

Roemer, John; Betts, Julian R.

**Working Paper**

## Equalizing educational opportunity through educational finance reform

Working Paper, No. 99-8

**Provided in Cooperation with:**

University of California Davis, Department of Economics

*Suggested Citation:* Roemer, John; Betts, Julian R. (1999) : Equalizing educational opportunity through educational finance reform, Working Paper, No. 99-8, University of California, Department of Economics, Davis, CA

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October 14, 1998

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“Equalizing Opportunity Through Educational Finance Reform”

by

Julian R. Betts  
Department of Economics  
UCSD, La Jolla, CA 92093-0508  
(619) 534-3369  
and  
The Public Policy Institute of California  
(415) 291-4474  
jbetts@ucsd.edu

and

John E. Roemer  
Department of Economics  
University of California  
One Shields Avenue  
Davis CA 95616  
(530) 752-3226  
jeroemer@ucdavis.edu

October 1998

The research for this project has been partially funded by the John D. and Catherine T. MacArthur Foundation and the Public Policy Institute of California. We also would like to thank Ron White for valuable research assistance.

## Abstract

We analyze the reallocations of educational expenditures required to equalize opportunities, according to the theory of Roemer (1998). Using the NLSYM data set, we find that implementing an equal-opportunity policy across men of different races, by using educational finance as the instrument, and holding per capita educational finance fixed, would require spending six to ten times as much on black students, per capita, as on white students. Implementing an equal-opportunity policy across men from different socio-economic backgrounds, but ignoring race, does almost nothing to equalize opportunities for men of different races. Raising the school-leaving age by one year, as opposed to increasing spending per pupil directly, is a relatively inexpensive way of reducing inequality of opportunity across races, but the reduction in opportunity inequality it achieves is very small.

JEL categories: D63, I22, I28

Key Words: Equal opportunity, educational finance, school quality

## 1. Introduction

Education is the means par excellence by which democracies attempt to equalize opportunities among citizens for economic success. It is commonly thought that opportunity equalization, in that dimension, is implemented by the provision of equal educational resources to all young citizens. We will argue here that that is not so, and we will attempt to compute the distribution of educational finance in the United States that would equalize opportunities for a measure of economic welfare, namely, earning capacity.

Specifically, we examine the relative effectiveness of changing educational expenditures along both the intensive and the extensive margin. By changes along the intensive margin we mean reallocating spending per pupil in public schools. Changes along the extensive margin, in contrast, entail increasing the school-leaving age so that many students would stay in school longer. Over the last twenty to thirty years, and in fact throughout the twentieth century, public school systems have radically increased spending per pupil in real terms. (See for instance Hanushek and Rivkin, 1997 or Betts, 1996.) This century has also seen significant tightening (or institution) of compulsory attendance laws. Lang and Kropp (1986) document that most increases in the school-leaving age, to their typical current level of 16 years, largely occurred before 1960. It is important from a policy perspective to learn about the relative effectiveness of these two types of educational reform for equalizing opportunity.

Significant bodies of empirical work examine the impact of school spending and years of education on adults' earnings. Relatively little work has used this literature to

estimate the magnitude of educational reform that would be required to equalize opportunities across workers from different backgrounds. Such an analysis requires estimates of the impact of the either type of reform on earnings for each type of worker in society, and an analysis of the required reallocation, or increase, in education dollars needed to level the playing field. The goal of this paper is to provide estimates of the extent to which increasing spending per pupil or raising the school-leaving age contribute to creating equality of opportunity.

The next section outlines the theory of equal opportunity, and discusses what equality of opportunity has come to mean in the United States over the last thirty years. Section 3 describes the data and presents regression estimates of the impact of school spending. Section 4 summarizes the algorithm used to compute the equal-opportunity policy. Section 5 discusses the optimal spending per pupil that we derive using this algorithm. Section 6 estimates the returns to an extra year of schooling, and compares the relative effectiveness of increasing the school-leaving age and reallocating spending per pupil. Section 7 concludes with a summary of the most important policy implications that emerge.

## 2. The theory of equality of opportunity

Our goal is to calculate the reallocation of educational spending needed to equalize opportunities among students for future earning capacity. To do so first requires a short review of a theory of equal opportunity that one of us has recently elaborated (Roemer [1998]), a theory that attempts to formalize the 'level the playing field' metaphor. The gulleys and mounds of the playing field are the advantages and disadvantages that

people enjoy or suffer, with regard to attaining some goal (here, the capacity to earn income), based on *circumstances* for which society believes they should not be held accountable -- such as their race, or the socio-economic status of their parents. In contrast to circumstances, an equal-opportunity ethic maintains that differences in the degree to which individuals achieve the goal in question that arise from their differential expenditure of *effort* are, morally speaking, perfectly all right. It is crucial to understand that by *effort* we mean not only the extent to which a person exerts himself or herself, but all the other background traits of the individual that might affect his or her success, but which we exclude from the list of *circumstances*. The partition of causes into circumstances and effort is the central move that distinguishes an equal-opportunity ethic from an equal- outcome ethic. Equal opportunity, in contrast, emphasizes the moral responsibility of the individual, holding that an individual has a claim against society for a low outcome only if he expended sufficiently high effort.

Five words constitute the vocabulary of the equal-opportunity theory: circumstances, type, effort, objective, and instrument. A *type* is the set of individuals with the same circumstances. The *objective* is the thing for which opportunities are to be equalized (the 'opportunity equalisandum'), and the *instrument* is the policy intervention - in our case, educational finance. We may state, verbally and somewhat imprecisely, that the equal-opportunity (EOp) policy is the value (or specification) of the instrument which makes it the case that an agent's expected value of the objective is a function only of his effort and not of his circumstances. Thus, educational finance, if it is to equalize opportunities for future earning capacity, should make it the case that a young person's expected wage be a function only of his effort and not of his circumstances.

We can formulate this in a precise manner as follows. We suppose that a list of circumstances has been specified, as has a unidimensional measure of effort. First, we partition the relevant population into  $T$  types. We suppose that the expected value of the objective for individuals in type  $t$  is a function  $u^t(x,e)$ , where  $x$  is the 'resource' that the individual is allocated by the policy instrument and  $e$  is the effort she expends. Suppose, for the moment that all those in type  $t$  are allocated an amount  $x^t$  of the resource -- in our case, educational finance. Then there will ensue a distribution of effort in that type, to be denoted by a probability distribution  $F^t(x^t)$ . ( $x^t$  is a parameter of the distribution; its support is an interval of effort levels.) These distributions will differ across types, even should different types be assigned the same amount of the resource. Note that the distribution functional  $F^t$  is a *characteristic of the type*, not of any individual. This apparently trivial remark is important.

Equality of opportunity holds that individuals should not be held responsible for their circumstances, that is, for their type. In constructing an inter-type comparable measure of effort, we must therefore take account of the fact that some individuals come from types with 'good' distributions of effort, and some from types with 'poor' distributions -- for coming from a type with a poor distribution of effort should not count against a person. We therefore take the inter-type comparable measure of effort to be the *centile* of the effort distribution in his type at which an individual sits. We say that all individuals at the  $\pi^{\text{th}}$  centile of their effort distributions, across types, have tried equally hard, or been equally responsible. It would be wrong to pass judgments on the quality of effort expended by individuals in different types by looking at their pure expenditure of effort, for those numbers are polluted, as far as the theory is concerned, by being drawn

from distributions for which we do not wish to hold the individuals responsible. Using the centile measure of effort, on the other hand, factors out the 'good' or 'bad' nature of the distribution of effort in the type.

Our task is therefore: To find that value of the policy which makes it the case that, *at each centile*, the expected values of the objective *across types*, are 'equal.' Since equality will virtually never be possible, we really mean 'maximinned' where we just wrote 'equal.' Unfortunately, this instruction is incoherent, for it amounts to maximizing many objectives simultaneously, and so some second-best approach must be taken.

Let  $v^t(\pi, x^t)$  be the (average) value of the objective for individuals in type  $t$ , at centile  $\pi$  of the effort distribution in type  $t$ , if the type is allocated  $x^t$  in resource by the policy instrument. If we fix a particular value of  $\pi$  in the interval  $[0,1]$ , there will be a policy  $x(\pi) = (x^1, x^2, \dots, x^T)$  that solves the following program:

$$\begin{aligned} & \underset{x^1, x^2, \dots, x^T}{\text{Max}} \underset{t}{\text{Min}} v^t(\mathbf{p}, \mathbf{x}^t) \\ & \text{subject to } (x^1, \dots, x^T) \in X \end{aligned}$$

where  $X$  is the feasible set of policies.  $x(\pi)$  is the policy that maximins the value of objective for all agents at effort centile  $\pi$ . If  $x(\pi)$  were the same policy for all  $\pi$ , that would be, unambiguously, the equal-opportunity policy. But that will never be the case in actual applications, and so we compromise as follows. We create an aggregate objective which gives each effort centile in the population its per capita weight in the aggregate objective: namely, we declare the equal-opportunity policy to be the one which solves the program:



$$\underset{(x^1, \dots, x^T)}{\text{Max}} \int_0^1 \underset{t}{\text{Min}} v^t(\mathbf{p}, \mathbf{x}^t) d\mathbf{p}, \quad (2.1)$$

subject to  $(x^1, \dots, x^T) \in X$ .

Thus, given a specification of the circumstances, the effort measure, the objective, and the instrument, and given the data necessary to calculate the functions  $v^t$ , we can solve for the equal-opportunity policy. Actually, we never completely equalize opportunities according to this formulation: what we do is find the policy that equalizes opportunities as much as possible, given the resource constraints (the feasible set of policies).

