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**Working Paper**

# Market Design with Correlated Valuations

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# Market Design with Correlated Valuations

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# Market Design with Correlated Valuations

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**Abstract**. The effects of information on market design are explored in a simple setting where firms have private information about their correlated fixed costs and the government aims to maximize its expected revenue conditional on achieving efficient allocations. Government revenues are higher when the costs are less correlated (or are more of a private value). The reduced correlation increases the firms' information rents, but a change in the information structure also changes the expected market structures with positive effects on government revenues. If the government faces the no-deficit constraint, there are situations where efficient allocations are achieved under asymmetric information but not under symmetric information.

*JEL Classification Numbers: D4, L1, H8.*

*Keywords: market structure, correlated values, market design, government revenue.*

## INTRODUCTION

Government agencies sometimes face the problem of how to allocate production licenses (or award production contracts) to firms. The production license or contract could be for the provision of health-care services, cable services, or recycling services; for the production of military systems; or, as in the recent FCC spectrum auction in the U.S., for the provision of new telecommunication services. An important consideration for the government in such situations is whether an allocation mechanism will choose the most efficient supplier(s). The theory of auctions and competitive bidding has offered useful insights in this respect. Another interesting issue, one that has received less attention, is how many production licenses to award, in addition to who should obtain the licenses. In other words, an allocation mechanism may also need to choose a desirable structure of the market. This is the issue of market design.[1]

This paper studies the allocation of production assets by government when the market structure is endogenously determined. The literature on this issue, pioneered by Dana and Spier (1994), generally assumes that firms have private information about their private valuations.[2] We depart from this approach by considering situations where firms have private information about a correlated value, such as the cost of building a network. Under our formulation pure private value and pure common value will be special cases of our model.[3] In addition, in stead of using a social welfare function that assigns some weight to government revenues, we assume that the government's objective is to obtain the highest revenue possible conditional on the allocation being efficient,[4] and we discuss what happens if the government cannot have negative revenues.

We consider a simple model where a government agency needs to decide whether to issue one, two, or zero production licenses and who may receive a license. Firms are initially uncertain about their own fixed costs but receive private signals about the possible values of the costs. We focus on two issues in this simple setting. First, how does the nature of information structure affect government revenues, conditional on the allocation being efficient? We show that, when the private signal is uniformly distributed, the expected government revenue is higher when different firms' costs are

more independent, and we identify three effects on government revenues when the correlation between different firms' costs decreases: it becomes more difficult for the government to obtain true reports of cost information from each firm (an incentive effect); decreased cost correlation across firms lowers the lowest cost realization among different firms, which reduces the expected cost of the firm(s) awarded the license (a cost effect); and monopoly becomes more likely, increasing government revenues (a market structure effect).

Second, how does asymmetric information affect market efficiency, if the government cannot have negative revenues and hence the equilibrium allocation may not be fully efficient?[5] Our analysis reveals a surprising result: there are parameter values under which full efficiency is not obtained under complete information but is obtained under asymmetric information! This result has a simple intuition: firms may have positive profits in some states of the world but negative profits in the other states, conditional on the allocation being efficient. Under complete information, the efficient allocation cannot be achieved if that leads to negative profits and if government revenues cannot be negative. Under asymmetric information, however, the firms' incentives are pooled and this relaxes the participation constraint, making it possible to have the efficient allocation without the need for government subsidy.

Our study is closely related to the literature on optimal auction design (cf., Myerson, 1981). When the goods being auctioned are productive assets or licenses, as are in our case, additional complications arise because the value of winning to a bidder may depend on who else wins and what is the nature of strategic interactions in a market. This is related to studies on auctions with endogenous valuations, such as Krishna (1993), Krishna and Rosenthal (1996), Rosenthal and Wang (1996), and Chen (2000).

In what follows, we conduct our analysis in Section 2, and conclude in Section 3. All proofs are relegated to the appendix.

# I THE MODEL AND ITS ANALYSIS

A government needs to decide whether to award 0, 1 or 2 licenses to two firms in a market. The market will be a monopoly if only one license is awarded, and firms will compete in a duopoly if both are awarded licenses.

Assume that Firm $i$ privately observes signal $x_i$ in the beginning of the game, for $i = 1, 2$, and the fixed costs of firms 1 and 2 are, respectively:

$$\begin{aligned} c_1 &= \alpha x_1 + (1-\alpha)x_2, \\ c_2 &= \alpha x_2 + (1-\alpha)x_1, \end{aligned}$$

where $\frac{1}{2} \leq \alpha \leq 1$. Thus, the firms' fixed costs are correlated and our formulation includes pure private value ($\alpha = 1$) and pure common value ($\alpha = \frac{1}{2}$) as special cases. The firms' other costs are normalized to zero. Each firm learns its (fixed) cost only after it receives the license.

Assume that $x_1$ and $x_2$ are i.i.d. with c.d.f. $F(x)$ and p.d.f. $f(x)$, where $x \in [\underline{x}, \bar{x}]$ and $0 \leq \underline{x} < \bar{x} < \infty$.

