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**Working Paper**

# The existence of informationally efficient markets when individuals are rational

CESifo Working Paper, No. 1295

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*Suggested Citation:* Muendler, Marc-Andreas (2004) : The existence of informationally efficient markets when individuals are rational, CESifo Working Paper, No. 1295, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:
https://hdl.handle.net/10419/18933

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# THE EXISTENCE OF INFORMATIONALLY EFFICIENT MARKETS WHEN INDIVIDUALS ARE RATIONAL

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CESIFO WORKING PAPER NO. 1295
CATEGORY 6: MONETARY POLICY AND INTERNATIONAL FINANCE
OCTOBER 2004

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CESifo *Working Paper No. 1295*

# THE EXISTENCE OF INFORMATIONALLY EFFICIENT MARKETS WHEN INDIVIDUALS ARE RATIONAL


## Abstract

A rational-expectations equilibrium with positive demand for financial information does exist under fully revealing asset price - contrary to a wide-held conjecture. Generalizing the common additive signal-return model with CARA utility to the family of distributions with moment generating functions, this paper shows that individual investors endowed with an average portfolio demand information in equilibrium if they can adjust portfolio size. More information diminishes the expected excess return of a risky asset so that investors who only have a choice of portfolio composition or whose asset endowments strongly differ from the average portfolio are worse off. Under fully revealing price, information market equilibria both with and without information acquisition are Pareto efficient.


JEL Code: D82, D83, G14.

Keywords: information, efficiency, financial markets, portfolio theory.

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I thank Vince Crawford, Mark Machina, Joel Sobel, Ross Starr, Joel Watson and seminar participants at UCSD for insightful suggestions. I owe the inspiration to Walter Novaes, whose casual remarks on CARA and moment generating functions at a meeting in Rio de Janeiro made me first ponder this approach. Any remaining mistakes are mine. A previous version of this paper circulated under the title "Demand for financial information under fully revealing asset price."

Contrary to a commonplace conjecture, a unique equilibrium with strictly positive demand for financial information does exist under fully revealing asset price—just as demand for public goods in other economic contexts is positive albeit not necessarily socially optimal. Existence merely requires a finite number of individual investors.

An equilibrium is called fully revealing if every investor can infer a sufficient statistic of all other investors' information from asset price. Similar to benchmarks in other economic fields—such as perfect competition in industrial organization, the welfare theorems in microeconomic theory, or perfect foresight in macroeconomics—, the benchmark case of fully revealing asset prices may be unrealistic but it is instructive.[1] Financial information figures prominently in recent research into financial markets and crises. However, research tends to treat information receipt as exogenous to the investors' decision problem.

This paper elucidates key properties of financial information and the incentives for its acquisition, clarifies the detrimental effect of information on the expected excess return of the asset to which it applies, underscores the crucial importance of an *intertemporal* decision for the value of information and investigates welfare properties of the equilibrium. The information market equilibrium is efficient in a Pareto sense both when investors choose to acquire information and when they don't.

To make the case, this paper generalizes the widely used 'additive signal-return model' to the family of distribution functions with a moment generating function.

**Assumption 1** (Additive signal-return structure). *The gross asset return $\theta$ of a risky asset is the sum of a fundamental $S$, which can become fully known through the signal realization $s$, and independent noise $\varepsilon$:*

$$\theta = S + \varepsilon. \tag{1}$$

Signal $S$ and noise $\varepsilon$ are drawn from some real-valued support. The certain gross return of a bond is $R \in (0, \infty)$.

[1] Some financial markets might be close to the benchmark. As Federal Reserve chairman Alan Greenspan remarked at the 21st Annual Monetary Conference Washington DC, November 20, 2003: "My experience is that exchange markets have become so efficient that virtually all relevant information is embedded almost instantaneously in exchange rates to the point that anticipating movements in major currencies is rarely possible."

Since jointly normally distributed random variables can be transformed into the sum of two independent normal random variables, all models with a normally distributed signal and asset return share the structure of (1). Strands of research that explicitly use additive signal-return models include, for instance, those on information acquisition (Grossman and Stiglitz 1980), delegated portfolio management (Bhattacharya and Pfleiderer 1985), or currency attacks (Morris and Shin 1998). Together with assumption 1, two further assumptions completely characterize the class of exchange economies in this paper.

**Assumption 2** (Common CARA). *Investors evaluate portfolios with intertemporally additive von Neumann-Morgenstern utility and share an identical degree of constant absolute risk aversion (CARA).*

**Assumption 3** (Single-price responses to information). *The equilibrium price of an asset only responds to signal realizations on its own return.*

Assumption 2 requires the return distribution to have a moment generating function. The present paper considers both an infinite number of investors and an arbitrarily large finite number of investors. However, an information equilibrium exists only if there are finitely many individuals.

**An Illustration.** Consider two assets, a bond $B$ with certain gross return $R$ and a stock $X$ with risky return $\theta$. The stock sells at price $P$. CARA utility gives rise to a risky asset demand function $X(RP)$ so it is convenient to refer to $RP$ as the asset price in opportunity cost terms (of holding a bond). In additive signal-return models with CARA utility, demand for the stock is zero if opportunity cost $RP$ equals the expected return $\mathbb{E}[\theta]$. As opportunity cost $RP$ falls below $\mathbb{E}[\theta]$, an investor demands more and more of the stock. Figure 1 depicts the resulting demand schedule $X(RP)$ with a solid curve.

An informed investors gets to observe realization $s$ of the signal $S$. With fully revealing price, there can only be two cases. Either no one acquires the signal $S$. Or one investor acquires the signal $S$ and everyone gets to know the signal realization $s$ through fully revealing price. An investor who anticipated not to act on realization $s$ would not acquire signal $S$ (the entitlement to receive $s$) in the first place. By analogy, fully revealing price gives every investor the choice to either push an information button and broadcast the

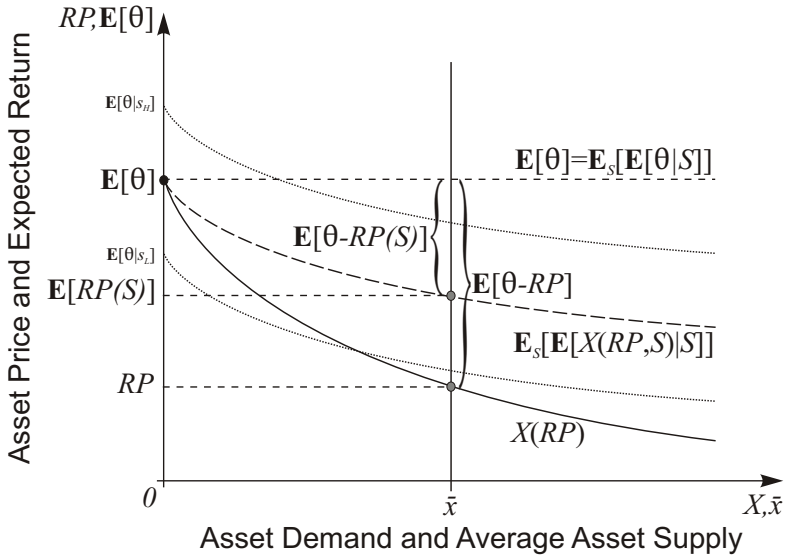


Figure 1: **Asset Demand and Expected Excess Return**

signal realization to everyone, or, alternatively, to keep everyone uninformed.[2] So, equilibrium price $P(s)$ depends on the signal realization if at least one investor acquires the signal, otherwise $P$ is independent of $S$.

In additive signal-return models with CARA utility and identical risk preferences, the unique equilibrium allocation of the stock is symmetric irrespective of the investor's wealth. No matter whether or not information is acquired, every investor ends up with the average amount of the stock $\bar{x}$ in her portfolio (the 'market portfolio'). Suppose no investor acquires the signal. Then the unique asset-market equilibrium results in an opportunity cost $RP$ as depicted in figure 1.

