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Preliminary Results from a Study to Estimate the Cost of Errors in a Firm's Aggregate Predictions

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PRELIMINARY RESULTS FROM A STUDY TO
ESTIMATE THE COST OF ERRORS IN A FIRM'S
AGGREGATE PREDICTIONS

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PRELIMINARY RESULTS FROM A STUDY TO ESTIMATE THE COST
OF ERRORS IN A FIRM'S AGGREGATE
PREDICTIONS

by

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"Unfortunately, microeconomics is probably not an elegant subject when really well done. In order to keep us going, every now and then it is a good idea to set almost all the parameters equal to one or zero, keep everything convex, and make a few simplifications here and there, and for this price we are rewarded with some nice convergence theorem. The Invisible Hand appears, pats us on the head, and we feel elegant. Beyond that it is necessary to go back to the miasmal swamp of reality."

Martin Shubik
 "A Curmudgeon's Guide to Microeconomics,"
 Journal of Economic Literature
 Vol. VIII, No. 2 (June 1970), p.421.

Introduction

Clearly there are many aggregate factors which affect all firms in the economy to some degree. Presumably there are others which differentially affect individual industries. Industry considerations aside, any particular firm would want to formulate "own-firm" expectations incorporating aggregate expectations at least insofar as it felt aggregate events important. Much of the analysis of earlier research concentrated on measuring the errors in these aggregate expectations per se (i.e. by using absolute error criteria (basically the mean absolute and mean square error criteria)).¹ As regards the firm making the forecasts, the important standard of "accuracy" might rather be economic consequences for its operations of these forecast errors (which may or may not be adequately approximated by **absolute error criteria**).

A great deal of work has been reported in the literature on the nature of expectations and their uses in economic models. The work of Muth and Mills, for

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1. See, for example, Henri Theil, Economic Forecasts and Policy, (1st edition), (Amsterdam: North-Holland Publishing Company, 1958; Victor Zarnowitz, An Appraisal of Short-term Economic Forecasts, National Bureau of Economic Research, Occasional Paper 104 (New York: Columbia University Press, 1967); Christopher Simms, "Evaluating Short-term Macro-Economic Forecasts: The Dutch Performance," Review of Economics and Statistics, 49 (May, 1967), p. 225; Donald Smyth, "How Well do Australian Economists Forecast?" Economic Record, 42 (June, 1966), p. 293 and Merv Daub, "An Appraisal of Canadian Short-term Aggregate Economic Forecasts," unpublished Ph.D Dissertation, University of Chicago, 1971, forthcoming.

example, is well known.² However the orientation is almost exclusively on the "industry-firm" question with much of the theoretical discussion centering on making a profit maximizing price selection under various assumptions about the structure of the market and the amount of uncertainty present. The specific question of the inaccuracy of aggregate expectations and firm activity has not been treated as extensively.

Before considering this question, and in order to point up some limitations, a brief examination of how a firm might forecast is in order. In forecasting aggregate economic activity the firm would have to examine its own past record of aggregate forecasting, the past record of other aggregate forecasts, as well as other "exogenous" aggregate information such as planned tax revisions, capital expenditure anticipations and the like. These would result in a firm's preliminary aggregate forecast for the next period and other preliminary aggregate forecasts as well. The firm would also consider the history of its forecasting of the industry and its own data, any past records of other forecasts of the industry and its own performance (e.g. by trade associations, other firms in the industry, security analysts) as well as relevant "exogenous", more micro-economic factors such as upcoming wage negotiations or preferential tariff expectations. This would result in a firm's preliminary forecast for the industry and itself as well as "other" preliminary industry and

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2. J.F. Muth, "Rational Expectations and the Theory of Price Movements," Econometrica, 29(July, 1961), pp. 315-335 and E.S. Mills, Price, Output and Inventory Policy, (New York: John Wiley and Sons, 1962). Other useful references in this regard include M.S. Bowman (ed.), Expectations, Uncertainty and Business Behavior, (New York: Social Science Research Council, 1958) and Mincer (ed.), Economic Forecasts and Expectations, (N.B.E.R.: Columbia University Press, 1969) on the nature of expectations; and J. Gould, "The Micro-economic Approach to the Demand for Physical Capital," Report 6633 (Center for Mathematical Studies in Business and Economics, University of Chicago, 1966) and R. Nelson, "Uncertainty, Prediction and Competitive Equilibrium," Quarterly Journal of Economics, 75(January 1961), pp. 41-62 for discussions of some investment and production problems which include the consideration of uncertainty.

that firm forecasts. Then there probably ensues a period of formal and informal comparison and feedback between the firm's own aggregate, industry and firm forecasts and those of others out of which come the final forecasts of all parties at various dates near the end of the current period. This is admittedly simplistic especially in the time dimension since the forecasting process is a continuing one of updating and change throughout any period but it probably does generally summarize the approach of those companies who do a comprehensive annual budget and forecast.

What this points out quite clearly is that there are several major types of inputs into final "own-firm" forecasts besides the firm's own aggregate forecast. And ideally one would want to consider all these factors explicitly in trying to assess the influence of the inaccurate aggregate forecasts on the firm's actions. Just as certainly some of these, such as the firm's own past preliminary and final industry and "own-firm" forecasts, for example, are unobservable or not available. As a result it was necessary to devise a method of estimating the role of inaccurate aggregate forecasts within such a limited framework all the time realizing that more generality would be desirable. Such an approach takes the form of examining several models which are based on different assumptions about how "own-firm" forecasts might have included these aggregate predictions. The necessarily simplistic nature of such an approach is, as is more generally the case also, an excellent indication of the infancy of the research into the subject and the exploratory nature of what follows.

Accordingly the next section presents a "native" model for introducing aggregate considerations which is estimated for three companies. Following it is a section dealing with a "market-share" model. Two sections are then devoted to presenting somewhat more elaborate models for two of the companies. A brief section comparing the various estimates of these models is then given. Finally a summary of the major findings concludes the paper.

A "Naive" Model

One simple method of introducing aggregate considerations is to assume that the firm felt the relationship of Gross National Product to various relevant firm variables was of the form (1) and it had estimated this relationship up to the end of the previous period on the basis of past data.

$$(1) \quad AD_t = \alpha_{1t} + \alpha_{2t} Y_t + u_t$$

where AD_t = actual firm accounting data up to the end of the previous period.

Y_t = actual Gross National Product (up to the end of the previous period).

Given the simple assumptions of the model, if the firm had made a perfect forecast of Gross National Product, the expected firm data in the next period would be the regression prediction:

$$(2) \quad \hat{AD}_t = \hat{\alpha}_{1t} + \hat{\alpha}_{2t} Y_t$$

However the firm did not make perfects and the result of this error is presumably incorporated in the actual change in firm data in the next period. Thus the error in forecasting AD_t :

$$(3) \quad e_t = AD_t - \hat{AD}_t$$

represents the "cost" to the firm of the inaccurate aggregate forecasts.³

Summary statistics of the results of estimating equation (3), for the four indicated firm variables of Companies A, B and C, are presented in Table 1.⁴

3. Some evidence in a related case of a similar approach using this model can be found in Philip Brown and Raymond Ball, "Some Preliminary Findings on the Association between Earnings of a firm, Its Industry and the Economy", Empirical Research in Accounting: Selected Studies, 1967 (Supplement to Volume 5 of the Journal of Accounting Research). pp. 55-77.

4. For a discussion of the reasons for not identifying these companies explicitly see M. Daub, "An Appraisal of Canadian Short-term Aggregate Economic Forecasts," pp.4-5

Table 1

Estimates of the Cost of Errors in Level Forecasts
of Gross National Product using a "Naive" Model

Firm Variable ^a	Total Revenue	Total Labour Costs	Total Other Costs	Net Income ^c
Company A: Mean	\$-9.6 million	\$+22.1 million	\$-14.5 million	\$-2.3 million
(1956-1969) Mean Absolute	11.6 (10.43) ^b	27.4 (20.71)	20.1 (21.87)	5.1 (4.62)
$\overline{R}_{AD_t, Y_t}^2$.98	.96	.96	.97
R_{AD_t, \hat{AD}_t}^2	-	-	-	.96
Company B: Mean	-1.3	-.9	-.1	-1.9
(1961-1969) Mean Absolute	3.7 (5.10)	1.1 (1.39)	2.1 (3.44)	8.1 (9.56)
$\overline{R}_{AD_t, Y_t}^2$.93	.97	.88	.80
R_{AD_t, \hat{AD}_t}^2	-	-	-	.46
Company C: Mean	+31.3	-8.3	-23.9	-1.3
(1964-1969) Mean Absolute	31.3 (29.3)	9.4 (10.6)	25.8 (27.1)	2.7 (2.1)
$\overline{R}_{AD_t, Y_t}^2$.74	.89	.63	.68
R_{AD_t, \hat{AD}_t}^2	-	-	-	.53

^a Total Revenue is measured by Net Sales; Total Labour Costs by Salaries, Wages and the like; Total Other Costs by the difference [Net Sales minus (Total Labour Costs plus Net Income)]; and Net Income by Operating Income (after taxes).

^b The figures in brackets are standard deviations.

^c The individual results for each company were weighted for the reporting period and the size of the average profits during the respective periods to arrive at weighted means. These combined weights were .51, .32 and .17 for Companies A, C and D respectively. The weighted mean (mean absolute) cost of error in terms of net incomes was accordingly \$-2.0 million (\$5.6 million).

Thus Company A's net income was \$2.3 million less on average with imperfect forecasts (3.2% of average profits in the period) than it would have been with perfect forecasts and using this model, Company B's \$1.9 million less (4.6% of average profits) and Company C's \$1.3 million less (8.1% of average profits). Overall the companies weighted net income was \$2.0 million lower on average (3.9% of the weighted average of net incomes) with imperfect forecasts than it would have been with perfect forecasts.

Clearly such a model is quite imperfect. Among other things, aggregate variables such as Total Personal Expenditures or New Non-Residential Construction may be more relevant to the firm's own expectations than Gross National Product.⁵ Also, some of the difference between actual income and the conditional expectations generated from this model undoubtedly come from firm decisions i.e. other variables not included in the model.⁶

Consider also in what sense it can be said that the degree of accuracy of the expectations about firm data correctly estimates the "cost" to the firm of the inaccurate aggregate forecasts. To say that the difference, for example, between the actual and predicted Total Revenue or Total Labor Costs obtained from this model represents the "cost" to the firm of inaccurate sales or labor expenditure forecasts

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5. In the case of Company A Gross National Product was replaced by Total Personal Expenditures as the independent variable and the statistics of Table 1 reestimated. In this case net income was \$2.1 million lower on average (3.0% of average profits) with imperfect Total Personal Expenditure forecasts than with perfect predictions ($R^2_{AD_t, Y_t} = .96$, $R^2_{AD_t, \hat{AD}_t} = .94$). Which variable is more relevant would appear to make

little difference in the estimates, given the model and techniques used.

