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## Staggered Price Setting and Endogenous Persistence

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**University of  
California, Davis**

**STAGGERED PRICE SETTING AND  
ENDOGENOUS PERSISTENCE**

**Paul R. Bergin  
Robert C. Feenstra**



**Working Paper 198-05**

**Department  
of Economics**

# Staggered Price Setting and Endogenous Persistence

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# Staggered Price Setting and Endogenous Persistence

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## **Abstract:**

This paper generates persistent effects of a monetary disturbance in the context of staggered price-setters. Previous research has been restricted by the CES functional form to price-setting rules that are constant markups over marginal costs. The present paper considers a translog form for preferences and an input-output structure for production in the context of a dynamic general equilibrium model of monopolistically competitive staggered price-setters. We derive a price-setting rule that is a function of marginal cost and also competitors' prices. This rule better captures the interaction of price-setters envisioned in Taylor (1980) and Blanchard (1983) in their early work on staggered contracts. The model is able to generate reasonable persistence, and also confirms the conjecture of Taylor and Blanchard that increasing the number of contracting groups increases the degree of persistence.

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## 1. Introduction

The real effects of monetary disturbances is a perennial question, which recently has received renewed attention. Several recent papers have worked to incorporate nominal rigidities into dynamic general equilibrium models (including Ohanian and Stockman (1994), Cho and Cooley (1995), King and Watson (1995), Woodford (1996), and Yun (1996)). These models easily can generate real effects of monetary shocks, but they have difficulty in generating persistence of these effects beyond the exogenously imposed rigidity. Such endogenous persistence is an essential feature of the data to replicate.<sup>1</sup>

This issue of persistence has been explored previously in the context of staggered contracts. Taylor (1980) showed that staggered wage contracts as short as one year could generate persistence similar to that observed in the data. Blanchard (1983) showed a similar result in the case of firms that set prices in a staggered fashion. The underlying intuition is as follows: when a firm sets its price, it is influenced by the prices set by other firms with which it must compete. Under staggering, a price-setter will not fully adjust its price to shocks, because the prices of some competing firms will still be fixed during part of the price-setters contract period. Taylor characterizes this mechanism for wage setters:

...when considering relative wages, firms and unions must look both forward and backward in time to see what other workers will be paid during their own contract period. In effect, each contract is written relative to other contracts, and this causes shocks to be passed on from one contract to another – a sort of "contract multiplier."

Taylor and Blanchard also both speculated, but did not demonstrate, that the degree of persistence would be more extreme for cases in which there were a larger number of overlapping price-setters.

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<sup>1</sup> See Christiano, Eichenbaum and Evans (1997) for a discussion of facts that models of monetary policy transmission should replicate

The price-setting rules of Taylor and Blanchard are ad hoc, however, and recent research has worked to derive price-setting rules for optimizing monopolistically-competitive firms, and then embed this in a general equilibrium model. Chari, Kehoe and McGrattan (1996) were unable in such a framework to generate reasonable endogenous persistence. Individual prices are adjusted so that once the last contracting period is over, the aggregate price level has fully adjusted and all real effects of a monetary disturbance disappear.

A limitation of the work to date is that the monopolistic competitors are assumed to face a demand with a constant elasticity of substitution form. This assumption dictates a price-setting rule in which price is a constant markup over marginal cost. Price is not set in response to the prices of competitors, as Taylor and Blanchard had in mind. As a result, when a monetary shock induces a rise in output, the rise in labor costs induces firms to raise their goods price, and this precludes persistence. Given the restriction of a constant markup, the existing literature has sought a solution by focusing on the role of marginal cost. Dotsey, King and Wolman moderate the rise in marginal costs by assuming an implausibly high degree of labor supply elasticity. Erceg (1997) moderates marginal costs by assuming wage stickiness. Kiley (1997) considers shifts in demand composition, in addition to an infinite labor supply elasticity.

The present paper moves away from the restrictive assumption of CES preferences for demanders and considers as an alternative a translog functional form. The result is that the endogenous price-setting rule is not a simple markup over marginal cost, but rather is significantly influenced by competitors' prices. This mechanism of staggered contracts is embedded in a dynamic model of imperfectly competitive firms. The production function incorporates an input output structure, as suggested in Basu (1995), in which firms use the output of other firms as inputs in their own production process. This structure introduces output prices as an important component of marginal costs

faced by firms, and is yet a further reason why the price-setting rule is influenced by competitors' prices.

Results suggest this framework can generate significant endogenous persistence. For the case of two overlapping contracts and reasonable parameters, nearly 40% of the initial impact of a monetary shock on output persists one year after the initial shock, the time at which all prices have been reset. Further, contrary to the findings of Chari, Kehoe and McGrattan (1996), the speculation of Taylor (1980) and Blanchard (1983) are confirmed, that increasing the number of contracting groups somewhat increases the degree of persistence.

The next section presents the basic two-group model, highlighting the use of translog preferences and the implications for the price-setting rule. Section three presents results. Section four extends the model to consider larger numbers of staggered groups.

## 2. The Model

### 2.1 Consumer's Problem

The consumer will allocate income intertemporally to maximize utility, defined over the consumption of differentiated products, real balances, and leisure:

$$\sum_{t=0}^{\infty} \beta^t \left[ \ln u_t + \ln \left( \frac{M_t}{P_t} \right) - \frac{L_t^{1+\sigma}}{1+\sigma} \right] \quad (1)$$

where  $u_t$  is the sub-utility obtained from consumption of the differentiated products,  $M_t$  is money balances.  $P_t$  is an aggregate price index, and  $L_t$  is labor. The parameter  $\sigma$  will equal the inverse of the labor supply elasticity.

Since the work of Dixit and Stiglitz (1977), a common choice for the sub-utility function defined over the differentiated products has been the constant elasticity of substitution (CES) form.<sup>2</sup> Despite its tractability this functional form has serious drawbacks for the analysis of firms' pricing. Since optimal prices are a constant markup over marginal costs, this means that the reaction functions of firms are completely independent of their competitors' prices. In the illustration used in oligopoly models, reaction functions become perfectly horizontal or vertical in price-space, so there is no strategic interaction between the firms.

