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## Efficient Location of Production in a Simple Neo-Classical Model of General Equilibrium

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Queen's Economics Department Working Paper No. 56

EFFICIENT LOCATION OF PRODUCTION IN A  
SIMPLE NEO CLASSICAL MODEL OF GENERAL  
EQUILIBRIUM

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1. Introduction

The theory of the location of the firm is characterized by a number of classic analyses. Each analysis has been concerned with developing a particular property of the equilibrium location of the firm, in general, for quite different models of markets. However, there is a consistent conceptual refocusing over time as we move from Weber's (7) analysis in 1919 through Hoover's (3) and Isard's (4) to Moses's (5) in 1958. The focus shifts from a transportation cost minimization locational equilibrium criterion to a profit maximization criterion with successively more general treatments of the technology of production.<sup>1</sup>

The purpose of this article is to indicate a basic equilibrium condition for the efficient location of a production process characterized by a neoclassical technology. Producers will locate at sites which maximize profits in the face of existing commodity and factor prices, and transportation costs. This condition was first presented by Moses (5), geometrically, in a partial equilibrium model.

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\*This paper was prepared while the author was associated with the Urban Affairs Ministry, Government of Canada.

We shall conduct the analysis in a Weber-like model of general equilibrium with a neo-classical technology.<sup>2</sup>

In Section 2, the model is developed in a context when all markets are geographically distinct and fixed in space. In Section 3, the location of producers is treated as a variable.

## 2. Equilibrium When Locations Are Fixed

Our economy will be comprised of three commodities produced with three factors. The commodity indexed 1 is a transportation good and this good is consumed in the process of transporting factors and the other two commodities. We assume that the transport of the transportation good requires no transportation good or takes place at zero cost.<sup>3</sup> The commodities indexed 2 and 3 are orthodox commodities desired by consumers as final demands. Commodity 1 in contrast does not directly enter the utility functions of consumers and is only produced in order to permit trade to take place.

The geography of our economy is illustrated in Figure 1 below.

