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DIFFERENCES IN THE RESPONSE OF THE DEMAND FOR LABOUR TO VARIATIONS IN OUTPUT AMONG CANADIAN REGIONS: A PRELIMINARY INTERPRETATION

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DIFFERENCES IN THE RESPONSE OF THE DEMAND FOR LABOUR TO VARIATIONS IN OUTPUT AMONG CANADIAN REGIONS: A PRELIMINARY INTERPRETATION

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Differences in the Response of the Demand for Labour to Variations in Output Among Canadian Regions: a Preliminary Interpretation*

1. Introduction

It is well known the unemployment rate varies much more in some regions of Canada than in others. Denton has documented this for the period 1947-1964 [4], and updating his data does not change the conclusions.

In principle the unemployment rate could fluctuate more in one region than another either because employment varied more around its trend or because the labour force varied more about its trend. In this paper we focus on the proximate reasons for employment fluctuating more about its trend in some regions than in others.

For a given wage rental ratio the demand for labour will vary if demand for output varies. As demand for output varies through the cycle demand for labour will vary correspondingly. Employment could then vary differently in different regions for two reasons.

First, aggregate demand for output might vary differently, rising and falling around its trend more in some regions than others. Second, even with similar variations about trend in aggregate output, the derived

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demand for labour might vary differently if the short run elasticity of demand for labour with respect to output differed among regions.

In this paper we try to sort out the relative importance of each of these proximate causes of variation in employment sensitivity to the cycle, region by region.

The next section outlines the conventional theory underlying the equations fitted to explain demand for labour in the regions; extends it slightly in one respect where it seems rather unsatisfactory; and adjusts it in another respect in order to allow for the lack of certain data on a regional basis.

Section 3 gives the results of fitting the equations.

Section 4 is the main part of the paper, and explains how the effects of regional differences in output cycles, and regional differences in the elasticities of demand for labour with respect to output, can be separated. The method is applied to a comparison of each region's employment cycle with that for Canada as a whole. The result is an estimate, for four of the regions (the Prairies are the maverick), of how much of the difference between each region and Canada is due to the region's output cycle being different from Canada's, and how much is due to the elasticity of demand for labour with respect to output in the region being different from Canada's.

The choice of Canada as a benchmark, as it were, is arbitrary, but the method could be applied to a comparison of any pair of regions if this were considered more appropriate.

Conclusions and implications for further research follow in section 5.

2. The Model

Several writers have fitted functions relating the demand for labour to output, [1],[2],[3],[6],[7],[8]. The theories underlying these functions vary somewhat, but the final equations used are usually very similar. We shall follow in this paper the theoretical derivation of Ball and St. Cyr [1] (B&S), with minor changes.

B&S specify a short-run relationship between output and man hours of the form

$$(1) \qquad Q_t = Ae^{\rho t}(Eh)_t^{\alpha}$$

where

- Q: output, taken as exogenously determined
- $e^{\rho t}$: a time trend to absorb the joint influence of increase in the capital stock and technical change
- E: level of employment, in men
- h: hours worked per man

They add a cost equation

$$(2) \qquad C_t = W_h(Eh)_t + F_t$$

where

C: total costs

F: fixed costs

W_h: effective wage per man hour

A discussion of the determinants of W_h culminates in the plausible view that W_h is a U-shaped function of h, with a minimum at some particular level of hours worked. Thus they write

$$W_h = a - bh + ch^2 \tag{3}$$

Minimisation of total costs in (2), subject to (1) and (3), with respect to E,h and W_h follows. Denoting the minimising value of E by E*, they derive

$$E_t^* = \frac{2c}{A^{1/\alpha}} e^{-\rho t/\alpha} Q_t^{1/\alpha} \tag{4}$$

They then postulate a short-run adjustment function of the form

$$E_t/E_{t-1} = (E_t^*/E_{t-1})^{\lambda} \qquad 0 < \lambda \leq 1 \tag{5}$$

Combining (4) and (5) and taking logarithms we have the estimating equation

$$\ln E_t = a_o - (\lambda\rho/\alpha)t + (\lambda/\alpha)\ln Q_t + (1-\lambda)\ln E_{t-1} + \ln \xi_t \tag{6}$$

Equation (6) includes the same variables as those

used by Brechling [2], Brechling and O'Brien [3], and Smyth and Ireland [5],[7]. Brechling, however, specifies it in linear form rather than logarithmic in [2], and Smyth and Ireland, while deriving the same form of equation and variables, use a different theoretical approach, explained in [5], and therefore interpret the fitted parameters differently.

We shall use equation (6), but will develop a little further the rationale of adjustment mechanism (5), which appears somewhat arbitrary, and which has not, to our knowledge been justified theoretically in this context.

The Adjustment Mechanism

Unless adjustment of employment is costly, it is clear that it should be instantaneous, and λ should be unity. If adjustment is costly, those costs should be included explicitly in the cost minimising process.