This special feature of the CES need not carry over to other choices of the sub-utility function. We will consider a sub-utility function defined by the dual expenditure function, which is assumed to have a translog form. That is, given nominal expenditure  $E_t$ , the sub-utility from consumption of the differentiated products  $1, \dots, N$  is  $u_t = E_t/e(p_t)$ , where the unit-expenditure function  $e(p_t)$  is defined by:

$$\ln e_t = \sum_{i=1}^N \alpha_i \ln p_{it} + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \phi_{ij} \ln p_{it} \ln p_{jt} \quad (2)$$

with  $\phi_{ij} = \phi_{ji}$ . In order for this function to be homogeneous of degree one, we need to impose the conditions:

$$\sum_{i=1}^N \alpha_i = 1 \text{ and } \sum_{i=1}^N \phi_{i3} = \sum_{j=1}^N \phi_{ij} = 0 \quad (3)$$

We can differentiate the unit-expenditure function to obtain the expenditure shares  $s_{it} = \partial \ln e_t / \partial \ln p_{it}$ ,

$$s_{it} = \alpha_i + \sum_{j=1}^N \phi_{ij} \ln p_{jt} \quad (4)$$

While (2), (3), and (4) are the general case of the translog function, we can consider a special case

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<sup>2</sup> The general class considered by Dixit and Stiglitz is a sub-utility function that is additively separable over the differentiated products. If we also impose symmetry over consumption of each differentiated product, and homotheticity, then the only additively separable function satisfying these properties is the CES.



where all goods enter symmetrically. In that case, the parameters become,

$$\alpha_i = \frac{1}{N}, \phi_{ii} = -\phi, \text{ and } \phi_{ij} = \frac{\phi}{N-1} \text{ for } j \neq i \quad (5)$$

## 2.2 Single-Period Firm Problem

To illustrate the usefulness of the translog functional form, we first consider a *single-period* problem faced by firms, where there are not staggered multi-period contracts. Denoting the marginal cost faced by the firm by  $c_t$  and the demand for product  $i$  as  $x_{it}$ , the firm problem may be stated:

$$\max (p_{it} - c_t) x_{it} \quad (6)$$

The (positive) elasticity of demand facing the firm for its differentiated product is computed as  $\eta_{it} = 1 - \frac{\partial \ln s_{it}}{\partial \ln p_{it}} = 1 - \frac{\phi_{ii}}{s_{it}}$ , where  $\phi_{ii} < 0$  is needed to ensure that demand is elastic. The first order condition then may be written:

$$p_{it} = c_t \left( 1 - \frac{s_{it}}{\phi_{ii}} \right) \quad (7)$$

The expenditure share can be substituted from (4), and (7) could be solved for the optimal price, in terms of marginal cost and the prices of competitors. This expression is nonlinear (involving  $p_{it}$  and  $\ln p_{it}$ ), so we will take an approximation to allow us to obtain a simple solution for the price. Taking logs of both sides of (7), using  $\ln \left( 1 - \frac{s_{it}}{\phi_{ii}} \right) \approx \frac{s_{it}}{\phi_{ii}}$  (which is valid for  $s_{it}$  small), and substituting for  $s_{it}$  from condition (4) we obtain,

$$\ln p_{it} = \frac{1}{2} \ln c_t - \sum_{j \neq i} \left( \frac{\phi_{ij}}{2\phi_{ii}} \right) \ln p_{jt} - \left( \frac{\alpha_i}{2\phi_{ii}} \right) \quad (8)$$

Or if we impose symmetry conditions (5):

$$\ln p_{it} = \frac{1}{2} \ln c_t + \frac{1}{2} \sum_{j \neq i} \left( \frac{1}{N-1} \right) \ln p_{jt} + \left( \frac{1}{2\phi N} \right) \quad (9)$$

Thus, an increase in marginal costs of 1% will increase the optimal price by 0.5%. This "pass through" coefficient of 0.5 is a feature of the translog demand equations. Empirically, this is not an unreasonable value for the response of price to a change in costs, while holding competitors prices constant.<sup>3</sup> Notice also that a rise in all the prices  $p_{jt}$ ,  $j \neq i$  by 1% will also increase the optimal price  $p_{it}$  by 0.5%, so that the pricing equation is homogeneous of degree one in marginal costs and all competitor's prices.

### 2.3 Multi-Period Firm Problem

Now we will consider the general case in which firms set their price for T periods. In addition, we will suppose that the translog aggregate of the differentiated products serves as both the final consumer good, and as an input into the production function of each firm producing a differentiated product. The aggregate is combined with labor in a Cobb-Douglas production function. The marginal costs of production are denoted by  $c_t = w_t^\theta P_t^{1-\theta}$ , where  $w_t$  is the nominal wage paid to labor, and  $P_t = e(p_t)$  is the aggregate price index. The profit-maximization problem for an individual firm is:

$$\max_{p_{it}} \sum_{\tau=t}^{t+T-1} (p_{i\tau} - c_\tau) x_{i\tau} \quad (10)$$

where the demand for product i,  $x_{i\tau}$ , includes both consumption from consumers, which equals  $s_{i\tau} E_\tau / p_{i\tau}$ , and demand from firms who use the differentiated products as intermediate inputs. Be-

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<sup>3</sup> The literature on exchange rate pass-through is useful here. See for example Hooper and Mann (1989), which find a pass-through coefficient in the range of 50-60% for U.S. importers of manufactures. Khosla (1991) finds a pass-through of 43% for Japanese data. See Moffet (1988) for a summary of other studies with estimates around 50%.

cause  $(1-\theta)$  of costs are devoted to purchases of intermediates, the firms' demand for each input is  $s_{i\tau} (1-\theta) E_\tau c_\tau / P_t$ , where we multiply revenue  $E_t$  by the ratio of marginal cost to price to obtain total costs. In calculating the elasticity of demand we treat  $E_t$  as constant, and also suppose that the firm ignored the impact of its own price on the aggregate price index  $P_t$  (which is reasonable if the number of products is large). Assuming symmetry (5) and using again the demand elasticity  $\eta_{it} = 1 + \frac{\phi}{s_{it}}$ , the first-order condition for the firm's problem may be written:

$$\sum_{\tau=t}^{t+T-1} \left[ \left( \frac{s_{i\tau}}{\phi} + 1 \right) \left( \frac{c_\tau}{p_{i\tau}} \right) - 1 \right] = 0 \quad (11)$$

To express this in a more convenient form, we use the approximations:

$$\left[ \left( \frac{s_{i\tau}}{\phi} + 1 \right) \left( \frac{c_\tau}{p_{i\tau}} \right) - 1 \right] \approx \ln \left[ \left( \frac{s_{i\tau}}{\phi} + 1 \right) \left( \frac{c_\tau}{p_{i\tau}} \right) \right] \approx \left( \frac{s_{i\tau}}{\phi} \right) + \ln \left( \frac{c_\tau}{p_{i\tau}} \right) \quad (12)$$

The first approximation holds if  $\left( \frac{s_{i\tau}}{\phi} + 1 \right) \left( \frac{c_\tau}{p_{i\tau}} \right)$  is close to unity, meaning that the first-order condition (7) for choosing price optimally for each single period is not too far from holding; in other words, we are assuming that the costs and demand conditions are not changing much over the  $T$  periods. The second approximation in (12) is valid for  $s_{it}$  small. Substituting (12) and (4) into (11), we can express the optimal price as:

$$\ln p_{it} = \left( \frac{1}{2T} \right) \sum_{\tau=t}^{t+T-1} \left[ \ln c_\tau + \sum_{j \neq i} \frac{1}{N-1} \ln p_{j\tau} + \left( \frac{1}{\phi N} \right) \right] \quad (13)$$

Note that if we were to set  $T = 1$ , we indeed would find the same pricing rule (9) found previously for the single-period problem.