North

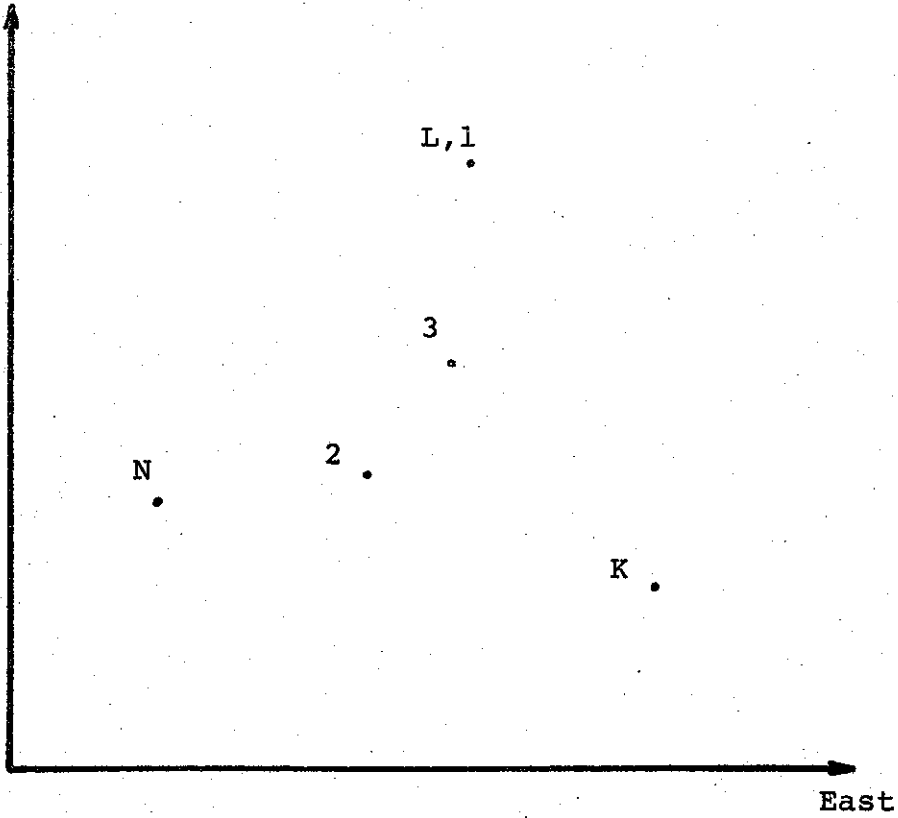


Figure 1

In Figure 1, there are three geographically distinct factor supply sites, labelled L, K, and N. These factor supply sites are geographically fixed. There are three geographically distinct commodity production sites, labelled 1, 2, and 3. In this section we shall be concerned with determining factor and commodity price vectors which results in efficient production in the economy illustrated in Figure 1. In Section 3, we shall let the sites for the production of commodities 2 and 3 be variable and determine conditions which cause efficient production and locations to obtain.

Transportation costs are defined on flows between all points except for flows of the transportation good. For example  $t_N^{N2}$  is the physical amount of transportation good required to transport a unit of the resources factor between supply point N and production point 2.  $t_N^{N2} p_1$  is the cost of transporting a unit of resource input between supply site R and production site 2, where  $p_1$  is the price of a unit of the transportation good.

We assume that labor commutes to the production sites at positive cost and consumes bundles of commodities 2 and 3 in town. Separate transportation costs are associated with the delivery of commodities 2 and 3 to town; that is, workers do not transport commodities 2 and 3 to town on their homeward commuting trip.

Let the production functions for the commodities 1, 2, and 3 be

$$(2.1) \quad y_i = f_i(l_i, k_i, n_i) \quad (i = 1, 2, 3)$$

where  $y_i$  denotes the output of commodity  $i$ ;  $l_i$  the labour input,  $k_i$  the capital input, and  $n_i$  the resource input in the production of commodity  $i$ .  $f_i$  are assumed to be concave, positively homogeneous of the first degree and differentiable.

The factors of production are assumed to be subject to the restrictions

$$(2.2) \quad \begin{aligned} l_1 + l_2 + l_3 &\leq l \\ k_1 + k_2 + k_3 &\leq k \\ n_1 + n_2 + n_3 &\leq n \end{aligned}$$

where  $l$  is the fixed endowment of labour in town,  $k$ , and  $n$  the endowments of capital and resources at their respective sites.

Factor-product coefficients will be denoted by

$$(2.3) \quad a_i = \frac{l_i}{Y_i}; \quad b_i = \frac{k_i}{Y_i}; \quad c_i = \frac{n_i}{Y_i} \quad (i = 1, \dots, 3)$$

When  $y_i > 0$ . From the assumed homogeneity,

$$(2.4) \quad f_i(a_i, b_i, c_i) = 1 \quad (i = 1, \dots, 3)$$

Let  $(w_1, r_1, s_1)$ ,  $(w_2, r_2, s_2)$  and  $(w_3, r_3, s_3)$  denote the vectors of factor prices at production sites 1, 2, and 3 respectively.  $w$  refers to the wage rate,  $r$  to the rental rate on capital, and  $s$  to the rental rate on resources.



The minimum unit cost functions are defined at production site  $i$  for non-negative  $w_i$ ,  $r_i$  and  $s_i$  by

$$(2.5) \quad g_i(w_i, r_i, s_i) = \min \{w_i a_i + r_i b_i + s_i c_i \mid f_i(a_i, b_i, c_i) = 1\} \quad (i=1,2,3)$$

These are also homogeneous of degree one, and concave (cf. Uzawa (6)). The factor-product ratios entering the minimum unit cost functions (2.5) depend on factor prices: when the dependence is unique, the functional relationship will be denoted by

$$(2.6) \quad a_i(w_i, r_i, s_i), b_i(w_i, r_i, s_i), c_i(w_i, r_i, s_i) \quad (i = 1, 2, 3)$$

where the functions are defined now for positive  $w_i, r_i$ , and  $s_i$ .