Let $\pi^m$ be the monopoly profit and $\pi^d$ the per-firm duopoly profit, excluding the fixed costs. Let $CS^m$ denote the consumer surplus in the case of monopoly, and $CS^d$ the consumer surplus in the case of duopoly. Then the social surplus in the case of firm $i$ (where $c_i \leq c_j$, $i \neq j$) being a monopoly is given by

$$W^{m_i} = \pi^m + CS^m - c_i,$$

and the social surplus in the case of duopoly is given by

$$W^d = 2\pi^d + CS^d - c_1 - c_2.$$

We assume

$$\bar{x} > 2\pi^d + CS^d - (\pi^m + CS^m) > \underline{x},$$

so that under some situations a duopoly market structure is efficient and under others a monopoly market structure is efficient. In addition, for convenience we assume that $\pi^m \geq \bar{x}$ so that at lease one license will be issued in any efficient allocation.

As a preliminary step, we characterize the efficient allocations, allocations that yield the highest social surpluses. We can divide the square of $(c_1, c_2) \in [\underline{x}, \bar{x}] \times [\underline{x}, \bar{x}]$ into three possible regions according to their social surpluses. (See Figure 1.) The first is the duopoly region $D$, where $W^d \geq \max\{W^{m_1}, W^{m_2}\}$, or

$$2\pi^d + CS^d - (\pi^m + CS^m) \equiv \hat{c} \geq \max\{c_1, c_2\}.$$

Since $2\pi^d + CS^d$ is generally larger than $\pi^m + CS^m$, region $D$ is a square area from $\underline{x}$ to $\hat{c}$, provided that $\hat{c} > \underline{x}$. In this region, two licenses should be issued under efficient allocation, because the increase in social benefits by issuing an additional license to the higher-cost firm exceeds the additional cost.

The second is the monopoly region $M_1$, where $c_1 \leq c_2$ and $W^d < W^{m_1}$. This is the area above both the 45 degree line and $c_2 = \hat{c}$. In this region, only one license should be issued, and it should be issued to firm 1 under efficient allocation.

The third is the monopoly region $M_2$, where $c_1 > c_2$ and $W^d < W^{m_2}$. This is the area below the 45 degree line and to the right of $c_1 = \hat{c}$. In this region, only one license should be issued, and it should be issued to firm 2 under efficient allocation.

**Insert Figure 1 about Here.**

Recall that $c_1 = \alpha x_1 + (1-\alpha)x_2$ and $c_2 = \alpha x_2 + (1-\alpha)x_1$. We can also depict regions $D$, $M_1$, and $M_2$ in $(x_1, x_2)$ space. It is easy to see that when $\alpha$ increases, $\max\{c_1, c_2\}$ increases, which means that $M_1$ and $M_2$ expand while $D$ shrinks in $(x_1, x_2)$ space. (See Figure 2.)

**Insert Figure 2 about Here.**

We assume that the government aims to maximize its revenue conditional on the allocation being efficient. This seems a realistic assumption in many situations, for instance in the case of the spectrum auction in the U.S. According to the Federal Communications Commission (FCC 1999), the regulator of radio spectrum in the U.S., the first core principle of effective spectrum management is to maximize the efficient use of radio spectrum. Furthermore, as another core principle, allocation and licensing assignments should be based on marketplace demands. Revenue maximization is regarded as less important than efficiency. In this regard, we model the government's objective as to maximize efficiency first, and then the revenue raised in the auction.[6]

## I.1 Government Revenues

We first derive the maximum revenue the government can achieve under efficient allocations. Since $x_i$ is firm $i$'s private information, we use techniques from the mechanism design literature and consider direct mechanisms.

Let $t_i(x_i)$, $i = 1, 2$, be the expected transfer from firm $i$ to the government when $x_i$ is reported by firm $i$. Then the incentive compatibility constraint for firm $i$ is given by

$$
\begin{aligned}
& \Pi(x_i, x_i) \\
(1) \qquad = \; & [\pi^d - E_{x_j}(c_i|(x_i, x_j) \in D, x_i)] \Pr\{D|x_i\} \\
& \quad + [\pi^m - E_{x_j}(c_i|(x_i, x_j) \in M_i, x_i)] \Pr\{M_i|x_i\} - t_i(x_i) \\
\geq \; & [\pi^d - E_{x_j}(c_i|(\tilde{x}_i, x_j) \in D, x_i)] \Pr\{D|\tilde{x}_i\} \\
& \quad + [\pi^m - E_{x_j}(c_i|(\tilde{x}_i, x_j) \in M_i, x_i)] \Pr\{M_i|\tilde{x}_i\} - t_i(\tilde{x}_i) \\
(2) \qquad \equiv \; & \Pi(\tilde{x}_i, x_i).
\end{aligned}
$$

Standard techniques show that $\Pi(\tilde{x}_i, x_i)$ is differentiable in $\tilde{x}_i$. We focus on the direct mechanisms where each firm reports its signal truthfully. Therefore, $\tilde{x}_i = x_i$

should be optimal for firm $i$ with signal $x_i$, and

$$\left.\frac{\partial \Pi(\tilde{x}_i, x_i)}{\partial \tilde{x}_i}\right|_{\tilde{x}_i = x_i} = 0. \tag{3}$$