When, on the other hand, one investor acquires the signal $S$, a message with the signal realization $s$ goes out to everyone before portfolios are chosen. Should an investor acquire the signal $S$ in the first place? *Post notitiam* (after the signal realization), price may go up in response to a good realization ($s_H$) or go down in response to bad news ($s_L$). Figure 1 shows these possibilities as dotted lines. *Ante notitiam* (before the signal realization), however, the intercept of the demand curve stays put since, by the law of iterated expectations, the expected return remains unaltered at $\mathbb{E}_S\left[\mathbb{E}\left[\theta|S\right]\right] = \mathbb{E}\left[\theta\right]$.

[2] Joel Watson pointed out this analogy.

Information (the ability to condition on $s$) reduces the risk *post notitiam*. So, for any given opportunity cost $RP(s)$, asset demand will be higher. This results in an upward turn of the demand curve. Figure 1 depicts the resulting expected demand curve $\mathbb{E}_S\left[\mathbb{E}\left[X(RP(S),S)|S\right]\right]$. The expected new equilibrium price is $\mathbb{E}\left[RP(S)\right] > RP$. The asset's expected excess return over opportunity cost just went down, falling from $\mathbb{E}\left[\theta - RP(S)\right]$ to $\mathbb{E}\left[\theta - RP\right]$. Although the stock has lost attractiveness relative to the bond, the investor will still have to put $\bar{x}$ in her portfolio since equilibrium is symmetric both with and without information. This strictly worsens her utility *ante notitiam*. But there are two benefits of information.

First, an investor who has information scheduled to arrive anticipates to reshuffle the asset composition of her portfolio (of given size). This reduces the expected variance of her future consumption *ante notitiam* (by variance decomposition $\mathbb{E}_S\left[\mathbb{V}\left(\theta|S\right)\right] = \mathbb{V}\left(\theta\right) - \mathbb{V}_S\left(\mathbb{E}\left[\theta|S\right]\right)$). As it turns out in additive signal-return models with CARA utility, the diminishing expected excess return just wipes out the benefits from improved asset composition. So, no signal will be acquired in equilibrium, and the absence of information is efficient in a Pareto sense.

Second, an investor who has information scheduled to arrive also anticipates to adjust her consumption path and the size of her portfolio. This benefit from improved intertemporal choice outweighs the costs from diminishing expected excess returns for an investor with a 'market endowment' of stocks. So, when investors are allowed to change their portfolio size in response to information, in addition to their portfolio composition, then there is a joint competitive equilibrium in asset and information markets under fully revealing price in which one, and only one, investor with close-to-average initial stock holdings acquires the signal. This equilibrium too is efficient in a Pareto sense.

The remainder of this paper is organized as follows. Section 1 reviews equilibrium conjectures for information demand under fully revealing asset price. Section 2 elaborates the model and establishes its fully revealing financial market equilibrium (under assumptions 1 through 3). In following Grossman and Stiglitz (1980), section 3 derives the information market equilibrium when investors only have a choice between assets. The unique equilibrium is one with no information. Section 4 presents the reason: More information diminishes the excess return of the risky asset over its opportunity costs. Section 5 revisits the information market equilibrium when investors can condition their intertemporal savings decision on the signal realization and

shows that the unique type of equilibrium is either one with or one without information acquisition. Section 6 concludes.

# 1 Equilibrium Conjectures in the Literature

There is an extensive literature on the generic existence of a rational expectations equilibrium with fully revealing asset prices (e.g. Radner 1979, Jordan 1983, Citanna and Villanacci 2000). However, papers in this literature generally stop short of investigating the incentives for investors to acquire information in the first place.

Grossman and Stiglitz (1976) first outlined and Grossman and Stiglitz (1980) later formulated the following no-equilibrium paradox for financial markets (Grossman and Stiglitz 1980, *conjecture 6*):

> In the limit, when there is no [exogenous] noise [in prices], prices convey all information, and there is no incentive to purchase information. Hence, the only possible equilibrium is one with no information. But if everybody is uninformed, it clearly pays some individual to become informed. Thus, there does not exist a competitive equilibrium.

There are numerous more instances of this 'no-equilibrium conjecture' in the literature. Romer (1993) argues, for instance, that no equilibrium exists for CARA utility and a normally distributed asset return if price is fully revealing because signal realizations enter price in a linear way. Barlevy and Veronesi (2000) remark more recently: "Finally, as Grossman and Stiglitz point out, we need to prevent prices from being fully revealing; otherwise an equilibrium will fail to exist."

Approaches to overcome the no-equilibrium paradox under fully revealing prices include Jackson (1991) with price setting investors, or Jackson and Peck (1999) with investors who submit demand functions in a Shapley-Shubik fashion. Routledge (1999) considers adaptive learning from past price so that investors cannot condition on current price. To my knowledge, the no-equilibrium conjecture has so far not been reconsidered in the original Walrasian equilibrium with rational expectations. Though that equilibrium concept has shortcomings, it remains a common framework in financial contexts. The present paper extends the model by an information acquisition stage and generalizes it to the family of distributions with moment generating

functions. Grossman and Stiglitz (1980) posed the 'no-equilibrium' conjecture in the context of infinitely many investors. An arbitrarily large but finite number of investors suffices for existence of a fully revealing equilibrium with well-defined information demand.

The 'no-equilibrium conjecture' lent support to the claim that financial markets could, by their mere logic, never be informationally efficient in a fully revealing sense. Admati (1991) summarizes this view succinctly: "[T]his impossibility result is important since it examines the theoretical and conceptual underpinnings of the frequently used notion of *efficient financial markets*. It ... shows that under some conditions it is logically impossible for financial markets to be efficient in the 'strong' sense that they reflect all the information in the market" (Admati's emphasis).

While reasons abound why financial markets may not be informationally inefficient, this paper argues that the sources of these inefficiencies are more subtle than outright impossibility when an arbitrarily large but finite number of investors is rational. If the unique equilibrium is one with no information as in section 3 of this paper, the outcome is efficient in the sense that a social planner would also allocate no information. Conversely, when the unique equilibrium entails positive information as in section 5, the information equilibrium is efficient even when some investors would be better off without information. A social planner obeying the Pareto criterion cannot take the indivisible signal from the acquiring investor since that would leave at least this investor worse off by revealed preference. In presenting information demand under fully revealing asset price, this paper revisits the overlooked benchmark case of informational efficiency.

# 2 Fully Revealing Equilibrium

This section shows that an asset-market equilibrium in an additive signal-return model with CARA utility is symmetric and fully reveals the signal realization $s$. It is unique if the share of informed investors is known at the time of the portfolio choice. Then the information equilibrium too is symmetric, given an indivisible signal $S$, in the sense that either all investors are informed or no investor is informed.

Consider a finite number $I$ of investors with arbitrary time preferences and arbitrary initial wealth. Investor $i$ holds initial wealth $W_0^i$ and chooses consumption $C_0^i$ today along with a portfolio $(B^i, X^i)$ to secure consumption

$C_1^i$ tomorrow. There is no income in period 1 other than asset returns. So, $C_1^i = RB^i + \theta X^i$ and $C_0^i = W_0^i - (B^i + P(s)X^i)$. Initial wealth is $W_0^i = B_0^i + P(s)X_0^i$ given asset price $P(s)$, which is known at the time of these choices. Under CARA, investor $i$'s period utility becomes $v(C) = -\exp\{-AC\}$, where $A > 0$ is the Pratt-Arrow measure of absolute risk aversion. So, under assumption 2,

$$V^i \equiv -\alpha \exp\{-AC_0^i\} - \beta^i \exp\{-AC_1^i\},$$

where either $\alpha = 0$ or $\alpha = 1$, and $\beta^i \in (0,1)$ is the time discount factor. Just as Grossman and Stiglitz (1980) do, sections 3 and 4 of this paper consider a terminal consumption maximization problem with $\alpha = 0$. Section 5 will consider the intertemporal consumption problem with $\alpha = 1$.