We shall assume that adjustment costs (AC) are zero only if there is no change in employment. Otherwise they are positive, and are taken as a function of the difference between employment last period and contemplated employment this period. Thus we have

$$(7) \qquad AC = G(E-E_{-1})$$

(with time subscripts dropped for convenience, and E_{-1} denoting lagged employment)

with the restrictions:-

$$G(0) = 0,\ G' > 0 \quad \text{if } E > E_{-1},\ G' < 0 \quad \text{if } E < E_{-1}.$$

Define $B = ((Q/A)e^{-\rho t})^{1/\alpha}$

Then from (1)

$$h = B/E$$

From (2) and (3) we have

$C = (a-bh+ch^2)(Eh) + F$, with C interpreted as production costs only,

or

$$(8) \qquad C = aB-bB^2/E + cB^3/E^2 + F$$

Define total costs (TC) as the sum of production costs (C) and adjustment costs (AC), so that

$$TC = aB-bB^2/E + cB^3/E^2 + F + G(E-E_{-1})$$

or, using H(E) for $ab-bB^2/E + cB^3/E^2 + F$,

$$(9) \qquad TC = H(E) + G(E-E_{-1})$$

Total costs are minimised for

$$(10) \qquad H'(E) + G'(E-E_{-1}) = 0$$

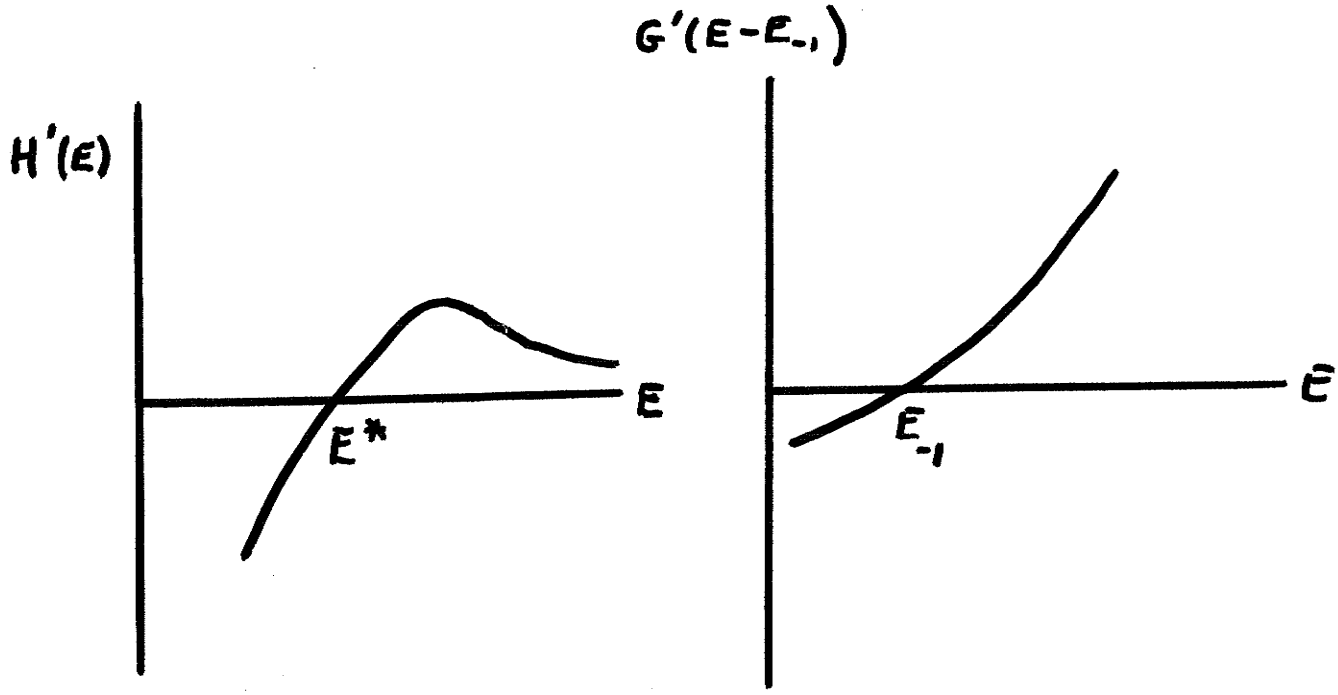
It will be useful to present the minimising choice graphically.

Let E* be the value of E for which H'(E) is zero. H' will be negative below E*, and positive above.

The function G' will be zero at E_{-1} (provided it

is continuous there), as the restrictions on G imply. It is negative below E_{-1} and positive above. Illustrative shapes for H′ and G′ are:-

Diagram 1



The shape given to G′ implies that adjustment costs resulting from hiring a given number of employees are rather greater than those resulting from firing the same number.

Putting the two diagrams together, and assuming for expositional purposes that $E^* > E_{-1}$, we have diagram 2 (diagram 3 should be ignored for a moment).