To gain some intuition into overlapping price setting, we will now focus on the two-period case. We will suppose that there are two groups of firm who set their prices in an overlapping fashion. Firms  $i = 1, \dots, N/2$  choose their price in period  $t$ , where  $t$  is an odd number, and this

price is then fixed for periods  $t$  and  $t + 1$ . Let us denote this price by  $p_{1t}$ , which is assumed to be the same for these firms. Similarly, the firms  $j = (N/2) + 1, \dots, N$  choose their price  $p_{2t}$  in even periods  $t$ , which is then fixed for  $t$  and  $t + 1$ . We apply expression (13) for all firms  $i = 1, \dots, N/2$  choosing their price  $p_{1t}$ , and use  $p_{1t}$  in place of  $p_{it}$ , for  $i = 1, \dots, N/2$ , and  $p_{2t}$  in place of  $p_{jt}$  for  $j = (N/2) + 1, \dots, N$ . Then assuming that  $N$  is large, we can solve for  $p_{1t}$  as:

$$\ln p_{1t} = \frac{1}{3} \left( \ln c_t + \frac{1}{2} \ln p_{2t} + \ln c_{t+1} + \frac{1}{2} \ln p_{2t+1} \right) \quad (14)$$

Of course, an analogous expression holds for  $p_{2t+1}$ , chosen when  $t + 1$  is even:

$$\ln p_{2t+1} = \frac{1}{3} \left( \ln c_{t+1} + \frac{1}{2} \ln p_{1t+1} + \ln c_{t+2} + \frac{1}{2} \ln p_{1t+2} \right) \quad (15)$$

In both of these expressions, the marginal costs are:

$$\ln c_t = \theta \ln w_t + (1 - \theta) \ln P_t \approx \theta \ln w_t + \frac{(1 - \theta)}{2} \ln p_{1t} + \frac{(1 - \theta)}{2} \ln p_{2t} \quad (16)$$

where the approximation holds provided that the prices  $p_{1t}$  and  $p_{2t}$  do not differ too much from each other.

Notice that in (14), the optimal choice for  $p_{1t}$  will depend on the predetermined value of  $p_{2t}$  (both directly and through marginal costs) and also on the future value of  $p_{2t+1}$ . The latter price depends on both  $p_{1t+1} = p_{1t}$  (chosen in the previous period) and  $p_{1t+2}$ . Thus, we can solve for  $p_{1t}$  by substituting (15) and (16) into (14), to obtain the forward looking expression:

$$\begin{aligned} \ln p_{1t} \left[ 1 - \frac{1}{4} \left( \frac{2-\theta}{2+\theta} \right)^2 \right] &= \frac{1}{2} \left( \frac{2-\theta}{2+\theta} \right) \ln p_{2t} + \frac{1}{4} \left( \frac{2-\theta}{2+\theta} \right)^2 \ln p_{1t+2} \\ &+ \left( \frac{\theta}{2+\theta} \right) \ln w_t + \left[ \left( \frac{\theta}{2+\theta} \right) + \frac{\theta(2-\theta)}{2(2+\theta)^2} \right] \ln w_{t+1} + \frac{\theta(2-\theta)}{2(2+\theta)^2} \ln w_{t+2} \end{aligned} \quad (17)$$

As an example, consider the case where  $\theta=1$ , so that the differentiated products are not used

as intermediate inputs. In that case, (17) reduces to:

$$\ln p_{1t} = \left(\frac{6}{35}\right) \ln p_{2t} + \left(\frac{1}{35}\right) \ln p_{1t+2} + \left(\frac{12}{35}\right) \ln w_t + \left(\frac{14}{35}\right) \ln w_{t+1} + \left(\frac{2}{35}\right) \ln w_{t+2} \quad (18)$$

This means that  $(6/35) \approx 0.17$  of the weight in the pricing equation is given to  $p_{2t}$ , which is pre-determined, and  $(1/35) \approx 0.03$  of the weight is given to the future price  $p_{1t+2}$ . The remaining  $(12+14+2)/35 = 0.80$  of the weight is given to wages, which are flexible. These conditions will lead to a substantial flexibility in prices  $p_{1t}$ , due to the large influence of wages. In contrast, suppose that marginal costs are heavily determined by the price of intermediates, so that we choose  $\theta$  rather small. In that case, the weight on wages becomes correspondingly small (approaching zero as  $\theta$  does), while the weight on the price  $p_{2t}$  becomes large, reaching a maximum value of  $2/3$ . This would indicate a large potential degree of price stickiness, as the firms choosing their prices are heavily influenced by those prices that are predetermined.

Another way to evaluate the pricing equation (17) is to compare it to that obtained when the demand for the differentiated product is obtained from a constant elasticity of substitution (CES) function, which is the case assumed previously in the literature. In that case the pricing rule would be:

$$\ln p_{1t} = \frac{1}{2} (\ln c_t + \ln c_{t+1}) \quad (19)$$

Price is set as a constant markup over marginal cost, with no role played by the price set by competitors. Previous papers have displayed a pricing function that looks very similar to our pricing rule (14), which apparently allows competitors' prices to affect price setting. Such a presentation is somewhat misleading, however, because the nominal price in these instances is specified as a function of the real marginal cost, not the nominal, as in (14). As a result the nominal price must

be deflated by the price index, which is an average over the prices set by the two groups. This indirectly introduces competitors' prices into the equation. But this role for competitors prices is very limited as well as indirect, and clearly does not reflect the motivation for staggered contracts in Taylor (1980) and Blanchard (1983).<sup>4</sup>

An equation analogous to (19) holds for  $p_{2t+1}$  in the *CES* case, and the marginal costs in (16) still apply. Combining these three equations, we obtain a forward-looking expression for  $p_{1t}$ :

$$\begin{aligned} \ln p_{1t} \left[ 1 - \frac{1}{4} \left( \frac{1-\theta}{1+\theta} \right)^2 \right] &= \frac{1}{2} \left( \frac{1-\theta}{1+\theta} \right) \ln p_{2t} + \frac{1}{4} \left( \frac{1-\theta}{1+\theta} \right)^2 \ln p_{1t+2} \\ &+ \left( \frac{\theta}{1+\theta} \right) \ln w_t + \left[ \left( \frac{\theta}{1+\theta} \right) + \frac{\theta(1-\theta)}{2(1+\theta)^2} \right] \ln w_{t+1} + \frac{\theta(1-\theta)}{2(1+\theta)^2} \ln w_{t+2} \end{aligned} \quad (20)$$

The weight given to the predetermined prices  $p_{2t}$  in this pricing equation is in general smaller than that given to  $p_{2t}$  in (17). An extreme example is provided when differentiated intermediate inputs are not used at all ( $\theta = 1$ ), in which case the predetermined prices  $p_{2t}$  receive zero weight, while current and future wages each receive weights of 0.5. This helps explain why previous papers have had limited success in generating endogenous persistence with staggered price setting.