CARA utility requires the return distribution to have a moment generating function (MGF). The MGF of a random variable $Z$ is defined as $M_Z(t) \equiv \mathbb{E}\left[\exp\{tZ\}\right] \in (0,\infty)$. So, a CARA investor's expected utility can be recast entirely in terms of MGFs:

$$\mathbb{E}\left[V^i\right] = -\alpha \exp\{-AC_0^i\} - \beta^i M_\theta(-AC_1^i).$$

MGFs exist for many distributions. An MGF $M_\theta(t)$ is continuously differentiable in $t$ by definition. The MGF of the sum of two independent random variables is the product of the two underlying MGFs (Casella and Berger 1990, Theorem 4.6.3). So, the MGF of the risky return $\theta = S + \varepsilon$ is $M_\theta(t) = M_S(t)M_\varepsilon(t)$. Similarly, the MGF of the conditional stock return given known signal realization $s$ is $M_{\theta|s}(t) = \exp\{st\}M_\varepsilon(t)$.

Irrespective of whether investor $i$ has a choice of the asset allocation only ($\alpha = 0$) or an intertemporal choice in addition ($\alpha = 1$), investor $i$'s demand for the stock must satisfy the same first order condition. Expected utility *post notitiam* is

$$\mathbb{E}\left[V^i|s\right] = -\beta^i \exp\{-AR\,W_0^i\} \exp\{A\,RP(s)\,X^i\}\, M_{\theta|s}\big(-A\,X^i\big) \tag{2}$$

when investor $i$ only has an inter-asset choice ($\alpha = 0$ so $C_0^i = 0$), and it is

$$\mathbb{E}\left[V^i|s\right] = -\exp\{-A(W_0^i - B^i - P(s)X^i)\} - \beta^i \exp\{-AR\,B^i\}\, M_{\theta|s}\big(-A\,X^i\big) \tag{3}$$

when investor $i$ also has an intertemporal choice ($\alpha = 1$). The Walrasian auctioneer presents $P(s)$ to every investor at the time of portfolio choice.

Table 1: Examples of Distributions in the Singleton Class

| Distribution | Equivalent to criterion $M''_Z(t)/M_X(t) > [M'_X(t)/M_X(t)]^2$ | Satisfied for |
|---|---|---|
| Binomial | $\frac{np\exp\{t\}\,[(1-p)+np\exp\{t\}]}{[(1-p)+p\,\exp\{t\}]^2} > \left(\frac{np\exp\{t\}}{(1-p)+p\,\exp\{t\}}\right)^2$ | $p < 1$ |
| Gamma[a] | $\left(1+\frac{1}{\alpha}\right)\frac{\alpha^2}{\beta^2}\left(1-\frac{t}{\beta}\right)^{-2} > \frac{\alpha^2}{\beta^2}\left(1-\frac{t}{\beta}\right)^{-2}$ | $\alpha > 0, t < \frac{1}{\beta}$ |
| Geometric | $\frac{1-(1-p)^2\exp\{2t\}}{[1-(1-p)\exp\{t\}]^3} > \left(\frac{1}{1-(1-p)\exp\{t\}}\right)^2$ | $t < \ln\frac{1}{1-p}$ |
| Laplace | $\frac{2\sigma^2(1+\sigma^2t^2)}{(1-\sigma^2t^2)^2} + \left(\mu+\frac{2\sigma^2t}{1-\sigma^2t^2}\right)^2 > \left(\mu+\frac{2\sigma^2t}{1-\sigma^2t^2}\right)^2$ | any $\mu, \sigma$ |
| Normal | $\sigma^2 + (\mu+\sigma^2t)^2 > (\mu+\sigma^2t)^2$ | any $\mu, \sigma$ |
| Poisson | $\lambda\,\exp\{t\} + \lambda^2\,\exp\{2t\} > \lambda^2\,\exp\{2t\}$ | $\lambda > 0$ |
| Uniform | $\frac{b^2\exp\{bt\}-a^2\exp\{at\}}{\exp\{bt\}-\exp\{at\}} - \left(\frac{b\exp\{bt\}-a\exp\{at\}}{\exp\{bt\}-\exp\{at\}}\right)^2 + \frac{1}{t^2} > 0$ | any $a, b$ |

[a]Includes special cases such as the exponential, $\chi^2$, and Erlang distributions.

Maximizing expected utility—i.e. maximizing either (2) over $X^i$, or (3) over $X^i$ and $B^i$—establishes the first order condition (see appendix A)

$$RP(s) = \frac{\mathbb{E}\left[\theta\exp\{-AX^i\,\theta\}|s\right]}{\mathbb{E}\left[\exp\{-AX^i\,\theta\}|s\right]} = \frac{M'_{\theta|s}(-AX^i)}{M_{\theta|s}(-A\,X^i)}, \tag{4}$$

where the prime in $M'_\theta(\cdot)$ denotes the first derivative of the MGF w.r.t. its argument $t$. Equation (4) can also be viewed as the inverse demand function for the stock. Bond demand $B^i \in \mathbb{R}$ satisfies the wealth constraint. Figure 1 depicts examples of stock demand. The inverse demand function intersects with the price axis at $RP = \mathbb{E}[\theta]$ for $X^i = 0$. Further properties depend on the underlying return distribution.

**Singleton Family of Distributions.** The first- and second-order conditions for a unique global equilibrium impose restrictions on the distribution of $\theta = S + \varepsilon$. There is a family of MGFs that satisfy the restrictions of both the first- and second-order conditions so that equilibrium is a singleton. Call the family of distributions with such MGFs the *singleton family.*

**Lemma 1** (Singleton Family of Distributions). *Under CARA utility, demand for the risky asset is a unique singleton if the MGF of the asset return $\theta$ satisfies*

$$\frac{M'_\theta(t)}{M_\theta(t)} > 0 \quad \textit{and} \quad \frac{M''_\theta(t)}{M_\theta(t)} > \left(\frac{M'_\theta(t)}{M_\theta(t)}\right)^2 \tag{5}$$

*for $t < 0$.*

**Proof.** Per-capita asset supply can take any value $\overline{x} \in (0, \infty)$, and so can $X^i$. Define $t \equiv -AX^i \in (-\infty, 0)$. The first inequality in (5) is a restatement of the first-order condition (4), while the latter condition is a restatement of the second-order condition equivalent (26) in appendix B. ■

Lemma 1 applies to CARA utility irrespective of whether $\theta$ stems from an additive signal-return distribution or not. The first inequality in lemma 1 states that interior asset demand is viable at non-zero price. The second condition implies that asset demand (4) strictly decreases in asset price (see figure 1), making it unique, and that the ratio $M'_\theta(t)/M_\theta(t)$ strictly increases in $t$—an important property for later derivations. The second condition becomes the variance of $\theta$ at $t=0$. It is straightforward to show that if both the distribution of $S$ and the distribution of $\varepsilon$ satisfy (5) individually then their sum satisfies (5).

Common distribution functions belong to the *singleton family* of lemma 1. The binomial, gamma, geometric, Laplace, normal (Gaussian), Poisson, and uniform distributions are examples of distributions that satisfy condition (5) (see table 1).

**Fully Revealing Financial Market Equilibrium.** If the share of informed investors $\lambda \in [0, 1]$ is known to all investors at the time of their portfolio choice, assumptions 1 and 2 suffice to make the fully revealing financial market equilibrium unique. Then, either $\lambda = 0$, or $\lambda = 1$ as soon as one investor acquires the signal.

Consider any $\lambda$. Define $\overline{x} \equiv \sum_{i=1}^{I} X_0^i / I$ as average asset supply per investor. A financial market equilibrium requires that the market for the risky asset clears

$$\lambda\, X_{\text{inf.}} + (1 - \lambda)\, X_{\text{uninf.}} = \overline{x}, \tag{6}$$

where $X_{\text{inf.}}$ and $X_{\text{uninf.}}$ denote demands of informed and uninformed investors, respectively.