In what follows, we apply this theory -- which the reader can find elaborated at more length, and philosophically justified, in Roemer (1998) -- to educational policy in the United States.

### *Equality of Opportunity in Practice*

To what extent does the theory of equal-opportunity outlined above correspond to the intent behind current legislation and practices related to affirmative action in the United States? As argued in Roemer (1998), one conception of what equal opportunity requires is the principle of non-discrimination. In labor markets, this approach says that employers should judge job applicants solely on their productivity, rather than upon characteristics such as race or nationality. The non-discrimination requirement lies at the heart of the Civil Rights Act of 1964.

But a second definition of equal opportunity, and the one that we use in this paper, argues that non-discrimination is insufficient for equalizing opportunities. One must

compensate for historical inequities to the extent that they adversely affect the circumstances of living individuals. This view has come to dominate the practical application of affirmative action in the United States in recent years.

Donohue (1994) argues persuasively that in recent years employment law has evolved from a 'non-discrimination' view, one that advocates 'intrinsic equality', toward an approach resembling our conception of equal opportunity, one he refers to as 'constructed equality'. For the two decades following the second world war, Donohue says that law -- particularly the Civil Rights Act of 1964 -- sought to establish intrinsic equality in the workplace, where wages of different workers are judged to be intrinsically equal if they are those that would be forthcoming in a perfectly competitive labor market. Thus, to the extent that low wages paid to black workers are the consequence of market imperfections, which allow discriminatory employer attitudes to survive, then law requiring that wages be 'intrinsically equal' will provide a remedy. Of course, a set of perfectly competitive markets is meritocratic -- employees, in particular, will be paid their marginal value products, and those marginal productivities reflect circumstances as well as effort. Thus, intrinsic equality does not implement equality of opportunity in our sense.

But Donohue writes that in recent years, law has sought to attain a higher goal that he calls 'constructed equality.' Under constructed equality, 'the dictates of law are defined no longer through some abstract market paradigm but rather through considering what steps would be necessary to define a fair society (Donohue[1994, p.2611]).' The American Disabilities Act (ADA), passed in 1991, is his primary example. The ADA does not require employers to pay disabled workers their (competitive) market values, but

rather to provide them with ‘reasonable accommodations’ that enable these workers to become more productive. As Donohue summarizes, ‘Thus, the transformation that has occurred in the realm of civil rights is that the ideal nondiscriminatory market solution, which previously was both the benchmark of intrinsic equality and what the law demanded, is now regarded as the obstacle to social justice (p. 2609).’ In our language, the ADA requires employers to supply extra resources to disabled workers on account of their disadvantageous circumstances, so that their productivity is more truly reflective of their effort. Donohue conjectures that economically disadvantaged classes may proceed, on the example of the ADA, to seek remedies from employers to compensate for their objectively lower productivity, due to economic and social circumstances<sup>1</sup>. If this indeed occurs, it will mark a transformation of employment law to an opportunity-equalizing device.

To summarize, the Americans with Disabilities Act specifically adopts the view that society must compensate for circumstances beyond a person’s control, as in the theory of equal opportunity outlined in this paper and in Roemer (1998).

Other examples of equal opportunity in the real world are provided by current educational practice. In 1975 the Education for All Handicapped Children Act put into place requirements for schools to provide additional services to handicapped children. This provides a clear example of equal-opportunity legislation, since it attempts to level

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<sup>1</sup> ‘The ADA has paved the way for the possibility that economically disadvantaged minorities such as blacks...will employ the ADA’s rationale to argue that the effects of the factors that have undermined their productivity -- including very poor schooling and broken families -- are now to be corrected by employers ( Donohue p.2612).’

the playing field by spending *more* than the average on students with learning or physical disabilities.<sup>2</sup>

The way in which American universities admit applicants provides a final example of how equal opportunity, rather than non-discrimination, has come into common use in the United States. Under a non-discriminatory admissions policy, a university would select students based on grades or test scores. But instead of using purely meritocratic procedures, admissions committees supplement students' grades and test scores with information on personal and family background. Typically, universities have set lower admissions standards for minorities in the belief that this could help to correct the racial imbalance still observed in many skilled occupations. This practice provides a clear example of how society in recent decades has pursued equal-opportunity policies with a view to compensating for disadvantageous circumstances.

Of course, in the last few years court decisions and voter initiatives have led public universities in Texas and California to end their policy of using race as a marker of disadvantage when making admission decisions. In both states, universities are now actively considering alternative forms of affirmative action in admissions, that, for instance, take into account whether either parent of a student has attended university. As will be shown below, a switch from a race-based equal-opportunity program to one that conditions on socioeconomic traits such as parental education leads to radically different recommendations. We consider this to be one of the important findings of the ensuing analysis.

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<sup>2</sup> For a description of this legislation, and its impact on overall educational spending between 1980 and 1990, see section IV of Hanushek and Rivkin (1997).

### 3. Data and regression results for spending per pupil

#### *Data*

We choose as objective the logarithm of an individual's weekly wages as a young adult. We model log weekly earnings from the National Longitudinal Survey of Young Men (NLSYM), computed as the log of the product of hours per week and hourly wages, and adjusted to 1990 prices using the Consumer Price Index. Spending per pupil in the student's district, gathered from a 1968 survey of high schools, is also included in the analysis as the policy instrument. The regression sample for each race consists of all wage observations between 1966 and 1981 for workers who were 18 or older and who were not enrolled in school or college in the given year. We drop a wage observation if weekly earnings are below \$50 or above \$5000 in 1990 prices.

#### *Outline of the Empirical Estimates on Spending per Pupil*

We will examine the reallocation of spending per pupil that would be necessary to equalize opportunities for (weekly) earnings. Such reallocations have been at the heart of court-mandated school reform over the last quarter century. We at first focus on reallocations of spending per pupil across types of student, given a fixed educational budget. However, since such reallocations are virtually guaranteed to reduce spending per pupil for certain types, we also calculate EOp solutions where the constraint is not a fixed budget but a requirement that no type receive less than a pre-specified amount per

pupil. Since no students become worse off in an absolute sense, this second approach is perhaps more politically realistic, but is potentially quite costly.

Recall that we partition each person's traits into two sets, those against which we wish to indemnify the person (circumstances), and those for which we hold the person accountable (effort). The former traits are used to partition people into types; the latter traits are treated as the person's choice variables. If we define many types, for instance by distinguishing people not only by race but also by marital status, geographic location and so forth, our EOp policy in general will call for a more differentiated allocation of expenditures.

With this in mind, we begin with a relatively conservative approach, in which we define only two types -- black and white -- thus holding each person in our sample accountable for all other traits, such as family background, geographic location (both region of the country and rural/urban/suburban residence). The use of two types also allows for a relatively intuitive discussion of the optimal policy. We then consider outcomes using parental education as an additional or alternative factor in determining type.

Our second task is to conceptualize and compute the percentile ranking of each person by effort, conditional upon type and spending per pupil. Recall that "effort" is just short-hand for what we more accurately called the aspect of autonomous volition in a person's behavior. In reality, effort is a multi-dimensional variable, which includes not only years of schooling but marital status, region, and other personal choices.

The approach we take is to define a person's effort ranking in his type as his ranking in the distribution of weekly earnings itself, conditional upon type and spending

per pupil. Whatever effort is, its increase should be reflected (on average) in an increase in the weekly wage. Not only will various personal choices be captured in effort, so measured, but so will be luck. An individual who earns a high wage simply by virtue of inheriting his father's good job will be classified as one who expended high effort. It is important to bear in mind the *conservative* nature<sup>3</sup> of this assumption when considering the estimates presented below of the extent to which school resources would be reallocated to maximize the EOp objective.

We use three different but related estimation techniques.

#### *Method 1*

Recall that the centile,  $\pi$ , of the effort distribution within type  $t$  is the observed effort level (measured by wages) conditional upon  $x$ , observed spending per pupil. To calculate  $\pi$  for each individual, we first partitioned people, within type, into 10 deciles based on spending per pupil in their school district. Within each of these spending ranges, we calculated the person's ranking in terms of the log weekly wage after de-meaning by mean log wages for workers of the same age and type. We then use this ranking to assign  $\pi$ . In the second step, we regressed log weekly earnings on a function of spending per pupil,  $x$ , and  $\pi$ . Since earnings tend to rise with age, it is necessary to adjust the wage data for variations in earnings that are due to variations in age of the workers. Accordingly, the model which we estimate separately for blacks and whites models log earnings after adjusting by the mean of log earnings of all males workers of the given race and age in the sample:

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<sup>3</sup> Conservative in that sense that Robert Nozick (1974) says that a person is morally entitled to benefit by virtue of luck -- the luck, for instance, of being born into a wealthy

$$\log w_{ia}^t - \overline{\log w_a^t} = a_1^t + a_2^t p_{ia}^t + a_3^t p_{ia}^t x_{ia}^t + a_4^t x_{ia}^t + m_i + e_{ia}^t \quad (3.1)$$

where  $t$  indexes the worker's type,  $i$  indexes the worker and the subscript  $a$  indicates the worker's age. The right-hand side of (3.1) is our function  $v^t(\pi, x^t)$ . Note that, in (3.1), the log of wages is modeled as a linear function of  $\pi$ , which is important for our optimization algorithm (see section 4). The compound error term includes a white noise error and a random effect  $\mu_i$  to account for our observing most individuals' earnings repeatedly. Failure to include the random effect in this grouped data would likely have led to inflated  $t$ -statistics (Moulton, 1989).