## 2.4 Equilibrium Conditions

Into (1) we substitute  $u_t = E_t/e(p_t)$  as the sub-utility from consuming differentiated products, where  $E_t$  is nominal expenditure and  $P_t = e(p_t)$  is the price index. Then the consumer's problem is to choose  $E_t$ ,  $M_t$ ,  $L_t$  and nominal bonds  $B_t$  to maximize:

$$\max \sum_{t=0}^{\infty} \beta^t \left[ \ln u_t + \ln \left( \frac{M_t}{P_t} \right) - \frac{L_t^{1+\sigma}}{1+\sigma} \right] \quad (21)$$

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<sup>4</sup> It should be noted that while Chari, Kehoe and McGrattan focus on a CES form for demand, they do also briefly consider a Stone-Geary functional form.

subject to the budget constraint:

$$E_t + B_{t+1} + M_t = w_t L_t + (1 + i_t) B_t + M_t + \Pi_t \quad (22)$$

where  $i_t$  is the nominal interest rate on bonds, and  $\Pi_t$  is profits received from the firms producing differentiated products. The first-order conditions for this problem are:

$$\frac{E_t}{E_{t-1}} = \beta(1 + i_t) \quad (23)$$

$$\frac{M_t}{E_t} = \frac{1}{i_t} \quad (24)$$

$$L_t^\sigma = \frac{w_t}{E_t} \quad (25)$$

Combining (23) and (24), we obtain  $E_t/E_{t-1} = \beta[1 + (E_t/M_t)]$ , which is a difference equation in  $E_t$ . The only stable solution for  $E_t$  (i.e. that does not approach zero or infinity) is:

$$E_t = [(1/\beta) - 1] (M_t) \quad (26)$$

Therefore, in response to an unanticipated 1% increase in the nominal money supply, nominal expenditure  $E_t$  will increase permanently by exactly 1%.

The response of the nominal wage  $w_t$  can be obtained from labor supply (25), in conjunction with labor demand. A simple way to obtain labor demand is to note that  $\theta$  percent of total costs go to pay labor, so that  $w_t L_t = \theta E_t c_t / P_t$  where we have multiplied total revenue by the ratio of marginal costs to price to obtain costs. Using  $c_t = w_t^\theta P_t^{1-\theta}$  and  $P_t = e(p_t)$ , we combine labor

demand with supply to obtain the equilibrium real wage:"

$$\ln w_t - \ln e(p_t) = \left[ \frac{1 + \sigma}{1 + \sigma(1 - \theta)} \right] (\ln E_t - \ln e(p_t)) \quad (27)$$

Note that  $\ln E_t - \ln e(p_t) = \ln u_t$ , where  $u_t$  is the sub-utility from consumption, which we shall use as a measure of real final output in the economy. Then (27) shows that the elasticity of the real wage with respect to output exceeds unity. The equilibrium conditions in the economy are (23), (24), (27) and the pricing equation (17) or (20). All equations are used in log-linearized form, and the model is solved as a perfect foresight equilibrium except for the initial period.

We calibrate the model as follows. We set the labor supply elasticity  $1/\sigma$ , to unity. this is the value most supported by empirical evidence. Previous studies have been able to achieve some endogenous persistence only by assuming very elastic labor supply, which is contrary to the body of empirical evidence.  $\beta$  will be set to 0.96. As discussed earlier, the benchmark case will set the share of inputs in marginal cost,  $\theta$ , at the value of 0.2 suggested by Basu(1995), although other values will also be considered in sensitivity analysis below.

### 3. Results

The experiment we consider is a permanent shock raising money supply one percent. We consider two price-setting groups and we will regard each period of the model as half a year, so that one year after the monetary shock both groups will have reset their prices.

Consider first a benchmark case in which preferences are CES and there is no input-output structure. Figure 1a shows the impulse response of real final output, in percent changes from the

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<sup>5</sup> We ignore an inessential constant in this equation



initial value. The figure shows that output rises 1 percent initially as a result of the monetary expansion. But there is no endogenous persistence. One year after the shock, when both groups of firms have been able to adjust their goods prices, output is below its long-run level. This replicates the findings of Chari, Kehoe and McGrattan (1996). Figure 1 b helps explain the result. The rise in output requires a large increase in labor input, in turn causing the wage rate to rise significantly. This rise in marginal costs induces firms to raise their price significantly when given the opportunity. In the first period after the shock, the price-setter raises his price in excess of the 1 percent money supply increase, so the aggregate price rises 0.58 percent. In the second period, once both groups have reset prices, aggregate prices are above their long-run level.

Some intuition into this result can be gained by solving the model analytically (following Chari, Kehoe and McGrattan, 1996). Beginning with the CES pricing rule (14), substituting for wage with (27), substituting out expenditure from the linearized form of (26), and writing the aggregate price index as the average of the two goods prices, we may write:

$$E_{t-1} [p_{t+1}] - \frac{2(1+\gamma)}{1-\gamma} E_{t-1} [p_t] + p_{t-1} = -\frac{\gamma}{1-\gamma} (M_t + M_{t-1}) \quad (28)$$

where  $p_t$  is the optimal goods price set in period  $t$ , and  $m_t$  is money, both in log deviations. Also note that in the above:

$$\gamma = \frac{(1+\sigma)\theta}{1+\sigma(1-\theta)} \quad (29)$$

Assuming a random walk process for money, a solution may be written for  $p_t$ :

$$p_t = a_{CES} p_{t-1} + (1 - a_{CES}) m_{t-1} \quad (30)$$

where:

$$a_{CES} = \frac{1 - \sqrt{\gamma}}{1 + \sqrt{\gamma}} \quad (31)$$

The variable,  $a_{CES}$ , may be interpreted as an index of persistence, as (30) implies the following, where we write  $y_t$  for the log of real final output:

$$y_t = a_{CES}y_{t-1} + (m_t - m_{t-1}) + \frac{1}{2}(1 - a_{CES})(m_{t-1} - m_{t-2}) \quad (32)$$

So  $a_{CES}$  represents the persistence of output deviations after the second period, when all price setters have had a chance to reset their price.<sup>6</sup> This formulation is identical to that of Chari, Kehoe and McGrattan (1996) and Kiley (1997), except that  $\gamma$  is a function of the deep parameters particular to our model.

To find persistence, we need  $a_{CES} > 0$  or equivalently  $\gamma < 1$ . But in the pure CES case with no input-output structure ( $\theta = 1$ ), it is true that  $\gamma = 1 + a$ , which is always greater than one. Under these assumptions there can never be positive persistence. This case is identical to that analyzed by Chari, Kehoe and McGrattan (1996), and we confirm their finding. A high value of labor substitutability ( $\sigma$  low) raises the degree of persistence, but even an infinite elasticity ( $a=0$ ) is not sufficient to generate persistence that is positive.

The introduction of translog preferences and input-output structure generates more persistence. Setting the input-output parameter in line with the estimates of Basu (1995) ( $\theta = 0.2$ ), figure 2a shows that output is significantly more persistent. Output rises again 1 percent in the initial period of the monetary shock, and one year afterward, output is still 0.375 percent above normal. Figure 2b suggests that price rises much more gradually in this case. Price setting is now a

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<sup>6</sup> Note that this indicates the fraction of the previous period's output that persists, not the fraction of the initial impact two period previous, which would be a preferable indicator of persistence in the two-period case.

function of competitors' prices, half of which are fixed in any period. Figure 2b also suggests that nominal wages are rising much less; this is due to the fact that labor is no longer the major input in production.

