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The Efficiency Effects of Discrete Tax Rate Changes Without Lump-Sum Taxes and Transfers

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The Efficiency Effects of Discrete Tax Rate Changes Without Lump-Sum Taxes and Transfers

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Changes Without Lump-Sum Taxes and Transfers

DISCUSSION PAPER #782

by

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Abstract

An exact expression for the normalized change in utility resulting from discrete changes in tax rates on economic activities is derived using certain linearity assumptions in preferences. This expression is used to consider alternative ways of identifying the resulting efficiency cost (or excess burden) when lump-sum taxes and transfers are assumed to be unavailable to compensate for revenue changes. Benchmark tax structures other than lump-sum taxation, specifically Ramsey optimal and uniform tax structures, are used to identify the efficiency cost of revenue neutral tax changes. The results are used to explain and extend various known results on the efficiency effects of such tax changes as well as to make an efficiency case for avoiding arbitrary departures from uniformity even when lump-sum taxes are infeasible. Finally, the theoretical shortcomings of considering revenue neutral tax changes are considered and an alternative methodology is introduced.

1. Introduction.

When analyzing the efficiency effects of "partial " tax reforms from a non-optimal point, it is commonly assumed that the residual revenue changes can be balanced with lump-sum taxes or transfers. In this case, a straightforward measure of the efficiency cost of a tax change can be obtained, at least if the fiction of a representative consumer is maintained. When lump-sum tax/transfers are precluded and an untaxable economic activity is allowed, the picture is far less clear. While well-known conditions describing an optimal structure of tax rates across taxable activities can be derived, methodologies for analysing and quantifying the effects of partial reforms are limited. Two approaches are available. The gradient projection approach (e.g., Tirole and Guesnerie [1981]) looks for differentially small tax changes that yield the greatest improvement. Revealed preference approaches (e.g., Pazner and Sadka [1981]) can be used to analyze discrete changes in tax rates but provide sufficient conditions only for tax changes to increase or decrease economic efficiency.

These developments have left an unsatisfactory state of affairs in the analysis of partial tax reforms. Caveats regarding the availability of lump-sum tax/transfers reduce the acceptability of applied analyses which rely on measured excess burdens, while an "anything can happen" philosophy undermines the traditional argument for neutral taxation. In this paper, I address these issues in a limited way. Primarily, I derive an exact measure (that is, one providing necessary and sufficient conditions for efficiency

gains) for the impact on utility of discrete tax changes when certain linearity assumptions are made on the compensated demands for taxable goods. While the analysis does require certain restrictions on preferences, it allows me to examine and quantify the effects of discrete tax changes in circumstances where lump-sum tax/transfers are ruled out. I use this measure to generalize some of the results obtained from the revealed preference approach mentioned above. I also use it to separate the primary and secondary burdens associated with a set of tax rates using "benchmark" tax structures other than those of the lump-sum type. Among other things I re-examine the efficiency case for uniform taxation. I argue that even where conditions imply that a uniform tax rate structure is not necessarily desirable, large variations in effective tax rates across different economic activities should be avoided unless there is positive evidence that they are in the "right direction". Finally, I examine a short-coming of what I call the "differential efficiency" approach most commonly used by analysts and suggest an alternative "balanced budget" approach.

2. The Formal Analysis.

In order to focus on efficiency considerations I use the case of a single representative consumer. Correspondingly I assume that preferences are quasi-homothetic (Gorman polar form) so the Engels curves are linear. As Diewert [1976] has noted, the demand functions implied by such preferences can be regarded as a local first-order approximation to any demand system (on this, also see Deaton and Muellbauer [1980, pp. 144-145]). I further assume

that the compensated demand functions for the taxed goods are linear (i.e. $\partial S_{ij}/\partial p_k = 0$ for $i, j, k = 1, \dots, N-1$) and parallel (i.e., $\partial S_{ij}/\partial U = 0$ for i and $j = 1, \dots, N-1$) where C_i is consumption of good i , S_{ij} is the Slutsky substitution term and U is utility of private consumption¹. As will be seen, these assumptions permit a simple solution to the usual index number problem that must be solved in order to consider discrete tax changes.

Further simplifying assumptions are invoked, none of which bears directly on the issues to which this paper is directed. I assume Ricardian technology so producer prices are fixed and can be normalized at unity. Also, the tax revenue is used to finance a public good which yields utility to the consumer in a form which is additively separable from the utility of private goods. In this way, variations in the level of revenue, should they occur, do not affect the level of demands for the private goods except through the income effect of the tax revenue extracted from the private sector.