### *Method 2*

Method 2 is identical to method 1, except that it uses a trimmed wage sample, in which we dropped observations for which weekly earnings were below \$150 or above \$1500. This provides a useful robustness check since a few outliers in the wage distribution could significantly influence the coefficients on the terms involving  $\pi$ .

### *Method 3*

The model in (3.1) provides an easy and obvious way to compute the  $v^t$  functions that our theory requires. But a practical problem is that  $\pi$  is defined as a wage rank (conditional upon the decile of spending per pupil into which workers fall), which is likely to be an endogenous regressor.

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family.



Our third estimation technique eliminates the need to include  $\pi$  as a regressor. In this method, we use quantile regression to estimate models of the form:

$$\log w_{ia}^t - \overline{\log w_a^t} = \mathbf{b}_p^t + \mathbf{g}_p^t \mathbf{x}_{ia}^t + \mathbf{e}_{ia}^t \quad (3.2)$$

such that  $Quantile_p(\log w_{ia}^t - \overline{\log w_a^t} | \mathbf{x}_{ia}^t) = \mathbf{b}_p^t + \mathbf{g}_p^t \mathbf{x}_{ia}^t$  is the conditional quantile for the age-demeaned wages for a given  $\pi$ . We estimate this model nine times for each type of worker for quantiles  $\pi=0.1, 0.2, \dots, 0.9$ . The coefficient estimates are calculated by minimizing the following objective function for the  $\pi$ -th centile for type t:

$$\sum_{i,a} \lambda_{ia} \left| \log w_{ia}^t - \overline{\log w_a^t} - \mathbf{b}_p^t - \mathbf{g}_p^t \mathbf{x}_{ia}^t \right| \quad (3.3)$$

where  $\lambda_{ia}$  are weights defined by

$$\lambda_{ia} = \begin{cases} 2p, & \text{if } \log(w_{ia}^t - \overline{\log w_a^t} - \mathbf{b}_p^t - \mathbf{g}_p^t \mathbf{x}_{ia}^t) > 0 \\ 2(1-p), & \text{otherwise} \end{cases} \quad (3.4)$$

After performing this quantile regression for each value of  $\pi$  between 0.1 and 0.9, we then model the set of nine estimated coefficients  $\hat{\mathbf{b}}_p^t$  for the various values of  $\pi$  as a linear function of an intercept and  $\pi$ . We perform similar estimates using the nine observed values of  $\hat{\mathbf{b}}_p^t$  for each worker type. Specifically, we estimate for each type of worker t:

$$\hat{\mathbf{b}}_p^t = a_1^t + a_2^t \pi + \mathbf{u} \quad (3.5)$$

and

$$\hat{\mathbf{g}}_p^t = a_3^t + a_4^t \pi + \mathbf{x} \quad (3.6)$$

Both of these equations are estimated by weighted least squares, using the inverse of the sample variance on the estimated coefficients  $\beta$  and  $\gamma$  for the given value of  $\pi$  and type t

as weights.<sup>4</sup> Together, these regressions yield estimates  $\tilde{a}_j^t$  corresponding to the four  $a_j^t$  terms in (3.1),  $j=1, \dots, 4$ :

$$\log w_{ia}^t - \overline{\log w_a^t} = \tilde{a}_1^t + \tilde{a}_2^t p_{ia}^t + \tilde{a}_3^t p_{ia}^t x_{ia}^t + \tilde{a}_4^t x_{ia}^t \quad (3.7)$$

This method has three distinct advantages. It is entirely consistent with the theory outlined earlier in that  $\pi$  is defined conditional upon  $x$ . Second, this two-step method avoids the need to include  $\pi$  as a potentially endogenous regressor in the model. Third, the pattern of coefficients obtained from the nine quantile regressions performed for each type of worker  $t$  allows for an informal evaluation of the extent to which the relation between  $v^t$  and  $\pi$  is indeed linear.

### *Regression Results*

#### *Method 1 – Random Effects*

Table 1 presents estimates for these equations for black and white men in the NLSYM, estimated using the Generalized Least Squares estimator for random effect models. For both samples, earnings are positively associated with spending per pupil, although for blacks the gains are limited to those in the bottom 90% of the effort distribution.<sup>5</sup> It is important to remember that in our terminology effort refers to all of the

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<sup>4</sup> In practice, we also estimated these models without weights as a robustness check, and obtained similar results.

<sup>5</sup> In a regression for black males without an interaction between  $x$  and  $\pi$ , spending per pupil enters positively and significantly, with a coefficient and t-statistic of 0.0314 and 3.79 respectively.

traits and actions of the person apart from those traits which we include in the list of circumstances, which in this typology is limited to the person's race.

Tests for white noise errors strongly reject this null against the alternative of random effects in this and later tables. The table also reports the probability value for a Hausman test of the consistency of the random effects models. For both models, the null of consistency is retained.

We next present estimates based on two different partitions of the sample of workers into types. First, we partition blacks and whites into two approximately equally sized groups, based on the years of schooling of the more highly educated parent. This typology yields four types in total -- it is an appropriate partition if society takes into account that more than race influences a young person's chances in life. For young black men, we partitioned the sample approximately in half by including men whose more highly educated parent had nine or fewer years of schooling in one type and those whose parental education was 10 or more years in the other type. The closest we could come to partitioning the white sample in half was to use "fewer than 12 years of education for the more highly educated parent" as the criterion for the less advantaged type.

Table 2 shows the regression results for the four types based on workers' race and their parents' level of education. Among blacks whose parents finished nine or fewer years of schooling, spending per pupil is significantly and positively related to earnings, at least for the bottom 72% of the type, when ranked by  $\pi$ . The estimates for blacks with at least one parent with 10 or more years of schooling are fairly imprecise, but suggest a

positive relation between school resources and earnings.<sup>6</sup> For whites with parents who did not finish high school, there is a positive and significant relation between spending per pupil and the worker's subsequent wages, with an apparently constant impact of spending per pupil as  $\pi$  varies. For whites with more highly educated parents, a non-linear relationship emerges, in which the effectiveness rises with the student's ranking  $\pi$ .

In a third typology, we partitioned workers into two types, based on whether either parent had obtained 12 or more years of schooling, without regard to race.<sup>7</sup> By not taking account of race, such a typology runs against the nature of recent affirmative action programs. But ballot and court decisions in California and Texas have led to prohibitions on the use of race as an identifying variable in affirmative action programs such as those related to college admissions. It therefore is salient to study the implications of a color-blind equal-opportunity policy.

Table 3 shows the results when white and black workers are pooled together, and only parental education is taken to be a circumstance. For both types, spending per pupil appears to have a positive and significant impact on earnings. Among workers whose parents dropped out of high school, spending is more effective for those with low  $\pi$ . The opposite is true for workers who had at least one parent with at least 12 years of schooling.

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<sup>6</sup> To guard against the possibility that the interaction term between  $\pi$  and spending was creating collinearity with the linear spending term, we re-estimated the model without the interaction. In this simpler model the coefficient on spending per pupil was 0.0208, with a t-statistic of 1.96, suggesting a statistically significant relationship at the 5% level.

*Method 2 – Random Effects on Trimmed Samples*

To guard against the effects of outliers in the above estimates, we re-ran the models after trimming the wage samples further by excluding observations for which weekly earnings were below \$150 or above \$1500. The relevant coefficients appear in Table 4. Omitting unusually high or low wages in general increases the t-statistics on the spending per pupil terms significantly. At the median effort value of  $\pi=0.5$ , the estimated effect of spending per pupil is very close to what is reported in Tables 1 through 3. However, the interaction term in most cases changes considerably, which indicates that extreme wage values have had a fairly large effect on the estimated variation in the effectiveness of spending per pupil across different values of  $\pi$ . We take no firm position on which set of estimates is the more reliable. In principle, we want to include all but the most flagrant outliers among the wage observations in order to obtain a representative picture of the effect of school spending. On the other hand, it is important to point out the sensitivity of the cross term to the inclusion of high and low wages.

*Method 3 – Quantile Regressions on Full Sample*

Our third estimation technique is to employ the two-step method based on quantile regressions (3.2), followed by weighted OLS regressions (3.5) and (3.6). The

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<sup>7</sup> This division comes the closest to dividing the sample of wage observations into two equally sized types.

coefficients from these regressions are then combined to create estimated equations of the form of (3.7). Since one of the chief benefits of quantile regression is that it is more robust to outliers than is OLS or random effects, we applied this technique to the full samples used in Method 1.

Space constraints prevent us from displaying all nine models for  $\pi=0.1, 0.2, \dots, 0.9$  for each type, but Table A-1 in the appendix presents results for each type of worker for quantiles 0.2, 0.5 and 0.8. The t-statistics in this table are based on bootstrapped standard errors, which were calculated using 100 repetitions.

The second-stage regressions (3.5) and (3.6) were run separately for each type. The resulting estimates of the  $a_j^t$  terms are presented for each type in the three typologies in Table 5. The coefficients are similar to those estimated by method 1, and also to those obtained by method 2, that is, random effects on the trimmed sample.

An additional benefit of the quantile regression approach is that it allows us to gauge the accuracy of the assumption that the  $v^t$  functions in (3.1) are linear in  $\pi$ . In most cases, plots of the coefficients against  $\pi$  revealed quite linear relations. Indeed, t-statistics on the coefficients for  $\pi$  in (3.5) and (3.6) were typically on the order of 3 to 30. The one major exception was the second stage regression (3.6) for whites, in which no relation to  $\pi$  was apparent, with the coefficient on  $x$  fluctuating for values of  $\pi$  between 0.1 and 0.7 and then declining toward zero as  $\pi$  increased further. This pattern is reflected in the very small negative coefficient on the interaction term for  $\pi x$  shown in Table 5.