Note that, for $D$,

$$\begin{aligned} E_{x_j}(c_i | (\tilde{x}_i, x_j) \in D, x_i) &= E_{x_j}(\alpha x_i + (1-\alpha) x_j | (\tilde{x}_i, x_j) \in D) \\ &= \alpha x_i \Pr\{D | \tilde{x}_i\} + (1-\alpha) E_{x_j}(x_j | (\tilde{x}_i, x_j) \in D), \end{aligned}$$

where

$$\Pr\{D | \tilde{x}_i\} = \Pr\{(\tilde{x}_i, x_j) \in D | \tilde{x}_i\},$$

and similarly for $M_i$. It is easy to verify that $\Pr\{D|x_i\} + \Pr\{M_i|x_i\}$ is decreasing in $x_i$. An increase in $x_i$ would increase firm $i$'s cost more than increasing firm $j$'s cost. It decreases the probability of firm $i$ being the monopoly, and at the same time, it decreases the probability of a duopoly due to the cost increase for both firms. Therefore,

$$\begin{aligned} \left.\frac{\partial \Pi(\tilde{x}_i, x_i)}{\partial x_i}\right|_{\tilde{x}_i = x_i} &= \left.\frac{\partial \Pi(\tilde{x}_i, x_i)}{\partial x_i}\right|_{\tilde{x}_i = x_i} \\ &= -\alpha \Pr\{D|x_i\} - \alpha \Pr\{M_i|x_i\} \end{aligned} \tag{4}$$

is increasing in $x_i$.

Now we verify the second order condition for the above optimization problem. Taking derivatives of both sides of equation (3) with respect to $x_i$, we have

$$\left.\frac{\partial^2 \Pi(\tilde{x}_i, x_i)}{\partial \tilde{x}_i^2}\right|_{\tilde{x}_i = x_i} + \left.\frac{\partial^2 \Pi(\tilde{x}_i, x_i)}{\partial \tilde{x}_i \partial x_i}\right|_{\tilde{x}_i = x_i} = 0.$$

As we argue above, the second term is positive. Therefore, the first term is negative. That is, $\Pi(\tilde{x}_i, x_i)$ is concave in $\tilde{x}_i$ at $\tilde{x}_i = x_i$, confirming the second order condition for the optimization.

To characterize a firm's profit function $\Pi(x_i, x_i)$, we derive

$$\begin{aligned}
\frac{d\Pi(x_i, x_i)}{dx_i} &= \left.\frac{\partial \Pi(\tilde{x}_i, x_i)}{\partial \tilde{x}_i}\right|_{\tilde{x}_i = x_i} + \left.\frac{\partial \Pi(\tilde{x}_i, x_i)}{\partial x_i}\right|_{\tilde{x}_i = x_i} \\
&= \left.\frac{\partial \Pi(\tilde{x}_i, x_i)}{\partial x_i}\right|_{\tilde{x}_i = x_i} \\
&= -\alpha \Pr\{D|x_i\} - \alpha \Pr\{M_i|x_i\} \qquad (5)
\end{aligned}$$

Integrating both sides from $\bar{x}$ to $x$, we have

$$\begin{aligned}
\Pi(x, x) - \Pi(\bar{x}, \bar{x}) &= -\alpha \int_{\bar{x}}^{x} (\Pr\{D|x_i\} + \Pr\{M_i|x_i\}) dx_i \\
&= \alpha \int_{x}^{\bar{x}} (\Pr\{D|x_i\} + \Pr\{M_i|x_i\}) dx_i
\end{aligned}$$

To maximize the transfers the government receives, we set $\Pi(\bar{x}, \bar{x}) = 0$. Noting equation (1), we have

$$\begin{aligned}
t_i(x) &= [\pi^d - E_{x_j}(c_i|(x, x_j) \in D, x)] \Pr\{D|x\} \\
&\quad + [\pi^m - E_{x_j}(c_i|(x, x_j) \in M_i, x)] \Pr\{M_i|x\} \\
&\quad - \alpha \int_{x}^{\bar{x}} (\Pr\{D|x_i\} + \Pr\{M_i|x_i\}) dx_i. \qquad (6)
\end{aligned}$$

The expression for $t_j(x)$ is identical. It is easy to verify that $t_i(x)$ is decreasing in $x$.