We can solve the translog system analytically as we did for the CES case. The solution is the same, except that the index of persistence is:

$$a_{TLog} = \frac{\sqrt{2} - \sqrt{\gamma}}{\sqrt{2} + \sqrt{\gamma}} \quad (33)$$

Again for persistence we need  $a_{TLog} > 0$ , which requires here that  $\gamma < 2$ , a more generous restriction than in the CES case. This requires that:

$$\theta < \frac{2 + 2\sigma}{1 + 3\sigma} \quad (34)$$

so that even under the assumption of a reasonable labor supply elasticity ( $\sigma = 1$ ), the model always produces positive persistence, regardless of the degree of input-output structure ( $\theta$  is restricted to the interval  $(0, 1)$ ).

What portion of this persistence is due to the translog preferences and what portion to the input-output structure? Consider what happens to the CES case when we consider an input-output structure. Condition (31) suggests that the CES persistence index should be a negative function of the level of  $\gamma$  and hence  $\theta$ . In particular,  $a_{CES} > 0$  when:

$$\theta < \frac{1 + \sigma}{1 + 2\sigma} \quad (35)$$

So under our labor-supply assumption of  $\sigma = 1$ , positive persistence would require that labor costs account for less than  $2/3$  of marginal costs ( $\theta < 2/3$ ). Figure 3 graphs the output responses to a 1 percent increase in money for various values of  $\theta$  in the CES case. The figure indeed shows

positive persistence for  $\theta = 0.5$ , and it shows this persistence increases as  $\theta$  falls and the role of labor costs decreases. Condition (31) suggests that in the extreme case where labor plays no role in marginal cost ( $\theta = 0$ ), then  $a_{CES} = 1$  and persistence would be complete, with real effects persisting indefinitely. Note also that there is a potential trade-off between the labor supply elasticity and the degree input-output structure. If we are willing to consider values of  $\alpha$  less than unity, then persistence can be generated with less extreme values for  $\delta$ .

Consider next how various degrees of input-output structure affect the translog case. Figure 4 shows output responses for various values of  $\delta$ . When goods play no role as inputs ( $\theta = 1$ ), output falls immediately to its long-run level one year after the shock. This suggests that the input-output structure is an important element of persistence here. Nevertheless, this persistence still represents an improvement over the CES case, in which output actually fell below its long-run level. Comparison of the impulse responses in figures 3 and 4 shows that for each of the values of  $\theta$  considered, persistence is greater for the translog case. This is confirmed by comparing conditions (31) and (33), which implies that:

$$a_{TLog} > a_{TCES} \tag{36}$$

for all  $\theta$ . Further, the gap between the two cases is not constant. Over most of the range of  $\delta$ , the additional benefit of using translog preferences grows as the role of input-output structure increases. Clearly both elements are important, and they appear to interact in generating persistence. This suggests that earlier research, which tended to consider only one element at a time, may have missed potential persistence generated by the interaction of multiple elements.

## 4. Increased Number of Groups

Finally, we wish to check the conjecture of Taylor and Blanchard that increasing the number of staggered groups would increase the degree of persistence.

Begin with the general T-period firm optimization problem (I0) and its first-order condition (13). Suppose the N firms are divided into G equally-sized groups, indexed by  $g = 1 \dots G$ , with each group setting their prices for  $T = G$  periods in staggered fashion. The price-setting equation for any firm in group one would be (replacing equation (14)):

$$\ln p_{1t} = \frac{1}{2G-1} \sum_{\tau=t}^{t+T} \left( \ln c_{\tau} + \frac{1}{G} \sum_{g \neq 1} \ln p_{g\tau} \right) \quad (37)$$

The weight of all other price groups is now  $(G-1)/(2G-1)$ , which becomes larger with G, the number of groups. In other words, now a larger share of competitors are not in one's own price-setting group, so a larger fraction of competitor prices are fixed in the current period. Using the analogous price setting rules for all groups, as well as the definition of costs and price index:

$$\ln c_t = \theta \ln w_t + (1-\theta) \ln P_t \approx \theta \ln w_t + \sum_{g=1}^G \frac{(1-\theta)}{G} \ln p_{gt} \quad (38)$$

we may write a rather lengthy price-setting equation for any G, which corresponds to equation (17) for the two-group case.

Figure 5 presents the impulse response to the experiment of a permanent one-percent increase in the money supply, in the context of various numbers of price-setting groups. With four groups rather than two, the degree of persistence after one year increases from 0.375 percent to 0.413 percent. Twelve staggered groups increases persistence to 0.423 percent. Figure 6 suggests the additional gains in persistence from multiple groups is not as large as the contributions of input-

output structure or translog preferences considered earlier. Nevertheless, this result does confirm the conjecture of Taylor and Blanchard, which was not the case in other subsequent studies. And it again points out the importance of interaction of model elements. Only in a framework where competitors' prices matter in the pricing rule, is there an additional effect of having a larger share of competitors whose price is currently fixed.

## **5. Conclusion**

A CES functional form is severely limiting in the context of monopolistically competitive price setters. It implies that the real price is set by firms as a constant markup over marginal costs, and is not affected by competitors' prices. This misses the interaction of price-setters envisioned in Taylor (1980) and Blanchard (1983) in their early work on staggered contracts. This paper has taken steps to move beyond the CES functional form and has consider the implications of a translog form for preferences. It has also taken seriously the notion of Basu (1995), assigning a significant role for goods as inputs in the production process. These two affects contribute to a pricing-setting rule that gives significant weight to competitors' prices. The result is a reasonable amount of persistence beyond the period of exogenously-imposed rigidity. Further, once firms begin to care about the prices set by competitors, it is found that increasing the number of staggered price-setting groups increases the degree of endogenous persistence.

The degree of persistence achieved in the present model is reasonable, but does not fully match the degree Taylor generated using his ad hoc pricing rule, which he considered necessary to fully reflect features of the data. The translog form used here probably cannot deliver further persistence, at least not without implying a degree of pass-through of marginal costs inconsistent with

empirical evidence. Further work will consider additional elements which may deliver persistence. The present model has demonstrated also that the interaction of various elements can be important in generating persistence, in a way not apparent when elements are considered individually.

Future work will also extend the model to a two-country case. Persistence is a vital question in international macroeconomics, in particular in explaining the comovement of real and nominal exchange rates. Further, we will explore our model's implications for the occurrence and effects of exchange rate overshooting. Finally, it is hoped that the translog specification developed here could augment the micro-foundations of dynamic models used for macroeconomic policy analysis.'