Since $M_\theta(t) = M_S(t)M_\varepsilon(t)$ by independence of $S$ and $\varepsilon$, stock demand of an uninformed investor must satisfy

$$RP = \frac{M'_\theta\left(-A\,X_{\text{uninf.}}\right)}{M_\theta\left(-A\,X_{\text{uninf.}}\right)} = \frac{M'_S\left(-A\,X_{\text{uninf.}}\right)}{M_S\left(-A\,X_{\text{uninf.}}\right)} + \frac{M'_\varepsilon\left(-A\,X_{\text{uninf.}}\right)}{M_\varepsilon\left(-A\,X_{\text{uninf.}}\right)}. \tag{7}$$

in equilibrium. For informed investors, $M_{\theta|s}(t) = \exp\{st\}M_\varepsilon(t)$ and stock demand must satisfy

$$RP(s) = \frac{M'_{\theta|s}\left(-A\,X_{\text{inf.}}\right)}{M_{\theta|s}\left(-A\,X_{\text{inf.}}\right)} = s + \frac{M'_\varepsilon\left(-A\,X_{\text{inf.}}\right)}{M_\varepsilon\left(-A\,X_{\text{inf.}}\right)}, \tag{8}$$

given the observed signal realization $s$. Note that $M'_S(0)/M_S(0) = \mathbb{E}[s]$ so that the stock demand schedules share the same intercept *ante notitiam* (before the signal realization is observed) as depicted in figure 1.

**Theorem 1** (Fully Revealing Financial Market Equilibrium). *In additive signal-return models with CARA utility and arbitrary initial endowments of the risky asset (assumptions 1 and 2), a symmetric and fully revealing financial market equilibrium exists. It is unique if the share of informed investors $\lambda$ is common knowledge at the time of the Walrasian auctioning process.*

**Proof.** Using $\lambda = 1$ and $X^i = \bar{x}$ in (6), (7) and (8) shows that a symmetric financial market equilibrium exists. Equilibrium price is fully revealing because $RP(s)$ is invertible in the signal realization by (8).

Suppose $\lambda \in (0, 1]$ is common knowledge at the time of the Walrasian auctioning process. Then rational stock demand of an informed investor satisfies (8), given the observed signal realization $s$, while an uninformed investor can infer each other uninformed investor's rational choice of $X_{\text{uninf.}}$ from her own choice (7). Hence, every uninformed investor can infer informed investors' demand $X_{\text{inf.}}$ from market clearing (6) and thus $s = \frac{M'_\varepsilon(-A\,X_{\text{inf.}})}{M_\varepsilon(-A\,X_{\text{inf.}})} - RP$ from (8). So, the symmetric and fully revealing equilibrium is unique if $\lambda$ is known. If $\lambda = 0$, the equilibrium is unique and symmetric, and fully revealing in the degenerate sense that nothing can be revealed. ■

Known risk aversion is a necessary condition for price to be fully revealing since the realization of $s$ cannot be inferred from an informed investor's demand unless his risk aversion is known. Jordan (1983) shows in addition that constant (absolute or relative) risk aversion, of which risk neutrality is a

limiting case, is *necessary* for a fully revealing equilibrium to exist under regularity conditions. With the additive signal-return structure (assumption 1), theorem 1 provides a *sufficient* condition for existence (and for uniqueness if $\lambda$ is known). Assumption 1 does not require the state space to be finite and is, in this regard, more general than other sufficient conditions (e.g. Citanna and Villanacci 2000). If the share of informed investors $\lambda$ is not known at the time of portfolio choice, or if there are different degrees of being informed in the presence of more than one signal, partially revealing equilibria could be supported alongside the fully revealing equilibrium, depending on the beliefs that investors are allowed to hold about other investors. The focus of this paper, however, lies on fully revealing equilibrium.

# 3 No-information Equilibrium

Does any investor $i$ have an incentive to acquire the signal under a fully revealing asset price? To investigate the answer, first consider the case where investors only face an *inter-asset* decision but have no intertemporal choice. Section 5 will extend the problem to an intertemporal setting.

Under fully revealing price, information is a public commodity. Two equilibrium definitions are commonly applied to public commodities: (i) Nash equilibria, or (ii) public goods equilibria in the style of Samuelson (1954). Both concepts rest on the following principle.

**Definition 1** (Competitive REE). *A competitive rational expectations equilibrium (REE) in an exchange economy is an allocation of commodities and assets to agents, along with a price for each unit of the commodities and assets, so that no agent wants to acquire amounts that differ from this allocation subject to the observed choice of other agents and a wealth constraint.*

The financial market equilibrium in theorem 1 satisfies this equilibrium definition for asset demand. It remains to establish the competitive equilibrium for the signal $S$. If at least one investor buys the signal $S$, everyone becomes fully informed of $s$ after its transmission and it is not rational for any other investor to acquire the signal again. So, there can be at most one investor to whom the indivisible signal $S$ is allocated in a competitive equilibrium under fully revealing price. Will there be one investor to acquire the signal?

Grossman and Stiglitz' (1980) concise solution strategy uses investors' indirect utility in financial equilibrium to determine the information equilib-

rium. Applying (7) and (8) to (2), we obtain an investor $i$'s indirect utility *post notitiam* in a financial market equilibrium with no information ($\lambda = 0$) and with full information ($\lambda = 1$). In the absence of an informed investor, indirect utility $\mathbb{E}[V^i]$ is

$$\mathbb{E}\left[V^i\right] = -\beta^i \exp\{-AR\,B_0^i - A\,RP\,(X_0^i - \overline{x})\}\,M_S(-A\,\overline{x})M_\varepsilon(-A\,\overline{x}), \quad (9)$$

where $RP = M_S'(-A\,\overline{x})/M_S(-A\,\overline{x}) + M_\varepsilon'(-A\,\overline{x})/M_\varepsilon(-A\,\overline{x})$ by (7).

*Post notitiam*, an informed investor's indirect utility $\mathbb{E}[V^i|s]$ in financial market equilibrium is

$$\mathbb{E}\left[V^i|s\right] = -\beta^i \exp\{-AR\,B_0^i - A\,RP(s)\,(X_0^i - \overline{x})\}\,\exp\{-A\,\overline{x}\,s\}M_\varepsilon(-A\,\overline{x}), \quad (10)$$

since $M_{\theta|s} = \exp\{-A\,\overline{x}\,s\}M_\varepsilon(-A\,\overline{x})$. In this equilibrium, $RP(s) = s + M_\varepsilon'(-A\,\overline{x})/M_\varepsilon(-A\,\overline{x})$ by (8).

At the time when investor $i$ chooses whether or not to acquire a costly signal $S$, its realization $s$ must still be unknown. So, the investor bases information demand on a comparison between the *ante notitiam* indirect utilities with and without the expected receipt of a signal realization. If the indirect utility ratio satisfies

$$\frac{\mathbb{E}_S\left[\mathbb{E}\left[V^i|S\right]\right]}{\mathbb{E}\left[V^i\right]} < 1, \quad (11)$$

information acquisition is worthwhile for investor $i$. Recall that $V^i < 0$ under CARA utility so that this ratio must fall below unity. For costly signals, (11) must hold with strict inequality.

For CARA utility and an additive signal-return distribution, condition (11) translates into a restriction on the MGF of the signal distribution (called criterion $Crit_{\text{IAC}}(t, t_0^i)$ for inter-asset choice IAC below). As it turns out, this restriction is never satisfied for signal distributions in the *singleton family*. So, no investor has an incentive to acquire the signal if the only choice is an *inter-asset* decision. The unique information equilibrium is one with zero information.