The government's expected revenue can be expressed as

$$R = \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^{\bar{x}} [t_i(x_i) + t_j(x_j)] f(x_i) f(x_j) dx_i dx_j = 2 \int_{\underline{x}}^{\bar{x}} t_i(x) f(x) dx.$$

Substituting the expression for $t_i(x)$ into above, we have shown:

**Proposition 1** *Conditional on the allocations being efficient, the highest expected revenue the government can obtain is*

$$\begin{aligned}
R &= 2\{[\pi^d - E(c_i|(x_i, x_j) \in D)] \Pr\{D\} + [\pi^m - E(c_i|(x_i, x_j) \in M_i)] \Pr\{M_i\} \\
&\quad - \alpha \int_{\underline{x}}^{\bar{x}} \int_{x}^{\bar{x}} (\Pr\{D|x_i\} + \Pr\{M_i|x_i\}) dx_i dx\}.
\end{aligned}$$

The first two terms in the braces for $R$ represent firm $i$'s profit (net of its costs) in the cases of duopoly and monopoly, respectively. The government captures these profits in the optimal mechanism. The third term represents the informational rent firm $i$ receives because of its private information. The larger the $\alpha$, the larger the informational rent. The government needs to pay this rent in order for firm $i$ to reveal its private information. It may appear that the higher the $\alpha$, the higher will be the information rents to the firms and the lower the expected government revenue. However, a change in $\alpha$ also changes the boundaries of the regions for duopoly and monopoly. The relationship between $\alpha$ and $R$ is thus more complex. In fact, as we shall show below, with some additional assumptions on $F(\cdot)$ and on the market demand, $R$ increases in $\alpha$.

## I.2 The Effect of Cost Correlation and Market Demand on Government Revenues

One interesting question is how $\alpha$, which measures the correlation between the two firms' costs, affects government revenues. A higher $\alpha$ means that the firms' costs are more independent, increasing the information rents of the firms. One may then think that the government revenue should decrease in $a$. However, as $\alpha$ increases, the cost realizations of the two firms are more likely to be asymmetric and the lower of these is more likely to be lower, which increases the probability that the monopoly market structure is chosen and that the cost of the monopolist is lower. These market structure and cost effects imply that an increase in $\alpha$ also has positive effects on government revenues. It is thus possible that an increase in $\alpha$ increases government revenues.

Another interesting question is how the market demand will affect government revenue. It is perhaps natural to think that an increase in market demand should increase government revenue. But this turns out to be false. An increase in demand may change the market structure and affect the government's revenue adversely.

We have the following proposition.

**Proposition 2** *Assume that $F(x)$ is $U[0,1]$, the market demand is given by $Q = \beta(1-P)$, where $\beta \in [4,14]$, and firms are Cournot competitors when in a duopoly. Then government revenue is increasing in $\alpha$ and decreasing in $\beta$.*

Proposition 2 sheds light on how the nature of information structure affects government revenues under efficient allocations. When a firm's cost is more dependent on its own private signal, in a sense the firm benefits more from its private information, which, if the value to be shared between the firm and the government is fixed, would imply a lower revenue for the government. But both the value and the number of licenses are endogenous here, and under the conditions we specify the expected government revenue is higher when each firm's costs depend more on its own private signal.

Proposition 2 also says that the expected government revenue is lower when market demand, measured by $\beta$, is higher. The endogenous nature of the market structure is crucial for this result: with higher market demand, it becomes more likely that the duopoly market structure is efficient; but firms' profits are lower under duopoly than under monopoly, which leads to lower expected government revenues under efficient allocations.

## I.3 Possible Higher Efficiency under Asymmetric Information

Due to their private information, firms will receive informational rents. Thus, conditional on achieving efficient allocations, the expected government revenues are lower under asymmetric information than under symmetric information. In this sense, the government should prefer to have symmetric information, a situation where both firms' costs are common knowledge. However, asymmetric information sometimes may also be desirable to a government who faces the constraint of non-negative revenues. In our model, when a firm does not know the other firm's private information, it knows neither the efficient allocation nor its own cost. The revenue maximizing

mechanism we analyzed earlier has already (implicitly) made used of this uncertainty. It pools a firm's incentives across different states in favor of the government's revenue.

In the proof of the following proposition, we provide an example in which, conditional on the licenses being allocated efficiently, government revenue is positive in the revenue maximizing mechanism with asymmetric cost information, but it is negative in some regions of the cost realizations if the cost information is common knowledge.

**Proposition 3** *Assume that government revenue must be non-negative. There are situations where efficient allocations are achieved under asymmetric information but not under symmetric information.*

Intuitively, firms have positive profits in some states of the world but negative profits in the others, conditional on the allocation being efficient. Under complete information, the efficient allocation cannot be achieved if that leads to negative firm profits and if government revenues cannot be negative. Under asymmetric information, however, the firms' incentives are pooled, which relaxes the participation constraint, making it possible to have the efficient allocation without the need for government subsidies.[7]

Figure 3 illustrates a region where duopoly is efficient but not profitable. This region exists as long as $\pi^d < \hat{c}$. The duopoly would not operate in that region unless they are subsidized by the government. Therefore, in that region, efficient allocation of licenses cannot be achieved under symmetric information and with the constraint of non-negative government revenue.

**Insert Figure 3 about Here.**

We can restate Proposition 3 in another way. Since efficient allocation cannot be achieved in some region with the non-negative revenue constraint, what can be achieved must be inefficient in that region. We have the following corollary.

**Corollary 1** *Assume that the government is constrained to receive non-negative revenues. Then there are situations in which efficiency is higher under asymmetric information than under no asymmetric information.*

## I.4 Auctions as Optimal Mechanisms: Theory and Practice

The optimal mechanism in our model resembles some of the familiar auction formats. Here, we consider two auction formats, each is equivalent to the optimal mechanism in subsection 2.1. We shall also discuss the use of such an auction for government procurement in practice.