We require sets of commodity and price vectors which not only sustain efficient production as in an aspatial economy, but which also cause interpoint flows to be shipped so as to sustain efficient production at the various production sites. In equilibrium non-negative profits will be made by shippers along routes where positive flows are carried and negative profits will be sustained along alternative routes where no flows are carried. We shall limit our analysis to situations where positive amounts of commodities 1, 2, and 3 are produced and positive amounts of the three factors are required in the production of the three commodities.

Factor transportation equilibrium requires

$$\begin{aligned}
 w_1^{-t_{L1}} p_1 &= w_2^{-t_{L2}} p_1 \\
 w_2^{-t_{L2}} p_1 &= w_3^{-t_{L3}} p_1 \\
 r_1^{-t_{K1}} p_1 &= r_2^{-t_{K2}} p_1 \\
 r_2^{-t_{K2}} p_1 &= r_3^{-t_{K3}} p_1 \\
 s_1^{-t_{N1}} p_1 & \quad s_2^{-t_{N2}} p_1 \\
 s_2^{-t_{N2}} p_1 & \quad s_3^{-t_{N3}} p_1
 \end{aligned}
 \tag{2.7}$$

Each equation in (2.7) indicates that the difference between the shipping price for a factor and the delivered price for the factor is equal to the cost of transport.<sup>4</sup>

Also, in an efficient equilibrium, minimum unit cost conditions in (2.5) will be met at each production site and non-negative profits (in fact zero profits) will be made. That is,

$$w_i a_i + r_i b_i + s_i c_i = p_i \quad (i = 1, 2, 3)
 \tag{2.8}$$

Equations (2.7) and (2.8) form a system of nine equations in nine unknowns and define a set of prices which are efficient in the sense that all factors will be utilized in minimum unit cost combinations. However, an efficient equilibrium requires in addition that there be no excess production of the transportation good over the exact requirements made by the economy. Thus we must limit the set of prices defined by (2.7) and (2.8) to those in a subset

defined such that the quantity of transportation good produced equals the quantity required. The quantity produced is  $y_1$ . The quantity required is the left side of (2.9) below.

$$(2.9) \quad y_2 t_2^{21} + y_3 t_3^{31} + l_2 t_L^{L2} + l_3 t_L^{L3} \\ + k_1 t_K^{K1} + k_2 t_K^{K2} + k_3 t_K^{K3} \\ + n_1 t_N^{N1} + n_2 t_N^{N2} + n_3 t_N^{N3} = y_1$$

Note  $l_1 t_L^{L1}$  is zero since the transportation good is produced in the town where labor is supplied.

The set of points sustained by our price vectors define an efficient set of output combinations. If we specify a social utility function defined on bundles of commodities 2 and 3, we could isolate the optimal production point for the economy.

### 3. Equilibrium When Locations Are Variable

Consider an efficient equilibrium as defined in Section 2 sustained by a particular set of mill prices  $(\bar{p}_1, \bar{p}_2, \bar{p}_3)$  and a corresponding set of delivered factor prices. Consider now the possibility of the producers of commodities 2 and 3 being able to shift their respective sites of production at zero cost. Let  $u_2$  and  $u_3$  be indices of sites (to be explained below) for the production of commodities 2 and 3, respectively.

At site  $u_2$ , the producers of commodity 2 will face unit mill revenue  $p_2(u_2)$  and unit mill factor prices  $w_2(u_2)$ ,  $r_2(u_2)$  and  $s_2(u_2)$ . Similarly for the producers of commodity 3 at  $u_3$ . At the perceptual level of the firm, these prices will vary only with transportation costs. Thus, producers of commodity 2, by shifting their site will observe changes in the mill prices as different transportation cost burdens are incurred. These changes in mill prices,  $\frac{dp_2}{du_2}$ ,  $\frac{dw_2}{du_2}$ ,  $\frac{dr_2}{du_2}$ , and  $\frac{ds_2}{du_2}$  will be precisely defined as functions of changed relative transportation costs for shifts in location for the firm.  $\frac{dp_2}{du_2}$  will be a change in unit revenue for the firm and

$$\bar{a}_2 \frac{dw_2}{du_2} + \bar{b}_2 \frac{dr_2}{du_2} + \bar{c}_2 \frac{ds_2}{du_2}$$

will be the change in unit costs, where barred values indicate efficient intensities of factor utilization defined in Section 2.

