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Increasing Pollution with Optimal Corrective Taxes

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Increasing Pollution
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by

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September 1990

Discussion Paper #795

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Abstract

In a simple two factor, two good, two person general equilibrium model with pollution, we indicate a case in which the introduction of optimal Pigovian corrective taxes results in an equilibrium with a higher level of pollution. Our argument turns on our two persons earning incomes from distinct input sources and having different tastes for the two goods, one of which causes pollution. The introduction of taxes causes one person's income to rise relative to the other person's and can induce a shift in total consumption toward the commodity causing pollution.

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Introduction

We examine the effects of introducing corrective taxes in an economy with pollution and no corrective taxes. Our model is a simple two good, two factor, two person general equilibrium system in which one output results in pollution and inhibits production of the other good. Our principal result is that the introduction of socially optimal (Pigovian) taxes in the untaxed economy can lead to a Pareto optimum with more pollution than was present in the corresponding no tax economy. Our result turns on their being two distinct consumers whose incomes arise from different factor incomes and whose tastes for the two outputs differ. The logic runs: the introduction of corrective taxes drives up the relative price of the good causing pollution and changes factor prices so as to increase the income of the person with a strong taste for the output that happens also to cause pollution. In some cases the result is a higher level of pollution persisting after the corrective taxes are introduced. We note again: the introduction of taxes causes one person's income to rise relatively more than the other (via Stolper-Samuelson like effects) and if that person is consumption-intensive in the output of the good which causes pollution, the economy can end up with a higher level of pollution after the corrective taxes have been introduced.

Our model is inspired by the smoky factory scenario. We have a steel industry producing steel and pollutants and farmers producing wheat and being visited by the pollutants. We draw on existing approaches to formalizing

these interactions. Thus in the absence of pollution effects the quantity of wheat Q produced with K units of machines and L units of labor might be $Q = f(K, L)$ with $\partial f/\partial K$ and $\partial f/\partial L > 0$. However with pollution level R , the production relationship might become $Q = g(K, L, R)$ with $\partial g/\partial K$ and $\partial g/\partial L > 0$ and $\partial g/\partial R < 0$. If pollution is caused by the production of another commodity such as steel, at say level Y , then we might have $Q = g(K, L, R(Y))$, with $dR/dY > 0$. The essential feature is given K and L fixed, more Y implies less Q . We make use of a particularly simple form of this relationship. We specialize to $Q = f(K, L) - \delta Y$ where Q is net output of wheat, and $\delta > 0$. We preserve the basic relation, given K and L , net output of wheat declines as Y increases, the increase in Y being associated with more pollution of the production process for wheat.

The Model

There are two commodities, steel and wheat subscripted s and w . The production of steel pollutes the soil via airborne emissions in such a way that each unit of steel produced "consumes" indirectly δ units of wheat. Each commodity employs labor L_i and machines K_i . The economy is endowed with L amount of labor and K of machines or capital. Outputs are

$$Q_w = f^w(K - K_s, L - L_s)$$

and

$$Q_s = f^s(K_s, L_s)$$

of wheat and steel respectively. We assume that the production functions f^w and f^s are constant returns to scale.

There are two consumers, denoted A and B , which each consume some of each output. Utility functions are

$$Q^A(C_s^A, C_s^A)$$

and

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for A and B respectively. We assume that A derives all income from capital (has income rK where r is the rental rate for machines) and taxes transferred and B derives all income from labor (has income vL where v is the wage rate) and taxes transferred. Taxes, when levied, are charged to steel producers for the damage they cause to wheat production (for the amount of wheat they "consume").

The price of wheat is the numeraire (equal to \$1.0). In terms of numeraire units, tax revenues are $t_K K_s + t_L L_s$, where t_K is the tax on capital in the steel industry and t_L is the tax on labor. γ is the fraction of tax revenue allotted to person A and $(1 - \gamma)$ the fraction to B. We will discuss optimal pollution taxes below. For now the taxes are arbitrary "wedges" on capital and labor driving up the cost of capital and labor to the steel (polluting) industry.

Equilibrium on the consumption side is defined by

$$\frac{u_s^A}{u_w^A} = \frac{u_s^B}{u_w^B} \quad (1)$$

$$\frac{u_s^A}{u_w^A} = p_s \quad (2)$$

$$c_s^A p_s + c_w^A = rK + \gamma \cdot [t_K K_s + t_L L_s] \quad (3)$$

where u_s^A is an abbreviation of $\partial u^A(c_s^A, c_w^A)/\partial c_s^A$ and so on. We can omit B's income equals expenditure equation because it is automatically satisfied,

given (3) and revenue equals costs in sectors s and w below (equations (4) and (5)).