Let us first consider a "modified" first-price, sealed-bid auction. Recall equation (6). Define $x = \omega(b)$ as the inverse function of

$$\frac{t_i(x)}{\Pr\{D|x\} + \Pr\{M_i|x\}} = b$$

We have the following:

**Remark 1** *The revenue maximizing mechanism characterized in Proposition 1 is equivalent to the following modified first-price, sealed-bid auction where a bidder pays (his own bid) only if he wins. The two firms submit bids $b_1$ and $b_2$ simultaneously. If $\alpha\omega(b_1) + (1-\alpha)\omega(b_2) \leq \hat{c}$ and $\alpha\omega(b_2) + (1-\alpha)\omega(b_1)\} \leq \hat{c}$, then each firm wins a license; if any of the above inequalities does not hold, then only firm 1 wins a license if $\omega(b_1) \leq \omega(b_2)$, and only firm 2 wins a license if $\omega(b_1) > \omega(b_2)$.*

This remark follows directly from the fact that the proposed auction corresponds exactly to the optimal mechanism in Proposition 1 in terms of expected transfers and winning probabilities.

Auctions similar to those in Remark 1 have often been employed by the government in procuring goods and services. The United States Department of Defense, for example, uses auctions to procure a significant portion of its weapons that explicitly leave open the number of producers.

According to the Defense Procurement and Acquisition Policy from the Department of Defense of the United States (Department of Defense, 2005), the procurement agency can "make multiple awards for the same indefinite requirement in situations where multiple firms are capable of delivering similar, but not necessarily identical, products to meet the needs of the Government and provide alternatives for ordering offices. Ordering offices then have the choice of selecting the product and firm that best meet their needs. ... ... [For] supplies or services other than advisory and assistance services, give preference to making multiple awards, unless [the procurement agency] determines that a single award is appropriate. " Furthermore, "award of requirements for an individual line item may be split between two or more sources. The size of each portion of the split or a method for calculating the split should be established in the solicitation. Every possible effort should be made to assure that any amount awarded is an economic production quantity. Multiple sourcing is necessary to maintain competitive sources for a product that would otherwise be available only from one source."

For example, the online newsletter "Contract" (Department of Defense, 1998) described a case of split offers. "Raytheon Systems Company, Ft. Wayne, Ind., is being awarded a $21,749,920 split, firm-fixed-price contract to procure 22,751 AN/SSQ-62E sonobuoys and associated data. ...Sparton Defense Electronics, DeLeon Springs, Fla., is being awarded a $5,258,478 split, firm-fixed-price contract to procure 3,000 AN/SSQ-62E sonobuoys and associated data.... [These contracts were] competitively procured with two proposals solicited and two offers received. This is a split procurement under Class Justification and Approval Number CR 01240, based on a Federal Acquisition Regulation 6.302-2 citing industrial mobilization. It is customary practice to split procurements in the sonobuoy program to meet industrial mobilization requirements. "

There are also cases where split awards do not materialize. In the Department of the Navy of U.S.A. (1998), a protest was logged by one of the bidders because the government agency did not make a split award in the procurement of waterborne hull cleaning and associated services. The protest was denied because the agency did not have to make a split award.

Alternatively to the auction in Remark 1, we can consider a more direct sealed-bid, all-pay auction. Let the inverse of the transfer function (6) be $x = \chi(b)$. Note that $x = \chi(b)$ is a decreasing function. We have the following:

**Remark 2** *The revenue maximizing mechanism characterized in Proposition 1 is also equivalent to the following sealed-bid, all-pay auction where bidders pay their own bids regardless of winning or not. The two firms submit bids $b_1$ and $b_2$ simultaneously. If $\alpha\chi(b_1) + (1-\alpha)\chi(b_2) \leq \hat{c}$ and $\alpha\chi(b_2) + (1-\alpha)\chi(b_1)\} \leq \hat{c}$, then each firm wins a license; if any of the above inequalities does not hold, then only firm 1 wins a license if $b_1 \geq b_2$, and only firm 2 wins a license if $b_1 < b_2$.*

Remark 2 also follows directly from the fact that the proposed auction corresponds exactly to the optimal mechanism in Proposition 1 in terms of transfers and winning probabilities.

When $\alpha = 1$, the mechanism in Remark 2 resembles a simple sealed-bid, all-pay auction with maximum price $\hat{t}$ – a bid of $\hat{t}$ or higher will be guaranteed a license. If a firm submits a lower price, it wins only if the other firm submit an even lower price. In this auction, there are three outcomes. First, both firms submit the maximum price $\hat{t}$. In this case, each firm wins a license. Second, one firm submits the maximum price, and the other firm submits a lower price. In this case, the first firm wins a license but the second firm does not. Third, no firm buys at or submits the highest price. In this case, the firm with the higher bid will win.

Note that there is a gap between the maximum price and the rest (which are at most $t_i(\hat{c})$ in (8)). This happens in equilibrium because bidding the maximum price guarantees winning a license with probability one, even if the other firm bids at the same price. However, bidding an amount lower than the maximum price implies a discrete drop in winning probability – the firm loses if the other firm bids at the maximum price (which occurs with a strictly positive probability).