I assume the government collects T in tax revenue by imposing taxes $t = (t_1, \dots, t_{N-1})$ on private economic activities $C = (C_1, \dots, C_{N-1}) \geq 0$ (which for concreteness shall subsequently be identified as consumption goods) and spends the proceeds on the public good, which is produced at a constant marginal cost of unity, and which yields utility to the consumer, if any, in

¹It is well known that these conditions cannot hold for all goods. The force of this assumption is to "hide" the curvature changes in the non-taxed composite good.

an additively separable form. There is an untaxed commodity activity $C_N > 0$ which is also the numeraire good. The consumer maximizes $U(C, C_N)$ subject to $p \cdot C + C_N = M$ where the value of the endowment at producer prices M is fixed and $p_i = 1 + t_i$ for $i = 1, \dots, N-1$. $U(\cdot)$ is strictly quasi-concave (and quasi-homothetic as noted) and the indirect utility function of private economic activities is denoted $V(p, M)$ and has the usual properties.

Let $C(p, U)$ denote the Hicksian demand system for the taxed activities. Producer prices are fixed so $\partial C_i / \partial p_j = \partial C_i / \partial t_j = S_{i,j}$ and $S = S_{i,j}$ for $\begin{Bmatrix} i=1, \dots, N-1 \\ j=1, \dots, N-1 \end{Bmatrix}$ denotes the $N-1$ by $N-1$ Slutsky matrix for the taxed goods with the usual properties. Also, $\partial C / \partial U = \{\partial C_1 / \partial U, \dots, \partial C_{N-1} / \partial U\}$ is constant for the purposes of analysis since the Engel's curves are linear and $\partial S_{i,j} / \partial U = 0$ for $i, j = 1, \dots, N-1$. Let $\lambda = \frac{\partial V}{\partial M}$ denote the marginal utility of income and $\partial C / \partial M = \lambda \cdot \partial C / \partial U$. Further let the superscript $h = 0, 1$ denote the pre- and post-tax change values of a variable respectively. The superscript is omitted where the value of the variable is unchanged.

Tax revenue T from non-lump-sum taxes is equal to $t \cdot C$, so we can write²

$$\Delta T = C^0 \cdot \Delta t + t^1 \cdot \Delta C \quad (1.1)$$

$$= C^1 \cdot \Delta t + t^0 \cdot \Delta C \quad (1.2)$$

where $\Delta x = x^1 - x^0$. Since by the linearity assumptions $\Delta C = S \cdot \Delta t + \left(\frac{\partial C}{\partial U}\right) \cdot \Delta U$, the change in tax revenue can be expressed

²Throughout, a vector is a column or row as needed to be conformable.

$$\Delta T = C^0 \cdot \Delta t + t^1 \cdot S \cdot \Delta t + \beta^1 \cdot (\lambda^1)^{-1} \cdot \Delta U \quad (2.1)$$

$$= C^1 \cdot \Delta t + t^0 \cdot S \cdot \Delta t + \beta^0 \cdot (\lambda^0)^{-1} \cdot \Delta U \quad (2.2)$$

where $\beta^h = t^h \cdot \left(\frac{\partial C}{\partial M}\right)^h$ for $h = 0, 1$ can be interpreted as the "tax recovery" rate on hypothetical lump-sum transfers under the initial and terminal tax structures respectively, and $(1-\beta^h)^{-1}$ is the "tax multiplier" which is assumed to be positive.

The level of utility can be chosen to equal the level of expenditure along a given income expansion path. For example, if one normalizes along the expansion path corresponding to pre-tax reform consumer prices p^0 so $\lambda(p^0)$ is a constant along the expansion path (i.e., $\frac{\partial \lambda(p^0)}{\partial U} = 0$) then $\Delta U / \lambda^0$ can be interpreted as the equivalent variation (see McKenzie and Pearce 1976). Expanding the indirect utility function in a Taylor's series yields

$$\Delta U = -\lambda^0 \cdot \left\{ C^0 \cdot \Delta t + \Delta M + \frac{1}{2} \left[(\Delta t \cdot S \cdot \Delta t - 2\gamma^0 \cdot (C^0 \cdot \Delta t + \Delta M)) \cdot \sum_{i=0}^{\infty} (-\gamma^0)^i \right] \right\} \quad (3)$$

where M is lump-sum transfer income and $\gamma^0 = \Delta t \cdot \left(\frac{\partial C}{\partial M}\right)^0$. It is assumed that $0 \leq |\gamma^0| < 1$. Since $\sum_i (-\gamma^0)^i = (1+\gamma^0)^{-1}$ we can write (3) as

$$(1+\gamma^0) \cdot \Delta U = -\lambda^0 \cdot \left[C^0 \cdot \Delta t + \Delta M + \frac{1}{2} \cdot \Delta t \cdot S \cdot \Delta t \right] \quad (3')$$

Further, we can expand the marginal utility of income function in a Taylor series at p^0, M^0 to get $\lambda^1 = \lambda^0 \cdot \sum_i (-\gamma^0)^i$ so $\lambda^1 / \lambda^0 = (1+\gamma^0)^{-1}$. It also follows that $(1-\beta^1) / (1-\beta^0) = (1+\gamma^0)^{-1}$.

