Schultz, Christian

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VIRTUAL CAPACITY AND COMPETITION

CHRISTIAN SCHULTZ

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VIRTUAL CAPACITY AND COMPETITION

Abstract

In several European merger cases competition authorities have demanded that the merging firm auctions off virtual capacity. The buyer of virtual capacity receives an option on an amount of output at a pre-specified price, typically equal to marginal cost. This output is sold in the market in competition with the merging firm. The paper compares sale of physical and virtual capacity by the merging firm and shows that virtual capacity leads to a less competitive outcome. The merging firm can build up a reputation for producing little, so that the output price increases in the market, and this increases the auction price on virtual capacity.

JEL Code: L40, L41, D44.

Keywords: virtual capacity, reputation, tacit collusion, antitrust, mergers, competition policy.

Christian Schultz
Centre for Industrial Economics
University of Copenhagen
Studiestraede 6
1455 Copenhagen
Denmark
cs@econ.ku.dk

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1 Introduction

It is common in merger cases that the competition authority requires dominant firms to sell off capacity, so that its market share does not grow (too much). Recently, several European merger cases in electricity markets have resulted in the sale of virtual capacity in the form of so called Virtual Power Plants (VVP). The aim of this paper is to investigate, whether the competitive effects of these requirements are as good as the selling of real physical capacity. In particular, we will be interested in reputation building and tacit collusion. Tacit collusion - or coordinated interaction - is a major worry in merger cases; see for instance the horizontal merger guidelines published by Department of Justice (1997) and the European Union (2004).

Virtual capacity is an option to buy products (e.g. electricity) at a predetermined price per unit (typically equal to marginal cost), which the buyer then sells in the final product market in competition with the producer. In the European electricity examples, auctions are held at regular intervals (several times a year).

When the virtual capacity is auctioned, the recipient of the revenue is the large merging firm. If the auction is efficient, the price for the virtual capacity will equal the expected profit - suitably adjusted for risk etc. - from having access to the capacity. So although the merging firm meets competition in the market from the virtual competitor, it will pocket the profits made by the competitor in the auction. This potentially has effects on the competition. The paper investigates this.

The paper provides a simple model of a market with one producing firm. The producing firm first auctions off virtual capacity and then the firm and the virtual producer competes in the market. For simplicity we consider a Cournot model, where the firms choose production and the price is set in
the market. We first consider a static market. There the competitive effects
of introducing the virtual producer are equivalent to those of introducing an
independent producer who owns his capacity. The reason is simple. Once
the auction is held, the payment in the auction is sunk and everything is as
if there are two independent firms in the market. In line with the European
elements alluded to, we consider the case where the virtual capacity is small
relative to the market, so that the virtual producer wants to market his whole
capacity. In principle one could conceive of large virtual producers, but this
has not been demanded by competition authorities yet and is left for future
research.

Typically, virtual capacity is not auctioned off once and for all, but a
sequence of auctions are held. We therefore consider long run effects and
reputation building when the merging firm has to auction off virtual capacity
repeatedly. Since the virtual capacity typically is auctioned off for relatively
short periods and there are many bidders, we first consider the case where the
virtual producers are short run players, who just market whatever capacity
they have. Short run players will not consider participating in tacit collusion.
Clearly, this makes it more difficult to maintain profits above the competitive
level. In the standard case with two independent firms, such profits cannot
be implemented without the cooperation of both firms. If one firm plays a
best response, the best the other can do is to play best response and this
results in the Cournot equilibrium. With virtual capacity this logic breaks
down. Since the big (merging) firm sells the virtual capacity it will pocket
the expected profit to be earned on the virtual capacity. If it acts moderately
in the market and lowers production, the price will rise. In fact it can induce
the monopoly price by reducing its own sales to the monopoly output minus
the virtual capacity it has sold off. Then the market price will equal the
monopoly price, and the virtual producer will net the monopoly price minus marginal (virtual) cost times virtual capacity. In a repeated game where this happens in each period, the large firm can thus build a reputation for inducing the monopoly price and the participants in the auction will realize this. The revenue of the large firm will equal the high earnings of the virtual producer. In this way the large firm can realize the whole monopoly profit. Of course, it has an incentive to deviate in a given period. When it has auctioned off the virtual capacity in a period, there is an incentive to produce more than the low level giving rise to the monopoly price. However, there will be a future punishment, as future bidders in the auction will realize that monopoly profits cannot be earned in the market; they will only bid the virtual producer’s Cournot profit. We show that this punishment is sufficient for the big producer to restrain production and maintain monopoly profits if the discount factor is sufficiently high. This is a very general result, which just assumes that the monopoly profit exceeds the Cournot profit.

