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RISK AVERSION AND THE VALUE OF RISK TO LIFE

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Abstract

This paper argues for an alternative methodology to estimate the value of risk to life. By relaxing the assumption of additive separability, we introduce risk aversion with respect to the length of life and show that the extended model better fits available data. This is crucial for the extrapolation stage that the evaluation of life-saving programs systematically requires. Current practice, we show, puts too little weight on the young. Our correction surpasses in magnitude that introduced by the switch from the notion of number of lives saved to the notion of years of life saved.

Keywords: value of statistical life; lifecycle behavior; cost-benefit analysis.

JEL: D61, D81, D91, I18.

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1 Introduction

Billions of dollars are spent every year on mortality reduction programs. Issues like the allocation of funds to medical research, the design of safety rules or environmental codes raise intense debate on the relevance of the choices made by governments and their agencies. For economists, the baseline is that alternative projects should be evaluated with objective criteria to avoid pure waste and, above all, dramatic underinvestment in less popular issues.

To back public decisions, some inquiry into individual valuation of life is indispensable. In practice, if we leave apart contingent valuation, the analysis of the wage-risk tradeoff is the major source of estimates of people behavior with respect to risk to life. This is informative basically about industry workers. Public programs touch wider population whose characteristics may vary considerably and some extrapolation of the available data is necessary. Whatever the extrapolation method retained, a structural lifecycle model is required to introduce minimum bias at the estimation stage and then to reconstruct the missing data with minimum error.

The standard life-cycle model assumes that individual preferences are separable additive. Although this model has been severely criticized in other branches of literature (see for example the literature on savings and references given in Subsection 2.1 below), it remains an almost universal assumption for the applied economics literature on the value of life. Nearly all mortality-related cost-benefit analysis rely, explicitly or not, on this assumption. The aim of the paper is to extend the theory to a broader class of models, to confront it to the data, and to draw practical conclusions on the methods being currently recommended.

The generalization we develop consists in introducing risk aversion with respect to the length of life. Although this extension increases the complexity of intermediate calculations, we can derive results that are almost as simple as those obtained with the standard additive model. This extension keeps therefore practical difficulties at a reasonable level. Moreover our theoretical results provide simple insights on the potential bias induced by the additive separability assumption.

We calibrate our model using the wage-risk tradeoff recently reported in Aldy and Viscusi
The data are hardly consistent with the additive model (unless one assumes that the rate of time preference equals $-8\%$) whereas the generalization proposed provides a much improved fit and more likely estimates.

In order to illustrate the practical interest of our study, we compare the benefits of different (fictitious) life saving policies under different specification choice. The magnitude of the bias caused by the additive separability assumption appears to be uncomfortably big. A practical conclusion is that the type of cost-benefit analysis that is currently recommended for life-saving programs is likely to be strongly biased in favor of the elderly.

The structure of the paper is as follows. Section 2 recalls the additive model and introduces more general preferences. Section 3 shows the consequence of alternative models for the individual and social valuations of statistical lives. Using an available hedonic regression of the value of statistical life, Section 4 searches the best fitting model and shows the performance of the nonadditive version. Section 5 contrasts quantitatively several evaluation procedures on typical life saving programs.

2 Lifetime preferences

2.1 The additive model

Most of the economics literature on the value of life is based on a particular model, whose standard version (e.g. Shepard and Zeckhauser 1984 or Rosen 1988) relies on elements developed in Yaari (1965). We refer to it thereafter as the “additive model”. According to that model, preferences are additively separable, time consistent and independent of past history. An individual of age $a$ maximizes the expected utility

\[ U_{a}^{add} = \int_{a}^{\infty} s_{a}(t)u(c(t))e^{-\lambda(t-a)}dt, \]  

where $s_{a}(t)$ is the probability of being alive at age $t$ conditional on being alive at age $a$, $u$ is the well-behaved instantaneous utility function and $\lambda$ is the subjective discount factor.
For discussing the tradeoff between present and future consumption under death risk, it will prove convenient to express the survival function $s_a(t)$ in terms of mortality rates, i.e.

$$s_a(t) = \exp(-\int_a^t \mu(\tau) d\tau),$$

(2)

where $\mu(\tau)$ is the hazard rate of death at age $\tau$. It will be assumed that $\mu(t)$ tends to infinity as $t$ tends to infinity. This is a purely technical assumption.

Additivity has been hardly discussed for the economic valuation of mortality changes,\(^1\) and the model keeps being used. A broader look at the literature shows intense criticism on both theoretical and empirical grounds. Even with fixed or absent mortality, theoretical arguments underlined unpleasant consequences of the additive separability assumption (e.g. Richard 1975, Deaton 1974 and 1992, Epstein and Zin 1991). Empirical studies repeatedly showed the additive model’s inability to fit intertemporal choice (Hayashi 1985, Muellbauer 1988, Browning 1991, and Carrasco, Labeaga and López-Salido 2002).

### 2.2 Generalization

The additive model is built upon three fundamental assumptions: time consistency, independence to past history, additivity. There is no rush to be slack on individual rationality and to introduce time inconsistencies, notwithstanding the fact that there is no solid foundation to social choice theory in that case, a severe shortcoming for the analysis of public policy. Rather than open the door for ad hoc assumptions (different tastes, direct age dependency), we maintain independence to the past and assume that agents have the same preferences and vary only with respect to the constraints they face (age-related mortality, wealth). So, we suggest to relax the less compelling assumption only, additivity.