Equilibrium on the production side is defined by

$$p_s f^s(K^s, L^s) = [r + t_k]K_s + [v + t_L]L_s \quad (4)$$

$$f^w(K - K^s, L - L^s) = (K - K_s)r + (L - L_s)v \quad (5)$$

$$\frac{f_K^s}{f_L^s} = \frac{f_K^w + t_k}{f_L^w + t_L} \quad (6)$$

$$\frac{f_K^w}{f_L^w} = \frac{r}{v} \quad (7)$$

where f_K^s is an abbreviation for $\partial f^s(K_s, L_s)/\partial K_s$ and so on. (4) and (5)

define the revenue equals expenditure relationships for industries s and w respectively.

The complete equilibrium system in the seven equations, (1) - (7) has seven unknowns, c_s^A , c_w^A , K^s , L^s , p_s , r and v .

Optimal Pollution Taxes

Optimal taxes satisfy the social planning problem

$$\begin{aligned} &\text{maximize} \quad u^A(c_s^A, c_w^A) \quad \text{subject to} \quad \bar{u}^B = u^B(f^s(K^s, L^s) - c_s^A, \\ &\{c_s^A, c_w^A, K^s, L^s\} \quad \quad \quad f^w(K - K^s, L - L^s) - \delta f^s(K^s, L^s) - c_w^A). \end{aligned}$$

The first order conditions reveal that at the optimum,

$$t_k = \delta f_{K^s}^s \quad (8)$$

$$t_L = \delta f_{L^s}^s \quad (9)$$

In our equilibrium system $t_k = t_L = 0$ corresponds to a economy with pollutants and no corrective taxes. We are accustomed to thinking that

$t_K > 0$ and $t_L > 0$ will lead to less pollution. Certainly for $0 < t_K \leq \delta f_K^s$ and $0 \leq t_L \leq \delta f_L^s$ we expect pollution to decline relative to the case of $t_K = t_L = 0$. Note first that corrective pollution taxes do not target pollution level δf^s per se but rather are instruments for augmenting u^A in our planning problem given \bar{u}^B . It is the maximization of welfare which corrective taxes contribute to, not the reduction or "minimization" of pollution. Though this is not our main point, it is a point worth keeping in mind. We turn now to an example involving more pollution under the optimal tax regime than exists at the corresponding zero tax regime.

An Example with More Pollution at the Optimally Taxed Equilibrium

Tractability invites us to work with Cobb-Douglas preferences and Leontief or fixed coefficient production functions. The former are attractive because expenditure on commodity i is a fixed proportion of income (linear income expenditure Engel curves). Leontief technologies appear restrictive but we will make clear below that they are not in our illustration. (In other words, our illustration goes through with "smooth" production functions and we will explain this below.) We will set out our equilibrium system and discuss the solution. Then we will solve the corresponding zero tax model. Person A has utility function $(c_s^A)^\alpha (c_w^A)^{1-\alpha}$ and person B has utility function $(Q_s - c_s^A)^\beta (Q_w - c_w^A - \delta Q_s)^{1-\beta}$. a_{Ks} is the amount of K required to produce a unit of steel. a_{Ls} , a_{Kw} and a_{Lw} are defined in the corresponding way. Our seven equation system is

$$\frac{\alpha c_w^A}{(1-\alpha) c_s^A} = \frac{\beta (Q_w - c_w^A - \delta Q_s)}{(1-\beta) (Q_s - c_s^A)} \quad (10)$$

$$\frac{\beta (Q_w - c_w^A - \delta Q_s)}{(1-\beta) (Q_s - c_s^A)} = p_s \quad (11)$$

$$(Q_s - c_s^A)p_s + (Q_w - c_w^A - \delta Q_s) = vL + (1 - \gamma)[t_K a_{Ks} Q_s + t_L a_{Ls} Q_s] \quad (12)$$

$$p_s Q_s = [r + t_K] a_{Ks} Q_s + [v + t_L] a_{Ls} Q_s \quad (13)$$

$$Q_w = r[K - a_{Ks} Q_s] + v[L - a_{Ls} Q_s] \quad (14)$$

$$a_{Ks} Q_s + a_{Kw} Q_w = K \quad (15)$$

$$a_{Ls} Q_s + a_{Lw} Q_w = L \quad (16)$$

Optimal taxes are

$$t_K = \delta / a_{Ks} \quad (17)$$

$$t_L = \delta / a_{Ls} \quad (18)$$

Our zero tax equilibrium occurs at a facet of the production possibility schedule with the capital constraint not binding. Hence $r = 0$ and person A drops out of the economy, so to speak. In this case equilibrium values of Q_s and Q_w can be obtained from

