

Lapham, Beverly J.

**Working Paper**

## A Dynamic, General Equilibrium Analysis of Deviations From the Laws of One Price

Queen's Economics Department Working Paper, No. 793

**Provided in Cooperation with:**

Queen's University, Department of Economics (QED)

*Suggested Citation:* Lapham, Beverly J. (1990) : A Dynamic, General Equilibrium Analysis of Deviations From the Laws of One Price, Queen's Economics Department Working Paper, No. 793, Queen's University, Department of Economics, Kingston (Ontario)

This Version is available at:

<https://hdl.handle.net/10419/189118>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



Queen's Economics Department Working Paper No. 793

# A Dynamic, General Equilibrium Analysis of Deviations From the Laws of One Price

Beverly J. Lapham

Department of Economics  
Queen's University  
94 University Avenue  
Kingston, Ontario, Canada  
K7L 3N6

9-1990

**A Dynamic, General Equilibrium Analysis of  
Deviations From the Laws of One Price**

by

Beverly J. Lapham

DISCUSSION PAPER #793

Department of Economics  
Queen's University  
Kingston, Canada K7L 3N6

September 1990

The author thanks Edward Prescott, Patrick Kehoe, Allen Head, and the participants of the Queen's Macro Workshop for helpful discussions and comments.

### Abstract

There are at least two versions of the Law of One Price under the assumptions of perfect competition, no trade barriers, and no transport costs. One version predicts equal prices of competing goods sold in the same country and manufactured by producers located in different countries. Another version predicts equal prices of a good manufactured by a single producer and sold in different countries when prices are converted to a common currency. Empirical studies suggest persistent, time-varying deviations from both versions of the Law of One Price. This paper explores a stochastic, dynamic, general equilibrium theoretical economy with explicitly strategic price setting behavior. The equilibrium processes generated by the economy exhibit properties which are generally consistent with persistent, time-varying deviations from the Laws of One Price.

## 1. INTRODUCTION

Relationships among the prices of internationally traded goods across national boundaries have been the subject of a wide range of economic research. Empirical and theoretical studies have focused on relative price levels (Purchasing Power Parity), relative prices of individual commodities (Law of One Price), and on the relationship between nominal exchange rate movements and resulting movements in traded goods' prices (exchange rate pass-through).

The version of Purchasing Power Parity developed by Cassel(1918) predicts that nominal exchange rates will move in response to relative price level movements so that real exchange rates will be constant over time<sup>1</sup>. Using price indices including GDP deflators, Consumer Price Indices, and Wholesale or Producer Price Indices, various studies have found significant and persistent deviations from Purchasing Power Parity. Officer(1976) and Shapiro(1983) provide surveys of the theoretical and empirical Purchasing Power Parity literature.

A portion of the deviations from Purchasing Power Parity in the data may be explained by discrepancies in the basket of goods measured by relative price indices due to, among other things, indexing differences and non-traded goods. This has led to examinations of the related theory of the Law of One Price applied to individual commodities sold in international markets. There are at least two versions of the Law of One Price under the assumptions of perfect competition, no trade barriers, and no transport costs. One version predicts equal prices of competing goods sold in the same country and manufactured by producers located in different countries. For example, the prices of U.S. produced

automobiles and German produced automobiles sold in Germany should be close. Another version predicts equal prices of a good manufactured by a single producer and sold in different countries when prices are converted to a common currency. For example, the Deutsche Mark prices of German produced automobiles sold in Germany and sold in the U.S. should be equal. Empirical studies, (e.g. Giovannini(1989), Kravis and Lipsey(1974,1977,1978), Lapham(1990), Marston(1990)), suggest persistent, time-varying deviations from both versions of the Law of One Price.<sup>2</sup>

In this paper, a dynamic theoretical economy incorporating explicitly strategic price setting behavior and uncertainty in a general equilibrium framework is investigated to explore these deviations. The equilibrium processes generated by the theoretical economy exhibit properties which are generally consistent with persistent, time-varying deviations from the Laws of One Price both across countries and across producers selling in the same market.

Deviations from the Laws of One Price in the absence of trade barriers and transport costs have been difficult to explain within perfectly competitive frameworks. Dornbusch(1987) generates deviations from the Law of One Price within a country in a variety of industrial organization models as imperfectly competitive firms with sticky wages set prices in response to exogenous exchange rate movements. Krugman(1988) suggests a number of static and dynamic approaches to studying the issue and concludes that it is best studied in dynamic models of imperfect competition.

Giovannini(1988) and Marston(1989) examine deviations from the Law of One Price in static, partial equilibrium models of a price discriminating

monopolist operating under exogenous exchange rate uncertainty. Froot and Klemperer(1989) explore international price setting in a two-period, partial equilibrium model with Cournot duopolists.<sup>3</sup>

The remainder of the paper is organized as follows. Section Two presents a theoretical economy in which differentiated oligopolists strategically set prices in a dynamic, stochastic game. Section Three describes market arrangements and agents' maximization problems. Section Four discusses the equilibrium concept and the solution technique. Section Five presents results of computational experiments and Section Six discusses conclusions.

## 2. THE ECONOMY

In the theoretical economy investigated below, international oligopolists producing differentiated products set prices under wage uncertainty in a dynamic Nash game. The world economy is composed of two countries, a home country denoted by  $h$  and a foreign country denoted by  $f$ . The set of countries is denoted  $J=\{h,f\}$ .

### Commodities

There are three classes of goods: a traded homogeneous good, traded differentiated goods, and non-traded goods produced in each country. The primary focus of this paper is the equilibrium pricing behavior of the commodities produced in a differentiated oligopoly. This industry is composed of home and foreign producers selling in both the home and the foreign market.

Following Salop(1979), the space of differentiated commodities is the unit-circumference circle. Differentiated commodities are indexed by  $k \in K=[0,1)$  with  $k=0$  corresponding to the top of the circle. Commodity  $k$

corresponds to the point which is distance  $k$  from the top of the circle in the clockwise direction.

Non-traded goods are produced in a perfectly competitive industry in each country. The traded homogeneous good is an endowment good.

### Preferences

Consumers in country  $j \in J$  are indexed by  $i \in [0,1)$  according to their most preferred commodity in the space of differentiated goods. Consumers in each country are uniformly distributed in the commodity space with unit density. Therefore, a consumer is indexed by his country,  $j \in J$  and his location on the unit-circumference circle,  $i \in [0,1)$ .

The consumption possibilities set for consumers in country  $j \in J$  is

$$X_j = \{m_j \in \mathbb{R}_+, c_j \in \mathbb{R}_+, \mu_j(A) \in \{0,1\} \forall A \in \mathcal{B}(K)\}$$

where  $\mathcal{B}(K)$  is the collection of Borel subsets of  $K$ . A consumption bundle for consumer  $i$  in country  $j$ , is represented by  $x_{ji} = (m_{ji}, c_{ji}, \mu_{ji}) \in X$ . Here  $m_{ji}$  denotes consumption of the homogeneous good,  $c_{ji}$  denotes consumption of the non-traded good produced in country  $j$  and  $\mu_{ji}$ , a nonnegative measure on  $(K, \mathcal{B}(K))$ , denotes consumption of the differentiated commodities. The restriction,  $\mu_{ji}(A) \in \{0,1\} \forall A \in \mathcal{B}(K)$ , limits consumers to purchase, at most, one unit of a single differentiated good and zero units of all other differentiated goods.

Consumer  $i \in [0,1)$  in country  $j \in J$  has preferences over consumption bundles ordered by the utility function

$$U_j(m_{ji}, c_{ji}, \mu_{ji}) = m_{ji} + c_{ji} + \int [u_j - d_j \Lambda(i, k)] \mu_{ji}(dk)$$

Here  $u_j$  is the utility associated with consumption of a consumer's ideal good.  $\Lambda(i, k)$  denotes the shortest distance around the circle between points  $i$  and  $k$ , and  $d_j \int \Lambda(i, k) \mu_{ji}(dk)$  is the disutility associated with



consuming a good other than the consumer's ideal good. Note that  $d_j$  is a measure of the degree of product substitutability of goods within the differentiated industry with lower  $d_j$  associated with products which are perceived to be highly substitutable.

### Endowments

Each consumer in country  $j \in J$  is endowed each period with  $\omega_j$  units of the homogeneous good and one unit of time. Time is supplied inelastically and is used to produce the non-traded good and the differentiated goods produced by firms in country  $j$ .