We can now substitute (2.1) into (3') and use $t^1 = t^0 + \Delta t$ to get

$$\frac{\Delta U}{\lambda^h} = \frac{1}{1-\beta^h} \cdot \left[t^0 \cdot S \cdot \Delta t + \frac{1}{2} \Delta t \cdot S \cdot \Delta t + \Delta M - \Delta T \right]. \quad (4)$$

The normalized change in utility is found to be equal to a tax multiplier times the term in square brackets which will henceforth be referred to as the "kernel" of expression (4).

There are several things to note about expression (4). First it is an exact expression under the linearity assumptions made on the compensated demand functions for the taxed goods which holds for any vector of changes in the tax vector. Remember, however, that when $\Delta M = 0$, the total change in the utility of the consumer must include the change in utility resulting from any changes in the amount of revenue collected. Second, if lump-sum transfers are feasible, we can separate the primary and excess burdens of the tax rates changes by setting $\Delta M - \Delta T = 0$. Further, if we set $t^0 = 0$ and evaluate the equivalent variation version of (4) (that is, $h = 0$) so $\beta^0 = 0$, we obtain the familiar quadratic loss or "Harberger expression" $\Delta U / \lambda^0 = \frac{1}{2} \cdot \Delta t \cdot S \cdot \Delta t$ which represents the efficiency cost of collecting the tax revenue using the "distortionary" tax vector t rather than using lump-sum taxation. Third, note that we can substitute $\Delta t = t^1 - t^0$ in the first two terms in the kernel of expression (4) so that becomes (after canceling terms) $\frac{1}{2} \cdot t^1 \cdot S \cdot t^1 - \frac{1}{2} \cdot t^0 \cdot S \cdot t^0 + \Delta M - \Delta T$. Thus the quadratic loss part of (4) acts as a linear operator. That is, the efficiency cost of changing the vector of tax rates from t^0 to t^1 is just the difference between the efficiency costs of each tax vector as compared to the no-tax case.

3. Analysing the Differential Efficiency Effects of Tax Changes.

The difficulty in analyzing the efficiency impact of tax rate changes without the revenue balancing effects of lump-sum taxation is evident from expression (4). Setting $\Delta M=0$, we see that the change in utility includes the "primary burden" of the change in the revenues collected ΔT . In order to concentrate on the efficiency impact of tax changes, it is common to consider only those tax changes that leave the level of tax revenue unchanged so $\Delta T = 0$. Because this methodology is similar to the "differential incidence" methodology, it is sometimes called "differential efficiency cost" approach. Continuing this analogy, I will also discuss briefly in a later section a "balanced budget efficiency cost" in which changes in tax rates are associated with changes in government spending.

The main problem with the differential efficiency cost analysis is that, assuming that positive tax revenue is desired, it makes no sense to assume $t^0 = 0$ in (4). Thus the efficiency impact of a revenue neutral change in tax rates depends on what the new tax rates are compared to. The most practical approach is to assume that t^0 is the existing tax structure. However, this benchmark appears to provide little in the way of general guidance about how to change the tax rates for the better or about the desirability of the absolute tax structures (that is, the t) themselves. The efficiency effects of adopting a particular t depend on "where you are starting from". (For example, Atkinson and Stiglitz [1980, pp. 382-5] show that when $t^0 \approx 0$,

utility may fall even if the tax changes reduce the revenue collected. This can be ascertained from (4)).

As it happens, this is an overly pessimistic assessment. First, even using the status quo benchmark some general statements regarding the efficiency impact of revenue neutral tax changes which are based on the partial revenue effects of the tax changes have been derived by Pazner and Sadka [1981] using revealed preference reasoning. I begin by using the analysis of this paper to review and extend these results. Second, one can measure and analyze the efficiency impact of revenue neutral tax rate changes relative to some abstract benchmark other than lump-sum taxation. In particular I will measure the efficiency impact of revenue neutral discrete tax rate changes away from the Ramsey optimal and the uniform (same tax rate on all taxable goods) tax structures respectively.

3.1. Efficiency Inferences From Partial Revenue Changes.

Pazner and Sadka [1981] demonstrate by the means of a revealed preference argument that $t^1 \cdot \Delta C \geq 0$ (that is the tax change would generate an increase in revenue from induced changes in private consumption at the post-change tax rates) is sufficient for a revenue neutral tax change to improve efficiency and make the consumer better off. This follows directly from the fact that the consumer could have purchased the initial consumption bundle at prices which are inclusive of the new tax rates.