Diagram 2

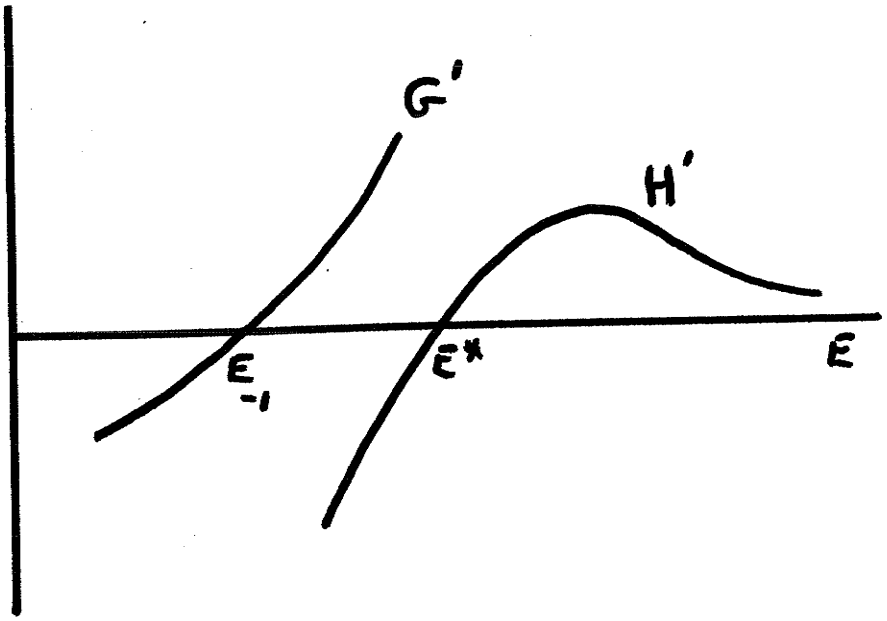
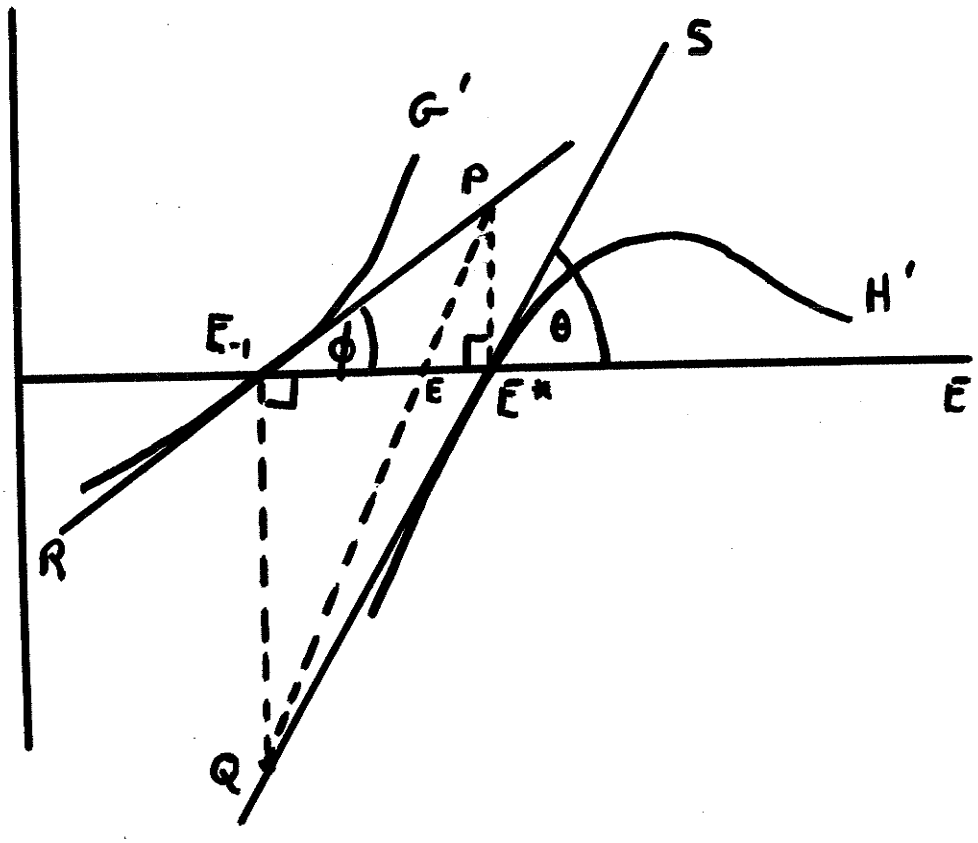


Diagram 3



The minimising value of E is where the vertical sum of the two curves is zero, taking account of sign, and will therefore lie between E_{-1} and E*. This is what one would expect: with adjustment costs not all the gap between E_{-1} and E* is made up in one period.

If the two curves are reasonably well approximated in the region between E_{-1} and E* by their tangents at E_{-1} and E*, we may use diagram 3 to derive the minimising E value geometrically.

The minimising value is found by drawing a perpendicular at E*, to meet RE_{-1} at P, and a perpendicular at E_{-1}, to meet SE* at Q. Where PQ intersects the E axis is the minimising value of E.

If the slopes of H′ and G′ at E* and E_{-1} are respectively denoted tan θ and tan ϕ, elementary geometry shows that

$$(11) \qquad \frac{E^*-E}{E-E_{-1}} = \frac{\tan \phi}{\tan \theta} = m \text{ (say)}$$

Provided that E^*/E_{-1} and E/E_{-1} are not too different from unity (as they will not be if demand for output never rises more than a few percentage points in a single period), we may write, to a close approximation:-

$$(12) \quad \ln(E^*/E_{-1}) = (E^*/E_{-1})-1, \text{ and } \ln(E/E_{-1}) = (E/E_{-1})-1.$$

Substituting these approximations into (11) and rearranging we find

$$\ln(E/E_{-1}) = \frac{1}{1+m} \ln(E^*/E_{-1})$$

Define $\frac{1}{1+m} = \lambda$, remove the logarithms, replace the time subscripts, and we have

$$(13) \qquad (E_t/E_{t-1}) = (E_t^*/E_{t-1})^{\lambda} \qquad 0 < \lambda \leq 1,$$

which is the adjustment already indicated in (5).

The derivation makes it clear that, though λ will not in general remain constant as the adjustment process proceeds, a sufficient condition for it to do so very nearly# would be that both $H(E)$ and $G(E-E_{-1})$ be quadratic functions of E. In that case their derivatives will be straight line functions of E, with constant slopes, so that m will be the ratio of two constants, and λ, which is $1/(1+m)$, will also be constant.