### Technologies

#### Non-Traded Goods Production

The technology for producing the non-traded good in country  $j$  is

$$y_{jt} = \theta_{jt} n_{jt} \quad \forall j \in J, \forall t$$

Here  $n_{jt}$  is labor input and  $\theta_{jt}$  is a stochastic production technology parameter.  $\theta_{jt}$  is independently distributed across countries. Specific distributions for these processes are specified in the computational experiments in Section Five.

#### Differentiated Goods Production

The technology for producing differentiated good  $k \in K$  is linear in labor input,  $\ell_{kt}$ . The technology for differentiated goods' production is given by

$$\chi_{kt} = \frac{1}{\phi} \ell_{kt} \quad \forall k \in K, \forall t, \quad \phi > 0$$

#### Distribution Technologies

The technology for producing changes in the sales of differentiated producers is captured by distribution technologies. These technologies

reflect expenditures of the homogeneous good required to alter firms' sales and distribution networks and lead to a dynamic price setting game. The distribution technologies are summarized by total cost functions.

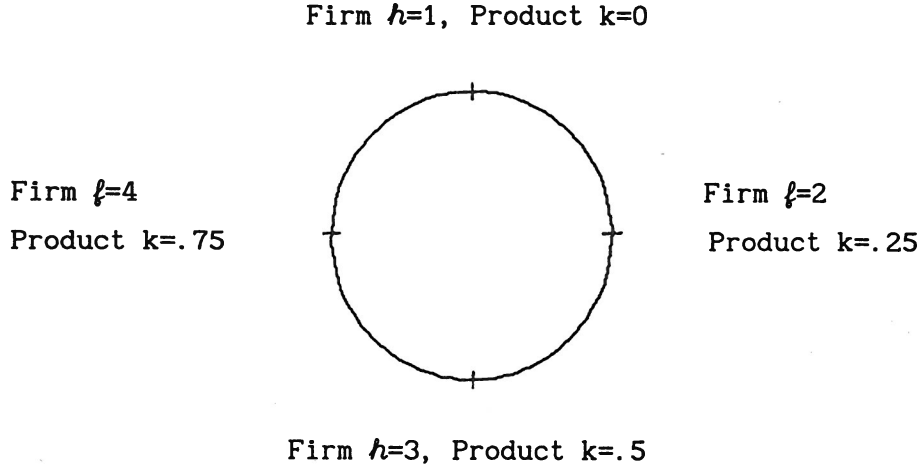
Distribution technologies are country specific and reflect possible differences in costs associated with altering sales in a particular market. Resources of the homogeneous good required to alter sales by  $\Delta X$  in country  $j \in J$  are given by:

$$C_j(\Delta X) = .5\alpha_j(\Delta X)^2$$

### 3. MARKET ARRANGEMENTS

There are an even number of firms,  $n$ , producing differentiated products;  $n/2$  firms are home producers and  $n/2$  are foreign producers. The set of home firms is denoted by  $\mathcal{H}=\{1,3,\dots,n-1\}$ , the set of foreign firms by  $\mathcal{F}=\{2,4,\dots,n\}$ , and the set of all firms in the differentiated industry by  $\mathcal{S}=\mathcal{H} \cup \mathcal{F}$ . Associated with each firm  $q \in \mathcal{S}$  is a product on the unit-circumference circle indexed by  $k=[(q-1)/n] \in K$ . This implies that firms are equally spaced at a distance of  $1/n$  around the unit circle with home and foreign firms alternating in location around the circle. Therefore, a home firm's closest competitor on either side is a foreign firm and a foreign firm's closest competitors are home firms. The symmetric nature of firms' locations allows analysis of representative home and foreign firms.

The following diagram illustrates the market arrangement in a country when  $n = 4$ .



Let  $\Pi_{qt}$ ,  $\forall q \in \mathcal{Q}$ , denote firm  $q$ 's profits at time  $t$ . Then aggregate profits in each country are given by

$$\Pi_{ht} = \sum_{h \in \mathcal{H}} \Pi_{ht} \quad \text{in the home country}$$

and

$$\Pi_{ft} = \sum_{f \in \mathcal{F}} \Pi_{ft} \quad \text{in the foreign country.}$$

Every consumer in each country owns an equal share of each firm in that country. Therefore, an individual consumer's share of equilibrium profits is aggregate profits divided by the measure of consumers in the country. Letting  $\lambda$  denote the Lebesgue measure, an individual consumer's share of equilibrium profits in country  $j \in J$  at time  $t$  is

$$(\Pi_{jt}) / (\lambda([0,1])) = \Pi_{jt}$$

as the Lebesgue measure of the unit interval is one. Note that profits in the non-traded goods sector are zero in equilibrium.

### Consumers' Maximization Problems

The value of a consumption bundle at time  $t$  expressed in units of the homogeneous good at time  $t$  in the home country is

$$v_h(m_{hit}, c_{hit}, \mu_{hit}) = m_{hit} + r_{ht} c_{hit} + \int p_{kt} \mu_{hit}(dk)$$

Here  $r_{ht}$  denotes the price of the home non-traded good and  $p_{kt}$  denotes the price in the home market of differentiated good  $k \in K$  at time  $t$ .

The value of a consumption bundle at time  $t$  expressed in units of the homogeneous good at time  $t$  in the foreign country is

$$v_f(m_{fit}, c_{fit}, \mu_{fit}) = m_{fit} + r_{ft} c_{fit} + \int q_{kt} \mu_{fit}(dk)$$

Here  $r_{ft}$  denotes the price of the foreign non-traded good and  $q_{kt}$  denotes the price in the foreign market of differentiated good  $k \in K$  at time  $t$ .

Consumers seek to maximize discounted utility subject to a date zero budget constraint. Letting  $\beta$  denote the subjective discount rate of consumers, the maximization problem of consumer  $i \in [0, 1)$  in country  $j$  is

$$\begin{aligned} \max_{\{m_{j1t}, c_{j1t}, \mu_{j1t}\}_s} \quad & \sum_{t=0}^T \beta^t \left\{ m_{j1t} + c_{j1t} + \int [u_j - d_j \Lambda(i, k)] \mu_{j1t}(dk) \right\} \\ \text{subject to} \quad & \sum_{t=0}^T \rho_t v_j(m_{j1t}, c_{j1t}, \mu_{j1t}) \leq \sum_{t=0}^T \rho_t (\omega_j + w_{jt} + \Pi_{jt}) \end{aligned}$$

Here  $\rho_t$  is the relative price of the homogeneous good at time 0 to the homogeneous good at time  $t$ , and  $w_{jt}$  is the equilibrium wage in country  $j$  at time  $t$ .

The quasi-linear structure of preferences implies that in an interior equilibrium,  $\rho_t$  and, therefore, the interest rate will be constant. Furthermore, this quasi-linearity implies that any intertemporal consumption smoothing requires varying consumption of the homogeneous

good. However, since utility is linear in the homogeneous good, the consumer is indifferent as to when the good is consumed. As consumers receive no benefit from intertemporal consumption smoothing, attention is restricted to interior equilibria with no borrowing or lending. Interior solutions are guaranteed by making endowments of the homogeneous good sufficiently large. Therefore, allocations resulting from period by period utility maximization subject to the period budget constraint are examined.

Parameters are restricted so that equilibrium prices induce all consumers to purchase a differentiated product. Using Salop's terminology, this guarantees that the industry is operating on the competitive portion of its demand curve. This implies that  $\mu$  will give zero measure to all commodities except one which will have measure one.

This, along with the restrictions on  $\mu$ , implies that a consumer's problem can be separated into a two stage problem. In the first stage, consumers choose which differentiated product to consume. In the second stage, consumers allocate their remaining wealth between the homogeneous good and the non-traded good produced in their country.

#### Non-Traded Goods' Production

Perfectly competitive firms in country  $j$  producing non-traded goods seek to maximize discounted profits. Since there are no dynamic factors affecting firms' profits, allocations resulting from maximizing profits each period will be the same as those resulting from maximizing discounted profits. Therefore, non-traded goods' producers face:

$$\begin{aligned} \max_{y_{jt}} \quad & r_{jt} y_{jt} - w_{jt} n_{jt} \\ \text{subject to} \quad & y_{jt} = \theta_{jt} n_{jt} \end{aligned}$$

In an interior equilibrium with all goods produced, the following conditions will hold:

$$r_{jt} = 1 \quad \forall j \in J, \forall t$$

$$r_{jt} \theta_{jt} = \theta_{jt} = w_{jt} \quad \forall j \in J, \forall t$$

Note that equilibrium wages are determined by the technology shocks to non-traded goods production in each country. In particular, the behavior of differentiated firms will have no affect on the equilibrium wage in an interior solution.