The result is easily ascertained from the analysis in section 2. Set $\Delta T = 0$ in equation (1.1) and substitute for $C^0 \cdot \Delta t$ from equation (3') to get

$$\Delta U / \lambda^1 = t^1 \cdot \Delta C - \frac{1}{2} \Delta t \cdot S \cdot \Delta t. \quad (5.1)$$

Equation (5.1) yields the Pazner and Sadka result immediately as $\Delta t \cdot S \cdot \Delta t \leq 0$ because S is negative definite. Furthermore, $\lambda^1 > 0$ so $t^1 \cdot \Delta C \geq 0$ implies $\Delta U \geq 0$. A complementary result, which also can be proved by a revealed preference argument, is also found in Pazner and Sadka³. Substitute $t^0 = t^1 - \Delta t$ in the first term on the right hand side of (5.1) and substitute for ΔC in the $\Delta t \cdot \Delta C$ term to get

$$\Delta U / \lambda^0 = t^0 \cdot \Delta C + \frac{1}{2} \Delta t \cdot S \cdot \Delta t. \quad (5.2)$$

Equation (5.2) implies that a tax change that would generate a loss in revenue from induced consumption changes at the initial tax rates, that is $t^0 \cdot \Delta C \leq 0$, is sufficient for $\Delta U \leq 0$ ⁴. Again this follows from the fact that S is negative definite and $\lambda^0 > 0$.

While (5.1) and (5.2) yield sufficient conditions for inferences about the efficiency effects of revenue neutral tax changes, we can obtain some necessary conditions by taking the contrapositives. Specifically, $t^1 \cdot \Delta C < 0$ is necessary for the tax change to reduce utility and $t^0 \cdot \Delta C > 0$ is necessary

3A related proposition is found in Harris [1979].

⁴The revealed preference reasoning is that when producer prices are fixed, $t^0 \cdot \Delta C \leq 0$ implies that the new consumption bundle was available at the initial tax inclusive prices.

for it to increase utility. Note that neither condition (5.1) or (5.2) by itself provides a necessary and sufficient condition for inferring the sign of ΔU . However, we can average (5.1) and (5.2) to get a necessary and sufficient condition for a revenue neutral tax reform to increase or decrease the utility of the representative consumer. That is,

$$\Delta U \cdot (1/\bar{\lambda}) = \bar{t} \cdot \Delta C \quad (5.3)$$

where $(1/\bar{\lambda}) = \frac{1}{2}(1/\lambda^0) + \frac{1}{2}(1/\lambda^1) > 0$ is a reciprocal average of the marginal utility of income and $\bar{t} = \frac{1}{2}t^0 + \frac{1}{2}t^1$ is an arithmetic average of the tax vectors. If the partial revenue effect of the tax change is to increase (decrease) revenue from induced consumption changes at the average of the initial and terminal tax rates, then utility increases (decreases) accordingly. The reader is warned that this result only holds for equal revenue tax reforms and does not have a revealed preference proof because it requires the linearity assumptions in its derivation.

Further, if we assume that $\Delta C_i \gtrless 0$ as $\Delta t_i \gtrless 0$ --that is, the own tax effect on the consumption of each good always dominates and is negative--we can establish that

$$t^0 \cdot \Delta C \geq \bar{t} \cdot \Delta C \geq t^1 \cdot \Delta C. \quad (6)$$

It follows then that $t^0 \cdot \Delta C > 0$ is both necessary and sufficient for a revenue neutral tax change to increase efficiency and that $t^1 \cdot \Delta C < 0$ is both necessary and sufficient for a revenue neutral tax change to reduce efficiency.

3.2 The Efficiency Effects of Changes From Benchmark Tax Structures.

In the conventional efficiency cost measurement analyses, the benchmark tax structure, either implicitly or explicitly, is lump-sum taxation. That is, the efficiency cost of a tax structure t is the loss in utility as compared to obtaining the same tax revenue using a lump-sum tax. This methodology of measuring efficiency cost by comparing a particular tax structure to an abstract one can be extended to the case where lump-sum taxation is precluded by utilizing a more "acceptable" benchmark. I consider the Ramsey optimal and the uniform benchmarks. In both cases I assume that the initial tax structure t^0 is the benchmark so that the efficiency cost measure can be interpreted as the cost of being away from the benchmark structure (which may be negative in the case of the uniform benchmark).

