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Investment Complementarities, Coordination Failure and Systemic Bankruptcy

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Investment Complementarities, Coordination Failure and Systemic Bankruptcyd'

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Investment Complementarities, Coordination Failure and Systemic Bankruptcy*

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Abstract

I argue that systemic bankruptcy of firms can originate from coordination failure in an economy with investment complementarities. This new explanation about the origin of systemic bankruptcy promotes better understanding of how financial fragility arises, and provides theoretical guidance for central banks to establish an “early warning system” to prevent the occurrence of financial crises. In a global game setup, investment decisions of firms are studied in the presence of uncertainty and investment complementarities. Uncertainty is twofold here: first, firms are uncertain about economic fundamentals; second, firms are also uncertain about other firms’ investment decisions. I demonstrate that even small uncertainty about economic fundamentals can be magnified through the uncertainty about other firms’ investment decisions and can lead to coordination failure, which may be manifested as systemic bankruptcy. Moreover, my model reveals that systemic bankruptcy tends to arise when economic fundamentals are in the middle range where coordination matters. High financial leverage of firms greatly increases the severity of systemic bankruptcy. Optimistic beliefs of firms and banks can alleviate coordination failure, but can also increase the severity of systemic bankruptcy once it happens.

Keywords: Systemic Bankruptcy, Financial Crises, Global Games

JEL Classification: D82, E44, G21

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1 Introduction

Financial stability has gradually become an important topic in economic literature since the 1980's, after the world witnessed a series of financial crises in both developed and developing countries. A large body of literature is devoted to explaining the origin and propagation of financial crises (Krugman (1979, 2000); Diamond and Dybvig (1983); Obstfeld (1996); Cole and Kehoe (1996); Chang and Velasco (2001); Chari and Kehoe (2003)).

Although most financial crises manifest themselves in a seemingly unique manner, a common feature is widespread bankruptcy among both nonfinancial firms and financial institutions. A study on how this kind of bankruptcy arises will help us understand the origin of financial crises and, furthermore, how central banks should tackle them.

Existing literature on systemic bankruptcy of nonfinancial firms and financial institutions primarily focuses on how the bankruptcy of an individual economic agent is spread to other economic agents through different financial contagion mechanisms, such as credit chains and herding behavior (Allen and Gale (2000); Kiyotaki and Moor (2002); Chen (1999)). The origin of systemic bankruptcy, that is, how the first economic agent goes bankrupt, is simply attributed in the literature to some exogenously given shock. By doing so, this literature fails to provide any economic rationale behind the origin of systemic bankruptcy.

This paper focuses on the origin of systemic bankruptcy of nonfinancial firms associated with uncertainty in real investment. A formal model is established to demonstrate that systemic bankruptcy of nonfinancial firms can endogenously originate from coordination failure in an economy with investment complementarities. This new explanation promotes better understanding of the origin of financial fragility. Moreover, the model can be used to identify economic situations associated with systemic bankruptcy, and therefore can provide theoretical guidance for central banks to establish an “early warning system” to prevent the occurrence of financial crises.

The model is established in a global game setup, where investment decisions of nonfinancial firms are studied under three conditions. In the first, investment com-

plementarities exist. Therefore, investment returns are determined not only by economic fundamentals, but also by the proportion of firms investing. With more firms investing, the investment return is higher. In the second, firms only have incomplete information about economic fundamentals. In the third, firms have to finance their investment by debts. In such a setup, firms face two kinds of uncertainties when investing: (1) firms are uncertain about the economic fundamentals, and (2) firms are also uncertain about the investment decisions of other firms. The second kind of uncertainty, ignored in the existing literature, is endogenously generated in an economy with investment complementarities. I demonstrate that a even small uncertainty about economic fundamentals can be magnified through the second kind of uncertainty, causing coordination failure, which may be manifested as systemic bankruptcy. Thus, the meaning of systemic bankruptcy is twofold in this paper: (1) it happens to a large number of firms in an economy, instead of to an individual firm, and (2) it is endogenously rooted in a decentralized credit economy, where coordination among firms matters.

One of the key assumptions in this paper is that investment complementarities exist, and therefore coordination among firms can be critical. Investment complementarities are widely observed in the economy. They can exist in industries with industry-specific externalities, that is, the externalities within an industry. The sources of such externalities can be the benefits of “within-industry specialization, conglomeration, indivisibility and public intermediate inputs such as roads” (Caballero and Lyons (1989)). The industries with network externalities are special examples of such industries. Network externalities are defined as a change in the utility that an agent derives from a good, when the number of other agents consuming the same kind of goods changes. For example, as the Internet is increasingly used as a communication tool, Internet users find it more valuable since they can make greater use of it. So any investment by a firm in this industry attracting more internet users will benefit other firms in the industry.

My model can be interpreted as a study of systemic bankruptcy in such an industry. Due to the externalities within the industry, one firm’s investment return will be higher with higher investment activities by other firms. Therefore, there are

investment complementarities among the firms in the industry.

Meanwhile, my model can also be interpreted as a study of systemic bankruptcy in the whole economy, if external economies of scale across industries, or cross-industry externalities, are taken into account. A source of cross-industry externalities can be the demand spillover effect. If other firms in the economy are investing more, they will be spending more too, and this leads to increased demand for the product of an individual firm, which will in general increase the investment return of the firm. In addition, if we drop the unrealistic assumption of a Walrasian auctioneer and admit that there are transaction frictions in an economy, a higher level of economic activity will lower trading costs and raise the average investment return due to trading externalities, or the “thick market” effect (Diamond (1982)). The empirical work of Caballero and Lyons (1989, 1990) reveals that cross-industry externalities are significant in the economy. Using the data of two-digit manufacturing industries in Belgium, West Germany, France, the U.K. and the U.S., they test external economies and internal returns to scale in those countries. Strong evidence of external economies is found in all the countries.

Using my model, economic situations where systemic bankruptcy tends to arise can be identified. In this way, my model provides some theoretical guidance for central banks to establish an “early warning system” for financial crises. According to my model, systemic bankruptcy tends to arise when economic fundamentals are in a middle range where coordination failure arises, which I call the coordination failure zone. More specifically, systemic bankruptcy tends to happen when economic fundamentals fall into a low to medium value in this zone. Comparative statics further reveals that higher financial leverage of firms can greatly increase the severity of systemic bankruptcy. Moreover, optimistic public beliefs of firms and optimistic beliefs of banks about the fundamentals can alleviate coordination failure. The range in which systemic bankruptcy arises is narrowed. However, systemic bankruptcy can be more severe once it occurs.

All of the above results are generally consistent with empirical observations on financial crises. A large body of empirical studies on financial crises finds that financial crises tend to arise at the economic downturn of a business cycle, which is shortly

after the economy reaches its peak (Gorton (1988); Kaminsky and Reinhart (1999)). This fact can be interpreted to be consistent with my model’s result that systemic bankruptcy tends to arise when economic fundamentals take low to medium values in the coordination failure zone. In my model, coordination failure will not arise when economic fundamentals are extremely high or low. So systemic bankruptcy will not arise at the peak or trough of a business cycle. Only when the fundamentals are in the middle range, especially when the fundamentals are deteriorating from medium to low level, which can be interpreted as the economy being at the downturn from a boom, does systemic bankruptcy arise.

According to my model, systemic bankruptcy tends to be more severe when the financial leverage of firms is high. This result is also consistent with the empirical observation that financial crises tend to happen when the credit/GDP ratio is higher than that in the tranquil time (Kaminsky and Reinhart (1999)).

Anecdotal observations on financial crises also reveal that financial crises tend to happen at the end of an economic boom, when both banks and firms are still sanguine about the economy, which is consistent with the results in my model that although optimistic public beliefs of firms and banks can alleviate the coordination failure, they will increase the severity of systemic bankruptcy once it occurs.

This paper provides a mechanism through which uncertainty in real investment leads to financial fragility, which is manifested as systemic bankruptcy of nonfinancial firms. In this sense this paper is in favor of the “fundamentalist” opinion that financial crises are caused by real economic factors. However, the mechanism in this paper can be easily combined with financial contagion theories. As mentioned before, the mechanism that I provide here can be regarded as an alternative explanation about the exogenously given shock in financial contagion theories. Bankruptcy caused by the mechanism in this paper can be further spread to other economic agents through financial contagion mechanisms, leading to more severe bankruptcy. An important message conveyed in my paper is that due to investment complementarities, the economy will be more vulnerable to financial crises. Systemic bankruptcy can occur even without significant economic shocks.

The rest of this paper is organized as follows. Section 2 gives literature survey. In

section 3 a basic model is presented. Section 4 analyzes how economic fundamentals, financial leverage, and public beliefs of firms affect systemic bankruptcy. Section 5 introduces banks without private signals into the basic model and examines the role that banks play in systemic bankruptcy. In section 6, banks with private signals are introduced. In section 7, conclusions and policy implications are given. Future research is also discussed.

2 Literature Survey

This paper is related to three strands of literature. The first strand of literature is on financial crises, especially on systemic bankruptcy in both nonfinancial firms and financial institutions.

Kiyotaki and Moore (2002) study systemic bankruptcy of nonfinancial firms originating from two contagion mechanisms. One is the trade credit chain, and the other is the fall of the price of a collateral asset. The spread of bank failure from one banking region to another due to the overlapping claims of the banks on each other is explored by Allen and Gale (2000). Chen (1999) models how bank failure in a few banks can cause runs on other banks due to asymmetric information.

The second strand of literature is on macroeconomic complementarities and their implications for the economy. Bryant (1983) uses a special form of production function to study how technological complementarities generate Pareto-ranked multiple equilibria. The business cycle implications of technology complementarities is explored by Baxter and King (1991) in a model whose structure is similar to a standard real business cycle model, where the business cycle generated by a demand shock and propagated through the technological complementarities is quantitatively modeled.

Diamond (1982) studies how trading externalities cause "thick market" effects in the presence of trade frictions. He finds that the return of an individual economic agent will be higher due to the reduced searching costs if more agents are in the market searching trading partners.

Cooper (1999) comprehensively surveys macroeconomic complementarities and their implications for macroeconomic behavior. He examines a variety of sources of

macroeconomic complementarities, such as technological complementarities, demand spillover effects and trading externalities, and studies their implications.

The third strand of literature is about global games and their applications in macroeconomic and financial stability.

Global games were first established by Carlsson and van Damme (1993). They introduce incomplete information into a traditional coordination game with perfect information. In the game each player observes his payoffs with some noise. By iterated elimination of strictly dominated strategies, they prove that when the noise gets infinitely small, there is a unique equilibrium in the game.

Morris and Shin (1998) study currency attacks in a global game setup. They find that when speculators need to coordinate their actions to successfully attack a fixed exchange rate regime, and meanwhile are only able to observe economic fundamentals with some small noise, there is a unique equilibrium in the game, determined by both economic fundamentals and the beliefs of speculators. This result differs from that of a traditional coordination game with perfect information, where a currency attack is solely determined by the self-fulfilling beliefs of speculators. Successfully overcoming the problem of indeterminacy of multiple equilibria models, their model allows the analysis of policy implications.

Morris and Shin (2000) summarize the applications of global games in macroeconomic modeling by explaining how global games can be used in the context of bank runs, currency crises, and debt pricing. They argue that global games are a useful approach for the analysis of many macroeconomic issues where players' payoffs are interdependent. They reckon that global games provide a more solid ground for policy analysis than multiple equilibria models due to their property of unique equilibrium.

How public information influences equilibrium allocation and social welfare in economies with investment complementarities is studied by Angeletos and Pavan (2004). They demonstrate that when coordination is socially desirable, an increase in the precision of public information will always increase social welfare, given that the complementarities are weak so that the equilibrium is unique. On the other hand, when the complementarities are strong, such that multiple equilibria are possible, the increase in public information may facilitate the coordination in both “bad” and

“good” equilibria.

Chamley (2004) gives a survey on coordination games and global games. A detailed summary about the theory and applications of global games is also given by Morris and Shin (2003).

3 The Basic Model

This model is based on Morris and Shin (2000). In their model, a continuum of depositors of mass 1 has to decide whether to run a bank or not, based on their beliefs about deposit returns, which are determined by both economic fundamentals and the actions of other depositors. I apply their model to investment decisions of nonfinancial firms in an economy with investment complementarities. The firms are analogous to the depositors, and investment returns are analogous to deposit returns. Technically, my model differs from theirs in that the firms are assumed to finance their investment by debts. Thus the payoff structure of the firms is asymmetric and the firms care about the upside risks only when their capital is positive.¹ This greatly complicates the calculation of the expected payoffs of the firms. Both the investment returns (given realized economic fundamentals levels) and the threshold level of economic fundamentals (above which the firms’ capital becomes positive) will change in the firms’ strategies. I prove that the main properties of global games are still held in such a situation. Moreover, in sections 5 and 6, two models with banks are established respectively. These two two-stage games in which banks move at the first stage and firms move at the second stage further differ from the one-stage game established by Morris and Shin (2000).

There is a continuum of risk-neutral firms with initial wealth w_0 who have to simultaneously decide whether to invest or not.

The gross return rate is 1 if a firm chooses not to invest. The gross return rate

¹Morris and Shin (2004) study the issue that creditors of a distressed borrower have to decide whether to withdraw their loans or not. The creditors in their model also have an asymmetric payoff structure. But since their model focuses on a totally different issue, the whole setup of their model is quite different from mine.

from investing is e^{r-l} . Here r denotes economic fundamentals of the investment, and l denotes the proportion of firms not investing. Thus the return of the investment is increasing both in economic fundamentals and in the proportion of the firms investing, $1 - l$. The latter introduces investment complementarities to the game.