$$\frac{\beta(Q_w - \delta Q_s)}{(1 - \beta)Q_s} = \frac{a_{Ls}}{a_{Lw}}$$

$$\text{and } a_{Ls} Q_s + a_{Lw} Q_w + \delta a_{Lw} Q_s = L.$$

Then v and p_s can be solved for. See Figure 1.

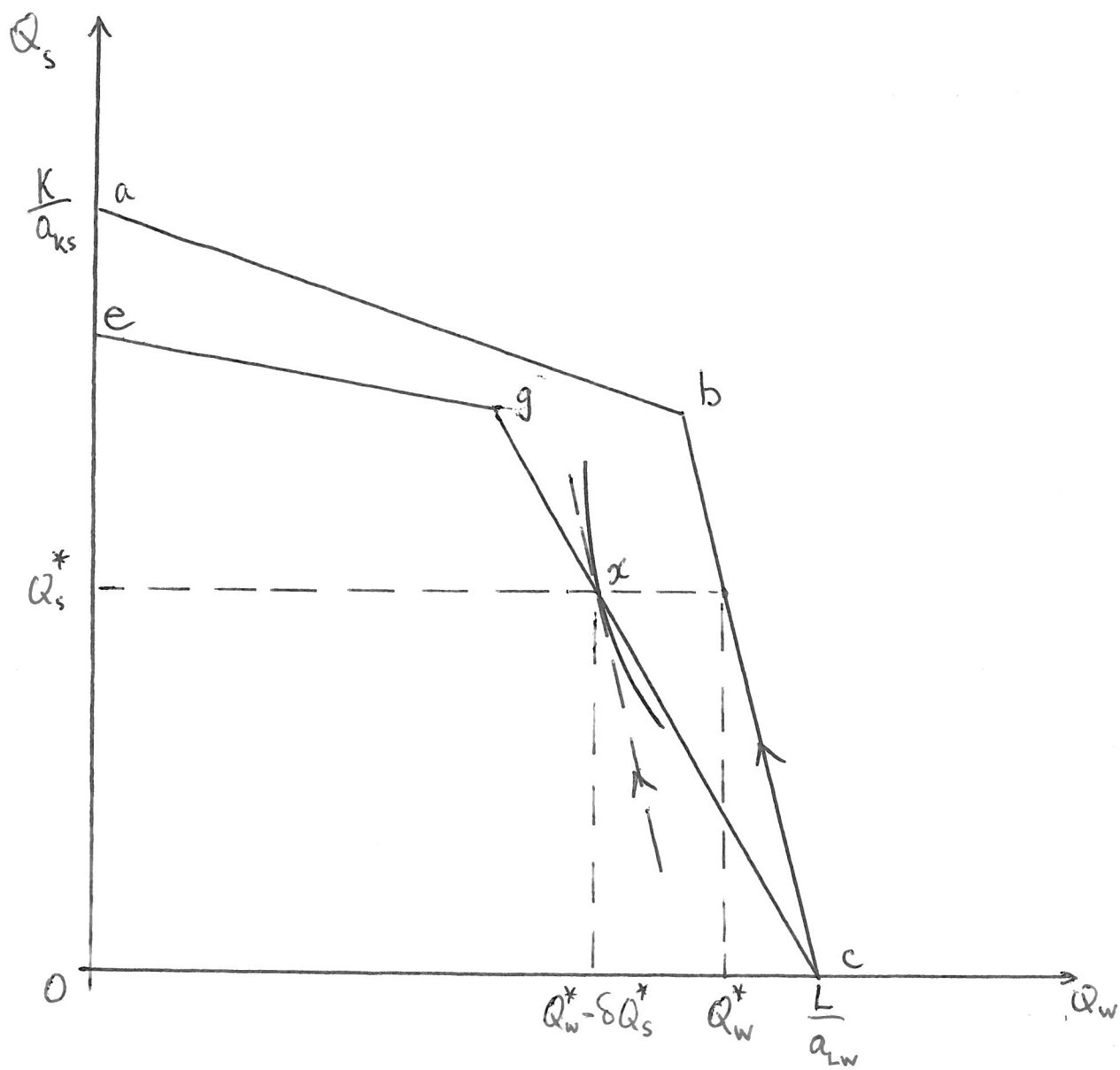


Figure 1

In Figure 1, abc is the (gross) production possibility frontier before pollution effects are netted out. $aegc$ is the net production possibility frontier.¹ The no tax equilibrium is at x with person B's indifference curve tangent to a line cutting the net production possibility curve at x and parallel to bc . The introduction of optimal taxes on r and v facing the polluting industry, induces p_s to rise.² Given a tax redistribution share γ positive for person A, her income becomes positive with the presence of taxes. If A is taste-intensive in the output from the polluting industry ($\alpha > 0.7$ for example) then the new equilibrium will have a new Q_w below that at x and a Q_s above that at x . Pollution δQ_s will also increase relative to its level at x , the no tax equilibrium. Point g is a plausible post optimal tax equilibrium. It is the equilibrium defined in equations (10) - (18).

The "strategy" involved in this example is as follows: if point x corresponds to the untaxed solution, it is inefficient. Thus the relative prices at x differ from those corresponding to the slope of the facet gc .

¹ Note that the production possibility frontier $aegc$, net of pollution effects, is not concave to the origin as Baumol and Bradford [1972] emphasized.

² Changing terms of trade in a neo-classical two good economy can be related to the Stolper-Samuelson result on changes in factor prices. In our two sector economy plus pollution the traditional Stolper-Samuelson results do not obtain. With pollution and the w and s sectors with Cobb-Douglas production functions, I have computed examples in which a rise in p_s/p_w is associated with both factor prices increasing in contrast to the well-known Stolper-Samuelson result. In our illustrative example, as p_s/p_w rises with the imposition of pollution taxes, the rental rate rises (from zero) and the wage rate declines. I note also that