6. While the $R^2_{AD_t, Y_t}$ and $R^2_{AD_t, \hat{AD}_t}$ are quite large (in other circumstances suggesting the model is appropriately chosen) there are undoubtedly multicollinearity problems present. In the case of Company A the equation used in Table 1 for the cost of aggregate errors in terms of net income was reestimated using percentage changes in Net Income and Gross National Product. The results suggest that the mean percentage change in Company A's income was 2.0% of the average percentage change in profits lower with imperfect forecasts than it would have been with perfect forecasts ($R^2_{AD_t, Y_t}$ dropping to .36).

is not legitimate. That is, inaccuracy does not directly translate into measured costs of that inaccuracy. Ideally one would want to solve simultaneously for all firm variables and their expectations, the costs of inaccuracy, in terms of profits, then being measured by the "net" difference in expectations. This is an extremely difficult problem in estimation, programming and the like and infeasible in the current circumstances. The closest approximation to such a result in this model would be to treat the difference between expected and actual net income as the relevant "cost" of inaccurate aggregate forecasts. The model is used here then as an example of a particularly simplistic naive device by which one might associate firm variables with aggregate expectations and their errors.

A "Market-Share" Model

Introducing some structural assumptions, suppose that in the past the actual sales of the j th firm have been some proportion k_j of Gross National Product where k_j is determined by industry i 's share of Gross National Product (c_i) and firm j 's share of industry sales (s_j). That is:

$$(4) \quad S_j = k_j Y = s_j c_i Y \quad \text{where } S_j \equiv \text{actual sales of firm } j$$

$$c_i \equiv \text{industry's } i\text{'s share of Gross National Product}$$

$$s_j \equiv \text{firm } j\text{'s share of industry } i \text{ sales}$$

$$Y \equiv \text{Gross National Product at current market prices}$$

Pursuing this approach, assume also that in the past if all expenses (excluding profits) can be summarized by the two major categories of labor and other non-labor costs, actual labor and other costs have been some proportions l_j and o_j of actual sales. That is:

$$(5) \quad L_j = l_j S_j \quad \text{and} \quad O_j = o_j S_j$$

This would imply that actual profits in the past have been some proportion $(1 - l_j - o_j) s_j c_i$ of Gross National Product. That is:

$$(6) \quad \pi_j = S_j - (TC_j) = S_j - (l_j + o_j)S_j = (1 - l_j - o_j)s_j c_i Y$$

If the firm had made a perfect prediction of Gross National Product (Y) for the next period and all the parameters (s_j , c_i , l_j and o_j) are assumed relatively stable overtime, this model implies that profits for the next period would have been predicted as π_j calculated in (6). However, the prediction of Gross National Product contained an error the result of which, among other things, is incorporated in realized profits (π_A). The difference then between equation (6) and realized profits, i.e. $\pi_j - \pi_A$, measures the cost of this aggregate error. Summary statistics of the results of evaluating this difference for the three companies concerned is given in Table 2.

Table 2

Estimates of the Cost of Errors in Level Forecasts of Gross National Product using a "Market-Share" Model^a

	c_i	s_j	Total Revenue	l_j	Total Labor Costs	o_j	Total Other Costs	Net Income
Company A: Mean	.0111 (.0061)	1.00 (.00)	\$-1.5 million	.38 (.05)	\$+5.9 million	.48 (.04)	\$-8.2 million	\$-.9 million
Mean Absolute			21.5 (11.8)		14.9 (14.3)		29.5 (15.3)	5.5 (3.8)
$R^2_{AD_t, \hat{AD}_t}$			-		-		-	.22
Company B: Mean	.0210 (.0020)	.42 (.01)	-3.5	.30 (.01)	+2.8	.61 (.01)	+5.1	-1.7
Mean Absolute			47.1 (38.8)		7.7 (5.6)		28.7 (24.4)	7.0 (8.1)
$R^2_{AD_t, \hat{AD}_t}$			-		-		-	.26
Company C: Mean	.0120 (.0001)	.58 (.01)	-4.3	.32 (.02)	-2.6	.63 (.001)	-.6	-4.7
Mean Absolute			9.7 (6.5)		3.0 (2.3)		5.8 (2.1)	5.0 (4.2)
$R^2_{AD_t, \hat{AD}_t}$			-		-		-	.13

^a For an explanation of the variables used see Table 1.

- b Company A is a government and regulated monopolist in the only market it serves.
- c The weighted mean (mean absolute) cost of errors in terms of net income was \$-1.8 million (\$5.8 million). For an explanation of the weighting scheme see Table 1.

Table 2 suggests that using this model Company A's net income was \$.9 million lower on average (1.3% of average profits in the period) with imperfect forecasts than it would have been with perfect forecasts, Company B's \$1.7 million lower (4.1% of average profits) and Company C's 4.7 million lower (29.6% of average profits). Overall the companies weighted net income was \$1.8 million lower on average (3.4% of the weighted average of net incomes) with imperfect forecasts than it would have been with perfect forecasts.

Perhaps the major difficulty with this model (as with the earlier naive model) is that some of the difference $(\pi_j - \pi_A)$ is due considerations other than the error in the aggregate forecasts R_{π_j, π_A}^2 is quite low, for example. One alternative approach is to assume that equations of the form of (4) - (6) apply on the basis of past data. Then the firm's predictions of Gross National Product for the next period (Y^*) can be substituted everywhere for (Y), for a perfect prediction of Gross National Product, and the anticipated profit (π_j^*) calculated. This could be compared to the solution of (4) - (6) which would have obtained if a perfect prediction of Y had been made (which is (6) as it has been calculated here), the difference between the two estimates, $(\pi_j - \pi_j^*)$, representing the "cost" of the inaccurate aggregate forecasts. That is, the "cost" of aggregate forecast errors would be:

$$(7) \quad C = (\pi_j - \pi_j^*) = (1 - l_j - o_j) s_j c_i (Y - Y^*)$$

The problem with such an approach is that it also ignores firm actions which would have taken place within the period as the error in predicting Gross

National Product was recognized. If this approach had been used it would have been more correct to assume that the firm would expect sales and therefore contract for labor and other expenses at the start of the period on the basis of predicted Gross National Product (Y^*). However by the end of the period realized sales would have been $S_j^A = s_j c_i Y_A$, not $S_j^* = s_j c_i Y^*$. Several questions now arise. Firstly how does this actual sales revenue come about? If s_j and c_i are assumed relatively stable then the difference in sales (caused by the error $(Y^* - Y_A)$) could have been the result of maintaining the chosen quantity of production which was based on Y^* but allowing the price to fluctuate, maintaining the price but allowing the quantity to fluctuate, allowing both price and quantity to fluctuate or adjusting neither but acknowledging the existence of inventories or backordering to clear the market. And if this approach is pursued what assumptions can be made about the nature of the reaction function to the error in Gross National Product predictions (for example, is 25% of the error recognized by midyear, 50%, 75% by the third quarter?). And what, if any, are the additional costs to adjusting labor and non-labor inputs from contracted - for levels? As an example of several somewhat more sophisticated models which consider some of these problems more directly, the following sections present individual models for two of the companies.⁷

A Model for Company A

As noted earlier Company A is a government-regulated monopolist in its major product market. This and other evidence from its operations suggested that

7. The data necessary for the estimation of the models which follow, while by no means necessarily adequate in the case of Companies A and B, definitely suggest as unwise the consideration of Company C whose aggregate predictions are only available for six years.

consideration be given to a monopoly model with the usual assumptions excepting the production decision which is made after demand is observed and price which is chosen before demand is observed and assumed not to change.

Suppose then that, as in Figure 1, if demand is assumed to be a function of prices (P) and income (Y) (with and error), demand is forecast as $D(Y^*)$ thereby suggesting an optimal price of $P_{Y^*}^*$. Realized or actual demand is then observed as $D(Y_A, e)$ indicating that, given $P_{Y^*}^*$, the actual amount produced will be $q^*(Y_A, e)$ at a profit (ABCD). If income (Y) had been perfectly forecast then demand would have been predicted as $D(Y^* = Y_A)$ with $P_{(Y^*=Y_A)}^*$ the implied optimal price suggesting after demand was observed an optimal quantity $q^{*'}(Y_A, e)$ would have been produced at a profit DEFG. Thus (DEFG-ABCD) is the difference in net income which would have resulted from accurate forecasts of income (Y).

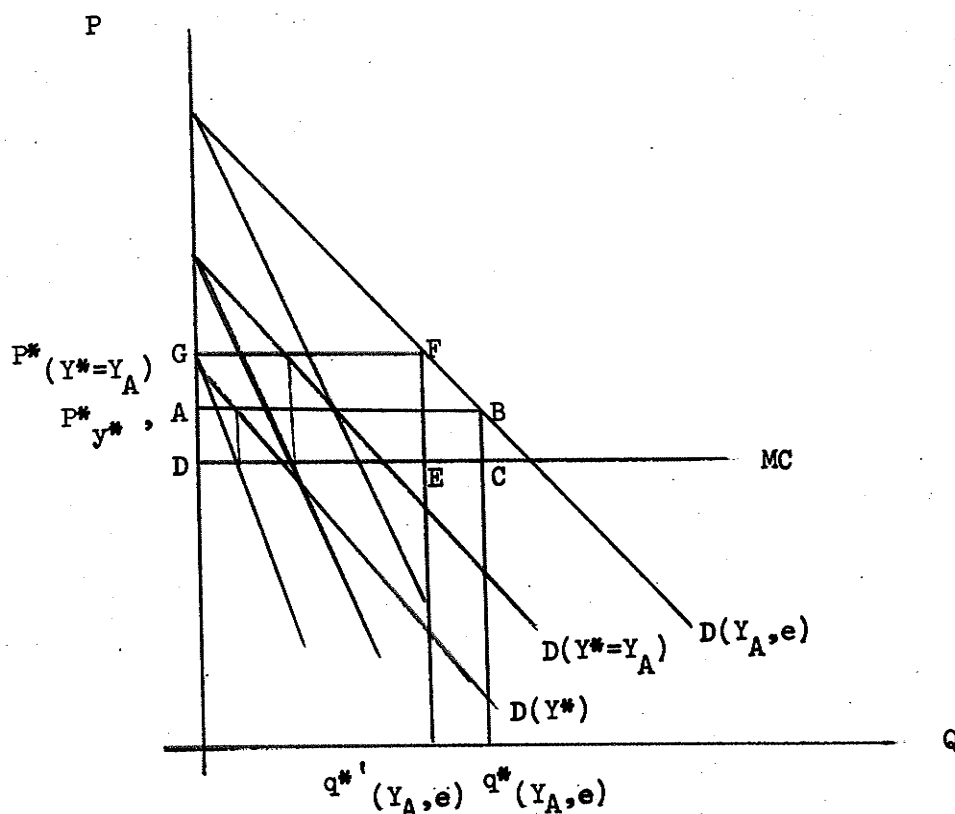


Figure 1
Model I for Company A

More particularly consider the monopoly model (Model I) where Company A's capital stock (K) and its price (P) are fixed before actual demand is observed. After the demand curve is observed the firm chooses labor (L) and the utilization rate of capital (U) to maximize profits.⁸

Formally assume:

$$(8) \quad D = A_p^{\gamma}, \quad \gamma, < -1, \quad A \sim (\bar{A}, 6_A^2) \quad \text{and} \quad A = A'(Y, e)$$

$$(8a) \quad D|_p = Q \quad \text{where } Q \equiv \text{quantity produced (the market must be cleared)}$$

$$(9) \quad Q = S^{\alpha} L^{1-\alpha} \quad \text{where } S = UK \text{ (utilization rate times the stock of capital)}$$

$$(10) \quad \gamma(U) = \frac{a}{2} U^2, \quad a > 0 \quad \text{(the rate of depreciation of capital in use is a function of the utilization rate).}$$

$$(11) \quad w(\text{wage rate}), G(\text{price of capital goods}), r(\text{interest rate}), \text{ and } d(\text{rate of time depreciation}) \text{ are given.}$$

The firm's problem is to:

$$(12) \quad \begin{array}{l} \text{Maximize } E(\pi) = \text{Maximize } E(pD - wL - (r+d)GK - (U)GK) \\ K, p \qquad \qquad \qquad K, p \end{array}$$

subject to (8) - (11) given that labor and the capital utilization are chosen optimally within the period.