The investment is assumed to have a fixed size of mw_0 , where $m > 1$ is exogenously given. So a firm investing has to borrow $(m - 1)w_0$. The gross borrowing rate is exogenously given by 1. Later it will be endogenized by introducing banks into the model.

3.1 The Case with Perfect Information

With perfect information about r , this game has three possible cases:

When $r > 1$, there is a unique equilibrium in which all firms invest. No bankruptcy occurs.

When $r < 0$, there is also a unique equilibrium in which all firms do not invest. No bankruptcy occurs either.

When $0 < r < 1$, there are two (stable) equilibria. One is that all firms invest, with $l = 0$ and $r - l > 0$. The other is that all firms do not invest, with $l = 1$ and $r - l < 0$. No bankruptcy occurs in both equilibria.

3.2 The Case with Incomplete Information

Now I introduce incomplete information into the model. Suppose that at the beginning of the game each firm has an identical prior belief about the fundamentals of the investment, $\tilde{r} \sim N(\bar{r}, 1/\alpha)$. Here α is the precision of \tilde{r} and $1/\alpha$ is its variance. This belief is also called the public belief. In addition, after economic fundamentals are realized, each firm has access to very precise but not perfect information about them before it makes its decision. More specifically, given the realization of \tilde{r} , r , firm i observes the realization of signal $x_i = r + \varepsilon_i$, where $\varepsilon_i \sim N(0, 1/\beta)$. ε_i is i.i.d across the firms.

After observing the private signal x_i , firm i updates its belief about the funda-

mentals according to Bayes' rule. Thus $(\tilde{r}|x_i)$ is also normally distributed with mean

$$\rho_i = \frac{\alpha\bar{r} + \beta x_i}{\alpha + \beta} \quad (1)$$

and precision $\alpha + \beta$. Let $\gamma = \frac{\alpha^2}{\beta}$.

Proposition 1 *Provided that $\gamma \leq 2\pi$, there is a unique symmetric trigger strategy equilibrium. In this equilibrium, firm i chooses to borrow and invest if and only if $\rho_i > \rho^*$, where ρ^* is the unique solution to*

$$\sqrt{\alpha + \beta} \int_{r^*}^{+\infty} \phi(\sqrt{\alpha + \beta}(r - \rho^*)) (m e^{r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}))} - m + 1) dr = 1,$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are respectively the PDF and CDF of a standard normal distribution with mean 0 and variance 1, and r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(\rho^* - r^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}))) = \ln \frac{m-1}{m}.$$

Otherwise, the firm chooses not to invest.

Proof:

There are two ways to prove the equilibrium in this game. One way is to use the iterated elimination of strictly dominated strategies. The other way is to confine attention to symmetric trigger strategy equilibria and to prove that there is such a unique equilibrium.

I will first give the proof that confines attention to symmetric trigger strategies. There are two steps involved. First, I pinpoint the unique value of ρ^* such that in equilibrium each firm i will invest if and only if $\rho_i > \rho^*$. Second, I demonstrate that this strategy is optimal for all the firms.

For ρ^* to be an equilibrium switching point, a firm with the private signal x^* and updated belief ρ^* must be indifferent between investing and not investing. Recall that the relationship between x^* and ρ^* is given by equation (1).

Let a^{NI} and a^I denote the actions of not investing and investing respectively. We know that

$$R(a^{NI}|x^*) = 1,$$

that is, the gross return of firm x^* (here I abuse the notation of x^* , the firm's private signal, to denote the firm) from not investing is always equal to 1.

The expected gross return rate of firm x^* from investing is given by:

$$ER(a^I|x^*(\rho^*)) = 0 \times \sqrt{\alpha + \beta} \int_{-\infty}^{r^*} \phi(\sqrt{\alpha + \beta}(r - \rho^*))dr + \sqrt{\alpha + \beta} \int_{r^*}^{+\infty} \phi(\sqrt{\alpha + \beta}(r - \rho^*)) [me^{r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r})))} - m + 1]dr, \quad (2)$$

where r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(\rho^* - r^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}))) = \ln \frac{m-1}{m}.$$

Equation (2) is the key equation in the whole paper, which is derived as follows.

First, the expected gross return rate of firm x^* is based on its belief on the fundamentals, which is $(\tilde{r}|x^*) \sim N(\rho^*, \frac{1}{\alpha + \beta})$. Thus the PDF of $(\tilde{r}|x^*)$ is given by $\sqrt{\alpha + \beta}\phi(\sqrt{\alpha + \beta}(r - \rho^*))$.

Second, given each realized value of \tilde{r} , r , and given that all the firms take the trigger strategy ρ^* , the total payoff of firm x^* from investing is a certain number, given by:

$$mw_0e^{r-l(r, \rho^*)} = mw_0e^{r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r})))},$$

where $\Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r})))$ is the proportion of the firms not investing. It is derived as follows:

$$l(r, \rho^*) = Prob(\tilde{x} < x^*) = Prob(\tilde{x} < \rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{r})) = \Phi(\sqrt{\beta}(\rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}) - r)).$$

That is, it is the proportion of the firms whose private signal \tilde{x} is less than $x^* = \rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{r})$. The CDF of \tilde{x} is $\Phi(\sqrt{\beta}(x - r))$, since given r , $\tilde{x} \sim N(r, \frac{1}{\beta})$.

When $mw_0e^{r-l} < (m-1)w_0$, or $r - l < \ln \frac{m-1}{m}$, or $r < r^*$, the firm loses all of its initial wealth, w_0 , and its gross return rate is 0. When $r - l > \ln \frac{m-1}{m}$, or $r > r^*$, the firm earns the gross return rate of $me^{r-l} - m + 1$.

Here r^* is the unique solution to

$$r - l = r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}))) = \ln \frac{m-1}{m}.$$

Notice that $r - l$ is strictly increasing in r . Thus there is a unique critical level of r , r^* , below which $r - l < \ln \frac{m-1}{m}$ and above which $r - l > \ln \frac{m-1}{m}$.

Until now, I define the PDF of firm x^* over \tilde{r} , which is $\sqrt{\alpha + \beta}\phi(\sqrt{\alpha + \beta}(r - \rho^*))$. In addition, I define the gross return rate of firm x^* given each realized value of \tilde{r} . It is straightforward to get the expected gross return rate of firm x^* , equation(2). Rearranging the above equation, I get the expected gross return rate of firm x^* :

$$ER(a^I|x^*(\rho^*)) = \sqrt{\alpha + \beta} \int_{r^*}^{+\infty} \phi(\sqrt{\alpha + \beta}(r - \rho^*)) (me^{r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r})))} - m + 1) dr. \quad (3)$$

Firm x^* will be indifferent between investing and not investing if and only if

$$ER(a^I|x^*(\rho^*)) = R(a^{NI}|x^*) = 1. \quad (4)$$

I can prove that given $\gamma < 2\pi$, the expected gross return rate of firm x^* is strictly increasing in ρ^* . Therefore, there is a unique solution of ρ^* to equation (4). The proof is given in appendix 1.

It is straightforward to demonstrate that the trigger strategy ρ^* is optimal for every firm. For firm x_i , its expected gross return rate from investing is given by:

$$ER(a^I|x_i(\rho_i)) = \sqrt{\alpha + \beta} \int_{r^*}^{+\infty} \phi(\sqrt{\alpha + \beta}(r - \rho_i)) (me^{r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r})))} - m + 1) dr.$$

From the above equation we can see that $\Phi(\sqrt{\alpha + \beta}(r - \rho_i))$ first order stochastically dominates $\Phi(\sqrt{\alpha + \beta}(r - \rho^*))$ when $x_i > x^*$, and is first order stochastically dominated when $x_i < x^*$. We also know that $me^{r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r})))} - m + 1$ is strictly increasing in r . Since the expected gross return rate is 1 if and only if $\rho_i = \rho^*$, the expected gross return rate is less than 1 when $\rho_i < \rho^*$, and is greater than 1 when $\rho_i > \rho^*$. Thus the trigger strategy ρ^* is optimal for all the firms.

In appendix 2, the iterated elimination of strictly dominated strategies is used to prove that this is the unique Bayesian Nash equilibrium.

Q.E.D

Notice that this game can be completely changed by introducing a coordinator, who asks each firm to submit its private signal and makes investment decisions for the firms. The Pareto optimal equilibrium can be at least one possible equilibrium

in such a setup. In this equilibrium, the private signals from all firms are collected. Thus the uncertainty about economic fundamentals vanishes. Moreover, since the coordinator can coordinate the investment actions between firms, the uncertainty about other firms' actions vanishes too. But in a decentralized economy without such a coordinator, the uncertainty about both economic fundamentals and other firms' actions leads to inefficiency in the equilibrium, which I will demonstrate later to be manifested as systemic bankruptcy. In this sense I argue that this model provides a new explanation about systemic bankruptcy caused by coordination failure.

4 The Analysis of the Basic Model

In this section, the basic model is used to study how systemic bankruptcy is influenced by different factors. Section 4.1 studies the relationship between realized economic fundamentals r and systemic bankruptcy; how firms' financial leverage influences systemic bankruptcy is analyzed in section 4.2; section 4.3 analyzes the impact of public beliefs about the fundamentals, \bar{r} , on systemic bankruptcy.

In this section, numerical examples are given to gain some qualitative insights. I choose $\alpha = 1$ and $\beta = 100$ such that $\gamma < 2\pi$. In addition, the uncertainty about economic fundamentals is assumed to be extremely small ($\beta = 100$) to demonstrate that systemic bankruptcy is mainly caused by the uncertainty about other firms' investment decisions. I choose $m = 2$, or capital/asset ratio = $1/m = 0.5$. This ratio varies from 0.2 to 0.6 in different countries in reality. Finally, $0 < \bar{r} = 0.5 < 1$ since I am interested in the coordination failure zone.

4.1 Realized Economic Fundamentals and Systemic Bankruptcy

This section studies how the realized economic fundamentals, r influences systemic bankruptcy. I find that *systemic bankruptcy only appears when economic fundamentals are in the middle range where coordination matters, which I call the coordination failure zone. More specifically, systemic bankruptcy begins to arise when r is lower than a threshold level. But the severity of bankruptcy is not monotonically decreasing*

in r . Instead it reaches its peak when r falls into a low to medium range.

First, bankruptcy appears only when r is lower than a threshold level, \underline{r} . Since $\tilde{x} \sim N(r, 1/\beta)$, the proportion of firms not investing is given by:

$$l(r) = Prob(x \leq x^*(\rho^*)) = \Phi(\sqrt{\beta}(x^* - r)).$$

A firm will go bankrupt if and only if $mw_0e^{r-l(r)} < (m-1)w_0$, or $r-l(r) < \ln(\frac{m-1}{m})$. Since $r-l(r)$ is strictly increasing in r , there is a unique \underline{r} satisfying

$$\underline{r} - l(\underline{r}) = \ln(\frac{m-1}{m}). \quad (5)$$

Second, given $r < \underline{r}$, the bankruptcy rate is $1-l(r)$, that is, the proportion of the firms investing. Since $l(r)$ is decreasing in r , the bankruptcy rate is increasing in r .

Third, the unpaid debts of an individual firm when $r < \underline{r}$, $(m-1)w_0 - e^{r-l}mw_0$, are decreasing in r .

In order to fully reflect the severity of systemic bankruptcy, I introduce a single creditor, who lends to all the firms. Its total loss from lending, defined by equation (6), is used to measure the severity of bankruptcy:

$$TL(r) = (1-l(r)) \times \max\{(m-1)w_0 - e^{r-l}mw_0, 0\}. \quad (6)$$

According to equation (6), TL is not monotonically decreasing in r . Instead, there are two opposite effects on TL when r is decreasing. On the one hand, the unpaid debts of an individual firm, $(m-1)w_0 - e^{r-l}mw_0$, are increasing. On the other hand, $1-l(r)$, the proportion of firms going bankrupt, is decreasing. Due to these two opposite effects, bankruptcy is the most severe when r is at some value lower than \underline{r} , where TL reaches its maximum value, which I call the maximum loss.

This result seems counter-intuitive, since we usually expect that bankruptcy is the most severe when the fundamentals are the worst. But it is not surprising in this model, since here bankruptcy will happen only when firms invest. A firm can always avoid loss by not investing. So it is not the adverse fundamentals, but the uncertainty about the adverse fundamentals and about the actions of other firms, that causes systemic bankruptcy. Later I will show that the latter uncertainty can

be the main cause of systemic bankruptcy, when the first uncertainty is assumed to be extremely small (firms have very precise private information). The uncertainty about the actions of other firms matters only when $0 < r < 1$, where coordination is in need. Lower r reduces the bankruptcy rate, leading to less total loss. When $r < 0$, the economy is out of the coordination failure zone and uncertainty about other firms' actions is vanishingly small. Therefore no systemic bankruptcy arises.

Table 1: A numerical example with $\alpha = 1$, $\beta = 100$, $m = 2$, $\bar{r} = 0.5$, and $w_0 = 100$

x^*	ρ^*	\underline{r}	r at maximum loss	maximum loss
0.4237	0.4244	0.2580	0.2226	0.1334

A numerical example with the parameter values given at the beginning of this section reveals the conclusions above. Table 1 tells us that firm i will invest if and only if its updated belief $\rho_i > 0.4244$, or its private signal $x_i > 0.4237$. Bankruptcy appears when $r < 0.2580$. The total loss reaches its maximum value of 0.1334 when $r = 0.2226$. Notice that r at the maximum loss is pretty high. That is because $1 - l$ rapidly decreases to 0 with the decrease of r , which effectively reduces the total loss.

