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Risk Premia in the Sovereign Loan Market

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RISK PREMIUM IN THE SOVEREIGN LOAN MARKET

by

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March 1992

RISK PREMIA IN THE SOVEREIGN LOAN MARKET

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DISCUSSION PAPER NO. 850

Abstract

This paper offers an explanation of how risk premia emerge in the sovereign loan market. The economy is composed of countries which borrow each period from private banks. In the event of default, the bank imposes a penalty by seizing the country's overseas assets. The country may also choose to appeal to an international authority, such as the IMF, which determines whether the country must repay the loan or pay the penalty. This process takes one period to be completed and its outcome is stochastic. In an environment with a deterministic penalty to default, the supply of funds schedule is upward sloping. The interest rate on sovereign loans is greater than the prime rate charged on private loans, implying the existence of risk premia.

I am indebted to Beverly Lapham and Mick Devereux for their supervision and helpful comments. All errors remain my own.

I. INTRODUCTION

When sovereign loans are extended at a higher interest rate than the prime rate charged on private loans, risk premia are said to be present in the sovereign loan market. Risk premia in this market are widely observed. For example, in 1981 and 1983, the risk premium for developing country debt stood at 1% and 1.5%, respectively, over the LIBOR¹. In the same years, the LIBOR averaged 16.5% and 10%, respectively. This paper offers an explanation of how the presence of an international authority (such as the International Monetary Fund or the World Bank) modifies the sovereign lending environment and gives rise to risk premia in the sovereign loan market.

Since sovereign loans are non-enforceable contracts, no lending occurs unless there exists an effective penalty for defaulting on loans. With an effective penalty to default, contract enforceability is restored. This implies that, since the probability of default is zero, lending takes place at the same interest rate charged on private loans, and risk premia are absent from the sovereign loan market. Also, loans for which the penalty to default is not binding are not extended, since the probability of default on these loans is one, and there exists credit rationing. In this paper, the presence of an international authority in the sovereign loan market introduces some uncertainty

¹The London Interbank Offer Rate is the interest rate on Eurodollar deposits. The Eurodollar market is a dollar-based financial market outside the U.S.. The dollar-denominated deposits can be made either at a foreign bank or a foreign branch of a U.S. bank.

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Figure 1. The effect of the concentration of the *Agrobacterium* suspension on the transformation efficiency of *Agrobacterium* strains. The number of transformed cells was determined by the number of colonies obtained on the selective medium. The results are the mean of three independent experiments. Error bars represent standard deviation.

• **Prevalence** = the proportion of a population that has a disease at a particular point in time

1

Proof: We set $\hat{i}^s = \frac{1}{p}(r^2)$ and rewrite the uncertainty range supply equation as

$$\alpha p K_2(\theta, \hat{i}^s) = (1+R)^2 - (1-p)(1+r^2).$$

Substituting in the default range supply equation, equation (23), yields

$$\alpha(1+R-p)K_2(\theta, \hat{i}^s) = (1-p)(1+r^2). \quad (24f)$$

From above, the upper frontier equation was rewritten as

$$(1+\delta-p)K_2(\theta, \hat{i}^s) = \beta(1-p)(1+r^2). \quad (15e)$$

Combining equations (24f) and (15e), we obtain

$$\alpha\beta(1+\hat{r}^s)(1+R-p) = (1+r^2)(1+\delta-p) \text{ or}$$

$$\alpha\beta(1+\hat{r}^s) = (1+r^2) \text{ if } \delta = R.$$

Since $\alpha\beta < 1$, $\hat{r}^s > r^2$ and supply is discontinuous between the uncertainty and default ranges. QED

over the terms of repayment of a loan, giving rise to risk premia in the sovereign loan market.

The economy is composed of countries which borrow each period from private banks to import and export to produce. A country's output is a function of the loan it receives. In the event of default, the bank imposes a penalty by seizing the country's overseas assets and the penalty is also a function of the loan. The country may also choose to appeal to an international authority or to a set of procedures (such as the Paris Club or the London Club) which determines whether the country must repay the loan or pay the penalty. This process takes an additional period to be completed and its outcome is stochastic. This option can be thought of as a rescheduling clause in the contract. From 1978 to 1984, twenty-nine countries appeared before the Paris Club to negotiate 56 debt-rescheduling agreements². Between 1984 and 1990, twenty-one African, Latin American, and East European countries rescheduled in the Paris Club and in 1989 alone, the Club concluded twenty-four rescheduling agreements³.

An environment with single-period loans and a deterministic penalty is examined. As a result of the IMF's mediating role, three sets of loan contracts arise: a repayment range in which the country repays its loan, an uncertainty range in which it appeals to the IMF, and a default range in which it defaults. The concavity

²See A. Riefel, "The Role of the Paris Club in Managing Debt Problems", 1985.

³"Multilateral Official Debt Rescheduling, Recent Experience", World Economic and Financial Surveys, November 1990, IMF, Washington D.C.

of the penalty function implies that the supply of funds schedule is upward sloping in the uncertainty range. If the demand function for loans intersects the supply function in its upward sloping range, then the interest rate on sovereign loans is greater than the prime rate charged on private loans, implying the existence of risk premia in the sovereign loan market.

Evidence of risk premia on the sovereign loan market is considerable. The following chart depicts the average U.S. prime rate, the six-month Eurodollar rate, and the spread between the sovereign loan rate and the Eurodollar rate.⁴

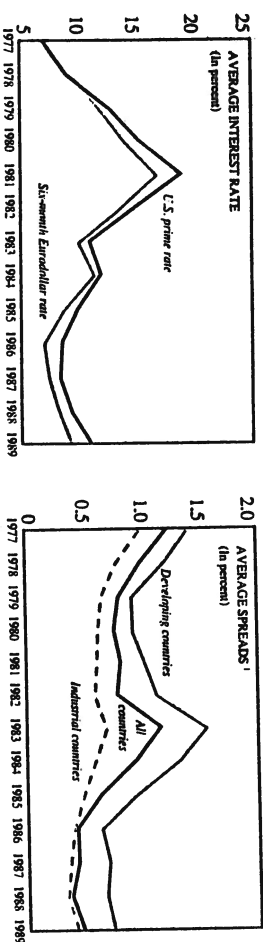


Chart 1.