That is, within the period, the firm must:

$$(13) \quad \begin{array}{l} \text{Maximize} \\ L, U \end{array} \quad \begin{array}{l} (pD - wL - (r+d)GK - \gamma(U)GK) \\ p, K, A \end{array}$$

Therefore within the period:

$$(14) \quad \frac{\partial E(\pi)}{\partial L} = 0 \Rightarrow p(1-\alpha)U^{\alpha}K^{\alpha}L^{-\alpha} = w$$

$$(15) \quad \frac{\partial E(\pi)}{\partial U} = 0 \Rightarrow paU^{\alpha-1}K^{\alpha-1}L^{1-\alpha} = aUG$$

$$(16) \quad U^{\alpha}K^{\alpha}L^{1-\alpha} = A_p^{\gamma}$$

8. This model is due Kenneth R. Smith, "The Effect of Uncertainty on Monopoly Price, Capital Stock and Utilization of Capital", Journal of Economic Theory. 1 (June 1969), pp. 48 - 59.

Solving (14) - (16) for L and U, then solving (15) for K and p (subject to these solutions for L and U) and taking logs implies:⁹

$$(21a) \log K = C_1 - \frac{m}{2} \log (r+d) + (\alpha(1-\gamma_1)-1) \log G + ((1-\gamma_1)(1-\alpha)) \log w \\ + \left[\frac{g(2-\alpha)(1-\gamma_1) - 2\gamma_1}{2} \right] \log Y^* - \gamma_1 \log E(A_0) \\ + \left[\frac{(2-\alpha)(1-\gamma_1)}{2} \right] \log E(A^{\frac{2}{2-\alpha}}) \text{ where } m = (2-\alpha(1+\gamma_1))$$

$$(22a) \log p = C_2 + \frac{\alpha}{2} \log (r+d) + \alpha \log G + (1-\alpha) \log w + \left[\frac{g(2-\alpha)}{2} \right] \log Y^* \\ - \frac{2-\alpha}{2} \log E(A^{\frac{2}{2-\alpha}})$$

$$(17a) \log U = \frac{1}{2-\alpha} (\log A_0 + \gamma_2 \log Y) + \frac{\gamma_1}{2-\alpha} \log p - \frac{1}{2-\alpha} \log K - \frac{1-\alpha}{2-\alpha} \log C \\ = C_3 + \frac{1}{2} \log (r+d) + \left[\frac{2\gamma_1 - g(2-\alpha)}{2(2-\alpha)} \right] \log Y^* + \frac{\gamma_2}{2-\alpha} \log Y - \frac{1}{2-\alpha} \log A_0 \\ + \frac{\gamma_1}{2-\alpha} \log E(A) - \frac{1}{2} \log E(A^{\frac{2}{2-\alpha}})$$

$$(18a) \log L = C_4 + \left[\frac{\alpha(1-\gamma_1)}{2} \right] \log (r+d) + (\alpha(\gamma_1+1) \log G + (\gamma_1(1-\alpha)-\alpha) \log w \\ + \left[\frac{\alpha(\gamma_1(2-\alpha g) + 2g(\alpha-1))}{2(2-\alpha)} \right] \log Y^* + \left[\frac{2\gamma_2}{2-\alpha} \right] \log Y + \frac{2}{2-\alpha} \log A_0 \\ - \frac{\alpha \gamma_1}{2-\alpha} \log E(A_0) + \left[\frac{2\gamma_1(1-\alpha) - \alpha}{2} \right] \log E(A_0^{\frac{2}{2-\alpha}})$$

Also since Total Revenue (R) is price times quantity then:

$$(23) \log R = \log p + \log Q = \log A_0 + (1+\gamma_1) \log p + \gamma_2 \log Y \\ = C_5 + \frac{(1-\gamma_1)\alpha}{2} \log (r+d) + \alpha(1+\gamma_1) \log G + (1+\gamma_1)(1-\alpha) \log w \\ + \left[\frac{g(1+\gamma_1)(1-\alpha)}{2} \right] \log Y^* + \gamma_2 \log Y + \log A_0 \\ + \left[\frac{(1+\gamma_1)(2-\alpha)}{2} \right] \log E(A^{\frac{2}{2-\alpha}})$$

9. See Appendix for details. Equations here are numbered to coincide with the ones in this appendix.

And Total Costs (TC):

$$\begin{aligned}
 (24) \quad \log TC = & C_6 + \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log(r+d) + \alpha(1+\gamma_1) \log G + (1+\gamma_1)(1-\alpha) \log w \\
 & + \left[\frac{\alpha\gamma_1 \frac{2-(2-\alpha)g}{2-\alpha} - g(2-\alpha)(\gamma_1-\alpha)}{2} \right] \log Y^* + \frac{2\gamma_2}{2-\alpha} \log Y \\
 & + \left[\frac{\gamma_1(2-\lambda) - \alpha}{2} \right] \log E(A^{\frac{2-\alpha}{2}}) + \frac{2\gamma_1}{2-\alpha} \log E(A_0) + \frac{2}{2-\alpha} \log A_0
 \end{aligned}$$

The cost of inaccurate aggregate forecasts can be obtained in the following manner. Equations (17a), (18a), (21a), (23) and (24) i.e. $\log U$, $\log L$, $\log K$, $\log R$ and $\log TC$ respectively, can be estimated from firm data. Then $(\gamma_2 \log Y)$ is substituted for $(\log Y^*)$ i.e. a perfect prediction for one which contains an error and an estimate obtained for U , L , K , R and TC . The difference between the estimates of R and TC represents expected profits, given perfect aggregate forecasts. These are compared with actual past profits (which occurred during the period of inaccurate aggregate forecasts) to determine the "costs" of inaccuracy. Similarly the difference between the estimated and actual values of U , L and K is then an estimate of the costs of the inaccurate aggregate forecasts in terms of the labor and capital inputs.

The estimates of equations (17a), (18a), (21a), (23) and (24) are as follows:

$$\begin{aligned}
 (23) \quad \log R = & \underset{(.280)}{.10} \log(r+d) + \underset{(-.120)}{-.05} \log G + \underset{(4.852)}{1.31} \log w + \underset{(.040)}{.001} \log Y^* \\
 & + \underset{(2.137)}{.09} \log Y
 \end{aligned}$$

$R^2 = .98$ $F = 124.3^a$ $D.-W. = 1.26$ $n = 14$ a significance at the .05 level.

$$(21a) \quad \log K = \begin{matrix} -.14 \log (r+d) \\ (-.32) \end{matrix} + \begin{matrix} .73 \log G \\ (-1.487) \end{matrix} + \begin{matrix} +1.00 \log w \\ (4.733)^a \end{matrix} \\ + \begin{matrix} .03 \log Y^* \\ (1.244) \end{matrix}$$

$R^2 = .96$ $F = 74.9^a$ $D.-W. = .87$ $n = 14$ a significance at the .05 level.

$$(18a) \quad \log L = \begin{matrix} -.13 \log (r+d) \\ (-.784) \end{matrix} + \begin{matrix} 1.09 \log G \\ (5.556)^a \end{matrix} - \begin{matrix} .34 \log w \\ (-2.668)^a \end{matrix} \\ + \begin{matrix} -.02 \log Y^* \\ (-1.869)^a \end{matrix} + \begin{matrix} .03 \log Y \\ (1.525) \end{matrix}$$

$R^2 = .79$ $F = 1.06^a$ $D.-W. = 1.45$ $n = 14$ a significance at the .05 level.

$$(17a) \quad \log U = \begin{matrix} -.05 (r+d) \\ (-1.848)^a \end{matrix} + \begin{matrix} .005 \log Y^* \\ (.998) \end{matrix} + \begin{matrix} .01 \log Y \\ (1.623) \end{matrix}$$

$R^2 = .43$ $F = 4.31^a$ $D.-W. = .7$ $n = 14$ a significance at the .05 level.

$$(24) \quad \log C = \begin{matrix} +.16 \log (r+d) \\ (.492) \end{matrix} + \begin{matrix} .04 \log G \\ (.099) \end{matrix} + \begin{matrix} 1.22 \log w \\ (4.909)^a \end{matrix} \\ - \begin{matrix} .002 \log Y^* \\ (-.081) \end{matrix} + \begin{matrix} .08 \log Y \\ (2.149)^a \end{matrix}$$

$R^2 = .98$ $F = 138.9^a$ $D.-W. = 1.33$ $n = 14$ a significance at the .05 level.

- Variables: $R \equiv$ Total Revenue (Net Sales)
 $K \equiv$ Capital (Total Assets at original cost deflated by the Implicit Non-residential Construction Price Index)
 $U \equiv$ Utilization Rate of Capital (percentage of the labor force employed)
 $L \equiv$ Labor (Total Man-hours per year (includes over time))

$C \equiv$ Total Costs (including taxes)

$r \equiv$ rate of discount (average yield on Canadian long-term government bonds)

$d \equiv$ proportional rate of discount (depreciation chargeable for tax purposes divided by Total Fixed Assets at original cost)

$G \equiv$ price of capital goods (Implicit Non-Residential Construction Price Index)

$W \equiv$ wages per man-hour (includes over time)

$Y^* \equiv$ predicted absolute change in Gross National Product¹⁰

$Y \equiv$ actual absolute change in Gross National Product

The signs are generally those expected from the theory. The number of observations was limited by the number of Gross National Product predictions which were available. And when compared to the number of independent variables found in most of the equations, this fact undoubtedly accounts for the high R^2 and F values (as well as in part for the non-significant t-values).

The estimates of the "cost" of the aggregate forecast errors obtained using these equations are summarized in Table 3.

Table 3

Estimates of the Costs of Errors (Using Model I) in
Forecasts of Gross National Product made by Company A

	Total Revenue	Labor	Capital	Utiliza- tion Rate of Capital	Total Costs	Net Income
Mean ^a	-\$1.5 million (.3%)	1.4 million man-hours (1.8%)	\$76.8 million (3.5%)	-.9% (1.0%)	\$1.6 million	-\$3.1 million (4.4%)
Mean Absolute ^b	17.4 (16.3)	1.8 (1.1)	120.8 (114.4)	.9 (.5)	13.1 (12.5)	6.5 (7.1)
Mean Absolute Value	512.8	79.1	2193.6	94.8	441.7	71.7
$R^2_{AD_t, AD_t}$	-	-	-	-	-	.99

^a The figure in brackets is the estimated mean cost of error as a percentage of the mean actual value

^b The figure in brackets is the standard deviation of the estimated mean absolute cost of error.

10. The usual demand equation incorporates some measure of income. When representing this income (Y) by Gross National Product here, it is assumed that some proportionate relationship between Gross National Product and disposable income for example, does exist.

Table 3 suggests that using this model Company A's net income was \$3.1 million lower on average (4.4% of average income during the period) with imperfect aggregate forecasts than it would have been with perfect forecasts. This result is due to the fact that Total Revenue would have been \$1.5 million higher on average (.3% of average Total Revenue) while Total Costs would have been \$1.6 million lower on average (.4% of average Total Costs) with perfect forecasts.