Bommier (2003) showed that if preferences are time consistent and independent of past histories, there are two functions $u$ and $v$ such that individuals of age $a$ maximize a utility

\(^1\)See Bommier (2001) and Bommier (2003).
function of the form

\[ U_a = \int_a^{\infty} s_a(t)u(c(t)) \exp \left( -\int_a^t v(c(\tau))d\tau \right) dt. \]  

(3)

In the particular case where \( v = \lambda = \text{Constant} \), we retrieve the additive model: \( u \) is the instantaneous utility and \( v \) is the (constant) pure rate of time preference.

However, as soon as we depart from the additive case, the meanings of \( u \) and \( v \) are not so clear. Uzawa (1968) considered preferences with a similar structure and he interpreted the integral \( \int_a^t v(c(\tau))d\tau \) as an “accumulated rate of time of preference”. This extrapolation from the additive model is misleading: it suggests that the rate of time discounting depends on past consumption whereas preferences are characterized by independence with respect to it. A preferable approach is to start from well defined marginal properties of individual preferences (MRS, elasticities of substitution) and to derive proper concepts of time discounting and intertemporal elasticity of substitution. This approach was initiated in Epstein (1987) in the case of an infinitely long lived agent, and pursued by Bommier (2003) for the preferences considered in this paper.\(^2\)

The first concept we need is a rate of discount that tells how individuals trade between present and future consumption:\(^3\)

**Definition 1 (RD)** The mortality adjusted rate of time discounting at age \( t \) is

\[ RD(c, t) = -\frac{d}{dt} \log \left( \frac{1}{s_a(t)} \cdot \frac{\partial U_a}{\partial c(t)} \right) \bigg|_{\frac{dc}{dt}=0}. \]  

(4)

If \( s_a(t) \) were constant (no mortality), this rate of time discounting would be nothing else than the logarithmic derivative of a marginal rate of substitution. The factor \( \frac{1}{s_a(t)} \) corrects the fact that mortality generates a risk on consumption.

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\(^2\)The paper offers a more extensive discussion of the methodology that we only summarize here.

\(^3\)Because of our continuous time modelling, we use Volterra derivatives. They measure utility changes when consumption (or mortality) varies by an infinitesimal value during an infinitesimally short lapse of time. For example \( \frac{dU}{d\mu} \frac{d\mu}{dt} \) gives the change in \( U_a \) when mortality rates increase by \( d\mu \) during \( dt \) around \( t \). A first application of Volterra derivatives to economics is Ryder and Heal (1973).
Calculations lead to the cumbersome expression:

\[ RD(c, t) = -v'(c(t))u(c(t)) + \mu(t)v'(c(t)) + \mu(t)v(c(t)) \int_{t}^{+\infty} s(t)u(c(t)) \exp(-R_{t}^{\tau} v(c(\tau))) \exp \left(-\int_{t}^{\tau} v(c(\tau)) d\tau \right) d\tau. \] (5)

For the additive model \((v = \lambda, \text{ a constant})\) the above formula simplifies to \(RD = \lambda\).

We measure how people compromise between survival probabilities at different ages with the rate of discount for life years:

**Definition 2 (RDLY)** The rate of time discounting for life years is defined by

\[ RDLY(c, t) = -\frac{d}{dt} \log \left( \frac{\partial U_{a}}{\partial s_{a}(t)} \right) \bigg|_{\frac{ds}{dt}=0} \] (6)

It is fairly simple to see that:

\[ RDLY(c, t) = v(c(t)). \] (7)

It is remarkable that for additive preferences, \(RDLY(c, t) = RD(c, t) = \lambda\) so that there is a possible confusion between the different rates of discount above. It is clear, however, from (5) and (7) that in general \(RDLY(c, t)\) and \(RD(c, t)\) are not equal. We shall show that the difference captures an important economic effect: risk aversion with respect to the length of life.

This third concept simply measures the utility loss (or gain) when a life of a given length is replaced with a infinitesimal lottery over life duration. See Bommier (2003) for a formal definition and the economic interpretation of this particular index of risk aversion. We only recall here its expression when preferences have the form specified in (3).

**Definition 3 (RAL)** Risk aversion with respect to the length of life is defined by

\[ RAL(c, t) = \frac{u(v(c(t))v'(c(t))}{u'(c(t)). \] (8)

6
In the additive case, \( v' = 0 \) and \( RAL = 0 \): the individual is risk neutral with respect to the length of life.

Among the utility functions (3), a few forms are characterized by convenient properties. We already mentioned the additive model (obtained when \( v \) constant). Another one is the multiplicative model, where \( v = \beta u \) with \( \beta \) a constant. This model illustrates the role of risk aversion with respect to the length of life as an alternative to the familiar notion pure time preference: according to the multiplicative model, individuals have no pure time preferences and time discounting is exclusively driven by the combination of mortality and risk aversion with respect to the length of life (Bommier 2003). Since this model is a serious alternative to the additive one, we shall mention its implications along the paper.