The spread between the sovereign loan rate and the Eurodollar rate⁵

⁴"International Capital Markets, Developments and Prospects", World Economic and Financial Surveys, April 1990, IMF, Washington, D.C..

⁵Perhaps a clarifying comment is useful at this point. The interest rate on Eurodollar deposits in chart 1 is the LIBOR, the London Interbank Offer Rate. Since the Eurodollar market is less regulated than the U.S. financial market, and therefore riskier, the LIBOR is usually higher than the U.S. deposit rate. The U.S. prime rate, which is the lending rate on private loans, is also higher than the U.S. deposit rate. The U.S. prime rate can either be slightly higher or lower than the LIBOR, as shown in chart 1. Sovereign loans are usually quoted at a premium over the LIBOR. Chart 1 shows the average

Proposition A5: Let the penalty function be given by $K(\theta, 1) = g(\theta)l^\beta$, where $\beta < 1$. Let R be the interest rate at which the lower frontier and the repayment range supply function intersect and let r^1 be the rate at which $l^1(R)$ and the uncertainty range supply intersect, i.e. $l^1(R) = l_p^s(r^1)$. If $\delta = R$, $r^1 > R$ and supply is discontinuous between the repayment and uncertainty ranges.

Proof: We set $l^1(R) = l_p^s(r^1)$ and rewrite the uncertainty range supply equation as

$$\alpha p K_2(\theta, l^1(R)) = (1+R)^2 - (1-p)(1+r^1). \quad (24e)$$

From above, the lower frontier equation was rewritten as

$$\alpha p K_2(\theta, l^1(R)) = \alpha \beta (\delta + p)(1+R). \quad (13e)$$

Combining equations (24e) and (13e), we obtain

$$(1+R)^2 - (1-p)(1+r^1) = \alpha \beta (\delta + p)(1+R) \text{ or} \\ (1-p)(1+r^1) = (1+R)[1 + R - \alpha \beta (\delta + p)].$$

Since $\alpha \beta < 1$, $(\alpha \beta - 1)p < (1 - \alpha \beta)R$.

If $\delta = R$, $(\alpha \beta - 1)p < (R - \alpha \beta \delta)$ and $\alpha \beta p + (1-p) < (1+R - \alpha \beta \delta)$.

Therefore, $(1-p) < [1 + R - \alpha \beta (\delta + p)]$ and $r^1 > R$. Supply is discontinuous between the repayment and uncertainty ranges. QED

Proposition A6: Let the penalty function be given by $K(\theta, 1) = g(\theta)l^\beta$, where $\beta < 1$. Let \hat{r}^s be the rate at which the upper frontier and the default range supply intersect, i.e. $\hat{l}^s = l^2(\hat{r}^s)$ and let r^2 be the rate at which \hat{l}^s and the uncertainty range supply intersect, i.e. $\hat{l}^s = l_p^s(r^2)$. If $\delta = R$, $\hat{r}^s > r^2$ and supply is discontinuous between the uncertainty and default ranges.

$$\alpha\beta(1+R-p) = (1+\delta-p).$$

Proof: a) the lower frontier

Rewrite the lower frontier equation, equation (13), for $r = R$:

$$pK(\theta, 1^1(R)) = (\delta+p)(1+R)1^1(R).$$

Since $K_2(\theta, 1) = \beta K(\theta, 1)/1$, this can be rewritten as

$$\alpha p K_2(\theta, 1^1(R)) = \alpha\beta(\delta+p)(1+R). \quad (13e)$$

If $\alpha\beta(\delta+p) = (R+p)$, it can also be written as

$$\alpha p K_2(\theta, 1^1(R)) = (1+R)^2 - (1-p)(1+R).$$

Note that this is the uncertainty range supply equation, equation (24), for $r = R$.

Thus, if $\alpha\beta(\delta+p) = (R+p)$, $1^1(R) = 1_p^s(R)$ and supply is continuous between the repayment and uncertainty ranges.

b) the upper frontier

Using the same method, we set $1^2(\hat{r}^s) = \hat{r}^s$ and rewrite equation (15), the upper frontier equation, as

$$(1+\delta-p) K(\theta, \hat{r}^s) = (1-p) (1+\hat{r}^s) \hat{r}^s.$$

Since $K_2(\theta, 1) = \beta K(\theta, 1)/1$, this equation can also be written as

$$(1+\delta-p) K_2(\theta, \hat{r}^s) = \beta(1-p) (1+\hat{r}^s). \quad (15e)$$

If $\alpha\beta(1+R-p) = (1+\delta-p)$, we can also write $\alpha(1+R-p) K_2(\theta, \hat{r}^s) = (1-p) (1+\hat{r}^s)$. Combining this equation with the default range supply equation, equation (23), we obtain

$$\alpha p K_2(\theta, \hat{r}^s) = (1+R)^2 - (1-p)(1+\hat{r}^s),$$

the uncertainty range supply equation, equation (24), for $r = \hat{r}^s$. Therefore, if $\alpha\beta(1+R-p) = (1+\delta-p)$, $1^2(\hat{r}^s) = \hat{r}^s = 1_p^s(\hat{r}^s)$ and supply is continuous between the uncertainty and default ranges. QED

The existence of risk premia has been difficult to explain in the sovereign debt literature. Other authors have assumed an infinitely elastic supply of loans schedule up to the level of a maximum loan, where it becomes infinitely inelastic. Moreover, models that treat explicitly of an upward sloping supply of funds schedule rely on an uncertain penalty (see for example Krugman (1985) and Aizenman (1986)) or on stochastic output (see Sachs(1983)) to obtain that result. On the other hand, Detragiache (1989) exploits a signalling model of sovereign debt, with a deterministic penalty to default, which enables banks to screen between good and bad borrowers through a risk premium; however, screening takes place in the first period of this infinite-horizon model and the risk premium disappears thereafter. Finally, Cole (1989) develops a model where the borrower's type can eventually switch, i.e. a bad borrower can become a good one once sufficient stability has been restored in the country's regime, however, there again, the borrower's "new" type is known to the banks and the risk premium therefore absent.