Overall, the similarity of the coefficient estimates across the three methods and the particularly close estimates between the trimmed-sample estimate and the full-sample quantile regressions suggest that the results are quite robust.

### *Other Robustness Checks*

The regressions in Tables 1 through 3 assume a linear relation between log wages and spending per pupil. There are no a priori reasons to expect a fully linear relationship. Suppose, for example, that there are diminishing returns to school spending. Using Census data on wages combined with rough state-level proxies of school inputs, Betts and Johnson (1997) find some evidence of diminishing returns, especially with respect to the teacher-pupil ratio. Hence, as an additional test of robustness, we re-ran all of the models in Tables 1 through 3 adding the square of spending per pupil ( $x^2$ ). In all regressions but two, the squared term was not significant at the 5% level. The two exceptions were whites (Table 1) and whites with at least one parent with 12 or more years of schooling (Table 2). In both cases, the regressions indicated that spending per pupil is subject to mildly diminishing returns (in the sense that the  $x^2$  term was negative).

As a final robustness check we returned to the two cases in Table 2 in which the interaction term between  $\pi$  and  $x$  was not statistically significant, and re-ran the models with only  $x$  by itself. Little changed in these models.

In our estimation, neither of these final robustness checks leads to sufficiently important changes to the results to merit re-calculation of the EOp policy. Accordingly, in the next section we place equal emphasis on EOp policies based on the results using

methods 1, 2 and 3, (random effects on untrimmed and trimmed samples, and results based on quantile regressions on the untrimmed sample).

#### 4. Solving the equal-opportunity program

Let  $(p^1, \dots, p^T)$  be the distribution of types in the population. Then our problem is to solve:

$$\underset{(x^1, \dots, x^T)}{\text{Max}} \int_0^1 \underset{t}{\text{Min}} v^t(\mathbf{p}, \mathbf{x}^t) d\mathbf{p} \quad (4.1)$$

$$\text{subject to } \sum_1^T p^t \mathbf{x}^t = r,$$

where  $r$  is the dollar amount of educational finance allocated per capita for the target population. Because the objective function is not differentiable, we cannot straightaway write down the Kuhn-Tucker conditions necessary for the solution. However, a simple variational analysis, presented in Roemer (1998, chapter 11), derives a set of first-order conditions, which we now describe. Although these conditions are general, they take a particularly simple form when the functions  $v^t$  are linear in  $\pi$ . As the reader notes, we have, for this reason, constructed these functions to be linear in  $\pi$  in our econometric estimation. That assumption will be maintained in this section.

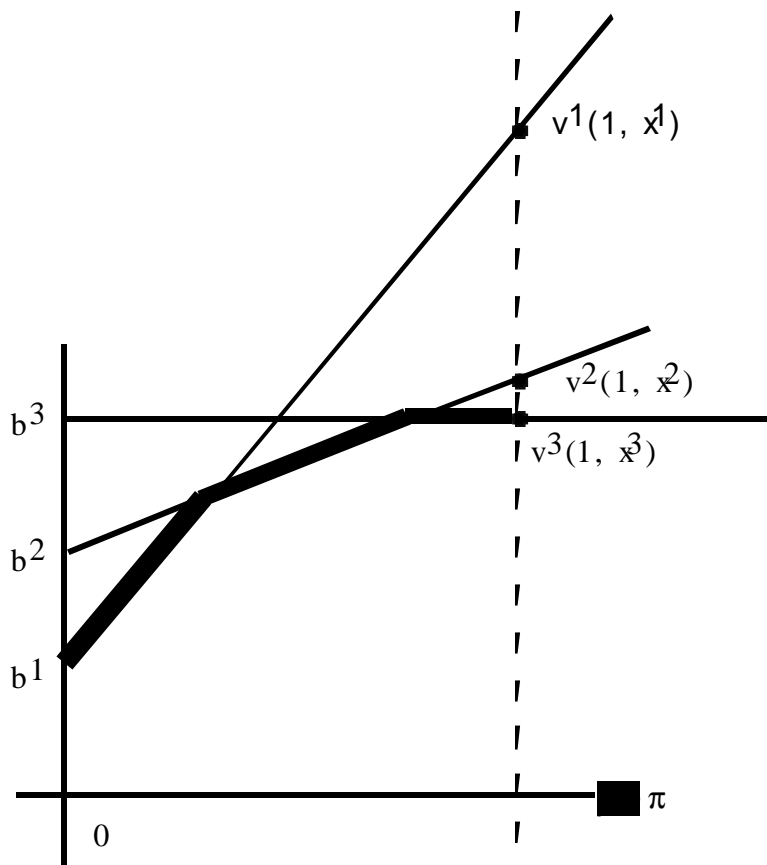
Define the function

$$V(\pi; x^1, \dots, x^T) = \underset{t}{\text{Min}} v^t(\mathbf{p}, \mathbf{x}^t).$$

$V$  is a piece-wise linear function in  $\pi$ : it is the lower envelope of the functions  $v^t$ , always viewed here as functions of  $\pi$ . Program (4.1) says to choose the instrument to maximize



the area under the lower envelope  $V$ . In figure 1, we illustrate the lower envelope for a hypothetical problem with three types.



**Figure 1:** An illustration for  $T = 3$

Let  $\pi_1, \dots, \pi_{T-1}$  be the  $T-1$  values of  $\pi$  at which there is a kink in the function  $V$ .

(In all our applications, every function  $v^t$  will in fact intersect the lower envelope in an interval, as in figure 1, and so there will be  $T-1$  such points.) Re-order the types, if necessary, so that  $v^1$  is the first segment on the lower envelope,  $v^2$  is the second segment, ..., and  $v^T$  is the  $T^{\text{th}}$  segment. Then at the solution to (4.1), the following conditions must hold:

$$\left. \begin{aligned}
 v^1(\pi_1, x^1) &= v^2(\pi_1, x^2) \\
 v^2(\pi_2, x^2) &= v^3(\pi_2, x^3) \\
 &\cdot \\
 &\cdot \\
 &\cdot \\
 v^{T-1}(\pi_{T-1}, x^{T-1}) &= v^T(\pi_{T-1}, x^T)
 \end{aligned} \right\} \quad (4.2)$$

Equations (4.2) simply state the fact that the various  $v^t$  graphs intersect at the points  $\{\pi_t\}$ .

In addition, we have the first-order conditions

$$\frac{\int_0^{p^1} \frac{\partial V^1}{\partial x} (p, x^1) dp}{\int_{p^t}^{p^{t+1}} \frac{\partial V^{t+1}}{\partial x} (p, x^{t+1}) dp} = \frac{p^1}{p^{t+1}}, \quad \text{for } t = 1, \dots, T-1. \quad (4.3)$$

Equations (4.2), (4.3), and the budget constraint of (4.1) together constitute  $2T-1$  equations in  $2T-1$  unknowns  $x^1, \dots, x^T, \pi_1, \dots, \pi_{T-1}$ . Since the problem is concave, any solution to these equations, which characterize a local solution, is also a global solution of (4.1).

The one remaining problem in solving (4.1) is to determine the order in which the functions  $v^t$  intersect the lower envelope at the optimum. There is an algorithmic technique for doing this which we shall not here describe. All that the reader must understand is that if we have determined a set of values  $x^1, \dots, x^T, \pi_1, \dots, \pi_{T-1}$  which satisfy (4.2) and (4.3) and the order of the functions  $\{v^t\}$  on the lower envelope is in fact the postulated order, then we have solved the program.

##### 5. Calculation of the spending allocations that implement equal opportunity

The next step is to calculate the optimal allocation of spending per pupil among types, subject to a budget constraint stipulating that average spending per pupil in the population is given. Beginning with the simply black/white typology in Table 1, we first calculated the optimal allocation of educational funding under the assumption that average spending per pupil ( $r$ ) is limited to \$2500 in 1990 prices, which is approximately the average in the NLSYM sample.<sup>8</sup> Using the procedure outlined earlier, we derive not only optimal spending subject to this budget constraint, but also the crossover point  $\pi_1$  at which the graphs of the indirect utility functions ( $v(\pi, x^1)$ ) of the two types of worker cross in  $\pi - v$  space.

Egalitarian policies are criticized for being ‘inefficient’, that is, decreasing output. It is possible, but not certain, that when one reallocates educational expenditures between different types, the overall wage bill will shrink, if the marginal product of educational resources is higher for the type from which funding is being removed. Therefore we also calculate the ratio of the wage bill that is predicted to result from the EOp policy to the wage bill under the *equal resource* policy, in which all students receive the same amount of the financial resource. Our calculations based on the black/white typology in Table 1

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<sup>8</sup> Taking all observations in 1966, the weighted mean spending per pupil, in 1990 prices, was \$2233. Spending per pupil has grown steadily since then. Current expenditures per pupil in American public schools during the 1990-91 school year were \$4847. (National Center for Education Statistics, 1991, p. 155).

assume that 12.0% of the population is black and that 88.0% is white, which matches the population frequencies in 1966 in the NLSYM.<sup>9</sup>

We also calculate the required aggregate budget which assures that, under the EOp policy, all types would receive at least \$2,500 per capita. This exercise assumes that such a 'no-lose' option might be politically necessary in order to implement an EOp policy in reality.

We report the results for three partitions of the sample into types: (A) black (B) and white (W), (B) low(L) and high (H) parental education, and (C) the four types obtained by crossing {B,W} with {L,H}.