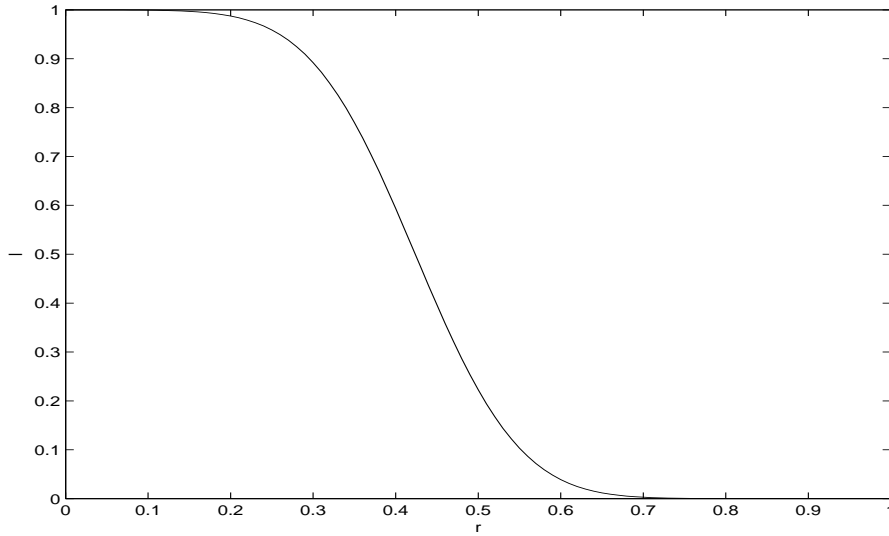


Figure 1: How proportion of firms not investing l changes with realized economic fundamentals r

Figure 1 shows that l goes to 1 when the realization of $r > 0.7$ and to 0 when the realization of $r < 0.2$. It is so because β is high and $l(r) = \Phi(\sqrt{\beta}(x^* - r))$ rapidly goes to 1 when r decreases and to 0 when r increases.

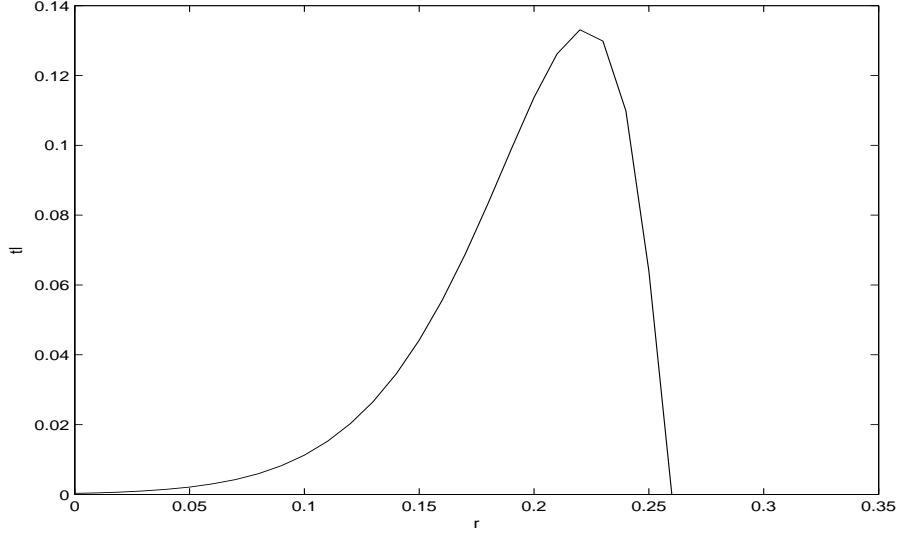


Figure 2: How total loss changes with realized economic fundamentals r

From figure 2, we can see that bankruptcy appears only when $0 < r < 1$, where coordination matters. More specifically, it begins to occur after the fundamental r reaches 0.2580, and the total loss rapidly increases to its maximum when r decreases to 0.2226. Then it rapidly decreases to 0 when r gets lower. The intuition behind this result is as follows: when r first decreases from the threshold level \underline{r} where bankruptcy begins to occur, the increase from the individual firm's unpaid debts, $(m-1)w_0 - e^{r-l}mw_0$, dominates the decrease in the bankruptcy rate, $1-l(r)$. However, since β is high, that is, the firms have a very precise private signal about r , the proportion of firms not investing, $l(r) = \Phi(\sqrt{\beta}(x^* - r))$, rapidly goes to 1 when r decreases. Therefore, the bankruptcy rate, $1-l(r)$, rapidly decreases to 0 with r 's further decrease, and its effect dominates the increase from the individual firm's unpaid debts, $(m-1)w_0 - e^{r-l}mw_0$.

In order to demonstrate that bankruptcy in this model is mainly caused by the uncertainty about other firms' investment decisions, I give an example without invest-

ment complementarities. Keeping all the parameter values unchanged in the above numerical example, I assume that the investment return is only determined by e^r . Table 2 gives the results.

Table 2: A numerical example with $\alpha = 1$, $\beta = 100$, $m = 2$, $\bar{r} = 0.5$, $w_0 = 100$ and no investment complementarities

x^*	ρ^*	\underline{r}	r at maximum loss	maximum loss
-0.01	-0.005	-0.6931	-0.7100	2.1389×10^{-12}

The maximum loss is almost equal to zero in this case, which is far less than that of 0.1334 in the case with investment complementarities. This example clearly reveals that the uncertainty about other firms' investment decisions can be an important source of systemic bankruptcy, even when the uncertainty about the fundamentals is extremely small and causes almost no bankruptcy.

4.2 Financial Leverage and Systemic Bankruptcy

This section analyzes how firms' financial leverage influences systemic bankruptcy. I find that *with higher financial leverage, systemic bankruptcy arises in wider economic fundamentals range, and is more severe once it happens.*

Observe that m influences bankruptcy in three ways. First, ρ^* , the equilibrium switching point, is a function of m . But the relationship between ρ^* and m is ambiguous. It depends on the distribution of $(\tilde{r}|x_i)$. With a higher m , the firms can earn more profits when $e^{r-l(r)} > 1$, or $r - l(r) > 0$. But the firms also lose more when $e^{r-l(r)} < 1$, or $r - l(r) < 0$. Second, we know that the threshold fundamentals level for bankruptcy \underline{r} is determined by equation (5). So when ρ^* is given, \underline{r} is increasing in m . Third, given ρ^* and $r < \underline{r}$, the unpaid debt of an individual firm, $(m-1)w_0 - e^{r-l}mw_0$, is increasing in m . Therefore, the second and third effects of a higher m will definitely increase the severity of bankruptcy.

It is difficult to get an unambiguous relationship between m and systemic bankruptcy analytically. A numerical example will help reveal the total effect of m on systemic bankruptcy. Keeping the values of all the other parameters unchanged, I want to see

how ρ^* , \underline{r} , r at the maximum loss, and the maximum loss change when m changes from 1.5 to 3.0.

It turns out that m has little impact on ρ^* . ρ^* is constant when m varies from 1.5 to 3.0. This is because the precision of the updated belief of firm x^* , $\alpha + \beta = 101$, is so high that its expected payoff is determined only by a small range of values of $me^{r-l} - m + 1$, where e^{r-l} is close to 1 and m has little impact.

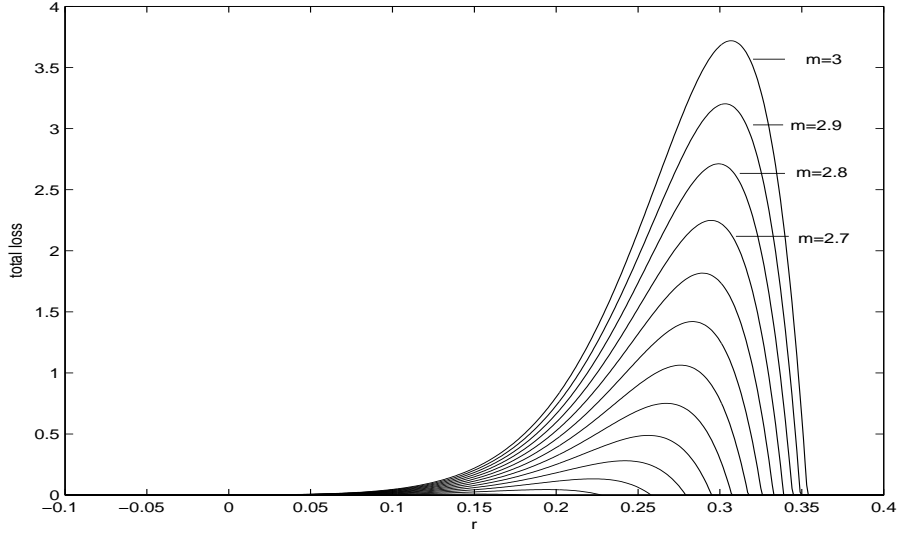


Figure 3: How total loss changes with financial leverage m

Figure 3 shows that systemic bankruptcy appears in wider fundamentals range with higher m . The threshold level of the fundamentals where systemic bankruptcy begins to appear, \underline{r} , is strictly increasing in m . The intuition is straightforward: the more a firm borrows, the more easily it is unable to repay its debts. The realized fundamentals r at the maximum total loss is also strictly increasing in m . In addition, the severity of systemic bankruptcy at each given realized economic fundamentals level is strictly increasing in m . The maximum total loss is trivial and close to 0 when m is around 1.5. Then it takes off and goes above 3.5 when $m = 3$.

So the impact of m on systemic bankruptcy works mainly through the rest of the two channels as long as the private signal of firms is highly precise. The severity of systemic bankruptcy increases rapidly with the increase in m .

4.3 Public Beliefs and Systemic Bankruptcy

This section examines the relationship between the mean of the public belief \bar{r} and systemic bankruptcy. It reveals that *a higher \bar{r} leads to more investment and alleviates coordination failure. The range of economic fundamentals where systemic bankruptcy arises is narrowed. However, systemic bankruptcy tends to be more severe once it happens.*

The public belief is given by $\tilde{r} \sim N(\bar{r}, 1/\alpha)$. Since

$$ER(a^I | x^*(\rho^*)) = \sqrt{\alpha + \beta} \int_{r^*}^{+\infty} \phi(\sqrt{\alpha + \beta}(r - \rho^*)) (me^{r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}))} - m + 1) dr$$

is strictly increasing in \bar{r} , $x^*(\rho^*)$ is decreasing in \bar{r} . Therefore, given each realized r , the proportion of firms investing, is increasing in \bar{r} , because

$$1 - l(r) = 1 - \Phi(\sqrt{\beta}(x^* - r)),$$

where x^* is decreasing in \bar{r} .

Coordination is easier with more optimistic public beliefs about the fundamentals, \bar{r} . This is because a firm observing a good public signal not only anticipates that the fundamentals are good, but also anticipates that other firms will believe that the fundamentals are good, and will tend to invest more.

Since $\underline{r} - l(\underline{r}, \bar{r}) = \ln(\frac{m-1}{m})$, a higher \bar{r} will decrease \underline{r} . Meanwhile, the unpaid debts of an individual firm, $(m-1)w_0 - e^{r-l}mw_0$, decrease at each given realized fundamentals level r due to higher proportion of firms investing. But at the same time, the bankruptcy rate given each $r < \underline{r}$, $1 - l(r, \bar{r})$ increases.

A numerical example is given to show the relationship. Keeping the values of all the other parameters unchanged, we want to see how ρ^* , \underline{r} , r at the maximum loss, and the maximum loss change when \bar{r} varies from 0 to 1.0.

Figure 4 shows that ρ^* is strictly decreasing in \bar{r} . That is, optimistic public beliefs about the economic fundamentals encourage more investment and alleviates coordination failure.