In the case of the Ramsey benchmark, I let $t^0=t^*$ where t^* satisfies $t^* \cdot S = \theta^* \cdot C^*$ where C^* is the consumption vector chosen by the private sector when the desired revenue is collected using the Ramsey optimal tax structure t^* , and $\theta^* = (1-\mu^* \cdot (1-\beta^*)) \cdot (\mu^*)^{-1}$. In this last expression, β^* is the tax recovery rate and μ^* is the marginal cost of revenue (or public funds) all evaluated at t^* . This is the standard first-order necessary condition for Ramsey optimal taxation. As usual, I assume that the sufficiency conditions are satisfied. We can now set $t^0=t^*$ and $\Delta T=0$ in (4) and substitute for $t^* \cdot S$ to get

$$\frac{\Delta U \cdot (1-\beta^*)}{\lambda^*} = \frac{1}{2} \left[t^1 \cdot S \cdot t^1 - \theta^* \cdot T \right]. \quad (4')$$

Thus the efficiency cost (i.e., the negative of the expression in (4')) of

imposing some tax structure t^1 rather than raising the same amount of revenue using the optimal tax structure t^* is just the Harberger quadratic loss expression less some fraction of the tax revenue collected. The latter represents the unavoidable efficiency cost (relative to the no taxation case) of collecting the revenue when lump-sum taxes are precluded.

Of course, λ^* , β^* and θ^* are unknowns. But as long as the efficiency costs of different tax structures are evaluated consistently with respect to the Ramsey benchmark, it doesn't matter. One needs only to evaluate the different tax rate structures that raise the given amount of tax revenue to be collected and evaluate them according to the standard quadratic loss measure. The lower the efficiency cost according to this measure, the better. Of course the magnitude of the quadratic loss efficiency cost measure is an overestimate of the potential improvements possible through tax reform because some fraction of it (i.e., $\frac{1}{2}\theta^* \cdot T$) is unavoidable⁵.

While the Ramsey tax structure is an attractive theoretical benchmark, it is considered by some to be "too abstract" for applied tax policy analysis. The only other natural benchmark that I can think of is the uniform tax rate structure in which all goods that can be taxed are taxed at the same *ad valorem* rate. Of course, an important implication of precluding

⁵While the unavoidable "efficiency cost" is unknown without determining the Ramsey optimal tax structure, it is bounded above by half of the tax revenue to be collected. Letting the marginal cost of revenue $\mu^* \rightarrow \infty$ in the expression given for θ^* above, we see that $-\frac{1}{2}\theta^*T \rightarrow \frac{1}{2}(1-\beta^*) \cdot T \leq \frac{1}{2}T$ since $\beta^* \geq 0$.

lump-sum tax/transfers is that uniform tax rates on those commodities that can be taxed need not, and generally will not, be demanded by economic efficiency. Nevertheless there are good administrative and political economy reasons for preferring the uniform structure apart from the efficiency objectives⁶. For these reasons alone it is a meaningful benchmark.

Sadka (1977) derives necessary and sufficient conditions under which uniform tax rates are optimal in the presence of non-taxable activities, but these results imply nothing about whether a particular deviation from uniform taxation is desirable or not. Pazner and Sadka [1981] provide a condition under which a revenue neutral tax reform to a uniform tax rate structure is efficiency improving. This condition is that, with constant producer prices, the unification of tax rates is (weakly) efficiency improving if the value added of taxed commodities does not fall. This is apparent from equation (5.1). Let $t_i^1 = u > 0$ for all $i=1, \dots, N-1$, where u is the uniform tax rate. Then $t^1 \cdot \Delta C = u \cdot \sum_i^{N-1} \Delta C_i$, so $\sum_i^{N-1} \Delta C_i > 0$ is sufficient for an efficiency improvement. Note that by a similar argument, equation (5.2) implies that $\sum_i^{N-1} \Delta C_i < 0$ is a sufficient condition for a movement away from uniform tax rates to reduce economic efficiency.

In order to consider further the efficiency implications of introducing revenue neutral variations in tax rates away from a uniform benchmark tax

⁶Primarily, the existence of tax rate differentials may induce undesirable product transformations as well as intense political lobbying for preferential rates on other (inappropriate from an efficiency sense) commodity groups.

rate structure, set $\Delta T = 0$ and $t^0 = u = \{u, \dots, u\}$ in the kernel of expression (4). The first term in the kernel can now be expanded as $u \cdot \sum_i \sum_j S_{ij} \cdot \Delta t_j$. Also, $\sum_i S_{ij} = -(1+u)^{-1} \cdot S_{Nj}$ by the zero rank condition. Multiplying and dividing $-(1+u)^{-1} \cdot S_{Nj}^0$ by C_j^0 , we can express the second term as $-u \cdot \sum_j \eta_{jN}^u \cdot C_j^u \cdot \Delta t_j$ where η_{jN}^u is the compensated cross elasticity of taxed good j with respect to the relative price of the untaxed numeraire good at C_j^u , the level of consumption of good j at the uniform tax equilibrium. Further, let $\bar{\eta}^u = \sum_j^{N-1} \omega_j \cdot \eta_{jN}^u > 0$ denote the compensated cross elasticity of the taxed good composite with respect to the price of the untaxed numeraire good, where $\omega_j = \frac{C_j^u}{\sum_i C_i^u}$ is the relative weight on taxed good j . The elasticity $\bar{\eta}^u$ must be positive because the composite taxed good and the untaxed good must be substitutes. Now add and subtract $u \cdot \bar{\eta}^u \cdot C^u \cdot \Delta t$. After collecting terms, substituting for $C^u \cdot \Delta t$ from (3'), and some rearrangement, (4) can be written