### Differentiated Goods' Production

The following notation is adopted.

$$h-1 \equiv \begin{cases} h-1 & \text{for } h \in \mathcal{H}, h \neq 1 \\ n & \text{for } h=1 \end{cases}$$

$$l+1 \equiv \begin{cases} l+1 & \text{for } l \in \mathcal{F}, l \neq n \\ 1 & \text{for } l=n \end{cases}$$

Define  $x_{ht}^+$  as the shortest distance around the unit circle between home firm  $h \in \mathcal{H}$  and the home consumer indifferent between purchasing firm  $h$ 's product and purchasing the product of the neighboring foreign firm,  $l=h+1$ . Therefore  $x_{ht}^+$  is implicitly defined by:

$$p_{ht} + d_h x_{ht} = p_{h+1t} + d_h (1/n - x_{h+1t})$$

or,

$$x_{ht}^+ = (p_{h+1t} - p_{ht} + d_h/n) / 2d_h.$$

$x_{ht}^+$  denotes the measure of home consumers in the clockwise direction around the circle from  $h$  who purchase the product of firm  $h$  at time  $t$ .

Similarly, let  $x_{ht}^-$  denote the measure of home consumers in the counter-clockwise direction around the circle who purchase the product of firm  $h$  at time  $t$ . Then  $x_{ht}^-$  is defined by

$$\bar{x}_{ht} = (p_{h-1t} - p_{ht} + d_h/n)/2d_h.$$

Finally, the measure of home consumers who purchase the product of the home firm  $h \in \mathcal{H}$  at time  $t$  is

$$x_{ht} = x_{ht}^+ + x_{ht}^- = (p_{h+1t} + p_{h-1t} - 2p_{ht})/2d_h + 1/n.$$

Similarly, the measure of home consumers who purchase the product of foreign firm  $f \in \mathcal{F}$  at time  $t$  is

$$x_{ft} = (p_{f+1t} + p_{f-1t} - 2p_{ft})/2d_h + 1/n.$$

The measure of foreign consumers who purchase the product of home firm  $h \in \mathcal{H}$  at time  $t$  is

$$z_{ht} = (q_{h+1t} + q_{h-1t} - 2q_{ht})/2d_f + 1/n.$$

The measure of foreign consumers who purchase the product of foreign firm  $f \in \mathcal{F}$  at time  $t$  is

$$z_{ft} = (q_{f+1t} + q_{f-1t} - 2q_{ft})/2d_f + 1/n.$$

The following chart summarizes the notation for prices and sales.

Prices, Sales		Market	
		Home	Foreign
Producer	Home	$p_{ht}, x_{ht}$	$q_{ht}, z_{ht}$
	Foreign	$p_{ft}, x_{ft}$	$q_{ft}, z_{ft}$

Firms in the differentiated sector play a dynamic Nash price setting game taking wages and the demand functions above as given. Firms choose Markov pricing functions as functions of last period prices and current and future expected wages to maximize expected discounted profits. These

maximization problems are described below.

Define the following vectors:

$$\begin{aligned} p_t &\equiv (p_{qt})_{q \in \mathcal{G}} & p_{t \setminus k} &\equiv (p_{qt})_{q \in \mathcal{G}, q \neq k} \\ q_t &\equiv (q_{qt})_{q \in \mathcal{G}} & q_{t \setminus k} &\equiv (q_{qt})_{q \in \mathcal{G}, q \neq k} \\ S_t &\equiv (p_{t-1}, q_{t-1}) & w_t &\equiv (w_{ht}, w_{ft}) \\ w_{\geq t} &\equiv \{w_s\}_{s \geq t} \end{aligned}$$

Let  $\Pi_q: \mathbb{R}_+^{4n+2} \rightarrow \mathbb{R}$  denote the one-period profit function of firm  $q \in \mathcal{G}$  located in country  $j \in J$ . Then

$$\begin{aligned} \Pi_q(S_t, w_t, p_t, q_t) &= (p_{qt} - \phi_{jt} w_{jt}) x_{qt} - .5 \alpha_h (x_{qt} - x_{qt-1})^2 \\ &\quad + (q_{qt} - \phi_{jt} w_{jt}) z_{qt} - .5 \alpha_f (z_{qt} - z_{qt-1})^2 \end{aligned}$$

Let  $R_{qt}: \mathbb{R}_+^{4n+2(T-t+1)} \rightarrow \mathbb{R}$  denote the objective function of firm  $q \in \mathcal{G}$  at time  $t$ . Then

$$R_{qt}(S_t, E_t w_{\geq t}, p_t, q_t) = \Pi_q(S_t, w_t, p_t, q_t) + \beta E_t V_{qt+1}(S_{t+1}, w_{\geq t+1})$$

Here  $V_{qt+1}(\dots)$  denotes the discounted value of future profits to firm  $q \in \mathcal{G}$  given that other firms play their Nash equilibrium strategies. That is, defining a Nash Equilibrium at stage  $t$  to be a set of strategy functions  $p_t^*(S_t, E_t w_{\geq t})$  and  $q_t^*(S_t, E_t w_{\geq t})$  that satisfy  $\forall q \in \mathcal{G}$

$$(p_{qt}^*(S_t, E_t w_{\geq t}), q_{qt}^*(S_t, E_t w_{\geq t})) = \operatorname{argmax}_{p_{qt}, q_{qt}} R_{qt}(S_t, E_t w_{\geq t}, p_{qt}, p_{t \setminus q}^*, q_{qt}, q_{t \setminus q}^*)$$

then

$$V_{qt}(S_t, E_t w_{\geq t}) = R_{qt}(S_t, E_t w_{\geq t}, p_t^*(S_t, E_t w_{\geq t}), q_t^*(S_t, E_t w_{\geq t})).$$

Therefore, firm  $q \in \mathcal{G}$  in the differentiated sector faces the following problem:  $\forall t$

$$\max_{\{p_{qs}, q_{qs}\}_{s \geq t}} R_{qt}(S_t, E_t w_{\geq t}, p_t, q_t)$$

A method for determining subgame perfect Markov equilibrium pricing



functions is described in the following section.

#### 4. EQUILIBRIUM

##### Definition

A subgame perfect equilibrium is pricing functions  $\{r_{jt}\}$ ,  $\{w_t, p_t, q_t\}$ ; allocations  $\{m_{jit}, c_{jit}, \mu_{jit}\}$ ,  $\{y_{jt}\}$ ,  $\{x_{qt}, z_{qt}\}$ ; and labor inputs  $\{n_{jt}\}$ ,  $\{\ell_{qt}\}$  such that

- 1)  $\forall j \in J, \forall i \in [0, 1), \{m_{jit}, c_{jit}, \mu_{jit}\}_t$  solves consumer  $ji$ 's problem.
- 2)  $\{p_t, q_t\}$  are Nash equilibrium pricing functions for each subgame of the dynamic game described above.
- 3) Goods markets clear:  $\forall t$

- i) Non-traded Goods

$$\int c_{jit} \lambda(di) = y_{jt} \quad \forall j \in J$$

- ii) Homogeneous Good

$$\sum_{j \in J} \left( \int m_{jit} \lambda(di) \right) + .5 \sum_{q \in \mathcal{G}} \left( \alpha_h (x_{qt} - x_{qt-1})^2 + \alpha_f (z_{qt} - z_{qt-1})^2 \right) = \sum_{j \in J} \omega_j$$

- iii) Differentiated Goods,  $\forall q \in \mathcal{G}$

$$a) \int \mu_{hit}(\{q\}) \lambda(di) = x_{qt}$$

$$b) \int \mu_{fit}(\{q\}) \lambda(di) = z_{qt}$$

- 4) Labor markets clear:  $\forall t$

$$i) n_{ht} + \sum_{h \in \mathcal{H}} \ell_{ht} = 1$$

$$ii) n_{ft} + \sum_{f \in \mathcal{F}} \ell_{ft} = 1$$

Because of the linear-quadratic structure of firms' objective functions, the firms' best response functions will be linear and equilibria, when they exist will be unique. For all economy parameter settings considered below, equilibria exist.

### Solution Method

To find the subgame perfect equilibrium pricing processes for the dynamic game, the game is solved backwards through time. Suppose the economy lasts until period T.