after much fiddling with Cobb-Douglas production functions and numerical examples, I was unable to obtain an example of the imposition of optimal taxes leading to an increase in pollution. The role of elasticities of substitution in production in our results was reported on in Hartwick [1989], particularly in the Appendix.

Optimal taxes correspond to an efficient point on frontier aegc. The introduction of optimal taxes must increase ratio p_s/p_w and the new equilibrium must be efficient in the sense it must lie on aegc and exist where relative prices do not cut (are tangent) the frontier aegc. If person A is consumption intensive in the pollution causing good, the positive tax equilibrium will be on cgea above x. Given relatively small pollution effects (small δ), the solution will move to interior point g. (Our algebraic solution (equations (10) to (18) with an interior solution) corresponds to point g. That is the equilibrium moves from a point at which the capital constraint is not binding to one at which both factor constraints are binding. This seemingly drastic shift arises because p_s/p_w must rise as taxes are imposed.

An Example:

$$a_{Ks} = 2, a_{Ls} = .2, a_{Kw} = .3, a_{Lw} = 1.0$$

$$K = 12, L = 10$$

Preferences: $\alpha = .8$ for person A and $\beta = .1$ for person B. The pollution parameter $\delta = 0.1$ and the tax sharing parameter $\gamma = 0.5$. At the no tax equilibrium, point x in Figure 1, $c_w^B = Q_w - \delta Q_s = 8.667$. $Q_s = c_s^B = 3.33$. $p_s = 0.1$ and $v = 0.9$. (Recall $r = 0$ since the capital constraint is non-binding.)

At the tax equilibrium, point g in Figure 1, $Q_s = 4.64$ and $Q_w = 9.07$. $Q_w - \delta Q_s = 8.606$. At point g, $p_s = 3.357$, $r = 1.524$ and $v = 0.543$. $C_s^A = 4.469$ and $C_s^B = 0.17$. $C_w^A = 3.751$ and $C_w^B = 4.855$. Observe that the imposition of optimal taxes raises the price of steel, lowers B's

non-tax income (and income gross of tax transfers), and raises A's non-tax income (and income gross of tax transfers). The total quantity of steel increases after optimal taxes are imposed as does the total quantity of pollution, namely δQ_s .