This model shows how risk premia will be part of the equilibrium sovereign loan contract when the penalty to default and output are non-stochastic. The paper is organized as follows. The model is outlined in section II and the demand for loans and supply of loans are analyzed in sections III and IV respectively. Equilibrium values are solved for in section V. The main result of this section is the presence of risk premia in equilibrium loan

spread between the sovereign loan rate and the LIBOR. This spread is used as a proxy for the spread between the sovereign loan rate and the U.S. prime rate. The risk premium varies over time and as a proportion of the LIBOR.

contracts. Finally, section VI offers concluding remarks to the paper and discusses possible extensions of the model.

II. THE ECONOMY

In this economy, countries borrow to import and import to produce. A final consumption good is produced with a nonstochastic technology given by $f(l_t)$, where l_t is the sovereign loan at time t extended at rate r_t .

$$f(0) = 0, \text{ and } f'(0) = \infty \quad (1a)$$

$$f''(l_t) < 0 < f'(l_t) \quad (1b)$$

There is a single representative consumer in the country, with preferences over the final good ordered by the following utility function.

$$U = \sum_{t=0}^{\infty} u(c_t) / (1+\delta)^t \quad (2)$$

where $u(c_t) = c_t$ and c_t is consumption at time t .

The consumption good is not storable. Note that consumers are risk neutral and do not borrow for consumption smoothing purposes. Their exogenous discount rate is given by δ , $\delta > 0$, and the price of the consumption good in period t is denoted p_t . Therefore, focusing on an interior solution, in equilibrium

$$p_t/p_{t+1} = (1+\delta).$$

The penalty to default is the seizure by the banks of the country's overseas assets : bank accounts, direct foreign investments, ships, aircraft, etc. The penalty a country incurs for defaulting is given by

$K(\theta, l_t)$, where θ is the aggregate value of the country's overseas assets.

Proof: We set $\hat{l}^d = l_p^d(r^1)$ and rewrite the uncertainty range demand equation, equation (18), as

$$(1-p)(1+r^1) = (1+\delta)^2 f'(\hat{l}^d) - pK_2(\theta, \hat{l}^d).$$

Substituting in the default range demand equation, equation (17), yields

$$(1-p)(1+r^1) = (1+\delta-p)K_2(\theta, \hat{l}^d). \quad (18d)$$

Also, we set $\hat{l}^2(r^2) = \hat{l}^d$ and write the upper frontier equation, equation (15), as

$$(1-p+\delta)K(\theta, \hat{l}^d) = (1-p)(1+r^2)\hat{l}^d.$$

Since $K_2(\theta, l) = \beta K(\theta, l)/l$, this can be rewritten as

$$(1-p+\delta)K_2(\theta, \hat{l}^d) = \beta(1-p)(1+r^2). \quad (15d)$$

Combining equations (18d) and (15d) yields

$$(1+r^1) = \beta(1+r^2).$$

Therefore, $r^2 > r^1$ and demand is a correspondance between the uncertainty and default ranges. QED

In what follows, parameters for which the supply function is continuous across ranges are derived, given a specific form of the penalty function.

Proposition A4: Let the penalty function be given by $K(\theta, l) = g(\theta)l^\beta$, where $\beta < 1$. Let R be the interest rate at which the lower frontier and the repayment range supply function intersect and let \hat{r}^s be the rate at which the upper frontier and the default range supply intersect, i.e. $\hat{l}^s = l^2(\hat{r}^s)$. Then supply is continuous between the repayment and uncertainty ranges for

$$\alpha\beta(\delta+p) = (R+p)$$

and continuous between the uncertainty and default ranges for

Proposition A2: Let the penalty function be given by $K(\theta, 1) = g(\theta)l^\beta$, where $\beta < 1$. Let r^1 be the interest rate at which the lower frontier and the uncertainty range demand intersect, i.e. $l^1(r^1) = l_p^d(r^1)$, and let r^2 be the rate at which the repayment range demand and $l^1(r^1)$ intersect, i.e. $l^1(r^1) = l^d(r^2)$. Then $r^1 > r^2$ and demand is a correspondance between the repayment and uncertainty ranges.

Proof: Since $K_2(\theta, 1) = \beta K(\theta, 1)/1$, we set $l^1(r^1) = l_p^d(r^1)$ and rewrite the uncertainty range demand equation, equation (18), as

$$(1+\delta)^2 f'(l^1(r^1)) = [(1-p) + \beta(\delta+p)] (1+r^1). \quad (18c)$$

We also set $l^1(r^1) = l^d(r^2)$ and rewrite the repayment range demand, equation (16), as

$$(1+\delta) f'(l^1(r^1)) = (1+r^2). \quad (16c)$$

Combining equations (16c) and (18c) yields

$$(1+\delta)(1+r^2) = [(1-p) + \beta(\delta+p)] (1+r^1).$$

Since $\delta(1-\beta) > p(\beta-1)$,

$$(1+\delta) > [(1-p) + \beta(\delta+p)].$$

Therefore, $r^1 > r^2$ and demand is a correspondance between the repayment and uncertainty ranges. QED

Proposition A3: Let the penalty function be given by $K(\theta, 1) = g(\theta)l^\beta$, where $\beta < 1$. Let r^2 be the interest rate at which the upper frontier and the default range demand intersect, i.e. $l^2(r^2) = \hat{l}^d$ and let r^1 be the rate at which the uncertainty range demand and \hat{l}^d intersect, i.e. $\hat{l}^d = l_p^d(r^1)$. Then $r^1 < r^2$ and demand is a correspondance between the uncertainty and default ranges.

$$K_2(\theta, l_t) > 0 > K_{22}(\theta, l_t) \quad (3)$$

$$K_1(\theta, l_t) > 0 \text{ and } K_{21}(\theta, l_t) > 0. \quad (4)$$

$$K(\theta, 0) = 0. \quad (5)$$

θ measures the effectiveness of the penalty to default. θ is exogenous and time invariant. This specification is similar to that of Bulow and Rugoff (1989) where banks seize the country's traded output in case of default and the value of output is given by the terms of trade.