### Period T Problems

At time T, firm  $q$ 's objective function is

$$R_{qT}(S_T, w_T, p_T, q_T) = \Pi_q(S_T, w_T, p_T, q_T)$$

A set of first order conditions for the time T problem of firm  $q$  located in country  $j$  is given by

$$(1) \quad x_{qT} + (p_{qT} - \phi_{jT})(\partial x_{qT} / \partial p_{qT}) - (\alpha_h(x_{qT} - x_{qT-1}))(\partial x_{qT} / \partial p_{qT}) = 0$$

$$z_{qT} + (q_{qT} - \phi_{jT})(\partial z_{qT} / \partial q_{qT}) - (\alpha_f(z_{qT} - z_{qT-1}))(\partial z_{qT} / \partial q_{qT}) = 0$$

The definitions of  $x_{qT}$  and  $z_{qT}$  guarantee that the second order conditions for a maximum are satisfied.

The first order conditions (1) implicitly define the period T best response functions for the firms:

$$\hat{p}_{qT}(S_T, w_T, p_{T \setminus q}) \quad \text{and} \quad \hat{q}_{qT}(S_T, w_T, q_{T \setminus q}) \quad \forall q \in \mathcal{G}$$

Combining the best response functions gives period T Nash equilibrium pricing functions for the firms as functions of last period prices and current wages:

$$p_{qT}^*(S_T, w_T) \quad \text{and} \quad q_{qT}^*(S_T, w_T) \quad \forall q \in \mathcal{G}$$

Substituting these Nash equilibrium prices and resulting Nash equilibrium sales into the firms' objective functions gives Nash equilibrium profits at period T as a function of  $S_T$  and  $w_T$  for each firm:

$$V_{q_T}(S_T, w_T) = R_{q_T}(S_T, w_T, p_T^*(S_T, w_T), q_T^*(S_T, w_T)), \quad \forall q \in \mathcal{Q}$$

### Period T-1 Problems

In period T-1, firm  $q$ 's objective function is given by

$$R_{q_{T-1}}(S_{T-1}, w_{T-1}, E_{T-1} w_T, p_{T-1}, q_{T-1}) = \Pi_q(S_{T-1}, w_{T-1}, p_{T-1}, q_{T-1}) + \beta E_{T-1} V_{q_T}(S_T, w_T)$$

A set of first order conditions for this problem is

$$\begin{aligned} x_{q_{T-1}} + (p_{q_{T-1}} - \phi w_{j_{T-1}})(\partial x_{q_{T-1}} / \partial p_{q_{T-1}}) \\ - (\alpha_h (x_{q_{T-1}} - x_{q_{T-2}}))(\partial x_{q_{T-1}} / \partial p_{q_{T-1}}) \\ + \beta E_{T-1} (\partial V_{q_T}(S_T, w_T) / \partial p_{q_{T-1}}) = 0 \end{aligned} \quad (2)$$

$$\begin{aligned} z_{q_{T-1}} + (p_{q_{T-1}} - \phi w_{j_{T-1}})(\partial z_{q_{T-1}} / \partial q_{q_{T-1}}) \\ - (\alpha_f (z_{q_{T-1}} - z_{q_{T-2}}))(\partial z_{q_{T-1}} / \partial q_{q_{T-1}}) \\ + \beta E_{T-1} (\partial V_{q_T}(S_T, w_T) / \partial q_{q_{T-1}}) = 0 \end{aligned}$$

The first order conditions (2) implicitly define the period T-1 best response functions for the firms:

$$\hat{p}_{q_{T-1}}(S_{T-1}, w_{T-1}, E_{T-1} w_T, p_{T-1} \setminus q) \text{ and } \hat{q}_{q_{T-1}}(S_{T-1}, w_{T-1}, E_{T-1} w_T, q_{T-1} \setminus q)$$

Combining these best response functions gives period T-1 Nash equilibrium pricing functions for the firms:

$$p_{q_{T-1}}^*(S_{T-1}, w_{T-1}, E_{T-1} w_T) \text{ and } q_{q_{T-1}}^*(S_{T-1}, w_{T-1}, E_{T-1} w_T)$$

Substituting these Nash equilibrium prices and resulting Nash equilibrium sales into the firms' objective functions gives the discounted stream of Nash equilibrium profits starting at period T-1 for each firm:

$$V_{q_{T-1}}(S_{T-1}, w_{T-1}, E_{T-1} w_T) = R_{q_{T-1}}(S_{T-1}, w_{T-1}, E_{T-1} w_T, p_{T-1}^*(\dots), q_{T-1}^*(\dots))$$

Continuing in this manner, the T-s period problems are solved for  $T \geq s \geq 1$ . The resulting equilibrium prices can be written as follows:

$$p_{qt}^*(S_t, E_t w_{\geq t}), \quad q_{qt}^*(S_t, E_t w_{\geq t}) \quad \forall q \in \mathcal{G}, \forall t$$

Imposing an initial condition that prices in each market are equal at time 0

$$p_{q0} = p_0 \quad \text{and} \quad q_{q0} = q_0 \quad \forall q \in \mathcal{G}$$

allows equilibrium prices to be expressed as a function of wages alone:

$$p_{qt}^*(E_t w_{\geq 1}), \quad q_{qt}^*(E_t w_{\geq 1}) \quad \forall q \in \mathcal{G}, \forall t$$

Since wages are determined by technology shocks in the non-traded sectors, equilibrium prices can be expressed as functions of technology shocks:

$$p_{qt}^*(E_t \{\theta_{hs}, \theta_{fs}\}_s), \quad q_{qt}^*(E_t \{\theta_{hs}, \theta_{fs}\}_s) \quad \forall q \in \mathcal{G}$$

Finally, equilibrium quantities can be specified as functions of technology shocks. Individual quantities for consumption of the endowment good and the non-traded goods cannot be determined. Only their sum,  $m_{jit} + c_{jit}$ ,  $\forall j, i, t$  will be determined.

## 5. COMPUTATIONAL EXPERIMENTS

Complexities associated with explicitly solving the dynamic game for time invariant Markov pricing functions as functions of last period's prices and current and future expected wages suggest using computational experiments to analyze the behavior of the economy in equilibrium. The focus of the analysis is the relationship among equilibrium prices and economy parameters: the number of firms, the degree of product differentiation, the size of adjustment costs, and the persistence of

technology shocks. Particular attention is given to properties of deviations from the Laws of One Price across countries and across producers.

Define the following relative prices:

$$R_{qt} = \ln(p_{qt}/q_{qt}) \quad \text{with mean} = \bar{R}_q \quad \text{for } q=h, f$$

$$S_{ht} = \ln(p_{ft}/p_{ht}) \quad \text{with mean} = \bar{S}_h$$

$$S_{ft} = \ln(q_{ft}/q_{ht}) \quad \text{with mean} = \bar{S}_f$$

Then  $\bar{R}_q$  is a measure of average deviations across countries for producer  $q \in \mathcal{Q}$  and  $\bar{S}_j$  is a measure of average deviations across producers selling in country  $j \in J$ . Non-zero values for these variables are defined to be deviations from the Laws of One Price.

In the following experiments, technology shocks to non-traded goods production are assumed to follow the following Markov processes,  $\forall j \in J$ :

$$\theta_{jt} = \rho \theta_{jt-1} + \varepsilon_{jt} \quad \text{where } \varepsilon \sim \log N(\bar{\theta}_j(1-\rho), .08) \text{ and is iid across time and across countries.}$$

Different values for the persistence parameter,  $\rho$ , are evaluated. In each of the following experiments,  $\bar{\theta}_h = 1$ . and  $\bar{\theta}_f = 1.2$ .

The results reported below are averages of five simulations resulting from five realizations of the  $\theta$  processes. The time horizon for each simulation was 100 periods with averages calculated from the results of periods 10-80. Truncating the simulations in this manner corrects for the artificial beginning and end of time effects.

In each of the diagrams which follow, curves labelled by "H"("F") denote home(foreign) producers in the figures depicting prices and deviations across countries. Curves labelled by "H"("F") denote the

home(foreign) market in the figures depicting deviations across producers.

### 5.1 Relationships Between Prices and the Number of Firms

Economy parameters for the following computational experiments are as follows:

$$\beta = .98, \quad \phi = .5$$

$$d_h = 1. \quad d_f = .5$$

$$\alpha_h = \alpha_f = 1.$$

$$\rho = .8$$

In this economy, firms price discriminate across countries according to the perceived substitution parameter,  $d_j$ , in each country. This will lead to deviations from the Law of One Price across countries. Furthermore, firms located in different countries and selling in the same market will price differently as the realizations of the  $\theta$  processes differ implying differing wages. This will lead to deviations from the Law of One Price across producers.