In the case considered neither function was quadratic in E, but it does seem likely that a quadratic could well reasonably approximate both functions over the relevant range of values of E.

Thus the adjustment function (5) is very close to the cost minimising one if both costs of production and costs of adjustment, given last period's employment, are

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#Not quite, because of approximation (12). In addition, firms might minimise costs over several periods together: the associated calculus of variations has proved, so far, intractable.

quadratic functions of employment in the current period.

Adjustment of Equation (6) for Data Problems

The employment data we are interested in explaining are total non-agricultural employment (it was not considered that the model would apply adequately to the link between output and employment in agriculture, since output may fluctuate independently of employment due to climatic variations). The output data should therefore be total non-agricultural output, including both commodity output and services output.

Unfortunately, on a regional basis, only commodity output is available - real value added in manufacturing, mining, construction, electricity, and some other minor commodity producing industries. Value added in the service industries, on a regional basis, is missing. Clearly with no data we can make no progress, unless some reasonable assumptions about the relationship of service output to commodity output can be made. I think this can be done.

Service output differs from commodity output in two ways: it generally grows at a different rate, and it does not fluctuate cyclically as much as commodity output. This suggests that some relationship between service output on the one hand, and commodity output and time on the other, may exist.

Denote service output by Q_s, commodity output by Q_c, and consider the relationship

$$Q_s = H(Q_c)^{\delta} e^{(g_s - \delta g_c)t}, \quad H \text{ a constant}$$

The relationship has the following properties.

(i) If Q_c grows at a steady rate g_c, without fluctuations, then Q_s grows at a steady rate g_s.

(ii) If the actual value of Q_c varies from steady state growth, up or down, by any given percentage μ, Q_s will vary in the same direction, but by a percentage $\delta\mu$, different from μ.

If we assume that this gets close to the facts, and write $g_s - \delta g_c = \tau$, we have

$$Q_s = HQ_c^{\delta} e^{\tau t}$$

Data for this relationship are available for Canada as a whole, even though not for the regions separately. The equation was fitted for data from 1949 to 1967. Both δ and τ were significant (t values of 4, and R^2 of .99) with values 0.54 and 0.024 respectively. This suggests that in Canada service output varies cyclically about half as much as commodity output, though this need not be true region by region. The values of δ and τ can differ regionally: a stable relationship is all we need.

Thus
$$Q = Q_c + Q_s$$
$$= Q_c + HQ_c^{\delta} e^{\tau t}$$

Substituting in (6) we have

$$(14) \quad \ln E_t = a_0 + (1-\lambda)\ln E_{t-1} - \frac{\rho\lambda t}{\alpha} + \frac{\lambda}{\alpha}\ln\left(Q_{ct}+HQ_{ct}^{\delta}e^{\tau t}\right)+\ln\xi_t$$

The term with coefficient λ/α is non linear, but may be approximated by a Taylor series expansion around the mean values of Q_{ct} and t. If a linear approximating expansion is used we find (recalling that $Q_{st} = HQ_{ct}^{\delta}e^{\tau t}$), and denoting the mean value of a variable by a bar over it, e.g. mean of $\ln Q_{ct} = \overline{\ln Q_{ct}}$,

$$\ln(Q_{ct}+HQ_{ct}^{\delta}e^{\tau t}) \doteq \overline{\ln(Q_{ct}+Q_{st})} + \frac{Q_{ct}-\bar{Q}_c}{\bar{Q}_c+\bar{Q}_s}\left(1 + \frac{\delta\bar{Q}_s}{\bar{Q}_c}\right)$$

$$+ (t-\bar{t})\,\tau\,\frac{\bar{Q}_s}{\bar{Q}_c+\bar{Q}_s}$$

Writing $r = \dfrac{\bar{Q}_c}{\bar{Q}_s+\bar{Q}_c}$, and noting that

$$\ln Q_{ct} \doteq \overline{\ln Q_{ct}} + \frac{Q_{ct}-\bar{Q}_c}{\bar{Q}_c} \quad ,$$

we get

$$\ln(Q_{ct}+HQ_{ct}^{\delta}e^{\tau t}) = \overline{\ln(Q_{ct}+Q_{st})} - [r+\delta(1-r)]\,\overline{\ln Q_{ct}}$$

$$- \bar{t}\,\tau\,(1-r) + [r+\delta(1-r)]\ln Q_{ct}$$

$$+ \tau\,(1-r)t$$

$$= \text{Constant} + [r+\delta(1-r)]\ln Q_{ct} + \tau(1-r)t$$

On substituting this last into (14) we find that

$$(15)\quad \ln E_t = \text{Constant} + (1-\lambda)\ln E_{t-1} + \left[\tau(1-r) - \frac{\rho\lambda}{\alpha}\right]t + \frac{\lambda}{\alpha}[r+\delta(1-r)]\ln Q_{ct} + \ln \xi_t$$

Or, putting a_o = Constant

$$a_1 = 1-\lambda$$

$$a_2 = \tau(1-r) - \rho\lambda/\alpha$$

$$a_3 = \frac{\lambda}{\alpha}[r+\delta(1-r)]$$

$$\ln \xi_t = u_t$$

$$(16)\quad \ln E_t = a_o + a_1 \ln E_{t-1} + a_2 t + a_3 \ln Q_{ct} + u_t$$

Some Comments on (16)

The coefficient a_3 is the short run elasticity of total employment with respect to commodity output. It depends on λ, the lag adjustment parameter; on $1/\alpha$, the elasticity of demand for man hours with respect to total output; on δ, the short run elasticity of service output with respect to commodity output; and on r, the ratio of commodity output on average to total output.