Figure 5 shows that with higher \bar{r} , systemic bankruptcy appears only in a narrower economic fundamentals range. The threshold fundamentals level where bankruptcy

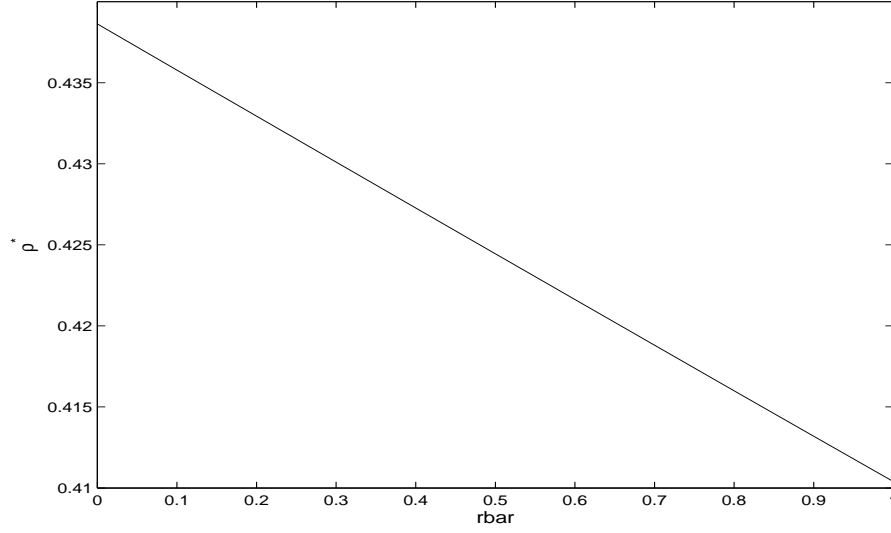


Figure 4: How firms' optimal trigger strategy ρ^* changes with public beliefs \bar{r}

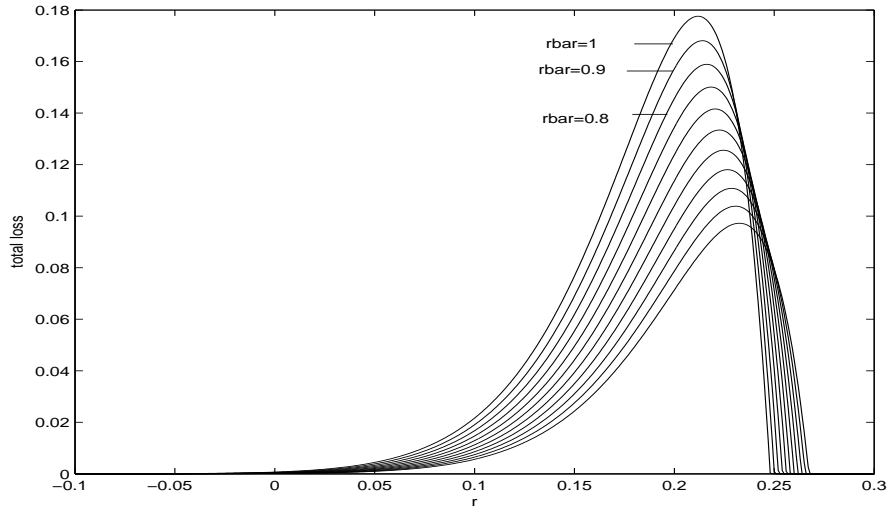


Figure 5: How total loss changes with public beliefs \bar{r}

begins to arise, \underline{r} , is decreasing in \bar{r} . This is because with the improvement in coordination, investment return is higher at any given level of economic fundamentals. For the same reason, the level of the fundamentals at which systemic bankruptcy is the severest is also lower with higher \bar{r} . It is interesting to see that higher \bar{r} leads

to more severe systemic bankruptcy once it occurs. The intuition is that optimistic public beliefs induce more firms to invest at each fundamentals level. Thus, when the fundamentals turn out to be weak, the bankruptcy rate is higher, leading to higher total loss.

5 A Model with Banks

In this section, banks are introduced into the basic model to endogenize the borrowing rate. Here I do not intend to explain why banks exist in an economy. I simply assume that the transaction costs between investors and firms are prohibitively high, and the firms have to finance their investment via banks.

5.1 The Model

N risk neutral banks compete over the borrowing rate to maximize their profits.

The banks are assumed to hold the public belief about economic fundamentals, $\tilde{r} \sim N(\bar{r}, 1/\alpha)$. This public belief is shared by the firms. Later I will introduce banks with their own private signals.

The banks are assumed to have limitless access to funds at the risk-free rate of 1.

The timing of the game is as follows. At the first stage, the banks offer $e^{\bar{r}_b}$, the gross borrowing rate, to the firms. At the second stage, given the borrowing rate and their own private signals, the firms decide whether to invest or not. The game of the firms is the same as before, except that now they face a different borrowing rate. Thus it is a sequential game with the banks as the leaders, and the firms as the followers.

I can prove that there is a unique subgame perfect Bayesian equilibrium in this game. Let $\gamma_0 = \frac{\alpha^2}{\beta}$.

Proposition 2 *Provided that $\gamma_0 \leq 2\pi$, and the public belief \bar{r} is high enough for the banks to make nonnegative profit, there is a unique subgame perfect Bayesian equilibrium. In this equilibrium, the banks offer the borrowing rate of $e^{\bar{r}^*}$. Given this*

borrowing rate, firm i chooses to borrow and invest if and only if $\rho_i > \rho^*$. Otherwise, the firm chooses not to invest. Given $\bar{r}_b = \bar{r}_b^*$, ρ^* is the unique solution to

$$\int_{r^*}^{+\infty} \sqrt{\alpha + \beta} \phi(\sqrt{\alpha + \beta}(r - \rho^*)) (me^{r-l(r, \rho^*)} - (m-1)e^{\bar{r}_b}) dr - 1 = 0,$$

and \bar{r}_b^* is the smallest positive solution to

$$\begin{aligned} & \int_{-\infty}^{r^*} \sqrt{\alpha} \phi(\sqrt{\alpha}(r - \bar{r})) (me^{r-l(r, \rho^*)} - (m-1))(1 - l(r, \rho^*)) dr \\ & + \int_{r^*}^{+\infty} \sqrt{\alpha} \phi(\sqrt{\alpha}(r - \bar{r})) (m-1)(e^{\bar{r}_b} - 1)(1 - l(r, \rho^*)) dr = 0, \end{aligned} \quad (7)$$

where

$$l(r, \rho^*) = \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}))),$$

and r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(\rho^* - r^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}))) = \ln \frac{m-1}{m} + \bar{r}_b.$$

Proof:

Backward induction is used to find the subgame perfect Bayesian equilibrium in this game. First, I can prove that there is a unique equilibrium in the game of the firms. In equilibrium, a firm will invest if and only if its updated belief $\rho > \rho^*$. The proof is basically the same as that in section 3 with few modifications. Here I only give the proof confined to symmetric trigger strategies. The method of iterated elimination of strictly dominated strategy can also be applied here to prove the unique equilibrium.

To find the unique equilibrium in the game of the firms, I need to pinpoint ρ^* first. Suppose that firm i is at the switching point, that is, $\rho_i = \rho^*$, then it must be indifferent about investing or not, which means

$$\begin{aligned} ER(a^I | x^*(\rho^*)) &= \\ & \int_{r^*}^{+\infty} \sqrt{\alpha + \beta} \phi(\sqrt{\alpha + \beta}(r - \rho^*)) [me^{r-\Phi(\sqrt{\beta}(x^*-r))} - (m-1)e^{\bar{r}_b}] dr \\ & = 1 = R(a^{NI} | x^*(\rho^*)), \end{aligned} \quad (8)$$

where r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(x^* - r^*)) = \ln \frac{m-1}{m} + \bar{r}_b.$$

By simplifying the above equation and substituting ρ^* for x^* , we get

$$\int_{r^*}^{+\infty} \sqrt{\alpha + \beta} \phi(\sqrt{\alpha + \beta}(r - \rho^*)) (m e^{r-l(r, \rho^*)} - (m-1) e^{\bar{r}_b}) dr = 1, \quad (9)$$

where

$$l(r, \rho^*) = \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}))).$$

Using the same method in appendix 1, I can prove that the above equation is strictly increasing in ρ^* , given $\frac{\alpha^2}{\beta} < 2\pi$. Here I omit the proof. Based on the above equation, I find the unique solution of $\rho^*(\bar{r}_b)$. It is easy to show that the symmetric trigger strategy ρ^* is optimal for every firm.

Now let us look at the first mover of this game, the banks. The banks fully understand the game among the firms and the equilibrium strategies of the firms. Taking the equilibrium strategies of the firms into consideration, the banks will set the lowest borrowing rate \bar{r}_b that makes zero expected profit in the banking sector. This is the unique equilibrium and no bank will deviate. By raising the borrowing rate above the equilibrium rate, a bank will not have any firm to lend to. On the other hand, by lowering the borrowing rate, the bank will make negative profits.

Given that the banks' belief about the fundamentals is $\tilde{r} \sim N(\bar{r}, 1/\alpha)$, the expected profit that the whole banking sector can make is given by:

$$\begin{aligned} E\Pi_b &= \int_{-\infty}^{r^*} \sqrt{\alpha} \phi(\sqrt{\alpha}(r - \bar{r})) (m w_0 e^{r-l(r, \rho^*)} - (m-1) w_0) (1 - l(r, \rho^*)) dr + \\ &\quad \int_{r^*}^{+\infty} \sqrt{\alpha} \phi(\sqrt{\alpha}(r - \bar{r})) (m-1) w_0 (e^{\bar{r}_b} - 1) (1 - l(r, \rho^*)) dr. \end{aligned}$$

The gross borrowing rate, $e^{\bar{r}_b^*}$, that a bank will charge is the smallest positive solution to $E\Pi_b = 0$, which can be simplified as

$$\begin{aligned} &\int_{-\infty}^{r^*} \sqrt{\alpha} \phi(\sqrt{\alpha}(r - \bar{r})) (m e^{r-l(r, \rho^*)} - (m-1)) (1 - l(r, \rho^*)) dr \\ &+ \int_{r^*}^{+\infty} \sqrt{\alpha} \phi(\sqrt{\alpha}(r - \bar{r})) (m-1) (e^{\bar{r}_b} - 1) (1 - l(r, \rho^*)) dr = 0. \end{aligned} \quad (10)$$

Notice that when \bar{r} is low enough, the expected profit of banks from lending will be always non-positive. Thus the banks will lend if and only if \bar{r} is large enough, such that $\max\{E\Pi_b(\rho^*, \bar{r}_b)\} \geq 0$. Or the banks will choose not to lend.

Q.E.D

5.2 The Analysis of the Model

In this section, numerical examples are given to examine how systemic bankruptcy will be influenced by the realization of economic fundamentals, firms' financial leverage, and public beliefs when the borrowing rate is endogenously given.

5.2.1 Systemic Bankruptcy and Realized Economic Fundamentals

In this model with banks, firms will still take the optimal trigger strategy ρ^* in equilibrium. Therefore, the relationship between systemic bankruptcy and realized economic fundamentals will be similar to what I found in the basic model without banks except that the level of ρ^* will be different due to different borrowing costs.

A numerical example with $\alpha = 1$, $\beta = 100$, $m = 2$, $\bar{r} = 0.5$, and $w_0 = 100$ is given. I find that banks will charge a borrowing rate of $e^{0.00008}$, which is extremely close to 1. The firms' optimal trigger strategy $\rho^* = 0.4245$ is also very close to 0.4244, the optimal trigger strategy in the case with an exogenously given borrowing rate. Therefore, the results in section 4.1 about the relationship between systemic bankruptcy and economic fundamentals still holds here.

The reason that the gross borrowing rate charged by bank is extremely close to 1 is that β is high and firms have very precise information about economic fundamentals. Thus banks expect that the proportion of firms borrowing, $1 - l(r)$, when bankruptcy arises ($r < r^*$) is extremely low, and that their expected loss from lending is also extremely low. On the other hand, when $r > r^*$, banks expect that the proportion of firms borrowing, $1 - l(r)$, is high, and their expected gain from lending is high too. Since banks only aim at zero profit, they will charge an extremely low borrowing rate. This feature will hold in the rest of the numerical examples. Therefore, in general, banks will not greatly change firms' equilibrium behavior through charging different

borrowing rates in this model.

5.2.2 Systemic Bankruptcy and Financial Leverage

Comparative statics reveals that *the equilibrium trigger strategy of firms will slightly increase in m . Meanwhile, the severity of systemic bankruptcy rapidly increases in financial leverage. With higher financial leverage, the range of economic fundamentals where systemic bankruptcy arises is wider, and the total loss at each level of economic fundamentals is higher.*

Financial leverage will influence systemic bankruptcy in the following ways: first, the equilibrium borrowing rate charged by the banks, $e^{\bar{r}_b^*}$, and the equilibrium trigger strategy by the firms, ρ^* , are functions of m . Second, given ρ^* , the threshold fundamentals level for bankruptcy, \underline{r} , is increasing in m . Third, the unpaid debt of an individual firm, $(m - 1)w_0 - e^{r-l}mw_0$, is increasing in m . The last two effects are exactly the same as those in the case without banks.

A numerical example with $\alpha = 1$, $\beta = 100$, $\bar{r} = 0.5$, $w_0 = 100$, and m changing from 1.5 to 3 is given.

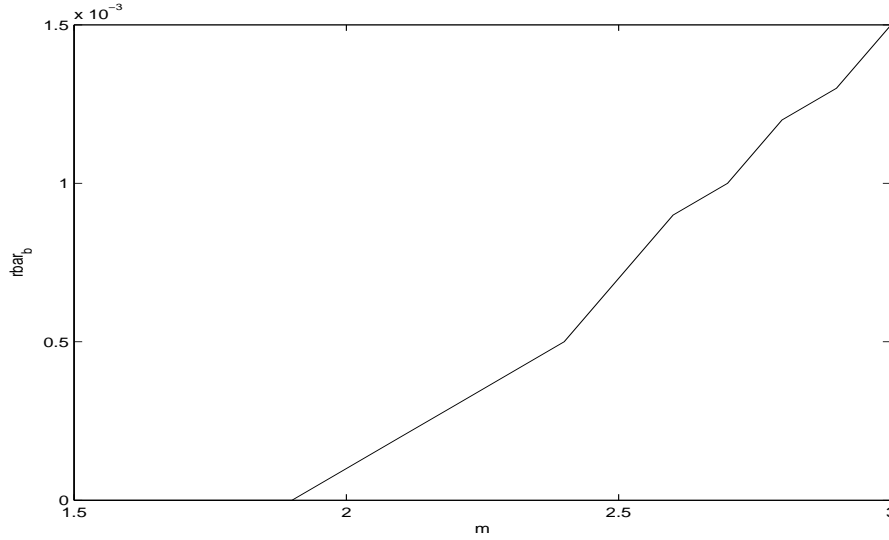


Figure 6: How borrowing rate \bar{r}_b^* changes with financial leverage m

Figure 6 reveals that the banks will charge higher borrowing rates with higher m .