$$\frac{\Delta U}{\lambda^1} = \frac{1}{1 - \beta^1 - u \cdot \bar{\eta}^u} \left[T \cdot \sum_j^{N-1} \omega_j (\bar{\eta}^u - \eta_{jN}^u) \cdot \Delta t_j + \frac{1}{2} (1 + u \cdot \bar{\eta}^u) \cdot \Delta t \cdot S \cdot \Delta t \right]. \quad (7)$$

While this expression appears quite complicated, it can be used to illustrate a couple of important points about the efficiency costs of deviating from a uniform tax rate structure. First note that the term multiplying the square bracket on the right-hand side is assumed to be positive. While $1 - u \cdot \bar{\eta}^u > 0$ if a rise in the uniform tax rate increases tax revenue (i.e., we are not in the backward-bending region of the "Laffer curve" for the taxed composite) and $1 - \beta^0 > 0$, we need a combination of these familiar conditions to hold.

It can now be seen that the efficiency effect of a revenue neutral change in tax rates away from a uniform tax rate structure consists of two parts. A preference interaction term (the first term in the kernel of (7)) and a multiple of the standard quadratic loss term (the second term). The first term is a weighted sum over all taxed goods of the "preference asymmetry" of good j which is given by $\bar{\eta}^u - \eta_{jN}^u$ and the change in the tax rate on good j , all times the tax revenue collected. Note that when the preference asymmetry and the change in the tax rate on good j both have the same sign, the product is positive. Thus utility is increased according to the preference interaction term if, predominantly, tax rates are raised (lowered) on individual taxable goods which have lower (higher) cross elasticities with respect to the untaxed good than does the taxed composite as a whole. The second term is unambiguously negative and increasing with the square of the deviations from uniformity⁷.

Several points can now be stressed. First if $\eta_{jN}^u = \bar{\eta}^u$ for all j then the preference interaction term is zero and any revenue neutral deviation from uniform rates reduces efficiency. This is just the standard case (see Sadka [1977]) where uniform tax rates happen to be Ramsey optimal. Second, generally it is true that $\eta_{jN}^u \neq \bar{\eta}^u$ for at least some goods j . Then, for

⁷This is most apparent in the case where taxed goods are independent so S is diagonal. More generally, S can be diagonalized by pre- and post multiplying by C and C^t respectively where C is an orthogonal matrix with columns equal to the normalized eigenvectors of S . The resulting matrix is diagonal with negative elements on the diagonal as S is negative definite. In this case, the tax variation vector Δt and the preference asymmetry vectors must be expressed in terms of an orthonormal basis.

sufficiently small changes in tax rates the quadratic loss term can be ignored while the preference interaction term is positive if the tax rate deviations "point in the right direction". Thus efficiency can be improved by a small deviation from uniformity. This is just an illustration of the famous Corlett and Hague [1953] result.

There are two features of the preference asymmetry interaction term in comparison to the quadratic loss term that should be noted. First, its sign depends on the direction in which the tax variations point unlike the loss term which is negative for any $\Delta t \neq 0$. Second, as $\bar{\eta}^u - \eta_{jN}^u$ is fixed, and ignoring changes in the weights ω_j , the preference interaction term is linear in Δt whereas the loss term is quadratic. This implies that if the deviations from uniform taxation are sufficiently large, economic efficiency will be reduced even if the deviations are in the right direction. Of course, if the deviations are in the wrong direction both terms are negative.

The above suggests a form of an "equal ignorance" argument for the desirability of uniform tax rate structure, or more precisely, for avoiding arbitrary tax rate variations. It is related to Lerner's [1944] famous argument for an equal distribution of incomes in the face of Bayesian uncertainty regarding the marginal utility of income schedules across households⁸. The point is that in face of the unambiguously negative

⁸An elegant proof of Lerner's theorem is found in Sen [1977]. Also, Bennett [1981] has recently extended Lerner's theorem to the case of variable production in which case equal ignorance about abilities implies a policy of equalizing wage rates.

quadratic loss term resulting from discrete tax rate variations around the uniform tax rate equilibrium, compelling evidence that the deviations point in the right direction is needed if an improvement in economic efficiency is to be likely⁹.