For a given r, a_3 rises with δ, i.e. the more closely service output follows commodity output through a cycle, the more sensitive is employment to commodity output.

If δ is zero, so that service output is not

subject to cyclical fluctuations, the sensitiveity of total employment with respect to commodity output is smaller the smaller is r, the ratio of commodity output to total output.

For given δ , λ, and α - if, say, these were the same across regions - a_3 would be larger the larger was r, the ratio of commodity output to the total.

The sign of a_2, the coefficient on time, is not predictable a priori, nor can we even make a plausible guess about it, since quite reasonable values of the underlying parameters on which it depends can be chosen to make it positive or negative.

The coefficient a_1 should be between zero and unity.

Estimation of (16)

There is a lagged dependent variable on the right, and there could very likely be autocorrelation of u_t. Under these circumstances only consistent estimators can be derived. The method chosen was the Hildreth-Liu technique.

It was not considered that the above theory would apply well to the link between output and employment in agriculture, since output may fluctuate independently of employment due to climatic variations. Equation (16) was therefore fitted using data excluding both agricul-

tural employment and agricultural production.

Only annual data are available on a regional basis. The regions were the Atlantic, B.C., Ontario, the Prairies and Quebec. The time period covered was from 1949 to 1967.

3. Results of Fitting the Employment Functions

The results are in Table I below. "t" statistics are in parentheses.

TABLE I

Region	Coefficients on			R^2	DW	Theil's "U" (based on first differences)
	$\ln E_{t-1}$ (a_1)	Time (t) (a_2)	$\ln Q_{ct}$ (commodity output) (a_3)			
Atlantic	.32 (2.4)	-.011 (2.5)	.58 (4.6)	.97	1.99	.40 (6.2)
B.C.	.33 (4.4)	.008 (2.6)	.35 (6.3)	.99	2.02	.25 (12.7)
Ontario	.23 (1.4)	.014 (2.5)	.21 (2.8)	.99	1.48	.25 (13.6)
Prairies	.00 (0.3)	.044 (7.0)	-.09 (0.9)	.99	1.47	.25 (12.7)
Quebec	-.01 (2.1)	.017 (3.1)	.41 (2.9)	.99	1.51	.25 (12.7)
All Canada	.00 (.10)	.016 (3.5)	.36 (3.9)	1.00	1.30	.17 (20.0)

The fit is generally very good. Coefficients on commodity output are between zero and unity, and four out of five are significant at 5% or better. The exception is the Prairies, where the estimated coefficient is almost

zero and not significantly different from zero. This implies that commodity output excluding agriculture plays no detectable role in fluctuations in the demand for labour in the non-agricultural sector. It is not that there are no variations in output or employment about their trends, but rather that the output fluctuations do not appear to generate corresponding employment fluctuations.

The coefficients on time are all significant, varying in sign as expected, and of the right order of magnitude.

Lagged employment appears to be important in three regions, the Atlantic, B.C. and Ontario. The coefficients are significant for the Atlantic and B.C., but not for Ontario. Nevertheless, in what follows, we retain the value of 0.23 for Ontario, even though we could not reject the hypothesis that the true value is zero. For the Prairies and Quebec, lagged employment has a zero coefficient for practical purposes. Though the Quebec coefficient is significantly negative, its actual value is so small (-.01), that we lose nothing by calling it zero.

The Durbin Watson statistics indicate that, after the Hildreth-Liu technique has been applied, autocorrelation is not a serious problem.

R^2 values are high, as is to be expected with time series data of this kind. A better measure of how well

or badly the theory explains employment fluctuations is Theil's "U", based on first differences. A value of zero for U would be perfect prediction of percentage changes in employment, in both direction and absolute size. A value of unity is a complete failure to predict. The "t" values shown refer to the hypothesis that U is unity, rather than zero as is customary for parameter values, since U=1 represents failure of prediction. All the U's are highly significant in this sense, indicating that the high R^2's are not, in fact, misleading. The theory does fit well.

4. Separation of Output and Elasticity Effects

The ranking of short run elasticities of employment with respect to commodity output (the a_3's) is the same as a ranking based on sensitivity of the unemployment rate to the aggregate unemployment rate, namely the Atlantic, Quebec, B.C., Ontario and the Prairies, (see Denton [4]). This result is encouraging, but not conclusive, in that the short run elasticity does not tell the whole story of the link between output and employment.

The relationship fitted was of the form

$$E_t = e^{a_0} E_{t-1}^{a_1} e^{a_2 t} Q_{ct}^{a_3}$$

A problem arises in that due to the presence of the lag

term it is difficult to assess the quantitative importance, as far as employment fluctuations are concerned, of differences among the regions in parameter values, except in very simple cases.

If, for example, $a_1 = a_2 = 0$ for each of two regions, differences in a_3 could be simply interpreted, as indicating the degree to which fluctuations in output would generate employment fluctuations. If, say, one region had $a_3 = 0.25$, and a second region had $a_3 = 0.50$, then the second region would have, for given percentage fluctuations in output, percentage fluctuations in employment that were twice as great as in the first region.