A Model for Company B

Company B is the major producer, supplying about fifty percent of the market (based on sales revenue), in one of Canada's larger basic industry's. This and other evidence from its operations suggested that consideration be given to a monopoly model given the usual assumptions with the exception that the production decision must be made before demand is observed and price flexibility exists (Model II). That is, in that by assumption price fully adjusts to clear the market, this model clearly abstracts from such questions as the existence of inventories, the ability to backorder and like considerations.

Suppose then that, as in Figure 2, if demand is assumed to be a function of prices and income (with an error), demand is forecast as $D(Y^*)$ suggesting $q^*_{Y^*}$ as the optimal quantity to produce. Realized demand is $D(Y_A, e)$ resulting in a profit (ABEF). If Y had been perfectly forecast then demand would have been predicted as $D(Y^* = Y_A)$ with $q^*(Y^* = Y_A)$ the optimal quantity implied. With the realized demand $D(Y_A, e)$ a profit of (ACDG) would have resulted. The difference, (ACDG - ABEF), is the pure difference in the net income which would have resulted from accurate forecasts of Y. Note that as with Figure 1 above this explanation treats the case of a period in which Y^* underestimates Y_A with realized demand containing some positive random error. Various other situations could also have been

considered. For example, relabeling $D(Y_A, e)$ as D_{Y^*} and D_{Y^*} as $D(Y_A, e)$ suggests a "cost" to inaccurate aggregate forecasts, in the case of an overestimation of actual income (which contains a negative random component), of (AHIL - ACJK)

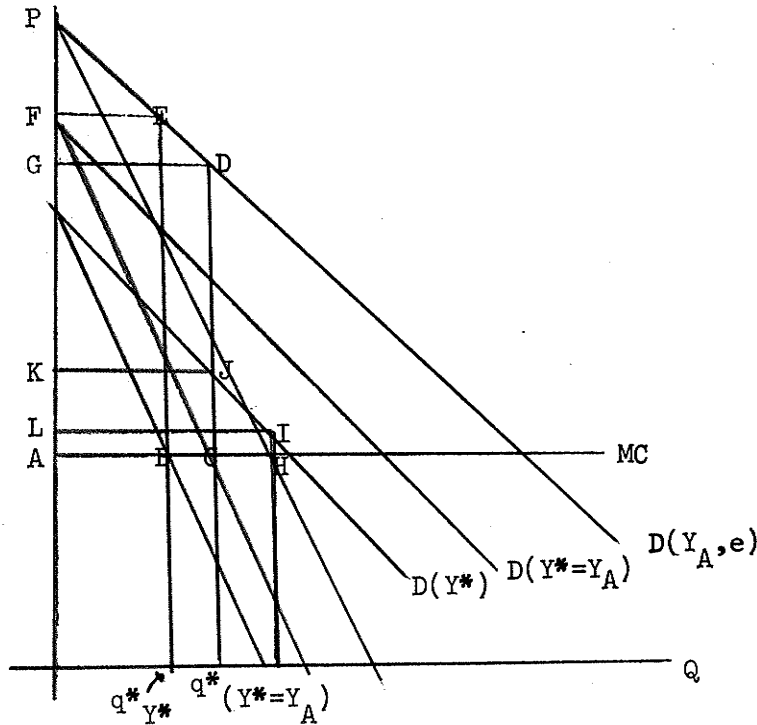


Figure 2

Model II for Company C

More particularly,¹¹ let:

$$(25) \quad p_t = \gamma_0 q_t^{\gamma_1} Y_t^{\gamma_2} e^{\epsilon_t} \quad \gamma_0, -\gamma_1, \gamma_2 > 0$$

$$(26) \quad q_t = AK^\alpha L^\beta e^{\rho t} \quad \text{where: } K \equiv \text{capital with a constant per unit cost } r, \\ L \equiv \text{labor with a constant per unit cost } w.$$

Then:

$$(27) \quad \pi_t = p_t q_t - wL - rK \\ = (\gamma_0 q_t^{\gamma_1} Y_t^{\gamma_2} e^{\epsilon_t}) (AK^\alpha L^\beta e^{\rho t}) - wL - rK$$

And:

$$(28) \quad E(\pi) = (\gamma_0 q_t^{\gamma_1} E(Y_t^{\gamma_2}) E(e^{\epsilon_t})) (AK^\alpha L^\beta e^{\rho t}) - wL - rK$$

11. This model is due J. Gould, "The Micro-economic Approach to the Demand For Physical Capital", pp. 14-18.

Assuming rational expectations and an error term in the demand function which is distributed log - normal, i.e.

$$(28a) \quad Y_t^* = E(Y_t^2)$$

$$(28b) \quad e^{\epsilon_t} \sim \log N(0, \sigma_\epsilon^2)$$

Then expected profit is:

$$(28) \quad E(\pi) = (\gamma_0 q_t^{\gamma, Y^*} e^{\sigma_\epsilon^2/2}) (AK^\alpha L^\beta e^{\rho t}) - wL - rK$$

$$= (\gamma_0 (AK^\alpha L^\beta)^{\gamma, Y^*} e^{\sigma_\epsilon^2/2} (AK^\alpha L^\beta e^{\rho t}) - wL - rK$$

Then:

$$(29) \quad \frac{\partial E(\pi)}{\partial K} = 0 \Rightarrow \alpha \gamma_0 (\gamma, +1) A^{(\gamma, +1)} K^{\alpha(\gamma, +1)} L^{\beta(\gamma, +1)} Y^* e^{\sigma_\epsilon^2/2} e^{\rho t} = rK$$

$$(30) \quad \frac{\partial E(\pi)}{\partial L} = 0 \Rightarrow \gamma_0 \beta (\gamma, +1) A^{(\gamma, +1)} K^{\alpha(\gamma, +1)} L^{\beta(\gamma, +1)} Y^* e^{\sigma_\epsilon^2/2} e^{\rho t} = wL$$

Solving (29) and (30) simultaneously for L and K, making certain parameter transformations and taking logarithms implies:¹²

$$(33) \quad \log K = \frac{1-\beta_2}{\beta_3} \log r + \frac{\beta_2}{\beta_3} \log w - \frac{1}{\beta_3} \log Y^* - \frac{\gamma_1+1}{\beta_3} \log A$$

$$- \frac{1}{2\beta_3} \sigma_\epsilon^2 + \rho t + \beta_4$$

$$(35) \quad \log L = \frac{\beta_2}{\beta_3} \log r + \frac{1-\beta_2}{\beta_3} \log w - \frac{1}{\beta_3} \log Y^* - \frac{\gamma_1+1}{\beta_3} \log A$$

$$- \frac{1}{2\beta_3} \sigma_\epsilon^2 + \rho t + \beta_4'$$

Since Total Revenue (R) is price times quantity, then:

$$(36) \quad \log R = \frac{\alpha(\gamma_1+1)}{\beta_3} \log r + \frac{\beta(\gamma_1+1)}{\beta_3} \log w - \frac{(\gamma_1+1)(\alpha+\beta)}{\beta_3} \log Y^*$$

$$+ \gamma_2 \log Y + \alpha(\gamma_1+1)\beta_4 + \beta(\gamma_1+1)\beta_4' + \rho t - \frac{\sigma_\epsilon^2 (\gamma_1+1)(\alpha+\beta)}{2\beta_3} + \epsilon_t$$

12. See the Appendix for details. Equations are numbered here to coincide with the ones in this appendix.

Further, since:

$$(26) \quad q_t = AK^\alpha L^\beta e^{\rho t}$$

$$(31) \quad L = \left(\frac{r}{w}\right)\left(\frac{\beta}{\alpha}\right) K$$

$$(37) \quad C = wL + rK$$

Solving (26), (31) and (37) for C, assuming constant returns to scale and taking logs implies:

$$(38) \quad \log C = \frac{\alpha(\gamma_1+1)(\alpha+\beta)}{\beta_3} \log r + \frac{\beta(\gamma_1+1)(\alpha+\beta)}{\beta_3} \log w - \frac{(\alpha+\beta)}{\beta_3} \log Y^* \\ - \frac{\alpha+\beta}{2\beta_3} \epsilon^2 + \alpha\beta_4 + \beta\beta_4' + 2\rho t$$

Now the "cost" of inaccurate aggregate forecasts in terms of profits can be obtained in the following manner. Equations (33), (35), (36) and (38), Log K, Log L, Log R and Log C respectively, can be estimated from firm data. Then everywhere ($\gamma_2 \log Y$) is substituted for ($\log Y^*$) i.e. a perfect prediction for one which contains an error and an estimate obtained for K, L, R and C. The difference between the estimates of R and C represents expected profits, given perfect aggregate forecasts. These are compared with actual past profits (which occurred during the period of inaccurate aggregate forecasts) to determine the "costs" of inaccuracy. Similarly the difference between the estimated and actual values of K and L is then an estimate of the costs of inaccurate aggregate forecasts in terms of the labor and capital inputs.

The estimates of equations (36), (33), (36) and (38) are as follows:

$$(36) \quad \log R = \underset{(1.187)}{-.46} \log r - \underset{(-.994)}{1.50} \log w + \underset{(.416)}{.13} \log Y^* + \underset{(.067)}{.01} \log Y + \underset{(1.261)}{.06} t \\ R^2 = .84 \quad F = 9.5^a \quad D.-W. = 2.46 \quad n = 9 \quad \text{significant at the .05 level}$$

$$(33) \quad \log K = \underset{(-1.973)}{-.38} \log r - \underset{(.944)}{.57} \log w + \underset{(.095)}{.02} \log Y^* + \underset{(2.603)^a}{.05} t \\ R^2 = .94 \quad F = 28.8^a \quad D.-W. = 2.42 \quad n = 9 \quad \text{significant at the .05 level.}$$

$$(35) \quad \log L = -.42 \log r - .81 \log w + .06 \log Y^* + .04 t$$

$$\quad \quad \quad (-2.979) \quad (-1.821) \quad (.459) \quad (3.034)^a$$

$$R^2 = .93 \quad F = 27.7 \quad D.-W. = 2.69 \quad n = 9 \quad a \text{ significant at the .05 level.}$$

$$(38) \quad \log C = -.41 \log r - 1.64 \log w + .12 \log Y^* + .07 t$$

$$\quad \quad \quad (-1.337) \quad (-1.723) \quad (.393) \quad (2.442)^a$$

$$R^2 = .89 \quad F = 17.6^a \quad D.-W. = 2.53 \quad n = 9 \quad a \text{ significant at the .05 level.}$$

Variables: $R \equiv$ Total Revenue (net sales)

$K \equiv$ Capital (Total Assets at original cost deflated by the Implicit Non-Residential Construction Materials Price Index).

$L \equiv$ Labor (Total Man - hours per year (includes overtime)).

$C \equiv$ Total Costs (pre - tax).

$r \equiv$ user cost of capital $q \frac{1 - \mu v}{1 - \mu} \delta - \frac{1 - \mu m}{1 - \mu} c$

where: $q \equiv$ price of capital goods (Implicit Non-Residential Construction Materials Price Index).

$\mu \equiv$ tax rate (taxes paid divided by pre - tax income).

$v \equiv$ tax-deductible portion of replacement expenditures (depreciation and depletion expenses chargeable to income for tax purposes divided by the change in depreciation allowances on the balance sheet).

$\delta \equiv$ proportional rate of depreciation (depreciation and depletion chargeable for tax purposes divided by total fixed assets at original cost).