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See Svensson (1998) for an example of such a model with micro-foundations

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Figure 1a : Output Response  
CES preferences, no I/O structure

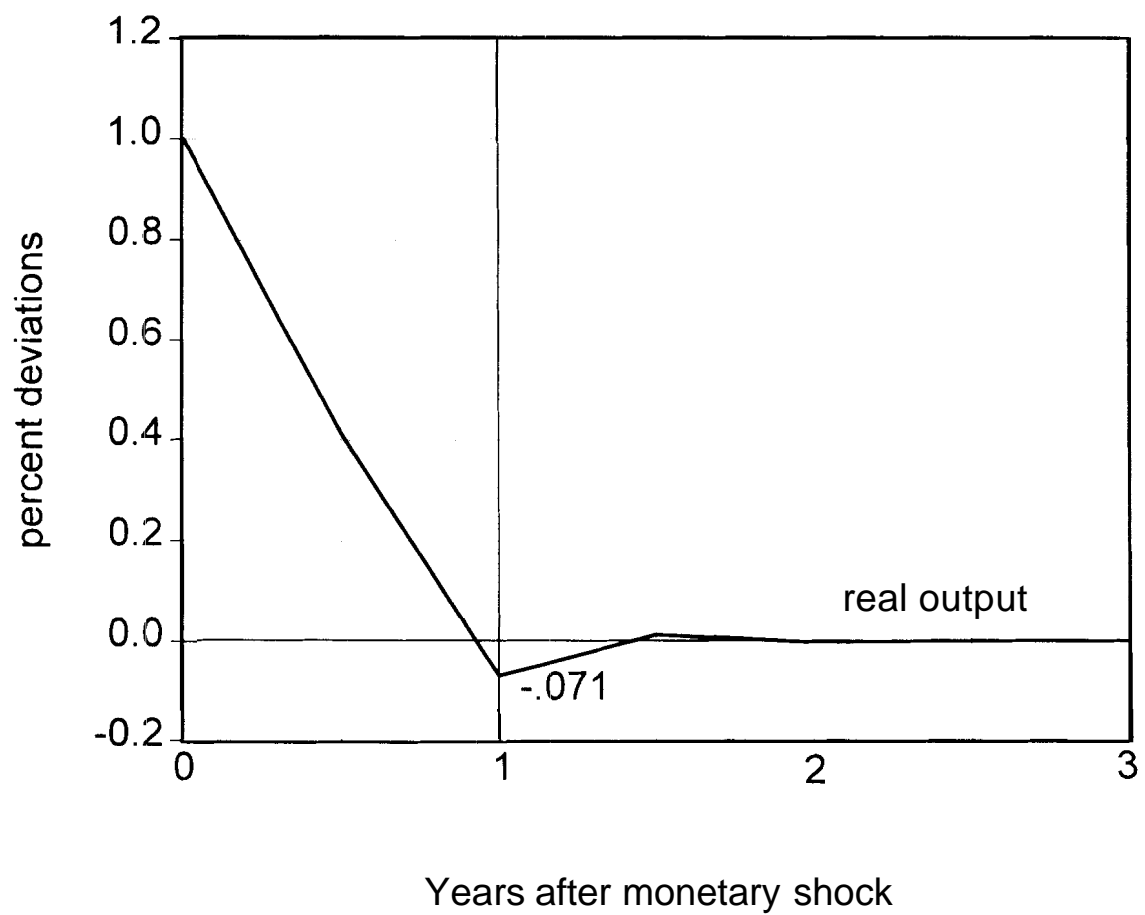


Figure 1b: Price and Wage Response  