In general, a firm's fixed cost depends on the other firm's signal. The allocation of licenses is according to Figure 2. When both firms' signals are low (so both firms' costs are low), each firm wins a license. When one firm's signal is low but the other

firm's signal is high (so one firm's cost is low and the other firm's cost is high), the firm with the lower signal (and thus lower cost) wins the license.

# II CONCLUSION

The design of a market is affected by the nature of information. This study explores this issue by considering a simple setting where firms have private information about their correlated fixed costs and the government aims to maximize its expected revenue conditional on achieving efficient allocations. We find that government revenues tend to be higher when the costs are less correlated (or a firm's private information is more about a private value). While the firms' information rents would increase as they can learn more from their private information, a change in the information structure also changes the expected market structures, and the latter effect dominates under certain conditions. We also find that, if the government faces the no-deficit constraint, there are situations where the allocation is efficient under asymmetric information but not under symmetric information, because there is "pooling" of profits under asymmetric information that can make government subsidies unnecessary.

When firms' private information is about a correlated value, the problem of market design is generally very complicated. Our modest goal in this study has been to consider a particular setting that allows us to gain some insights about the possible effects of information on the outcomes of resource allocations in a government-designed market. To study these effects in more general settings remains an interesting area for future research.

# III APPENDIX: PROOFS

**Proof of Proposition 2** It is straight-forward to show that $\pi^d = \frac{\beta}{9}$, $\pi^m = \frac{\beta}{4}$,

$p^d = \frac{1}{3}$, $p^m = \frac{1}{2}$, $CS^d = \frac{2\beta}{9}$, and $CS^m = \frac{\beta}{8}$. The duopoly region $D$ is characterized by

$$\begin{aligned} D &= \{(c_1, c_2) : c_1 \geq c_2,\ 2\pi^d + CS^d - c_1 - c_2 \geq \pi^m + CS^m - c_2\} \\ &\quad \cup \{(c_1, c_2) : c_1 < c_2,\ 2\pi^d + CS^d - c_1 - c_2 \geq \pi^m + CS^m - c_1\} \\ &= \{(x_1, x_2) : x_1 \geq x_2,\ 2\pi^d + CS^d - \pi^m - CS^m \geq \alpha x_1 + (1-\alpha) x_2\} \\ &\quad \cup \{(x_1, x_2) : x_1 < x_2,\ 2\pi^d + CS^d - \pi^m - CS^m \geq \alpha x_2 + (1-\alpha) x_1\}. \end{aligned}$$

The monopoly region $M_1$ is characterized by

$$\begin{aligned} M_1 &= \{(c_1, c_2) : c_1 < c_2,\ \pi^m + CS^m - c_1 > \pi^d + CS^d - c_1 - c_2\} \\ &= \{(x_1, x_2) : x_1 < x_2,\ 2\pi^d + CS^d - \pi^m - CS^m < \alpha x_2 + (1-\alpha) x_1\}. \end{aligned}$$

Therefore,

$$\begin{aligned} &\frac{R}{2} \\ = &\int_0^{\frac{5\beta}{72}} \int_{x_1}^{-\frac{1-\alpha}{\alpha} x_1 + \frac{5\beta}{72\alpha}} \left( \frac{4}{9} - 2\alpha x_1 - (1-\alpha) x_2 \right) dx_2 dx_1 \\ &+ \int_0^{\frac{5\beta}{72}} \int_{x_2}^{-\frac{1-\alpha}{\alpha} x_2 + \frac{5\beta}{72\alpha}} \left( \frac{4}{9} - 2\alpha x_1 - (1-\alpha) x_2 \right) dx_1 dx_2 \\ &+ \int_0^{\frac{5\beta}{72}} \int_{-\frac{1-\alpha}{\alpha} x_1 + \frac{5\beta}{72\alpha}}^{1} (1 - 2\alpha x_1 - (1-\alpha) x_2)\, dx_2 dx_1 \\ &+ \int_{\frac{5\beta}{72}}^{1} \int_{x_1}^{1} (1 - 2\alpha x_1 - (1-\alpha) x_2)\, dx_2 dx_1 \\ = &-\frac{25\beta^2 (5\beta - 96\alpha + 5\alpha^2\beta)}{2239\,488\alpha^2} - \frac{25\beta^2 (15\beta + 5\alpha\beta - 96)}{2239\,488\alpha} \\ &+ \frac{5\beta \left(15\,552\alpha^2 - 1080\alpha\beta + 15\,552\alpha^3 + 25\beta^2 - 1080\alpha^2\beta - 2160\alpha^3\beta + 50\alpha^2\beta^2 + 75\alpha^3\beta^2\right)}{2239\,488\alpha^2} \\ &- \frac{(5\beta - 72)^2 (5\beta + 15\alpha\beta - 72)}{2239\,488} \\ = &\frac{\left(373\,248\alpha - 600\beta^2 - 375\beta^3 - 125\alpha\beta^3\right)}{2239\,488\alpha}. \end{aligned}$$