$m \equiv$ proportion of interest chargeable against income tax purposes (interest chargeable against income tax purposes divided by the product of Total Outstanding Debt times the average yield on Canadian long - term government bonds).

$c \equiv$ rate of discount (average yield on Canadian long - term government bonds).

$w \equiv$ wages per man - hour (includes overtime).

$Y^* \equiv$ predicted absolute change in Gross National Product

$Y \equiv$ actual absolute change in Gross National Product

$t \equiv$ time

Again the number of observations that could be used was limited by the number of Gross National Product predictions available (in this case less than was

true of Company A). And as before, when compared with the number of independent variables found in most of the equations this probably accounts for the high R^2 and F values and non-significant t-values as well.

The estimates of the "cost" of aggregate forecasting errors obtained using these equations are summarized in Table 4.

Table 4

Estimates of the Costs of Errors (Using Model II) in Forecasts of Gross National Product made by Company B.

	Total Revenue	Labor	Capital	Total Costs	Net Income
Mean ^a	-\$69.8 million (15.2%)	-2.8 million man hours (7.0%)	-\$37.9 million (4.4%)	-\$62.8 million (15.0%)	\$-7.0 million (17.9)
Mean Absolute ^b	69.8 (53.5)	2.8 (3.0)	46.6 (33.2)	62.8 (43.1)	11.3 (7.9)
Mean Actual Value	457.8	40.1	860.0	418.9	38.9
$R^2_{AD_t, AD_t}$	-	-	-	-	.88

^a The figure in brackets is the estimated mean cost of error as a percentage of the mean actual value.

^b The figure in brackets is the standard direction of the estimated mean absolute cost of errors.

The results of Table 4 suggest that using this model Company B's net income was \$7.0 million lower on average (17.9% of average income) during the period with imperfect forecasts than it would have been had the forecasts been perfect. In this case Total Revenue would have been \$69.8 million higher on average (15.2% of average Total Revenue) while Total costs would have been \$62.8 million higher on average (15.0% of average Total Costs) with perfect forecasts than they were

with imperfect forecasts.

A Brief Comparison of the Models

Three models (two in the case of Company C) have now been presented and the "cost" of the errors in the predictions of Gross National Product made by Companies A and estimated for each. These estimates are summarized in Table 5. They range from \$.9 million (1.3% of average profits) for the market-share model to \$3.1 million (4.3% of average profits) for Model I in the case of Company A; \$1.7 million (4.1% of average profits) for the market-share model to \$7.0 million (17.9% of average profits) for Model III in the case of Company B and \$1.3 million (8.1% of average profits) for the Naive model to \$4.7 million for the market-share model in the case of Company C.

Table 5

A Comparison of the Estimates of the Cost (in terms of Net Income) in the Predictions of Gross National Product made by Companies A, B and C

	Company A			Company B			Company C	
	Naive Model	Market Share Model	Model I	Naive Model	Market Share Model	Model II	Naive Model	Market Share Model
Mean Cost	\$-2.3 million	\$-.9 million	\$-3.1 million	\$-1.9 million	\$-1.7 million	\$-7.0 million	\$-1.3 million	-.4.7 million
Mean Cost as a Percentage of Average Net Income	3.2%	1.3%	4.3%	4.6%	4.1%	17.9%	8.1%	29.2%
Mean Absolute Cost ^a	\$5.1 million (4.6)	\$5.5 million (3.8)	\$6.5 million (7.1)	\$8.1 million (9.5)	\$7.0 million (8.1)	\$11.3 million (7.9)	\$2.7 million (2.1)	\$5.0 million (4.2)

^a The figure in brackets is the standard deviation of the estimated absolute cost of errors.

One question which naturally arises is which of these estimates is most "correct" for each of the companies. Without a complete model for each firm, and as a result of the simplifying assumptions of the models used, these estimates provide only approximations to the "true" cost of aggregate errors.

Certainly there are means available for comparing the models, and thus by implication their estimates. For example, one method of comparison is on the basis of correlation coefficients (i.e. how well correlated are the actual firm data with the estimates obtained from the various models). Earlier results suggest that using this criteria the "best" model (based on net income measures only) was Model I for Company A, Model II for Company B and the naive model for Company C. This superiority of the more complex models (in the case of companies A and B) may however be due to estimation procedures in that the number of independent variables relative to the number of observations may be causing the rather larger correlation coefficients.

In an attempt to investigate the appropriateness of Models I and II, the following model (Model III) was also used. Consider the same monopoly model as in the case of Model I with the exception in this instance that the price (P) is not fixed before demand is observed (capital (K) however still remains so assumed) but, like labor and the utilization rate, is chosen to maximize profits. That is, assuming as before that equations (11) - (14) apply, the firm's long run decision is to choose capital (K) so as to maximize profits while the short run, or within period decision, is, given K, to choose L, U and P so as to maximize profits, i.e.

$$(40) \quad \max_{L, U, P} E(\pi) \Big|_{K, A} = \max E \{ pD - wL - (r+d)GK - \gamma(U)GK \}$$

Therefore:

$$(41) \quad \partial E(\pi) / \partial L = 0 \Rightarrow p \left[1 + \frac{1}{\gamma_L} \right] (1-\alpha) U^\alpha K^\alpha L^{-\alpha} = w$$

$$(42) \quad \partial E(\pi) / \partial U = 0 \Rightarrow p \left[1 + \frac{1}{\gamma_L} \right] \alpha U^{\alpha-1} K^\alpha L^{1-\alpha} = aUG$$

$$(43) \quad U^\alpha K^\alpha L^{1-\alpha} = Ap^\gamma,$$

Solving (41) - (43) for L, U and P, then solving (28) for K (subject to these solutions for L, U and P) and taking logs implies:¹³

$$(49a) \log K = C' - \frac{m}{2} \log (r+d) + \left[\frac{\alpha(1+\gamma_1)-1}{2} \right] \log G + \left[(1+\gamma_1)(1-\alpha) \right] \log w \\ + \frac{m}{2} \log Y^* + \frac{m}{2} \log E(A_0^{\frac{m}{2}}) \quad \text{where } m = \left[2-\alpha(1+\gamma_1) \right]$$

$$(44a) \log L = C'_1 + \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log (r+d) + \alpha(1+\gamma_1) \log G + \left[\gamma_1 - \alpha(1+\gamma_1) \right] \log w \\ - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log Y^* + \frac{2\gamma_2}{m} \log Y + \frac{2}{m} \log A_0 \\ - \left[\alpha(1+\gamma_1)/2 \right] \log E(A_0^{\frac{m}{2}})$$

$$(45a) \log U = C'_2 + \frac{1}{2} \log (r+d) - \frac{1}{2} \log Y^* + \frac{\gamma_2}{m} \log Y + \frac{1}{m} \log A_0 \\ - \frac{1}{2} \log E(A_0^{\frac{m}{2}})$$

$$(46a) \log P = C'_3 + \frac{\alpha}{2} \log (r+d) + \alpha \log G + (1-\alpha) \log w - \frac{\alpha}{2} \log Y^* \\ + \frac{2\gamma_2}{m} \log Y + \frac{\alpha}{m} \log A_0 - \frac{\alpha}{2} \log E(A_0^{\frac{m}{2}})$$

Since Total Revenue (R) equals Price (P) times Quantity (Q):

$$(50) \log R = \log P + \log Q = \log P + \log A_0 + \gamma_1 \log P + \gamma_2 \log Y \\ = C'_4 + \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log (r+d) + \alpha(1+\gamma_1) \log G + \left[(1-\alpha)(1+\gamma_1) \right] \log w \\ - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log Y^* + \left[\frac{\gamma_2(2-\alpha\gamma_1)}{m} \right] \log Y + \frac{2}{m} \log A_0 \\ - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log E(A_0^{\frac{m}{2}})$$

Further, since: Total Cost (TC) = $wL + (r+d)GK + \alpha(U)GK$

$$(51) \log TC = C'_5 + \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log (r+d) + \left[\alpha(1+\gamma_1) \right] \log G + \left[(1+\gamma_1)(1-\alpha) \right] \log w \\ - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log Y^* + \left[\frac{2\gamma_2}{m} \right] \log Y + \frac{2}{m} \log A_0 \\ - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log E(A_0^{\frac{m}{2}})$$

13. See the appendix for details. Equations here are numbered to coincide with those in the appendix.

By comparing this model with Model I for Company A it can be seen that the variables entering the regressions in both models are identical but that the theoretical values of the coefficients differ. The reason for these differences lies in the differing assumptions each model makes about what the firms aggregate predictions represent. In the case of Model I the assumption is that $Y^* = E(Y^2)$, in the case of Model III that $Y^* = E(Y^{2\gamma_2/m})$ where $m = [2 - \alpha(1 + \gamma_1)]$. And hence the cost of aggregate prediction errors will differ depending on which model is assumed in analyzing the regression results. Using Model III the cost of Company A's aggregate forecast errors (in terms of net income) was estimated at \$3.2 million on average (4.5% of average profits) as compared to an estimated cost using Model I, reported in Table 3 above, of \$3.1 million. These estimates are quite close (so were accompanying correlation coefficients). Another means of investigating the appropriateness of the two models is on the basis of which interpretation of the coefficient estimates is more consistent with the theoretically calculated coefficients. For example, it would appear that in interpreting the estimate of the value of the coefficient of $\log(r+d)$ in the $\log R$ equation one would be better to assume that Model I applies as opposed to III in that the estimate, +.10, is more consistent with Model I's calculated coefficient, $[\alpha(1 - \gamma_1)/2]$, than it is with Model III's (which is $[\alpha(1 + \gamma_1)/2]$).¹⁴ More generally however, for some of the other ten cases in which coefficients differed on a theoretical basis coefficient estimates obtained from regression were more consistent with Model III specifications than with Model I's (e.g. the $\log(r+d)$ coefficient in the $\log L$ equation), in several cases it was difficult to determine

14. Since $\gamma_1 < -1$, from equation (8), it suggests that $[\alpha(1 - \gamma_1)/2] > 0$ and $[\alpha(1 + \gamma_1)/2] < 0$. As the coefficient estimate is +.10 it is accepted on the basis of sign agreement as more consistent with Model I requirements.

which was more consistent (e.g. the $\log Y^*$ coefficient in the $\log K$ equation). Because the results from the two models failed to give distinctly different results on a number of bases, it is likely that the estimation problems due the number of available observations relative to the number of variables used in the model may be a major problem in the case of Company A.

In the case of Company B, Model III is quite different from Model II in that, for example the interest rate variables are defined differently and several equations contain an independent variable ($\log Y$) which does not appear in the same equation using Model II. Thus one method of evaluating the appropriateness of Models II and III in the case of Company B is on the basis of the belief that if theory indicates a variable appearing in the equation for one model (Model III) which does not appear in the same equation of the other model (Model II) and this variable's coefficient estimate appears and is significant, then this is evidence in favor of the model which contains that variable in a particular equation (i.e. Model III). In the two equations where this does apply, $\log L$ and $\log TC$, the coefficient estimates exist but were not significantly different from zero at the .05 level. In addition, using Model III, the estimated "cost" of aggregate forecast errors in terms of net income was \$28.3 million (72.8% of average profits) with the accompanying correlation coefficient equal to .70. While the size of the sample relative to the number of independent variables is a major problem (more so here than in the case of Company A) and the different manner of measuring the variables associated with the interest rate may be a confounding factor, this evidence (non-significant coefficient estimates, an extremely large estimate of the cost of aggregate errors and a lower correlation coefficient) would seem to suggest that Model II is a more appropriate representation of Company B than is Model III.