We also show that if the discount factor is lower than the crucial value, which allows the firm to reap the monopoly profit, the highest obtainable price and profit are increasing in the discount factor. We show that reputation building is not affected if the small firm is a long run player. Since the small firm’s future profits are zero - all future profits are spent in the auction buying the virtual capacity - there is no punishment available if it deviates from collusive play. Therefore there are no equilibria, where the small firm participates in tacit collusion when it has virtual capacity.

The analysis here suggests that reputation building is something the competitive authorities should worry about. Reputation building in theory takes an infinitely repeated game. In a finite game, the effects analysed in this paper do not arise in equilibrium. However, it is well known from experiments
- see e.g. Selten and Stoeker (1986) - that even in finite - but long games - reputation and tacit collusion are observed. Hence, there are good reasons to believe that the reputation building analysed in this paper can take place in reality even when the horizon is finite. Experimental evidence also shows that as the end game approaches, reputation and tacit collusion breaks down. The temptation to deviate becomes too high. For practical purposes, competition authorities are therefore advised not to make the period on which the adverse effects of a merger is countered by virtual capacity too long.

The longer the duration of a contract for virtual capacity, the longer can the merging firm reap the benefits of deviating before it is punished in the upcoming auctions. This incentive to deviate undermines the reputation building. In order to hinder reputation building it is therefore advisable that the authorities make contracts for virtual capacity with a long duration.

In conclusion the analysis here lends support to the view that contracts for virtual capacity should have a relatively long duration and that it is not advisable to continue with auctions for virtual capacity indefinitely.

If the small firm is an independent producer, who owns his own capacity, and the firm is short sighted, tacit collusion as well as reputation building are impossible. It therefore follows that virtual capacity in itself facilitates reputation building if the small firm is a short run player.

Of course, one may argue that if the small firm owns its own capacity, it will be more reasonable to consider the case where it participates in tacit collusion. In this case it will be in the market for many periods, and be interested in future profits. A non-trivial question here is how the firms share production - and profits - when they collude on the monopoly outcome. We first assume that they split the market in the same proportion as they do in the Cournot equilibrium. This would for instance be the case in a split
the surplus bargain where the Cournot outcome is the threat point. This complicates the model somewhat, and we resort to a linear specification. For this case we show that the minimum discount factor allowing collusion for monopoly profits is higher than the discount factor which allows the merging firm to build a reputation for maintaining monopoly profits when capacity is virtual. So in this comparison, virtual capacity also facilitates collusion. However, one may object that the firms may choose another sharing rule for the profits, if they cannot uphold collusion. We therefore consider the case where the firms share the market such that collusion is most easy to sustain. We show that it is not possible to maintain collusion on the monopoly profit when the discount factor is as low as the lowest allowing reputation building for monopoly profits with virtual capacity.

As stated above, virtual capacity has been introduced in a number of recent European merger cases. In relation to Electricité de France’s (EDF) purchase of 34.5% of the shares in the German utility EnBW, EDF agreed to make 6,000 MW of virtual capacity available in France by November 2003 in order to increase competition in the market. EDF was at the time selling to around 90% of the so-called free costumers in the French market. The virtual capacity is to be auctioned to companies who will act as sellers in the French power market. The contracts for virtual capacity have durations of 3, 6, 12, 24 and 36 months. The first auction for 1,200 MW took place in September 2001. As of April 2004 11 auctions have been held. The auctions are organized as ascending clock auctions. Around 30 energy traders and suppliers competed in previous auctions, which were conducted over the Internet, with approximately 20 bidders emerging as successful purchasers.

According to agreement with the European Commission, EDF shall provide virtual capacity for a period of five years. The French electricity market
is then expected to have developed so that sufficient competition will be present without the Virtual Capacity (see Electricité de France, 2004).

Due to the Electricity Supply Board’s (ESB) dominance in the Irish power market, the Irish government has initiated the Virtual Independent Power Producer Auction (VIPP), a form of virtual capacity auction as in France. The auctions - where independent suppliers can bid for 600 MW out of a total of 4.500 MW - are intended to reduce ESB’s market power until more independent suppliers enter the market (see European Commission, Madrid Forum, 2002).

In 2003, the Belgian Competition Council approved that a subsidiary of Electrabel became the default supplier for the customers of several inter-municipal distribution companies. As Electrabel has a very large market share in Belgium it was agreed that Electrabel should offer, via auctions, up to a maximum of 1,200 MW of virtual power plant (VPP) capacity in Belgium. The terms are to a large extent similar to the French, in particular capacity shall be offered for a period of five years (see Konkurrencestyrelsen, 2004).