For each of these type partitions, we performed four computations of the optimal solution to the EOp program. First, we compute the EOp solution when the resource endowment is \$2,500 per capita ( $r = 2.5$ ). Second, we compute what the value of the aggregate endowment would have to be for no type to be allocated less than \$2,500 per capita. Further, we do each of these two computations for both the untrimmed and the trimmed samples.

#### A. Type partition: Black and white

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<sup>9</sup> The bottom portion of Tables 1, 2 and 3 show estimates of the distribution of the population of men in 1966 by type, and mean spending per pupil by type. Both of these were calculated using sampling weights from 1966, on all available 1966 observations. The bottom of these tables also show weekly earnings by type averaged over all wage observations in all years, using sample weights. The frequencies of worker types in Table 2 do not exactly add up within races to the frequencies reported in Table 1, due to a slightly smaller sample once parental education is added to the race variable.

The results are reported in Table 6. Under each of the three methods, in the EOp solution, blacks must receive approximately ten times what whites receive when  $r = 2.5$ . If  $r$  increases to the point where whites receive \$2,500 per capita, then this ratio falls, so that blacks receive approximately six times as much per capita. The penultimate column in the table reports the ratio of the average wage in the sample, according to our regression equation, at the EOp allocation, to the average wage in the 'equal resource (ER)' allocation, when both types receive exactly  $r$  amount of the resource. That statistic is a measure of the 'inefficiency' of the EOp policy, in the precise sense of output lost due to implementing it. We get slightly greater 'inefficiency' of EOp with the untrimmed OLS method than with the other two methods; the loss is on the order of 2%-3%.

Figure 2 compares the graphs of the  $v^t$  functions, for  $r = 2.5$ , in the EOp and ER solutions, based on the trimmed OLS method. The dashed heavy and light lines are the  $v^{\text{black}}$  and  $v^{\text{white}}$  functions, respectively, in the equal-resource solution. The solid heavy and light lines are the  $v^{\text{black}}$  and  $v^{\text{white}}$  functions in the EOp solution. We see, graphically, how EOp narrows the differential in outcomes in comparison to the ER solution<sup>10</sup>.

Figure 3 illustrates the lower envelope of the  $v^t$  functions for  $r = 4.08$ .

The reallocation of school resources needed to equalize opportunity between black and white men is substantial. Note, though, that our wage sample is drawn from the years 1966-1981. To check whether it is possible that today smaller reallocations would be required, we examined data on usual weekly earnings of full-time male workers

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<sup>10</sup> One should add a caveat. In the data, the highest per capita educational expenditure on a black individual was \$4372, far less than we contemplate in the EOp policy. The usual caveat applies about the credibility of the extrapolation of our estimated wage function.

by race, as reported for the year 1996 in the Current Population Survey. Strikingly, the size of the wage gap between black and white men is almost identical in 1996 to the average value observed in the NLSYM data. The ratio of blacks' earnings to those of whites in 1996 was 71.0%, compared to 72.2% in our sample of wages over the period 1966-1981. In absolute terms, the black-white wage gap in the NLSYM data was \$149 per week in 1990 prices (Table 1). In 1996, the same gap was \$140.<sup>11</sup> Some readers may be surprised that the ratio and absolute gap in wages between black and white male workers changed so little between 1966-81 and 1996, although a number of researchers have documented the slowing of the convergence in wages between blacks and whites during the 1980's.

The implication for our analysis is simple. Although our wage observations are centered in the 1970's, the black-white wage gap has changed so little over the last two decades that our results would be virtually unchanged if we used recent wage distributions.

#### B. Type partition: Low and High parental education

Table 7 reports the results for the partition of the sample into two socio-economic types. Again, the three regression methods yield quite similar results.

It is obvious that opportunities are ex ante less unequal for the two SES types than for the two racial types. This is seen both in the ratio of  $x_{Lo}$  to  $x_{Hi}$ , compared to  $x_B$  to  $x_W$ ,

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<sup>11</sup> Data for 1996 earnings by race and data for the Consumer Price Index required to deflate to 1990 prices were taken from U.S. Bureau of the Census. (1997, pages 431, 497).

and in the fact that a smaller increment in aggregate resources is needed than in Table 6 to bring the more advantaged type up to an EOp resource allocation of \$2,500 per capita.

Furthermore, the efficiency cost of implementing EOp for the SES type partition is only on the order of 1 - 1.5%.

Figure 4 displays the graphs of  $v^{\text{Hi}}$  (light line) and  $v^{\text{Lo}}$  (heavy line) at the EOp solution, for  $r = 2.5$ . We see the lines are almost coincident: that is, the EOp policy succeeds in virtually equalizing opportunities completely.

C. Type partition: Low-Black (LB), High-Black (HB), Low-White (LW), High-White (HW)

There are two observations of similarity among the three regression methods displayed in Table 8. First, the black types receive much more than the white types; second, in the case of  $r = 2.5$ , the three most disadvantaged types (LB, HB, and LW) all receive more than their per capita share of the resource at the EOp solution -- only the HW type receives less. The second statement, however, is not true at the higher level of resource endowment. In those experiments (for the trimmed and untrimmed OLS), both white types receive less than their per capita share and both black types more. There is some variation among the EOp solutions of the three methods.

Figure 5 shows the graph of the EOp solution for  $r = 2.5$ , based on the trimmed OLS method. Notice that the two white types have  $v^{\text{t}}$  graphs that are virtually coincident at the EOp solution, and the Hi-Black type is not too far away: it is the Lo-Black type that is the real outlier.

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Figure 6 graphs the EOp solution for the quantile regression method. It is remarkable how similar this graph is to figure 5.

One singularity of Table 8 deserves comment : in the untrimmed sample,  $x_{HB}$  is greater than  $x_{LB}$ . We attribute this to the weight of very low income observations in the Lo-Black sample -- it is inefficient, from the viewpoint of our EOp objective, to spend too much on LB. When these very low income observations are trimmed away, then  $x_{LB}$  is greater than  $x_{HB}$ , which is perhaps more intuitive as a policy recommendation.

D. Type partition:  $E_1=\{PE<8\}$ ,  $E_2=\{8 < PE < 12\}$ ,  $E_3=\{ PE =12\}$ ,  $E_4=\{PE. < 12\}$

We were surprised by the very limited redistribution implied by division of students into two categories based on parental education. Would a more radical reallocation result if we divided students into four groups based on parental education? Accordingly, we partition the sample into four types based on the level of parental education (PE), as noted above. Table 9 and Figure 7 present the EOp solution, for the untrimmed OLS method. Again we observe that the disadvantage associated with coming from a very poorly educated family appears to be less than the disadvantage of being black. Figure 7 shows that, in the EOp solution, we succeed in equalizing opportunities for the more advantaged three types almost completely (their  $v^l$  lines are almost coincident); it is the  $E_1$  who remain the outliers.

An interesting inference follows from the results reported here for the on-going debate on affirmative action. As we wrote earlier, the emerging view in the United States seems to be that affirmative action, at least with regard to university admissions, is desirable when it is used to favor students of low socio-economic status, but not when it



is used to favor students of color<sup>12</sup>. In our language, this view holds that the type partition into low and high SES is ethically acceptable, but not the one into black and white. The natural question is, to what extent will opportunities be equalized in our society by recognizing differential socio-economic, but not differential racial, circumstances?

Our results suggest a pessimistic conclusion. The four-type partition (Tables 8 and 9) says that, to equalize opportunities, blacks of both low and high socio-economic status must be allocated substantially more resources than whites of low and high socio-economic status. If we recognize only SES types, we have the allocations of Tables 6 and 7, which do little to compensate blacks in comparison to what they receive in Tables 8 and 9.

Evidently, the effects of race and racism in American society upon economic prospects of blacks are only very partially captured by the socio-economic status of their parents -- at least by our measure of parental education. To formalize this idea, in Table 10 we show the percentage of black men in the regression samples in each of the earnings quintiles before and after our various EOp policies are put into place. We focus on the EOp allocations calculated using the trimmed samples. In order to put workers of different ages on an equal footing, we adjusted the weekly wages of each worker by subtracting the difference between mean earnings for all workers of his age and the mean earnings of workers of age 30.

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<sup>12</sup> Indeed, Ward Connerly, who spear-headed the initiative on the University of California Board of Regents to abolish race-based affirmative action admissions holds this view. He said, "UC should use economic status and other genuine hardships when making special admissions, not race. (Sacramento Bee, May 20, 1995, p. B1)"

The top row shows that in the raw data, blacks predominantly occupy the bottom two earnings quintiles. For the sake of comparison, we also show the result when all students receive spending of 2.5. We predict earnings under this scenario using the parameter estimates from the top two rows of Table 4. The results are quite similar to the raw data. We then estimate the wages each worker would earn if various reallocations were put into effect. Since the crossover point is so high, our EOp policy (B/W,  $r=2.5$ ) greatly improves the earnings of blacks relative to whites, so that the median black now occupies the second highest quintile. The alternative EOp policy, with  $r=3.95$ , pushes blacks toward the second and third quintiles.

A quite remarkable result is shown in the next two rows: when type is defined independently of race, and only parental education is used, the EOp reallocations leave the distribution of black workers across earnings quintiles *virtually unchanged from the status quo*. Even though 74% of blacks in the regression sample are in the type with low parental education, and so receive spending of 3.628, this is a small reallocation relative to the more advantaged type, which receives 2.502. This limited reallocation, combined with the fact that 35% of whites also fall into the bottom socioeconomic group, implies that the gap in mean earnings between blacks and whites is virtually unchanged after the EOp policy is implemented.