Producers will shift sites in the face of existing prices until no additional positive profits can be made or until

$$\frac{dp_i}{du_i} - \bar{a}_i \frac{dw_i}{du_i} - \bar{b}_i \frac{dr_i}{du_i} - \bar{c}_i \frac{ds_i}{du_i} = 0 \quad (i = 2, 3)$$

that is until zero profits obtain.

In full general equilibrium with variable sites the  $a_i$ 's,  $b_i$ 's, and  $c_i$ 's will change as sites are shifted and factor proportions are changed. With variable sites,

the physical transportation use coefficients defined as constants in Section 2 will now be variables depending on the location of the producers. Let these then be variables depending on the site indices  $u_2$ , and  $u_3$ . Now an efficient point in the economy ( on the production possibility frontier ) will be one which simultaneously satisfies (2.7) (2.8) (2.9) and the following condition (3.1).

$$(3.1) \quad \frac{dp_i}{du_i} - a_i \frac{dw_i}{du_i} - b_i \frac{dr_i}{du_i} - c_i \frac{ds_i}{du_i} = 0 \quad (i = 2, 3)$$

The  $a_i$ 's,  $b_i$ 's, and  $c_i$ 's must be unit cost minimizing intensities corresponding to solution values of the factor price vectors. Observe (2.7) (2.8) and (3.1) form a system of eleven equations in eleven unknowns, given the values of the  $a_i$ 's,  $b_i$ 's, and  $c_i$ 's. This system of equations is analogous to the system defining efficient points in an aspatial world.<sup>5</sup>

The site index or location index must be defined on a straight line in order to have the derivatives mathematically defined.  $u_i$  then can be defined as the geographic distance as the crow flies from any fixed point in the plane to the site of production of commodity  $i$ . A shift in the site is represented by changes in location along the line drawn from the arbitrary fixed point to the existing production site. We require, however, perturbations in the site to take place in any direction around

a possible site. Thus a site index must be an arbitrarily fixed distance to a possible production site where the direction of the distance measure is defined over  $2\pi$  radians. Figure 2 below provides an illustration.