The Dutch electricity producer Nuon agreed with competition authorities that it would auction 900 MW virtual capacity in order to be allowed to buy Reliant and its 3500 MW capacity. Again there is a five year limit on the requirement. The Dutch market size is around 20.000 MW, (Konkurrencestyrelsen, 2004)

In March 2004 the large Danish producer Elsam agreed to auction off 600 MW virtual capacity in order to be allowed to make an indirect purchase of 36% of the shares in the other big Danish producer E2, see Konkurrencestyrelsen (2004). The total Danish market size is about 7000 MW. As in the other countries auctions are to be held regularly, and for varying durations all
below three years. The Danish rules specify that a single buyer at most must acquire 300 MW. The agreement with the competition authorities stipulates that the virtual producer can buy electricity at the lowest marginal cost obtainable in the different plants owned by Elsam. Contrary to the previously mentioned cases, the Danish competition authorities required that the virtual capacity should be provided indefinitely. This makes worries about reputation building potentially more important as there will be no end game effects.

There is a long literature on tacit collusion, see Tirole (1991) for an overview. The detection of tacit collusion in electricity markets have been the subject of a number of papers including Green and Newbury (1992), von der Fehr and Harbord (1993), Borenstein and Bushnell (1999), Wolfram (1999), and Fabra and Toro (2004). To the best of my knowledge the issue of virtual capacity and tacit collusion has not been considered in the literature.

The organization of the paper is as follows. Section 2 describes the basic model and derives the static solution. Tacit collusion with virtual capacity and short run players is considered in section 3. Section 4 contains the case where the small producer is independent. Section 5 offers some concluding remarks.

## 2 Basics: The static market

We consider a market with two firms, a big firm 1, who sells some virtual capacity $q_2$. The buyer of the capacity, firm 2, is also called the virtual firm. Both firms sell in a final market. The amount sold by firm 1 in the final market is denoted $q_1$. The price in the final market is given by the inverse demand curve $p(Q)$ where $Q$ is total production. For some - but not all - of
the results we will rely on the linear specification

\[ p(Q) = a - bQ, \]

where \( a \) and \( b \) are two positive parameters. Although Theorem 1 below is valid under general concavity assumptions, we introduce the linear specification already from the start and give the results for the linear model as we go along in order to shorten on the presentation. We will state explicitly, when a result depends on the linear specification.

In this section we consider a single period, where the timeline is as follows. First virtual capacity in the amount \( q_2 \) is sold in an auction. After the auction, the big firm, firm 1, and the buyer of the virtual capacity, firm 2, competes a la Cournot in the final market. We assume that there are sufficiently many potential bidders and the auction format is such that the price of the virtual capacity equals the profit which can be earned in the final market with the virtual capacity. This is for instance the case if the auction is an open English auction with at least two independent bidders. The virtual capacity allows the buyer to request up to \( q_2 \) units at firm 1’s marginal cost \( c \geq 0 \).

We assume that the amount of virtual capacity, \( q_2 \), is so small that firm 2 is capacity constrained and wants to utilize all capacity. This assumption is motivated by the examples discussed in the Introduction, where the virtual producers indeed are small.

We solve the static model for the subgame perfect equilibrium, as usual by solving backwards. After the auction, firm 2 possesses virtual capacity \( q_2 \) and it sells all \( q_2 \) units in the final market. We will verify below that this is indeed optimal. Given firm 2 sells \( q_2 \) units, the problem of firm 1 is

\[ \max_{q_1} (p(q_1 + q_2) - c) q_1. \]
The best reply is (using superscript "c" for Cournot),

\[ q^c_1 (q_2) = \frac{a - c}{2b} - \frac{1}{2} q_2, \tag{1} \]

and the total production is therefore

\[ q^c_1 (q_2) + q_2 = \frac{a - c}{2b} + \frac{1}{2} q_2. \]

The final price in the market is

\[ p^c = \frac{a + c}{2} - \frac{b}{2} q_2. \tag{2} \]

Equation (2) clearly shows that the introduction of virtual capacity lowers the market price.

The profit to each firm is

\[ \pi^c_1 = \frac{(a - c - bq_2)^2}{4b} \quad \text{and} \quad \pi^c_2 = \frac{(a - c - bq_2) q_2}{2} \]

As mentioned above, we assume that \( q_2 \) is so low that the virtual firm is capacity constrained, i.e. that \( q_2 \) is less than firm 2’s best reply to \( q^c_1 (q_2) \).