Consider relaxing the assumption of Leontief technology. Since the constant elasticity of substitution (CES) production function has the Leontief production function as a special case when the elasticity of substitution is zero, we can replace the two Leontief production functions in our example with two CES production functions, each with an elasticity of substitution almost zero. With appropriate share values in the two production functions, our example will go through almost unchanged. In place of $r = 0$ at the no tax equilibrium, r will be small and positive. Thus person A will not drop out at the no tax equilibrium; rather person A will have a very small income, since r will be small. There will be an optimal tax equilibrium almost identical to that at g in Figure 1. Thus the Leontief technology is not essential to our result.³

What is essential is that persons A and B have different tastes. With different tastes, the implicit aggregate demand curves for steel and wheat

³ There is a case involving no efficiency losses from no corrective taxes peculiar to the Leontief (zero substitutability) technology. It is possible that the taxed and zero tax equilibria both occur at g . In this case socially optimal taxes simply change incomes and utilities of persons A and B and have no effect on the pollution level or the amounts of steel and wheat available for aggregate consumption. But this curiosum is an artifact of the assumption of the Leontief technology. This contrasts with our case above involving more pollution at the taxed equilibrium which goes through even with some substitutability among inputs in the production of steel and wheat.

can have seemingly strange properties.⁴ Our example requires that at a higher price for steel (taxed equilibrium) more steel in aggregate is purchased. In terms of Figure 1, the shift from point x (no tax equilibrium) to point g (optimal tax equilibrium) involved person A's income rising dramatically and person A had a strong taste for steel (large fraction of her income spent on steel). Person B had a strong taste for wheat.

Concluding Remarks

In many person economies, the demand curves of the complete group of demanders can have seemingly unusual properties in general equilibrium. One consequence is that corrective taxes in a polluted economy can in some cases induce more consumption of the good whose production is causing pollution and result in an increase in pollution. The result turns on differences of tastes and here differences of sources of income for different individuals. Such seeming paradoxes do not occur in one person economies in which essentially demand curves slope downward.⁵ The empirical relevance of the

⁴ That differences in tastes in a multi-person economy can lead to "strange" demand schedules was implicit in Johnson [1959]. In Hartwick [1989] I discuss the phenomenon to illustrate a contraction in exports for an economy facing an improvement in its terms of trade. A graphical analysis should convince the reader that in a regular single consumer economy, the imposition of optimal pollution taxes, starting from the no tax position, always leads to less pollution. This is because the rise in p_s from the imposition of taxes induces a decline in the amount of steel consumed and hence a decline in pollution.

⁵ The Edgeworth excise tax paradox (e.g., Hotelling [1932] or Bailey [1954]) involves, in its original form, the introduction of a tax on commodity i provided by a monopolist, also providing substitute j, resulting in a lower price for i. Hotelling argued that such a paradox could also arise in a competitive industry. This contributor sees no link between the Edgeworth tax paradox and our tax anomaly, either in substance or in methods of derivation but there may well be a Stolper-Samuelson Effect - Rybczynski Effect duality which could be teased out at some future date.

paradoxical cases is unclear but it is reasonable to infer that we are not dealing with an implausible family of cases. In the case of pollution control, one implication of our analysis is to suggest more reliance be put on encouraging clean technologies rather than on simply charging polluters for the damages that they cause. In our model cleaner technologies would involve R & D and investment policies directed to shrinking δ .

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Abstract

In a simple two factor, two good, two person general equilibrium model with pollution, we indicate a case in which the introduction of optimal Pigovian corrective taxes results in an equilibrium with a higher level of pollution. Our argument turns on our two persons earning incomes from distinct input sources and having different tastes for the two goods, one of which causes pollution. The introduction of taxes causes one person's income to rise relative to the other person's and can induce a shift in total consumption toward the commodity causing pollution.

Increasing Pollution with Optimal Corrective Taxes

Introduction

We examine the effects of introducing corrective taxes in an economy with pollution and no corrective taxes. Our model is a simple two good, two factor, two person general equilibrium system in which one output results in pollution and inhibits production of the other good. Our principal result is that the introduction of socially optimal (Pigovian) taxes in the untaxed economy can lead to a Pareto optimum with more pollution than was present in the corresponding no tax economy. Our result turns on their being two distinct consumers whose incomes arise from different factor incomes and whose tastes for the two outputs differ. The logic runs: the introduction of corrective taxes drives up the relative price of the good causing pollution and changes factor prices so as to increase the income of the person with a strong taste for the output that happens also to cause pollution. In some cases the result is a higher level of pollution persisting after the corrective taxes are introduced. We note again: the introduction of taxes causes one person's income to rise relatively more than the other (via Stolper-Samuelson like effects) and if that person is consumption-intensive in the output of the good which causes pollution, the economy can end up with a higher level of pollution after the corrective taxes have been introduced.