3 The impact of structural assumptions

The search for the best public programs requires the calculation of basic indices like the value of a statistical life (VSL) or the welfare equivalent of a statistical like (WE). To evaluate counterfactual effects on mortality, the choice of the structural model is critical. We show in the following subsection the systematic bias that the additive model introduces.

3.1 The individual value of a statistical life

A natural concept to deal with choices involving mortality changes is the opposite of the marginal rate of substitution between mortality and consumption:

**Definition 4 (VSL)** The value of a statistical life at age \( t > \alpha \) is defined by

\[
VSL(c, t) = -\frac{\partial U}{\partial \mu(t)} \cdot \frac{\partial U}{\partial c(t)}.
\]  

(9)

By definition, \( VSL(c, t) \cdot d\mu \cdot dt \) is the willingness to pay of an agent of age \( t \) to reduce his mortality rate from \( \mu(\tau) \) to \( \mu(\tau) - d\mu \) between age \( t \) and \( t + dt \). In other words, an agent of age is ready to give up \( VSL(c, t) \cdot d\mu \cdot dt \) consumption to save \( d\mu \cdot dt \) statistical lives.

7
From there comes our use of the terminology “Value of Statistical Life”, although it may differ from other definitions of the VSL that can be found in the economic literature. By derivation of (3) one obtains

\[
VSL(c, t) = \frac{U_t}{u'(c(t)) - v'(c(t))U_t},
\]

with \(U_t\) defined as in (3).

When the consumption profile is constant, a fairly simple expression relates VSL to survival probabilities and discount rates.

**Proposition 1** For any constant consumption profile

\[
VSL(c, t) = \frac{u(c)}{u'(c)} \int_t^{+\infty} s\tau() e\left(-\int_t^{\tau} RD(c, \tau_1)d\tau_1\right)e^{\frac{u'(\tau-t)}{u'}d\tau}. (11)
\]

**Proof.** See appendix. ■

In the additive case, this expression simplifies to

\[
VSL(c, t) = \frac{u(c)}{u'(c)} \int_t^{+\infty} s\tau() e^{-\lambda(\tau-t)}d\tau. (12)
\]

The result for the additive case has been known for years. It is considered as very convenient since, if we abstract from consumption variations, VSL is proportional to a discounted sum of life years. The relation between age and VSL is simply computed from a standard life table and a discount rate. This way to account for age heterogeneity in VSL was initially introduced by Moore and Viscusi (1988) and is now used and recommended by agencies like the Environmental Protection Agency (EPA) and the Office of Management and Budget (OMB) for cost-benefits analyses.

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4As discussed in Johansson (2002), various definitions of VSL have been suggested, depending on the modelling (continuous, discrete, etc.). Our definition is consistent with Johansson’s (2002). The reader can just remember that in this paper, VSL is only a MRS (a well defined economic concept) between mortality risk and consumption.
The simplicity of the formula is an attractive feature of the additive model. Proposition 1 shows that the extension we suggest is only associated with a minor increase in complexity. Although the generalization makes intermediate calculations more fastidious, we eventually find that the benefit of saving the life of an individual of a given age is also proportional to the discounted sum of years saved.

There are however two differences between equations (11) and (12). First, in the general model the mortality adjusted rate of discount is not constant. Instead of using a discount function given by $e^{-\lambda(\tau-t)}$, as in the additive case, we have to use $e^{-\int_{t}^{\tau} RD(c,\tau)d\tau_1}$. Actually, when we calibrate the model, we find that the variations in the rate of discount remain limited until advanced ages, so this first difference can be considered as minor. The second difference is much more important. Equation (11) requires a discount factor $e^{\delta \mu (\tau-t)}$ which does not appear in the additive case since $\nu' = 0$. Remember that $\frac{\delta \mu}{\tau}$ is nothing else than the risk aversion with respect to the length of life. Therefore, years of life have to be discounted with the mortality adjusted rate of discount (the first discount function) minus risk aversion with respect to the length of life (second discount function). In particular, the greater risk aversion with respect to the length of life, the faster VSL declines as a function of age. The additive model, which assumes away risk aversion with respect to the length of life, may underestimate the speed at which VSL declines with age.

Our results are fairly intuitive. A risk averse agent is willing to pay more to avoid the chance of a major loss. In terms of mortality, a loss would be an early death and the concept of risk aversion with respect to the length of life is pertinent. The additive model neglects this effects and the magnitude of the bias clearly depends on the value of RAL. The calibration in Section 4 estimates the bias.

3.2 The welfare equivalent of a statistical life

To compare the “weights” given to mortality reduction at different ages in social welfare, it is useful to give a simple expression of the derivative of individual utility with respect to instantaneous mortality rate. We follow Arthur’s (1981) terminology.
Definition 5 (WE) The welfare equivalent of a statistical life is defined by

\[ WE(c, t) = -\frac{\partial U_0}{\partial \mu(t)}. \] (13)

WE has a fairly simple expression.

Proposition 2 In the general case,

\[ WE(c, t) = \int_1^{+\infty} s_0(\tau) u(c(\tau)) e^{-\int_0^\tau RDLY(c, \tau_1) d\tau}. \] (14)

Proof. See appendix. 