The best reply to \( q^c_1 (q_2) \) is given by (1) with \( q^c_1 (q_2) \) inserted for \( q_2 \) on the right hand side. We therefore get that 2 is capacity constrained if

\[ q_2 < \frac{a - c}{2b} - \frac{1}{2} \left( \frac{a - c}{2b} - \frac{1}{2} q_2 \right) \]

or

\[ q_2 < \frac{1}{3} \frac{a - c}{b} \]

This condition says that \( q_2 \) should be less than the production level of each firm in the symmetric Cournot equilibrium. With two firms in the market this implies that \( q_2 \) should be less than 50% of the market. This is clearly fulfilled in the examples discussed in the Introduction. In the model, we only include one successful bidder in the auction, whereas in the examples
discussed in the introduction, there typically were many. As long as the virtual producers use all capacity, the results derived here would not change if we introduced more virtual producers with total capacity $q_2$. This would just lead to a more cumbersome notation, so we refrain from that.

Now we look at the auction stage. The prospective buyers are rational and foresee that the Cournot equilibrium will arise and that the profit, which can be earned from the virtual capacity is $\pi_2^c$. Under the assumption that the auction is competitive, the price of the capacity will equal this profit.

The total profit to firm 1 from own sales and the sale of the virtual capacity therefore equals

$$\pi_1^c + \pi_2^c = \frac{(a - c)^2 - (bq_2)^2}{4b}$$

We see that the larger the virtual capacity, the lower is the total profit of firm 1. This is of course just a mirror of the lower price. Virtual capacity enhances the competitiveness of the static market.

## 3 Reputation with virtual capacity

In this section we consider the case where the merging firm auctions off virtual capacity repeatedly and consider reputation building by the large firm. There are infinitely many periods $t = 0, ..., \infty$.

At first we will assume that the auction format and the many participants in the auction means that the winner of the virtual capacity has a short horizon. She can not be sure to win the next auction and will therefore not be willing to reduce supply in order to raise the price. She will seek to gain as much as possible and for $q_2$ sufficiently small, this means that she will wish to supply $q_2$. In short, the owner of the virtual capacity is not willing
to collude, she is a short run player in the language of Fudenberg, Kreps and Maskin (1990).

Firm 1, however, is a long run player and has an incentive to keep prices high in the market. We assume that firm 1 discounts future profits with the discount factor \( \delta \), where \( 0 < \delta < 1 \), and the firm is interested in the sum of discounted future profits.

The participants in the auction have an expectation about the market price and therefore about the profit, which can be earned using the virtual capacity. The participants observe previous prices and as time passes, the expectation for period \( t \) may depend on these previous prices. At time \( t \) the expectation about prices for period \( t \) is a function of previous prices. In equilibrium, these expectations are rational, which in this non-stochastic model means that they are correct\(^1\).

A subgame perfect, rational expectations equilibrium of the repeated game consists of an expectation function for the participants in the auction, which is correct for all possible histories - including out of equilibrium histories - and a strategy for the big producer which is sequentially rational in all subgames.

First, we will find the condition under which an equilibrium, where firm 1 earns monopoly profits in the market, exists.

Suppose that the auction participants have the following expectations function (where they implicitly take into account that the winner of the

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\(^1\)In principle the expectation for period \( t \)'s price can depend on the whole history of the game (productions, profits, and prices of all previous periods). As will be clear, the more simple formulation chosen here just simplifies the exposition and does not affect the results.
The auction participants expect that the price will be the monopoly price as long as this has been the case in the past (or it is the very first period). If they ever see another price, they expect the Cournot price, $p^c$, in all future. These are trigger expectations, which punishes firm 1 if it ever floods the market and makes the price go below $p^m$. If the participants in the auction are unable to collude on bidding zero, this is the hardest punishment available. If they can collude on bidding zero for the virtual capacity an even harder punishment is available\(^2\). We will assume that the number of participants in the auction is sufficiently large that such collusion is not possible. Notice, however, that if such collusion is possible, and firm 1 thus can be punished even harder than assumed here, this would just make reputation building easier. In this sense our assumption stacks the deck against reputation building.