$K(\theta, l_t)$ is concave in l_t , as some overseas assets, e.g. ships and aircraft, increase less than proportionally in l_t . In addition, there are costs to seizure so that banks collect only a fraction α of the assets they have seized. In the event of default, banks collect $\alpha K(\theta, l_t)$, where $0 < \alpha < 1$.

Following Sachs (1984), we focus on environments where debt repayment is always feasible, therefore the production and penalty technologies are restricted such that

$$f(1) > \max [K(\theta, 1), (1+r)1] \quad \forall \theta, \forall 1. \quad (6)$$

In other words, the only type of default considered is debt repudiation.

The following condition is imposed to ensure, as will be shown below, that second order conditions for loan demand are satisfied.

$$(1+\delta)f''(l_t) - K_{22}(\theta, l_t) < 0 \quad \forall t \quad (7)$$

The type of contract available to debtor countries and banks is as follows : for a one-period loan extended in period t , the debtor country chooses to either

- 1) repay in period $t+1$,
- 2) default in period $t+1$, or
- 3) take its contract to an international organization, let this be

the IMF. The IMF evaluates the contract for one period and,

dictates whether the country must repay or default in period $t+2$.

The debtor country's decision between these three options is endogenous. If the country chooses to default, lenders impose the penalty contemporaneously on the repudiating country. If a country chooses to appeal, the IMF's decision process is described as follows. The IMF's decision variable at time $t+2$ is denoted by d_{t+2} . The variable d_{t+2} can either take value 0, in which case the debtor country must repay l_t , or value 1, in which case the country is allowed to default. The outcome of this third option is exogenous, i.e. neither the bank nor the country can influence the mediating international organization's decision. This option can be thought of as a rescheduling clause in the contract. The distribution of d_t is known: d_t takes value zero with probability $(1-p)$ and value one with probability p .

When the country chooses to approach the IMF, the penalty for not complying with the random outcome of this third option is the exclusion from future borrowing. Clearly, due to its technical capacity and data availability, the IMF can coordinate such exclusion from international loan markets. Individual banks however, do not necessarily have the ability to coordinate their actions with that of other banks to exclude a country from capital markets, nor do they have the authority the IMF has in imposing this exclusion. This ensures that the probability of compliance with the IMF's decision is always one. Note, however, that the probability of repayment is smaller than one in the event of appeal because the IMF's decision is stochastic.

Consider the sovereign lending market at time t . In period t , a

APPENDIX

Proposition A1: For penalty functions $K(\theta, 1)$ which satisfy the following condition

$$2[K(\theta, 1)/1 - K_2(\theta, 1)] + K_{22}(\theta, 1) \geq 0,$$

the upper and lower frontiers of the uncertainty range are convex. In particular, if the penalty function is given by $K = g(\theta)l^\beta$, where $\beta < 1$, the upper and lower frontiers of the uncertainty range are convex.

Proof: a) the lower frontier

The sign of (d^2l / dr^2) hinges on the sign of

$$2(\delta + p) - [pK_{22}(\theta, 1)dl/dr] \text{ or} \quad 2[K(\theta, 1)/l^1(r) - K_2(\theta, 1)] + K_{22}(\theta, 1)l^1(r). \quad (13'a)$$

If $K(\theta, 1)$ is of the form $g(\theta)l^\beta$, where $\beta < 1$, then

$$K_2(\theta, 1) = \beta K(\theta, 1)/l^1(r) \text{ and} \\ K_{22}(\theta, 1)l^1(r) = (\beta - 1)\beta K(\theta, 1)/l^1(r).$$

Equation (13'a) therefore reduces to

$$(1 - \beta)(2 - \beta)K(\theta, 1)/l^1(r) \geq 0.$$

Therefore, the lower frontier of the uncertainty range is convex.

b) the upper frontier

Once more, the sign of (d^2l / dr^2) is given by equation (13'b) $2[K(\theta, 1)/l^2(r) - K_2(\theta, 1)] + K_{22}(\theta, 1)l^2(r)$. The particular form of $K(\theta, 1)$ ensures it is zero or positive.

Therefore, the upper frontier of the uncertainty range is convex.

QED

VI. CONCLUSION AND EXTENSIONS

The analysis has shown that an environment with a deterministic penalty to default, where countries have the option of appealing to an international organization and possibly reschedule their loans, gives rise to risk premia in the sovereign loan market. The three options a country considers of repaying, appealing and defaulting, imply the existence of three sets of loan contracts, namely a repayment range, an uncertainty range and a default range. These ranges are time invariant.

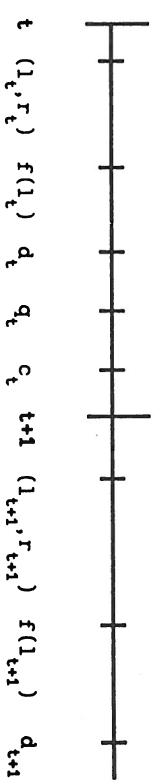
A concave penalty function yields an upward sloping supply of loans schedule within the uncertainty range. When the penalty to default is proportional to output, an equilibrium in the uncertainty range exists, given a certain range of parameters, and the equilibrium interest rate charged on sovereign loans exceeds the prime rate charged on private loans. Hence, risk premia are part of the equilibrium contract.

When the aggregate value of the country's overseas assets is stochastic and revealed in each period after the equilibrium loan is extended, the penalty to default is unknown a period in advance. In that case, the upper and lower frontiers of the uncertainty range may vary from period to period. In the context of multi-period loans, this additional uncertainty may give rise to buybacks.

loan contract (r_t, l_t) is determined on the basis of θ , the aggregate value of the country's overseas assets. Then $f(l_t)$ is produced. Then the country chooses between its three options with its choice denoted q_t . q_t takes value 0 if the country decides to repay l_{t-1} , 1 if the country defaults, and p if the country appeals to the IMF, where p denotes the probability that the IMF will choose default. The variable q_t is chosen to maximize the country's discounted consumption stream.