To see whether a “race-blind” policy could do more to equalize opportunity between blacks and whites if the EOp policy were based on a finer disaggregation of workers by parental education, we checked the distribution of blacks across earnings quintiles that would follow from the optimal solution in our typology with four ‘parental education’ types. Even here, the distribution of black workers across earnings quintiles

would change little. The bottom two rows of Table 10 show that the new distributions with  $r=2.5$  or  $r=3.48$  do virtually nothing to change the distribution of black workers across earnings quintiles.

Our analysis focuses on grade school, and so cannot speak directly to the postsecondary issue. But it seems clear that using proxies for race, such as parental education, will at best lead to equality-of-opportunity policies that are a weak imitation of what would be required to equalize opportunity across races. Our calculations suggest that an equality-of-opportunity policy based on two or even four levels of parental education does almost nothing to improve the income share of black men. Indeed, the most dramatic change is that in the fixed-budget calculation, the share of blacks in the bottom earnings quintile is predicted to drop from 38.1% to 35.3% once school spending is reallocated to equalize opportunity across workers based on parental education only.

Overall, our results suggest that reallocation of spending per pupil can significantly alter the distribution of earnings. However, the marginal effect of spending per pupil on adult wages is very small. The broader literature on school quality, as reviewed in Betts (1996), confirms this fact. Consequently, if society were to take equality of opportunity seriously, radical reallocations of educational expenditures would be required. These reallocations go far beyond merely equalizing spending across student types. This fact is noteworthy, since court-mandated reforms in school finance over the last 30 years have typically ordered at most equalization of spending across schools.

## 6. Comparing Changes in School Spending and Changes in the School-Leaving Age as Alternative Means of Equalizing Opportunity

In this section we examine the impact of increasing the school-leaving age from 16 to 17 years. We do this by estimating a wage model that conditions upon years of schooling, and then predicting the gains in wages that would result if every person in the sample who has less than 11 years of schooling stayed in school one year longer.

We ran random effect models of the natural log of weekly wages on a constant, dummies for all but one of the age levels in the sample (18 to 40), and years of schooling. This specification is similar to the one used to model the impact of school spending, except that now we do not condition on  $\pi$  since 'effort' is in part determined by a person's choice of how long to stay in school. Also, instead of de-meaning log wages by age we add age dummies on the right hand side, which is more appropriate given that years of schooling do tend to increase for individuals slightly with age.

We ran these regressions for the trimmed black and white samples. The coefficients (and t-statistics) on years of schooling were 0.0385 (6.98) for blacks and 0.0256 (10.94) for whites. These estimates are similar to what earlier work with the NLSYM has produced. For instance, Griliches (1977) obtained a coefficient of 0.022 when using the log hourly wage in 1969 as the dependent variable. These estimates are likely to appear low to many readers. The main reason for this is that our model and the model described from the Griliches paper condition on age rather than potential experience, typically defined as (age-years of schooling-6). Griliches (1977) reports that the returns to education rise from 0.022 to 0.065 when age controls are replaced by experience controls. Similarly, when we re-ran our models after replacing age dummies with potential experience, the coefficient on education for blacks and whites rose to 0.06262 and 0.06295 respectively. However, in these models, the marginal impact of

increased schooling on log wages holding constant age is the *difference* between the coefficients on education and potential experience. These marginal effects were 0.0403 and 0.0319, respectively, in these models, similar to what we report above. In the following analysis, we use the estimates from the model that conditions on age dummies rather than experience.

We now compare the costs and the benefits of increasing the school-leaving age and increasing spending per pupil. We work with the typology {B,W}. We measure benefits as the value of the EOp objective function, that is, the mean of the lower envelope of the earnings: $\pi$  functions by type. Using the trimmed sample, we ranked all wage observations within race using age-demeaned earnings (i.e. the dependent variable in (3.1)) and assigned each person to one of 100 centiles for each race. We then calculated mean earnings by centile for each race, and made them comparable between races by adding in the natural log of mean earnings as reported in Table 1.

Table 11 shows the value of the EOp objective function for various scenarios. The table presents this mean in dollar terms to aid understanding. The “base case” scenario is one in which mean  $x$  is \$2500 ( $r=2.5$ ).<sup>13</sup> The value of (the exponential) of the mean along the lower envelope, which in the base case consists exclusively of blacks, is \$399.07 per week. The second row (“equal resources”) shows the gains that would result if all schools spent exactly \$2500 per pupil. As shown, the average gain in earnings for workers on the lower envelope is \$2.25 per week, or about 0.5%. The next two rows

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<sup>13</sup> We use \$2500 to provide comparability with the simulations based on the EOp solutions presented in the previous section. Since the actual mean spending per pupil was slightly below \$2500 in the sample, we increased spending per pupil proportionately

show the (exponential of) the mean value of log wages on the lower envelope for the two EOp solutions, first where average spending is held constant at \$2500 per week, and then the cost-increasing intervention in which both types receive at least \$2500 per week. The gains in average earnings along the lower envelope are very large in both cases, between \$111 and \$131 per week, increases well over 25% above the base case.

The final experiment is an increase in the school-leaving age from 16 to 17. Predicted wage gains for individuals were calculated as the coefficient on years of schooling from the random effects regressions reported at the start of this section if the person had finished ten or fewer years of schooling, and zero otherwise. In the trimmed wage sample, 6.66% of whites and 16.63% of blacks held ten or fewer years of schooling. The predicted gain in the mean log wage of those on the lower envelope was \$2.57, slightly more than the gains that we predict to result from equalization of spending.<sup>14 15</sup>

One lesson is already clear: full implementation of EOp does far more than simple equalization of spending across schools or an increase in the school-leaving age to raise the earnings of those along the lower envelope. This raises an important question: what

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across workers, and calculated the predicted gain in earnings using the trimmed regression results in Table 4.

<sup>14</sup> The effect of raising the school-leaving age from 16 to 18 would be just under twice as big, given that an additional 11.91% of blacks in the sample held exactly 11 years of schooling.

<sup>15</sup> We also estimated the returns to education for the {High, Low} typology and the typology {HW, LW, HB, LB}. Due to space constraints, we do not show these results. But calculations of the impact on the mean value of the empirically derived lower envelope told a similar story to that told above for the {B,W} typology. The equal resources program (at  $r=2.5$ ) the compulsory attendance program each increased the objective function by \$1 to \$3 per week. These figures were dwarfed by either type of EOp program based on reallocation of spending per pupil, for which gains in the objective function ranged from roughly \$35 (for the {High, Low} typology) to \$130 for the four-way typology.

are the relative sizes of the costs of implementing the various programs? Starting from a base of \$2500 per pupil, equalizing spending at that level or implementing the EOp plan with mean spending  $r=2.5$  have no impact on costs. Of course, even equalization of spending across schools, let alone the radical reallocation suggested by EOp with  $r=2.5$ , may not be politically feasible, since some types (whites, in the present analysis) face lower spending per pupil after the reallocation.

Consider next the cost of the EOp program with minimum spending of \$2500 per person of either type. To evaluate its cost per pupil, we assume that any change in spending occurs from kindergarten through the year in which the pupil leaves school, which is appropriate since our measure of spending per pupil is measured for the school district in which the student attended school. Using the empirical distribution of years of schooling, we then calculate the cumulative change in spending per pupil from kindergarten up to the year in which the student left school (or Grade 12 in the case of those with more than 12 years of schooling). We convert all expenditures to their value in the year in which the student would have been in Grade 12, using a discount rate of 2.67%, which is the mean real interest rate over the period 1953 to 1997.<sup>16</sup>

The cost of increasing the school-leaving age by one year will depend on how actual spending per pupil is divided into fixed and variable costs. Classroom expenditures typically account for about 60 percent of total educational expenditures, with overhead and administrative costs taking up much of the remaining costs. For

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<sup>16</sup> This real interest rate was calculated as the yield on ten-year federal bonds minus the percentage change in the Consumer Price Index (for all urban consumers). The period 1953 to 1997 represents the widest time span possible with the available data. Sources

instance, in 1987-88 instructional expenditures accounted for 61.7 percent of total current expenditures on public K-12 education (National Center for Education Statistics, 1991, p. 154). If additional students stay in high school longer, classroom spending should rise proportionately, but overhead and capital spending might have to rise as well, as construction of additional classroom space might be required, bussing costs might rise and so on. Accordingly, in a manner similar to Betts (1996), we assume that the addition of one student to a school for a year will raise total spending by 80% of average district spending per pupil.

We then calculate the average cost per person of increasing the school leaving age as the sum across the two races of: 80% of mean spending per pupil by race from Table 1, multiplied by the fraction of each race with fewer than 11 years of schooling, multiplied by the shares of each race in the population from Table 1. Finally, we multiply this cost by 1.0267 to inflate the cost, incurred when the student is 17, to the year in which the student would have reached Grade 12 or age 18.

An informal cost-benefit comparison of increasing the school-leaving age and implementing EOp through differential increases in spending per pupil is intriguing. The EOp plan increases the mean earnings along the lower envelope by \$131.13, or about fifty times more than does increasing the school-leaving age. But the costs of achieving EOp in this way are extremely large: in terms of present value of spending in the year in which the person turns 18, the cost is over \$21,000 per person. Note that this figure is obtained by dividing total program cost by the number of people in the entire population. All of

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are the Economic Report of the President (Council of Economic Advisers, 1998) and the Bureau of Labor Statistics respectively.



this additional spending is directed toward blacks, who on average receive an extra \$184,000 while in school. This is spread out over the entire population, bringing the cost down to roughly \$21000 per person. In contrast, increasing the school-leaving age costs only \$142 per person.