Given the expectations, firm 1 essentially has two options if the monopoly price $p^m$ is expected for a period. Either it can choose $q^m_1 \equiv Q^m - q_2$, where

$$Q^m = \frac{1}{2} \frac{a - c}{b},$$

is the monopoly output and get the advantage that the price expectation for the next period will be high. Alternatively firm 1 can deviate to the best equilibrium can be sustained. See Abreu (1988).
possible production, which equals $q^*_1(q_2)$. Then the price will fall, and price expectations for the future periods will be $p^c$. If firm 1 chooses $q^m_1$, and the price becomes $p^m$, the profit of the virtual firm becomes

$$\pi^m_2 = (p^m - c)q_2 = \frac{a - c}{2}q_2.$$ 

This will be the price of the virtual capacity in a period where $p^m$ is expected. If firm 1 chooses $q^m_1$ its total profit, from own production and selling the virtual capacity, becomes the total monopoly profit

$$\pi^m = \pi^m_1 + \pi^m_2 = \frac{1}{4} \frac{(a - c)^2}{b}.$$ \hspace{1cm} (4)

If firm 1 chooses $q^m_1$ in each period, it will then realize the monopoly profit in each period. If firm 1 deviates to $q^*_1(q_2)$, total production in the period will be total Cournot production $q^*_1(q_2) + q_2$ and the firm’s total profit from the period will be $\pi^c_1 + \pi^c_2$, as the virtual producer expected $p^m$ and the winning bid in the auction has been $\pi^m_2$. In the next period, however, expectations will be that $p = p^c$, so the winning bid will be $\pi^c_2$ and the profit of firm 1 from that period and onwards will be $\pi^c_1 + \pi^c_2$. The no-deviation constraint for firm 1 therefore reads

$$\frac{1}{1 - \delta} \pi^m \geq \pi^c_1 + \pi^c_2 + \frac{\delta}{1 - \delta} (\pi^c_1 + \pi^c_2),$$ \hspace{1cm} (5)

which requires that firm 1’s discount factor $\delta$ is no less than

$$\hat{\delta} = \frac{\pi^c_1 - \pi^m_1}{\pi^c_2 - \pi^m_2} > 0.$$ \hspace{1cm} (6)

(Recall that when firm one produces $q^m_1$ it restricts output below $q^*_1$, which is the best reply, so $\pi^m_1 < \pi^c_1$).

In general, as long as the monopoly profit exceeds the sum of Cournot profits, i.e. as long as

$$\pi^c_1 + \pi^c_2 < \pi^m_1 + \pi^m_2,$$
we have that

\[ \hat{\delta} = \frac{\pi^c_1 - \pi^m_1}{\pi^m_2 - \pi^c_2} < 1. \]

So if the discount factor \( \delta \geq \hat{\delta} \), it is possible to realize the monopoly profit through reputation building. Notice, importantly, that this result \textit{does not} depend on demand and cost being linear. It holds true under general concavity assumptions ensuring existence of Cournot equilibrium and optimum for the monopolist, as long as monopoly profit exceeds Cournot profit.

Finally, notice that if the discount factor is high and firm 1 chooses \( q_1^m \), the expectation that \( p = p^m \) in each period is correct. If in the past another price has been observed and the expectation becomes \( p = p^c \), then the big firm can only earn \( \pi^2 \) in all future auctions regardless of its action in a period \( t \).

The optimal level of production is therefore \( q^c_1 (q_2) \) and the price becomes \( p^c \) as expected. Hence the price expectation is rational also off the equilibrium path. Summarizing the result.

\textbf{Theorem 1} For any demand and cost function such that the monopoly profit exceeds the total profit of the Cournot equilibrium the following is true: If the virtual firm is a short run player, there exists a subgame perfect equilibrium, where the firm 1’s total earnings equals the monopoly profit in each period, if the discount factor \( \delta \) exceeds \( \hat{\delta} \) given in equation (6).

In the model, the duration of a period equals the duration of the contract for the virtual capacity. For given time preferences, discount rate \( r \), and duration of contract for virtual capacity, \( \Delta t \), the relevant discount factor is

\[ \delta = \exp (-r \Delta t). \]

This is smaller, the longer duration of the contract. A longer contract therefore makes it more difficult to fulfill the requirement that the discount factor
exceeds the crucial discount factor \( \hat{\delta} \). In this sense a longer contract makes reputation building more difficult.

In the linear model, we can get an explicit expression for \( \hat{\delta} \). Inserting the relevant values gives

\[
\hat{\delta} = \frac{(a-c-bq_2)^2}{4b} - \left( \frac{1}{4} \left( \frac{(a-c)^2}{b} - \frac{a-c}{2} q_2 \right) \right) = \frac{1}{2}
\]

(7)

This crucial discount factor does not depend on the amount of virtual capacity, \( q_2 \). Hence, in the linear model, if full reputation building on the monopoly price is feasible, a larger amount of virtual capacity does not make reputation building more difficult. While interesting in itself, it is not a general result, it depends on the linear specification.