CES preferences, no I/O structure

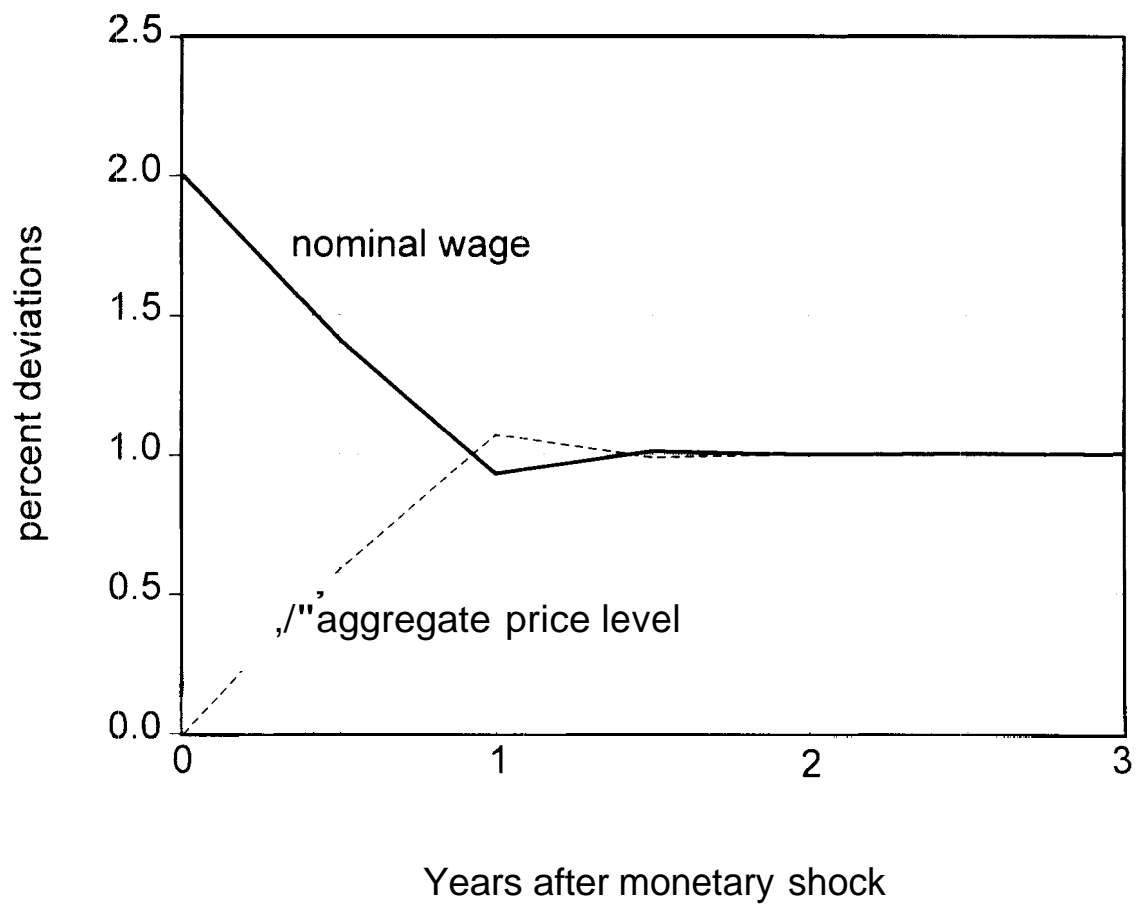


Figure 2a: Output Response  
Translog preferences,  $\theta=0.2$

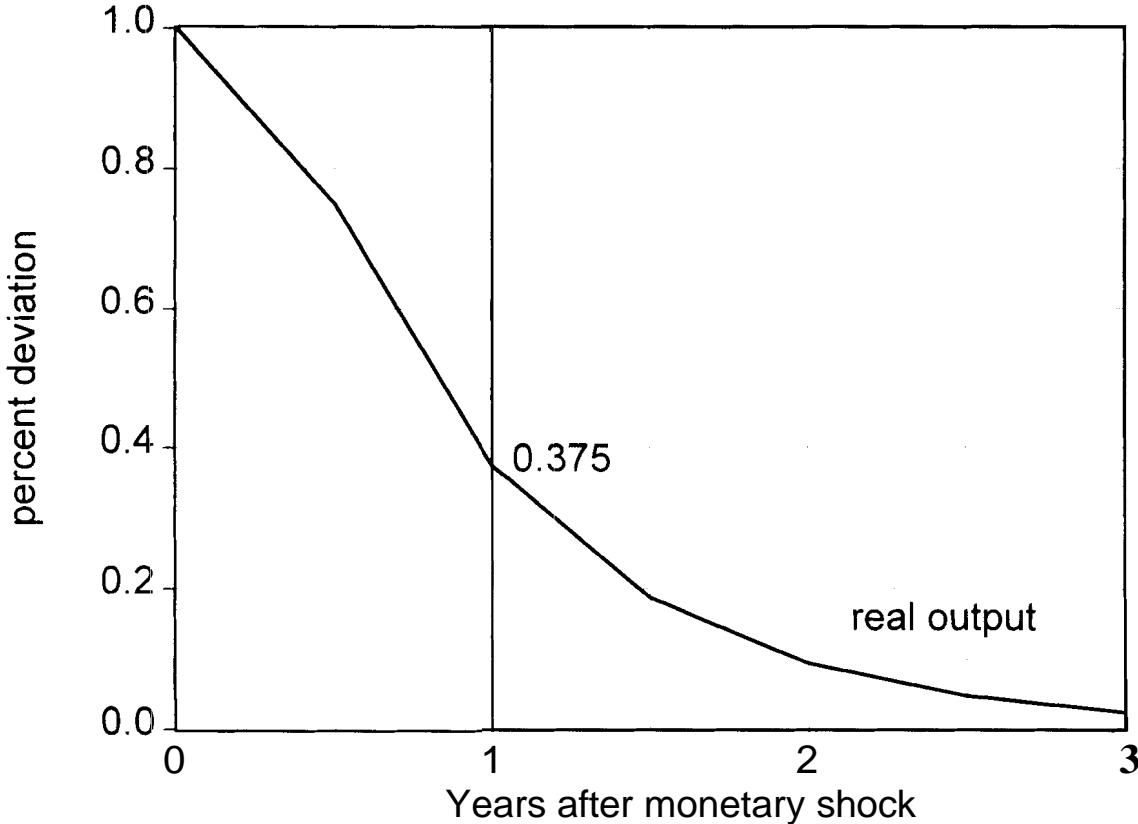


Figure 2b: Price and Wage Response  
Translog preferences,  $\theta=0.2$

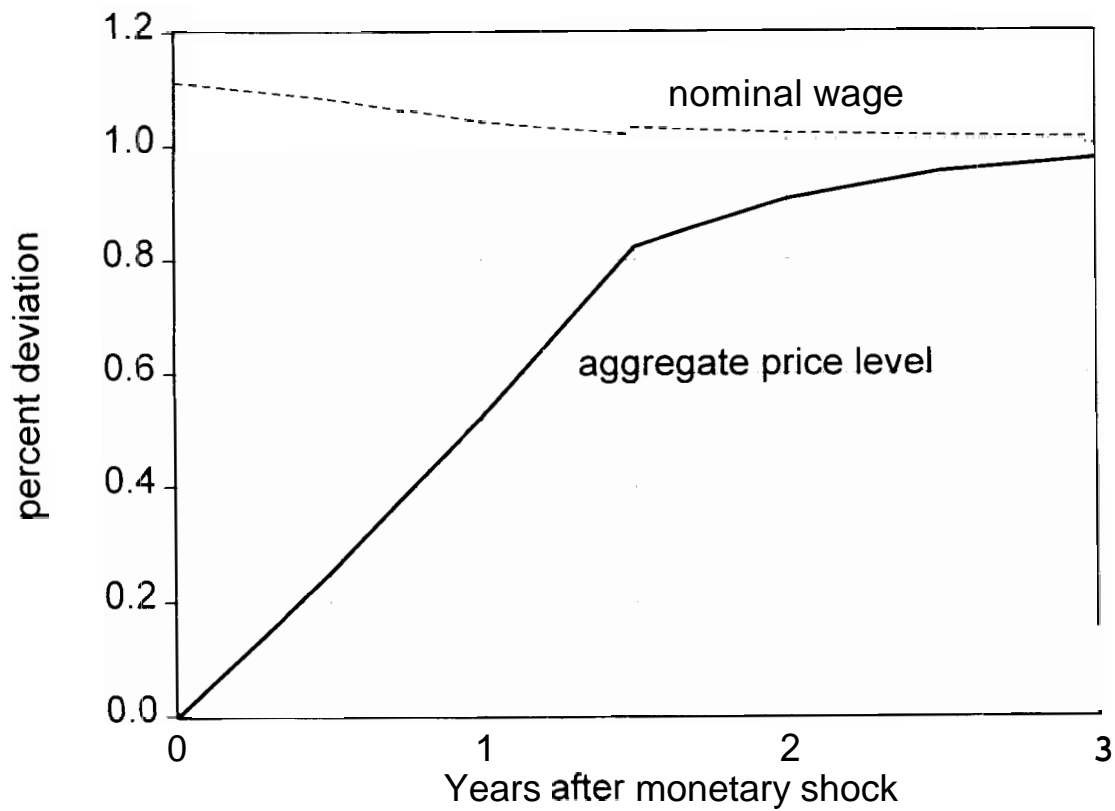


Figure 3: CES Output Response  