In addition, if the country appealed at time $t-1$, i.e. $q_{t-1} = p$, then the country must abide by the IMF's decision, d_t . If $d_t = 0$, the country repays $(1+r_{t-2})l_{t-2}$ in period t while if $d_t = 1$, it pays the penalty to default, i.e. $K(\theta, l_{t-2})$, in period t . Were the country not to comply with the IMF's decision, it would be excluded from future borrowing. Therefore the country complies with probability one.

The chronology of events can be summarized by the following time line:



The repayment, uncertainty and default ranges are defined as the three sets of loan contracts for which the country repays its loan, appeals to the IMF, and defaults, respectively. These ranges are derived in the following section and it is demonstrated that they are time invariant. This implies that q_t is constant

throughout all periods, i.e. $q \in \{0,1,p\}$. Moreover, given the supply of and demand for loans schedules, this implies a time invariant equilibrium level of the loan, l , and a time invariant equilibrium interest rate, r . Sections III and IV, by deriving and analyzing these schedules, clarify these points. Although we will establish that l_t and r_t are time invariant, time subscripts remain present in the following derivation in order to allow the reader to index the loans by time periods.

Consumption may not be time invariant :

If $q = p$, then $c_t = f(l_t) - K(\theta, l_{t-2})$ with probability p

and $c_t = f(l_t) - (1+r_{t-2})l_{t-2}$ with probability $(1-p)$.

If $q = 1$, then $c_t = f(l_t) - K(\theta, l_{t-1})$.

If $q = 0$, then $c_t = f(l_t) - (1+r_{t-1})l_{t-1}$.

Note that even if l_t and r_t are time invariant, c_t is in general not time invariant since d_t is random. The distribution of c_t is time invariant. Also, since

$$f(l_t) > \max [K(\theta, l_t), (1+r_t)l_t], \\ c_t > 0 \quad \forall t.$$

$$\frac{K(\theta, l^2(r^*))}{l^2(r^*)} < \frac{K(\theta, l^1)}{l^1} < \frac{K(\theta, l^1(r^*))}{l^1(r^*)} \\ \text{or } \frac{(1-p)(1+r^*)}{(1-p+\delta)} < \frac{K(\theta, l^1)}{l^1} < \frac{(\delta+p)(1+r^*)}{p}$$

Or, dividing by $(1-p)(1+r^*)$ and substituting in the uncertainty range supply function, equation (24),

$$\frac{1}{(1-p+\delta)} < \frac{K(\theta, l^1)}{[(1+r)^2 - \alpha p K_2(\theta, l^1)]l^1} < \frac{(\delta+p)}{(1-p)p}$$

Or, since $K_2(\theta, l) = \beta K(\theta, l)/l$,

$$\frac{1}{(1-p+\delta + \alpha \beta p)} < \frac{K(\theta, l^1)}{(1+r)^2 l^1} < \frac{(\delta+p)}{(p-p^2 + \alpha \beta p(\delta+p))} \quad (11)$$

By equation (1), the left hand side of equation (11) holds if

$$g(\theta) > \left[\beta(1+\delta)^2 / (1-p+\delta+p\beta) \right] = \bar{A} \quad (111)$$

Similarly, by equation (1), the right hand side of (11) holds if

$$g(\theta) < \left[\beta(\delta+p)(1+\delta)^2 / [(1-p)p + p\beta(\delta+p)] \right] = \bar{A} \quad (112)$$

Equations (111) and (112) define a range for $g(\theta)$. This range is

$$\bar{A} < g(\theta) < \bar{A}, \text{ and } \bar{A} < \bar{A} \text{ holds for } \delta > 0.$$

Therefore, for $\bar{A} < g(\theta) < \bar{A}$, an equilibrium in the uncertainty range exists. QED

Equilibrium in the uncertainty range is illustrated below.

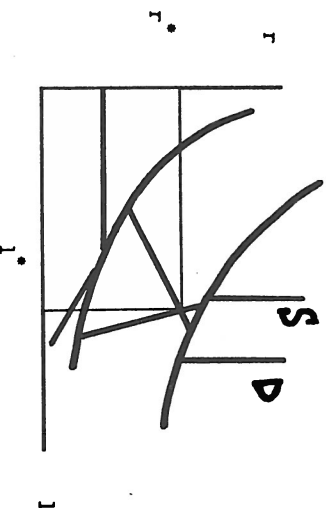


Figure 4. Equilibrium

Thus, for $K(\theta, 1) \leq f(1)$, r^* must be greater than R . QED
This proposition implies that risk premia are part of the equilibrium contract within the uncertainty range.

Proposition 5: Let the penalty and production functions be given by $K(\theta, 1) = g(\theta)^{1/\beta}$ and $f(1) = 1^\beta$ respectively, where $\beta < 1$ and $g(\theta) < \bar{A}$. If $\bar{A} < g(\theta) < \bar{A}$, where

$$\bar{\bar{A}} = \left[\beta(1+\delta)^2 / (1-p+\delta+p\beta) \right],$$

$$\bar{A} = \left[\beta(\delta+p)(1+\delta)^2 / [(1-p)p + p\beta(\delta+p)] \right],$$

and $\bar{A} < \bar{\bar{A}} \quad \forall \quad \beta, \delta, p,$

then an equilibrium in the uncertainty range exists.

Proof: First, 1^* is solved for using the equilibrium condition, equation (25'). Rewriting equation (25') given the functional forms of $f(1)$ and $K(\theta, 1)$ yields

$$(1+\delta)^2 \beta_1^{* \beta-1} - (1-\alpha) p \beta g(\theta)_1^{* \beta-1} = (1+R)^2.$$

Therefore, 1^* and $K(\theta, 1^*)/(1+R)^2 1^*$ are given by

$$I^* = \left[(1+R)^2 / [(1+\delta)^2 - (1-\alpha)pg(\theta)]\beta \right]^{1/\beta-1}$$

$$K(\theta, 1)^* / 1^* = g(\theta) \left[(1+R)^2 / [(1+\delta)^2 - (1-\alpha)pg(\theta)]\beta \right]$$

$$K(\theta, 1^*) / (1+R)^2 1^* = g(\theta) / [(1+\delta)^2 - (1-\alpha)pg(\theta)]_B. \quad (1)$$

Next, we calculate the upper and lower frontiers of the uncertainty range for $r = r^*$. If $l^1(r^*) < l^* < l^2(r^*)$ holds, then an uncertainty range equilibrium obtains.