The welfare equivalent is a discounted sum of life years. The rate of discount to be used is the rate of discount with respect to life years (RDLY). In the additive model, RDLY = RD, thus it is correct to use the discount rate inferred from empirical studies on consumption smoothing to estimate the welfare equivalent of a statistical life. In the more general case, RDLY ≠ RD and when consumption is constant and mortality increases with age, the difference between RDLY and RD has the same sign as v'. In other words, when risk aversion with respect to the length of life is considered, the rate of discount to be used is greater than the discount rate estimated in studies on consumption smoothing (RD). Thus, omission of the risk aversion with respect to the length of life generates a pro-old bias in the welfare evaluation of mortality risk reduction. The illustrative examples developed in Section 5 provides insights on the size of the bias.

4 Data fitting

In order to evaluate the power of the extension we propose, we calibrate the model to fit the empirical estimates of VSL reported in A&V. To do so, a first step shows the relationship between empirical VSL and the model above (Subsection 4.1), then, through distance minimization, we find the structural parameters that fit empirical VSL (Subsection 4.2).
4.1 Wage-risk tradeoff

Assume that, at all ages, an individual has to choose between jobs that differ with respect to wages and instantaneous fatality risk. Let \( \mu_0(t) \) be the exogenous baseline mortality rate at age \( t \). For an extra instantaneous mortality \( \mu(t) \) (total mortality \( \mu_0(t) + \mu(t) \)), the wage is denoted by \( w(t, \mu(t)) \). Labor income can be used for consumption or savings. We denote by \( c(t) \) the consumption at age \( t \) and by

\[
k(t) = w(t, \mu(t)) - c(t),
\]

the saving flow at age \( t \). For our purpose, we do not need to fully specify the lifetime budget constraints that are related to the intertemporal markets and their possible imperfections. We shall simply assume that these constraints (possibly infinitely many) only bear on function \( k(\cdot) \). We denote this set of constraints by \( \mathcal{K}(k) \).

We may think of different kinds of constraints. With non storable commodities and no intertemporal markets, \( k(t) = 0 \) for all \( t \). Another possibility would be a single constraint of the form \( \int_0^\infty h(t)e^{-rt}k(t)dt = 0 \) with \( r \) the rate of interest and \( h(t) \) an exogenous function. This includes the case of perfect intertemporal markets (including life annuities). We could also imagine that the constraints \( \mathcal{K}(k) \) have the form \( \int_0^t e^{-r \tau}k(\tau)d\tau \geq 0 \) for all \( t \). That would be the case in a world where there is no annuity market, no borrowing and a rate of return on savings equal to \( r \). More complex market imperfections can be thought of. Undoubtedly, allowing any kind of constraints on \( k \) leaves us with a fairly high degree of generality, although certain cases are not covered, like a nonlinear consumption tax.

Using (2), we rewrite the lifetime utility function of an agent of age \( a \) as

\[
U_a = \int_a^{\infty} e^{-\int_a^\tau (\mu(\tau_1) + \mu_0(\tau_1))d\tau_1} e^{-\int_a^\tau u(c(\tau))d\tau} u(c(t))dt.
\]

(16)
A rational agent solves the maximization program

\[
\max_{\mu,c} U_a \text{ s.t. } \begin{cases} 
    k(t) = w(t, \mu(t)) - c(t) \text{ for all } t, \\
    \mathcal{K}(k).
\end{cases}
\] (17)

Let \( c^* \) and \( \mu^* \) denote the optimal consumption and mortality paths. We relate the derivative \( w_\mu = \frac{\partial w}{\partial \mu} \) at the optimum to the parameter of the utility function and to \( c^* \) and \( \mu^* \). Following the terminology of A&V, we call \( w_\mu \) the “wage-risk tradeoff.”

Determination of VSL at the optimal choice can be done without having an explicit formulation of the constraints \( \mathcal{K}(k) \). Indeed, differentiating (15), for all \( t, \tau \), we have:

\[
\left( \frac{\partial}{\partial \mu(t)} + w_\mu \frac{\partial}{\partial c(t)} \right) k(\tau) = 0.
\] (18)

As we assumed that all constraints can be written as functions of \( k \), the first order conditions also ensure that

\[
\left( \frac{\partial}{\partial \mu(t)} + w_\mu \frac{\partial}{\partial c(t)} \right) U_a(\mu, c) = 0,
\] (19)

and therefore

\[
w_\mu = -\frac{\partial U_{\mu, c}}{\partial \mu(t)} = \text{VSL}(c^*, t).
\] (20)

The observation of the wage-risk tradeoff reveals the preferences and makes the calibration of the utility function possible. The strength of the result is that this possibility does not depend on the existence of complete markets. The following explains how we proceed in practice.

### 4.2 Fitting the data

As explained in Viscusi and Aldy (2003), the hedonic regression fits the envelope of the choices made by the workers in the sample. Since the tangents of the individual indifference curve and of the envelope are the same, estimates provided by hedonic regressions can be interpreted as the VSL for the corresponding worker.
that are considered as reasonable), we would at best explain 58% of the age-related variance.