Once a_1 is non zero interpretation becomes much more difficult. To take a simple example: suppose that $a_2 = 0$, but a_1 and a_3 are not zero, and that Q_{ct} follows the very special cycle given by

$$\ln Q_{ct} = (-1)^t k.$$

Then the mean value of Q_{ct} is unity, and it varies by $\pm 100k\%$ about that value. It is not difficult to show that E_t will then vary about its mean value by $\pm 100k\ a_3/(1+a_1)\%$.

Thus, if k were 0.05 for each of two regions, and region 1 had $a_3 = 0.60$ and $a_1 = 0.50$, whereas for region 2, $a_3 = 0.40$ and $a_1 = 0.00$, employment in both regions would fluctuate around the mean value by $\pm 2\%$. Even though

region 1 has a short run employment output elasticity half as large again as region 2, its employment fluctuations are no more severe, due to a dampening effect from the lag in adjustment of employment to output.

More generally, fluctuations in Q_{ct} will generate fluctuations in E_t, but the relationship between the two sets of fluctuations will depend in a rather complex way on the actual pattern of fluctuations in Q_{ct}, and on the values of both a_1 and a_3.

The situation is more difficult yet, because if we want to compare two regions, or one region with Canada as a whole, differences in the degree to which employment fluctuates will be traceable not only to differences in the values of a_1, a_2 and a_3 between the regions, but also to differences in the way output, Q_{ct}, fluctuates in the two regions.

The problem may be posed as follows. Suppose we have two regions, one of which may be the whole of Canada. For each region we have

(i) estimated parameter vectors,

$$a_{0i},\ a_{1i},\ a_{2i},\ a_{3i},\ i = 1,2$$

(ii) observed time paths of commodity output -

$$Q^i_{c1},\ Q^i_{c2},\ \ldots . Q^i_{ct} \ldots ,\ i=1,2$$

Each region will have a different cycle of employment.

Given this, can we say how much of the difference is due to the parameter vectors being different, and how much is due to the output paths being different?

Consider first separating the cyclical component of employment from its trend. Suppose that output can be multiplicatively decomposed into a trend factor, and a cyclical factor which varies around unity, i.e.

$$Q_{ct} = Q^T_{ct} \cdot Q^C_{ct},$$

where the superscripts indicate "trend" and "cycle" respectively. The data are such that this can in fact be done by fitting a time trend to the logarithms of Q_{ct}.

Then, denoting the trend component of employment by E^T_t, we have that

$$E^T_t = e^{a_0}(E^T_{t-1})^{a_1} e^{a_2 t}(Q^T_{ct})^{a_3} .$$

Since

$$E_t = e^{a_0} E_{t-1}^{a_1} e^{a_2 t} Q_{ct}^{a_3}$$

we obtain on dividing

$$\frac{E_t}{E^T_t} = \left[\frac{E_{t-1}}{E^T_{t-1}}\right]^{a_1} \left[\frac{Q_{ct}}{Q^T_{ct}}\right]^{a_3}$$

Defining $E^C_t = E_t/E^T_t$, we have

$$E^C_t = E^{C\ a_1}_{t-1}\ Q^{C\ a_3}_{ct}$$

Write $\ln E_t^C = r_t$, $\ln Q_{ct}^C = q_t$, and we get

$$r_t = a_1 r_{t-1} + a_3 q_t$$

Define the operator notation $Dr_t = r_{t-1}$, $D^2 r_t = r_{t-2}$, etc., so that

$$r_t = \frac{a_3}{1-a_1 D} q_t$$

Put $\frac{a_3}{1-a_1 D} = x$, so that

$$r_t = x q_t$$

Now take a Taylor series approximation around (x^*, q_t^*), so that

$$r_t \doteq x^* q_t^* + (x-x^*) q_t^* + x^*(q_t - q_t^*).$$

Replacing "x" and "x*" by the appropriate operators we have then the result we shall use:-

$$(17) \quad r_t \doteq \frac{a_3^*}{1-a_1^* D} q_t^* + \left[\frac{a_3}{1-a_1 D} - \frac{a_3^*}{1-a_1^* D}\right] q_t^* + \frac{a_3^*}{1-a_1^* D}(q_t - q_t^*)$$

We shall interpret the starred values (*) as being those for Canada as a whole, and the unstarred values as being those for a particular region.

The left hand side of (17), r_t, is the natural logarithm of the (multiplicative) cyclical component of employment. We shall call it the "logarithmic deviation from trend". It will vary around zero, and, apart from a

scale factor of 100, can be interpreted as the percentage deviation of employment from its trend (since $\ln(1+y)$ is well approximated by "y" if "y" is not too far from zero, say less than ± 0.1).

Equation (17) shows that, to a Taylor series linear approximation, the regional (percentage) deviation of employment from trend can be additively decomposed into three components.

The first component is the deviation of Canada wide employment from its own trend. Notice it could be of opposite sign to r_t, though usually it will be the same sign.

The second component is the difference between the deviations Canada would have had if it had had the region's parameters and the deviations with its own parameters. It represents that part of the region's deviations traceable to parameter differences exclusive of product cycle differences. It may be of the same sign or of opposite sign to r_t.

The third component is the difference between the deviations Canada would have had if it had had its own parameters but the region's product cycle and the deviations if it still had its own parameters and product cycle. It represents that part of the region's deviations traceable to product cycle differences exclusive of parameter differences. It too may be of the same or

opposite sign to r_t.