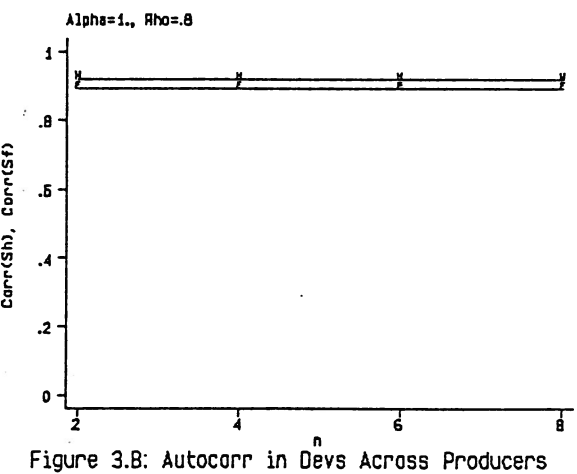
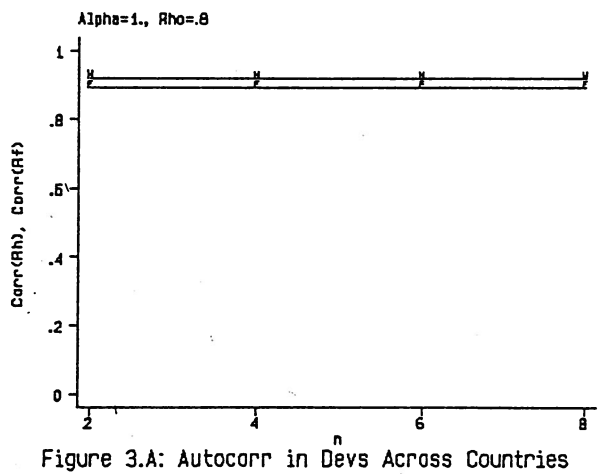
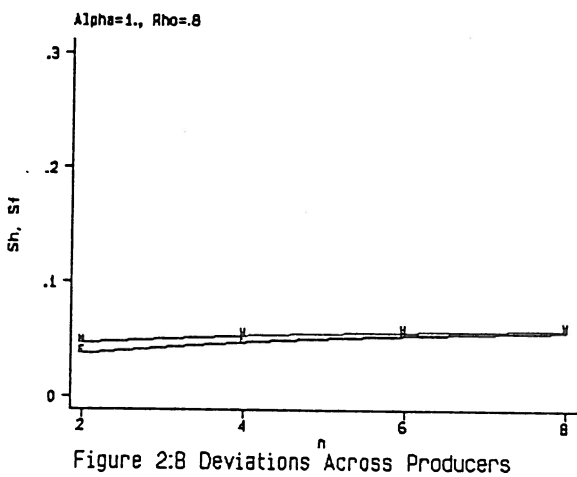
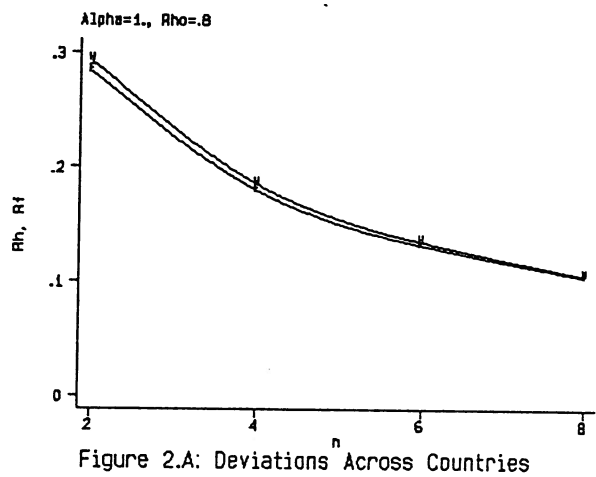
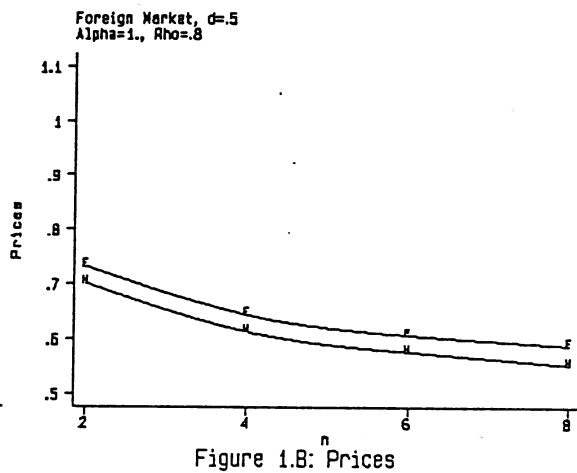
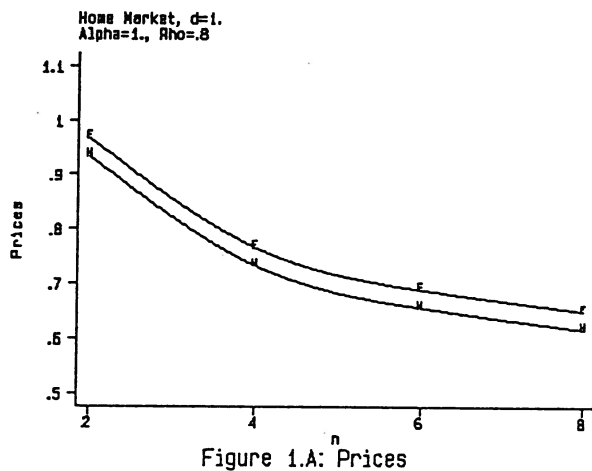
Figures 1.A and 1.B depict the relationship between price levels and the number of firms operating in the industry. These diagrams illustrate that industries which are characterized by a large number of firms will exhibit lower prices. Furthermore, foreign firms, with higher average wages, set higher prices than home firms.

Figures 2.A and 2.B depict the relationship between deviations (both across countries and across producers) and the number of firms. Figure 2.A illustrates that industries which are characterized by a large number of firms will exhibit lower deviations across countries. That is, in an industry with a large number of competitors, firms have less ability to

price discriminate across countries. Furthermore, foreign firms, facing higher costs, have less ability to price discriminate and exhibit deviations which are slightly lower than home firms.

Figure 2.B shows that a large number of firms is associated with slightly higher deviations across producers within a market. This result can be explained as follows. In industries with a large number of competitors, firms price closer to marginal cost as is clear from Figures 1.A and 1.B. However, as the number of firms increase, foreign firms decrease their prices by proportionally more than do home firms, widening the spread between prices in any market.

Figures 3.A and 3.B depict the relationship between the autocorrelation properties of deviations and the number of firms. The number of firms has a negligible effect on the persistence of deviations.





## 5.2 Relationships Between Prices and Product Substitutability

Economy parameters for the following computational experiments are as follows:

$$\beta = .98, \quad \phi = .5$$

$$d_h = d_f = d$$

$$\alpha_h = 0. \quad \alpha_f = 1.$$

$$\rho = .8$$

$$n = 2$$

In this economy, firms price discriminate across countries according to the costs of adjusting sales in that market,  $\alpha_j$ . This will lead to deviations from the Law of One Price across countries.

Figures 4.A and 4.B depict the relationship between price levels and the degree of product substitutability. These diagrams illustrate that industries which are characterized by products which are close substitutes (low  $d$ ) will exhibit lower prices.

Figures 5.A and 5.B depict the relationship between deviations (both across countries and across producers) and the degree of product substitutability. Figure 5.A illustrates that industries which are characterized by products which are close substitutes will exhibit lower deviations across countries. That is, in industries with closely substitutable products, firms have less ability to price discriminate across countries.

Figure 5.B shows closely substitutable products are associated with slightly higher deviations across producers within a market. This result can be explained by similar reasoning as the relationship between the number of firms and deviations across producers.

Figures 6.A and 6.B depict the relationship between the autocorrelation properties of deviations and the degree of product substitutability. Industries with products which are perceived to be close substitutes exhibit slightly lower persistence of deviations across countries but substitutability does not significantly effect the persistence of deviations across producers.