The general model, which introduces risk aversion with respect to the length of life, is more flexible. Nevertheless, we have to check that it fits (21) without assuming implausible values of $RD$. We explored the case where $u(c) = \frac{c^{1-\gamma} - c^{1-\gamma_0}}{1-\gamma}$ and $v = \lambda + \beta u$. This specification covers both the additive model ($\beta = 0$) and the multiplicative one ($\lambda = 0$) evoked in Subsection 2.2. In Figure 2, we report the minimum distance between the theoretical predictions and the empirical estimates, the survival weighted average $RD$ being constrained to take particular values given on the horizontal axis. The results obtained with the additive and the multiplicative models are also reported. The distance on the vertical axis has been normalized so that the distance between the empirical VSL and its mean equals 1.

Unsurprisingly, the general model always provides a better fit. Even if we constrain the mortality adjusted rate of discount to take reasonable positive values we still obtain an excellent fit. If we constrain the survival weighted average $RD$ to lie between 3 and 7%, we are able to explain more than 95% of age-related variability of the wage-risk tradeoff. That is much better than the additive model which only explains from 42 to 58% of the age-related variability. Table 1 reports the model’s performance (variance explained and parameters) for a range of discount factors. Figure 3 illustrates the fits obtained in the case where the average mortality adjusted rate of discount is constrained to equal 3%.

<table>
<thead>
<tr>
<th>Model: Additive</th>
<th>Non additive</th>
<th>Multiplicative</th>
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</thead>
<tbody>
<tr>
<td>RD</td>
<td>Average RD</td>
<td>Average RD</td>
</tr>
<tr>
<td>3%</td>
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<td>58%</td>
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<td>46%</td>
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<tr>
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<td>0</td>
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<td>9.6%</td>
<td>10.7%</td>
</tr>
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<td>8.3%</td>
<td>10.4%</td>
<td>12.0%</td>
</tr>
<tr>
<td>Average RDLY</td>
<td>3%</td>
<td>5%</td>
</tr>
<tr>
<td>8.3%</td>
<td>9.3%</td>
<td>10.5%</td>
</tr>
<tr>
<td>7.9%</td>
<td>9.7%</td>
<td>11.1%</td>
</tr>
</tbody>
</table>

*The elasticity of substitution is constrained to be non-negative.

Table 1: Calibration and performance.
Figure 1: Yearly individual consumption

- Empirical estimates from Lee and Tuljapurkar
- Smoothed profile used in the paper
Figure 2: Goodness of fit

Distance between theoretical predictions and A&V

Average RD
Figure 3: Age dependent value of a statistical life

- **Empirical estimates from Aldy and Viscusi**
- **Additive model, with RD=0.03**
- **General model with average RD=0.03**
utility of consumption, and therefore very low values of statistical lives. This is hard to buy. To circumvent this difficulty, we maintain the assumption that preferences are independent of age and artificially assume that for children, consumption is the same as at 20. Of course this option is arbitrary, one of its merit is that most of the difference between A and B is based on effects on the adults, for which estimates are more reliable.

Intuitively, it is not very clear whether A or B should be preferred. On the one hand A saves more lives. On the other hand B saves younger people, who still have many years of life. We compare the conclusion that we would draw from five types of benefit evaluation.

Method 1: *The number of lives saved.* Though there is no economic support for this method, it has been frequently used in the past. EPA and OMB still recommend to report the number of lives saved.

Method 2: *Utilitarianism with the additive utility function.* It assumes that all agents have the same additive utility function, with a rate of time preference of 3, 5 and 7%, the other parameters being drawn from estimates of Subsection 4.2. It also assumes that the government has a utilitarian social welfare function and that social and individual rates of discount are equal.

Method 2': *Aggregate WTP with additive utility function.* The benefits of a program is evaluated by the sum of the individual willingness to pay for such a program. Individual willingness to pay are estimated under the assumption that individual preferences are separable additive with the parameters estimated in Subsection 4.2.

Method 2’ amounts to method 2 if one assumes that the marginal social value of consumption is identical across people of different ages, in other words, redistribution is either perfect or neglected.

Redistribution being in general far from perfect, many papers argue that aggregate willingness to pay cannot be considered as a relevant policy indicator. The issue is not specific to saving live programs but general to any cost benefit analysis (see for example the dis-
Actually, the combination of parameters that optimally fit the data is difficult to say. Though the model is statistically identifiable, the regression is not reliable numerically. This is not surprising since we know from equation (11) that, at least when consumption is constant, what matters for determining the variations of $w_\mu$ along the life cycle is the combination of two elements: the mortality adjusted rate of discount minus the risk aversion with respect to the length of life. The difference between the two is correctly estimated, which suffices for a better performance than that of the additive model, but the estimate of each component is unstable. Ultimately, to discriminate more sharply between the several likely possibilities, we should integrate data on behaviors that go beyond the wage-risk tradeoff. A possibility would be to look at consumption smoothing behavior, but we leave that aside for lack of adequate data. Results thereafter are systematically reported for the values of 3, 5 and 7%.

Interestingly enough, one can see, from Table 1 or Figure 2, that when RD is constrained to plausible positive values, the multiplicative model does a better job than the additive one, although it has the same number of degrees of freedom. Therefore even if one is reluctant to increase the complexity of the model, an efficiency gain can be obtained by passing from the additive model to the multiplicative model.