Yet another question of concern is that without supporting data from these firms it is difficult to determine if the forecasts were the ones actually used in planning the firms operations. One method for examining if these forecasts supplied were the operationally relevant ones was attempted. The forecasts of Company A and B used in Models I and II were replaced by the minimum mean square error naively generated Gross National Product absolute change forecasts (Model N2 above). The relevant equations were then reestimated using these forecasts and the estimates of the cost of error so obtained compared with those above which used the forecasts. If the estimated cost of error associated with the naively generated forecasts is lower than that calculated above for the judgemental forecasts it can be questioned if the judgemental sets supplied were in fact the ones used in the firms' operation. The cost of the inaccurate naive aggregate forecasts in terms of net income for Company A was \$7.0 million on average (9.8% of average income) as compared with the earlier estimates of \$3.1 million (4.4% of average income) using the judgemental forecasts supplied. In the case of Company B the respective figures were \$12.0 million (30.7% of average income) using the naive forecasts versus \$7.0 million on average (17.9% of average income) as presented above. Since the cost of error was lower in the case of the judgemental forecasts, it suggests that the forecasts may well be the operationally relevant ones. Again it should be emphasized that in view of the underlying estimation problems it's difficult to accept this conclusion as anything but suggestive.

In fact all these "diagnostic checks" are at best suggestive. In the presence of the limitations indicated above it is certainly best to conclude that until such time as a sufficiently lengthy record of predictions (perhaps ten

years of quarterly data) accompanied by comparable data on firm actuals and specifications by the forecasts of the methods used to incorporate aggregate expectations into "own-firm" forecasts is available, it will be very difficult to establish with confidence whether the source of present problems is due to the data, estimation or the inappropriateness of the models. Until such time these estimates should be considered as approximations to the "true" cost of errors and the problems identified as indicative of questions to be answered before acceptable estimates can be obtained.

Summary

Much of the analysis of the earlier research concentrated on measuring the errors in the aggregate forecasts of firms per se. To the extent that the firms incorporate these aggregate forecasts into "own-firm" expectations, it is important to consider the consequences for their operations of these aggregate forecast errors. Realizing that a more general approach would be desirable, because of the limitations of the data, the role of inaccurate aggregate forecasts is considered in a limited framework for three companies. Several models are presented as examples of how firm variables might be associated with aggregate expectations and their errors. Given the assumptions of the models, estimates of the cost of these aggregate errors are obtained.

The first, a "naïve" model which assumes a direct, unspecified relationship between firm variables and Gross National Product, suggested that, using this model, Company A's net income was \$2.3 million lower on average (3.2% of average profits in the period) with imperfect forecasts of Gross National Product it would have been with perfect forecasts, Company B's \$1.9 million less (4.6% of average profits in the period) and Company C's \$1.3 million less (8.1% of average profits). An overall weighted mean of these estimates suggested that the net income of the three companies was \$2.0 million lower on average (3.9%

of weighted mean profits) with imperfect forecasts than it would have been with perfect predictions. Such a model was quite imperfect for several reasons and serves as an example of a particularly simplistic device by which firm variables might be associated with aggregate expectations.

One step removed from the "naive" model was a simple market-share model which added assumptions about how other factors (e.g. the industry's share of Gross National Product and the market share of the firm) influenced the cost of the aggregate forecast errors to the firm. Using this model, the net income of Company A was \$.9 million less on average (1.3% of average profits in the period) with imperfect Gross National Product forecasts, Company B's \$1.7 million less (4.1% of average profits during the period) and Company C's \$4.7 million lower (29.2% of average profits in the period). An overall weighted mean of these estimates suggested that the net income of the three companies was \$1.8 million lower on average (3.4% of weighted mean profits) with imperfect forecasts than it would have been with perfect predictions.

As with the earlier "naive" model the major criticism of this model was its failure to explicitly consider other important factors which the firm would consider during the period to which the forecasts applied.

Accordingly two different models which introduced additional assumptions about the structure of the market, the nature of the production decision and the like were then presented, one each for Companies A and B.

The first, a model for Company A (Model I), was a fixed-price, variable-quantity monopoly model where price and capital decisions are made before demand is observed with labor and the capital utilization rate chosen to maximize profits, aggregate factors being introduced via the demand function. Using this model, it

was estimated that the net income of Company A was \$3.1 million lower on average (4.3% of average profits in the period) with imperfect forecasts than it would have been with perfect predictions.

The model chosen for Company B (Model II) was a fixed-quantity, price-flexibility monopoly model with aggregate factors introduced, as before, via the demand function. Using this model Company B's net income was estimated to have been \$7.0 million lower on average (17.9% of average profits) with imperfect forecasts than it would have been with perfect predictions.

Thus three models (two in the case of Company C) were presented and the "cost" of the errors in the predictions of Gross National Product made by Companies A and B estimated for each model. The estimates range from \$.9 million (1.3% of average profits) for market share model to \$3.1 million (4.3% of average profits for Model I in the case of Company A; \$1.7 million (4.1% of average profits) for the market share model to \$7.0 million (17.9% of average profits) for Model II in the case of Company B and \$1.3 million (8.1% of average profits) for the naive model to \$4.7 million (29.2% of average profits) for the market share model in the case of Company C.

This range of estimates may be due several factors. First the models chosen may be inappropriate. In an effort to consider this question an additional model (Model III) was introduced and used a means of comparison in the case of the two complex models (I and II) estimated for Companies A and B. In the case of Company A models I and III did not give distinctly different results. However, in the case of Company B the evidence suggested that as between Models II and III model II was a better choice. The second factor which should be noted is that the predictions supplied by the forecasters may not be the operationally relevant ones. The results of a limited investigation of this question suggested however

that the forecasts supplied may well be those used in the firms operations. Throughout this analysis repeated reference has been made to the third factor, and perhaps the major one, which may be causing the range of estimates, namely the amount of data available for use in estimation. When compared to the number of independent variables appearing in the equations of Models I - III it definitely suggests that estimation may be a problem in all the work done here.

Thus it was concluded that these "diagnostic checks" as methods for indicating a preference for one of the model estimates of the cost of errors could be at best suggestive. In the presence of the limitations discussed it certainly seemed best to conclude that until such time as a sufficiently lengthy record of predictions accompanied by comparable data on firm actuals and specifications by the forecaster of the methods used to incorporate aggregate expectations into "own-firm" forecasts are available, it will be difficult to establish with confidence whether the source of present variance in the estimates is due to the data, estimation problems or the inappropriateness of the models. Until such time these estimates should be considered as approximations to the "true" cost of the errors and the problems identified as indicative of the questions to be answered before acceptable estimates can be obtained.

APPENDIX.

DETAILS OF MODELS I-III FOR COMPANIES A AND B

This appendix provides details on the algebraic derivations for the equations estimated in connection with models I - III for Companies A and B.

Model I: Company A

Assume capital stock (K) and price (P) are fixed before the actual demand is observed. After demand is observed the firm chooses labor (L) and the utilization rate (U) to maximize profits. That is:

$$\text{Assume: } (8) \quad D = Ap^{\gamma}, \quad \gamma_1 < -1 \quad A \sim (\mu=A, 6_A^2)$$

$$(8a) \quad D|_P = Q \quad \text{where } Q \equiv \text{quantity produced.}$$

$$(9) \quad Q = S^{\alpha} L^{1-\alpha} \quad \text{where } S = UK \text{ (utilization rate times the stock of capital)}$$

$$(10) \quad \gamma(U) = \frac{a}{2} U^2, \quad a > 0 \quad \text{(the rate of depreciation of capital in use is a function of the utilization rate).}$$

$$(11) \quad w(\text{wage rate}), G(\text{price of capital goods}), r(\text{interest rate}), \text{ and } d(\text{rate of time depreciation}) \text{ are given.}$$

The long run decisions then are to choose K and P so as to maximize expected profits, i.e.

$$(12) \quad \begin{aligned} \text{Max}_{K,P} E(\pi) &= \text{Max}_A E(pD - wL - (r+d)GK - \gamma(U)GK) \\ \text{s.t.} \quad &(8) - (11) \end{aligned}$$

The short run, or within period decision, is, given K and P, to choose L and U so as to maximize profits, i.e.

$$(13) \quad \text{Max}_{L,U} \left| \begin{aligned} &pD - wL - (r+d)GK - \gamma(U)GK \\ &p, K, A \end{aligned} \right|$$

Therefore: $(14) \quad \frac{\partial E(\pi)}{\partial L} = 0 \Rightarrow p(1-\alpha)U^{\alpha}K^{\alpha}L^{-\alpha} = w$

$$(15) \quad \frac{\partial E(\pi)}{\partial U} = 0 \Rightarrow paU^{\alpha-1} K^{\alpha-1} L^{1-\alpha} = aUG$$

$$(16) \quad U^{\alpha} K^{\alpha} L^{1-\alpha} = Ap^{\gamma_1}$$

Solving (14) - (16) for L and U gives:

$$(17) \quad U = \left[\frac{Ap^{\gamma_1}}{K C^{1-\alpha}} \right]^{\frac{1}{2-\alpha}} \quad \text{where } C = \frac{aG(1-\alpha)}{wa}$$

$$(18) \quad L = \left[\frac{C^{\alpha} A^2 p^{2\gamma_1}}{K^{\alpha}} \right]^{\frac{1}{2-\alpha}}$$

Therefore the long run decision is now:

$$(12) \quad \text{Max}_{K,P} \quad E(pD - wL - (r+d)GK - \gamma(U)GK)$$

$$\text{s.t.} \quad (17) \quad \text{and} \quad (18)$$

Therefore:

$$(19) \quad \frac{\partial E(\pi)}{\partial K} = 0 \Rightarrow E((r+d)G + \gamma(U)G) = E(aU^2 G)$$

$$\text{or} \quad \frac{2(r+d)}{a} = E(U^2)$$

$$(20) \quad \frac{\partial E(\pi)}{\partial p} = 0 \Rightarrow E(\gamma, Ap^{\gamma_1-1} (1 + \frac{1}{\gamma_1}) p) = E(\gamma, Ap^{\gamma_1-1} \lambda) \quad \text{where } \lambda = \frac{d(TC)}{dQ}$$

$$\text{or} \quad E(\frac{d(TR)}{dp}) = E(\frac{d(TC)}{dp})$$

Solving (19) and (20) for p and K:

$$(21) \quad K = \left[\frac{a}{r+d} \right]^{\frac{2-\alpha(1+\gamma_1)}{2}} \frac{S^{\gamma_1}}{C^{(1+\gamma_1)(1-\alpha)}} E(A)^{-\gamma_1} (E(A)^{\frac{2}{2-\alpha}})^{\frac{(2-\alpha)(1+\gamma_1)}{2}}$$

$$\text{where } S = \frac{aG\gamma_1}{\alpha(1-\gamma_1)}$$

or assuming $D = A_0 Y^{\gamma_2} p^{\gamma_1}$, $Y^* = E(Y^{\gamma_2})$, $E(Y^{\frac{2\gamma_2}{2-\alpha}}) = (Y^*)^{\frac{2}{2-\alpha}}$ and taking logs of both sides:¹⁵