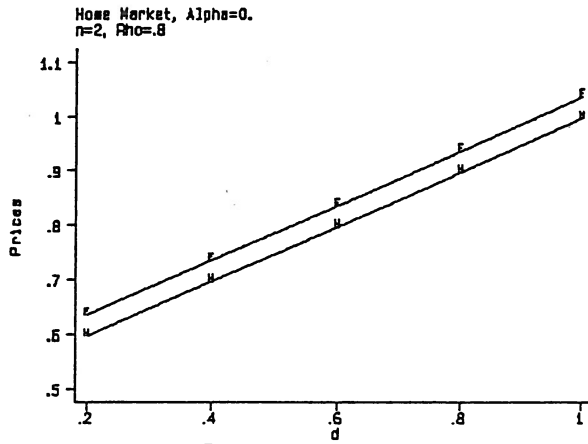


Figure 4.A: Prices

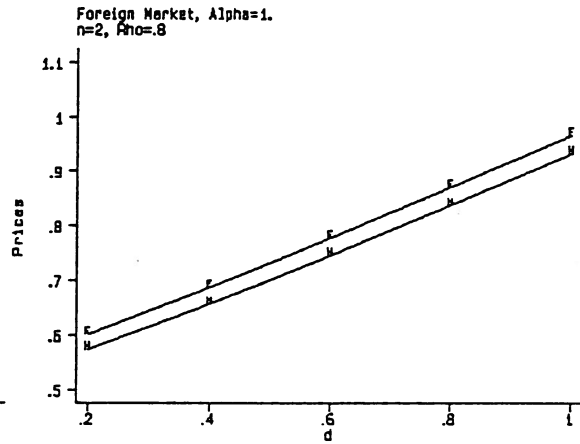


Figure 4.B: Prices

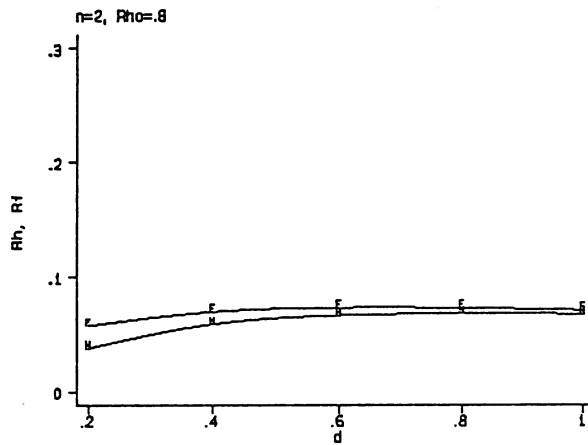


Figure 5.A: Deviations Across Countries

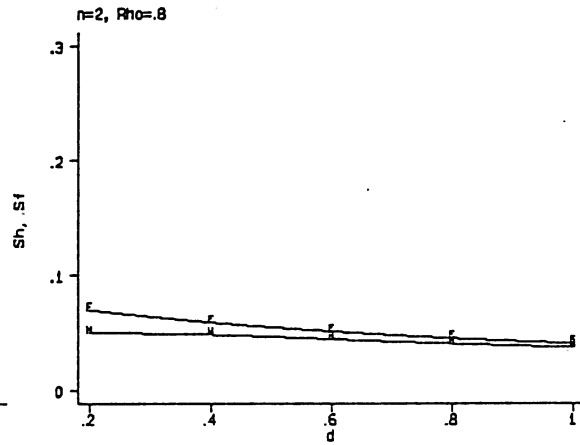


Figure 5.B: Deviations Across Producers

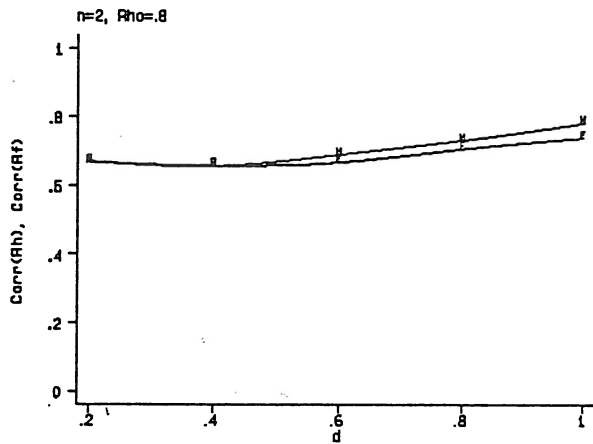


Figure 6.A: Autocorr in Devs Across Countries

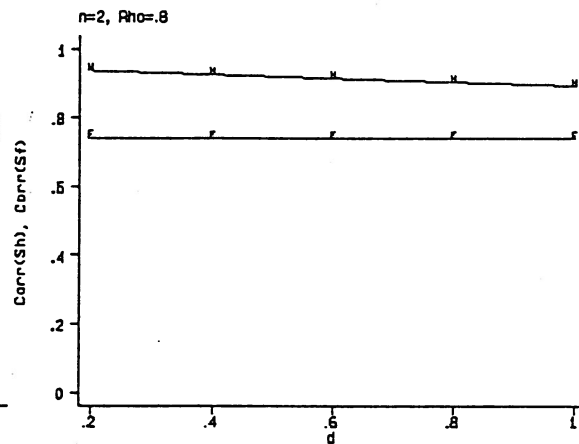


Figure 6.B: Autocorr in Devs Across Producers

### 5.3 Relationships Between Prices and Adjustment Costs

Economy parameters for the following computational experiments are as follows:

$$\beta = .98$$

$$d_h = 1. \quad d_f = .5$$

$$\alpha_h = \alpha_f = \alpha$$

$$\rho = .8$$

$$n = 2$$

Figures 7.A and 7.B depict the relationship between prices and costs associated with altering sales. The diagrams illustrate that industries which are characterized by high distributional costs will exhibit lower prices than industries with lower costs.

Figures 8.A and 8.B depict the relationship between deviations (both across countries and across producers) and costs associated with altering sales. The diagram illustrates that industries which have high adjustment costs will exhibit slightly lower deviations of both types than will industries with lower costs.

Figures 9.A and 9.B depict the relationship between autocorrelations of deviations and costs associated with altering sales. Industries with higher costs will exhibit deviations across producers which are more highly serially correlated. However, these costs have a negligible effect on the persistence of deviations across countries in this economy.