This result is intuitive. With higher m , firms more easily go bankrupt and the banks have to charge a higher borrowing rate to gain zero expected profit.

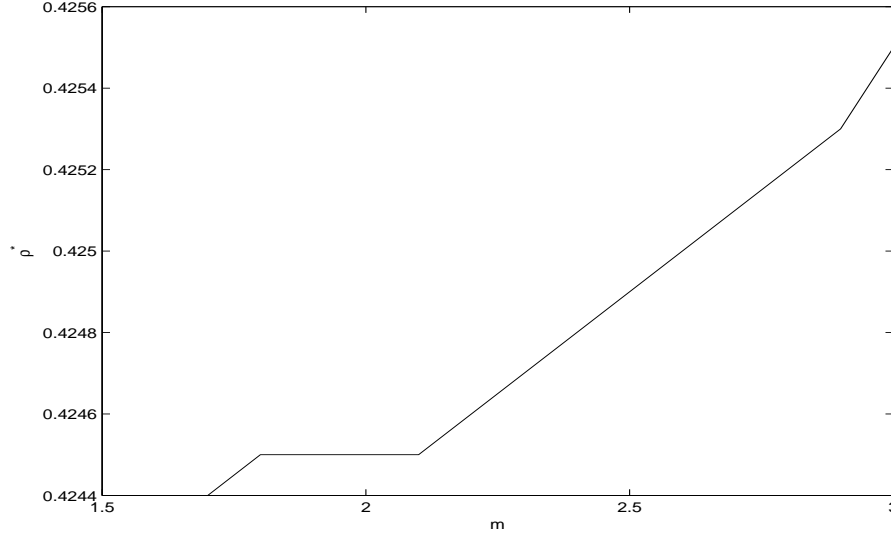


Figure 7: How optimal trigger strategy ρ^* changes with financial leverage m

Figure 7 shows that ρ^* , the optimal trigger strategy of the firms, is increasing in m . This is because the higher borrowing rate decreases the expected payoff of the firms from investing. Now firms will invest only when they have higher updated beliefs about the fundamentals.

Figures 8, 9 and 10 illustrate that higher financial leverage m greatly increases the severity of systemic bankruptcy. With higher m , the range of economic fundamentals where systemic bankruptcy arises is wider, and the total loss is higher at any given level of economic fundamentals.

5.2.3 Systemic Bankruptcy and Public Beliefs

Comparative statics reveals that higher public beliefs can alleviate coordination failure. With higher public beliefs, more firms invest at each level of economic fundamentals, leading to higher investment return. *The range of economic fundamentals where systemic bankruptcy arises is narrower with higher public beliefs. But once systemic bankruptcy happens, the total loss is higher with higher public beliefs.*

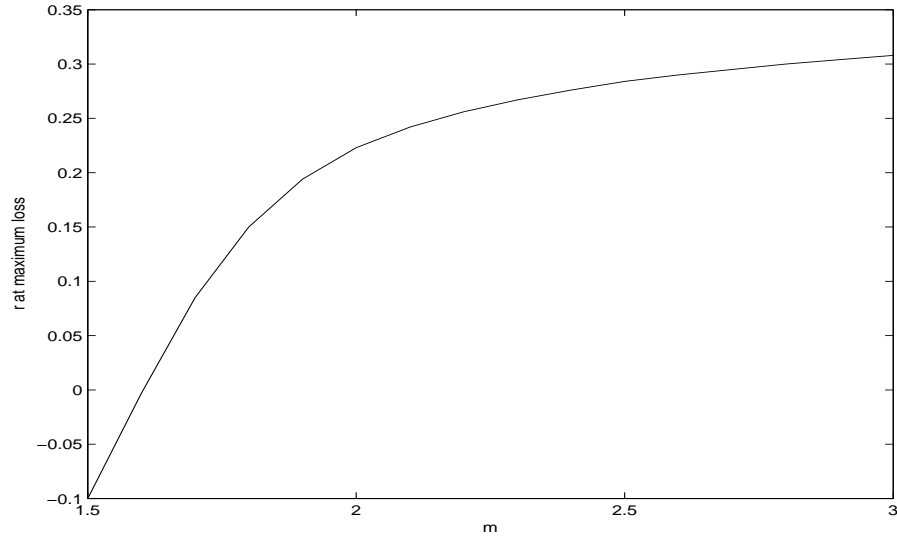


Figure 8: How economic fundamentals r at maximum loss changes with financial leverage m

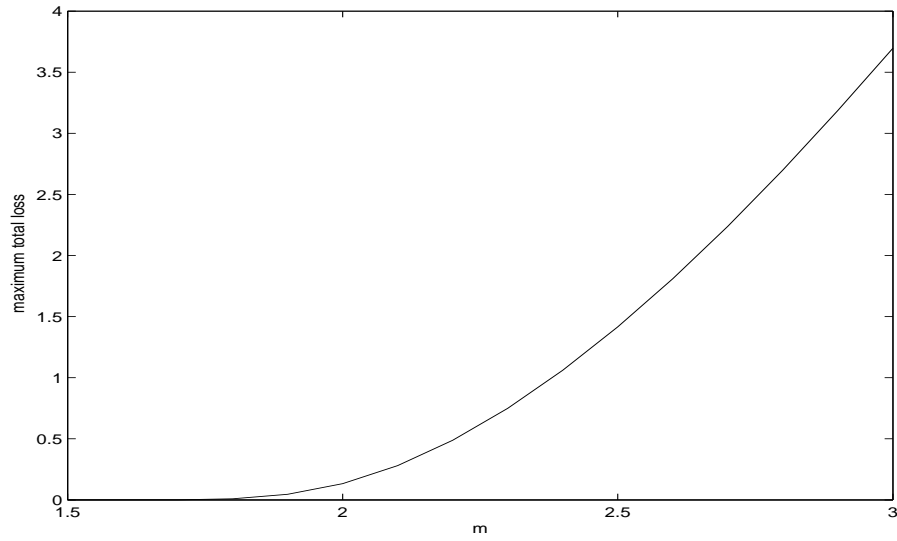


Figure 9: How maximum loss changes with financial leverage m

Analytically, public beliefs will influence systemic bankruptcy in the following way: first, given the borrowing rate $e^{\bar{r}b}$, public beliefs will influence the equilibrium outcome in the same way as it does in the case without banks. Optimal strategy ρ^* is lower with

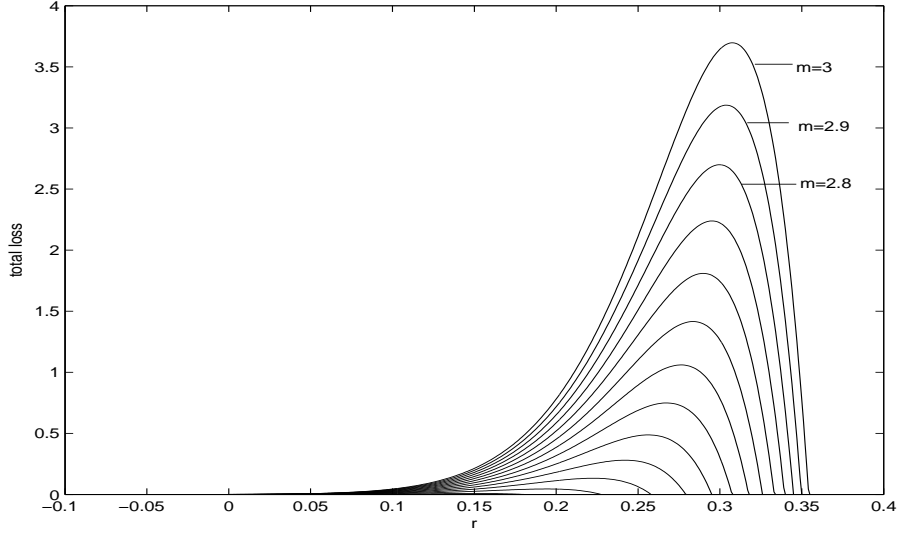


Figure 10: How total loss changes with financial leverage m

higher public beliefs \bar{r} , inducing more firms to invest at each economic fundamentals level. The threshold economic fundamental level where systemic bankruptcy begins to arise, \underline{r} , is lower. Given $r < \underline{r}$, bankruptcy rate $1 - l(r)$ increases, and the unpaid debt of an individual firm, $(m - 1)w_0 - e^{r-l}mw_0$, decreases.

Second, with banks in the model, the public belief \bar{r} is also the banks' belief and its change will influence banks' expected profits and consequently the borrowing rate they charge, $e^{\bar{r}b}$. This will work through two channels: first, higher \bar{r} leads to higher expectations about r , and higher return at each realized economic fundamentals level due to lower expected ρ^* , leading to higher expected profits of the banks. Second, higher \bar{r} leads to higher investment when bankruptcy occurs, leading to lower expected profits for the banks.

It is difficult to find the analytical solution to define the relationship between public beliefs and systemic bankruptcy. Here I will give a numerical example with $\alpha = 1$, $\beta = 100$, $m = 2$, $w_0 = 100$, and \bar{r} varying from 0.1 to 1.

The quantitative relationship between public beliefs and systemic bankruptcy is similar to that in the case without banks. However, theoretically, in this model with banks, public beliefs will influence firms' optimal strategy ρ^* through one more

channel: changing the borrowing rate $e^{\bar{r}_b}$ that the banks offer. The numerical example reveals that with higher public beliefs, the banks will charge lower borrowing rates, which will further lower the firms' optimal strategy ρ^* compared to the case without banks. Thus optimistic public beliefs have three functions in this case: first, they induce optimistic beliefs of firms about economic fundamentals. Second, they induce optimistic beliefs of firms about other firms' beliefs about economic fundamentals. Third, they induce optimistic beliefs of banks about investment returns. All will lead to lower optimal strategy ρ^* and higher investment returns in equilibrium.

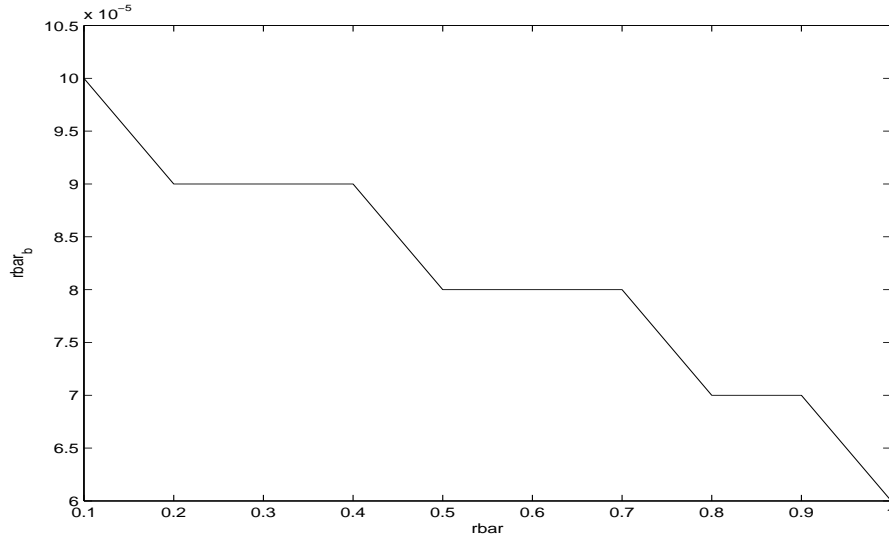


Figure 11: How borrowing rate \bar{r}_b^* changes with public beliefs \bar{r}

Figure 11 shows that the borrowing rate is lower with higher public beliefs. This is because the banks with higher beliefs about economic fundamentals have higher expected payoffs from lending, and only need to charge a lower borrowing rate to gain zero profit.

In figure 12, higher public beliefs lead to lower optimal trigger strategy of the firms, ρ^* for two reasons. First, the banks are charging a lower borrowing rate. Second, higher public beliefs raise the beliefs of the firms about the return from investing. Both will encourage firms to invest.