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Optimal Pollution Taxes

Optimal taxes satisfy the social planning problem

$$\begin{aligned} & \text{maximize} \quad u^A(c_s^A, c_w^A) \quad \text{subject to} \quad \bar{u}^B = u^B(f^s(K^s, L^s) - c_s^A, \\ & \{c_s^A, c_w^A, K^s, L^s\} \quad f^w(K - K^s, L - L^s) - \delta f^s(K^s, L^s) - c_w^A). \end{aligned}$$

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$$\frac{\alpha c_w^A}{(1-\alpha)c_s^A} = \frac{\beta(Q_w - c_w^A - \delta Q_s)}{(1-\beta)(Q_s - c_s^A)} \quad (10)$$

$$\frac{\beta(Q_w - c_w^A - \delta Q_s)}{(1-\beta)(Q_s - c_s^A)} = p_s \quad (11)$$

$$(Q_s - c_s^A)p_s + (Q_w - c_w^A - \delta Q_s) = vL + (1 - \gamma)[t_K a_{Ks} Q_s + t_L a_{Ls} Q_s] \quad (12)$$

$$p_s Q_s = [r + t_K] a_{Ks} Q_s + [v + t_L] a_{Ls} Q_s \quad (13)$$

$$Q_w = r[K - a_{Ks} Q_s] + v[L - a_{Ls} Q_s] \quad (14)$$

$$a_{Ks} Q_s + a_{Kw} Q_w = K \quad (15)$$

$$a_{Ls} Q_s + a_{Lw} Q_w = L \quad (16)$$

Optimal taxes are

$$t_K = \delta / a_{Ks} \quad (17)$$

$$t_L = \delta / a_{Ls} \quad (18)$$

Our zero tax equilibrium occurs at a facet of the production possibility schedule with the capital constraint not binding. Hence $r = 0$ and person A drops out of the economy, so to speak. In this case equilibrium values of Q_s and Q_w can be obtained from

$$\frac{\beta(Q_w - \delta Q_s)}{(1 - \beta)Q_s} = \frac{a_{Ls}}{a_{Lw}}$$

$$\text{and } a_{Ls} Q_s + a_{Lw} Q_w + \delta a_{Lw} Q_s = L.$$

Then v and p_s can be solved for. See Figure 1.