From the last two rows of Table 1, it is possible to have a first idea on the potential bias generated by the additive assumption. While the additive model constrains the risk aversion with respect to the length of life to be null, our estimates with the general model gives estimates that range from 8.9% to 10.7%. In other words, when people discount consumption with rates of 3, 5 and 7%, life years in VSL should be discounted with rates of $-5.9\%$, $-4.6\%$ or $-3.7\%$ respectively. Needless to say that the additive model, which imposes the same rate of discount for consumption and life years, is likely to cause a huge bias.

Should that lead to a major shift in policy recommendations? A first answer comes from the estimates of $RDLY$ that, we know from Proposition 2, is the rate of discount to be used for estimating the welfare equivalent of a statistical life. While the additive model constrains
**RDLY** to equal the rate of discount, the estimates we obtain with the more general model show values of **RDLY** that exceed those of **RD** by several percentage points. This means, that the additive model puts to much relative weight on the old. The following section illustrates the magnitude of the shift.

## 5 Application to program comparison

To show the magnitude of the distortion in the evaluation of safety programs, we consider two alternative scenarios. One decreases mortality rates proportionally. Control of air quality can be seen as an intervention of this kind (Pope et al. 1995 make this assumption). For the same cost, the other decreases mortality rates uniformly. That might be the case of a regulation that limits human fatalities in case of an earthquake.

We denote these hypothetical interventions as A and B. Policy A is characterized by a reduction of mortality rates

$$\mu(t) \rightarrow (1 - \varepsilon_A)\mu(t),$$

(22)

and policy B by a reduction of mortality rates

$$\mu(t) \rightarrow \mu(t) - \varepsilon_B.$$  

(23)

where \(\varepsilon_A\) and \(\varepsilon_B\) are positive constant. We take the age structure of the population and the baseline mortality rates observed in the USA in 1999. We shall also assume that A saves twice as many (statistical) lives than B. Policy A is mostly effective for the older people (and babies) while policy B saves lives with a uniform rate. Figure 4 shows the age distribution of lives saved (it has been scaled so that A saves 2000 statistical lives while B saves only 1000). We assume that the consumption profile is \(c^*\) (see Subsection 4.2), for ages above 20. For ages below 20, and especially for babies and young children, the assumption that preferences are independent of age becomes problematic. The low levels of consumption that are typically observed in the very first years of life would then imply very high marginal
Discussion in Blackorby and Donaldson 1990). In the case of mortality reduction, Pratt and Zeckhauser (1996) stressed that because of the strong heterogeneity in mortality rates, aggregating individual willingness to pay may actually be a particularly misleading indicator. Nevertheless, perhaps for lack of convincing alternatives, method 2' indisputably remains the most commonly used in the applied literature.

**Method 3:** *Utilitarianism with the general utility function.* Similar to method 2, with the difference that we take a general utility function as estimated in Subsection 4.2. Again we constrain the average survival weighted RD and the social rate of discount to equal, in turn, 3, 5 and 7%.

**Method 3':** *Aggregate WTP with the general utility function.* Similar to method 2', with the general utility function as estimated in Subsection 4.2. Method 3' suffers the same theoretical drawback as method 2'.

The results are synthesized in Table 2. By assumption, A is twice as efficient as B from the viewpoint of method 1. The additive model provides an age-adjusted value of a statistical life, thus methods 2 and 2' lead to different conclusions. Actually, when we use the parameters estimated in section 4, methods 2 and 2' predict that the benefits of A and B are of about the same size. The fact that B saves less lives than A is approximately compensated by the fact that it saves younger people. The question is whether this age adjustment and this conclusion are indeed correct. The results of methods 3 and 3' suggest that it is not the case. When using the more general model the benefits of B appear to be much greater than those of A. The correction related to the introduction of risk aversion with respect to the length of life is anything but negligible. Actually, passing from the additive model to the nonadditive one is a bigger step than passing from the traditional method (number of lives saved) to those based on the additive model.\(^6\)

\(^6\)We could also define two additional methods that parallel methods 2 and 2' but make use of the multiplicative model. However, as it happens that the general model estimated in Subsection 4.2 is practically multiplicative, the results would be very close to those obtained with methods 3 and 3'.
Figure 4: Distribution of lives saved

Number of statistical lives saved (per one year age class)

Age

- Intervention A
- Intervention B
Discount rate

<table>
<thead>
<tr>
<th>Method for benefit evaluation</th>
<th>3%</th>
<th>5%</th>
<th>7%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Number of lives saved)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>2 (Utilitarianism with additive utility function)</td>
<td>1.11</td>
<td>0.97</td>
<td>0.88</td>
</tr>
<tr>
<td>3 (Utilitarianism with general utility function)</td>
<td>3.23</td>
<td>2.64</td>
<td>2.18</td>
</tr>
<tr>
<td>2' (Aggregate WTP with additive utility function)</td>
<td>0.94</td>
<td>0.82</td>
<td>0.75</td>
</tr>
<tr>
<td>3' (Aggregate WTP with general utility function)</td>
<td>1.95</td>
<td>1.75</td>
<td>1.72</td>
</tr>
</tbody>
</table>

Table 2: Benefits of B/Benefits of A.