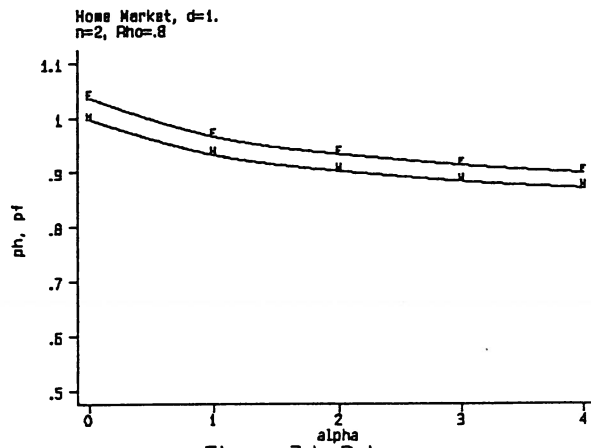


Figure 7.A: Prices

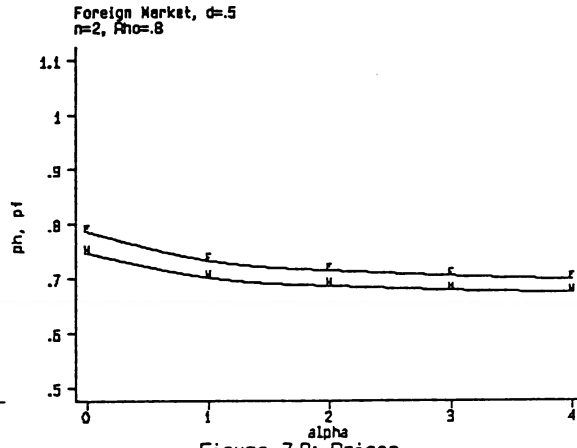


Figure 7.B: Prices

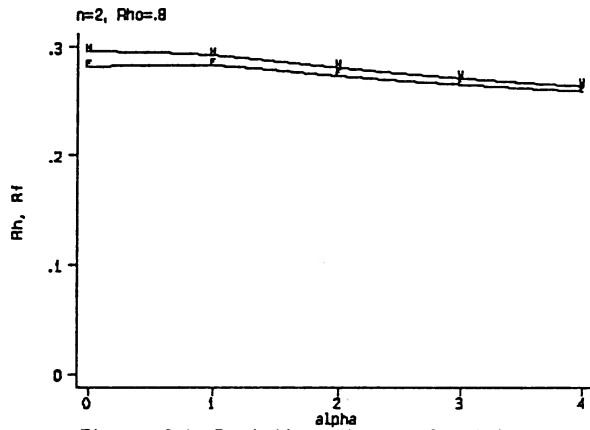


Figure 8.A: Deviations Across Countries

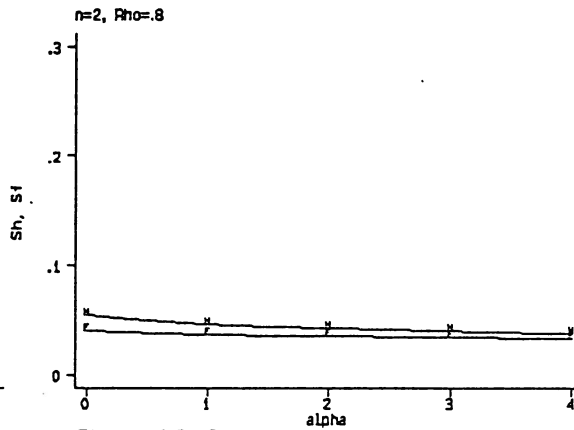


Figure 8.B: Deviations Across Producers

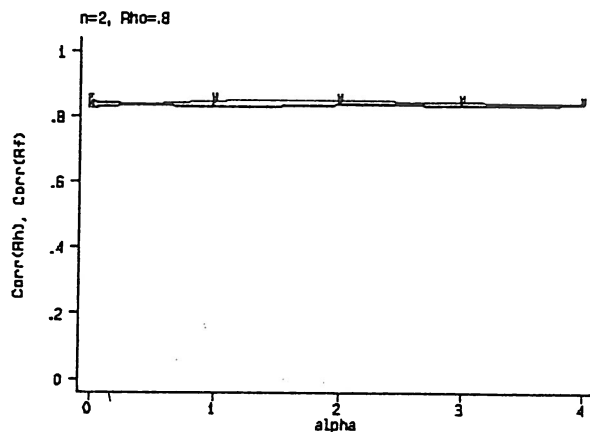


Figure 9.A: Autocorr in Devs Across Countries

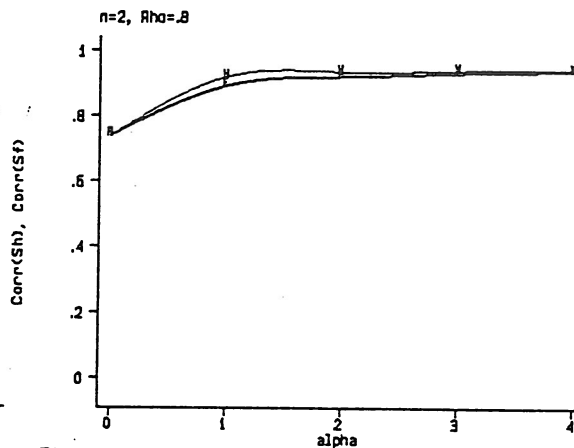


Figure 9.B: Autocorr in Devs Across Producers

#### 5.4 Relationships Between Prices and Persistence of Technology Shocks

Economy parameters for the following computational experiments are as follows:

$$\beta = .98$$

$$d_h = 1. \quad d_f = .5$$

$$\alpha_h = \alpha_f = 1.$$

$$n = 2$$

Figures 10.A and 10.B depict the relationship between prices and the persistence parameter on technology shocks to non-traded goods,  $\rho$ . These diagrams do not exhibit a monotonic relationship between persistence of these shocks and price levels.

Figures 11.A and 11.B depict the relationship between deviations (both across countries and across producers) and the persistence of technology shocks. The diagrams illustrate that the degree of persistence has a negligible effect on the size of both types of deviations.

Figures 12.A and 12.B depict the relationship between autocorrelations of deviations and the persistence of technology shocks. Countries which face more highly persistent technology shocks and, therefore, persistent wage series, will exhibit deviations of both types which are more persistent.

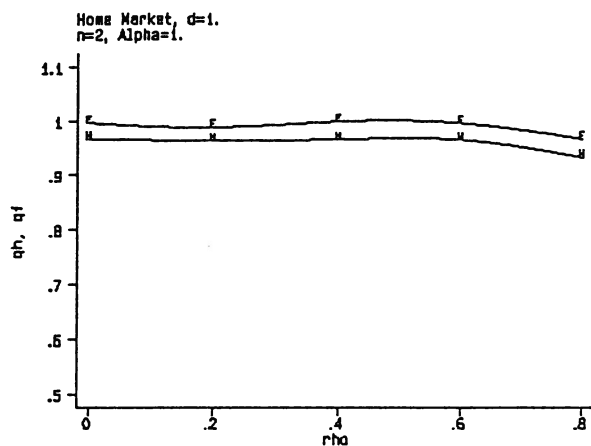


Figure 10.B: Prices

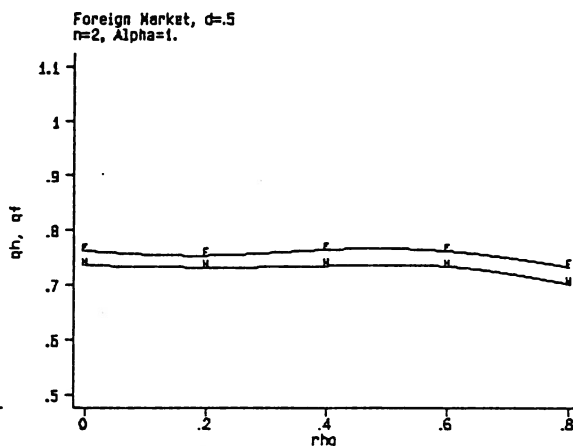


Figure 10.B: Prices

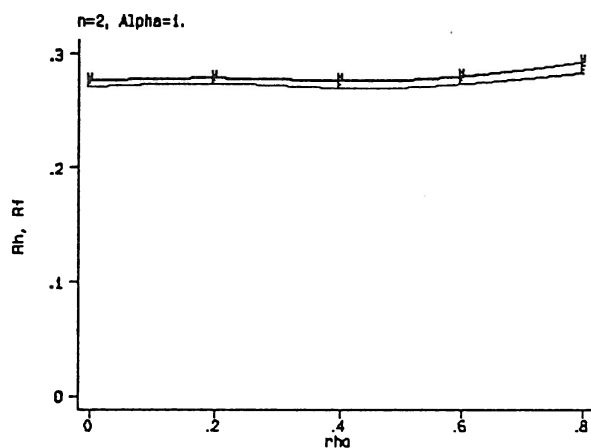


Figure 11.A: Deviations Across Countries

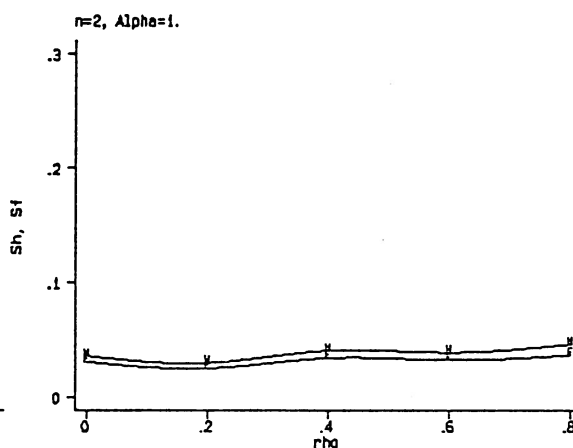


Figure 11.B: Deviations Across Producers

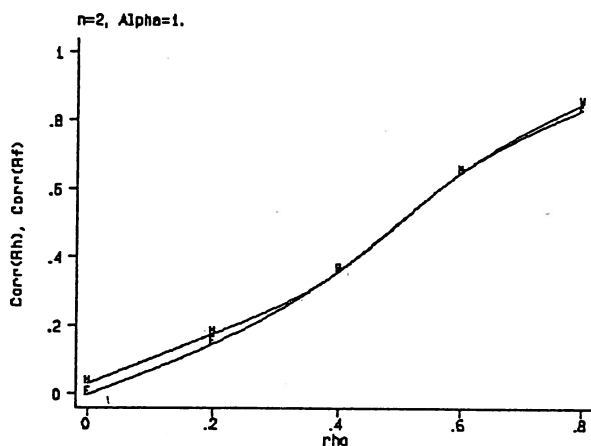


Figure 12.A: Autocorr in Devs Across Countries

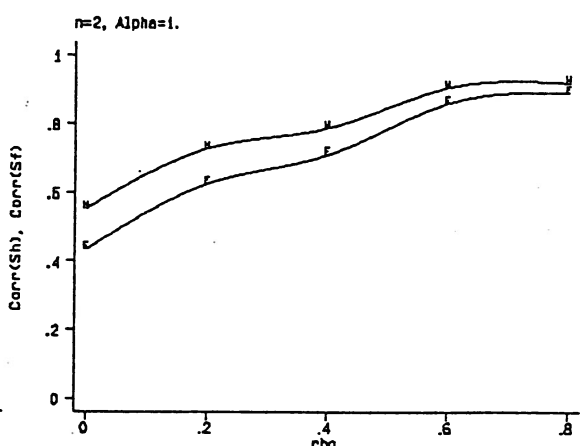


Figure 12.B: Autocorr in Devs Across Producers

## 5.5 Summary

The above computational experiments for the economy in equilibrium exhibit persistent deviations from the Laws of One Price across countries and across producers selling within a country. Deviations across countries result from price discrimination by producers as markets differ. Deviations across producers result as producers, located in different countries, facing differing wages, price differently.

