in the following manner:

$$\begin{aligned} [G_{MD}(V^{t+1})]^2 &= [G_{MD}(V_{WS}^{t+1})]^2 + [G_{MD}(V_{ST}^{t+1})]^2 + [G_{MD}(V^t)]^2 \\ &+ 2G_{MD}(V_{WS}^{t+1})G_{MD}(V_{ST}^{t+1})Y_{V_{WS}^{t+1}V_{ST}^{t+1}} - 2G_{MD}(V_{WS}^{t+1})G_{MD}(V^t)Y_{V_{WS}^{t+1}V^t} \\ &- 2G_{MD}(V_{ST}^{t+1})G_{MD}(V^t)Y_{V_{ST}^{t+1}V^t}. \end{aligned} \quad (1)$$

Equation (1) closely resembles the variation decomposition expression for the sum of three random variables. Using the covariance definition (Lerman and Yitzhaki 1984), we can write the Gini coefficient of  $V^t$  as  $G^t(V^t) = \frac{Cov(V^t, F(V^t))}{E(V^t)}$ , where  $V^t$  is labor productivity in period  $t$ ,  $F$  is the cumulative distribution of  $V^t$ , and  $E(V^t)$  is the expectation of  $V^t$ . This yields the following relationship between  $G_{MD}$  and  $G^t(V^t)$ :  $G_{MD} = 4E(V^t)G^t(V^t)$ . Plugging this back into equation (1), we obtain an expression for equation (1) in terms of the Gini indices:

$$\begin{aligned} [E(V^{t+1})G^{t+1}(V^{t+1})]^2 &= [E(V_{WS}^{t+1})G^{t+1}(V_{WS}^{t+1})]^2 + [E(V_{ST}^{t+1})G^{t+1}(V_{ST}^{t+1})]^2 \\ &+ [E(V^t)G^t(V^t)]^2 + 2E(V_{WS}^{t+1})G^{t+1}(V_{WS}^{t+1})E(V_{ST}^{t+1})G^{t+1}(V_{ST}^{t+1})Y_{V_{WS}^{t+1}V_{ST}^{t+1}} \\ &- 2E(V_{WS}^{t+1})G^{t+1}(V_{WS}^{t+1})E(V^t)G^t(V^t)Y_{V_{WS}^{t+1}V^t} \\ &- 2E(V_{ST}^{t+1})G^{t+1}(V_{ST}^{t+1})E(V^t)G^t(V^t)Y_{V_{ST}^{t+1}V^t}. \end{aligned} \quad (2)$$

If we assume that the  $Y$ s are equal to 1, then equation (2) can be transformed into

$$\begin{aligned} [E(V^{t+1})G^{t+1}(V^{t+1})]^2 &= [E(V_{WS}^{t+1})G^{t+1}(V_{WS}^{t+1}) \\ &+ E(V_{ST}^{t+1})G^{t+1}(V_{ST}^{t+1}) - E(V^t)G^t(V^t)]^2, \end{aligned} \quad (3)$$

where the right-hand side becomes a squared term of a linear relationship with three variables. Depending on whether the square-root term is positive or negative, we get two expressions for equation  $G^{t+1}(V^{t+1})$ . Since the value of the Gini coefficient lies between 0 and 1 and it can be plausibly assumed that  $|G^{t+1}(V_{WS}^{t+1}) + G^{t+1}(V_{ST}^{t+1})| > |G^t(V^t)|$ , we consider only the positive root and express equation (3) with an approximation error term ( $\varphi$ ), written in implicit form as

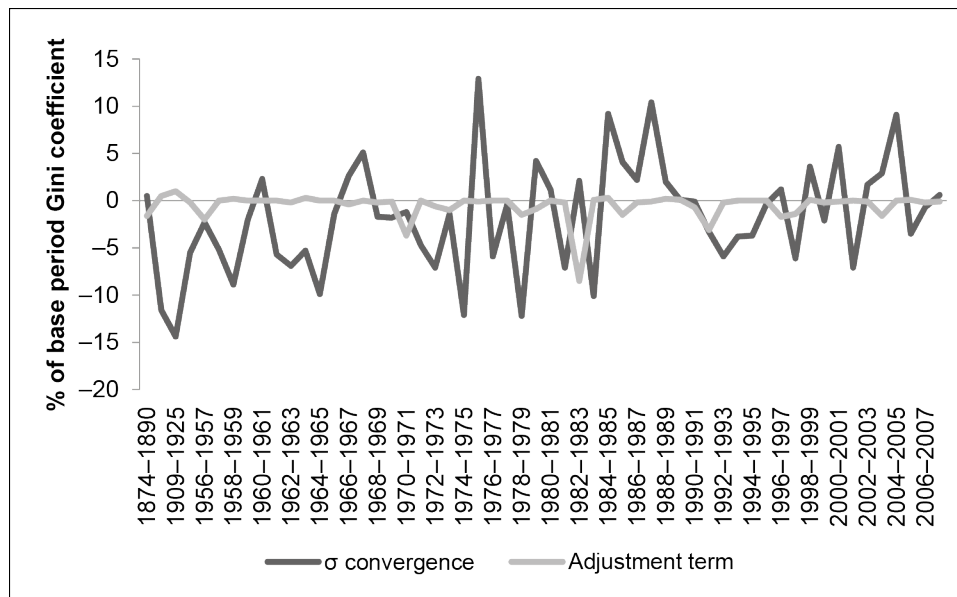
$$G^{t+1}(V^{t+1}) = G^{t+1}(V_{WS}^{t+1}) + G^{t+1}(V_{ST}^{t+1}) - G^t(V^t) + \varphi. \quad (4)$$

Subtracting  $G^t(V^t)$  from both sides, we get

$$\varphi = [G^{t+1}(V^{t+1}) - G^t(V^t)] - [G^{t+1}(V_{WS}^{t+1}) - G^t(V^t)] + [G^{t+1}(V_{ST}^{t+1}) - G^t(V^t)]. \quad (5)$$

## APPENDIX 2: DISTRIBUTION OF THE ADJUSTMENT TERM

Distribution of the Adjustment Term and  $\sigma$ -Convergence



Note: The figure shows that the distribution of the adjustment term is close to zero except in a few periods. A  $t$ -test accepts the null hypothesis that  $\varphi = 0$  at the 10% significance level. Empirically, the value of  $\varphi$  for each period can be calculated for any period as long as  $G^{t+1}(V^{t+1}) - G^t(V^t)$ ,  $[G^{t+1}(V_{WS}^{t+1}) - G^t(V^t)]$ , and  $[G^{t+1}(V_{ST}^{t+1}) - G^t(V^t)]$  are measured separately. We use these values to test the above hypothesis about  $\varphi$  using the benchmark years from 1874 to 1955 and then annual figures for the rest of the period from 1955 to 2008.