By the strict concavity of $K(\theta, 1)$, this is equivalent to

The following is a section of the game tree played by banks, the country and the IMF. It summarizes the three options facing the country:



information sets, respectively. The first payoffs are those of the bank, and the second those of the country. The IMF does not derive payoffs from what it plays. It plays $d = 0$ with probability $(1-p)$ and $d = 1$ with probability p .

Turning to the role played by banks in this economy, it is assumed banks are risk neutral. We observe, for small banks, that managers and owners are identical so that risk neutrality obtains. For large banks, managers are distinct from ownership. However, managers' compensation schemes are often designed to make both their interests coincide. To assume risk neutrality for large banks

is therefore not significantly restrictive.

Commercial banks are price-takers: no individual bank or group of banks can significantly influence the interest rate in the market for sovereign loans. Banks choose the levels of sovereign loans they extend to maximize their profits. The marginal cost to issue a loan is zero. Alternatively, banks can earn an interest rate R on their funds, where R is the prime rate. Hence, the bank's opportunity cost to issue a loan is the prime rate.

We examine a continuum of banks with measure one. The aggregate loan, l , is given by

$$l = \int_0^1 l_i d_i,$$

since banks are uniformly distributed over the unit interval. Under symmetry, $l_i = l$ $\forall i$.

Under symmetry, therefore, the following must hold:

$$l \int_0^1 d_i = l = 1.$$

This enables us to consider the supply of funds of the representative bank as the aggregate supply of funds.

III. THE UNCERTAINTY RANGE AND THE DEMAND FOR LOANS

Let the equilibrium loan contract in each period be denoted by

(r, l) . Then in any period :

If the country repays, it pays $(1+r)l$.

If the country defaults, it pays $K(\theta, l)$.

If the country appeals, in the following period it expects to pay in present value terms $[(1-p)(1+r)l + pK(\theta, l)]/(1+\delta)$.

Following Bulow and Rogoff (1989), it is assumed that interest does not compound on the loan, and also the penalty, when $q = p$. The

Since $0 < \alpha < 1$ and $0 < p < 1$, then the maximum value of the function $p + (1-p)\alpha$ must be smaller than one, $\forall \alpha, \forall p$. Also, since $0 < \delta < 1$, then $(1+\delta)^2 > p + (1-p)\alpha$ must hold.

Therefore, $px(1-p) < p[(1+\delta)^2 - p]$ holds, implying $px(1-p) - p[(1+\delta)^2 - p] + [(1+\delta)^2 - p] < [(1+\delta)^2 - p]$.

Rearranging the left hand side yields

$$[(1+\delta)^2 - (1-\alpha)p] (1-p) < [(1+\delta)^2 - p],$$

which, from equation (26) implies $r_1^* > R$.

The remainder of the proof consists of showing that for $K(\theta, l) < f(1)$, the equilibrium value of r , r_2^* , is greater than r_1^* and therefore greater than R .

Suppose not, $r_2^* < r_1^*$.

If $K(\theta, l)$ and $f(1)$ are both concave and $K(\theta, l) < f(1)$, then $K_2(\theta, l) < f'(1)$. Let $K_2(\theta, l) = f'(1) - t(1)$, where $t(1) > 0$.

From the demand function, equation (18), we derive

$$(1+r_2^*)(1-p) = [(1+\delta)^2 - p][f'(l_2^*) + pt(l_2^*)] \quad (18b)$$

while from the equilibrium condition, equation (25'), we derive

$$[(1+\delta)^2 - (1-\alpha)p][f'(l_2^*) + (1-\alpha)pt(l_2^*)] = (1+\alpha)^2. \quad (25'b)$$

From equations (25'a) and (25'b), we know the following must hold

$$[(1+\delta)^2 - (1-\alpha)p][f'(l_1^*) - f'(l_2^*)] = (1-\alpha)pt(l_2^*). \quad (26')$$

Since $[(1+\delta)^2 - (1-\alpha)p] > 0$, and $t(1) > 0$, $[f'(l_1^*) - f'(l_2^*)] > 0$.

From equations (18a), (18b) and (26'), for $r_2^* < r_1^*$ to prevail, we require

$$\begin{aligned} (1-\alpha)[(1+\delta)^2 - p][f'(l_2^*) - f'(l_1^*)] &< [(1+\delta)^2 - (1-\alpha)p][f'(l_2^*) - f'(l_1^*)], \\ \text{or } (1-\alpha)[(1+\delta)^2 - p] &> [(1+\delta)^2 - (1-\alpha)p], \\ \text{or } \alpha &< 0, \end{aligned}$$

which is a contradiction.

V. EQUILIBRIUM

When an equilibrium exists within the uncertainty range, it is given in all periods by

$$l_p^d(r) = l_p^s(r). \quad (25)$$

Therefore, combining equations (18) and (24), the equilibrium level of loans, l^* , satisfies

$$(1+\delta)^2 f'(l^*) - (1-\alpha) p K_2(\theta, l^*) = (1+R)^2 \quad (25')$$

Substituting l^* in equation (18) or (24) yields r^* , the equilibrium interest rate at which l^* is extended. In particular, when there are no costs to seizure, $\alpha = 1$, and equation (25') reduces to $(1+\delta)^2 f'(l^*) = (1+R)^2$. Then, a higher prime rate will depress the level of l^* while a more impatient country will be granted a higher level of l^* .

Proposition 4: When an equilibrium exists within the uncertainty range, banks charge a higher interest rate on sovereign loans than the prime rate in all periods: $r^* > R$.