The intuitive reason for this result is as follows. At the uniform tax equilibrium, the introduction of tax rate variations may offset or augment any "distortions" that result from a failure to impose an optimally varying tax rate structure. If compensated demand curves are linear, or approximately so, the efficiency loss that will be created when a tax rate variation augments the existing distortion is greater than the efficiency gain that occurs if it offsets it. This is the analogue to the idea that efficiency costs rise in proportion to the square of the distortion, roughly speaking¹⁰. The argument implies that although a uniform tax rate structure is not usually optimal, when the direction of the optimal tax structure is unknown the existing administrative and political economy arguments for a uniform tax rate structure can be augmented with an efficiency rationale even if lump-sum taxation is not feasible.

⁹Like the Lerner equal ignorance argument the reasoning is symmetric when considering movement towards uniformity. Substitute $t^1 - \Delta t$ for t^0 in the kernel of (4) to get $t^1 \cdot S \cdot \Delta t - \frac{1}{2} \Delta t \cdot S \cdot \Delta t$. Now the quadratic term is subtracted so it is unambiguously positive. Substituting t^* for t^1 , the first term can be solved as the preference interaction term which may be negative or positive depending on whether the movement towards uniformity is away from desirable or undesirable deviations.

¹⁰I am indebted to David Bradford for pointing out the relationship between my argument and Lerner's equal ignorance argument.

4. Toward a "Balanced Budget" Efficiency Cost Methodolgy.

The methodology of comparing the efficiency effects of revenue neutral tax changes, whether with respect to a lump-sum benchmark, other benchmarks, or the status quo tax structure, is widely if not universally adopted by analysts and comparable to the differential approach used in tax incidence analysis. An alternative methodology which I call the balanced budget approach is to allow for changes in the amount of revenue collected consistent with a second-best determination of the level of government spending. This latter approach is better suited, I think, to separating the efficiency costs of inappropriate tax policies from that of inappropriate spending policies when lump-sum taxes are not feasible, but its informational requirements appear prohibitive.

The difference between the two approaches is shown in Figure 1. In this figure I compare the balanced budget approach to the differential approach for the case of the Ramsey optimal and lump-sum benchmarks. Along the horizontal axis I measure tax revenue collected (T) which is equal to government spending (G). $\mu^*(T)$ and $\mu^1(T)$ represent the marginal cost of revenue (or public funds) when T units of revenue are collected using the Ramsey optimal tax structure t^* and an arbitrary tax structure t^1 respectively. The meaning of the marginal cost of revenue schedule when taxes are not set optimally is explained below. The locus labeled MWPG gives the marginal willingness to pay for the public good. If the government

chooses its spending policies optimally, it would choose level G^1 and G^m under tax structures t^1 and t^m respectively.

If the government actually chose G^t level of revenue to collect and spend, the efficiency cost according to the conventional lump-sum tax benchmark would be the area of abd . Under the Ramsey optimal benchmark, the efficiency cost is abc with the area of acd being the unavoidable loss due to the fact that taxation is necessarily distortionary in the absence of lump-sum taxation. Of course, if the government chose to collect and spend a different level of revenue, the efficiency cost would be different. For example, if the government collects and spends G^m , the efficiency costs under the two benchmarks would be the larger amounts $ae'f$ and $ae'e$ respectively.

But the question arises as to whether it would not be better to attribute the additional loss to the government's inappropriate spending policies rather than its taxation policies. The only consistent way to separate the efficiency cost from the primary burden of taxation is to calculate the efficiency cost assuming the amounts of revenue collected and spent are those which are optimal do so under the circumstances. Since changing the tax structure can be expected to change the amount which is optimal for the government to collect and spend, particularly if large scale tax reforms are being considered, the revenue neutral approach is inadequate. The amounts of revenue collected should be allowed to change consistent with it being desirable for the government to do so.

Thus, the efficiency cost of setting tax structure t^1 is identified as the area abe under the Ramsey benchmark and abg under the lump-sum benchmark. In each case, the efficiency cost of the tax structure includes a loss attributable to the fact that the best the government can do on the spending side is reduced by setting an inefficient tax structure, while any efficiency cost which is avoidable by having the government choose a better level of spending is excluded. For example, if the government chose level G^m when tax structure t^1 is picked, the area $be'e$ is an efficiency loss on the spending side and should not be attributable to the choice of t^1 . For arbitrary levels of revenue and spending, the differential efficiency methodology may give a larger or smaller measure of efficiency cost than the balanced budget methodology. If, however, the government actually chooses the best level of spending at the chosen tax structure, the balanced budget measure is always larger.