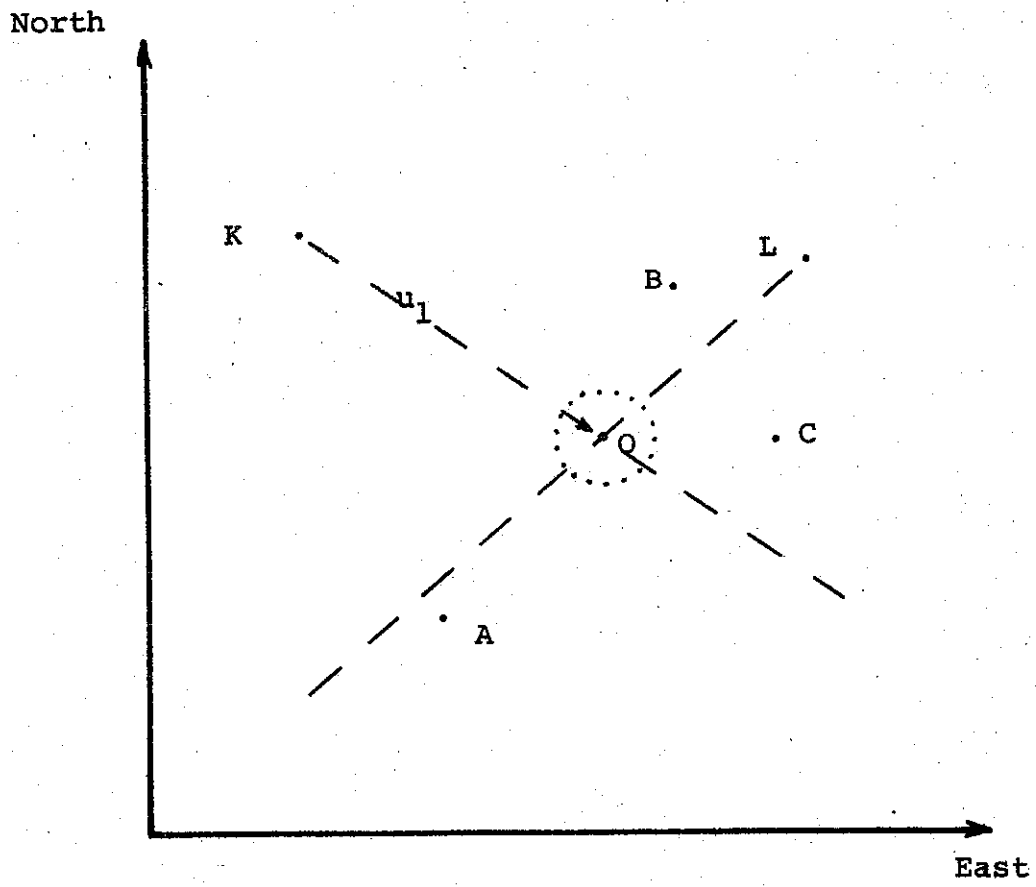


Figure 2

Assume production takes place at 0 lying among markets indicated by points A, B, and C. Let the dotted line indicate the region of possible perturbation of production point 0. If K is an arbitrary point then we can perturb site 0 along the line from K through 0. Maximum profits must result at 0 in equilibrium.<sup>6</sup> But in order to be the unique site of maximum profits we must perturb 0 throughout the region bounded by the dotted line or along, say, another line from arbitrary point L through 0. A site index is the distance from an arbitrary point K or L through 0 and this arbitrary point must rotate  $2\pi$  radians around 0 to insure that the region bounded by the dotted line is covered by possible perturbations.



FOOTNOTES

1. See the comments in Moses (5).
2. In Hartwick (2) I treated Weber's location problem in full general equilibrium with endogenous transportation costs by means of Samuelson's "spatial price equilibrium" approach or economic rent maximization approach. In that model it was necessary to take demand and supply functions as given whereas in this model, we shall be more general and start from factor endowments and production functions.
3. This simplifying assumption could be relaxed without changing the qualitative aspects of the model significantly. We would have a certain fraction of the transportation good evaporate in transit and as in Section 3, optimize the location of production of the transportation good.
4. We require in addition to the zero profit conditions on routes along which commodities are shipped, inequalities to hold for prices and transportation costs on all alternative routes not used. That is, negative profits must be made on all routes not used.
5. See Chipman (1).
6. We can illustrate the analytics of shifting positions along the line from K diagrammatically. Consider three alternative sites along the line, say at  $u_1$ ,  $u_2$ , and  $u_3$ . Given his unit mill price or unit revenue  $p(u_1)$  (which the producer considers a parameter) at  $u_1$  for instance, he will allocate the revenue among the two inputs so as to maximize the output of the commodity. This is equivalent to profit maximization with free entry. An equilibrium level of output corresponding to isoquant  $I_1$  will be arrived at for the site with index  $u_1$ . See  $I_1$ , and the revenue constraint in Figure F1 below.