Proof: First consider the case where $K(\theta, 1) = f(1)$. In this case, from the uncertainty range demand function, equation (18),

$$[(1+\delta)^2 - p] f'(l_1^*) = (1-p)(1+r_1^*) \quad (18a)$$

where r_1^* denotes the equilibrium value of r when $K(\theta, 1) = f(1)$. From the equilibrium level of loans equation, equation (25'), we derive

$$[(1+\delta)^2 - (1-\alpha)p] f'(l_1^*) = (1+R)^2 \quad (25'a)$$

Setting $f'(l_1^*)$ in equation (18a) equal to $f'(l_1^*)$ in equation (25'a), we obtain

$$[(1+\delta)^2 - p] (1+R)^2 = [(1+\delta)^2 - (1-\alpha)p] (1-p) (1+r_1^*) \quad (26)$$

delay the country introduces when it appeals is costly for banks.

To maximize utility, the country chooses the least costly option from its viewpoint. The country selects

$$Q(1) = \min [(1+r)1, [(1-p)(1+r)1 + pK(\theta, 1)] / (1+\delta) , K(\theta, 1)].$$

1) If $Q(1)$ is equal to $(1+r)1$, then $q = 0$.

2) If $Q(1)$ is equal to $K(\theta, 1)$, then $q = 1$.

3) If $Q(1) = [(1-p)(1+r)1 + pK(\theta, 1)] / (1+\delta)$, then $q = p$.

The country's objective function can thus take one of three forms.

1) For $q = 0$, the objective function is

$$\max_{\{l_t\}} f(l_0) + \sum_{t=1}^{\infty} [f(l_t) - (1+r_{t-1})l_{t-1}] / (1+\delta)^t \quad (8)$$

where δ is the country's discount rate.

2) For $q = 1$, the objective function is

$$\max_{\{l_t\}} f(l_0) + \sum_{t=1}^{\infty} [f(l_t) - K(\theta, l_{t-1})] / (1+\delta)^t. \quad (9)$$

3) For $q = p$, the objective function is

$$\max_{\{l_t\}} f(l_0) + \sum_{t=2}^{\infty} [f(l_t) - pK(\theta, l_{t-2}) - (1-p)(1+r_{t-2})l_{t-2}] / (1+\delta)^t \quad (10)$$

The uncertainty range is defined as the set of contracts $(r, 1)$ for which there is a probability p the country will default and a probability $(1-p)$ the country will repay, i.e. it is the range of contracts for which $q = p$. The uncertainty range is calculated by the banks using the country's objective function. It will be shown

that the uncertainty range is the same in all periods.

The lower frontier of the uncertainty range in period zero, $l_0^1(r)$, is obtained by equating V_0^p , the expected payoff of always repaying ($q = p$), to V_0^0 , the payoff of always repaying ($q = 0$).

Setting $\rho = 1/(1+\delta)$ gives

$$V_0^p = f(l_0^1) + \rho f(l_1^1) + \sum_{t=2}^{\infty} \rho^t [f(l_t^1) - pK(\theta, l_{t-2}^1) - (1-p)(1+r)l_{t-2}^1] \quad (11)$$

Given that $l_t^1 = 1$ for all t ,

$$V_0^p = (1+p)f(1) + \rho^2 [f(1) - pK(\theta, 1) - (1-p)(1+r)]/(1-p) \quad (12)$$

$$V_0^0 = f(1) + \rho[f(1) - (1+r)]/(1-p) \quad (13)$$

Therefore $l_0^1(r)$ is implicitly defined by

$$pK(\theta, l_0^1(r)) = (\delta + p)(1+r)l_0^1(r) \quad (14)$$

The upper frontier of the uncertainty range in period zero, $l_0^2(r)$, is calculated by setting V_0^p to V_0^1 , where V_0^1 is the payoff of always defaulting ($q = 1$).

$$V_0^1 = f(1) + \rho[f(1) - K(\theta, 1)]/(1-p) \quad (15)$$

The following equation implicitly defines $l_0^2(r)$:

$$[1 + \delta - p]K(\theta, l_0^2(r)) = (1-p)(1+r)l_0^2(r) \quad (16)$$

The set of contracts $(r, l(r))$ such that $l_0^1(r) < l(r) < l_0^2(r)$ defines the uncertainty range in period zero.

When $q = p$, loan supply $l_p^s(r)$ is implicitly defined by

$$\alpha p K_2(\theta, l_p^s(r)) = (1+r)^2 - (1-p)(1+r) \quad (24)$$

Since $K_{22}(\theta, 1) < 0$, the supply schedule is upward sloping in the uncertainty range.

In general, the supply schedule will not be continuous across ranges. Parameters for which supply is continuous, given a specific form of the penalty function, are derived in the appendix. For $K(\theta, 1) = g(\theta)l^\beta$, where $\beta < 1$, supply is continuous between the repayment and uncertainty ranges for $\alpha g(\delta+p) = (R+p)$, which implies $\delta > R$; and continuous between the uncertainty and default ranges for $(1+R-p)\alpha g = (1+\delta-p)$, which implies $R > \delta$. For $\delta = R$, supply is discontinuous across ranges. The nature of the discontinuity is discussed in the appendix.

The supply of loans schedule is depicted below.

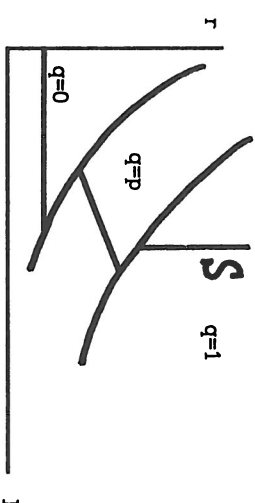


Figure 3. Supply

We have shown that when the penalty to default depends on known parameters, the supply of loans schedule is upward sloping in $r-1$ space within the uncertainty range, given that the penalty function is strictly concave in l .

IV. THE SUPPLY OF LOANS

The bank's objective function can take one of three forms.