Various degrees of I/O structure

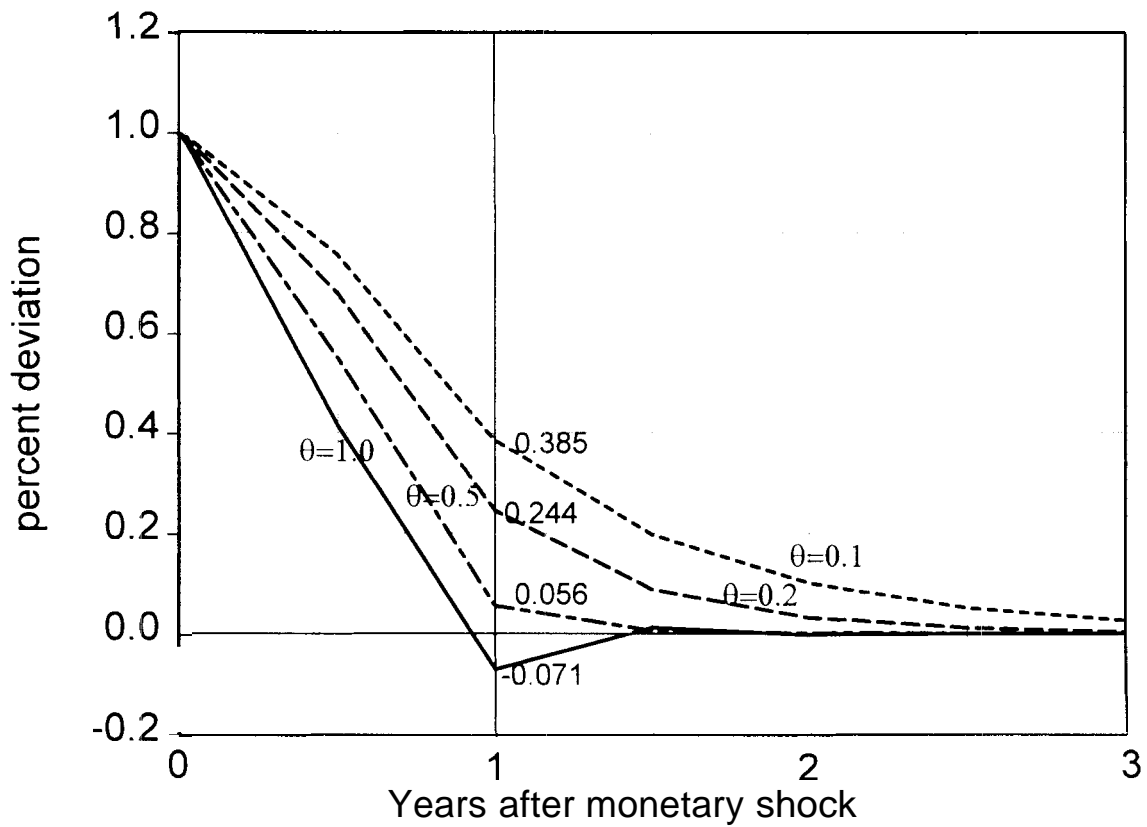


Figure 4: Translog Output Response  
Various degrees of I/O structure

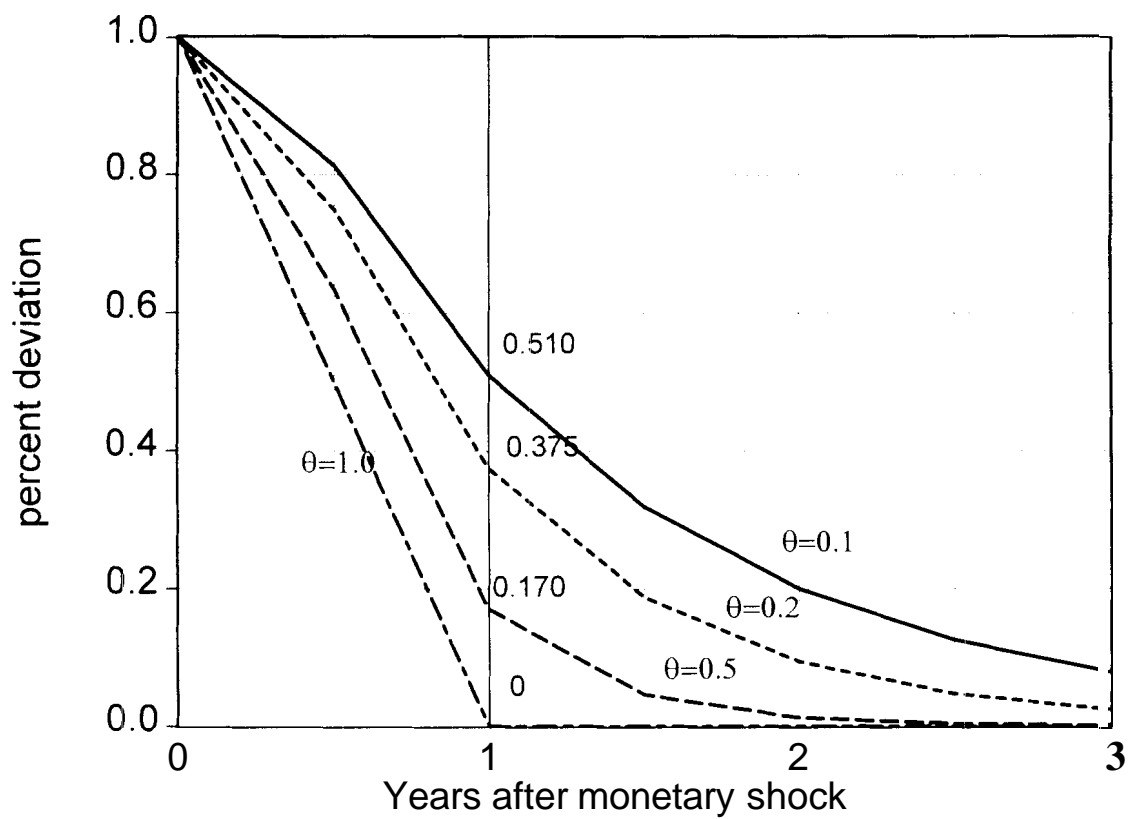


Figure 5: Output Response  
various numbers of staggered groups

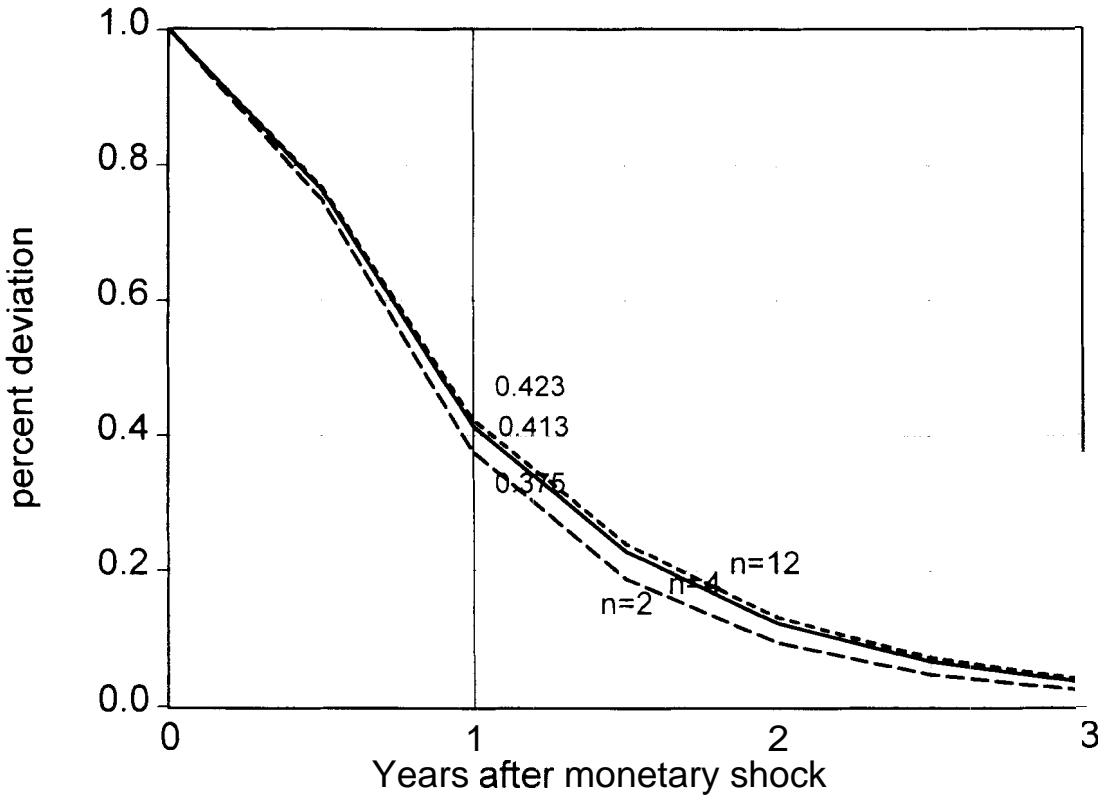




Figure 6: Summary of Output Response Contributions