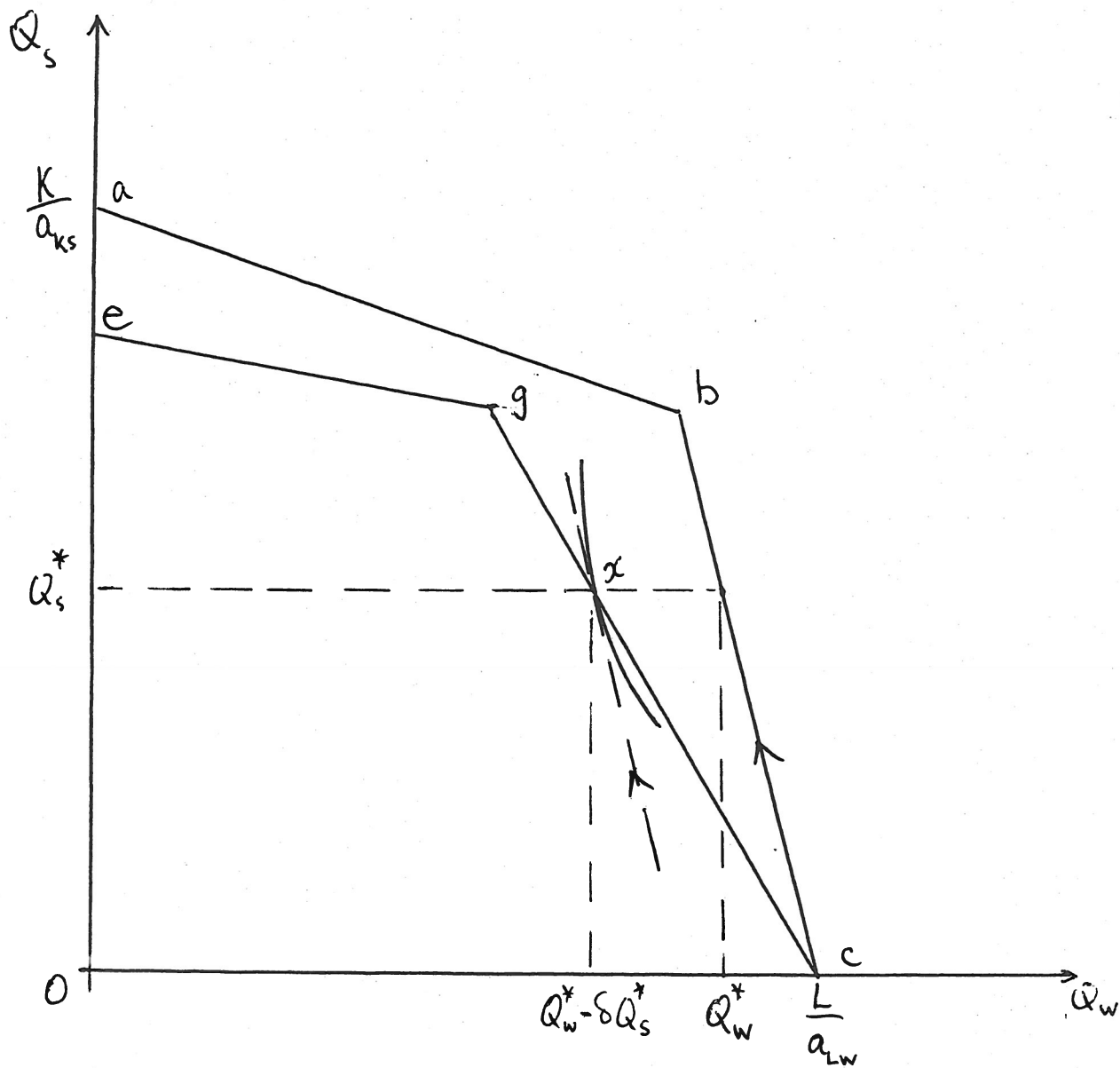


Figure 1

In Figure 1, abc is the (gross) production possibility frontier before pollution effects are netted out. $aegc$ is the net production possibility frontier.¹ The no tax equilibrium is at x with person B's indifference curve tangent to a line cutting the net production possibility curve at x and parallel to bc . The introduction of optimal taxes on r and v facing the polluting industry, induces p_s to rise.² Given a tax redistribution share γ positive for person A, her income becomes positive with the presence of taxes. If A is taste-intensive in the output from the polluting industry ($\alpha > 0.7$ for example) then the new equilibrium will have a new Q_w below that at x and a Q_s above that at x . Pollution δQ_s will also increase relative to its level at x , the no tax equilibrium. Point g is a plausible post optimal tax equilibrium. It is the equilibrium defined in equations (10) - (18).

The "strategy" involved in this example is as follows: if point x corresponds to the untaxed solution, it is inefficient. Thus the relative prices at x differ from those corresponding to the slope of the facet gc .

¹ Note that the production possibility frontier $aegc$, net of pollution effects, is not concave to the origin as Baumol and Bradford [1972] emphasized.

² Changing terms of trade in a neo-classical two good economy can be related to the Stolper-Samuelson result on changes in factor prices. In our two sector economy plus pollution the traditional Stolper-Samuelson results do not obtain. With pollution and the w and s sectors with Cobb-Douglas production functions, I have computed examples in which a rise in p_s/p_w is associated with both factor prices increasing in contrast to the well-known Stolper-Samuelson result. In our illustrative example, as p_s/p_w rises with the imposition of pollution taxes, the rental rate rises (from zero) and the wage rate declines. I note also that after much fiddling with Cobb-Douglas production functions and numerical examples, I was unable to obtain an example of the imposition of optimal taxes leading to an increase in pollution. The role of elasticities of substitution in production in our results was reported on in Hartwick [1989], particularly in the Appendix.

Optimal taxes correspond to an efficient point on frontier aegc. The introduction of optimal taxes must increase ratio p_s/p_w and the new equilibrium must be efficient in the sense it must lie on aegc and exist where relative prices do not cut (are tangent) the frontier aegc. If person A is consumption intensive in the pollution causing good, the positive tax equilibrium will be on cgea above x. Given relatively small pollution effects (small δ), the solution will move to interior point g. (Our algebraic solution (equations (10) to (18) with an interior solution) corresponds to point g. That is the equilibrium moves from a point at which the capital constraint is not binding to one at which both factor constraints are binding. This seemingly drastic shift arises because p_s/p_w must rise as taxes are imposed.