1) For $q = 0$, the objective function is

$$\max_{\{l_t\}} \sum_{t=0}^{\infty} [(1+r_t)l_t - l_{t+1}]/(1+R)^t \quad (19)$$

2) For $q = 1$, the objective function is

$$\max_{\{l_t\}} \sum_{t=0}^{\infty} [\alpha K(\theta, l_t) - l_{t+1}]/(1+R)^t \quad (20)$$

3) For $q = p$, the objective function is

$$\max_{\{l_t\}} \sum_{t=0}^{\infty} [(1-p)(1+r_t)l_t - l_{t+2} + \alpha p K(\theta, l_t)]/(1+R)^t \quad (21)$$

where R = the prime rate

α = the fraction of seized assets accruing to banks when they

assess the default penalty.

Note that, since banks are risk neutral, they discount their profits by the prime interest rate R .

Loan supply for each set of loan contracts is derived below. When $q = 0$, loan supply is defined by

$$r = R \quad (22)$$

Note that the supply schedule is horizontal in the repayment range.

When $q = 1$, loan supply \hat{l}^s is implicitly defined by

$$\alpha K_2(\theta, \hat{l}^s) = (1+R) \quad (23)$$

Note that the supply schedule is vertical in the default range.

Proposition 1: The repayment, uncertainty, and default ranges are time invariant.

Proof: In period one, since the expected payoffs V_1^q , $q \in \{0, p, 1\}$, are given by

$$\begin{aligned} V_1^0 &= V_0^0 - (1+r_0)l_0 \text{ if } q_1 = 0, \\ V_1^1 &= V_0^1 - K(\theta, l_0) \text{ if } q_1 = 1, \\ V_1^p &= V_0^p - \rho [(1-p)(1+r_0)l_0 + pK(\theta, l_0)] \text{ if } q_1 = p. \end{aligned}$$

Then by setting $V_1^0 = V_1^p$ and $V_1^1 = V_1^p$, we recover the same equations we had solving for $l_0^1(r)$ and $l_0^2(r)$, respectively. Therefore, $l_1^1(r)$ equals $l_0^1(r)$ $\forall i \in \{1, 2\}$.

Thus, the repayment, uncertainty, and default ranges are the same in period zero and period one. By the same argument, setting $V_2^0 = V_2^p$ and $V_2^1 = V_2^p$ will yield $l_2^1(r) = l_1^1(r) = l_0^1(r)$ for $i \in \{1, 2\}$ and all three ranges in period two will be the same as in periods zero and one. Clearly, this holds for all periods. **QED**

Proposition 2: Ruling out negative prices, a necessary and sufficient condition for the existence of an uncertainty range is that the borrowing country's discount rate, δ , be greater than zero.

Proof: Since $K(\theta, l)$ is concave in l , for $l^1(r)$ to lie everywhere below $l^2(r)$ and therefore ensure the existence of an uncertainty range, we require

$$p(1-p) < (1+\delta-p)(\delta+p)$$

or $\delta > 0$, ruling out negative prices. **QED**

Proposition 3: The upper and lower limits of the uncertainty range are downward sloping in $r-1$ space.

Proof: a) the lower frontier $l^1(r)$

From equation (13), and since $K(\theta, 1)$ is concave in 1

$$\frac{d l^1(\delta+p) l^1(r)}{d r} = \frac{p K_2(\theta, 1) - p K(\theta, 1) / l^1(r)}{d r} < 0.$$

b) the upper frontier $l^2(r)$

From equation (15), and given that $K(\theta, 1)$ is concave in 1

$$\frac{d l^2(1-p) l^2(r)}{d r} = \frac{(1+\delta-p) [K_2(\theta, 1) - K(\theta, 1) / l^2(r)]}{d r} < 0. \text{ QED}$$

The upper and lower frontiers of the uncertainty range are convex for $K(\theta, 1)$ satisfying

$$2 [K(\theta, 1) / 1 - K_2(\theta, 1)] + K_{22}(\theta, 1) 1 > 0.$$

A proof of this proposition is provided in the appendix.

The uncertainty range at any time t is depicted below.

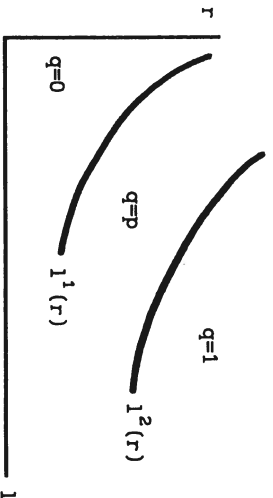


Figure 1. The uncertainty range

Loan demand for each set of loan contracts is derived below.

When $q = 0$, loan demand $l^d(r)$ is implicitly defined by

$$f'(l^d(r)) = (1+r) / (1+\delta). \quad (16)$$

Since $f''(1) < 0$, the demand schedule is negatively sloped in the repayment range.

When $q = 1$, loan demand \hat{l}^d is implicitly defined by

$$f'(\hat{l}^d) = K_2(\theta, \hat{l}^d) / (1+\delta). \quad (17)$$

Equation (7) ensures that the second order condition is satisfied.

Note that the demand schedule is vertical in the default range.

When $q = p$, loan demand $l_p^d(r)$ is implicitly defined by

$$(1+\delta)^2 f'(l_p^d) - p K_2(\theta, l_p^d) = (1-p)(1+r). \quad (18)$$

Equation (7) ensures that the second order condition is satisfied. Moreover, it guarantees a negatively sloped demand schedule within the uncertainty range. Finally, it implies that demand is steeper in the uncertainty range than in the repayment range.

In general, the demand schedule will not be continuous across ranges. Given a specific form of the penalty function, demand is a correspondence between the repayment and uncertainty ranges and between the uncertainty and default ranges. A proof of this proposition is provided in the appendix.

The demand for loans schedule is depicted below.

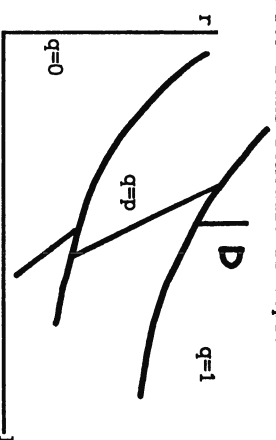


Figure 2. Demand