Suppose that the discount factor is less than \( \hat{\delta} \), so that reputation building on the monopoly price is impossible. The firm must then settle for partial reputation building. The best profit, which can be realized, fulfills the incentive constraint (5) with equality. This is equivalent to

\[
\pi_1 + \delta \pi_2 = \pi_1^c + \delta \pi_2^c
\]

In order to proceed, we focus on the linear model. Then, we get the condition

\[
(a - b (q_1 + q_2) - c) (q_1 + \delta q_2) = ((a - b (q_1^c (q_2) + q_2) - c) (q_1^c (q_2) + \delta q_2))
\]

which has two solutions, \( q_1 = q_1^c (q_2) \) and

\[
q_1 = \frac{a - c}{b} - (1 + \delta) q_2 - q_1^c (q_2)
\]

Inserting into the profit functions and manipulating a bit, we get that the total profit to firm 1 from partial reputation building is

\[
\pi_1 + \pi_2 = \frac{(a - c)^2 - (bq_2 (1 - 2\delta))^2}{4b}
\]

(8)
For \( \delta = \frac{1}{2} \), this gives the monopoly profit as it should. It is increasing in \( \delta \) and decreasing in \( q_2 \) for \( \delta < \frac{1}{2} \). For \( \delta = 0 \), it equals the sum of Cournot profits.

The price is

\[
p = \frac{a + c}{2} - \left( \frac{1}{2} - \delta \right) bq_2
\]

which is lower than the monopoly price and decreasing in \( q_2 \) for \( \delta < \frac{1}{2} \) and increasing in \( \delta \). Thus for discount factors below \( \frac{1}{2} \), larger virtual capacity is pro-competitive, it lowers the market price and it lowers the total profits of the firms.

**Theorem 2** Larger virtual capacity is pro-competitive if full reputation building is not possible. In the linear model with virtual capacity, the highest profit firm 1 can obtain from partial reputation building when \( \delta < \delta = \frac{1}{2} \) is given by (8) and the price is given by (9). Both is increasing in the discount factor, \( \delta \), and decreasing in the amount of virtual capacity, \( q_2 \).

We conclude that when the discount factor is so low that it is impossible to realize the monopoly profit, and the firm has to practise partial reputation building, then the market becomes more competitive, the larger the virtual capacity is and the lower the discount factor is.

We have assumed that firm 2 is a short run player, who will not even contemplate participating in tacit collusion. As we see, even in this case it is possible for firm one to build a reputation. However, the results will not change if firm 2 is a long run player. Suppose that it is a long run player, and the firms try to coordinate tacitly on an equilibrium with high profits. Suppose, in particular, that firm 2 is supposed not just to play a best response. In such an equilibrium there has to be a punishment if firm 2 deviates from collusive play. However, since firm two’s future profits are zero
- all future profits are spent in the auction buying the virtual capacity - there is no punishment available if it deviates from collusive play. Therefore there are no equilibria, where the small firm partipates in tacit collusion when it has virtual capacity.

Hence the results of this section do not change even if firm 2 is a long run player.

4 Tacit collusion when firm two owns its capacity

In this section we consider the case where firm 2 owns its capacity. If it is a short run player, then tacit collusion as well as reputation building is not possible. In this case firm 1 only receives profits from its own sale, and if firm 2 always plays a best response, firm 1 cannot do better than playing a best response, which results in the Cournot equilibrium. However, the assumption that player 2 is a short run player may appear questionable if indeed firm 2 owns its own capacity and is in the market indefinitely. We therefore now assume that both firms are long run players. As we assume that the firms are not symmetric (firm 2 is the small firm) it is an important question how the firms share the monopoly profit in the collusive phase. A natural benchmark is when the firms share the market in the collusive phase in the same way as they do in the absence of collusion. This would be the case if one conceives of equal split bargaining taking as the threat point the non-collusive profits. We will first focus on this case below. In this case the market share of firm 1 is

\[ s_1 = \frac{q_1^c (q_2)}{q_1^c (q_2) + q_2}, \]

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and the production of each firm in the collusive-monopolistic phase is $q_{ml}^1 = s_1 Q^m$; $q_{ml}^2 = (1 - s_1) Q^m$. The profits are correspondingly $\pi_{ml}^1 = s_1 \pi^m$; $\pi_{ml}^2 = (1 - s_1) \pi^m$.