¹⁵ It should be noted that this "rational expectations" assumption is of a specialized form. The assumption is that the predicted value of income, Y_t^* , is equal to $E(Y_t^{\gamma_2})$ not $E(Y_t)$. That is, it is assumed that when asked for their predictions of income the firms provide the "firm - relevant" estimate not just the expected value of the aggregate

$$(21a) \log K = C_1 - \frac{m}{2} \log (r+d) + (\alpha(1-\gamma_1)-1) \log G + ((1-\gamma_1)(1-\alpha)) \log w \\ + \left[\frac{g(2-\alpha)(1-\gamma_1) - 2\gamma_1}{2} \right] \log Y^* - \gamma_1 \log E(A_0) \\ + \left[\frac{(2-\lambda)(1-\gamma_1)}{2} \right] \log E(A^{\frac{2}{2-\alpha}}) \text{ where } m = (2-\alpha(1+\gamma_1))$$

And since:

$$(22) p = \left[\frac{a}{2(r+d)} \right]^{\frac{-\alpha}{2}} \frac{s}{c^{1-\alpha}} (E(A^{\frac{2}{2-\alpha}}))^{\frac{2-\alpha}{2}}$$

Then:

$$(22a) \log p = C_2 + \frac{\alpha}{2} \log (r+d) + \alpha \log G + (1-\alpha) \log w + \left[\frac{g(2-\alpha)}{2} \right] \log Y^* \\ - \frac{2-\alpha}{2} \log E(A^{\frac{2}{2-\alpha}})$$

Therefore:

$$(17a) \log U = \frac{1}{2-\alpha} (\log A_0 + \gamma_2 \log Y) + \frac{\gamma_1}{2-\alpha} \log p - \frac{1}{2-\alpha} \log K - \frac{1-\alpha}{2-\alpha} \log C \\ = C_3 + \frac{1}{2} \log (r+d) + \left[\frac{2\gamma_1 - g(2-\alpha)}{2(2-\alpha)} \right] \log Y^* + \frac{\gamma_2}{2-\alpha} \log Y - \frac{1}{2-\alpha} \log A_0 \\ + \frac{\gamma_1}{2-\alpha} \log E(A) - \frac{1}{2} \log E(A^{\frac{2}{2-\alpha}})$$

$$(18a) \log L = \frac{\alpha}{2-\alpha} \log C + \frac{2}{2-\alpha} (\log A_0 + \gamma_2 \log Y) + \frac{2\gamma_1}{2-\alpha} \log p - \frac{\alpha}{2-\alpha} \log K \\ = C_4 + \left[\frac{\alpha(1-\gamma_1)}{2} \right] \log (r+d) + (\alpha(\gamma_1+1) \log G + (\gamma_1(1-\alpha)-\alpha) \log w \\ + \left[\frac{\alpha(\gamma_1(2-\alpha g) + 2g(\alpha-1))}{2(2-\alpha)} \right] \log Y^* + \left[\frac{2\gamma_2}{2-\alpha} \right] \log Y + \frac{2}{2-\alpha} \log A_0 \\ - \frac{\alpha \gamma_1}{2-\alpha} \log E(A_0) + \left[\frac{2\gamma_1(1-\alpha) - \alpha}{2} \right] \log E(A_0^{\frac{2}{2-\alpha}})$$

Also since Total Revenue equals price times quantity

$$(23) \log R = \log p + \log Q = \log A_0 + (1+\gamma_1) \log p + \gamma_2 \log Y \\ = C_5 + \frac{(1-\gamma_1)\alpha}{2} \log (r+d) + \alpha(1+\gamma_1) \log G + (1+\gamma_1)(1-\alpha) \log w \\ + \left[\frac{g(1+\gamma_1)(1-\alpha)}{2} \right] \log Y^* + \gamma_2 \log Y + \log A_0 \\ + \left[\frac{(1+\gamma_1)(2-\alpha)}{2} \right] \log E(A^{\frac{2}{2-\alpha}})$$

And Total Costs (TC) = $wL + (r + d)GK + \gamma(U)GK = K(U^2(G(\frac{a}{2} + (r + d))))$:

$$(24) \log TC = \log K + 2 \log U + \log G + \log(\frac{a}{2} + (r + d))$$

$$\begin{aligned} &= C_6 + \frac{\alpha(1+\gamma_1)}{2} \log(r+d) + \alpha(1+\gamma_1) \log G + (1+\gamma_1)(1-\alpha) \log w \\ &\quad + \left[\frac{\alpha\gamma_1 [2-(2-\alpha)g] - g(2-\alpha)(\gamma_1 - \alpha)}{2-\alpha} \right] \log Y^* + \frac{2\gamma_2}{2-\alpha} \log Y \\ &\quad + \left[\frac{\gamma_1(2-\lambda) - \alpha}{2} \right] \log E(A^{\frac{2}{2-\alpha}}) + \frac{2\gamma_1}{2-\alpha} \log E(A_0) + \frac{2}{2-\alpha} \log A_0 \end{aligned}$$

Model II: Company B

Assume demand is a function of price and income (with an error) and the quantity produced is based on a Cobb - Douglas production function. Then:

$$(25) p_t = \gamma_0 q_t^{\gamma_1} Y_t^{\gamma_2} e^{\epsilon_t} \quad \gamma_0, -\gamma_1, \gamma_2 > 0$$

$$(26) q_t = AK^\alpha L^\beta e^{\rho t} \quad \text{where: } K \equiv \text{capital with a constant per unit cost } r, \\ L \equiv \text{labor with a constant per unit cost } w.$$

Then:

$$\begin{aligned} (27) \pi_t &= p_t q_t - wL - rK \\ &= (\gamma_0 q_t^{\gamma_1} Y_t^{\gamma_2} e^{\epsilon_t}) (AK^\alpha L^\beta e^{\rho t}) - wL - rK \end{aligned}$$

And:

$$(28) E(\pi) = (\gamma_0 q_t^{\gamma_1} E(Y_t^{\gamma_2}) E(e^{\epsilon_t})) (AK^\alpha L^\beta e^{\rho t}) - wL - rK$$

Assuming rational expectations of income and error term distributed log-normal, i.e.

$$(28a) \quad Y_t^* = E(Y_t^2)$$

$$(28b) \quad e^{\epsilon_t} \sim \log N(0, \sigma_\epsilon^2)$$

Then expected profit is:

$$(28) \quad E(\pi) = (\gamma_0 a_t^{\gamma, Y^*} e^{\sigma_\epsilon^2/2}) (AK^\alpha L^\beta e^{\rho t}) - wL - rK \\ = (\gamma_0 (AK^\alpha L^\beta)^{\gamma, Y^*} e^{\sigma_\epsilon^2/2}) (AK^\alpha L^\beta e^{\rho t}) - wL - rK$$

Then:

$$(29) \quad \frac{\partial E(\pi)}{\partial K} = 0 \Rightarrow \alpha \gamma_0 (\gamma, +1) A^{(\gamma, +1)} K^{\alpha(\gamma, +1)} L^{\beta(\gamma, +1)} Y^* e^{\sigma_\epsilon^2/2} e^{\rho t} = rK$$

$$(30) \quad \frac{\partial E(\pi)}{\partial L} = 0 \Rightarrow \gamma_0 \beta (\gamma, +1) A^{(\gamma, +1)} K^{\alpha(\gamma, +1)} L^{\beta(\gamma, +1)} Y^* e^{\sigma_\epsilon^2/2} e^{\rho t} = wL$$

Solving simultaneously:

$$(31) \quad \frac{\alpha}{\beta} = \frac{rK}{wL} \quad \text{or} \quad L = \left(\frac{r}{w} \right) \left(\frac{\beta}{\alpha} \right) K$$

Thus from (29):

$$(32) \quad \alpha \gamma_0 (\gamma, +1) A^{(\gamma, +1)} K^{\alpha(\gamma, +1)} \left(\frac{r\beta}{w\alpha} K \right)^{\beta(\gamma, +1)} Y^* e^{\sigma_\epsilon^2/2} e^{\rho t} = rK$$

$$\text{i.e.} \quad \alpha \gamma_0 (\gamma, +1) A^{(\gamma, +1)} \left(\frac{\beta}{\alpha} \right)^{\beta(\gamma, +1)} \left(\frac{r}{w} \right)^{\beta(\gamma, +1)} K^{(\gamma, +1)(\alpha+\beta)-1} Y^* e^{\sigma_\epsilon^2/2} e^{\rho t} = r$$

Let:

$$\beta_0 = \gamma_0 A^{(\gamma, +1)}, \quad \beta_1 = \alpha(\gamma, +1), \quad \beta_2 = \beta(\gamma, +1), \quad \beta_3 = (\gamma, +1)(\alpha+\beta)-1$$

Then (32) becomes:

$$\beta_0 \beta_1 \left(\frac{\beta}{\alpha} \right)^{\beta_2} \left(\frac{r}{w} \right)^{\beta_2} K^{\beta_3} Y^* e^{\sigma_\epsilon^2/2} e^{\rho t} = r$$

Solving in log form for Log K:

$$(33) \quad \log K = \frac{1-\beta_2}{\beta_3} \log r + \frac{\beta_2}{\beta_3} \log w - \frac{1}{\beta_3} \log Y^* - \frac{\gamma, +1}{\beta_3} \log A \\ - \frac{1}{2\beta_3} \sigma_\epsilon^2 + \rho t + \beta_4$$

$$\text{where:} \quad \beta_4 = \frac{\beta_2-1}{\beta_3} \log \beta_1 - \frac{\beta_2}{\beta_3} \log \beta_2 - \frac{1}{\beta_3} \log \gamma_0$$

Similarly from (30):

$$(34) \quad \gamma_0 A^{(\gamma, +1)} \beta^{(\gamma, +1)} L^{(\alpha + \beta)(\gamma, +1) - 1} \left(\frac{\alpha}{\beta}\right)^{\alpha(\gamma, +1)} Y^* e^{6\epsilon^2/2} e^{\rho t} = w$$

$$(\text{where } K = \frac{w\alpha}{r\beta} L)$$

$$\text{Let: } \beta_0 = \gamma_0 A^{(\gamma, +1)}, \beta_1' = (\gamma, +1), \beta_2' = (\gamma, +1), \beta_3 = (\alpha + \beta)(\gamma, +1) - 1$$

Thus (34) becomes:

$$\beta_0 \beta_1' \left(\frac{\alpha}{\beta}\right)^{\beta_2'} \left(\frac{w}{r}\right)^{\beta_2'} L^{\beta_3} Y^* e^{6\epsilon^2/2} e^{\rho t} = w$$

and solving for Log L:

$$(35) \quad \log L = \frac{\beta_2'}{\beta_3} \log r + \frac{1 - \beta_2'}{\beta_3} \log w - \frac{1}{\beta_3} \log Y^* - \frac{\gamma, +1}{\beta_3} \log A - \frac{1}{2\beta_3} 6\epsilon^2 + \rho t + \beta_4'$$

$$\text{where: } \beta_4' = \frac{\beta_2' - 1}{\beta_3} \log \beta_1' - \frac{\beta_2'}{\beta_3} \log \beta_2' - \frac{1}{\beta_3} \log \gamma_0$$

Total Revenue is then:

$$\begin{aligned} R = pq &= (\gamma_0 (AK^{\alpha} L^{\beta})^{\gamma} Y^{\gamma_2} e^{\epsilon t}) (AK^{\alpha} L^{\beta} e^{\rho t}) \\ &= \gamma_0 A^{(\gamma, +1)} K^{\alpha(\gamma, +1)} L^{\beta(\gamma, +1)} Y^{\gamma_2} e^{\epsilon t} e^{\rho t} \end{aligned}$$