EPA guidelines advise to perform sensitivity analysis by calculating the results of both methods 1 and 2. As the results of method 2 are known to depend on the rate of discount, about which there is no general agreement, they advise to report the results for different rates lying in the 3-7 % interval, in order to provide a reasonable confidence interval. Unfortunately, the additive model is so restrictive that the truth may be way out this interval. The methods currently used by EPA and OMB (and indirectly by policymakers) are likely to be significantly distorted in favor of the old.

6 Conclusion

Most economists would agree that predicting saving behavior under the assumption of risk neutrality would make little sense. They would also vehemently criticize a fund manager that decides to “optimize” investment under the assumption that members are risk neutral.

The economic literature on the value of a statistical life has however endorsed a similar embarrassing choice. Mortality makes our life akin to an extraordinary lottery. Bad luck and we die young, good luck and we spend hours playing with our grand children. Is it reasonable to assume that individuals are risk neutral with respect to the length of life? And to evaluate life saving programs under this assumption?

These questions have been addressed in this paper. On the theoretical side, the story is rather clear. Risk aversion with respect to the length of life makes individual willingness
to pay for mortality risk reduction decline more rapidly with age. Actually, although intermediate calculations are sometimes fastidious, we eventually found that accounting for risk aversion with respect to the length of life is fairly simple. Just like with the standard additive model, one simply has to use appropriate rates of discount accounting both for time preferences and for risk aversion with respect to the length of life.

The key issue is therefore to estimate the coefficient of risk aversion with respect to length of life. The difficulty of the task should not be underestimated. Since Arrow’s and Pratt’s seminal articles, about 40 years have passed and a number of empirical studies tried to measure the standard Arrow-Pratt index of absolute risk aversion. Surveys and experiments have even been designed for that purpose. Still, our knowledge of the magnitude of individual risk aversion is limited. Individual preferences with respect to lotteries on wealth remain the object of intense investigations. There is no reason to believe that individual preferences with respect to lotteries on the length of life will be easier to assess. It would be excessively optimistic to expect that a single study provides a robust estimate of risk aversion with respect to the length of life. Rather this should be seen as a long term objective that will probably require the collection of specific data.

However, in order to fix ideas, we used results from a recent empirical study on the relation between VSL and age to estimates plausible values of risk aversion with respect to the length of life. The theoretical extension neatly improved the quality of fit. Actually we found that the index of risk aversion with respect to the length of life is likely to be positive and greater than the rate of time discounting. In other words, accounting for risk aversion with respect to the length of life may even be more important than accounting for time preferences.

The contrast between our findings and the dominant economic approach is striking. While the notion of time preferences has been pointed out as being a major element to estimate the value of a statistical life, the standard method simply rules out the existence of risk aversion with respect to the length of life although it plays a more important role. It seems that “the paradigm of optimizing a simple functional form” (to take Rubinstein’s 2003 words)
Using a hedonic regression, A&V report several estimates of the VSL and of its variations with age. We use the parameters they give in their Table 4

\[ w_{\mu}^{AV}(t) = -1.92 \times 10^7 + 1.88 \times 10^6 t - 4.54 \times 10^4 t^2 + 335.24 t^3 \tag{21} \]

for \( t \in [20, 60] \).

The calibration strategy we pursue consists in estimating preference parameters that fit best equation (21). We only discuss what would be the most likely parameters, if the equation was actually exact. This is consistent with the objective of the paper: showing that the additive model biases substantially the econometric valuation of longevity gains. Actually, A&V stress that hedonic regressions exhibit uncertainty on the estimated parameters. One should therefore be cautious that equation (21) gives the most likely relation between age and VSL that emerges from the data that A&V had in hands, but it should not be considered as reporting an indisputable truth. By the same token, we cannot argue that we provide robust estimates of the true preferences parameters.

In order to calibrate the model, we also need the age-specific consumption profile \( c^* \), which is not available in the dataset used by A&V. The optimal consumption profile cannot be deduced from the theoretical model without specification of the constraints \( K(k) \) on which we have limited knowledge. Rather than posing specific constraints, we assumed that \( c^* \) corresponds with a smoothed version of the age specific individual consumption profile reported in Lee and Tuljapurkar (1997) (see Figure 1 for the original estimates and the smoothed profile that we use).\(^5\)

The first question that we may address is whether we can reproduce the relation (21) with the standard additive model (namely, \( v = \lambda \) = Constant and \( u(c) = \frac{c^{1-\gamma}-c^{-\gamma}}{1-\gamma} \) for some \( \gamma \geq 0 \)). The answer is yes, but with very implausible parameters. Indeed the distance minimizing discount rate is \(-8.1\%\), which explains 94\% of the age-related variance in equation (21). Had we constrained the rate of discount to be greater than or equal to 3\% (to approach values

\(^5\)Lee and Tuljapurkar (1997) is one of the few recent studies that provide individual (and not household) age-specific consumption profiles.
lead economists to ignore a key ingredient of individual preferences. The consequence is that cost-benefit analysis produced for the allocation of public money across saving lives programs is likely to be strongly distorted.