Figures 13, 14 and 15 reveal that with higher public beliefs, the range of economic

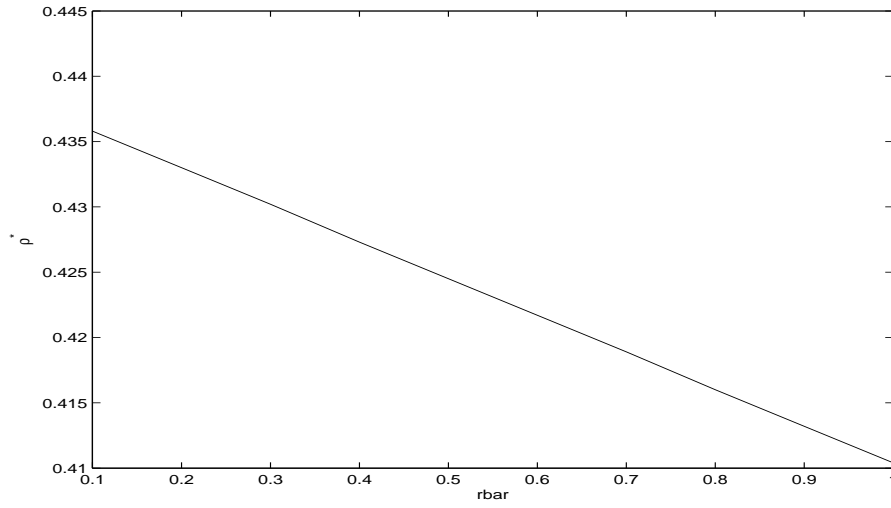


Figure 12: How optimal trigger strategy ρ^* changes with public beliefs \bar{r}

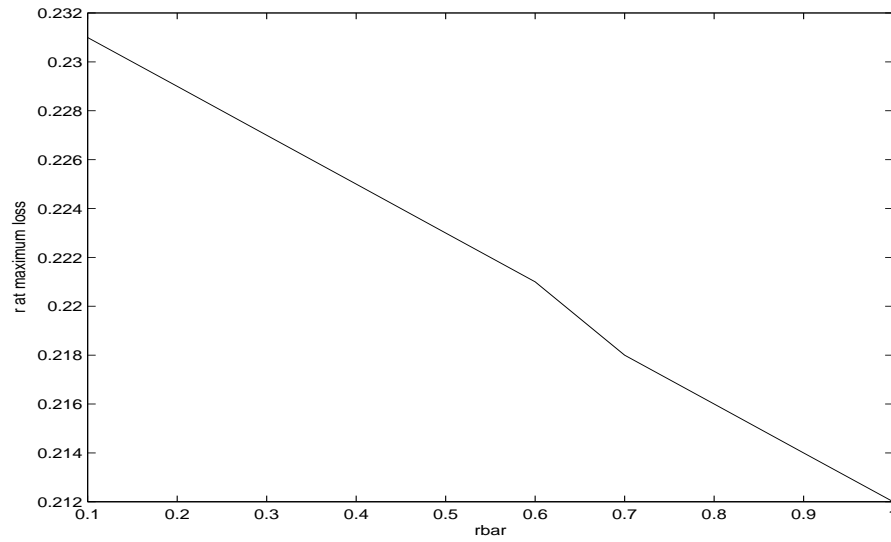


Figure 13: How economic fundamentals r at maximum loss changes with public beliefs \bar{r}

fundamentals where systemic bankruptcy arises is narrower. But the total loss is generally higher once systemic bankruptcy occurs.

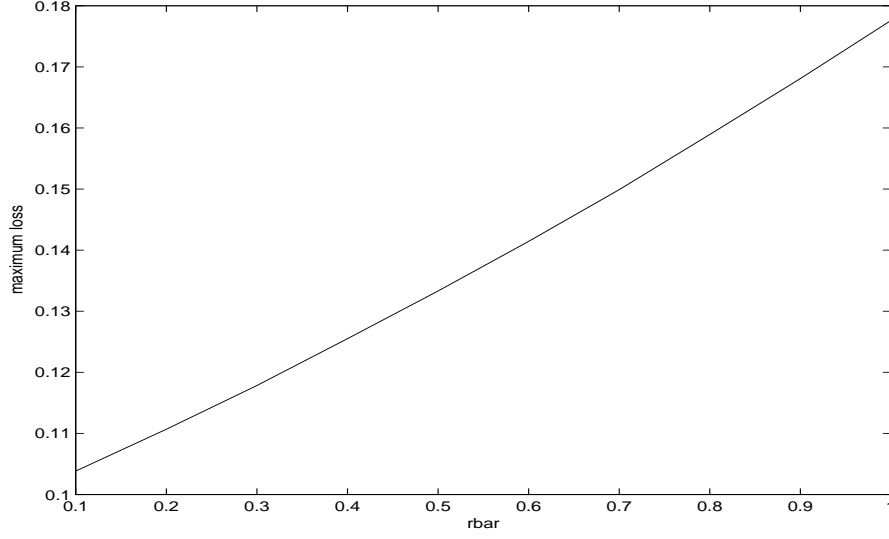


Figure 14: How maximum loss changes with public beliefs \bar{r}

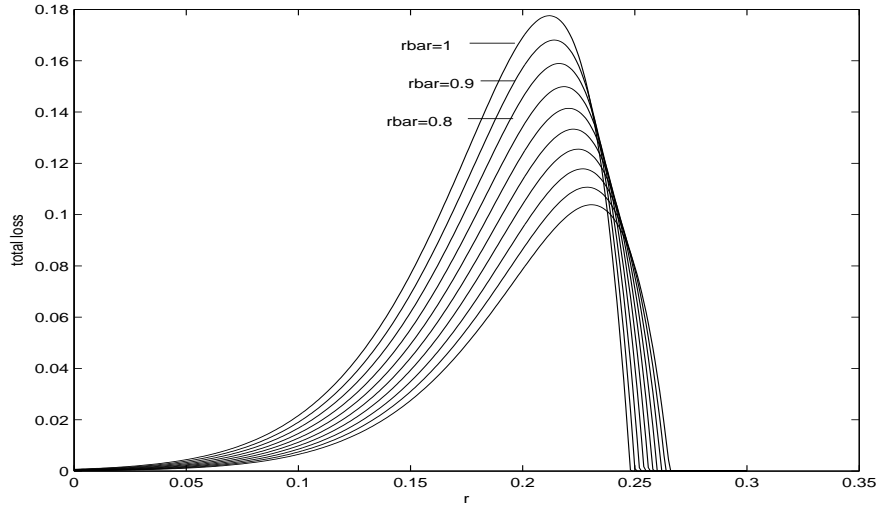


Figure 15: How total loss changes with public beliefs \bar{r}

6 The Model Where Banks Have Private Signals

6.1 The Model

This section studies the case when the banks have their own private signals. The prior belief of banks about economic fundamentals is still $\tilde{r} \sim N(\bar{r}, 1/\alpha)$. This public belief

is also shared by the firms. In addition, all the banks receive the same realization of private signal about economic fundamentals, $x_b = r + \varepsilon_b$, where ε_b is normally distributed with mean 0 and precision β_b . This signal is also observed by the firms.

I can prove that there is a unique subgame perfect Bayesian equilibrium in this game. Let $\gamma_1 = \frac{(\alpha + \beta_b)^2}{\beta}$.

Proposition 3 *Provided that $\gamma_1 \leq 2\pi$ and the private signal of the banks, x_b , is high enough for them to make zero expected profit from lending, there is a unique subgame perfect Bayesian equilibrium. In this equilibrium, the banks offer the borrowing rate of $e^{\bar{r}_b}$. Given this borrowing rate, firm i chooses to borrow and invest if and only if $\rho_i > \rho^*$. Otherwise, the firm chooses not to invest. Given $\bar{r}_b = \bar{r}_b^*$, ρ^* is the unique solution to*

$$\int_{r^*}^{+\infty} \sqrt{\alpha + \beta_b} \phi(\sqrt{\alpha + \beta_b}(r - \rho^*)) (me^{r-l(r, \rho^*)} - (m-1)e^{\bar{r}_b}) dr = 1,$$

and \bar{r}_b^* is the smallest positive solution to

$$\begin{aligned} & \int_{-\infty}^{r^*} \sqrt{\alpha + \beta_b} \phi(\sqrt{\alpha + \beta_b}(r - \rho_b)) (me^{r-l(r, \rho^*)} - (m-1))(1 - l(r, \rho^*)) dr \\ & + \int_{r^*}^{+\infty} \sqrt{\alpha + \beta_b} \phi(\sqrt{\alpha + \beta_b}(r - \rho_b)) (m-1)(e^{\bar{r}_b} - 1)(1 - l(r, \rho^*)) dr = 0, \end{aligned}$$

where

$$l(r, \rho^*) = \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}) + \frac{\beta_b}{\beta}(\rho^* - x_b))),$$

and r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(\rho^* - r^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}) + \frac{\beta_b}{\beta}(\rho^* - x_b))) = \ln \frac{m-1}{m} + \bar{r}_b.$$

Proof: See appendix 3.

6.2 Banks with Private Signals and Systemic Bankruptcy

This section analyzes the role that banks play in systemic bankruptcy when the banks have their own private signal. Given the setup in the model, systemic bankruptcy is influenced by banks in two ways. First, the belief of the banks about economic

fundamentals will determine their lending condition, $e^{\bar{r}_b^*}$, which will influence the expected payoffs of the firms. I call it the payoff effect. Second, the belief of the banks about economic fundamentals will also influence the beliefs of the firms through Bayesian updating rule. The second effect influences systemic bankruptcy in the same way as public beliefs do, which I analyze in sections 4 and 5. I call this effect the information effect.

Now let us look at a numerical example with $\alpha = 1$, $\beta = 100$, $\beta_b = 10$, $\bar{r} = 0.5$, $m = 2$, $w_0 = 100$, and x_b varying from 0.1 to 1.

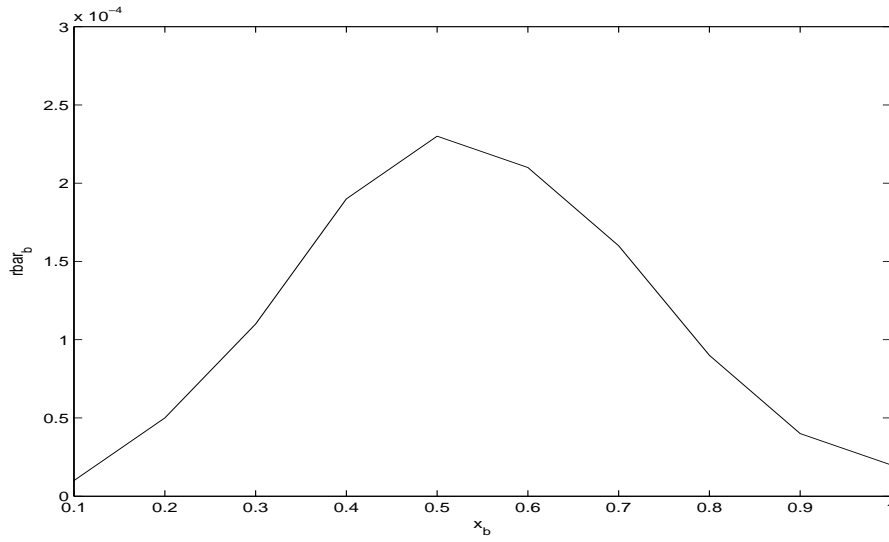


Figure 16: How borrowing rate \bar{r}_b changes with the banks' private signal x_b

Figure 16 shows a non-monotonic relationship between the banks' beliefs on the fundamentals and the borrowing rates they charge. When x_b is lower than 0.5, the borrowing rate is increasing in x_b , but when x_b is greater than 0.5, the borrowing rate is decreasing in x_b . This is because two opposite effects determine the banks' expected profits. Higher x_b makes the banks put more weight on the higher level of the fundamental r , leading to higher expected profits. But higher x_b also makes the banks expect more firms to borrow when the fundamentals r is low and nonperforming loans arise, inducing lower expected profits.

Figure 17 reveals that ρ^* is very sensitive to x_b . When x_b is lower than a certain

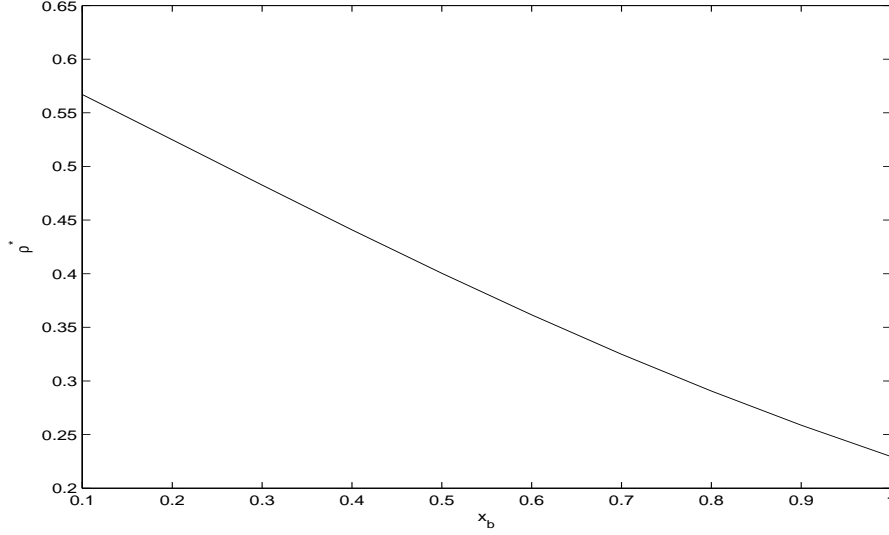


Figure 17: How firms' optimal trigger strategy ρ^* changes with the banks' private signal x_b

level, the banks always have negative expected return on the investment, no matter what \bar{r}_b is. No banks will lend, and naturally no bankruptcy will occur.

When x_b is high enough for banks to lend, the firms' optimal trigger strategy ρ^* rapidly decreases in the banks' private signal x_b . This impact is attained mainly through the information effect, instead of the payoff effect, because we can see that in general the borrowing rate is very close to zero and has little impact on the firms' optimal trigger strategy.

The following figures reveal how systemic bankruptcy is influenced by the banks' private signal x_b .

From the figures above we can see that pessimistic banks will make coordination more difficult, but at the same time will curb systemic bankruptcy. In the extreme case, the banks will not lend at all. Therefore, no bankruptcy occurs. On the other hand, optimism among banks will lead to easier coordination, but systemic bankruptcy also tends to be more severe once it happens.

The policy implication derived from this model is that an optimistic sentiment among banks will be an important indicator of possible severe systemic bankruptcy.