As the formulas become a little heavy-handed, we focus on the linear model in the special case where $a = 1$, $b = 1$ and $c = 0$. Then

$$q_1^c (q_2) = \frac{1 - q_2}{2}; \quad \text{and} \quad Q^m = \frac{1}{2},$$

and

$$s_2 = 2 \frac{q_2}{1 - q_2 + q_2} = 2 \frac{q_2}{q_2 + 1}$$

$$\pi_1^c (q_2) = \frac{(1 - q_2)^2}{4}; \quad \pi_2^c (q_2) = \frac{(1 - q_2) q_2}{2}; \quad \pi^c (q_2) = \frac{(1 - q_2^2)}{4}$$

The productions in the collusive phase are

$$q_{ml}^1 = \frac{1 - q_2}{1 + q_2} \frac{1}{2} \quad \text{and} \quad q_{ml}^2 = \frac{q_2}{1 + q_2} < q_2.$$

If a firm decides to deviate from the collusive phase, the best reply is found using (1)

$$q_1 (q_{ml}^2) = \frac{1}{2 (1 + q_2)}; \quad \text{and} \quad q_2 (q_{ml}^1) = q_2,$$

and the associated profits are

$$\pi_{dl}^1 = \frac{1}{4 (1 + q_2)^2}; \quad \text{and} \quad \pi_{dl}^2 = \frac{1}{2 (1 + q_2 - 2q_2^2)} \frac{q_2}{1 + q_2}.$$

If firm 2 has own capacity, the non-deviation constraint of firm 1 is

$$\frac{1}{1 - \delta} s_1 \pi^m \geq \pi_{dl}^1 + \frac{\delta}{1 - \delta} s_1 \pi^c,$$

which we can rewrite

$$\frac{1}{1 - \delta} \pi^m \geq \pi_{dl}^1 \frac{1}{s_1} + \frac{\delta}{1 - \delta} \pi^c,$$
while that of firm 2 is
\[
\frac{1}{1 - \delta} \pi^m \geq \frac{\pi^d_2}{s_2} + \frac{\delta}{1 - \delta} \pi^c.
\]
As
\[
\frac{\pi^d_1}{s_1} \frac{1}{\pi^d_2 \frac{1}{s_2}} = \frac{1}{(1 - q_2^2) (1 + q_2 - 2q_2^2)} < 1 \text{ for } q_2 < .321
\]
we have that
\[
\frac{\pi^d_1}{s_1} < \frac{\pi^d_2}{s_2},
\]
so that the constraint for firm 2 is the most binding constraint.

Inserting the relevant values we get
\[
\frac{1}{1 - \delta} \pi^m \geq \frac{\pi^d_2}{s_2} + \frac{\delta}{1 - \delta} \pi^c
\]
\[
\frac{1}{1 - \delta} \frac{1}{4} \geq \frac{1}{2 \frac{q_2}{q_2 + 1}} \frac{1}{2} \left(1 + q_2 - 2q_2^2\right) \frac{q_2}{1 + q_2} + \frac{\delta}{1 - \delta} \frac{(1 - q_2^2)^2}{4},
\]
which is fulfilled if \( \delta \) exceeds
\[
\delta_i = \frac{1 - 2q_2}{1 - q_2^2}.
\]
We have that \( \delta_i \in [\frac{1}{2}, 1] \), is decreasing and \( \delta_i(0) = 1 \) and \( \delta_i(\frac{1}{3}) = \frac{1}{2}, \) where \( \frac{1}{3} \) is the Cournot output for firm 2 in this example.

We see that lowest the crucial discount factor allowing full collusion on the monopoly profit is higher than the lowest discount factor \( (\delta = \frac{1}{2}) \) allowing reputation building on the monopoly profit with virtual capacity.

The above analysis rests on the assumption that the firms share the market in the collusive phase in the same way as they do in the absence of collusion. However, one could object, that if tacit collusion cannot be maintained under this sharing rule and there are other ways of sharing the market, which makes tacit collusion easier to sustain, the firms will choose to do that. The
non-deviation constraint is most binding for firm 2 in the above analysis. The higher is the market share of firm 2, the less is its incentive to deviate. We will therefore now allow the firms to allocate market shares to facilitate collusion.