References


A Appendix

A.1 Proof of Proposition 1

Since we only consider constant consumption profiles, dependency on consumption is not detailed (e.g. $u'(c(t))$ will be simply written $u'$). We will also note $\kappa = \frac{\nu u}{w}$ the coefficient of risk aversion with respect to the length of life. Equation (5) rewrites

$$RD(c, t) = \frac{-\kappa + \nu + \mu(t)\kappa \int_t^{+\infty} s(t) e^{-u(\tau-t)} d\tau}{1 - \kappa \int_t^{+\infty} s(t) e^{-u(\tau-t)} d\tau},$$

(24)

and equation (10) becomes

$$VSL(c, t) = \frac{u}{w} \frac{\int_t^{+\infty} s(t) e^{-u(\tau-t)} d\tau}{1 - \kappa \int_t^{+\infty} s(t) e^{-u(\tau-t)} d\tau}.$$

(25)
Let us denote

\[ G(c, t) = \frac{u}{u'} \int_t^{+\infty} s_t(\tau) \exp \left( - \int_t^\tau RD(c, \tau_1) d\tau_1 \right) e^{\kappa(\tau - t)} d\tau. \]  

(26)

The result that we aim at proving is that \( VSL(c, t) = G(c, t) \).

First, let us show that \( VSL \to 0 \) and \( G \to 0 \) as \( t \to \infty \). By assumption, the mortality rate is going to infinity. Thus the discounted expected length of life \( \int_t^{+\infty} s_t(\tau) e^{-\kappa(\tau - t)} d\tau \) tends to zero as \( t \) tends to infinity. This implies that \( VSL \to 0 \) as \( t \to \infty \). This also implies that, for \( t \) large enough, \( \kappa \int_t^{+\infty} s_t(\tau) e^{-\kappa(\tau - t)} d\tau < 1/2 \). Combined with (24), it provides a lower bound on \( RD \). Namely \( RD > -2|v - \kappa| \). Consequently, \( \mu(t) + RD(c, t) - \kappa \to \infty \) as \( t \to \infty \). Given that \( G \) can be rewritten as

\[ G(c, t) = \frac{u}{u'} \int_t^{+\infty} \exp \left( - \int_t^\tau (\mu(\tau_1) + RD(c, \tau_1) - \kappa) d\tau_1 \right) d\tau, \]  

(27)

we also conclude that \( G \to 0 \) as \( t \to \infty \). The functions \( VSL \) and \( G \) have therefore the same limit when \( t \to \infty \).

Second, we show that they are solutions of the same differential equation. Using the fact that \( \frac{d}{dt} s_t(\tau) = \mu(t) s_t(\tau) \), derivation of (26) gives

\[ \frac{\partial G(c, t)}{\partial t} = [\mu(t) + RD(c, t) - \kappa] G(c, t) - \frac{u}{u'}. \]  

(28)

Now denote \( I = \int_t^{+\infty} s_t(\tau) e^{-\kappa(\tau - t)} d\tau \) and remark that

\[ \frac{dI}{dt} = \mu(t) I + \nu I - 1. \]  

(29)

Since \( VSL(c, t) = \frac{u}{u'} \frac{I}{1 - \kappa I} \), it follows that

\[ \frac{\partial}{\partial t} VSL(c, t) = \frac{u}{u'} \frac{[(\mu(t) + \nu) I - 1] [1 - \kappa I] + I \kappa [(\mu(t) + \nu) I - 1]} ([1 - \kappa I]^2}, \]  

(30)

24
which simplifies to
\[
\frac{\partial}{\partial t} VSL(c, t) = \frac{u}{u'} \frac{[\mu(t) + vI - 1]}{[1 - \kappa I]^2}. \tag{31}
\]
But, as \(RD(c, t) = \frac{-\kappa + v + \mu(t)\kappa I}{1 - \kappa I}\), we have \(v(t) = (1 - \kappa I)RD(c, t) + \kappa - \mu(t)\kappa I\), which plugged into (31) leads to
\[
\frac{\partial}{\partial t} VSL(c, t) = \frac{u}{u'} \frac{[\mu(t) + (1 - \kappa I)RD(c, t) + \kappa - \mu(t)\kappa I]I - 1}{[1 - \kappa I]^2} \tag{32}
\]
and eventually
\[
\frac{\partial}{\partial t} VSL(c, t) = (\mu(t) + RD(c, t) - \kappa)VSL(c, t) - \frac{u}{u'}. \tag{33}
\]
This linear first order differential equation being the same as (28) we obtain that \(G - VSL\) is solution to the differential equation
\[
y' = (\mu(t) + RD(c, t) - \kappa)y. \tag{34}
\]
Now remember that \(\mu(t) + RD(c, t) - \kappa\) goes to infinity as \(t \to \infty\). Thus any non null solution of (34) diverges at \(\infty\). Since we know that \((G - VSL) \to 0\) as \(t \to \infty\), it is necessarily the case that \(G = VSL\).

**A.2 Proof of Proposition 2**

From (2), it follows that
\[
\frac{\partial s_a(\tau)}{\partial \mu(t)} = 0 \text{ if } \tau < t, \tag{35}
\]
\[
\frac{\partial s_a(\tau)}{\partial \mu(t)} = -s(\tau) \text{ if } \tau \geq t. \tag{36}
\]
Derivating (3) then gives (14).
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