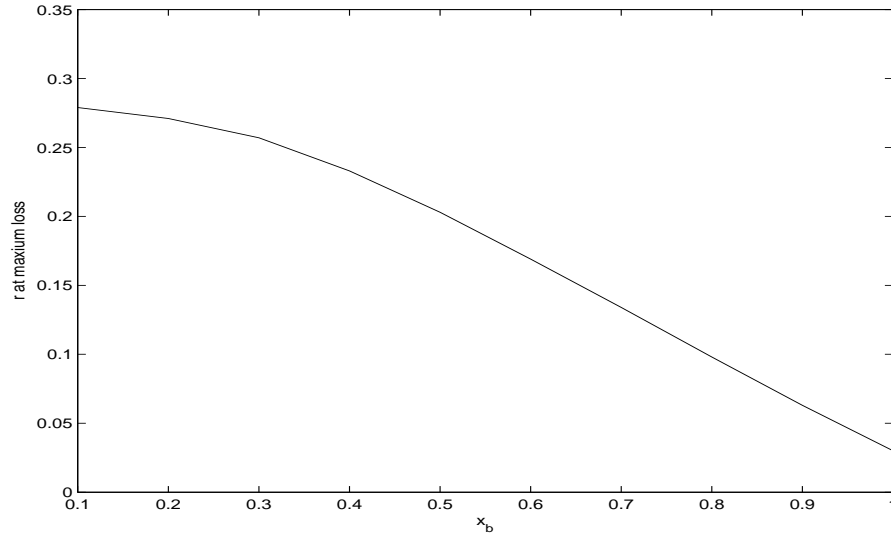


Figure 18: How economic fundamentals r at maximum loss changes with banks' private signals

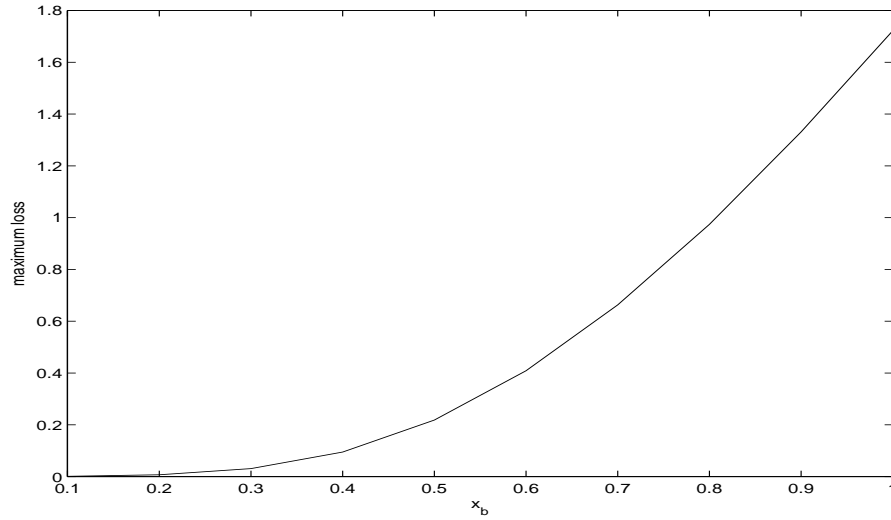


Figure 19: How maximum loss changes with the banks' private signal x_b

This finding is also consistent with the anecdotal observation that severe financial crises usually break out shortly after an economic boom when the banks are still optimistic.

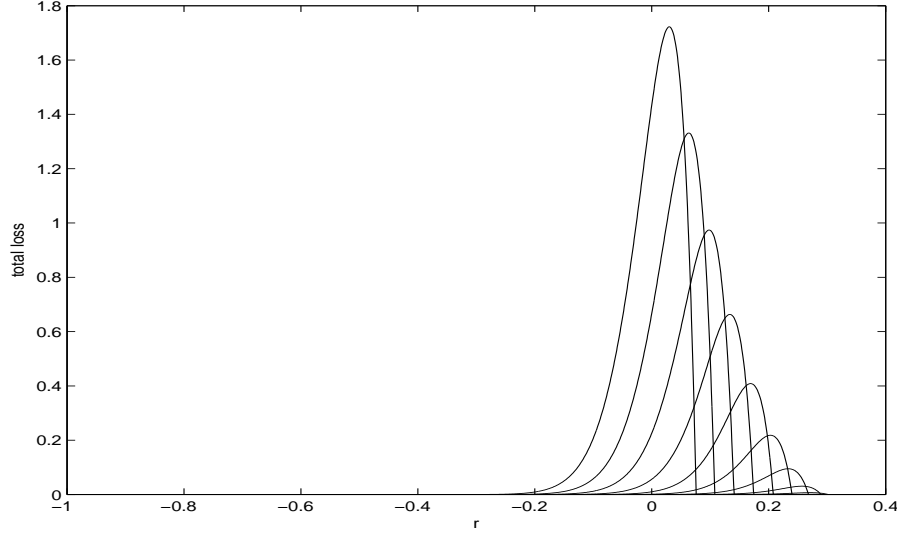


Figure 20: How total loss varies with the banks' private signal x_b

7 Conclusions, Policy Implications and Future Research

This paper explains the origin of systemic bankruptcy of nonfinancial firms by coordination failure in a decentralized credit economy with investment complementarities. By doing so, I provide a new explanation about how volatility in real investment can cause financial fragility. I hope that my research can promote a better understanding of the origin of financial fragility, and provide theoretical guidance for central banks to establish an “early warning system” to prevent the occurrence of financial crises.

The main conclusions and policy implications from this paper are as follows:

1. Systemic bankruptcy can originate from coordination failure in a decentralized credit economy with investment complementarities. Due to investment complementarities, an economy can be more vulnerable to systemic bankruptcy.
2. Systemic bankruptcy becomes possible when economic fundamentals fall into a middle range, where coordination is critical. Moreover, it tends to break out when the fundamentals are taking a low to medium value in this range.

3. Financial leverage of firms is an important indicator revealing the fragility of a financial system. Higher financial leverage of firms greatly increases the possibility and severity of systemic bankruptcy.
4. Optimistic public beliefs of firms and banks can alleviate coordination failure. Systemic bankruptcy will happen with lower economic fundamentals. But once it happens, it tends to be more severe.

In summary, systemic bankruptcy caused by coordination failure will hit an economy most severely when economic fundamentals fall into the coordination failure zone with a lending boom generated by high financial leverage and optimistic public beliefs of firms and optimistic beliefs of banks. Central banks should be highly alert about financial fragility in such economic situations.

A future direction to extend this paper is to put it in a General Equilibrium framework where uncertainty, investment complementarities and borrowing-lending relationships are present. In the current model, banks are assumed to have limitless access to some funds at an exogenously given cost. In a General Equilibrium model, I will be able to endogenize the size and cost of the funds that banks have. By doing so, I will be able to study investment fluctuations generated by uncertainty, investment complementarities and borrowing-lending relationships. In my another thesis paper, I have established an Overlapping Generation model where both uncertainty and investment complementarities exist to study the relationship between economic growth and volatility. It will be interesting to see what difference it will make to introduce the borrowing-lending relationship into the model.

APPENDIX 1

This appendix gives the proof that firm x^* 's expected gross return rate from investing, $ER(a^I|\rho^*)$, is strictly increasing in ρ^* , given $\frac{\alpha^2}{\beta} < 2\pi$.

We know that:

$$ER(a^I|\rho^*) = \sqrt{\alpha + \beta} \int_{r^*}^{+\infty} \phi(\sqrt{\alpha + \beta}(r - \rho^*)) (me^{r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r})))} - m + 1) dr,$$

where r^* is the unique solution to:

$$r^* - \Phi(\sqrt{\beta}(\rho^* - r^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}))) = \ln \frac{m - 1}{m}.$$

Let $r' = r - \rho^*$. The above function can be transformed into:

$$ER(a^I|\rho^*) = \sqrt{\alpha + \beta} \int_{r'^*}^{+\infty} \phi(\sqrt{\alpha + \beta}r') (me^{r' + \rho^* - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho^* - \bar{r})))} - m + 1) dr',$$

where r'^* is the unique solution to:

$$r'^* + \rho^* - \Phi(\sqrt{\beta}(-r'^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}))) = \ln \frac{m - 1}{m}.$$

Now pick up any $\rho^* \in R$ and let $\rho' = \rho^* + \Delta\rho$, where $\Delta\rho$ is a small positive number. Then we get:

$$ER(a^I|\rho') = \sqrt{\alpha + \beta} \int_{r''^*}^{+\infty} \phi(\sqrt{\alpha + \beta}r') (me^{r' + \rho' - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho' - \bar{r})))} - m + 1) dr',$$

where r''^* is the unique solution to:

$$r''^* + \rho' - \Phi(\sqrt{\beta}(-r''^* + \frac{\alpha}{\beta}(\rho' - \bar{r}))) = \ln \frac{m - 1}{m}.$$

Given $\frac{\alpha^2}{\beta} < 2\pi$, we have $r''^* < r'^*$. Let $\Delta r = r'^* - r''^*$, we can rewrite $NR(a^I|\rho')$ as:

$$\begin{aligned} ER(a^I|\rho') &= \sqrt{\alpha + \beta} \int_{r''^*}^{r'^* + \Delta r} \phi(\sqrt{\alpha + \beta}r') (me^{r' + \rho' - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho' - \bar{r})))} - m + 1) dr' \\ &\quad + \sqrt{\alpha + \beta} \int_{r'^*}^{+\infty} \phi(\sqrt{\alpha + \beta}r') (me^{r' + \rho' - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho' - \bar{r})))} - m + 1) dr'. \end{aligned} \quad (11)$$

Observe that $\sqrt{\alpha + \beta} \int_{r''_*}^{r''_* + \Delta r} \phi(\sqrt{\alpha + \beta} r') (me^{r' + \rho' - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho' - \bar{r})))} - m + 1) dr' > 0$ since it is the integral of a positive function over a normal distribution. This property holds when $\Delta \rho \rightarrow 0$.

Now let us look at the item $\sqrt{\alpha + \beta} \int_{r'_*}^{+\infty} \phi(\sqrt{\alpha + \beta} r') (me^{r' + \rho' - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho' - \bar{r})))} - m + 1) dr'$. I need to compare it with $\sqrt{\alpha + \beta} \int_{r'_*}^{+\infty} \phi(\sqrt{\alpha + \beta} r') (me^{r' + \rho^* - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho^* - \bar{r})))} - m + 1) dr'$.

Define a function $f(\rho)$, which is given by

$$f(\rho) = \sqrt{\alpha + \beta} \int_a^{+\infty} \phi(\sqrt{\alpha + \beta} r') (me^{r' + \rho - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho - \bar{r})))} - m + 1) dr',$$

where $a \in R$ is a constant.

Then I find that:

$$\frac{\partial f}{\partial \rho} = \sqrt{\alpha + \beta} \int_a^{+\infty} \phi(\sqrt{\alpha + \beta} r') me^{r' + \rho - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho - \bar{r})))} (1 - \phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho - \bar{r})))) \frac{\alpha}{\sqrt{\beta}} dr'.$$

Since $\phi(\cdot)$ is the PDF of a standard normal distribution, $0 < \phi(\cdot) \leq \sqrt{\frac{1}{2\pi}}$. So I get $\frac{\partial f}{\partial \rho} > 0$ if $\frac{\alpha^2}{\beta} < 2\pi$. Then it is straightforward to see that:

$$\begin{aligned} & \sqrt{\alpha + \beta} \int_{r'_*}^{+\infty} \phi(\sqrt{\alpha + \beta} r') (me^{r' + \rho' - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho' - \bar{r})))} - m + 1) dr' \\ & > \sqrt{\alpha + \beta} \int_{r'_*}^{+\infty} \phi(\sqrt{\alpha + \beta} r') (me^{r' + \rho^* - \Phi(\sqrt{\beta}(-r' + \frac{\alpha}{\beta}(\rho^* - \bar{r})))} - m + 1) dr' \end{aligned}$$

if $\rho' > \rho^*$.

Since the first item in $ER(a^I | \rho')$ is greater than 0 and the second item is greater than $ER(a^I | \rho^*)$, I prove that $ER(a^I | \rho') > ER(a^I | \rho^*)$. Let $\Delta \rho \rightarrow 0$, I prove that the objective function is strictly increasing in ρ^* given $\frac{\alpha^2}{\beta} < 2\pi$.

APPENDIX 2: The Proof of Proposition 1

This appendix shows how the unique trigger strategy equilibrium can be obtained by using the iterated elimination of strictly dominated strategies.

The expected gross return rate from investing of a firm receiving a private signal ρ given all the others follow the switching strategy $\hat{\rho}$, which is denoted by $ER(\rho, \hat{\rho})$, is given by

$$ER(\rho, \hat{\rho}) = \sqrt{\alpha + \beta} \int_{r^*}^{+\infty} \phi(\sqrt{\alpha + \beta}(r - \rho)) (me^{r - \Phi(\sqrt{\beta}(\hat{\rho} - r + \frac{\alpha}{\beta}(\hat{\rho} - \bar{r}))} - m + 1) dr.$$

where r^* is the unique solution to:

$$r^* - \Phi(\sqrt{\beta}(\hat{\rho} - r^* + \frac{\alpha}{\beta}(\hat{\rho} - \bar{r})) = \ln \frac{m - 1}{m}.$$

Notice that $ER(\rho, \hat{\rho})$ is increasing in ρ , and decreasing in $\hat{\rho}$.