**Theorem 2** (Unique No-information Equilibrium under Inter-asset Choice). *In additive signal-return models with CARA utility and arbitrary initial endowments of the risky asset (assumptions 1 and 2), when a finite number of*

*investors has an* inter-asset *choice, price is fully revealing and information demand criterion (11) is satisfied if and only if*

$$Crit_{IAC}(t, t_0^i) :\equiv \ln M_S(t) - \ln M_S(t_0^i) - (t - t_0^i)\,\frac{M_S'(t)}{M_S(t)} > 0. \qquad (12)$$

*Signal distributions in the* singleton family *(5) strictly violate this criterion so that the unique equilibrium is one in which no investor acquires a signal.*

**Proof.** Since investors' signal choices are known under equilibrium definition 1, price is fully revealing by theorem 1. Appendix C derives the information demand criterion. ■

So, if the signal has a common distribution from table 1 or falls into the singleton family (5) more generally, the unique equilibrium entails no information acquisition.[3] Worse, investors would pay not to receive information. However, in an investor's view, the *ante notitiam* variance of the asset return falls $\mathbb{E}_S\left[\mathbb{V}\left(\theta|S\right)\right] = \mathbb{V}\left(\theta\right) - \mathbb{V}_S\left(\mathbb{E}\left[\theta|S\right]\right)$ by a common decomposition result (Casella and Berger 1990, Theorem 4.4.2). Why can signal acquisition be undesirable for every signal investor although *ante notitiam* indirect utility would rise with reduced risk?

# 4 Diminishing Expected Excess Return

There is no demand for information when investors merely have an *inter-asset* choice because, *ante notitiam*, information diminishes the expected excess return of the asset

$$\mathbb{E}_S\left[\mathbb{E}\left[\theta - RP(S)\,|S\right]\right] = \mathbb{E}_{ante}\left[\theta\right] - R\,\mathbb{E}_{ante}\left[P(S)\right]$$

over its opportunity cost. The expected excess return falls because investors facing less uncertainty *post notitiam* will bid up the asset price. The expected

[3] It is conceivable that a distribution of $S$ exists outside the singleton family so that both the distribution of $\varepsilon$ and the distribution of $\theta = S + \varepsilon$ fall into the singleton family. This possibility, which would satisfy the requirements of a unique and well-defined financial market equilibrium both in the full-information and the no-information case, is neither ruled out nor confirmed here.

higher asset price more than offsets anticipated utility gains from lower risk when there is only an *inter-asset* choice.

While information does not affect $\mathbb{E}_S[\mathbb{E}[\theta|S]] = \mathbb{E}_{ante}[\theta]$ by the law of iterated expectations, the anticipated asset price $\mathbb{E}_{ante}[P(S)]$ is higher in the equilibrium with information than $\mathbb{E}_{ante}[P]$ without information. Information lowers risk, increases asset demand and raises asset price. This reduces the value of the risky asset relative to the bond. Figure 1 illustrates this effect. Ultimately, the diminishing excess return decreases expected consumption tomorrow because $C_1^i$ depends positively on the excess return of the stock over its opportunity cost $(\theta - RP)$. In additive signal-return models with CARA utility and fully revealing price, the losses from diminishing excess returns outweigh the gains from better information when there is only an *inter-asset* choice.

The negative effect of information on excess return occurs under general conditions. Any signal distribution function in the singleton family satisfies the following condition (13) for diminishing excess returns.

**Theorem 3** (Diminished Expected Excess Return). *In additive signal-return models with CARA utility and arbitrary initial endowments of the risky asset (assumptions 1 and 2), when asset price is anticipated to fully reveal a signal realization $s$, the signal $S$ strictly reduces the* ante notitiam *excess return of the risky asset $\mathbb{E}_S[\mathbb{E}[\theta - RP(S)|S]]$ if and only if*

$$\frac{M_S'(t)}{M_S(t)} < M_S'(0) \tag{13}$$

*for $t < 0$. This condition is satisfied for any distribution of $S$ in the* singleton family *(5).*

**Proof.** Define $t \equiv -A\overline{x}$. By the law of iterated expectations, the difference between the excess returns with and without information acquisition is $-\mathbb{E}_S[RP(S) - RP]$. Using the first-order conditions for informed investors (8) and uninformed investors (7) and taking prior expectations, the difference becomes $-\mathbb{E}[S] + M_S'(t)/M_S(t)$, which, by the properties of an MGF, is strictly negative if and only if (13) is satisfied. At $t = 0$, condition (13) turns into an equality. MGFs of distributions in the singleton family result in strictly falling asset demand by (5). So, as $t$ falls, the left-hand side of (13) is reduced further below zero. ■

In additive signal-return models with CARA utility and for common signal distributions, the information equilibrium exists but entails *zero information demand* if investors only have an *inter-asset* choice in a portfolio of given size. This resolves one part of Grossman and Stiglitz' (1980) no-equilibrium paradox. Grossman and Stiglitz (1980, *conjecture 6*) write (our emphasis):

> ... *But if everybody is uninformed, it clearly pays some individual to become informed.* Thus, there does not exist a competitive equilibrium.

The emphasized part of this conjecture can fail. Theorem 3 shows that information diminishes a risky asset's excess returns in general. So, it may never pay any investor to become informed in an additive signal-return model with CARA utility under a normal distribution of the signal or other common distributions. A competitive equilibrium does exist. It is unique and entails zero demand for information by theorem 2 when investors only have an inter-asset choice. Furthermore, the equilibrium is efficient in the sense that a benevolent social planner would not want any investor to acquire information.

If there were infinitely many investors, each investor with a measure zero, then individual demand would not affect the Walrasian price finding process and individual information would not be revealed. The expected excess returns would remain unaltered. However, a full measure of investors has an incentive to acquire the information so that information would be revealed. This non-concavity from the assumption of infinitely many investors, where each individual investor has no price impact while the full measure has the full impact, lies behind the no-equilibrium paradox. With a finite number of investors, the value of information is well defined. The value of information is strictly negative in additive signal-return models when investors only have an *inter-asset* choice.

However, the diminishing excess return does not exert a sufficiently negative utility effect to prevent information acquisition if investors have an *intertemporal* consumption choice in addition to the mere inter-asset choice.

# 5 Information Equilibrium

This section shows that a unique equilibrium with strictly positive information demand does exist under fully revealing asset price when information arrives before investors take their *intertemporal* consumption decision. With

an intertemporal choice, investors can adjust the size of their portfolio in response to the signal realization. The anticipation of information raises the *ante notitiam* utility of investors further than when their response to information is limited to an inter-asset choice. Loosely speaking, investors anticipated ability to condition both $C_0^i(s)$ and $C_1^i(s)$ on the signal realization $s$ (and not only $C_1^i(s)$) presents enough benefits to acquire the signal $S$. One, and only one, investor with close-to-average initial stock holdings has a strictly positive incentive to acquire the signal in a joint asset and information market equilibrium under fully revealing price. Consequently, everybody becomes informed.

When investors have an intertemporal consumption choice, the first order conditions for the bond and the stock imply that *post notitiam* indirect utility (3) becomes

$$\mathbb{E}\left[V^i|s\right] = -\delta \, \exp\left\{-ARB_0^i - ARP(s)(X_0^i - \overline{x})\right\}^{\frac{1}{1+R}} \, M_{\theta|s}(-A\,\overline{x})^{\frac{1}{1+R}} \quad (14)$$

in financial market equilibrium (see appendix D), for $\delta \equiv \frac{1+R}{R}(\beta^i R)^{\frac{1}{1+R}}$. Whereas the bond return $R$ was a parameter in the final consumption maximization problem, $R$ now serves to clear the bond market (see appendix E). *Ante notitiam*, $R$ is given and independent of signal acquisition by the law of iterated expectations. However, the bond return can respond to the signal realization *post notitiam* and correlate with other payoffs in indirect utility. To keep the analysis to closed-form solutions, I assume that the signal realization $s$ alters $R$ negligibly little. This assumption can be justified for an economy with a small stock endowment compared to the remaining asset markets (see appendix E for a formal derivation). So, I consider $R$ as insensitive to information on the small stock market. This assumption makes the present intertemporal model more closely comparable to Grossman and Stiglitz' (1980) inter-asset choice under deterministic $R$.