An Example:

$$a_{Ks} = 2, a_{Ls} = .2, a_{Kw} = .3, a_{Lw} = 1.0$$

$$K = 12, L = 10$$

Preferences: $\alpha = .8$ for person A and $\beta = .1$ for person B. The pollution parameter $\delta = 0.1$ and the tax sharing parameter $\gamma = 0.5$. At the no tax equilibrium, point x in Figure 1, $c_w^B = Q_w - \delta Q_s = 8.667$. $Q_s = c_s^B = 3.33$. $p_s = 0.1$ and $v = 0.9$. (Recall $r = 0$ since the capital constraint is non-binding.)

At the tax equilibrium, point g in Figure 1, $Q_s = 4.64$ and $Q_w = 9.07$. $Q_w - \delta Q_s = 8.606$. At point g, $p_s = 3.357$, $r = 1.524$ and $v = 0.543$. $C_s^A = 4.469$ and $C_s^B = 0.17$. $C_w^A = 3.751$ and $C_w^B = 4.855$. Observe that the imposition of optimal taxes raises the price of steel, lowers B's

non-tax income (and income gross of tax transfers), and raises A's non-tax income (and income gross of tax transfers). The total quantity of steel increases after optimal taxes are imposed as does the total quantity of pollution, namely δQ_s .

Consider relaxing the assumption of Leontief technology. Since the constant elasticity of substitution (CES) production function has the Leontief production function as a special case when the elasticity of substitution is zero, we can replace the two Leontief production functions in our example with two CES production functions, each with an elasticity of substitution almost zero. With appropriate share values in the two production functions, our example will go through almost unchanged. In place of $r = 0$ at the no tax equilibrium, r will be small and positive. Thus person A will not drop out at the no tax equilibrium; rather person A will have a very small income, since r will be small. There will be an optimal tax equilibrium almost identical to that at g in Figure 1. Thus the Leontief technology is not essential to our result.³

What is essential is that persons A and B have different tastes. With different tastes, the implicit aggregate demand curves for steel and wheat

³ There is a case involving no efficiency losses from no corrective taxes peculiar to the Leontief (zero substitutability) technology. It is possible that the taxed and zero tax equilibria both occur at g . In this case socially optimal taxes simply change incomes and utilities of persons A and B and have no effect on the pollution level or the amounts of steel and wheat available for aggregate consumption. But this curiosum is an artifact of the assumption of the Leontief technology. This contrasts with our case above involving more pollution at the taxed equilibrium which goes through even with some substitutability among inputs in the production of steel and wheat.

can have seemingly strange properties.⁴ Our example requires that at a higher price for steel (taxed equilibrium) more steel in aggregate is purchased. In terms of Figure 1, the shift from point x (no tax equilibrium) to point g (optimal tax equilibrium) involved person A's income rising dramatically and person A had a strong taste for steel (large fraction of her income spent on steel). Person B had a strong taste for wheat.

Concluding Remarks

In many person economies, the demand curves of the complete group of demanders can have seemingly unusual properties in general equilibrium. One consequence is that corrective taxes in a polluted economy can in some cases induce more consumption of the good whose production is causing pollution and result in an increase in pollution. The result turns on differences of tastes and here differences of sources of income for different individuals. Such seeming paradoxes do not occur in one person economies in which essentially demand curves slope downward.⁵ The empirical relevance of the

⁴ That differences in tastes in a multi-person economy can lead to "strange" demand schedules was implicit in Johnson [1959]. In Hartwick [1989] I discuss the phenomenon to illustrate a contraction in exports for an economy facing an improvement in its terms of trade. A graphical analysis should convince the reader that in a regular single consumer economy, the imposition of optimal pollution taxes, starting from the no tax position, always leads to less pollution. This is because the rise in p_s from the imposition of taxes induces a decline in the amount of steel consumed and hence a decline in pollution.

⁵ The Edgeworth excise tax paradox (e.g., Hotelling [1932] or Bailey [1954]) involves, in its original form, the introduction of a tax on commodity i provided by a monopolist, also providing substitute j, resulting in a lower price for i. Hotelling argued that such a paradox could also arise in a competitive industry. This contributor sees no link between the Edgeworth tax paradox and our tax anomaly, either in substance or in methods of derivation but there may well be a Stolper-Samuelson Effect - Rybczynski Effect duality which could be teased out at some future date.

paradoxical cases is unclear but it is reasonable to infer that we are not dealing with an implausible family of cases. In the case of pollution control, one implication of our analysis is to suggest more reliance be put on encouraging clean technologies rather than on simply charging polluters for the damages that they cause. In our model cleaner technologies would involve R & D and investment policies directed to shrinking δ .

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