When ρ is sufficiently low, not investing will be the dominant strategy for a firm, no matter what strategies other firms will take. Let us denote it by $\underline{\rho}_0$. All firms realize this and rule out any strategy for firms to invest below $\underline{\rho}_0$. Then investing cannot be optimal for a firm when it receives a private signal below $\underline{\rho}_1$, which solves

$$ER(\underline{\rho}_1, \underline{\rho}_0) = 1.$$

This is because the trigger strategy around $\underline{\rho}_1$ is the best respond to the trigger strategy around $\underline{\rho}_0$, and all firms believe that other firms will not invest when their private signals are below $\underline{\rho}_1$. Since the firms' expected return is decreasing in the second argument, this rule out any strategy for firms to invest below $\underline{\rho}_1$. Proceeding this way, I get an increasing sequence:

$$\underline{\rho}_0 < \underline{\rho}_1 < \cdots \underline{\rho}_k < \cdots,$$

where any strategy of investing when $\rho < \underline{\rho}_k$ does not survive k rounds of deletion of dominated strategies. The sequence is increasing because $ER(\cdot, \cdot)$ is increasing in the first argument and decreasing in the second one. The smallest solution $\underline{\rho}$ to the equation $ER(\rho, \rho) = 1$ is the least upper bound of this sequence, and hence its limit. Any strategy of investing below $\underline{\rho}$ cannot survive iterated dominance.

Similarly, I can have an analogous argument beginning with the case that ρ is large enough and the strategy to invest is dominant no matter what strategies other firms will take. If ρ is the largest solution to $ER(\rho, \rho) = 1$, any strategy of not investing when the signal is higher than ρ cannot survive the deletion of dominated strategies.

I have proved that given $\gamma < 2\pi$, there is a unique solution to $ER(\rho, \rho) = 1$. So the smallest solution is equal to the largest solution. There is only one strategy surviving the iterated elimination of dominated strategies, which is the unique equilibrium strategy in this game.

Q.E.D

APPENDIX 3: The Proof of Proposition 3

Given the borrowing rate $e^{\bar{r}_b}$ offered by the banks, the private signal of the banks, x_b and its own private signal x_i , firm i will update its belief on the fundamental based on Bayes' rule. Thus we get $(\tilde{r}|x_b, x_i) \sim N(\frac{\alpha\bar{r} + \beta_b x_b + \beta x_i}{\alpha + \beta_b + \beta}, \frac{1}{\alpha + \beta_b + \beta})$, where $\rho_i = \frac{\alpha\bar{r} + \beta_b x_b + \beta x_i}{\alpha + \beta_b + \beta}$ is the mean and $\alpha + \beta_b + \beta$ is the precision.

I can prove that there is a unique equilibrium in the firms' game. In equilibrium, a firm will invest if and only if its belief ρ_i is greater than some critical value ρ^* . The proof is basically the same as that in section 3 except some small modifications.

First, I need to pin down ρ^* . Suppose that firm i is at the switching point, that is, $\rho_i = \rho^*$, then it must be indifferent about investing or not, which means

$$\begin{aligned} ER(a^I|x^*(\rho^*)) &= \\ &\int_{r^*}^{+\infty} \sqrt{\alpha + \beta_b + \beta} \phi(\sqrt{\alpha + \beta_b + \beta}(r - \rho^*)) [m e^{r - \Phi(\sqrt{\beta}(x^* - r))} - (m - 1)e^{\bar{r}_b}] dr \\ &= 1 = R(a^{NI}|x^*(\rho^*)), \end{aligned} \quad (12)$$

where r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(x^* - r^*)) = \ln \frac{m - 1}{m} + \bar{r}_b.$$

By simplifying the above equation and substitute x^* by ρ^* , we get

$$\int_{r^*}^{+\infty} \sqrt{\alpha + \beta_b + \beta} \phi(\sqrt{\alpha + \beta_b + \beta}(r - \rho^*)) (m e^{r - l(r, \rho^*)} - (m - 1)e^{\bar{r}_b}) dr = 1, \quad (13)$$

where

$$l(r, \rho^*) = \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}) + \frac{\beta_b}{\beta}(\rho^* - x_b))),$$

With the same method I use in the appendix 1, I can prove that the above equation is strictly increasing in ρ^* , given $\frac{(\alpha + \beta_b)^2}{\beta} < 2\pi$. Here I omit the proof. Based on the above equation, we find the unique solution of $\rho^*(\bar{r}_b, x_b)$.

Now let us look at the first mover of this game, the banks. The banks fully understand the game among the firms and the equilibrium strategies of the firms. Taking the equilibrium strategies of the firms into consideration, the banks will set the lowest borrowing rate \bar{r}_b that makes the zero expected profit in the banking sector.

This is the unique equilibrium and no bank will deviate. By raising the borrowing rate, a bank cannot have any firm to borrow. While by lowering the borrowing rate, the bank will make negative profit.

After observing x_b , a bank updates its believe about the fundamental, \tilde{r} . The mean of $(\tilde{r}|x_b)$ is

$$\rho_b = \frac{\alpha\bar{r} + \beta_b x_b}{\alpha + \beta_b}, \quad (14)$$

and the precision is $\alpha + \beta_b$.

Therefore the expected profit of a bank is given by:

$$\begin{aligned} E\Pi_b = & \int_{-\infty}^{r^*} \sqrt{\alpha + \beta_b} \phi(\sqrt{\alpha + \beta_b}(r - \rho_b)) (mw_0 e^{r-l(r, \rho^*)} - (m-1)w_0)(1-l(r, \rho^*)) dr + \\ & \int_{r^*}^{+\infty} \sqrt{\alpha + \beta_b} \phi(\sqrt{\alpha + \beta_b}(r - \rho_b)) (m-1)w_0(e^{\bar{r}_b} - 1)(1-l(r, \rho^*)) dr, \end{aligned}$$

The borrowing rate, $e^{\bar{r}_b^*}$, that a bank will charge is the smallest positive solution to $E\Pi_b = 0$, which can be simplified as

$$\begin{aligned} & \int_{-\infty}^{r^*} \phi(\sqrt{\alpha + \beta_b}(r - \rho_b)) (m e^{r-l(r, \rho^*)} - (m-1))(1-l(r, \rho^*)) dr \\ & + \int_{r^*}^{+\infty} \phi(\sqrt{\alpha + \beta_b}(r - \rho_b)) (m-1)(e^{\bar{r}_b} - 1)(1-l(r, \rho^*)) dr = 0. \end{aligned} \quad (15)$$

Notice that when x_b is low enough, the expected profit of banks from lending will be always negative. Thus the banks will lend if and only if x_b is large enough such that $\max\{E\Pi_b(\rho^*, \bar{r}_b)\} \geq 0$. Or the banks will choose not to lend.

Q.E.D

References

- [1] Allen, Franklin, and Douglas Gale, 1998, "Optimal Financial Crises," *Journal of Finance*, 53, 1245-1283.
- [2] Allen, Franklin, and Douglas Gale, 2000, "Financial Contagion," *Journal of Political Economy*, 108(1), 1-33.
- [3] Angeletos, George-Marios, and Alessandro Pavan, 2004, "Transparency of Information and Coordination in Economies with Investment Complementarities," *American Economic Review*, 94(2), 91-98
- [4] Baxter, M. and R. King, 1991, "Productive Externalities and Business Cycles," *Institution for Empirical Macroeconomics*, Federal Reserve Bank of Minneapolis, Discussion Paper No.53.
- [5] Bernanke, B. S., M. Gertler, and S. Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," in J. Taylor, and M. Woodford (eds), *Handbook of Macroeconomics*, Amsterdam.
- [6] Borio, Claudio, and Philip Lowe, 2004, "Securing Sustainable Price Stability: Should Credit Come Back from the Wilderness," *BIS Working Papers No 157*.
- [7] Bryant, J., 1983, "A Simple Rational Expectation Keynes-Type Model," *Quarterly Journal of Economics*, 97, 525-29.
- [8] Caballero, Richardo J., and Richard K. Lyons, 1989, "The Role of External Economies in U.S Manufacturing," *Mimeo*, Columbia.
- [9] Caballero, Richardo J., and Richard K. Lyons, 1990, "Internal versus External Economies in European Industry," *European Economic Review*, 34, 805-830.
- [10] Calomiris, Charles, and Gary Gorton, 1991, "The origins of Banking Panics: Models, Facts and Bank Regulation," in *Financial Markets and Financial Crises*, edited by R. Glenn Hubbard, The University of Chicago Press, Chicago, US.

- [11] Chang, Roberto, and Andres Velasco, 2001, "A Model of Financial Crises in Emerging Markets," *Quarterly Journal of Economics*, 116, 489-517.
- [12] Carlsson, H., and van Damme, 1993, "Global Games and Equilibrium Selection," *Econometrica*, 61, 989-1018.
- [13] Chamley, Christophe P., 2004, *Rational Herding: Economic Models of Social Learning*, Cambridge University Press.
- [14] Chari, V. V., and Patrick J. Kehoe, 2003, "Hot Money," *Journal of Political Economy*, 111, no.6, 1262-1292.
- [15] Chen, Nan-Kuang, 2001, "Bank Net Worth, Asset Prices and Economic Activity," *Journal of Monetary Economics*, 48, 415-436.
- [16] Chen, Yehning, 1999, "Banking Panics: The Role of the First-Come, First-Served Rule and Information Externalities," *Journal of Political Economy*, vol. 107, no. 5, 946-968.
- [17] Cole, Harold L., and Timothy J. Kehoe, 1996, "A Self-fulfilling Model of Mexico's 1994-1995 Debt Crises," *Journal of International Economics*, 41, 309-330.
- [18] Corsetti, Giancarlo, Amil Dasgupta, Stephen Morris and Hyun Song Shin, 2004, "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders," *Review of Economic Studies*, 71(1):87-113.
- [19] Cooper, Russell W., 1999, *Coordination Games-complementarities and macroeconomics*, Cambridge University Press.
- [20] Demirguc-Kunt, Asli, and Enrica Detragiache, 1998, "The Determinants of Banking Crises in Developing and Developed Countries," *IMF Staff Papers*, Vo. 45, No.1 (March 1998).
- [21] Diamond, Douglas W., and Philip H. Dybvig, 1983, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91(3), 401-419.
- [22] Diamond, Peter A., 1982, "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 90(5), 881-894.

- [23] Demirguc-Kunt, Asli, and Enrica Detragiache, 1998, "The Determinants of Banking Crises in Developing and Developed Countries, " *IMF Staff Papers*, Vol. 45, No.1 (March 1998).
- [24] Goodhart, Charles, and Gerhard Illing (eds.), 2002, *Financial Crises, Contagion, and the Lender of Last Resort*, Oxford University Press.
- [25] Gorton, Gary, 1988, "Banking Panics and Business Cycles," *Oxford Economic Papers*, 40, 751-781.
- [26] The IMF, 1998, *IMF World Outlook*..
- [27] Kaminsky, Graciela L., and Carmen M. Reinhart, 1999, "The Twin Crises: The Cause of Banking and Balance-of-Payments Problems," *American Economic Review*, Vol.89(3), 473-500.
- [28] Kiyotaki, N., and J. Moore, 1997, "Credit Cycle," *Journal of Political Economy*, 105(2), 211-248.
- [29] Kiyotaki, N., and J. Moore, 2002, "Balance-Sheet Contagion," *American Economic Review*, Vol. 92(2), 46-50.
- [30] Krugman, Paul, 1979, "A Model of Balance of Payment Crises," *Journal of Money, Credit and Banking*, 11(3), 311-325.
- [31] Krugman, Paul, 2000, "Balance Sheets, the Transfer Problem, and Financial Crises, in Peter Isard, Assaf Razin and Andrew K. Rose (eds)," *International Finance and Financial Crises, Essays in Honor of Robert P. Flood Jr.*, Boston, MA: Kluwer Academic Publishers, 31-43.
- [32] Krugman, Paul, 2001, "Out of the Loop," *New York Times*, March 4, 2001, Sec. 4, 15.
- [33] Langlois, Richard N., 1992, "External Economies and Economic Progress: The Case of the Microcomputer Industry," *The Business History Review*, Vol.66 (1), 1-50.

- [34] Mehrling, Perry, 1999, "The Vision of Hyman P. Minsky," *Journal of Economic Behavior and Organization*, Vol.39, 129-158.
- [35] Minsky, Hyman P., 1982, *Can It Happen Again?* M. E. Sharpe Inc.
- [36] Morris, Stephen, and Hyun Song Shin, 1998, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *The American Economic Review*, Vol.88, 587-597.
- [37] Morris, Stephen, and Hyun Song Shin, 2000, "Rethinking Multiple Equilibria in Macroeconomic Modelling," *NBER Macroeconomics Annual 2000*.
- [38] Morris, Stephen, and Hyun Song Shin, 2002, "Social Value of Public Information," *The American Economic Review*, Vol.92, 1521-1533.
- [39] Morris, Stephen, and Hyun Song Shin, 2003, "Global Games: Theory and Applications," Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky (eds.), *Advances in Economics and Econometrics*, Cambridge University Press, 56-114.
- [40] Obstfeld, M., 1996, "Models of Currency Crises with Self-fulfilling Features," *European Economic Review*, 40(3-5), 1037-1047.
- [41] Vives, Xavier, 2005, "Complementarities and Games: New Developments," *Journal of Economic Literature*, Vol. XLIII (June, 2005), 437-479.