It is straight-forward to verify that $R$ is increasing in $\alpha$ and decreasing in $\beta$. ■

**Proof of Proposition 3** Suppose that the market demand is $Q = \beta(1-P)$, where $\beta \in (4\bar{x}, 8\bar{x})$, $0 \leq \underline{x} < \frac{1}{2}\bar{x}$, and firms are Bertrand competitors if the market structure is a duopoly. Then $\pi^m = \frac{\beta}{4}$, $\pi^d = 0$, $\hat{c} = \frac{\beta}{2} - \left(\frac{\beta}{4} + \frac{\beta}{8}\right) = \frac{\beta}{8} \in (\underline{x}, \bar{x})$ and $\pi^d < \hat{c}$. Thus there are situations where efficient allocation cannot be achieved under symmetric information.

To show that for some of these situations efficient allocation can nevertheless be achieved under asymmetric information, consider the case of $\alpha = 1$; that is, firms' costs are completely independent, with $c_1 = x_1$ and $c_2 = x_2$. We shall show that it is possible in this case to achieve efficient allocations and yet $t_i(x) > 0$ for all $x$ .

Since $\alpha = 1$ and $c_i$ $(= x_i)$ is independent of $x_j$, we have

$$t_i(c_i) = [\pi^d - c_i]\Pr\{D|c_i\} + [\pi^m - c_i]\Pr\{M_i|c_i\} - \int_{c_i}^{\bar{x}} (\Pr\{D|\tilde{c}_i\} + \Pr\{M_i|\tilde{c}_i\})d\tilde{c}_i. \tag{7}$$

Note that, for $\tilde{c}_i \leq \hat{c}$,

$$\Pr\{D|\tilde{c}_i\} = F(\hat{c}), \qquad \Pr\{M_i|\tilde{c}_i\} = 1 - F(\hat{c}), \qquad \Pr\{D|\tilde{c}_i\} + \Pr\{M_i|\tilde{c}_i\} = 1,$$

and that, for $\tilde{c}_i > \hat{c}$,

$$\Pr\{D|\tilde{c}_i\} = 0, \qquad \Pr\{M_i|\tilde{c}_i\} = 1 - F(\tilde{c}_i), \qquad \Pr\{D|\tilde{c}_i\} + \Pr\{M_i|\tilde{c}_i\} = 1 - F(\tilde{c}_i).$$

Therefore, for $c_i > \hat{c}$,

$$\begin{aligned} t_i(c_i) &= [\pi^d - c_i]0 + [\pi^m - c_i][1 - F(c_i)] - \int_{c_i}^{\bar{x}} [1 - F(\tilde{c}_i)]d\tilde{c}_i \\ &= [\pi^m - c_i][1 - F(c_i)] + c_i[1 - F(c_i)] - \int_{c_i}^{\bar{x}} \tilde{c}_i[1 - F(\tilde{c}_i)]d\tilde{c}_i \\ &= \int_{c_i}^{\bar{x}} [\pi^m - \tilde{c}_i]f(\tilde{c}_i)]d\tilde{c}_i \\ &> 0. \end{aligned} \tag{8}$$

For $c_i \leq \hat{c}$,

$$\begin{aligned}
t_i(c_i) &= [\pi^d - c_i]F(\hat{c}) + [\pi^m - c_i][1 - F(\hat{c})] \\
&\quad - \int_{c_i}^{\hat{c}} 1 d\tilde{c}_i - \int_{\hat{c}}^{\bar{x}} [1 - F(\tilde{c}_i)] d\tilde{c}_i \\
&= [\pi^d - c_i]F(\hat{c}) + [\pi^m - c_i][1 - F(\hat{c})] \\
&\quad -(\hat{c} - c_i) - \left\{ -\hat{c}[1 - F(\hat{c})] + \int_{\hat{c}}^{\bar{x}} \tilde{c}_i f(\tilde{c}_i)] d\tilde{c}_i \right\} \\
&= \pi^d F(\hat{c}) + \pi^m [1 - F(\hat{c})] - \hat{c}F(\hat{c}) - \int_{\hat{c}}^{\bar{x}} \tilde{c}_i f(\tilde{c}_i) d\tilde{c}_i \\
&= \int_{\underline{x}}^{\hat{c}} [\pi^d - \hat{c}] f(\tilde{c}_i) d\tilde{c}_i + \int_{\hat{c}}^{\bar{x}} [\pi^m - \tilde{c}_i] f(\tilde{c}_i) d\tilde{c}_i.
\end{aligned}$$

Note that $\pi^d$, $\pi^m$, and $\hat{c}$ are all independent of the p.d.f. $f(\cdot)$. Note also that $\pi^d - \hat{c} < 0$, and that $\pi^m - c > 0$ for all $c \in [\underline{x}, \bar{x}]$. Therefore, if the density on $[\hat{c}, \bar{x}]$ is sufficiently high and the density on $[\underline{x}, \hat{c}]$ is sufficiently low, we must have $t_i(c_i) > 0$ for $c_i \leq \hat{c}$. ■

# IV ACKNOWLEDGEMENTS

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# Notes

[1]Privatization is another case where the issue of market design arises. The question may then be whether a public enterprise should be sold off as a single firm or as several firms.