Applying (7) and (8) to (14), we obtain an investor $i$'s *post notitiam* indirect utility in a financial market equilibrium with no information ($\lambda = 0$) and with full information ($\lambda = 1$). In the absence of an informed investor, expected indirect utility $\mathbb{E}\left[V^i\right]$ is

$$\mathbb{E}\left[V^i\right] = -\delta \exp\left\{-ARB_0^i - ARP(X_0^i - \overline{x})\right\}^{\frac{1}{1+R}} M_S(-A\overline{x})^{\frac{1}{1+R}} \, M_\varepsilon(-A\overline{x})^{\frac{1}{1+R}}, \quad (15)$$

where $RP = M_S'(-A\,\overline{x})/M_S(-A\,\overline{x}) + M_\varepsilon'(-A\,\overline{x})/M_\varepsilon(-A\,\overline{x})$ by (7).

*Post notitiam*, an informed investor's expected indirect utility $\mathbb{E}\left[V^i|s\right]$ in financial market equilibrium is

$$\mathbb{E}\left[V^i|s\right] = -\delta \exp\left\{-ARB_0^i - ARP(s)(X_0^i - \overline{x}) - A\overline{x}\, s\right\}^{\frac{1}{1+R}} M_\varepsilon(-A\overline{x})^{\frac{1}{1+R}}, \tag{16}$$

since $M_{\theta|s} = \exp\{-A\,\overline{x}\,s\}M_\varepsilon(-A\,\overline{x})$. In this equilibrium, $RP(s) = s + M'_\varepsilon(-A\,\overline{x})/M_\varepsilon(-A\,\overline{x})$ by (8).

As discussed in section 3 before, investor $i$ bases information choice on a comparison between the *ante notitiam* indirect utilities with and without the expected receipt of a signal realization. If the indirect utility ratio satisfies (11)

$$\frac{\mathbb{E}_S\left[\mathbb{E}\left[V^i|S\right]\right]}{\mathbb{E}\left[V^i\right]} < 1,$$

then information acquisition is worthwhile.

Under CARA and an additive signal-return distribution, condition (11) translates into a restriction on the MGF of the signal distribution (criterion $Crit_{\text{ITC}}(t, t_0^i)$ for intertemporal choice ITC below). Contrary to the earlier finding in section 3, this restriction can be satisfied for signal distributions in the *singleton family*. An investor with close-to-average endowments of the risky asset (a 'market portfolio' endowment) has an incentive to acquire the signal on the stock return if she can take the *intertemporal* consumption decision after observing the signal realization. However, at most one investor can optimally acquire the indivisible signal in a competitive equilibrium of definition 1. Then the unique information equilibrium is one with full information.

**Theorem 4** (Unique Information Equilibrium under Intertemporal Choice). *In additive signal-return models with CARA utility, arbitrary initial endowments of the risky asset and single-price responses to signal realizations (assumptions 1 through 3), when a finite number of investors has an* intertemporal *choice in addition to the* inter-asset choice*, price is fully revealing and information demand criterion (11) is satisfied if and only if*

$$Crit_{ITC}(t, t_0^i) :\equiv \ln M_S(t) - (1+R)\ln M_S\left(\frac{t_0^i}{1+R}\right) - (t - t_0^i)\,\frac{M'_S\,(t)}{M_S\,(t)} > 0. \tag{17}$$

*Signal distributions in the* singleton family *(5) satisfy this criterion for $R > 0$ and a sufficiently small difference $t - t_0^i$. Then the unique type of equilibrium*

*is one in which one and only one investor with a strictly positive initial risky asset endowment, sufficiently close to the average endowment, acquires the costly signal. Otherwise the unique equilibrium is one in which no investor acquires the signal.*

**Proof.** Since investors' signal choices are known under equilibrium definition 1, price is fully revealing by theorem 1. Appendix F derives the information demand criterion. ■

The average investor with a 'market portfolio' endowment $X_0^i = \overline{x}$ has a strict incentive to acquire the signal for $R > 0$, which is satisfied since $R$ is bound away from zero in intertemporal equilibrium (see appendix E). Other investors with endowments $X_0^i$ close to $\overline{x}$ may also demand information. So, multiple equilibria can exist in the sense that it is indeterminate who exactly acquires the signal. However, the information level is unique: There is full information, irrespective of who bears the cost of acquiring the indivisible signal.

The intertemporal choice allows investors to adjust their portfolio size in response to the signal realization. Anticipating this additional choice, investors value information more than they do if they only have a choice between assets. The expected portfolio size is larger in the presence of information: The optimal portfolio value is $\frac{1}{1+R}[B_0^i - RP(t_0^i/R + t^i) + \frac{1}{A}\ln \beta^i R M_\theta(t)]$ (see (23) in appendix A), so the *ante notitiam* difference between the expected portfolio values with and without information becomes

$$\mathbb{E}_S\left[\pi^i_{\lambda=1} - \pi^i_{\lambda=0}\right] = -\left(\frac{t_0^i}{R} + t\right)\left(M_S'(0) - \frac{M_S'(t)}{M_S(t)}\right) > 0.$$

The difference is strictly positive by $t, t_0^i < 0$ and diminishing-excess-return theorem 3. Better information leads every investor to save more. In fact, the portfolio value increases more strongly than the stock price due to the wealth effect of the price increase (reflected in the term $t_0^i/R$).

**A Gaussian example.** For a normally distributed signal with mean $\mu_S$ and variance $\sigma_S^2$ the MGF is $M_S(t) = \exp\{\mu_S t + (\sigma_S^2/2)t^2\}$ so that criterion (17) of theorem 4 becomes

$$\tfrac{R}{1+R}\tfrac{\sigma^2}{2}A^2(X_0^i)^2 - \tfrac{\sigma^2}{2}A^2(X_0^i - \overline{x})^2 > 0.$$

The average investor with 'market portfolio' endowment $X_0^i = \overline{x}$ has a strict incentive to acquire the signal. So, the unique type of equilibrium is one with full information. Would a social planner implement this equilibrium outcome? An equilibrium with full information must be Pareto optimal in an additive signal-return model since the acquiring investor is better off with the signal by revealed preference. However, while any equilibrium with information must be Pareto optimal in this sense, signal acquisition can reduce overall welfare.

Suppose, for instance, that investor $i$ initially holds the entire endowment of stocks while the $I-1$ remaining investors have their initial wealth in bonds only. The single stock owner acquires the signal because the expected price increase awards her with the wealth effect of a more valuable initial portfolio. In particular, for a total of three investors $R > \frac{5}{4}$ satisfies criterion (17); for a total of four investors $R > \frac{9}{7}$ is needed. A social planner who follows the Pareto criterion cannot improve on this equilibrium outcome since taking away the signal would make investor $i$ worse of. However, overall welfare may fall with signal acquisition. In the example, the unweighted sum of the logs of *ante notitiam* indirect utilities is equal to the sum of criterion (17) over all investors. The sum over criterion (17) becomes $R/(1+R) - (I-1)$, which is negative for three investors and any $R \geq 0$ (though it exceeds $-\frac{13}{9}$ by $R > \frac{5}{4}$). Although the Pareto criterion judges information acquisition in additive signal-return models necessarily as socially optimal, there can yet be cases when overall welfare drops with more information as it diminishes the excess return for everyone.

Theorem 4 addresses the first part of Grossman and Stiglitz' (1980) no-equilibrium conjecture. Grossman and Stiglitz (1980, *conjecture 6*) write (our emphasis):

> In the limit, when there is no [exogenous] noise [in prices], prices convey all information, and *there is no incentive to purchase information* ...

Theorem 4 refutes this part of the conjecture. It does pay an investor with 'market portfolio' endowments of the risky asset to become informed in an additive signal-return model with CARA utility under common signal distributions. For the individual decision to acquire a public good, given other agents' choice of zero, only individual incentives matter. However, at most one investor will find it optimal to acquire the signal.