Deviations across countries will be higher in industries which are characterized by a small number of firms, and/or products which are not highly substitutable, and/or low costs of adjusting sales volume. The persistence of these deviations is primarily determined by the persistence of technology shocks and is virtually unaffected by the number of firms, the degree of product substitutability, and the level of adjustment costs.

Deviations across producers will be higher in industries which are characterized by a large number of firms, and/or products which are highly substitutable, and/or low costs to adjusting sales volume. The persistence of these deviations is positively correlated with persistence of technology shocks and with adjustment costs, but are unaffected by the number of firms and the degree of product substitutability.

## 6. CONCLUSIONS

Dynamic, strategic, equilibrium pricing behavior of international differentiated oligopolists was examined in a stochastic general equilibrium framework. Preferences and technology in this economy are such that the behavior of differentiated firms has no effect on



equilibrium wages.

The price setting behavior of these firms leads to persistent deviations from the Laws of One Price across countries and across producers. The results of the computational experiments are generally consistent with the empirical regularities discussed in the introduction. Relationships between industry characteristics and properties of both types of deviations were suggested by results of computational experiments.

Extensions include altering preferences and technology so that the behavior of differentiated oligopolists affects equilibrium wages. Such an economy is explored in Lapham(1990) and appears to lead to deviations across countries which are more highly correlated than in the economy examined in the present paper. Incorporating currencies and endogenous exchange rates into the present framework may also provide an analysis of the issues discussed in the exchange rate pass-through literature.

## NOTES

1. The real exchange rate in this work is defined as the rate at which a basket of goods produced in one country trades for that basket of goods produced in another country.
2. It should be noted that exchange rate pass-through, an issue which has received a great deal of attention in recent work, is an application of the second version of the Law of One Price for evaluating price movements resulting from nominal exchange rate movements. Investigating the amount of an exchange rate movement which is "passed through" to prices is simply an evaluation of how well the Law of One Price holds. Incomplete pass-through is a deviation from the Law of One Price while complete pass-through indicates that price movements have directly offset exchange rate movements so as to maintain the Law of One Price.
3. For related work in this area, see Feenstra(1987), Fisher(1987), Flood(1987), Foster(1986), and Mann(1986).

## REFERENCES

- Baldwin, Richard, "Some Empirical Evidence on Hysteresis in Aggregate U.S. Import Prices," NBER Working Paper No. 2483, January 1988.
- , "Hysteresis in Import Prices: The Beachhead Effect," First Boston Working Paper, February 1987.
- , "Hysteresis in Trade," unpublished manuscript, Massachusetts Institute of Technology, 1986.
- , and Krugman, Paul, "Persistent Trade Effects of Large Exchange Rate Shocks," NBER Working Paper No. 2017, September 1986.
- Cassel, Gustav, "Abnormal Deviations in International Exchanges," Economic Journal, December 1918, 28: 413-415.
- Crouhy-Veyrac, Liliane; Crouhy, Michel; and Mélitz, Jacques, "More About the Law of One Price," European Economic Review, 18, 1982, 325-344.
- Dohner, Robert S., "Export Pricing, Flexible Exchange Rates, and Divergence in the Prices of Traded Goods," Journal of International Economics, 1984, 16: 79-101.
- Dornbusch, R., "Exchange Rates and Prices," American Economic Review, 77, 1987, 93-106.
- Dixit, Avinash, "Hysteresis, Import Penetration, and Exchange Rate Pass-Through," Princeton mimeo, November, 1987.
- , "Entry and Exit Decisions of Firms Under Fluctuating Real Exchange Rates," Princeton mimeo, October, 1987.
- Feenstra, Robert, "Symmetric Pass-Through of Tariffs and Exchange Rates under Imperfect Competition: An Empirical Test," NBER Working Paper No. 2453, December 1987.
- Fisher, Eric, "Exchange Rate Pass-Through and the Relative Concentration of German and Japanese Manufacturing Industries," Cornell Working Paper, November 1988.
- , "A Model of Exchange Rate Pass-Through," unpublished manuscript, Cornell University, 1987.
- Flood, Eugene, "An Empirical Analysis of the Effect of Exchange Rate Changes on Goods Prices," unpublished manuscript, Stanford University, 1987.
- Foster, Harry, "The Relationship Between U.S. Import Prices and the Exchange Rate: An Empirical Investigation," MIT mimeo, December, 1986.

- , and Baldwin, Richard, "Marketing Bottlenecks and the Relationship between Exchange Rates and Prices," MIT mimeo, October, 1986.
- Froot, Kenneth and Klemperer, Paul, "Exchange Rate Pass-Through When Market Share Matters," American Economic Review, 79, September 1989, 637-654.
- Giovannini, Alberto, "Exchange Rates and Traded Goods Prices," Journal of International Economics, 24, 1988, 45-68.
- Isard, Peter, "How Far Can We Push the 'Law of One Price'?", American Economic Review, 67, 1977, 942-948.
- Jones, R.W. and D.D. Purvis, "International Differences in Response to Common External Shocks: The Role of Purchasing Power Parity," In E. Classen and P. Salin, eds., Recent Issues in the Theory of Flexible Exchange Rates, Amsterdam: North-Holland, 1983.
- Kalter, Eliot, "The Effect of Exchange Rate Changes Upon Matched Domestic and Export Prices," Unpublished Ph.D. Dissertation, University of Pennsylvania, 1978.
- Knetter, Michael M., "Price Discrimination by U.S. and German Exporters," American Economic Review, March 1989.
- Kravis, Irving B. and Lipsey, Robert E. "Price Behavior in the Light of Balance of Payments Theories," Journal of International Economics, 1978, 8: 193-246.
- , and ———, "Export Prices and the Transmission of Inflation," American Economic Review, 1977, 67: 155-163.
- , and ———, "International Trade Prices and Price Proxies," in Nancy D. Ruggles, ed., The Role of the Computer in Economic and Social Research in Latin America, NBER, New York, 1974, 253-268.
- Krugman, Paul, "Pricing to Market When the Exchange Rate Changes," in J. David Richardson and Sven Arndt, eds., Real Financial Linkages Among Open Economies, MIT Press, 1988.
- , and Baldwin, Richard, "The Persistence of the U.S. Trade Deficit," Brookings Papers on Economic Activity, No. 1, 1-43, 1987.
- Lapham, Beverly, "An Empirical Examination of the Properties of Deviations from the Laws of One Price Using West German Data," mimeo, August 1990.
- , "Further Analysis of Theoretical Deviations from the Law of One Price," mimeo, August 1990.
- Mann, Catherine, "Prices, Profit Margins, and Exchange Rates," Federal Reserve Bulletin, 72, 1986, 366-379.

Marston, Richard C., "Pricing to Market in Japanese Manufacturing," NBER Working Paper No. 2905, March 1989.

Officer, Lawrence H., "The Purchasing Power Parity Theory of Exchange Rates: A Review Article," IMF Staff Papers, March 1976, 23: 1-60.

Protopapadakis, Aris and Stoll, Hans, "The Law of One Price in International Commodity Markets: A Reformulation and Some Formal Tests," Journal of International Money and Finance, 5, 1986, 335-360.

———, and ———, "Spot and Futures Prices and the Law of One Price", The Journal of Finance, 5, December 1983, 1431-1455.

Richardson, David, "Some Empirical Evidence on Commodity Arbitrage and the Law of One Price," Journal of International Economics, 8, May 1978, 341-351.

Salop, Steven, "Monopolistic Competition with Outside Goods," Bell Journal of Economics, 1979, 141-56.

Shapiro, Alan, "What Does Purchasing Power Parity Mean?," Journal of International Money and Finance, December, 1983, 2: 295-318.