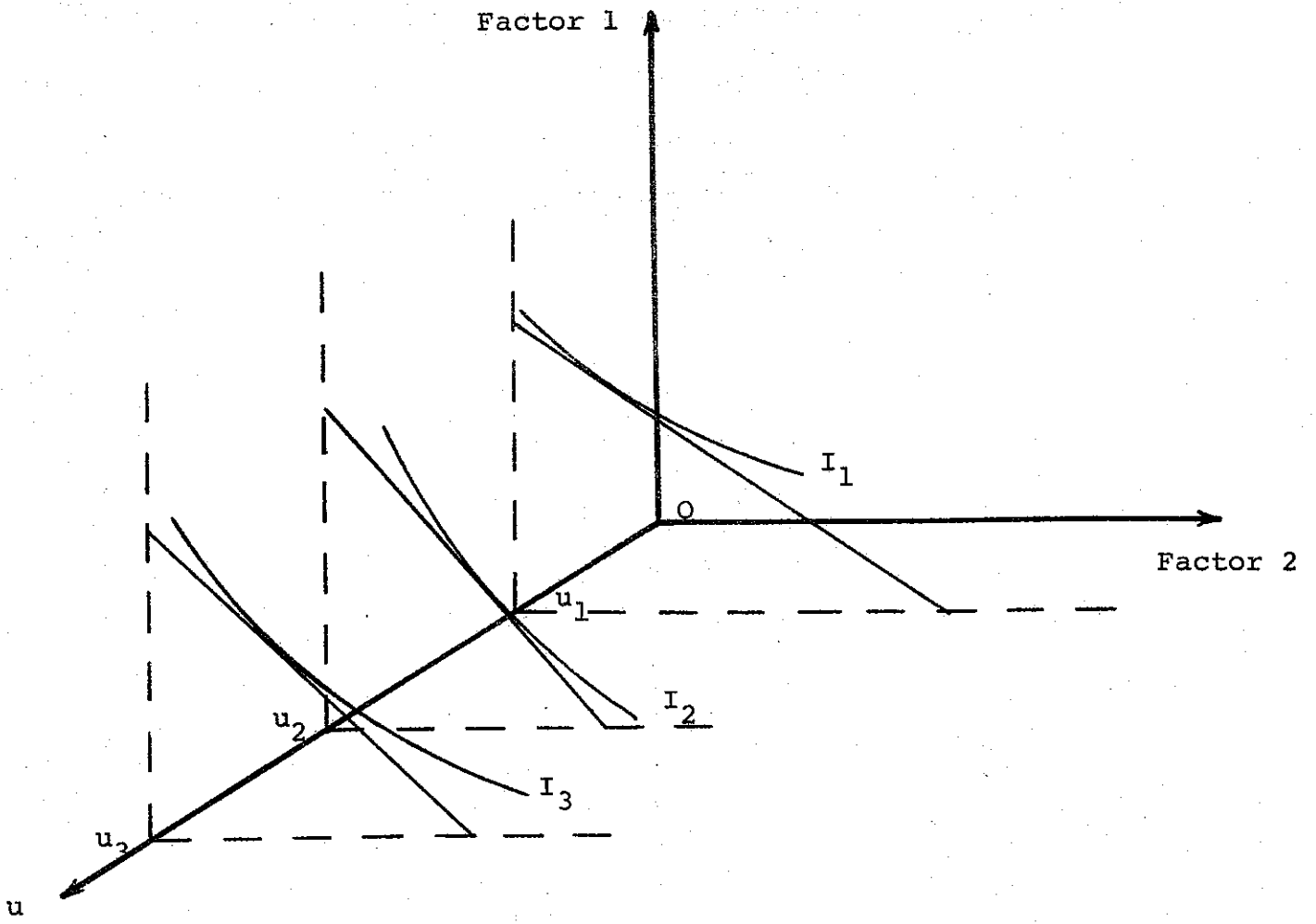


Figure F1

Because there will be different relative transportation costs on inputs and the output at different locations, different unit revenue and factor prices will face the producer at different locations. We have indicated the different isoquants,  $I_2$  and  $I_3$ , corresponding to different sites  $u_2$  and  $u_3$  in Figure 2 above. Efficient location in a partial equilibrium content will be at  $u_1$  where the highest isoquant is obtained.

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