If there were infinitely many investors, each investor with a measure zero, then a strictly positive measure of investors would be needed to acquire the same signal so that price can reveal information. However, individual demand would not affect the Walrasian price finding process and every single investor who acquired a duplicate of the signal would be better off not acquiring it. Again, this non-concavity from the assumption of infinitely many investors, where each individual investor has no price impact while an arbitrarily small but strictly positive measure has the full impact needed for price to be revealing, lies behind the no-equilibrium paradox.

When there is a finite number of investors, no matter how many investors have an incentive to acquire information, an allocation of the indivisible signal to any of the investors with positive information demand is a competitive equilibrium. Given the signal allocation, the paying investor would be worse off without the signal as long as the signal cost is low enough, no other investor with positive information demand wants to pay for the duplicate of fully revealed information, and those investors who prefer no information have no choice in a competitive equilibrium.

# 6 Conclusion

Contrary to a common no-equilibrium conjecture, an information market equilibrium does exist in additive signal-return models with CARA utility for all distribution functions with moment generating functions. Investors acquire information to a socially desirable degree under fully revealing price and, whenever no information is acquired, a social planner agrees with that market outcome. However, information is not beneficial to every investor. Even when acquired, there may be investors who would prefer that there were less information in the market because information diminishes the excess return of a risky asset.

Additive signal-return structures are common to many brands of research into information effects in financial markets. While investors' receipt of information is often treated as exogenous, results of the present paper are reassuring. Rational investors demand financial information to a Pareto efficient extent even in the extreme benchmark case of a fully revealing asset price.

# Appendix

## A First-order conditions and portfolio value

Define $t \equiv -AX^i \in (-\infty, 0)$.

Maximizing (2) over $X^i$ yields the first order condition

$$\frac{\partial \mathbb{E}\left[V^i|s\right]}{\partial X^i} = A\mathbb{E}\left[V^i|s\right]\left(RP(s) - \frac{M'_{\theta|s}(t)}{M_{\theta|s}(t)}\right) = 0. \tag{18}$$

Equation (4) in the text is an equivalent condition.

Maximizing (3) over $X^i$ and $B^i$ yields the first order conditions

$$\frac{\partial \mathbb{E}\left[V^i|s\right]}{\partial B^i} = A\left(\beta^i R\,\mathbb{E}\left[\exp\{-AC_1^i\}|s\right] - \exp\{-AC_0^i\}\right) = 0 \tag{19}$$

and

$$\frac{\partial \mathbb{E}\left[V^i|s\right]}{\partial X^i} = A\left(\beta^i\,\mathbb{E}\left[\theta\exp\{-AC_1^i\}|s\right] - P(s)\exp\{-AC_0^i\}\right) = 0, \tag{20}$$

respectively. These conditions are equivalent to

$$\frac{1}{\beta^i R} = \mathbb{E}\left[\exp\{-A(C_1^i - C_0^i)\}\right] = H^i\,M_{\theta|s}(t) \tag{21}$$

and

$$\frac{P(s)}{\beta^i} = \mathbb{E}\left[\theta\exp\{-A(C_1^i - C_0^i)\}\right] = H^i\,M'_{\theta|s}(t), \tag{22}$$

respectively, where $H^i \equiv \exp\{-A[(1+R)B^i + P(s)X^i - W_0^i]\}$. Dividing the latter by the former equation implies equation (4) in the text as a necessary condition. Note that $H^i$, $W_0^i$, $C_1^i$ and $C_0^i$ are functions of $s$ since $RP(s)$ is.

With the definition of $H^i$, the optimal portfolio value can be written

$$\begin{aligned} B^i + P(s)X^i &= \tfrac{1}{1+R}\left(W_0^i + RP(s)\,X^i - \tfrac{1}{A}\ln H^i\right) \\ &= \tfrac{1}{1+R}\left(B_0^i + RP(s)(X_0^i/R + X^i) + \tfrac{1}{A}\ln\beta^i R M_{\theta|s}(-AX^i)\right), \end{aligned} \tag{23}$$

where the second line follows from (21).

# B Second-order conditions

Define $t \equiv -AX^i \in (-\infty, 0)$.

When investor $i$ maximizes terminal consumption only and $\alpha = 0$, the second-order condition for a utility maximum follows from the first derivative of condition (18), or

$$\frac{\partial^2 \mathbb{E}[V^i|s]}{(\partial X^i)^2} = \left( ARP(s) - \frac{M'_{\theta|s}(t)}{M_{\theta|s}(t)} \right) \frac{\partial \mathbb{E}[V^i|s]}{\partial X^i} + A^2 \mathbb{E}\left[V^i|s\right] \left[ \frac{M''_{\theta|s}(t)}{M_{\theta|s}(t)} - \left( \frac{M'_{\theta|s}(t)}{M_{\theta|s}(t)} \right)^2 \right] < 0. \tag{24}$$

When investor $i$ also has an *intertemporal* choice ($\alpha = 1$), the matrix of cross-derivatives for the two assets $B^i$ and $X^i$ becomes

$$\mathbf{A} = -A^2 \beta^i \exp\{-ARB^i\} \begin{vmatrix} R(1+R)M_{\theta|s}(t) & \cdot \\ (1+R)M'_{\theta|s}(t) & P(s)M'_{\theta|s}(t) + M''_{\theta|s}(t) \end{vmatrix} \tag{25}$$

by (19) and (20). If $\mathbf{A}$ is negative definite, a unique global utility maximum results. Equivalently, we require $-\mathbf{A}$ to be positive definite and all upper-left sub-matrices must have positive determinants. Since the upper-left entry in $\mathbf{A}$ is strictly positive, negative definiteness of $\mathbf{A}$ is equivalent to

$$\det(-\mathbf{A}) = A^4 (\beta^i)^2 \exp\{-2ARB^i\} R(1+R) \left[ M''_{\theta|s}(t) M_{\theta|s}(t) - M'_{\theta|s}(t)^2 \right] > 0,$$

which is equivalent to

$$\frac{M''_{\theta|s}(t)}{M_{\theta|s}(t)} - \left( \frac{M'_{\theta|s}(t)}{M_{\theta|s}(t)} \right)^2 > 0 \tag{26}$$

since $M_{\theta|s}(t) > 0$. This condition implies that $M'_{\theta|s}(t)/M_{\theta|s}(t)$ is strictly monotonically increasing in $t$, or strictly monotonically decreasing in $X^i$ for $t \equiv -AX^i$.

Similarly, by (18) and $\mathbb{E}\left[V^i|s\right] < 0$ for CARA utility, condition (24) is equivalent to (26).

# C Proof of theorem 2

*Ante notitiam*, expectations of (10) are

$$\mathbb{E}_S \left[ \mathbb{E}\left[V^i|S\right] \right] = -\beta^i \exp\left\{ -ARB_0^i + (t_0^i - t) \frac{M'_\varepsilon(t)}{M_\varepsilon(t)} \right\} M_S(t_0^i) M_\varepsilon(t)$$

for $t \equiv -A\overline{x} < 0$ and $t_0^i \equiv -AX_0^i < 0$. Using this result and (9) in information demand criterion (11), and rearranging, yields the information-demand criterion under *inter-asset* choice

$$M_S(t_0^i)/M_S(t) < \exp\left\{ (t_0^i - t) \frac{M'_S(t)}{M_S(t)} \right\}.$$

Since $M_S(t) > 0$ for finite $t$ by definition of an MGF, taking logs is permissible and yields (12).

Taking the first derivative of $Crit_{\text{IAC}}(t, t_0^i)$ with respect to $t$

$$\partial Crit_{\text{IAC}}(t, t_0^i)/\partial t = -\left(t - t_0^i\right)\left(\frac{M_S''(t)}{M_S(t)} - \left(\frac{M_S'(t)}{M_S(t)}\right)^2\right)$$

shows that criterion (12) is strictly increasing in $t$ for $t < t_0^i$ and strictly decreasing in $t$ for $t > t_0^i$ since $M_S''(t)/M_S(t) > [M_S'(t)/M_S(t)]^2$ by (5), while it attains the value $Crit_{\text{IAC}}(t, t) = 0$ in its global maximum at $t = t_0^i$. This establishes the first statement: No single investor has an incentive to acquire a costly signal. Uniqueness of the no-information equilibrium follows since no more than one investor can optimally acquire an indivisible public commodity by definition 1.