Note that in Table 11 it is inappropriate to compare the costs and benefits directly since the costs are the present value of accumulated spending for all students in all grade levels, while our measure of benefit focuses on workers who are on the lower envelope only, and represents the gains during a typical year, rather than over the entire working lifetime. But as a rough measure of the relative cost effectiveness of the two programs, note that the ratio of the benefits gained in the EOp policy to the benefits from increasing the school-leaving age is  $\$131.13/\$2.57$ , or 51.0. But the ratio of the costs is fully 149.4. By this comparison, increasing the school-leaving age appears to be approximately three times as cost effective as the EOp program. Of course, this must be balanced against the realization that raising the school-leaving age does very little to equalize opportunities. This is shown by the very small gain in the mean of the lower envelope for this intervention of only \$2.57 per week.

There are two reasons why increasing the school-leaving age appears to be a much more cost-effective, if limited, way of equalizing opportunity. The first reason is that spending per pupil has a very modest impact on students' subsequent earnings. For instance, in a review of the literature, Betts (1996) calculates that using even the most optimistic estimates of the impact of increasing spending per pupil, the internal rate of return to this type of expenditure is only about 23% as high as the internal rate of return to having a student spend an extra year in high school. The second reason for the relatively

low cost effectiveness of increasing school spending is that under the “no-lose” EOp plan average spending rises dramatically. Furthermore, the value of the EOp objective at its optimum, viewed as a function of  $r$  (the per capita resource endowment), is a concave increasing function, and the ratio of the this ‘value function’ (our ‘benefit’) to  $r$  is a convex, decreasing function. Therefore, the benefit-cost ratio of an EOp program that increases dramatically the resources spent on education will be small. This fact provides a second reason why the pseudo-benefit-cost ratio of the relatively modest reform that increases the school-leaving age by one year is greater than the benefit-cost ratio of the very costly school-spending reform in which no type receives less than \$2500 per capita.

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## 7. Concluding Comments

We conclude by briefly reiterating some of the more important policy implications of this analysis.

First, even though court battles on educational finance have typically centered on the goal of equalizing spending across schools, our analysis suggests that this alone will do little to equalize opportunity, especially across races. The underlying reason is quite simple: the impact of school spending on students’ subsequent wages is rather modest.

Second, in order to equalize opportunity across races at a given budget, government would have to reallocate spending radically. Our results vary depending on

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<sup>17</sup> It might be more salient to note that we could achieve the benefit that the former reform achieves by *decreasing* resources spent on education, were we to allocate them according to the EOp recommendation. (This follows immediately from the fact that we can

whether overall spending is held constant, or spending is increased such that no type experiences a decrease in school funding. In the first case, equalizing opportunity between races entails spending ten times as much on blacks as on whites. In the second case, six times as much must be spent on blacks.

Third, it matters enormously whether a program to equalize opportunity takes race into consideration. This insight is important given recent moves in California and Texas to eliminate race as a factor that is considered in university admissions. We found that a “color-blind” EOp program that equalizes opportunities between types of student differentiated only by parental education does virtually nothing to change the distribution of blacks across earnings quintiles. In the language of our model, given such a race-neutral policy, any variations in earnings that are correlated with race would be attributed to variations in “effort” rather than “circumstance”. Thus, a color-blind EOp program based on socioeconomic traits other than race costs relatively little, but achieves relatively little as well. This has important implications for the affirmative action debate: affirmative action programs that do not take race into account explicitly are likely to do little to reduce variations in outcomes between races.

Fourth, we compared the relative effectiveness of equalizing opportunities across races using differential per capita spending on the races, while increasing the national education budget by approximately 60%, and, alternatively, of increasing the school-leaving age by one year. The former approach increases earnings along the lower envelope by \$131 per week, while the latter increases those earnings by only \$2.57 per

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achieve an *increase* in the benefit of \$111.01 at zero cost under the EOp program in which  $r=2.5$ .)

week . On the other hand, it would cost about \$21,000 per person in the economy to finance the former program, compared to just \$140 per person for increasing the school-leaving age. While the naive benefit-cost ratio for the latter program looks better, that comparison is not terribly salient, because we are comparing a huge program to a tiny one, and we can expect that the benefits will be a concave (increasing) function of costs. We note that in theory it is possible to reallocate spending per pupil at zero cost, by reducing spending for whites while increasing it for blacks. Of course, such a reform is likely to be much less politically feasible than a more expensive one which guarantees that no student sees a reduction in school spending.

A final remark concerning the practical implementation of our educational financial reforms is in order. We realize that implementing such reforms, which allocate more money to disadvantaged types than to advantaged ones, is a distant possibility, in a society that has not yet even fully implemented the 'equal resource' allocation. Nevertheless, we believe it is important to separate the normative analysis from a discussion of what reforms are politically feasible. Our analysis has centered on what equality of opportunity requires. Moreover, as we have emphasized, we have taken a quite minimalist, and therefore conservative view, of what characteristics of an individual's environment constitute his circumstances. Knowing what theory recommends, we can then begin the process of compromising as political reality requires.

**Table 1**  
**Estimates of Impact of Spending per Pupil on Earnings by Race**

<b>RACE:</b>	<b>Blacks</b>	<b>White</b>
Constant	-0.8501	-0.7360
	(-33.52)	(-65.08)
$\pi$	1.5865	1.2528
	(42.10)	(72.19)
( $\pi$ )(Spending per pupil)	-0.0751	0.0296
	(-4.20)	(3.94)
Spending per pupil (x)	0.0678	0.0342
	(5.69)	(7.00)
Number of Obs	3298	15451
R-squared - within	0.8586	0.7946
- between	0.8948	0.9017
- overall	0.8932	0.8741
P-value OLS vs. Random Effects (R.E.)	0.0000	0.0000
P-value: Hausman: R.E. vs. Fixed Effects	0.4294	0.4001
Estimated share of population, 1966	12.0%	88.0%
Estimated mean earnings of workers in this type, 1966-81	385.34	533.96
Mean spending per pupil, ( `000s)	2.091	2.243

Weekly wages and spending per pupil are expressed in 1990 prices. Spending per pupil is expressed in thousands of dollars.

**Table 2**  
**Estimates of Impact of Spending per Pupil on Earnings by Race and Parental Education**

RACE:	Black	Black	White	White
PARENTAL EDUCATION:	ED<10	ED <sup>3</sup> 10	ED<12	ED <sup>3</sup> 12
Constant	-0.8791	-0.7583	-0.7742	-0.6758
	(-20.87)	(-17.22)	(-37.54)	(-47.43)
$\pi$	1.6471	1.4364	1.2735	1.1770
	(33.12)	(19.76)	(42.82)	(53.17)
$(\pi)$ (Spending per pupil)	-0.1336	-0.0276	-0.0064	0.0639
	(-5.70)	(-0.82)	(-0.50)	(6.66)
Spending per pupil (x)	0.0961	0.0350	0.0650	0.0074
	(4.82)	(1.71)	(7.32)	(1.20)
Number of Obs	1470	1278	5095	9518
R-squared - within	0.8542	0.8223	0.8009	0.7845
- between	0.8415	0.9139	0.8848	0.8966
- overall	0.8673	0.8748	0.8635	0.8674
P-value OLS vs. Random Effects (R.E.)	0.0000	0.0000	0.0000	0.0000
P-value: Hausman: R.E. vs. Fixed Effects	0.0022	0.0701	0.1717	0.1787
Estimated share of population, 1966	5.6%	5.0%	30.4%	59.0%
Estimated mean earnings of workers in this type, 1966-81	369.94	409.55	508.36	546.35
Mean spending per pupil, ('000s)	2.017	2.148	2.248	2.242

Weekly wages and spending per pupil are expressed in 1990 prices. Spending per pupil is expressed in thousands of dollars.

**Table 3**  
**Estimates of Impact of Spending per Pupil on Earnings by Parental Education**

<b>PARENTAL EDUCATION:</b>	<b>ED &lt; 12</b>	<b>ED<sup>3</sup>12</b>
Constant	-0.9188	-0.6983
	(-50.36)	(-50.29)
$\pi$	1.4875	1.2157
	(56.98)	(55.68)
( $\pi$ )(Spending per pupil)	-0.0608	0.0570
	(-5.27)	(6.01)
Spending per pupil (x)	0.1113	0.0123
	(13.82)	(2.04)
Number of Obs	7177	10318
R-squared - within	0.8125	0.7844
- between	0.8855	0.8969
- overall	0.8757	0.8672
P-value OLS vs. Random Effects (R.E.)	0.0000	0.0000
P-value: Hausman: R.E. vs. Fixed Effects	0.0084	0.4762
Estimated share of population, 1966	38.0%	62.0%
Estimated mean earnings of workers in this type, 1966-81	482.07	541.70
Mean spending per pupil, ('000s)	2.213	2.244

Weekly wages and spending per pupil are expressed in 1990 prices. Spending per pupil is expressed in thousands of dollars.