And:

$$\begin{aligned} (36) \quad \log R &= \log \gamma_0 + (\gamma, +1) \log A + \alpha(\gamma, +1) \log K + \beta(\gamma, +1) \log L \\ &\quad + \gamma_2 \log Y + \rho t + \epsilon_t \\ &= \log \gamma_0 + (\gamma, +1) \log A + \alpha(\gamma, +1) \left[\frac{1 - \beta_2'}{\beta_3} \log r + \frac{\beta_2'}{\beta_3} \log w - \frac{1}{\beta_3} \log Y^* \right. \\ &\quad \left. - \frac{\gamma, +1}{\beta_3} \log A - \frac{1}{2\beta_3} 6\epsilon^2 + \beta_4' \right] + \beta(\gamma, +1) \left[\frac{1 - \beta_2'}{\beta_3} \log w \right. \\ &\quad \left. + \frac{\beta_2'}{\beta_3} \log r - \frac{1}{\beta_3} \log Y^* - \frac{\gamma, +1}{\beta_3} \log A - \frac{1}{2\beta_3} 6\epsilon^2 + \beta_4' \right] \\ &\quad + \gamma_2 \log Y + \rho t + \epsilon_t \end{aligned}$$

$$\begin{aligned}
&= \log \gamma_0 + \left[(\gamma_1 + 1) - \frac{(\gamma_1 + 1)^2 (\alpha + \beta)}{\beta_3} \right] \log A + \frac{\alpha (\gamma_1 + 1)}{\beta_3} \log r \\
&\quad + \frac{\beta (\gamma_1 + 1)}{\beta_3} \log w - \frac{(\gamma_1 + 1) (\alpha + \beta)}{\beta_3} \log Y^* + \gamma_2 \log Y + \alpha (\gamma_1 + 1) \beta_4 \\
&\quad + \beta (\gamma_1 + 1) \beta'_4 + \rho t - \frac{6\epsilon^2 (\gamma_1 + 1) (\alpha + \beta)}{2\beta_3} + \epsilon_t
\end{aligned}$$

Further, since

$$(26) \quad q_t = AK^\alpha L^\beta e^{\rho t}$$

$$(31) \quad L = \left(\frac{r}{w} \right) \left(\frac{\beta}{\alpha} \right) K$$

$$(37) \quad C = wL + rK$$

Solving (31) and (37) for K and L gives:

$$K = \frac{\alpha C}{r(\alpha + \beta)} \quad \text{and} \quad L = \frac{\beta C}{w(\alpha + \beta)}$$

Substituting in (26) and solving for C, the total cost,

$$\text{i.e.} \quad q = A \left(\frac{\alpha C}{r(\alpha + \beta)} \right)^\alpha \left(\frac{\beta C}{w(\alpha + \beta)} \right)^\beta e^{\rho t}$$

Then:

$$\begin{aligned}
(37a) \quad C &= \frac{r^{\frac{\alpha}{\alpha+\beta}} w^{\frac{\beta}{\alpha+\beta}} (\alpha+\beta)}{A^{\frac{1}{\alpha+\beta}} \alpha^{\frac{\alpha}{\alpha+\beta}} \beta^{\frac{\beta}{\alpha+\beta}}} q^{\frac{1}{\alpha+\beta}} e^{\frac{\rho t}{\alpha+\beta}} \\
&= \frac{r^\alpha w^\beta e^{\rho t}}{A^\alpha \alpha^\beta \beta^\beta} AK^\alpha L^\beta e^{\rho t} \quad \text{where } (\alpha + \beta) = 1
\end{aligned}$$

Implying:

$$\begin{aligned}
(38) \quad \log C &= \alpha \log r + \beta \log w - \alpha \log \alpha - \beta \log \beta + \alpha \log K + \beta \log L + 2\rho t \\
&= \alpha \log r + \beta \log w + \alpha \left[\frac{1-\beta_2}{\beta_3} \log r + \frac{\beta_2}{\beta_3} \log w - \frac{1}{\beta_3} \log Y^* \right. \\
&\quad \left. - \frac{\gamma_1+1}{\beta_3} \log A - \frac{1}{2\beta_3} 6\epsilon^2 + \beta_4 \right] + \beta \left[\frac{1-\beta'_2}{\beta_3} \log w + \frac{\beta'_2}{\beta_3} \log r \right. \\
&\quad \left. - \frac{1}{\beta_3} \log Y^* - \frac{\gamma_1+1}{\beta_3} \log A - \frac{1}{2\beta_3} 6\epsilon^2 + \beta'_4 \right] - \alpha \log \alpha \\
&\quad - \beta \log \beta + 2\rho t
\end{aligned}$$

$$= \frac{\alpha(\gamma+1)(\alpha+\beta)}{\beta_3} \log r + \frac{\beta(\gamma+1)(\alpha+\beta)}{\beta_3} \log w - \frac{(\alpha+\beta)}{\beta_3} \log Y^* \\ - \frac{\alpha+\beta}{2\beta_3} \epsilon^2 + \alpha\beta_4 + \beta\beta_4' + 2pt$$

$$\text{where: } \beta_4'' = \left[\frac{\beta_2' - 1 - \beta\beta_3}{\beta_3} \right] \log \beta_1' - \left[\frac{\beta_2' - \alpha\beta_3}{\beta_3} \right] \log \beta_2' - \frac{1}{\beta_3} \log \gamma_0$$

Model III: Companies A and B

Assume capital stock (K) is fixed before the actual demand is observed. After demand is observed the firm chooses labor (L), the capital utilization rate (U) and price (P) to maximize profits. That is, assuming as before that equations (8) - (11) apply, the long run decision is to choose capital (K) so as to maximize profits, i.e.

$$(39) \quad \max_K E(\pi)_A = \max E \{ PD - WL - (r+d)GK - \gamma(U)GK \}$$

The short run, or within period decision, is, given K, to choose L, U and P so as to maximize profits, i.e.

$$(40) \quad \max_{L,U,P} E(\pi) \Big|_{K,A} = \max E \{ pD - WL - (r+d)GK - \gamma(U)GK \}$$

Therefore:

$$(41) \quad \partial E(\pi) / \partial L = 0 \Rightarrow p \left[1 + \frac{1}{\gamma_1} \right] (1-\alpha) U^\alpha K^\alpha L^{-\alpha} = w$$

$$(42) \quad \partial E(\pi) / \partial U = 0 \Rightarrow p \left[1 + \frac{1}{\gamma_1} \right] \alpha U^{\alpha-1} K^\alpha L^{1-\alpha} = aUG$$

$$(43) \quad U^\alpha K^\alpha L^{1-\alpha} = A p^{\gamma_1}$$

Solving (27) - (29) for L, U and P gives:

$$(44) \quad L = \left[\frac{A^2 S^{2C}}{K^{\alpha(1+\gamma_1)} C^{2\gamma_1 - (1+\gamma_1)}} \right]^{\frac{1}{2-\alpha(1+\gamma_1)}} \quad \text{where: } S = \left[\frac{aG\gamma_1}{(1+\gamma_1)} \right]$$

$$\text{and } C = \left[\frac{aG(1-\alpha)}{w\alpha} \right]$$

$$(45) \quad U = \left[\frac{A S^{\gamma_1}}{K C^{(1+\gamma_1)(1-\alpha)}} \right]^{1/(2-(1+\gamma_1))}$$

$$(46) \quad P = \left[\frac{A^{\alpha} S^{2-\alpha}}{K^{\alpha} C^{2(1-\alpha)}} \right]^{1/(2-(1+\gamma_1))}$$

Therefore the long run decision is now:

$$(47) \quad \max_{K, A} E(\pi) = \max E \{ pD - wL - (r+d)GK - \gamma(U)GK \}$$

$$\text{s.t. } (4.30) - (4.32)$$

Therefore:

$$(48) \quad \partial E(\pi)/\partial K = 0 \Rightarrow 2(r+d)/a = E(U^2)$$

Or:

$$(49) \quad K = \left[\frac{a}{2(r+d)} \right]^{2-\alpha(1+\gamma_1)/2} \frac{S^{\gamma_1}}{C^{(1+\gamma_1)(1-\alpha)}} \cdot E \left[A^{2/[2-\alpha(1+\gamma_1)]} \right]^{[2-\alpha(1+\gamma_1)]/2}$$

Assuming $D = A_0 Y^{\gamma_2} P^{\gamma_1}$, $Y^* = E \left[Y^{2\gamma_2/[2-\alpha(1+\gamma_1)]} \right]$ and taking logs of both sides:

$$(49a) \quad \log K = C' - \frac{m}{2} \log(r+d) + \left[\alpha(1+\gamma_1) - 1 \right] \log G + \left[(1+\gamma_1)(1-\alpha) \right] \log w \\ + \frac{m}{2} \log Y^* + \frac{m}{2} \log E(A_0^{\frac{m}{2}}) \quad \text{where } m = [2-\alpha(1+\gamma_1)]$$

Therefore:

$$(44a) \quad \log L = C'_1 + \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log(r+d) + \alpha(1+\gamma_1) \log G + \left[\gamma_1 - \alpha(1+\gamma_1) \right] \log w \\ - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log Y^* + \frac{2\gamma_2}{m} \log Y + \frac{2}{m} \log A_0 \\ - \left[\alpha(1+\gamma_1)/2 \right] \log E(A_0^{\frac{m}{2}})$$

$$(45a) \quad \log U = C'_2 + \frac{1}{2} \log(r+d) - \frac{1}{2} \log Y^* + \frac{\gamma_2}{m} \log Y + \frac{1}{m} \log A \\ - \frac{1}{2} \log E(A_0^{\frac{m}{2}})$$

$$(46a) \quad \log P = C'_3 + \frac{\alpha}{2} \log(r+d) + \alpha \log G + (1-\alpha) \log w - \frac{\alpha}{2} \log Y^* \\ + \frac{2\gamma_2}{m} \log Y + \frac{\alpha}{m} \log A_0 - \frac{\alpha}{2} \log E(A_0^{\frac{m}{2}})$$

Since Total Revenue (R) equals Price (P) times Quantity (Q):

$$\begin{aligned}
 (50) \quad \log R &= \log P + \log Q = \log P + \log A_0 + \gamma_1 \log P + \gamma_2 \log Y \\
 &= C_4' + \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log(r+d) + \alpha(1+\gamma_1) \log G + [(1-\alpha)(1+\gamma_1)] \log w \\
 &\quad - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log Y^* + \left[\frac{\gamma_2(2-\alpha\gamma_1)}{m} \right] \log Y + \frac{2}{m} \log A_0 \\
 &\quad - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log E(A_0^{\frac{2}{m}})
 \end{aligned}$$

Further, since: Total Cost (TC) = $wL + (r+d)GK + \alpha(U)GK$

$$\begin{aligned}
 (51) \quad \log TC &= C_5' + \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log(r+d) + [\alpha(1+\gamma_1)] \log G + [(1+\gamma_1)(1-\alpha)] \log w \\
 &\quad - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log Y^* + \left[\frac{2\gamma_2}{m} \right] \log Y + \frac{2}{m} \log A_0 \\
 &\quad - \left[\frac{\alpha(1+\gamma_1)}{2} \right] \log E(A_0^{\frac{2}{m}})
 \end{aligned}$$

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