# D Indirect utility *post notitiam* under intertemporal choice

Under the definition $H^i \equiv \exp\{-A[(1+R)B^i + P(s)X^i - W_0^i]\}$, bond income can be written as

$$RB^i = \tfrac{R}{1+R}\left(W_0^i - P(s)X^i - \tfrac{1}{A}\ln H^i\right).$$

Note that $H^i$ and $W_0^i$ are functions of $s$ since $RP(s)$ is. Using this fact along with (23) in (3) yields

$$\begin{aligned}\mathbb{E}\left[V^i|s\right] &= -\exp\{-\tfrac{A}{1+R}\left[RW_0^i - RP(s)X^i + \tfrac{1}{A}\ln H^i\right]\}\left(1 + \beta^i H^i M_{\theta|s}(-A\,X^i)\right)\\ &= -\exp\left\{-A\tfrac{R}{1+R}W_0^i\right\}\left(\frac{e^{ARP(s)\,X^i}}{H^i}\right)^{\frac{1}{1+R}}\left(1 + \frac{1}{R}\right)\end{aligned}$$

for $\alpha = 1$. The second step follows from the first order condition (21) for the bond, substituting it for $H^i M_\theta(-A\,X^i)$. Indirect utility (14) in the text follows using (21) once more. Function (14) is a proper indirect utility function since the asset price $P(s)$ in equilibrium reflects the first order condition (22) for the risky asset.

# E Bond return response to stock return information

Taking logs of both sides of the bond first order condition (21) yields

$$A(1+R)B^i - AB_0^i + AP(s)(X^i - X_0^i) = \ln \beta^i R M_\theta(-A\,X^i),$$

a permissible operation since $\beta^i, R, M_\theta(\cdot) > 0$ by their definitions. Summing up both sides over investors $i$ and dividing by their total number (measure) yields

$$\exp\{AR\bar{b}\}/\beta^i R = M_\theta(t), \tag{27}$$

after exponentiating both sides, where $\bar{b}$ is the average initial bond endowment per investor, $\bar{b} \equiv \sum_{i=1}^{I} B_0^i/I$, and $t \equiv -A\overline{x}$. Equation (27) implicitly determines the gross bond return $R$. By the implicit function theorem,

$$\partial R/\partial \bar{b} = -AR^2/(A\bar{b}R - 1).$$

So, the bond return increases in response to a higher bond endowment if $\bar{b} < 1/(AR)$ but it would fall if $\bar{b} > 1/(AR)$. Since $R$ falls arbitrarily (infinitely) strongly at $\bar{b} = 1/(AR) + \epsilon$ for an arbitrarily small $\epsilon > 0$, the equilibrium is not well defined for $\bar{b} > 1/(AR)$. This restricts the model to $\bar{b} \leq \bar{b}_0 = 1/(AR_0)$ for $\bar{b}_0 = \beta^i M_\theta(t)/(Ae)$, $R_0 = e/(\beta^i M_\theta(t))$ satisfying (27).

Since $\mathbb{E}_S\left[M_{\theta|s}(t)\right] = M_\theta(t)$ by the law of iterated expectations, information acquisition on the stock return does not affect the bond return *ante notitiam*. *Post notitiam*, however, $M_{\theta|s}(t) = \exp\{st\}M_\varepsilon(t)$ and $R$ does respond to the signal realization. Applying the implicit function theorem to (27) for $M_{\theta|s}(t)$, we find that

$$\frac{\partial R}{\partial s} = A\overline{x}R^2 \exp\{A\overline{x}s - A\bar{b}R\} \frac{\beta^i M_\varepsilon(-A\overline{x})}{A\bar{b}R - 1} \leq 0.$$

The bond return falls in response to a favorable signal realization $s$ since $\bar{b} \leq 1/(AR)$. In principle, $R$ too is a function of the signal realization $s$. For small initial stock endowments, however,

$$\lim_{\overline{x}\to 0} \partial R/\partial s = +0.$$

So, $R \approx R_{\lambda=1} \approx R_{\lambda=0}$ in the presence of a small stock endowment relative to the bond endowment.

# F Proof of theorem 4

*Ante notitiam*, expectations of (16) are

$$\mathbb{E}_S\left[\mathbb{E}\left[V^i|S\right]\right] = -\delta \exp\left\{-ARB_0^i + (t_0^i - t)\tfrac{M'_\varepsilon(t)}{M_\varepsilon(t)}\right\}^{\frac{1}{1+R}} M_S\left(\tfrac{t_0^i}{1+R}\right) M_\varepsilon(t)^{\frac{1}{1+R}}$$

for $t \equiv -A\overline{x} < 0$ and $t_0^i \equiv -AX_0^i < 0$. Using this result and (15) in information demand criterion (11), and rearranging, yields the information-demand criterion under *intertemporal* choice

$$M_S\left(\tfrac{t_0^i}{1+R}\right)/M_S(t)^{1/(1+R)} < \exp\left\{(t_0^i - t)\,\tfrac{M'_S(t)}{M_S(t)}\right\}^{1/(1+R)},$$

where $R \in (0, \infty)$. Since $M_S(t) > 0$ for finite $t$ by definition of an MGF, taking logs is permissible and yields (17).

Taking the first derivative of $Crit_{\text{ITC}}(t, t_0^i)$ with respect to $t$

$$\frac{\partial Crit_{\text{ITC}}(t, t_0^i)}{\partial t} = -(t - t_0^i)\left(\frac{M_S''(t)}{M_S(t)} - \left(\frac{M_S'(t)}{M_S(t)}\right)^2\right)$$

shows that criterion (17) is strictly increasing in $t$ for $t < t_0^i$ and strictly decreasing in $t$ for $t > t_0^i$ since $M_S''(t)/M_S(t) > [M_S'(t)/M_S(t)]^2$ by (5), while it attains its global maximum at $t = t_0^i$. To prove the first statement that distributions in the *singleton family* (5) satisfy this criterion for a sufficiently small difference $t - t_0^i$, it remains to establish that the maximum strictly exceeds zero.

The fact that

$$\hat{t}M_S'(\hat{t})/M_S(\hat{t}) > \ln M_S(\hat{t}) \tag{28}$$

for $\hat{t} < 0$ is a useful property of distributions in the *singleton family* (5). Observe that both the left-hand and the right-hand side of (28) vanish for $\hat{t} = 0$. So, to establish (28), it suffices to show that its left-hand side increases faster than the right-hand side for all $\hat{t} < 0$ as $\hat{t}$ falls. Taking the first derivative of either side with respect to $\hat{t}$ shows that the increase in the left-hand side exceeds the increase in the right-hand side by $-\hat{t} \cdot [M_S''(\hat{t})/M_S(\hat{t}) - (M_S'(\hat{t})/M_\theta(\hat{t}))^2] > 0$ as $\hat{t}$ falls, which is a positive amount by $\hat{t} < 0$ and (5).

We know from theorem 2 that criterion (17) attains a maximum value of zero for $R = 0$. So, if we can show that (the maximum of) criterion (17) strictly increases in $R$, the first statement that distributions in the *singleton family* satisfy (17) for sufficiently large $R$ is proven. Taking the first derivative of (17) with respect to $R$ yields

$$\partial Crit_{\text{ITC}}(t, t_0^i)/\partial R = -\ln M_S(\hat{t}) + \hat{t}M_S'(\hat{t})/M_S(\hat{t}) > 0$$

for $\hat{t} \equiv t_0^i/(1+R) < 0$. The derivative is strictly positive by fact (28). So, if $R > 0$, criterion (17) holds for $t_0^i > 0$ in a neighborhood around $t$ but fails otherwise. Uniqueness of the information and the no-information equilibrium follows since no more than one investor can optimally acquire an indivisible public commodity by definition 1.

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