In short, the regional employment cycle is approximately made up of a component due to the Canada-wide cycle, a component due to parameter differences between the region and Canada, and a component due to product cycle differences between the region and Canada. The decomposition is only approximate because there will usually be some interaction between parameter and product cycle differences.

Measuring the Contribution of Each Component

In order to measure the relative contribution of each of the three components to a region's cycle we need some acceptable measure of "a cycle". How could we express, for example, in quantitative terms, the statement that the Atlantic's employment cycle is more severe than Ontario's?

Amplitude would be a possibility, but is subject to the drawback that one extreme peak or trough could give a region with a generally mild cycle a misleadingly high measure.

The measure we propose to use for the magnitude of the cycle in r_t is the mean absolute value:-

$$A = \frac{1}{n} \sum_{t=1}^{n} |r_t|$$

"A" is dimensionless. 100A is interpretable, if

A is of the order of 0.1 or less, as the average percentage by which employment deviates from its trend according to the fitted relationship between employment and output, lagged employment and time. It will, of course, differ somewhat from the actual average percentage deviation in that the fitted relationship is not a perfect explanation of employment.

Next, define, for i=1,2,3:-

$$\phi_{it} = +1 \quad \text{if either} \quad r_t \geq 0 \text{ and } r_{it} \geq 0 \quad \text{or} \quad r_t < 0 \text{ and } r_{it} \leq 0$$

$$= -1 \quad \text{if either} \quad r_t > 0 \text{ and } r_{it} < 0 \quad \text{or} \quad r_t < 0 \text{ and } r_{it} > 0$$

and then define:-

$$A_i = \frac{1}{n} \sum_{t=1}^{n} \phi_{it} |r_{it}| \quad \#$$

Then it follows that

$$A = A_1 + A_2 + A_3$$

This somewhat complex procedure ensures that, in time periods when the value of a component of r_t is of opposite sign to r_t itself, the absolute value of that component is, for that time period, subtracted rather than added in getting the total contribution of A_i to A. Thus, for example, A_2 would be negative if r_{2t} was generally of opposite sign to r_t.

A_i measures the contribution of the i'th component to the mean of the absolute value of the r_t's, taking appropriate account of whether r_{it} is of the same or opposite sign to r_t at time t.

The cycle measure has one further advantage apart from being separable into components, namely that, being dimensionless, it can be used to compare two different cycles in terms of how much they fluctuate. If, for example, the measure applied to the output cycle was .040, and to the employment cycle was .020, we could say that the output cycle was twice as severe as the employment cycle to which it gave rise.

Application to Cycles in the Regions

We shall illustrate the procedure with respect to the Atlantic region and then summarise results.

For Canada we have

$$a_3^* = 0.36 \text{ and } a_1^* = 0.00$$

and for the Atlantic

$$a_3 = 0.58 \text{ and } a_1 = 0.32$$

Thus

$$(18) \quad r_t = 0.36q_t^* + \left[\frac{0.58}{1-.320} - 0.36\right] q_t^* + 0.36(q_t - q_t^*)$$

The values of q_t^* are found as the logarithmic

deviations of Canadian output from trend.[#] Putting

$$\ln Q_{ct}^{CAN} = b_0^* + b_1^* t + b_2^* t^2 + u_t^*$$

and denoting estimated values by ′, we have

$$q_t^* = u_t^{*\prime} = \ln Q_{ct}^{CAN} - b_0^{*\prime} - b_1^{*\prime} - b_2^{*\prime} t^2$$

The values of q_t are found as the logarithmic deviations of Atlantic output from trend, so that

$$q_t = \ln Q_{ct}^{ATL} - b_0' - b_1' t - b_2' t^2$$

The first and third terms in r_t from (18) are straightforward. The centre term may be expanded as

$$r_{2t} = 0.58(1+.32D+.32^2D^2+.32^3D^3+....)q_t^* - .36q_t^*$$

$$= .22q_t^* + (.58)(.32)q_{t-1}^* + (.58)(.32)^2 q_{t-2}^* + (.58)(.32)^3 q_{t-3}^* + ...$$

Terms beyond q_{t-3}^* will be small enough to ignore safely.

For the Atlantic the value of A turns out to be 0.0229, indicating that the fitted relationship between employment and output would predict, given the actual path of output in the Atlantic, that employment would deviate

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Quadratic time trends used throughout.

from its trend by an average of 2.29%. The actual cycle in employment in the Atlantic may be measured (by averaging the absolute multiplicative deviations of employment from a fitted time trend) and turns out to be 0.0202, quite close to the predicted value. Of the theoretical value of 0.0229, A_1 contributed 0.0133, i.e. over half the total Atlantic employment cycle was due to the Canada-wide employment cycle. The contribution of A_2, the portion of the Atlantic's cycle due to parameter differences between the Atlantic and Canada, was 0.0148, about two-thirds. The contribution of A_3, the portion due to differences between the Atlantic's output cycle and Canada's, was -.0025, indicating that the output cycle in the Atlantic was slightly less severe than in the country as a whole (as can be confirmed by measuring it, see below, Table II). This factor compensated somewhat in the Atlantic's employment cycle for its less favourable parameter values. The sum of A_1, A_2 and A_3 is not exactly equal to A, since the breakdown is only approximate. The remaining "error", which is actually the interaction effect between parameter and output cycle differences, was -0.0027. We shall refer to the interaction term in what follows as A_4.