**Table 4**  
**Estimated Coefficients and t-statistics from Repetition of Models in**  
**Tables 1 to 3 Using a Trimmed Sample**

<b>Sample</b>	<b>(p)(x)</b>	<b>t-statistic</b>	<b>x</b>	<b>t-statistic</b>	<b>Sample Size</b>
<b>Black</b>	-0.0619	-5.62	0.0668	9.09	3219
<b>White</b>	-0.0066	-1.32	0.0522	16.41	15090
<b>Black Parental Ed &lt; 10</b>	-0.1114	-7.58	0.0866	7.16	1430
<b>Black Parental Ed <sup>≥</sup> 10</b>	-0.0442	-2.15	0.0514	4.11	1249
<b>White Parental Ed &lt; 12</b>	-0.0354	-3.99	0.0797	12.46	5013
<b>White Parental Ed <sup>≥</sup> 12</b>	0.0239	3.79	0.0277	6.95	9258
<b>Parental Ed &lt; 12</b>	-0.0749	-9.93	0.1176	22.20	7043
<b>Parental Ed <sup>≥</sup> 12</b>	0.0164	2.61	0.0329	8.40	10037



**Table 5**  
**Estimated Coefficients from Repetition of Models in Tables 1 to 3 Using**  
**Quantile Regressions on the Untrimmed Sample**

<b>Sample</b>	<b>Constant</b>	<b>p</b>	<b>(p)(x)</b>	<b>x</b>
<b>Black</b>	-0.750248	1.377013	-0.0861565	0.0778601
<b>White</b>	-0.6144554	1.006137	-0.0032941	0.0549204
<b>Black Parental Ed &lt; 10</b>	-0.7839297	1.487014	-0.138636	0.092617
<b>Black Parental Ed <sup>≥</sup> 10</b>	-0.7286847	1.356248	-0.1302618	0.1034774
<b>White Parental Ed &lt; 12</b>	-0.6865997	1.090531	-0.0559029	0.0963129
<b>White Parental Ed <sup>≥</sup> 12</b>	-0.5566784	0.9498873	0.0329285	0.0260043
<b>Parental Ed &lt; 12</b>	-0.7796187	1.249683	-0.0801068	0.1169024
<b>Parental Ed <sup>≥</sup> 12</b>	-0.5646592	0.9681708	0.0277492	0.0292607

Note: These coefficients derive from the second-stage regressions. Coefficients derived from 9 quantile regressions for each type, performed at  $\pi=0.1$  through 0.9, were regressed using weighted least squares on a constant and  $\pi$  to yield the above coefficients. See the text for fuller information.

**Table 10**  
**The Percentage of Black Workers in Each Earnings Quintile in Raw Data and After Various Types of Reallocation of Educational Expenditure**

Note: Earnings data are adjusted for variations in earnings by age. Quintile 5 refers to the fifth of the population with the lowest earnings. Calculations are based on EOp policies calculated using trimmed samples. Changes in earnings for blacks and whites are predicted using coefficients in the first two rows of Table 4.

Description of Allocation	Earnings Quintile				
	5	4	3	2	1
<b>Raw Data</b>	38.1	23.2	17.2	13.7	7.8
<b>r=2.5 for All Workers</b>	38.4	23.0	17.8	13.5	7.3
<b>EOp B/W r=2.5</b>	7.0	12.3	23.7	34.4	22.6
<b>EOp B/W <math>x_{\min}=2.5</math>, <math>r=3.95</math></b>	4.1	9.9	24.2	40.3	21.5
<b>EOP H/L (parental education only) r=2.5</b>	37.4	23.0	17.7	14.1	7.8
<b>EOp H/L <math>x_{\min}=2.5</math>, <math>r=2.93</math></b>	38.0	23.6	17.8	13.6	7.0
<b>EOp (4-type parental education) r=2.5</b>	35.3	24.0	18.1	14.5	8.0
<b>EOp (4-type as above) <math>x_{\min}=2.5</math>, <math>r=3.48</math></b>	37.2	24.9	18.0	13.5	6.4

**Table 11**  
**Estimated Gains in the Objective Function and Costs per Student of**  
**Various Interventions Using the Black-White Typology**

Note: Estimated cost per person is calculated as total program cost divided by the number of persons in the sample, where costs are calculated as a present value in the year in which the person reaches age 18. The “value of objective function” is derived from the average value of the lower envelope in log wage: $\pi$  space, re-expressed in average earnings per week for workers on the envelope. N/A: “not applicable”.

Policy Description	Value of objective function (\$)	Change Relative to Base Case	Estimated Cost per Person
Base Case $r=2.5$	\$399.07	N/A	N/A
Equal resources, $r=2.5$	\$401.32	\$2.25	0
EOp $r=2.5$	\$510.09	\$111.02	0
EOp $x_w=2.5$ ( $r=4.08$ )	\$530.20	\$131.13	\$21,258.80
Raise School-Leaving Age one yr	\$401.64	\$2.57	\$142.25

**Table A-1**  
**Quantile Regressions at p=0.2, 0.5 and 0.8 for Each Type, on Full Sample**

Sample	quantile	constant	t-stat	x	t-stat
<b>Black</b>	0.2	-0.44557	-10.358	0.049266	2.394
	0.5	-0.08007	-2.298	0.043867	2.758
	0.8	0.35683	11.651	0.001883	0.143
<b>White</b>	0.2	-0.40729	-24.743	0.055311	7.824
	0.5	-0.10347	-8.482	0.053912	10.143
	0.8	0.189309	15.204	0.048481	8.771
<b>Black Parental Ed &lt; 10</b>	0.2	-0.43113	-11.014	0.045762	2.444
	0.5	-0.0947	-2.23	0.042737	2.248
	0.8	0.428868	8.983	-0.03212	-1.701
<b>Black Parental Ed <sup>3</sup> 10</b>	0.2	-0.41234	-4.177	0.048784	1.112
	0.5	-0.06836	-1.825	0.054158	2.99
	0.8	0.357287	7.546	-0.00957	-0.461
<b>White Parental Ed &lt; 12</b>	0.2	-0.49281	-21.156	0.096718	10.274
	0.5	-0.13427	-7.291	0.067177	9.012
	0.8	0.163213	7.285	0.055425	5.759
<b>White Parental Ed <sup>3</sup> 12</b>	0.2	-0.35405	-18.29	0.030472	3.419
	0.5	-0.08426	-6.058	0.046748	8.124
	0.8	0.196769	11.757	0.048449	7.238
<b>Parental Ed &lt; 12</b>	0.2	-0.54065	-21.276	0.107332	9.694
	0.5	-0.14182	-6.796	0.077519	8.851
	0.8	0.19724	9.231	0.054903	5.717
<b>Parental Ed <sup>3</sup> 12</b>	0.2	-0.36591	-18.76	0.035691	3.884
	0.5	-0.07274	-5.664	0.043259	7.999
	0.8	0.202303	10.691	0.047069	5.65



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Table 6 EOp solutions, Typology = {Black, White}

r	$x_B$	$x_W$	$\pi$	order	wEOp/wER	method
2.5	12.65	1.115	.59	W,B	.962	untrimmed OLS
4.08	15.56	2.50	.59	W,B	.957	untrimmed OLS
2.5	12.00	1.204	.68	W,B	.976	trimmed OLS
3.95	14.54	2.51	.68	W,B	.974	trimmed OLS
2.5	11.215	1.31	.58	W,B	.971	quantile regress
3.81	13.44	2.50	.58	W,B	.974	quantile regress



Table 7 EOp solutions, Typology = {Lo, Hi}

Lo = parental ed  $\leq$  high school

Hi = parental ed  $>$  high school

r	$x_{Lo}$	$x_{Hi}$	$\pi$	order	wEOp/wER	method
2.5	3.57	1.85	.74	Hi, Lo	.984	untrimmed OLS
3.115	4.11	2.51	.74	Hi, Lo	.983	untrimmed OLS
2.5	3.30	1.85	.70	Hi, Lo	.989	trimmed OLS
2.93	3.63	2.50	.70	Hi, Lo	.992	trimmed OLS
2.5	3.63	1.81	.68	Hi, Lo	.983	quantile regress
3.13	4.16	2.50	.68	Hi, Lo	.986	quantile regress

Table 8 EOp solutions, Typology = {LB, LB, LW, HW}

LB = black and parental ed  $\leq$  10 years  
 HB = black and parental ed  $>$  10 years  
 LW = white and parental ed  $\leq$  12  
 HW = white and parental ed  $>$  12

r	x <sub>LB</sub>	x <sub>HB</sub>	x <sub>LW</sub>	x <sub>HW</sub>	$\pi_1, \pi_2, \pi_3$	order	wEOp/wE R	method
2.5	10.64	12.22	2.69	.81	.06,.39,.43	LW,HW,HB,LB	.972	untrimmed OLS
3.88	11.85	14.39	3.36	2.50	.35,.40,.42	HW, LW, HB, LB	.968	untrimmed OLS
2.5	12.91	9.41	2.53	.91	.38,.48,.51	HW, LW, HB, LB	.980	trimmed OLS
3.898	14.69	11.25	3.41	2.50		HW, LW, HB, LB	.977	trimmed OLS
2.50	10.17	6.06	2.62	1.41	.26,.31,.32	HW, LW, HB, LB	.982	quantile regress

Table 9 EOp solutions, Typology = { E<sub>1</sub>, E<sub>2</sub>, E<sub>3</sub>, E<sub>4</sub> }

E<sub>1</sub> = parental education less than eight years

E<sub>2</sub> = 8 < parental education < 12

E<sub>3</sub> = parental education = 12

E<sub>4</sub> = parental education > 12

r	x <sub>E1</sub>	x <sub>E2</sub>	x <sub>E3</sub>	x <sub>E4</sub>	π <sub>1</sub> , π <sub>2</sub> , π <sub>3</sub>	order	wEOp/wER	method
2.5	4.77	3.03	2.20	.53	.06, .52, .76	E <sub>2</sub> , E <sub>4</sub> , E <sub>3</sub> , E <sub>1</sub>	1.024	untrimmed OLS
3.48	5.68	3.37	2.92	2.49	.45, .52, .76	E <sub>4</sub> , E <sub>2</sub> , E <sub>3</sub> , E <sub>1</sub>	1.015	untrimmed OLS