[2]See also McGuire and Riordan (1995), Riordan (1996), and Wolinsky (1997). Chen and Li (1999) contains a study where firms' information is about a pure common value.

[3]Following the tradition of this literature, we adopt a normative approach and study optimal mechanisms. An alternative to the normative approach, beginning with the work of Wilson (1979) on share auctions, is to conduct positive analysis of equilibrium in a specific auction format with endogenous number of awards. See, for instance, Anton and Yao (1989, 1992) for split-award auctions and their use in government procurement.

[4]Krishna and Perry (2000) have used this objective function in their study of efficient mechanisms where agents' private information may be multi-dimensional.

[5]Full efficiency can always be achieved if government revenue is not constrained. When the government faces the no-deficit constraint and full efficiency cannot be obtained, we assume that the government aims to achieve the highest level of efficiency under the constraint that its revenue is non-negative.

[6]In Dana and Spier (1994), the government assigns a higher weight on government revenue than on other components of social welfare, taking into account the taxation distortion in raising public funds. If we use the same objective function as in Dana and Spier, monopoly licenses would be issued more often, similar to their result. This is because under monopoly, firm profits are higher, and hence more government revenues can be raised.

[7]Here, we implicitly assume that a firm is obligated to provide the service (product) if it is assigned a production license; or, alternatively, the firm learns its cost realization only after it has made the production decision.

# References


[1] ANTON, J.J. and YAO, D.A. (1992). Coordination in Split Award Auctions. *Quarterly Journal of Economics*, **107**, 681-707.

[2] —— and —— (1989). Split Awards, Procurement and Innovation. *RAND Journal of Economics*, **20**, 538-552.

[3] BIKHCHANDANI, S. (1988). Reputation in Repeated Second-Price Auctions. *Journal of Economic Theory*, **46**, 97-119.

[4] CHEN, Y. (2000). Strategic Bidding by Potential Competitors: Will Monopoly Persist. *Journal of Industrial Economics*, **XLVIII**, pp. 161-175.

[5] —— and LI, C. (1999). Designing a Market under Unknown Common Values. Working paper, University of Colorado at Boulder.

[6] DANA, J.D and SPIER, K.E. (1994). Designing a Private Industry: Government Auctions with Endogenous Market Structure. *Journal of Public Economics*, **53**, 127-147.

[7] DEPARTMENT OF DEFENSE, U.S.A. (1998). Contract. Online newsletter, http://www.defenselink.mil/contracts/1998/c03121998_ct108-98.html

[8] —— (2005). Defense Procurement and Acquisition Policy. http://www.acq.osd.mil/dpap/contractpricing/vol1chap4.htm#4.5

[9] DEPARTMENT OF THE NAVY, Office of the General Consel, U.S.A. (1998). REVIEW OF RECENT GAO BID PROTEST DECISIONS by James H. Haag. http://ogc.navy.mil/ogcwww/briefbank/020228.asp

[10] FEDERAL COMMUNICATIONS COMMISSION: Connecting the Globe, Section VII (1999). http://www.fcc.gov/connectglobe/sec7.html.

[11] KRISHNA, K. (1993). Auctions with Endogenous Valuations: The Persistence of Monopoly Revisited. *American Economic Review*, **83**, 147-160.

[12] KRISHNA, V. and ROSENTHAL, R.W. (1996). Simultaneous Auctions with Synergies. *Games and Economic Behavior*, **17**, 1-31.

[13] —— and PERRY, M. (2000). Efficient Mechanism Design. Working paper, Pennsylvania State University and Hebrew University.

[14] MCGUIRE, T.G. and RIORDAN, M.H. (1995). Incomplete information and Optimal Market Structure: Public Purchases from Private Providers. *Journal of Public Economics*, **56**, 125-41.

[15] MYERSON, R. (1981). Optimal Auction Design. *Mathematics of Operations Research*, **6**, 58-73.

[16] RIORDAN, M. (1996). Contracting with Qualified Suppliers. *International Economic Review*, **37**, 115-128.

[17] —— and SAPPINGTON, D. (1987). Awarding Monopoly Franchises. *American Economic Review*, **77**, 375-387.

[18] ROSENTHAL, R.W. and WANG, R. (1996). Simultaneous Auctions with Synergies and Common Values. *Games and Economic Behavior*, **17**, 32-55.

[19] WILSON, R. (1979). Auctions of Shares. *Quarterly Journal of Economics*, **XCIII**, 675-679.

[20] WOLINSKY, A. (1997). Regulation of Duopoly: Managed Competition vs Regulated Monopolies. *Journal of Economics and Management Strategies*, **6**, 821-847.

Figure 1. The optimal allocation of licenses in $(c_1, c_2)$ space.

Figure 2. The optimal allocation of licenses in $(x_1, x_2)$ space.

Figure 3. The optimal allocation of licenses when $\alpha = 1$.
(The shady area indicates the duopoly-efficient but non-profitable region.)