Recall that the monopoly profit is \( \pi^m = \frac{1}{4} \), while monopoly production is \( q^m = \frac{1}{2} \). Firm 2’s capacity is \( q_2 \), while marketed production from 2 in the collusive phase is now denoted \( x_2 \) and firm 1’s marketed production is \( \frac{1}{2} - x_2 \). In this case the collusive profits are

\[
\begin{align*}
\pi^m_1 (x_2) &= \frac{1}{4} \left( \frac{1}{2} - x_2 \right) = \frac{1}{4} - \frac{1}{2} x_2 \\
\pi^m_2 (x_2) &= \frac{1}{4} \left( \frac{1}{2} - x_2 \right) = \frac{1}{2} x_2
\end{align*}
\]

Using (1) we find that best replies are

\[q^c_1 (q^L_2) = \frac{1}{2} - \frac{1}{2} x_2 \text{ and } q^c_2 (q^L_1) = q_2\]

since we still assume that firm two’s capacity is below its best reply. The associated profits are

\[
\begin{align*}
\pi^{dL}_1 (x_2) &= \left( \left( 1 - \left( \frac{1}{2} - \frac{1}{2} x_2 + x_2 \right) \right) \left( \frac{1}{2} - \frac{1}{2} x_2 \right) \right) = \left( \frac{1}{2} - \frac{1}{2} x_2 \right)^2 \quad (11)
\end{align*}
\]

and

\[
\begin{align*}
\pi^{dL}_2 (x_2) &= \left( 1 - \left( \frac{1}{2} - x_2 \right) - q_2 \right) q_2 = q_2 \left( x_2 - q_2 + \frac{1}{2} \right)
\end{align*}
\]

We therefore get that if the firms collude on the monopoly outcome, firm two’s discount factor should at least be higher than \( \delta_2 (x_2) \) fulfilling

\[
\delta_2 (x_2) = \frac{\pi^{dL}_2 (x_2) - \pi^m_2 (x_2)}{\pi^{dL}_2 (x_2) - \pi^c_2 (x_2)} = \frac{(2q_2 - 1) (x_2 - q_2)}{q_2 (2x_2 - q_2)}
\]

which - when it is positive - is decreasing in \( x_2 \).

Recall that with virtual capacity, a reputational equilibrium, where firm 1 earned the monopoly profit could be sustained if \( \delta \geq \frac{1}{2} \). We will now show
that tacit collusion on the monopoly outcome can not be sustained if \( \delta \) is as 
low as \( \frac{1}{2} \). If firm two should not deviate when \( \delta = \frac{1}{2}, x_2 \) should at least fulfill

\[
\delta_2 (x_2) = \frac{(1 - 2q_2)(q_2 - x_2)}{q_2(2x_2 - q_2)} = \frac{1}{2}
\]

which gives

\[
x_2 = \frac{1}{2q_2 - 2}(3q_2^2 - 2q_2) = \frac{2 - 3q_2^2}{2 - 2q_2}q_2
\]

If firm one shall not deviate, its discount factor should not be less than \( \delta_1 (x_2) \). Inserting for \( x_2 \) we get get

\[
\delta_1 = \frac{\pi^m_1 (x_2) - \pi^m_1 (x_2)}{\pi^L_1 (x_2) - \pi^c_1} = \frac{x_2^2}{(1 - x_2)^2 - (1 - q_2)^2}
\]

\[
= \frac{\left( \frac{2 - 3q_2}{2 - 2q_2}q_2 \right)^2}{\left( 1 - \frac{2 - 3q_2}{2 - 2q_2}q_2 \right)^2 - (1 - q_2)^2} = \frac{9q_2^2 - 12q_2 + 4}{5q_2^2 - 8q_2 + 4}
\]

For \( q_2 \in [0, \frac{1}{3}] \), we have that \( \delta_1 \) belongs to \( [\frac{a}{17}, 1] \). We conclude, that it is not possible to sustain tacit collusion at the monopoly profit, if \( \delta = \frac{1}{2} \). Hence we conclude, that while it is possible to sustain a reputation and earn monopoly profits for firm 1, when capacity is virtual, and the discount factor equals \( \frac{1}{2} \), it is not possible to sustain tacit collusion when firm 2 is a long run player and has its own capacity when the discount factor is this low. It follows that reputation building is easier when capacity is virtual.

### 5 Concluding remarks

Virtual capacity has been a new and interesting feature in major merger cases. Competition authorities have tried to mitigate the anti-competitive effects of mergers by requiring that the merging firm auctions off virtual capacity to prospective competitors in the final market. We have shown
that indeed in a static setting this has the expected competitive effects. As regards the longer run effects the picture is more blurred. The fact that the merging firm pockets the profit of the competitor through the initial auction, gives incentives to moderate production so that the price is kept high, the value of the virtual capacity is enhanced and so is auction revenue. This allows it to build a reputation for maintaining monopoly profits and the competitive effects of virtual capacity vanishes. The analysis points to that a long duration for the contract for virtual capacity increases competitiveness and it points to that it would be good to limit the period in time for which the competitive remedies rely on virtual capacity.

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