We may conclude, for the Atlantic region, that most of the difference between the Atlantic and Canada in the amplitude of the employment cycle is traceable to differences in parameter values, i.e. to the combination

TABLE II

Cycle Measures of Output and Employment in Canadian Regions

Region	Output Cycle	Employment Cycle		Components of the Predicted Cycle due to:-				Short Run Employment Output Elasticity	Lag Adjustment Parameter
		Actual	Predicted	Canada-wide Cycle*	Parameter Differences	Output Cycle Differences	Interaction between A_2 and A_3		
				A_1	A_2	A_3	A_4	(a_3)	(a_1)
Atlantic	.0316	.0202	.0229	.0133	.0148	-.0025	-.0027	.58	.32
B.C.	.0656	.0226	.0283	.0131	.0032	.0105	.0015	.35	.33
Ontario	.0366	.0126	.0087	.0113	-.0032	.0018	-.0012	.21	.23
Quebec	.0359	.0153	.0147	.0122	.0017	.0008	.0000	.41	.00
Canada	.0354	.0134	.0137	.0137	0	0	0	.36	.00

* Would be .0137 if the region's cycle and Canada's were exactly in phase, -.0137 if the two employment cycles were of exactly opposite phase, otherwise A_1 lies between those limits.

of a higher short run employment/output elasticity (a_3) and a different lag adjustment parameter (a_1). There are lesser effects, and in the opposite direction, from differences in the output cycles, and from interaction between parameter differences and output cycle differences.

The results for other regions, except the Prairies, can now be given, along with those for the Atlantic already discussed. They are in Table II above, which also contains data on the output cycles and parameter values in the regions.

The first column of figures in Table II shows that the output cycle is about the same in Ontario and Quebec as in Canada as a whole#, and slightly smaller in the Atlantic. In B.C., however, the output cycle is nearly twice as great as the Canadian average.

These differences among the output cycles are reflected in the values of A_3 in column 6, which show contributions to the regional employment cycles of differences between the regional output cycle and the national one. For the Atlantic, Ontario and Quebec these contributions both absolutely, and in comparison to B.C.,are small. For B.C. the contribution is 0.0105, which is 37% of the total cycle, and no less than 69% of the difference between the

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#Not surprisingly, of course, in view of their heavy weight in the total.

total cycle and the part due to the Canada cycle. B.C.'s employment cycle is therefore much worse than the Canada average simply because its output fluctuates much more.

The column headed A_2 shows the contribution of parameter differences to each region's employment cycle. The relevant parameters from Table I, a_3 and a_1, are reproduced in the last two columns of Table II for convenience.

Parameter differences are of major importance in the Atlantic, as already noted. In B.C. they are moderately important. At 0.0032 they add about a third as much to B.C.'s employment cycle as the output cycle differences already noted. For Ontario the situation is reversed as compared with the Atlantic and B.C.. The negative entry of -0.0032 indicates that Ontario's employment cycle is moderated by parameter differences, and they account for the major part of the (negative) difference between Ontario's employment cycle and Canada's. In Quebec parameter differences worsen the employment cycle slightly, and also account for most of the (positive) small difference between Quebec's cycle and Canada's.

The column headed (A_1) shows the contribution of the Canada-wide employment cycle to the employment cycle in each region. Only in Ontario does this differ more than slightly from the Canada-wide employment cycle itself: it does so because the employment cycles in Canada and

Ontario are not precisely in phase, so that fluctuations in Canadian employment are not invariably accompanied by fluctuations in the same direction in Ontario.

5. Conclusions

The Atlantic's employment cycle is one and a half times as severe as the average for Canada. This is not because its output fluctuates more (it fluctuates less) but rather because the demand for labour varies more, in percentage terms, for given percentage variations in output than it does in Canada as a whole.

The situation in B.C. is quite different, and in sharp contrast to the Atlantic. The employment cycle is even more severe than in the Atlantic, being about 70% greater than the Canadian average. Unlike the Atlantic, however, this is mainly because B.C.'s output fluctuates more than Canada's, in fact nearly twice as much. The demand for labour also varies a little more in response to given output fluctuations in B.C. than it does in Canada as a whole, though the importance of this in explaining B.C.'s employment cycle is relatively small. In addition, the interaction of parameter and output cycle differences exacerbates B.C.'s employment cycle, rather than moderating it, as in the Atlantic.

Ontario's employment cycle is about 6% less severe than the average for Canada. While its output fluctuates

a little more than the Canadian average, this is more than offset by the fact that the demand for labour varies less, in percentage terms, for given percentage variations in output than it does in Canada as a whole.

Quebec's employment cycle is about 14% greater than the Canadian average. Its output cycle differs only very slightly from Canada's, and its more severe employment cycle is due mostly to a moderately less favourable response of employment to output variations than obtains for Canada as a whole.

The Prairies are different again: the evidence here suggests that within the range of cyclical output and employment variations that have occurred there is no cyclical effect of output on demand for labour at all.#

Direction of Future Research

As mentioned at the beginning, cycles in unemployment can differ regionally not only because demand for labour varies differently, but also because supply does so. The next major step is therefore to try and explain the supply of labour through the cycle in each region.

On the demand side, prime questions to be answered are: why does B.C.'s output fluctuate more than in any

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#Since agriculture was excluded from both the output and employment series the explanation presumably lies there.

other region? Why is the short run elasticity of employment with respect to output so high in the Atlantic? Why is this elasticity so low in Ontario? What does generate the mild employment cycle in the Prairies?

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