A difficulty associated with the balanced budget approach is determining the $\mu^1(T)$ schedule, both conceptually and quantitatively. In this paper, I only address the former. The $\mu^m(T)$ locus is conceptually clear from the optimal tax literature. In particular it is equal to $C_j^m / ((1-\beta^m) \cdot C_j^m + t^m \cdot S_j)$ for any good j where S_j is the j th column of the Slutsky matrix. The marginal cost of revenue is the same and greater than unity at t^m regardless of how the taxes are incremented. However, at any non-optimal tax structure, the marginal cost of revenue depends on how the taxes are incremented. Since the tax structure is non-optimal, the marginal cost of revenue can even be

less than unity for some patterns of tax increments, in particular those that move the tax structure toward the Ramsey optimal.

It seems to me that the spirit of the second-best theory suggests that the marginal cost of revenue at an arbitrarily chosen tax structure should be defined for tax increments that are proportional to the existing tax structure and which therefore, in some sense, hold the distortions constant. That is, $dt = \alpha \cdot t$. In this case, it is relatively straightforward to derive the marginal cost of revenue at t^1 as $\mu^1(T) = T / ((1-\beta^1) \cdot T + t^1 \cdot S \cdot t^1)$. It can be ascertained that substituting $t^1 = t^m$ and $\beta^1 = \beta^m$ yields $\mu^m(T)$ definitionally. In general $t^1 \cdot S \cdot t^1 < t^m \cdot S \cdot t^m$ so we expect that $\mu^1(T) > \mu^m(T)$ for all T as shown in Figure 1, but this is by no means necessary because the possibility that $\beta^1 < \beta^m$ cannot be ruled out. We can now use the $\mu^h(T)$ function along with the MWPG function to solve for the balanced budget efficiency cost measure. Although this methodology seems free of conceptual shortcomings, it is obvious that its informational requirements are substantial.

5. Conclusions.

The traditional methodologies for measuring the efficiency effects of tax changes are heavily dependent on the assumption of revenue balancing lump-sum taxes. I have derived expressions that, subject to the assumptions made about preferences, are suitable for determining the efficiency impact of discrete tax rate changes under alternative benchmarks. I use this analysis to discuss and extend the "partial revenue effect" criteria for evaluating efficiency effects which have been established using revealed preference

reasoning. I also use it to present an efficiency argument for setting uniform tax rates on taxable commodities under conditions of ignorance about the direction of the Ramsey optimal tax structure. Finally, I consider an alternative to the prevailing "differential efficiency" approach that considers only revenue neutral tax changes. It is called the "balanced budget" efficiency cost method, by analogy to tax incidence analysis, and it provides a theoretically superior way of separating the excess (efficiency) and primary burdens of taxation.

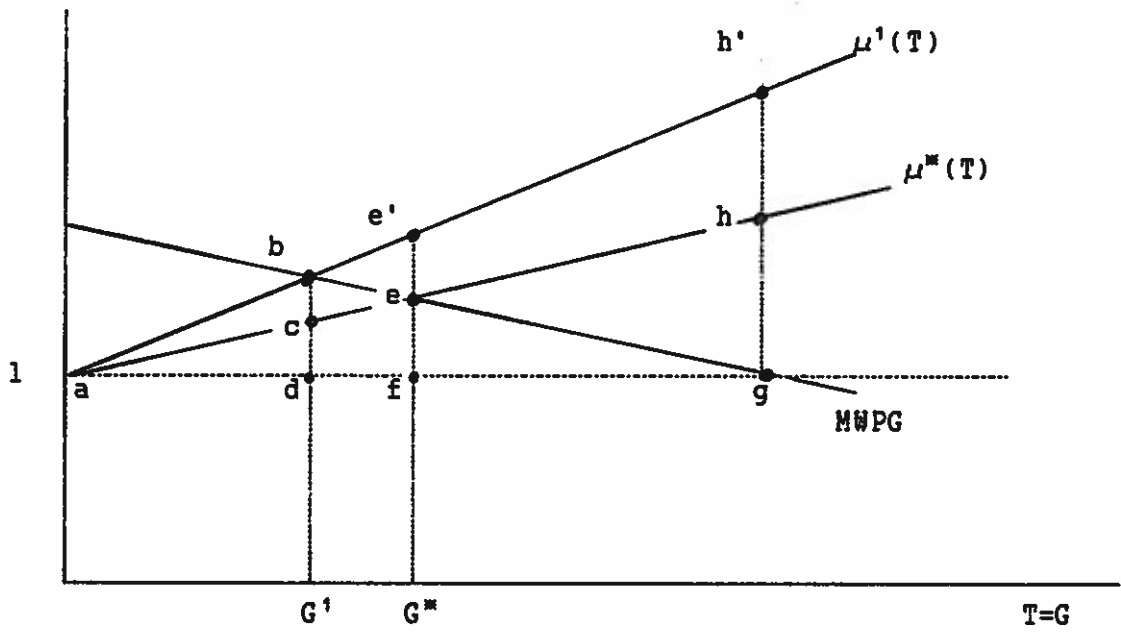


FIGURE 1

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